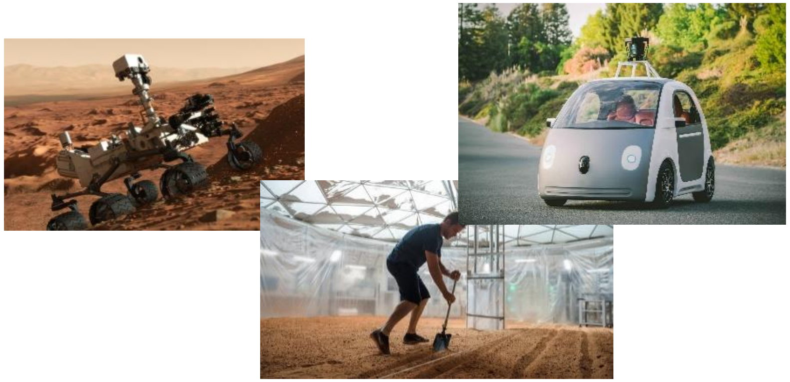
Lecture 1: Introduction

Decision making methodologies

- Machine Learning
- Artificial Intelligence
- Optimization
- Optimum Control
- Planning
- Markov Decision Process
- Influential Diagram
- Decision Tree
- Dynamic Control
- **Game Theory**
- Search
- Stochastic Programming
- Dynamic programming
- Reinforcement Learning
- Bandit problem

Engineering is all about decision makings



What are the differences in these decision-making strategies

What are the common aspects in these decision-making strategies?

Why do we have to learn many decision making methodologies?

engineers' creativity depends on the diversity of tools that he or she has

- Diversify tools for decision making
- Understand the usages of your tools
- Sharpen your tools
- Organize your tools

Be ready for solving any problem that might appear in the future!

Core idea

What type of decision making framework will be used?

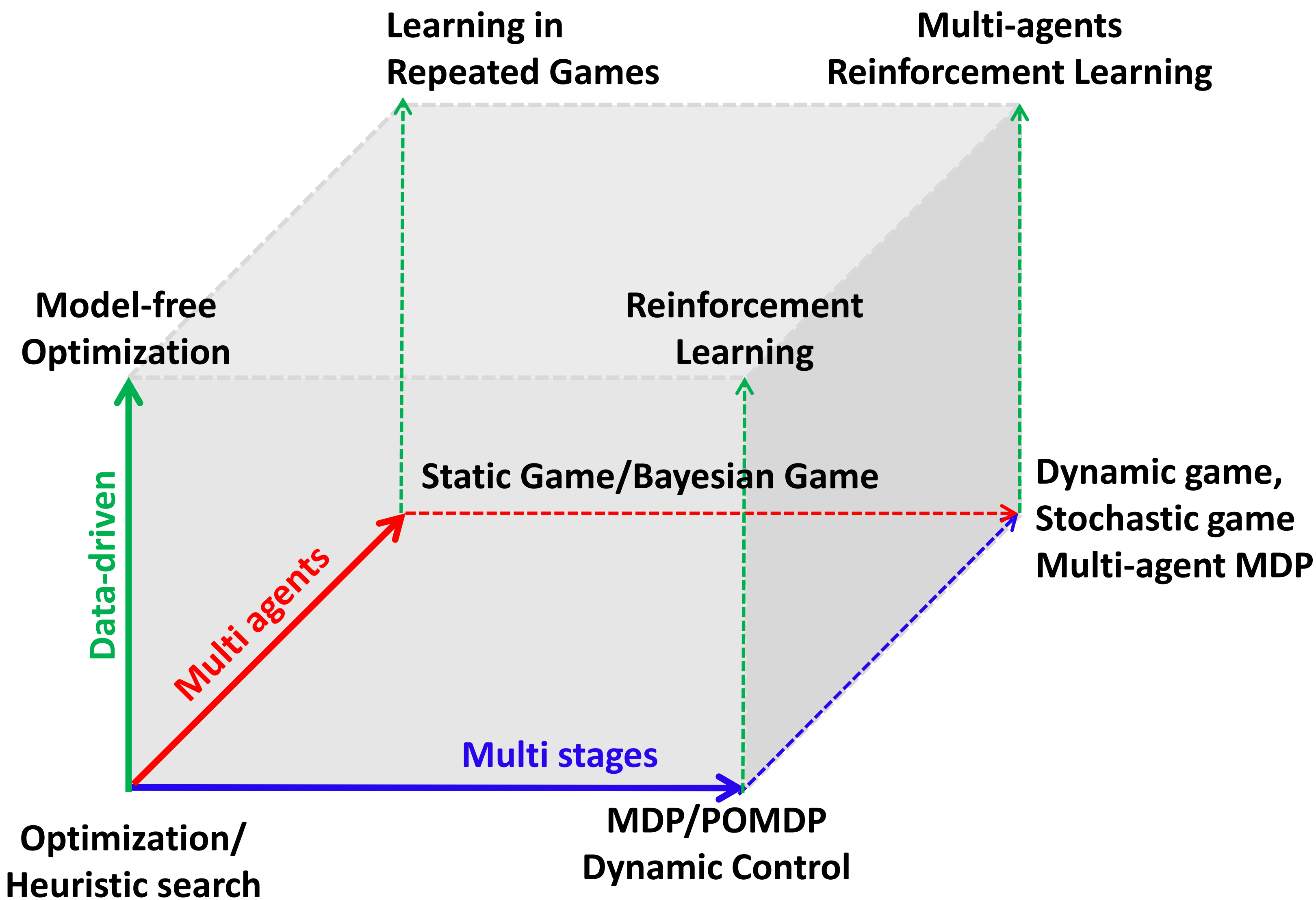
- Single stage or multi stages
- Single decision maker or many decision makers
- Model based or model-free

"Decision makings under uncertainties"

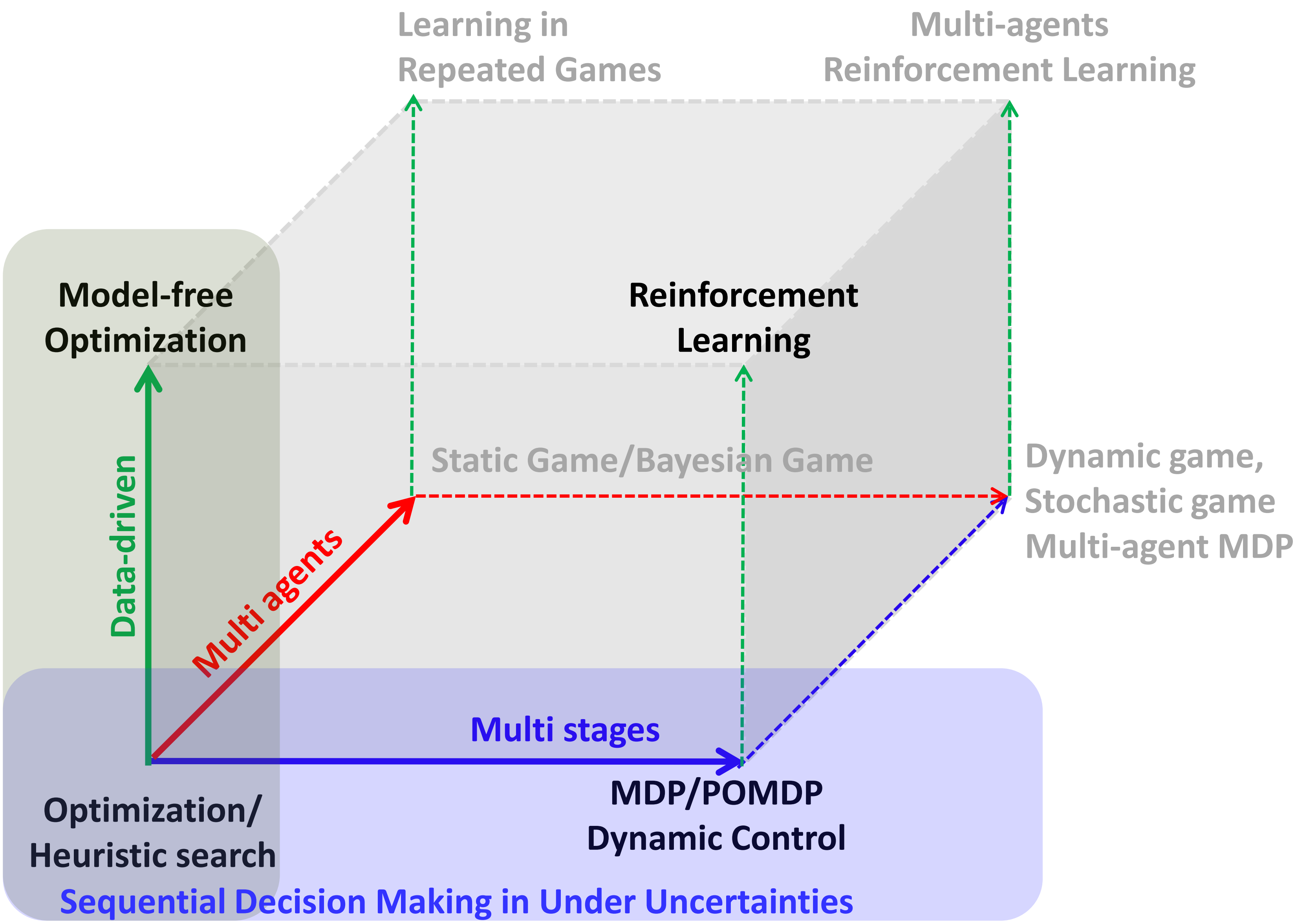
How to model uncertainties?

- **Epistemic Uncertainty** (systemic uncertainty) :
  Uncertainty arising through lack of knowledge
  - Model uncertainty
  - State uncertainty
- **Aleatoric uncertainty** (statistical uncertainty):
  Uncertainty arising through an underlying stochastic system

