IE241 Engineering Statistics 1 Homework 7

Due date : May 29

1. Let $Y \sim Binomial(n, p)$. Two estimators of p are $\hat{p}_1 = \frac{Y}{n}$, and $\hat{p}_2 = \frac{Y+1}{n+2}$
   A. Find the bias of $\hat{p}_1$, and $\hat{p}_2$.
   B. Find the MSE of $\hat{p}_1$, and $\hat{p}_2$.
   C. Find the condition on p such that $\mathrm{MSE}(\hat{p}_1) < \mathrm{MSE}(\hat{p}_2)$.

2. Let $Y \sim Geometric(p)$.
   A. Find the unbiased estimator for $Var(Y)$.
   B. Suggest a 2-standard-error bound on the error of estimation.

3. Let $Y_1, \dots, Y_n$ denote a random sample of size n from a population with a uniform distribution on the interval $(0, \theta)$.
   A. Show that $\frac{Y_{(n)}}{\theta} = \frac{1}{\theta}\max(Y_1, \dots, Y_n)$ is a pivotal quantity. That is, its distribution does not depend on θ.
   B. Find 95% lower confidence interval for θ.

4. Solve Exercise 8.62. in the text.

5. We try to estimate the average amount of time that students in KAIST spend in the gym every day. Assume that σ is in the neighborhood of 0.5 hour and we want to estimate to lie within 0.1 of μ with probability near 0.95. Approximately how many students should be included in our sample? Would it be valid to select all of students from a single department (e.g. Industrial & Systems Engineering)?

6. Suppose two new gyms were open in KAIST and the students were randomly assigned to either one of the gym to work out (students cannot change the gym once they were assigned). The amount of time that 16 students attending the first gym was increased by 11 points, with a standard deviation of 6 points. On the other hand, the amount of

time that 20 students attending the second gym was increased by 12 points, with a standard deviation of 8 points. Determine a 95% confidence interval for the difference in the mean enhancements in the average amount of time that students spend to work out. Assume that the measurements are normally distributed with equal variances.

7. The ages of a random sample of five university professors are 39, 54, 61, 72, and 59. Using this information, find a 99% confidence interval for the population standard deviation of the ages of all professors at the university, assuming that the ages of university professors are normally distributed.