IE241 Engineering Statistics 1 Homework 6

Due date : May 15

1. Suppose that

$$X_i \sim^{i.i.d.} Normal(\mu_1, \sigma_1^2), \qquad i = 1, \dots, m$$
$$Y_j \sim^{i.i.d.} Normal(\mu_2, \sigma_2^2), \qquad j = 1, \dots, n$$
$$(i.i.d = \text{ independent and identically distributed})$$

Let the difference between the sample means be $(\bar{X} - \bar{Y})$.

(a) Find $\mathrm{E}(\bar{X} - \bar{Y})$
(b) Find $\mathrm{V}(\bar{X} - \bar{Y})$
(c) Suppose $\sigma_1^2 = 2,\ \sigma_2^2 = 2.5$ and $m = n$. Find the sample sizes so that $(\bar{X} - \bar{Y})$ will be within 1 unit of $(\mu_1 - \mu_2)$ with probability 0.95

2. Suppose that

$$X \sim \text{Poisson}(25)$$

Use the central limit theorem (CLT) to calculate the following probabilities.

(a) $p(19 < X \leq 33)$ (without continuity correction)
(b) $p(19 < X \leq 33)$ (with continuity correction)

3. A machine is shut down for repairs if a random sample of 100 items selected from the daily output of the machine reveals at least 18% defectives. (Assume that the daily output is a large number of items.) If on a given day the machine is producing only 12% defective items, what is the probability that it will be shut down?

4. Suppose that $X_1, X_2, \dots, X_n$ and $Y_1, Y_2, \dots, Y_n$ are independent random samples from populations with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively. Show that the random variable

$$U_n = \frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2 + \sigma_2^2)/n}}$$

is asymptotically normally distributed with mean 0 and variance 1. That is, the distribution function of U_n converges to a standard normal distribution function as $n \to \infty$.

5. Shear strength measurements for spot welds have been found to have standard deviation 9 pounds per square inch (psi). If 81 test welds are to be measured, what is the approximate probability that the sample mean will be within 1 psi of the true population mean?