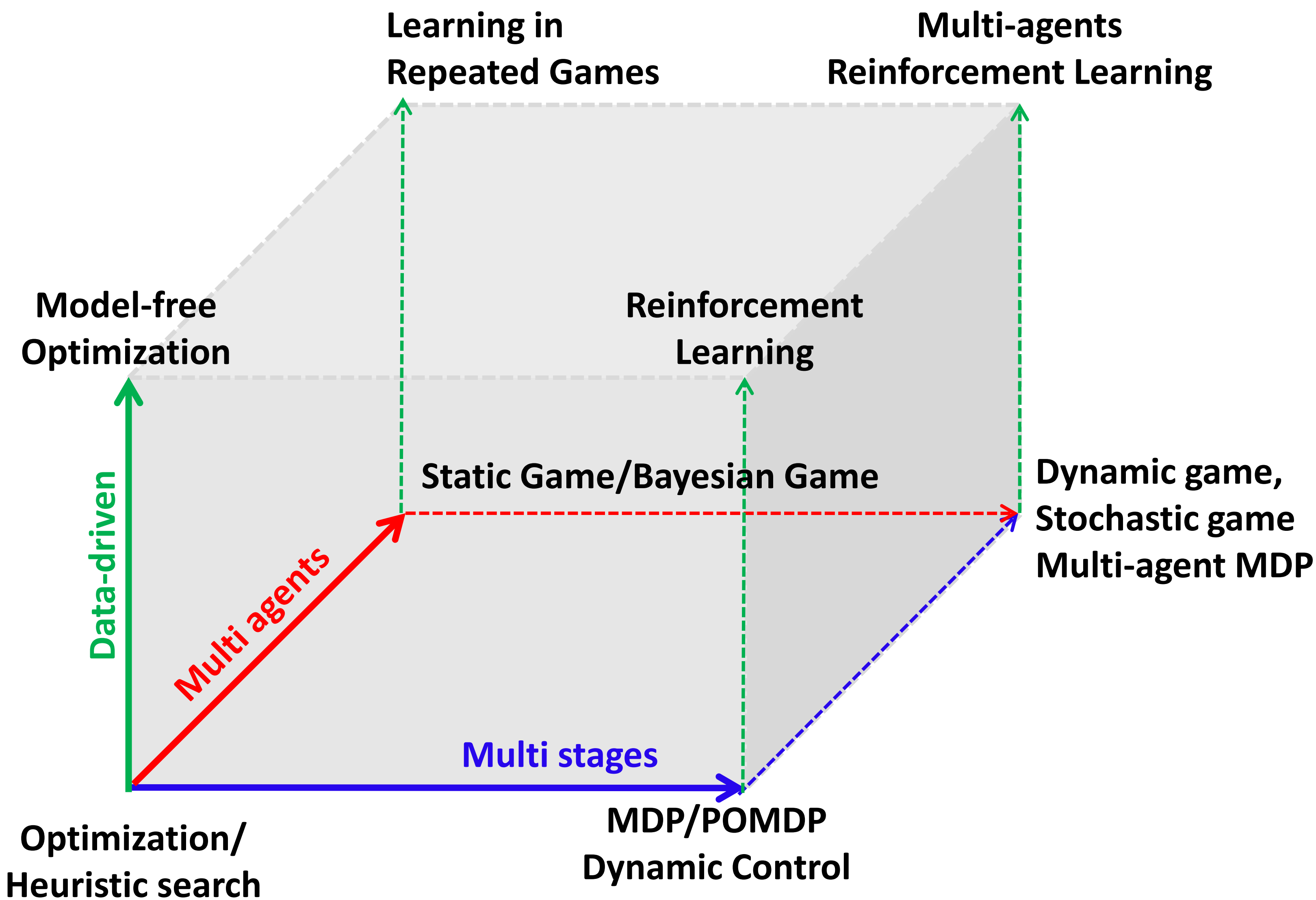
Classified decision making methods



Classified decision making methods



Classified decision making methods



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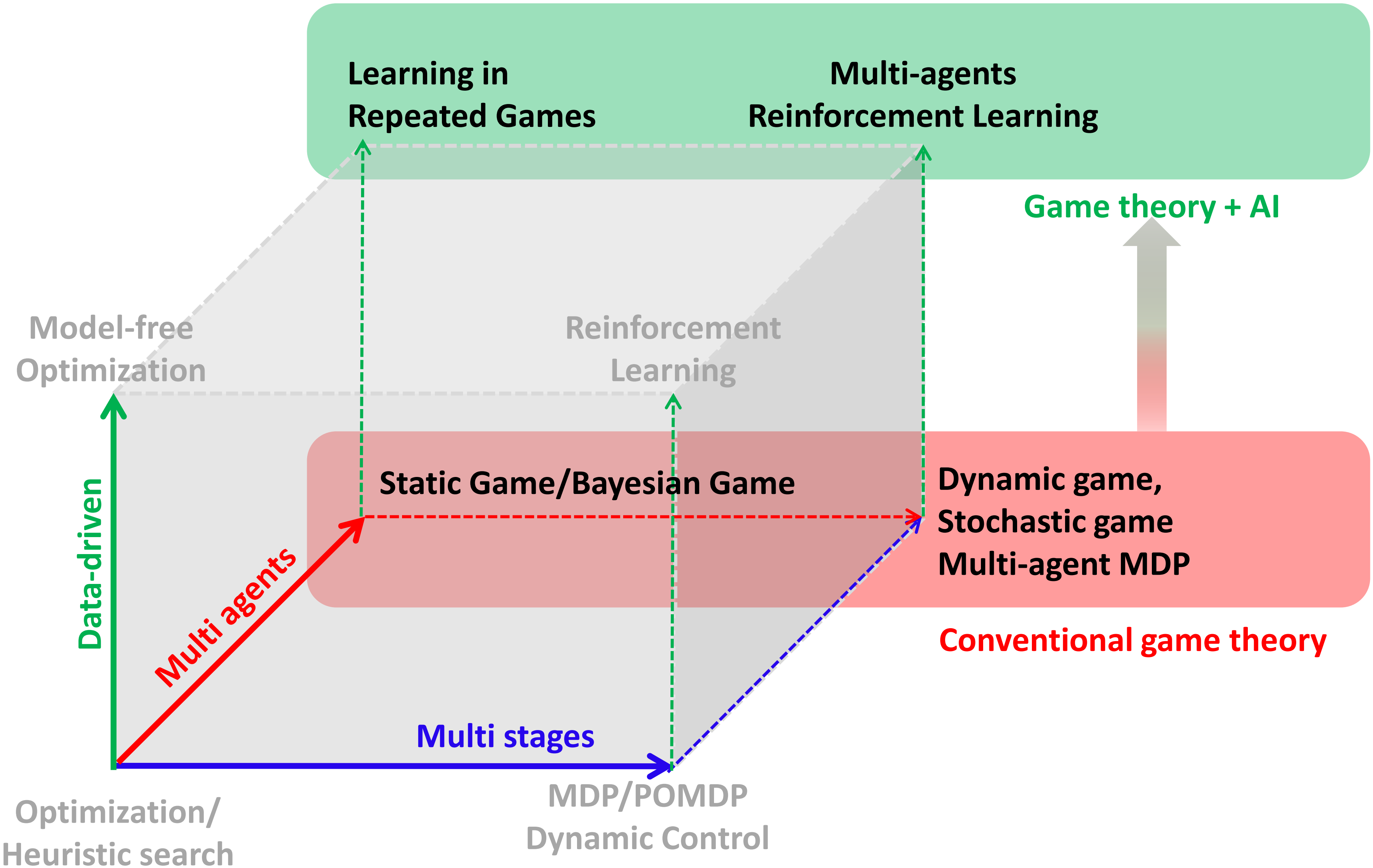
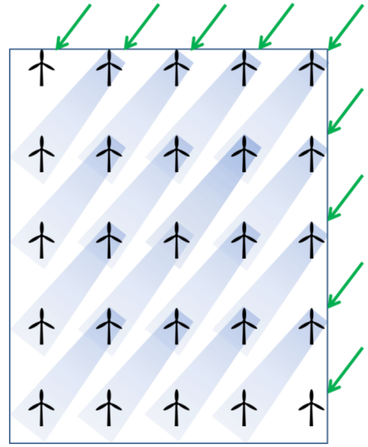


Problem solving

Real-world task

Modeling

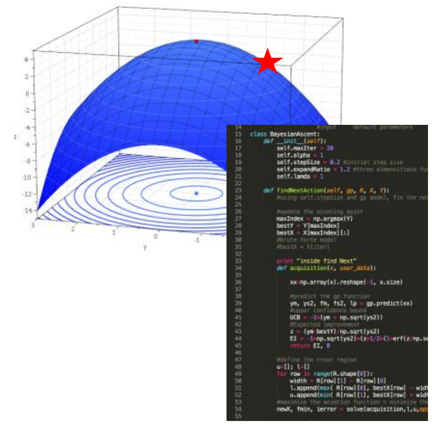
Formal task (model)



$$\underset{\boldsymbol{x}}{\text{maximize}} \sum_{i=1}^{N} P_i(\boldsymbol{x}; \theta, U)$$

Solving

Algorithm (program)



Validation

Are we building the right model?

Verification

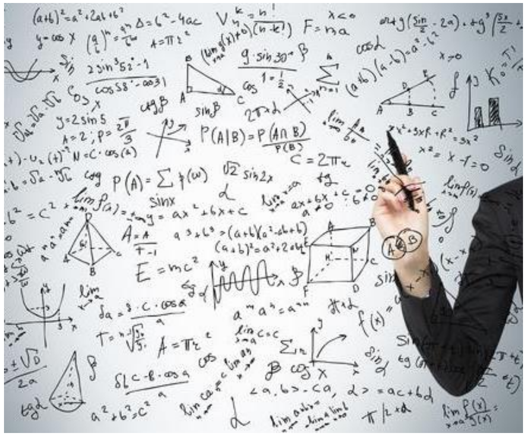
Does the algorithm capture all the essential aspects of the model?

Is this solution is good for the target system?

Data can help to model a target problem more realistically and derive more accurate solution!

Key elements of this course

Data analytics
- Bayesian Statistics
- Machine learning
- Artificial intelligence



Modeling
- Optimization
- Markov Decision Process
- Game Theory

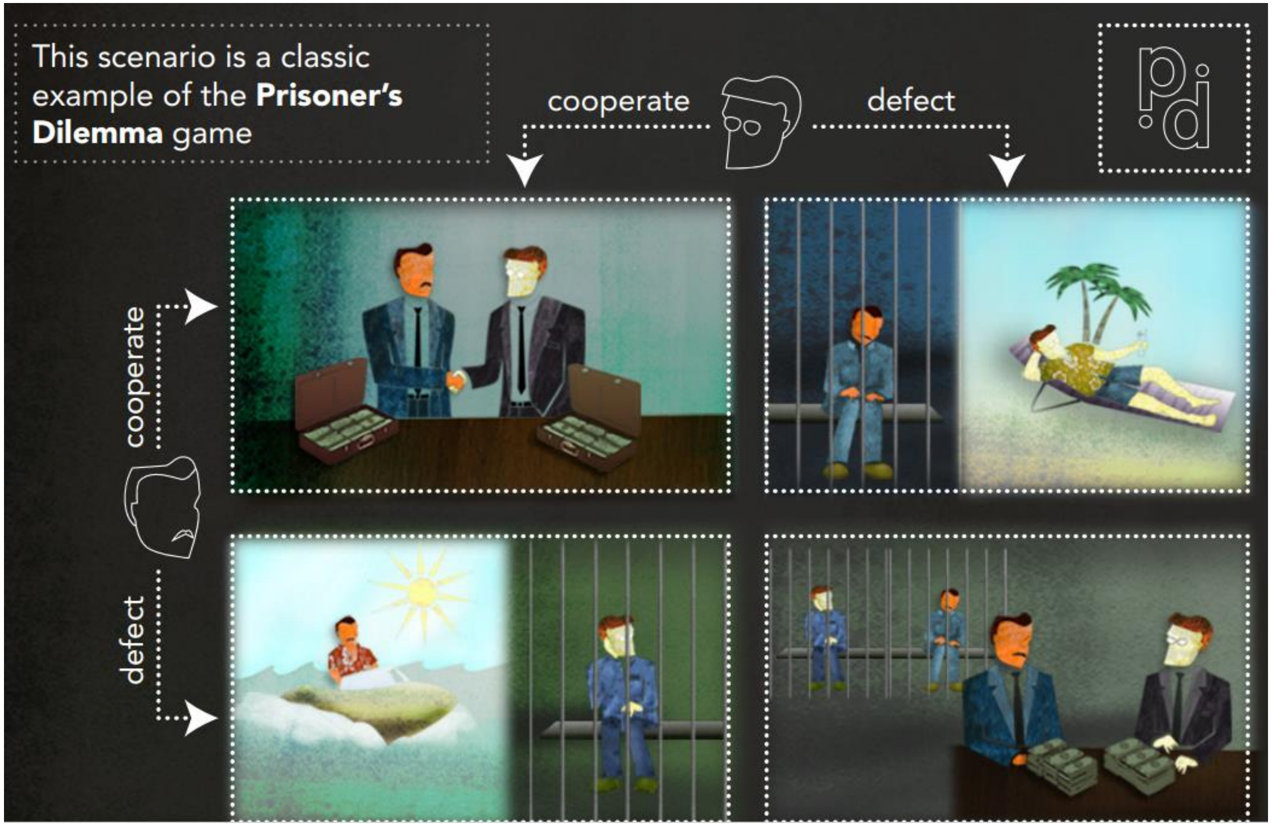
Decision Making
- Mathematical Programming
- Dynamic Programming
- Reinforcement Learning

Course objectives

Upon successful completion of the course, you are able to

- *understand* various mathematical models describing decision making problems.
- *formulate* real-world decision making problems in a mathematical form.
- *implement* key algorithms and approaches to solving various decision making problems.
- *Interpret* the results of decision-making problems.

