IE241 Engineering Statistics 1 Homework 8

Due date : June 14

1. Let $Y_1, Y_2, \dots, Y_n$ denote a random sample of size n from an exponential distribution with density function given by

$$f(y) = (1/\theta)e^{-y/\theta}1_{\{y>0\}}$$

   A. Show that $\hat{\theta}_1 = \bar{Y}$ is an unbiased estimator of θ.

   B. Let $\hat{\theta}_2 = nY_{(1)}$. Find $\text{eff}(\hat{\theta}_1, \hat{\theta}_2)$.

2. Let $Y_1, Y_2, \dots, Y_n$ denote a random sample of size n from a uniform distribution with support $(0, \theta)$. Show that $Y_{(n)}$ is consistent estimator of θ.

3. Let $Y_1, Y_2, \dots, Y_n$ denote a random sample of size n from normal distribution with mean μ and variance σ^2. Find the sufficient statistics for (μ, σ^2)

4. Let $Y_1, Y_2, \dots, Y_n$ denote a random sample of size n from normal distribution with mean μ and variance 1.

   A. Find the $\widehat{\mu^2} =$ MVUE of μ^2

   B. Find $Var\left(\widehat{\mu^2}\right)$

5. Let $Y_1, Y_2, \dots, Y_n$ denote a random sample of size n from Poisson distribution with mean λ.

   A. Show that $T = \begin{cases} 1, & if\ Y_1 = 0 \\ 0, & otherwise \end{cases}$ is an unbiased estimator of $e^{-\lambda}$.

   B. Find the MVUE of $e^{-\lambda}$.

6. Let $Y_1, Y_2, \dots, Y_n$ denote a random sample of size n from Poisson distribution with mean λ. Find the method-of-moment estimator of λ.

7. Let $Y_1, Y_2, \dots, Y_n$ denote a random sample of size n from a uniform distribution with support $(\theta, 0)$. Find the MLE of θ.