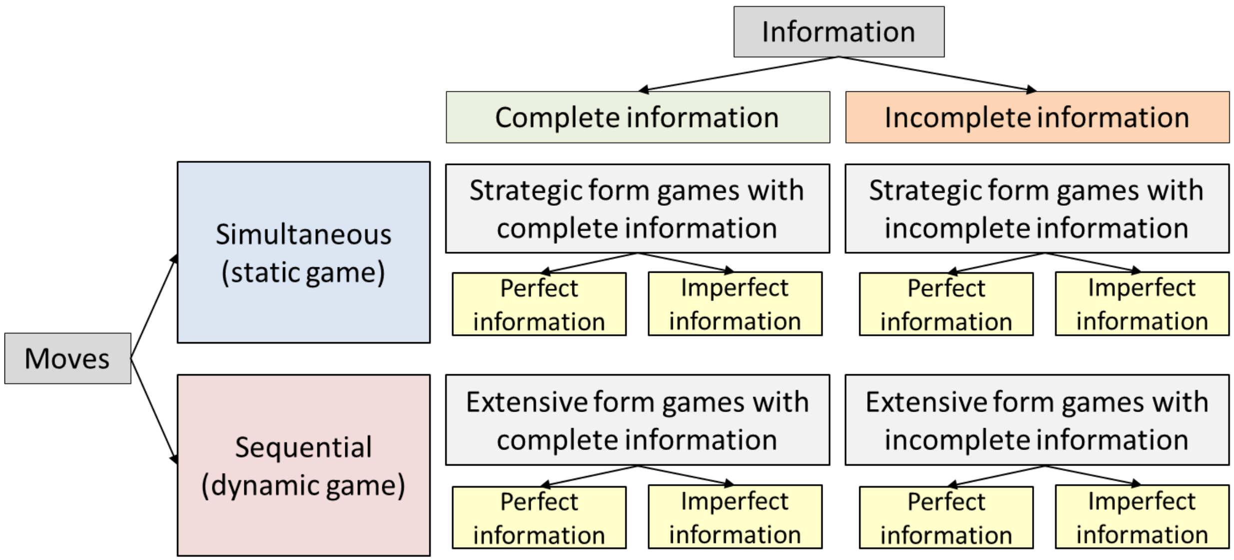
Prisoner's Dilemma



	C	D
C	-1, -1	-4, 0
D	0, -4	-3, -3

Course schedule

1. Static Games of Complete Information
2. Dynamic Games of Complete Information
3. Static Game of Incomplete Information
4. Dynamic Games of Incomplete Information



- **Incomplete Information**: Players do not have complete information about the game being played (regarding payoffs)
- **Imperfect Information**: Players do not perfectly observe the actions of other players or forget their own actions (regarding actions)
- **We will study, how each of these four classes of games can be mathematically described**

Course schedule

1. Static Games of Complete Information
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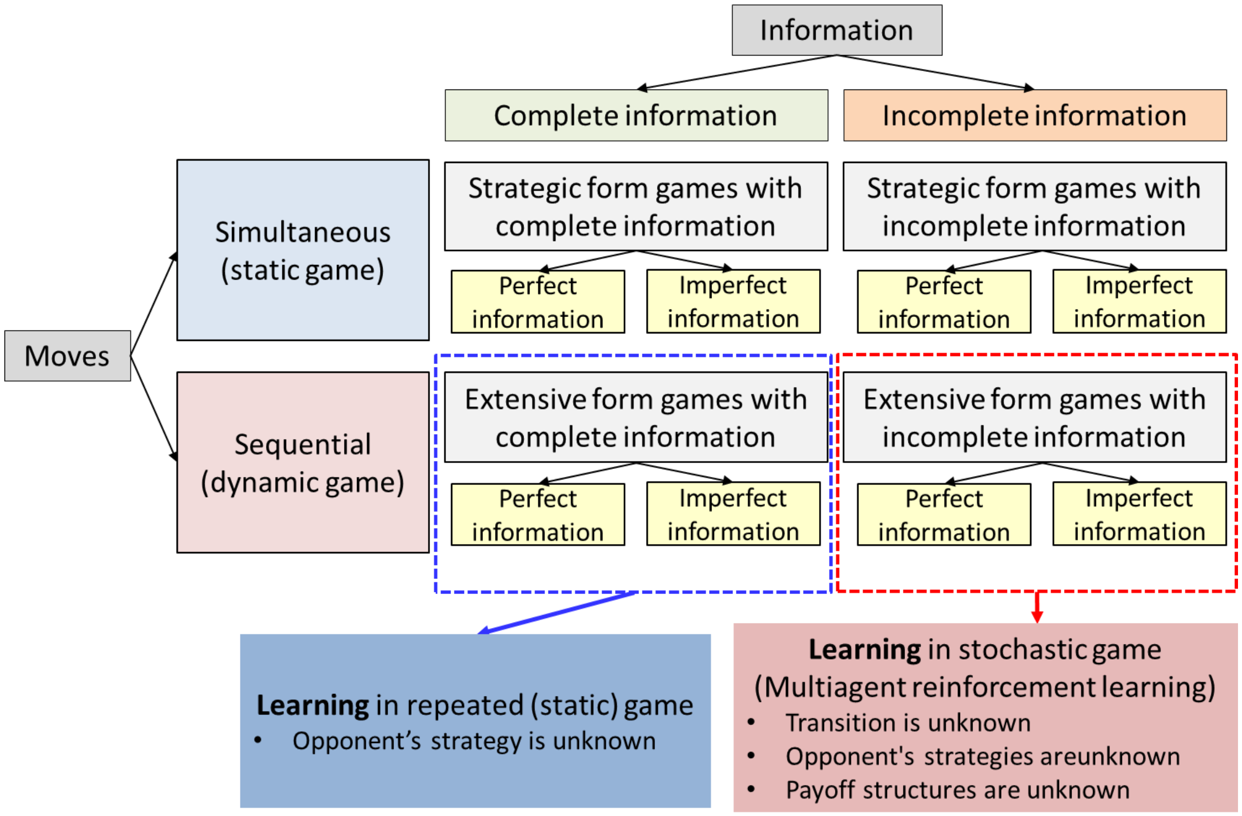
3. Static Game of Incomplete Information
4. Dynamic Games of Incomplete Information
5. Learning in Repeated Game
6. Learning Stochastic Games
7. Optimal Control and Differential Game (if time allows)



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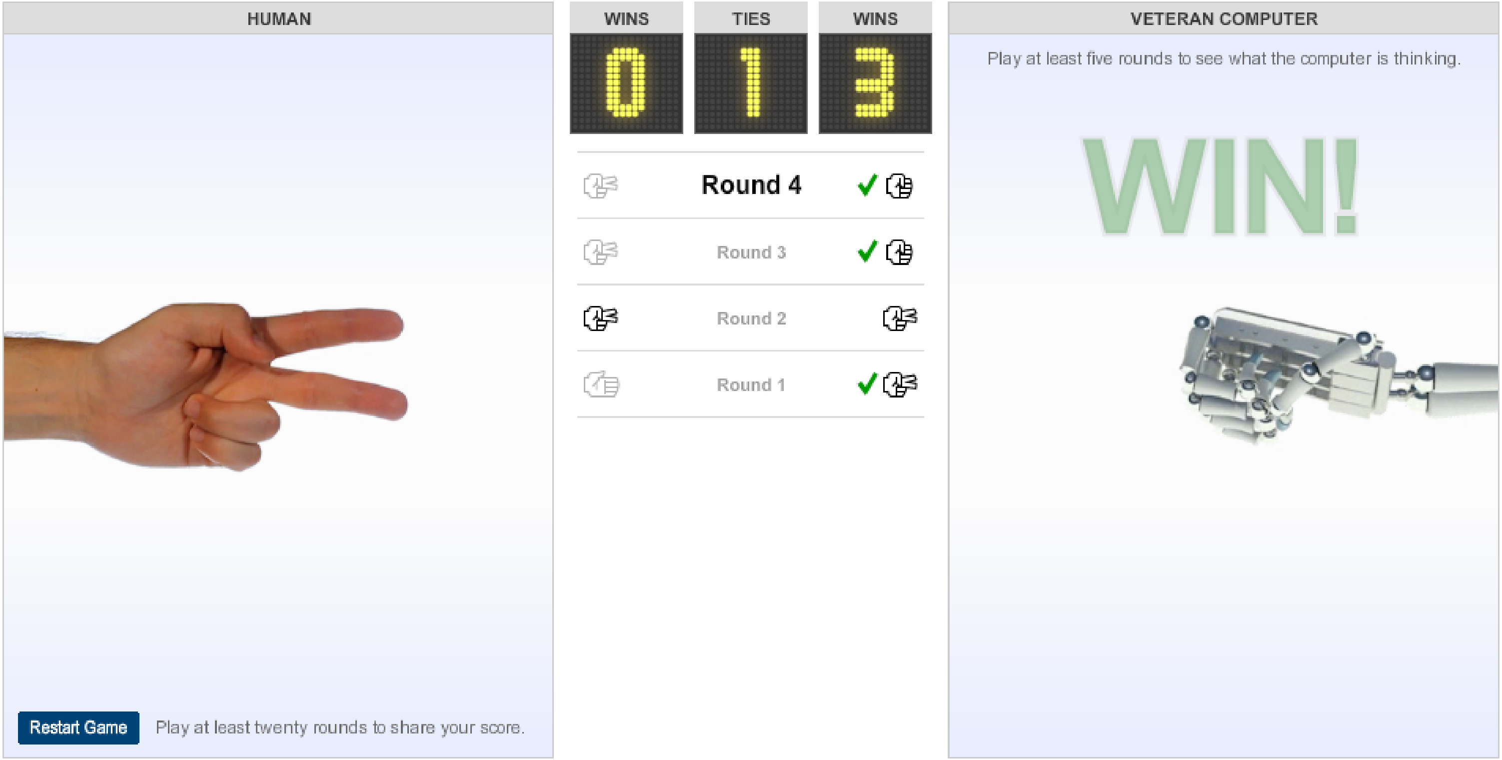


http://www.nytimes.com/interactive/science/rock-paper-scissors.html?_r=0

Course grades

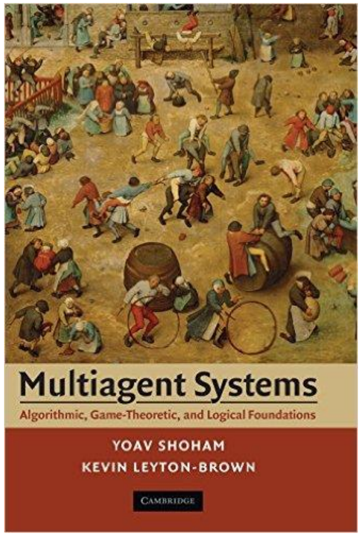
Undergraduate students	Graduate students
Homework (1,2,3,4)	
Midterm	
Course Project (term paper by a group)	Course Project (Individual final report in a conference paper style)

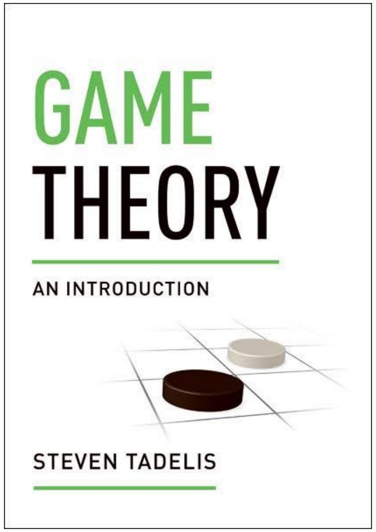
Separate grades for under and grad

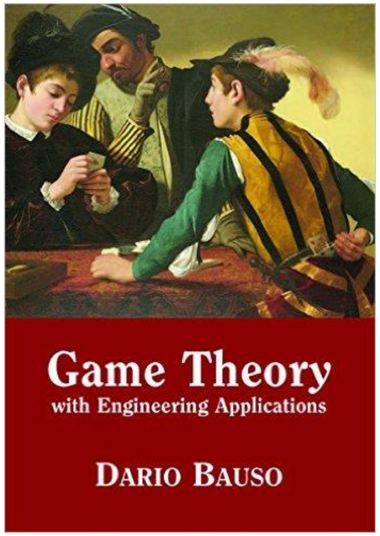
Course project

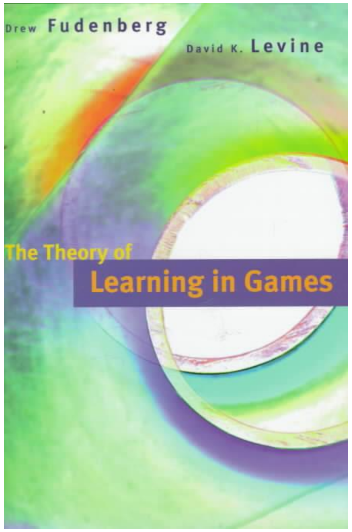
- The objective of the project is to encourage students to define their own problems of interests and formulate them in a formal mathematical way. The topic should be related to the general theme of the course. As part of the project you should:
  - formulate a target problem
  - apply a decision making methodology to solve the formulated problem
  - analyze and interpret the results obtained
  - present the result and derived insights to other people

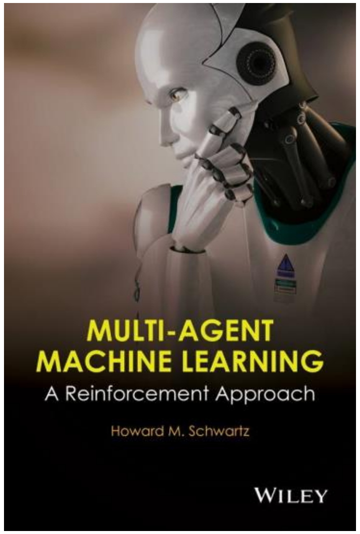
Text books

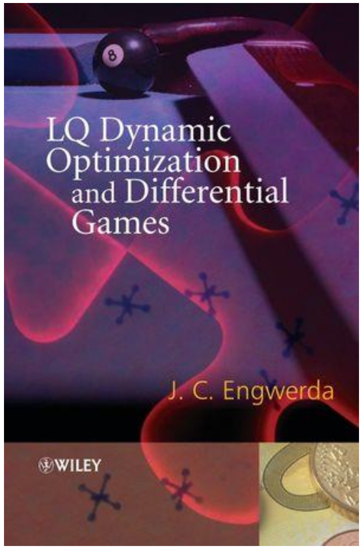

Multiagent Systems
Algorithmic, Game-Theoretic, and Logical Foundations
YOAV SHOHAM
KEVIN LEYTON-BROWN
CAMBRIDGE


GAME
THEORY
AN INTRODUCTION
STEVEN TADELIS


Game Theory
with Engineering Applications
DARIO BAUSO


Drew Fudenberg
David K. Levine
The Theory of
Learning in Games


MULTI-AGENT
MACHINE LEARNING
A Reinforcement Approach
Howard M. Schwartz
WILEY


LQ Dynamic
Optimization
and Differential
Games
J. C. Engwerda
WILEY

Why game theory is useful?

What is game theory?

Definitions of game theory found in literature….

- Game theory is described as “the study of mathematical models of conflict and cooperation between intelligent rational decision-makers.”

- Game theory is the study of the behavior of decision makers whose decisions affect each other

- Provides the languages and framework for the discussion of problems in economics, social science, evolutionary biology, etc.

- Mathematical study of interaction between rational, self-interested agents

What is game theory?

- **Cooperative/coalitional games**
  - The basic modeling unit is the group
- **Non-cooperative games**
  - The basic modeling unit is the individual (including his beliefs, preferences, and possible actions)
  - while it’s most interested in situations where agents’ interests conflict, it’s not restricted to these settings
  - the individuals pursue their own interests

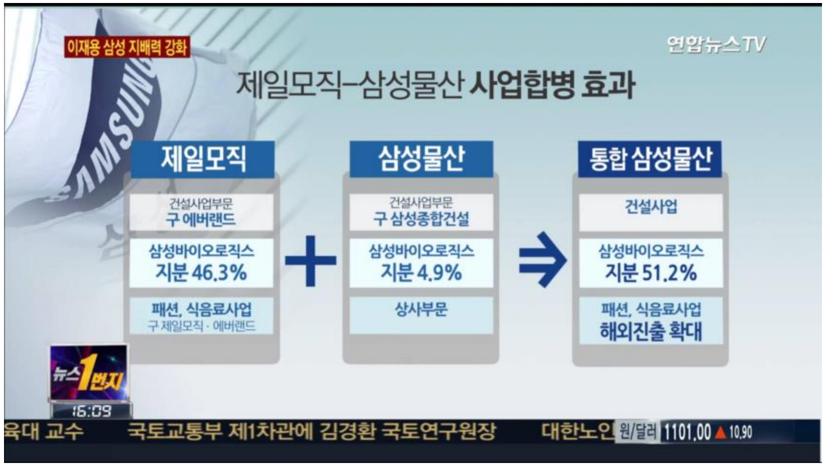
From optimality to equilibrium

	1 payoff	n payoffs
1 player	**Optimization (Optimality)**	Multi-objective optimization
n players	Team theory	**Game theory (Equilibrium)**

- In optimization problems, a solution can be determined by a single player
  - **Optimality condition** can be used

- In game theory, a set of strategies can be determined simultaneously by multiple players
  - Optimal condition with respect to a single player does not work (other players affect each agent's payoff)
  - Instead, **equilibrium concept** can be used

- **In many business situation**, strategic player has to determine his strategy without knowledge of what the rival is doing at the same time
  - Product design
  - Pricing and marketing some new product
  - Mergers and acquisitions competition
  - Auctions



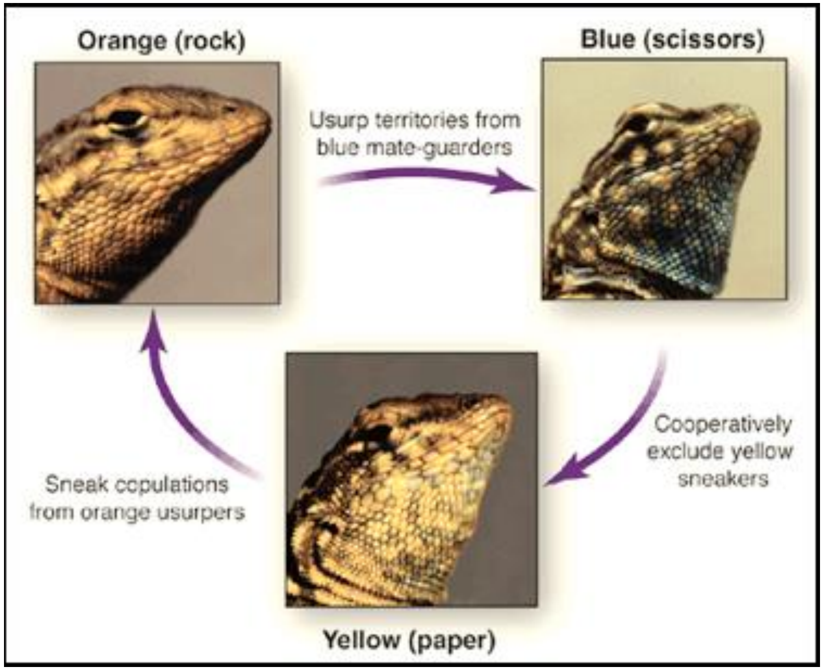
- **For international relations and politics**, there are also many conflictions that can be modeled by game theory
  - Voting
  - International organizations
  - Custom duty on imported goods
  - OPEC
  - Territory conflictions
  - Religion


Motivation

- **In Biology**, game theory has been used to describe
  - Evolution of spices
  - Behaviors of animals

Leadership in fish affected by previous experiences and linked to personality

Leadership behaviour is affected by social experiences from previous partners and depends on an individual's personality, as shown by our latest study with three-spined stickleback fish, now published in Behavioral Ecology.



- orange color: very aggressive and defend large territories;
- blue color: less aggressive defense smaller territories;
- yellow color: not aggressive, opportunistic mating behavior.

http://jollejolles.com/leadership-in-fish-linked-to-personality-and-previous-social-experience/

https://evolhappens.wordpress.com/2013/05/07/survival-of-the-sexiest-male-competition/

Motivation

- **In everyday life**, we are also using game theory
    - Rock paper scissor game
    - Team project
    - Getting a good GPA in a class
    - Applying for graduate schools and labs
    - Sports

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Motivation

- **In engineering,** the concept of game theory is being widely used to model the interaction among sub-systems that are designed to achieve global objectives
  - Drone formation
  - Driverless cars
  - Smart grid operation
  - Distributed water channel
  - Machine learning
  - Communication

https://www.youtube.com/watch?v=wFi5SkhUjR8&feature=youtu.be

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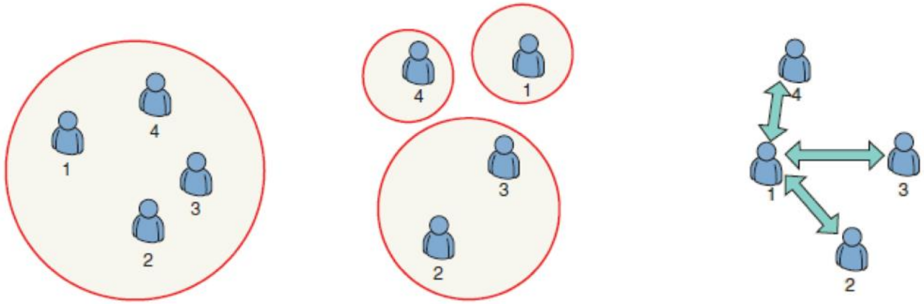


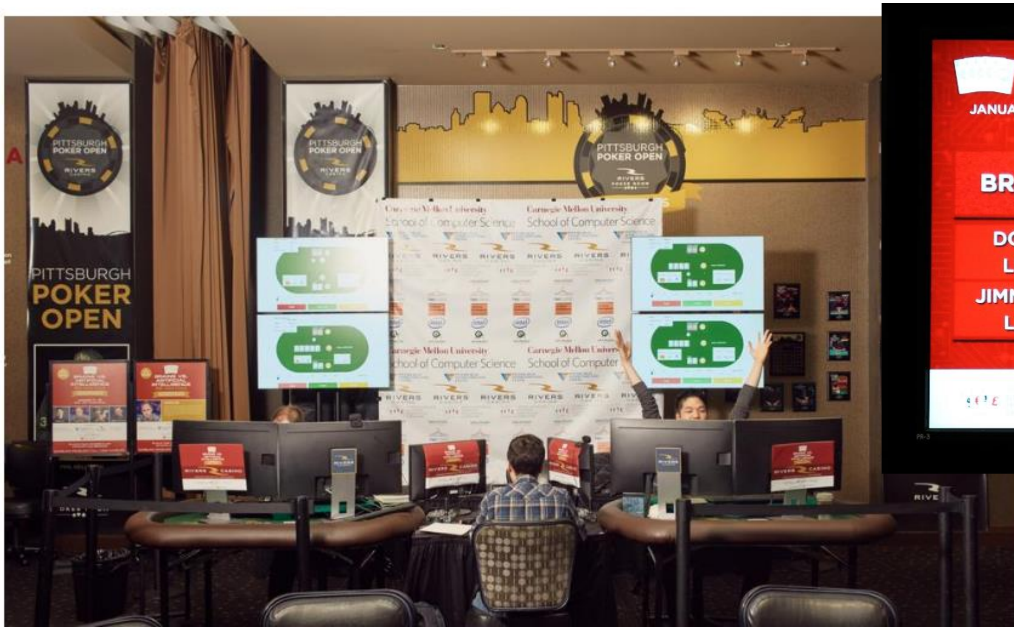
Community	Coordination chain	Value	division
$C_1 = \{1\}$	[1]	20	[20]
$C_2 = \{2\}$	[2]	10	[10]
$C_3 = \{3\}$	[3]	30	[30]
$C_4 = \{1, 2\}$	[1, 2]	34	[23, 11]
$C_5 = \{2, 3\}$	[3, 2]	44	[32, 12]
$C_6 = \{1, 2\}$	[2, 1]	44	[31, 13]
$C_7 = \{3, 1\}$	[1, 3]	49	[20, 29]
$C_8 = \{1, 2, 3\}$	[1, 2, 3]	46	[6, 9, 31]

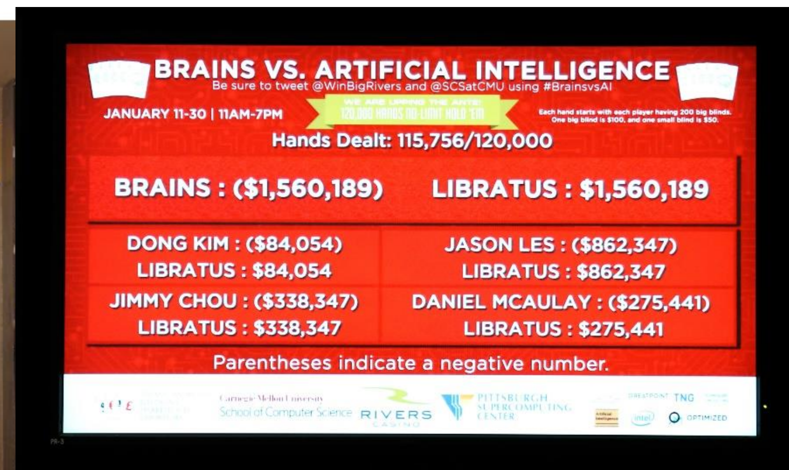
Game theory now and future

Inside the 20-Year Quest to Build Computers That Play Poker

Recent breakthroughs in artificial intelligence research raise questions about the threat that bots pose to the online gambling industry.

by **Joshua Brustein**
2017년 1월 31일 오후 7:00 GMT+9





https://www.engadget.com/2017/02/10/libratus-ai-poker-winner/

Can game theory help solve the problem of climate change?

Applying the mathematical principle of studying models of conflict and cooperation between groups could help us rein in global warming

A mother and her children wear masks for protection as smoke billows from a coal-fired power plant in Shanxi, China. Photograph: Kevin Frayer/Getty Images

https://www.theguardian.com/science/blog/2016/apr/13/can-game-theory-help-solve-the-problem-of-climate-change

Game theory now and future

Game Theory reveals the Future of Deep Learning

Credit: https://unsplash.com/search/game?photo=B4op5oZ4x5Q

3 different ways game theory plays in Deep Learning.
(1) As a means of describing and analyzing new DL architectures.
(2) As a way to construct a learning strategy and
(3) A way to predict behavior of human participants.

https://medium.com/intuitionmachine/game-theory-maps-the-future-of-deep-learning-21e193b0e33a#.84ftsyarg

Fake news on Facebook is no game, but it can explain game theory

December 02, 2016 | By Clare Milliken

Economist Jeff Ely says game theory can help explain the fake news phonomenon.

Trending

Myopia cell discovered in retina
February 6, 2017 – *Health*

Northwestern Athletics unveils Trienens Performance Center
February 7, 2017 – *University*

Change in astronaut's gut bacteria attributed to spaceflight
February 3, 2017 – *Science & Tech*

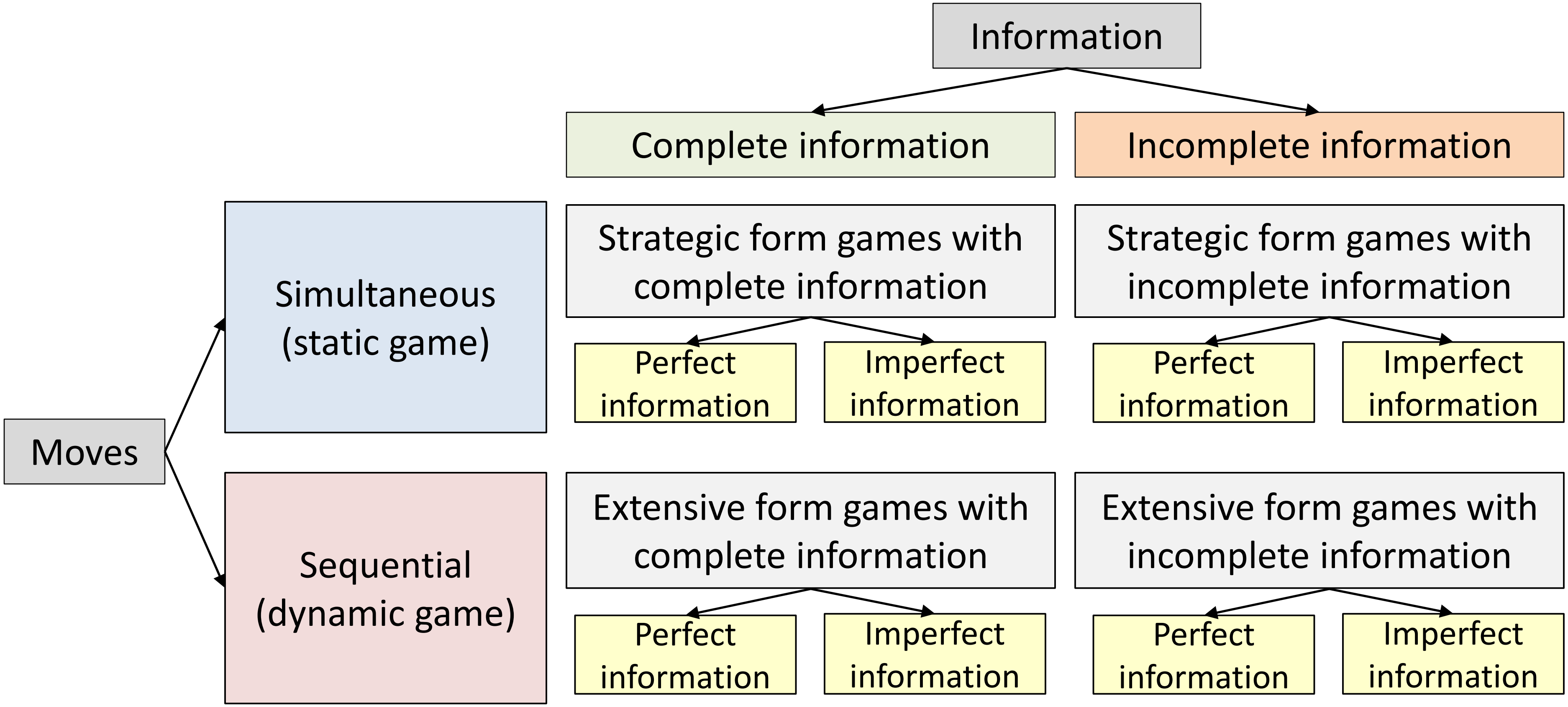
ALL STORIES >

- Player 1: person or group fabricating news stories
- Player 2: the reader who encounters stories and isn't immediately able to discern whether a story is fake or not
- Player 3: Mark Zuckerberg

Will filtering fake news help stop spreads of fake news?

https://news.northwestern.edu/stories/2016/12/economics-explainer-using-fake-news/

Scope of this course



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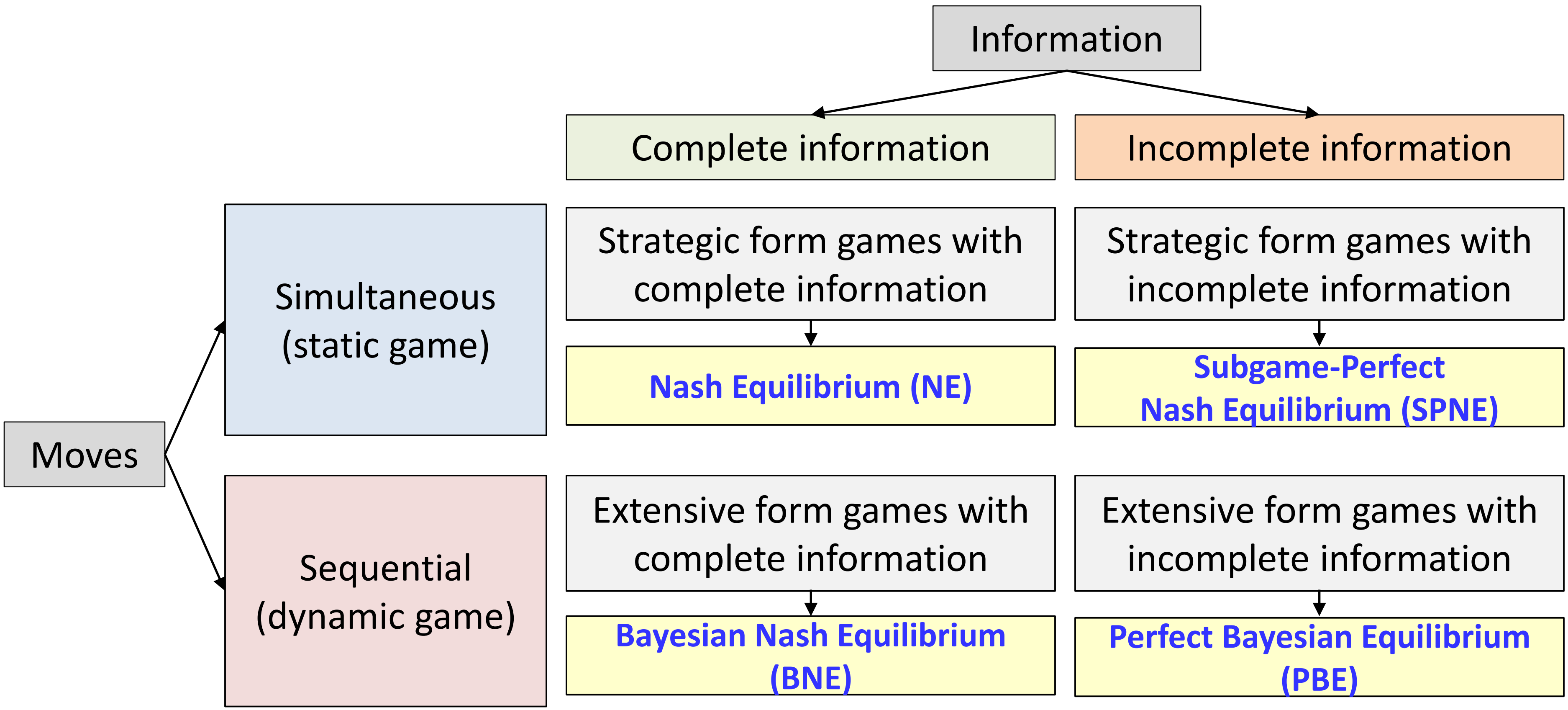
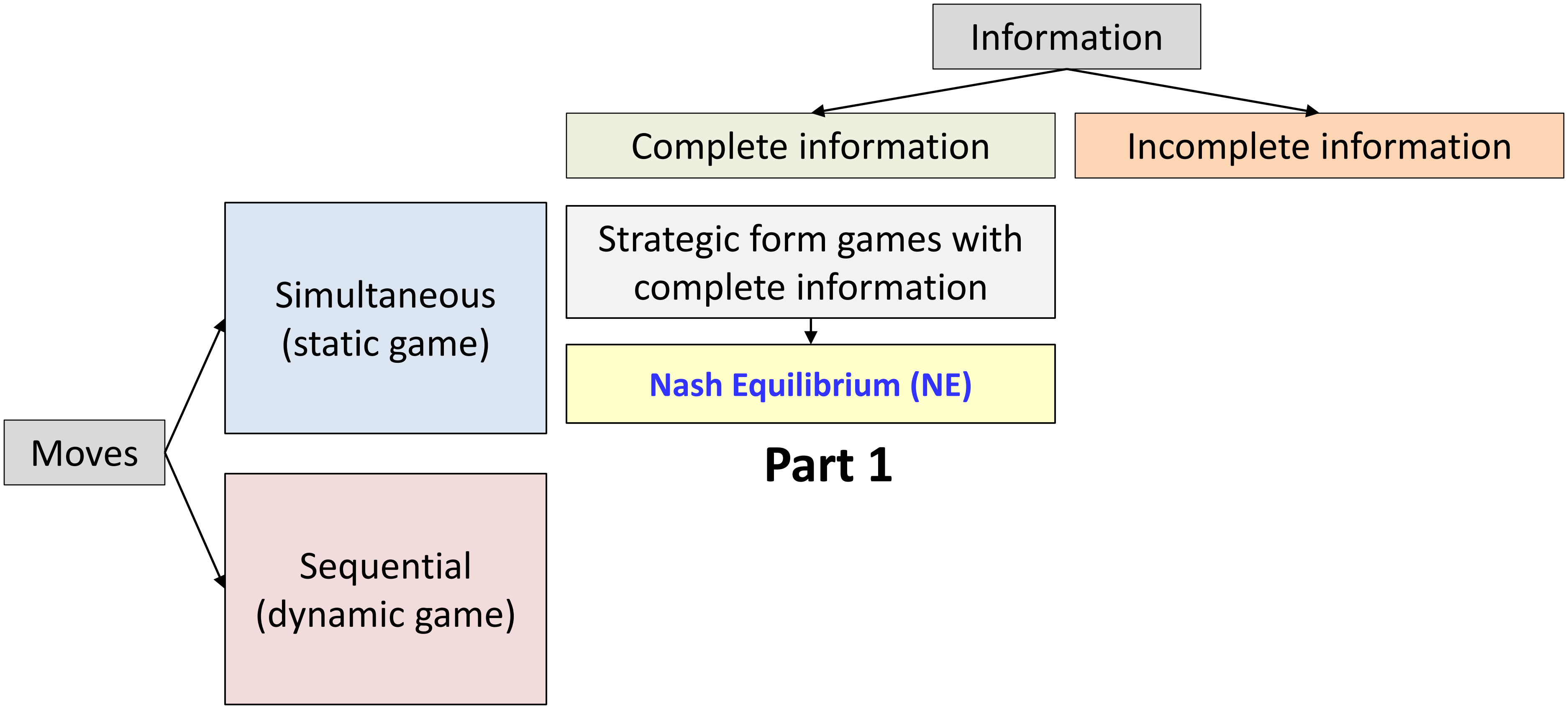


- Corresponding to these four games, we will discusses four notions of equilibrium
- We will also study how these four classes of games used to model various problems

Scope of this course



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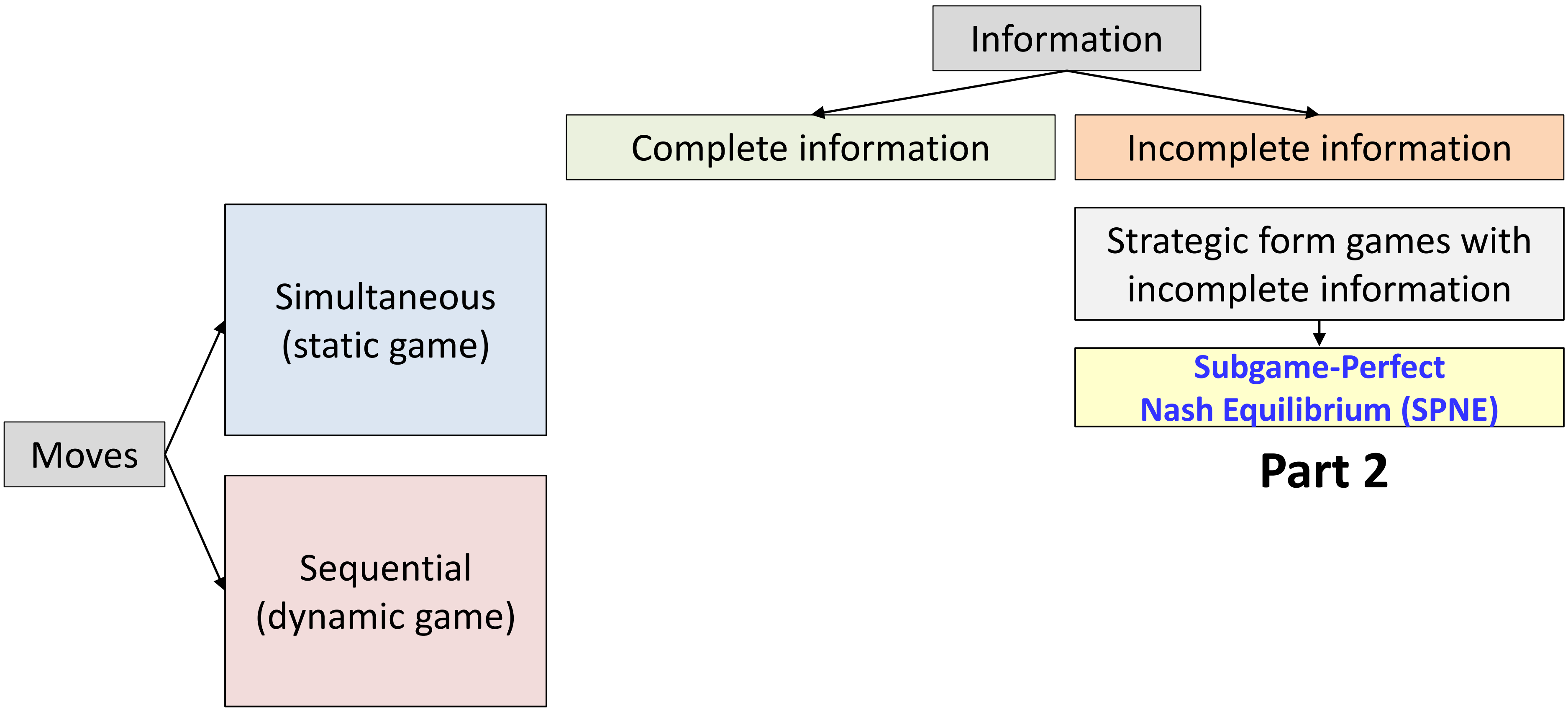

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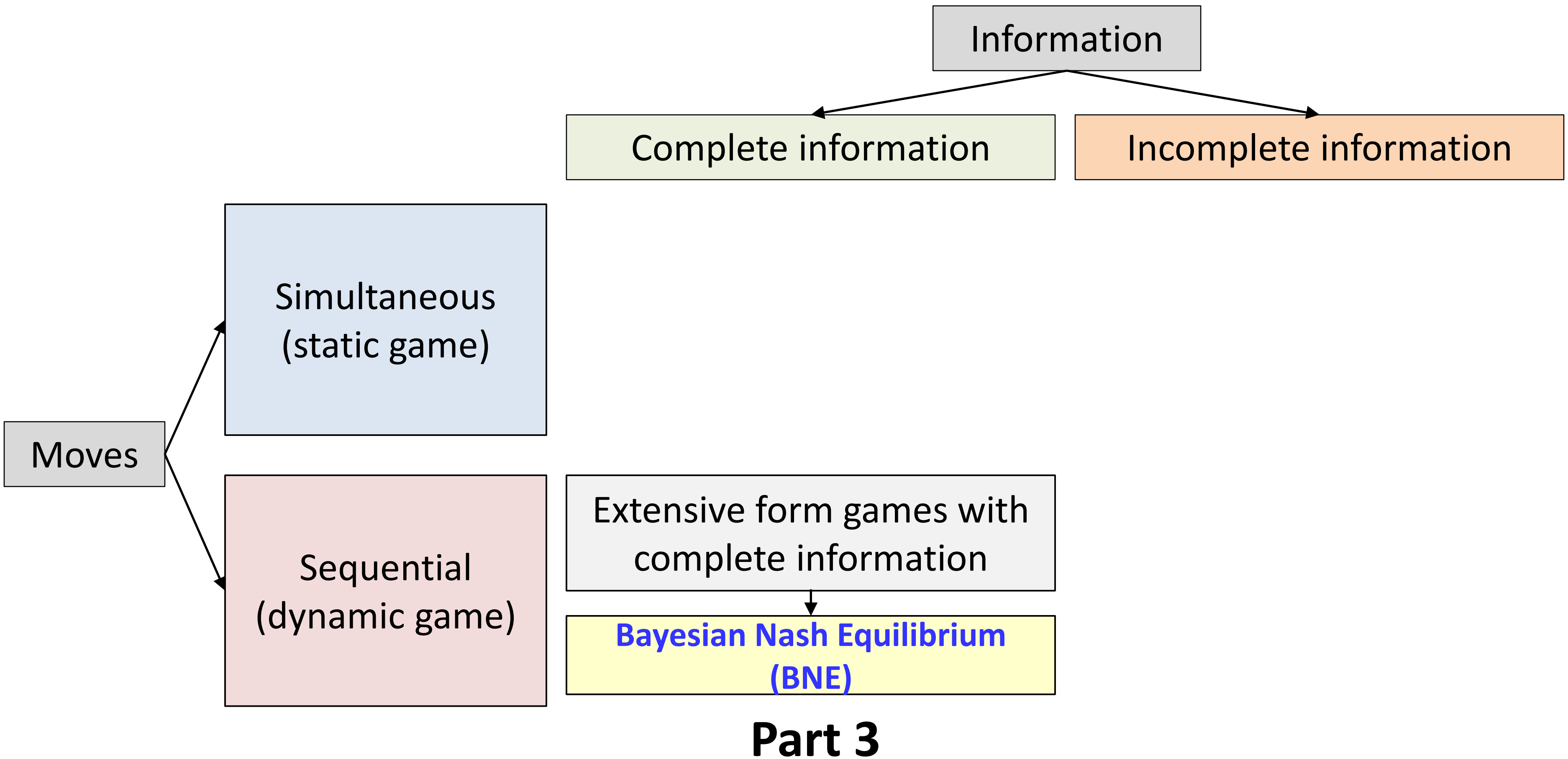

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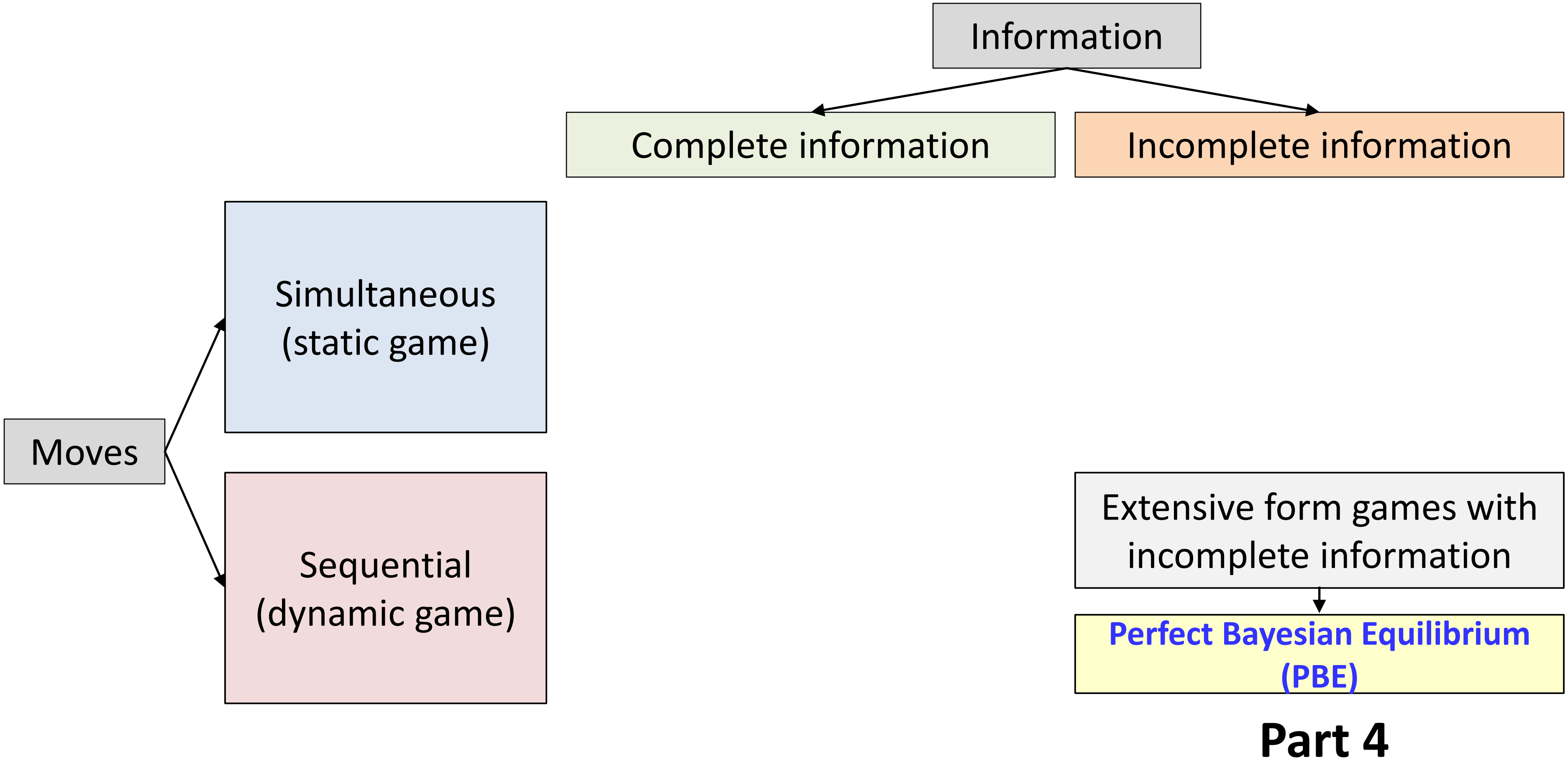

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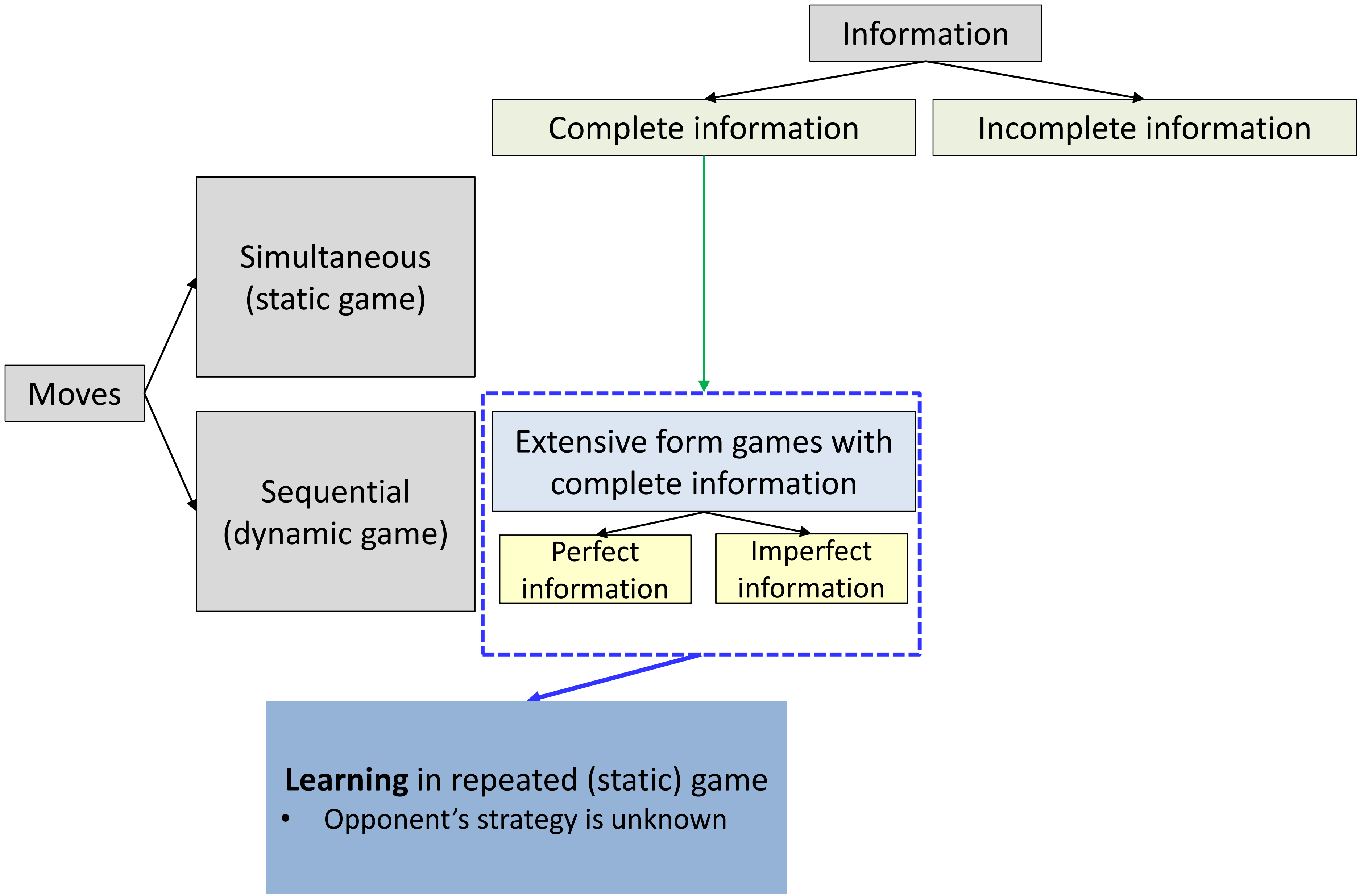
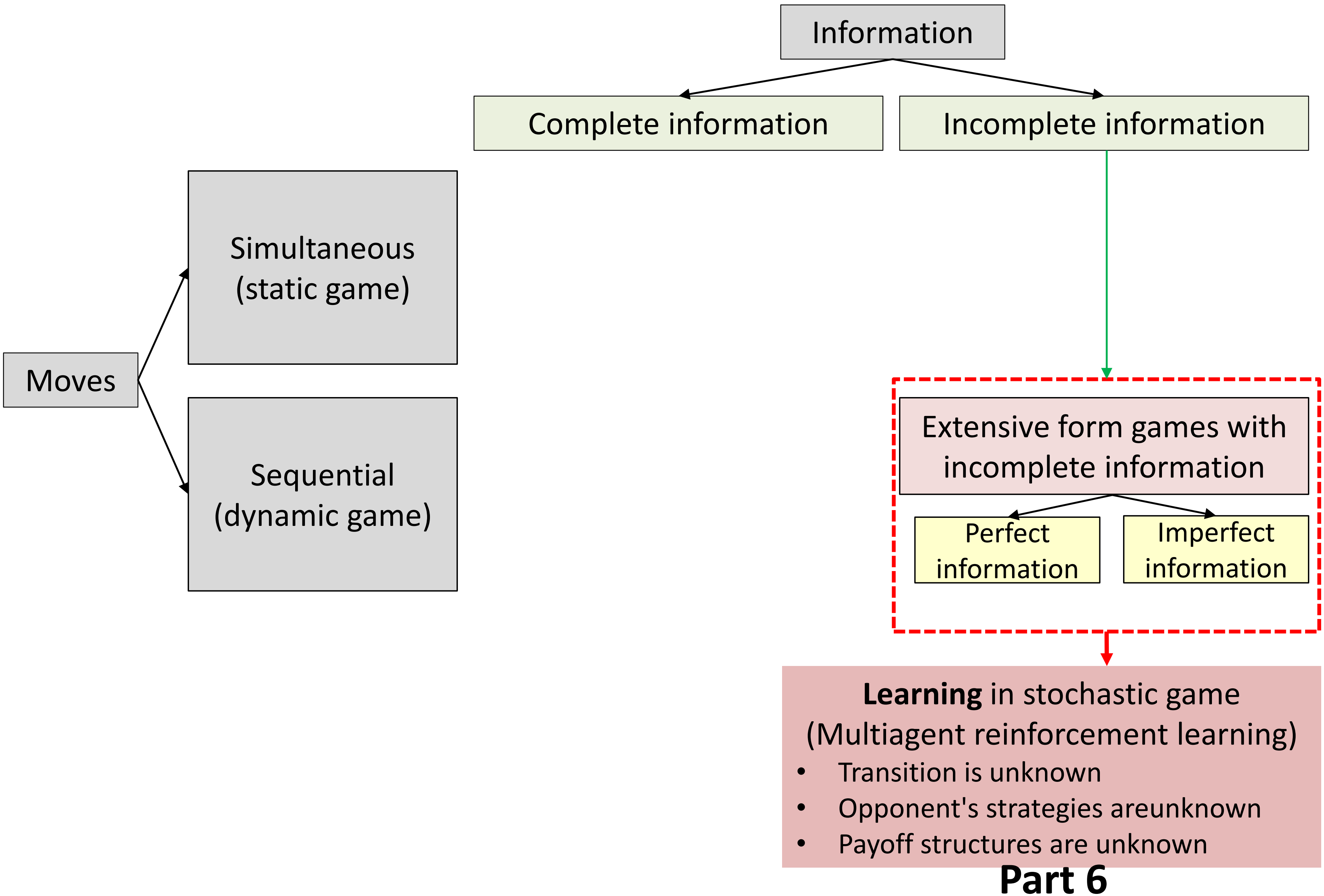
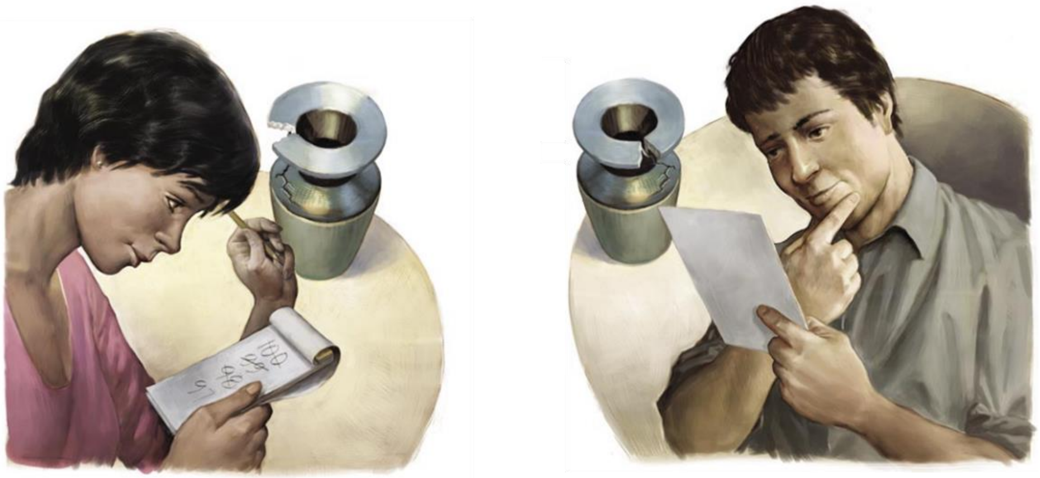


Part 5

Scope of this course



Fun game: Traveler's dilemma

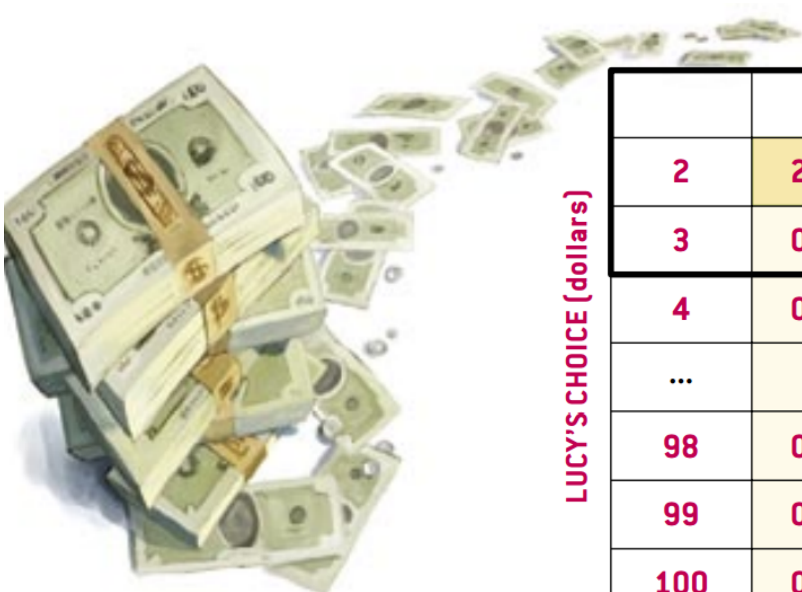


- A and B find that the airline has damaged the identical antiques that each had purchased
- An airline manager says that he is happy to compensate them but is handicapped by being clueless about the value of these strange objects.
- An airline manager asks each of them to write down the price of the antique as any dollar integer between 2 and 100 without conferring together.
  - ✓ If $A = B$, both will get the same money
  - ✓ If $A > B$, A will get $A - 2$ (penalty for cheating) and B will get $B + 2$ (bonus for being diligent)
  - ✓ If $A < B$, A will get $A + 2$ (bonus for being diligent) and B will get $B - 2$ (penalty for cheating)

Play this game with a person sitting next to you

http://www.cs.virginia.edu/~robins/The_Travelers_Dilemma.pdf

Fun game: Traveler's dilemma



PETE'S CHOICE (dollars)

LUCY'S CHOICE (dollars)	2	3	4	...	98	99	100
2	2 2	4 0	4 0	...	4 0	4 0	4 0
3	0 4	3 3	5 1	...	5 1	5 1	5 1
4	0 4	1 5	4 4	...	6 2	6 2	6 2
...	...	...	...		...	...	...
98	0 4	1 5	2 6	...	98 98	100 96	100 96
99	0 4	1 5	2 6	...	96 100	99 99	101 97
100	0 4	1 5	2 6	...	96 100	97 101	100 100

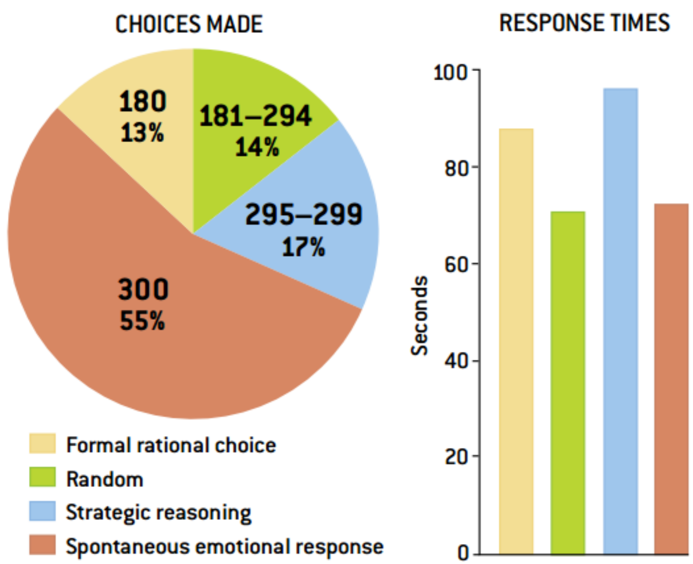
- The equilibrium reaches by two rational decision maker is (\$2, \$2)
- The example shows the paradoxes of rationality that plague game theory

Fun game: Traveler’s dilemma

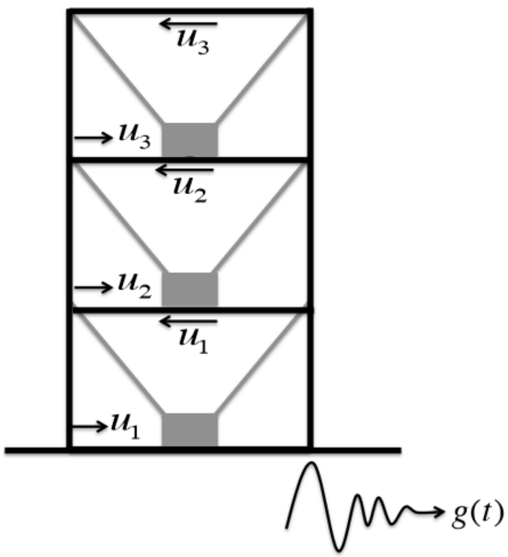
How actually people paly this game? (180-300) and a penalty and bonus are $5 each

WHAT WERE THEY THINKING?

Researchers hypothesized that different thought processes lay behind different types of choices that people made playing a version of Traveler’s Dilemma with the options ranging from 180 to 300 (*pie chart*): a spontaneous emotional response (*choosing 300*), a strategically reasoned choice (*295–299*) or a random one (*181–294*). Players making the formal rational choice (*180*) might have deduced it or known about it in advance. As expected, people making “spontaneous” or “random” selections took the least time to choose (*bar graph*).



Example: Structural control using game theory



State dynamics

$$\dot{x} = Ax + B_1 u_1 + B_2 u_2 + B_3 u_3 + Eg$$

Accumulated cost for each controller

$$J_1 = \int_0^\infty \left\{ x^T Q_1 x + u_1^T R_{11} u_1 + u_2^T R_{12} u_2 + u_3^T R_{13} u_3 \right\} dt$$

$$J_2 = \int_0^\infty \left\{ x^T Q_3 x + u_1^T R_{21} u_1 + u_2^T R_{32} u_2 + u_3^T R_{23} u_3 \right\} dt$$

$$J_3 = \int_0^\infty \left\{ x^T Q_3 x + u_1^T R_{31} u_1 + u_2^T R_{32} u_2 + u_3^T R_{33} u_3 \right\} dt$$

Example: Structural control using game theory

Nash control

$$J_1(F_1, F_2, F_3, x_0) = \int_0^\infty \{x^T(Q_1 + F_1^T R_{11} F_1 + F_2^T R_{12} F_2 + F_3^T R_{13} F_3)x\}\, dt$$

$$J_2(F_1, F_2, F_3, x_0) = \int_0^\infty \{x^T(Q_2 + F_1^T R_{21} F_1 + F_2^T R_{22} F_2 + F_3^T R_{23} F_3)x\}\, dt$$

$$J_3(F_1, F_2, F_3, x_0) = \int_0^\infty \{x^T(Q_3 + F_1^T R_{31} F_1 + F_2^T R_{32} F_2 + F_3^T R_{33} F_3)x\}\, dt$$

$$J_1(F_1^*, F_2^*, F_3^*, x_0) \le J_1(F_1, F_2^*, F_3^*, x_0)$$
$$J_2(F_1^*, F_2^*, F_3^*, x_0) \le J_2(F_1^*, F_2, F_3^*, x_0)$$
$$J_3(F_1^*, F_2^*, F_3^*, x_0) \le J_3(F_1^*, F_2^*, F_3, x_0)$$

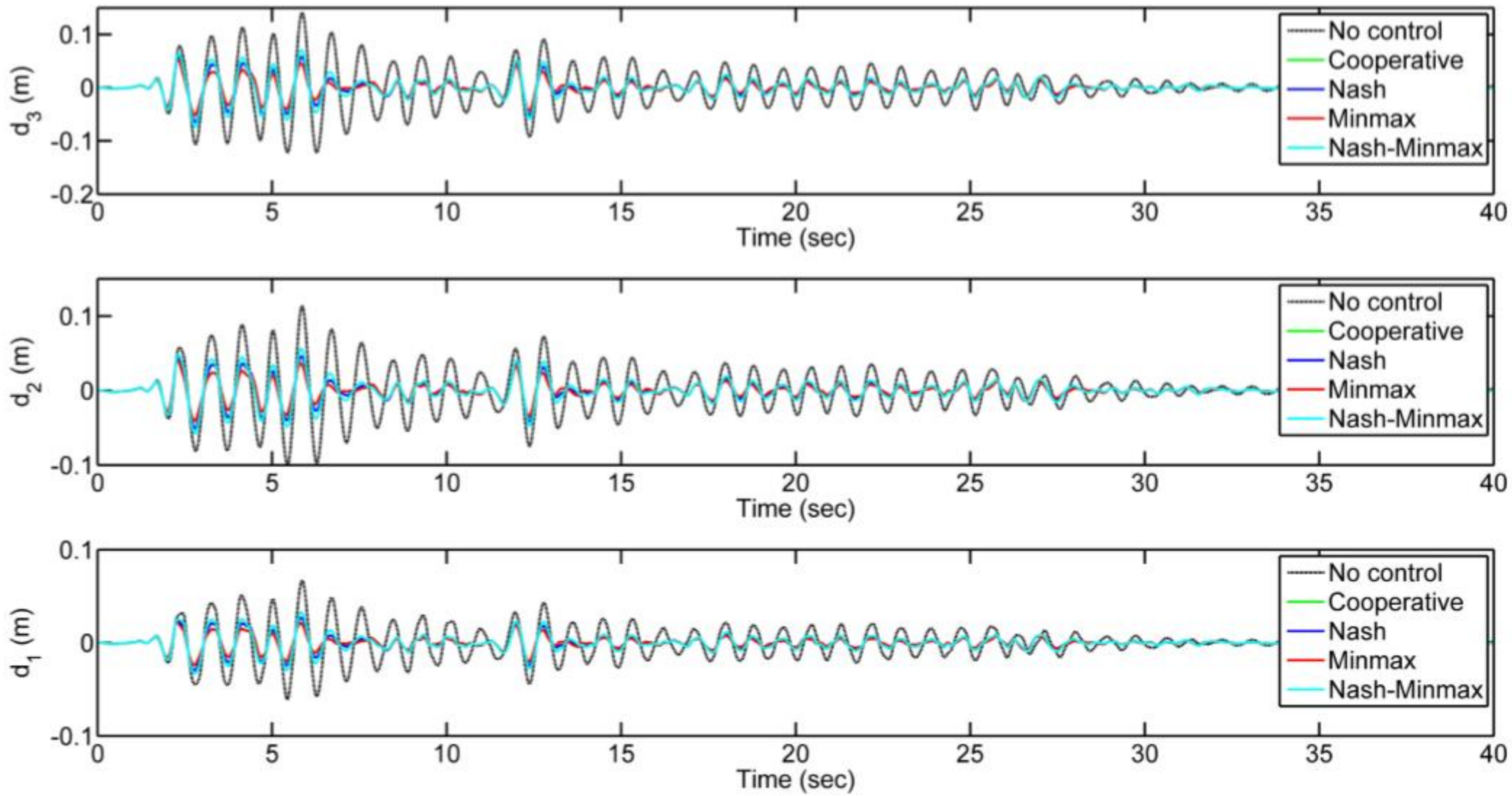
Minmax control

$$J_c(u, g, x_0) = \int_0^\infty \{x^T Q x + u^T R u - g^T V g\} dt$$

$$J_g(u, g, x_0) = -J_c(u, g, x_0) = \int_0^\infty \{-x^T Q x - u^T R u + g^T V g\} dt$$

$$F^* = \min_{F \in F} \sup_{g \in L_2^q(0,\infty)} J_c(F, g, x_0) = \int_0^\infty \{x^T (Q + F^T R F) x - g^T V g\} dt$$

Example: Structural control using game theory



(a) Story displacement

Example: Data-driven wind farm power maximization

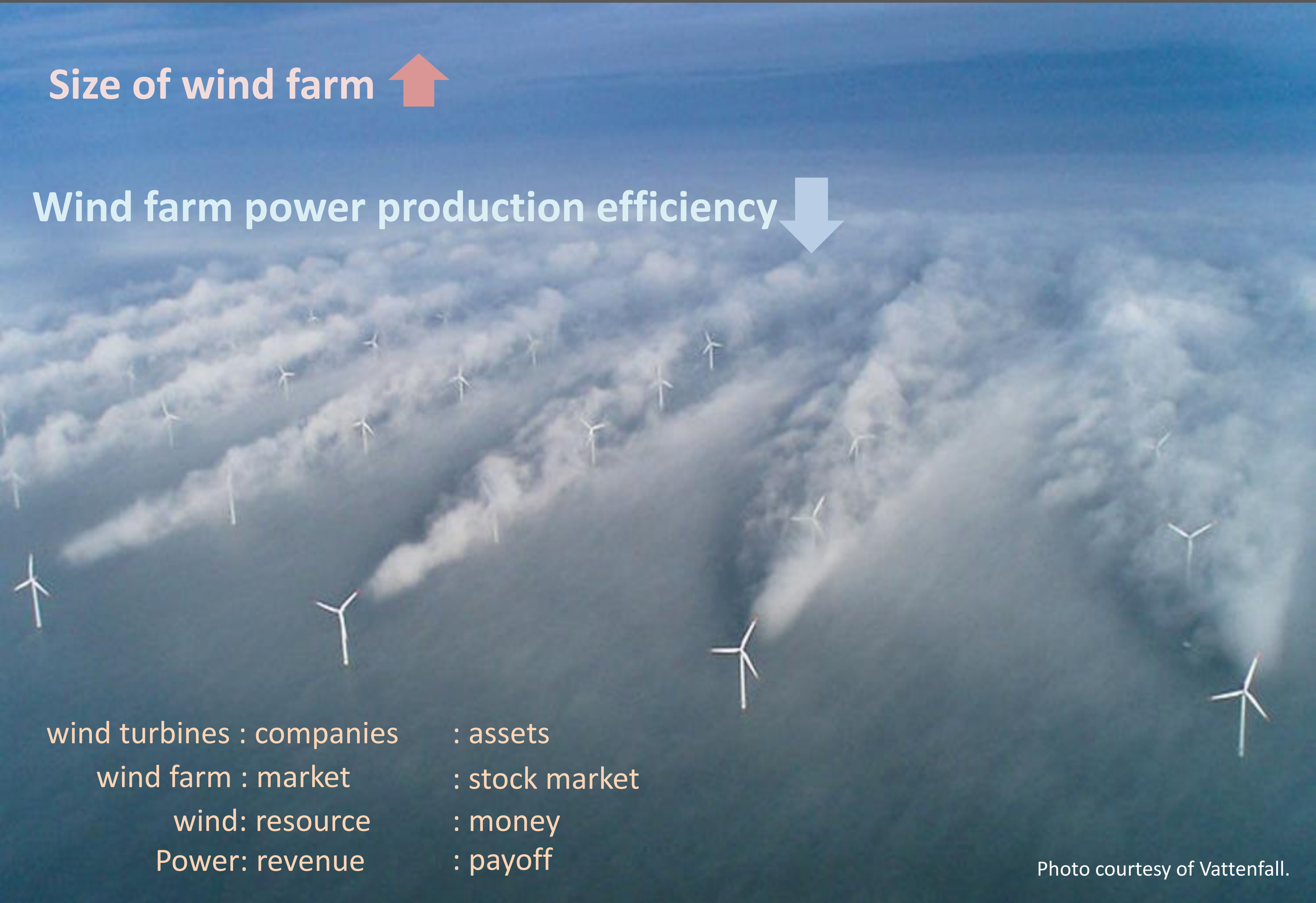
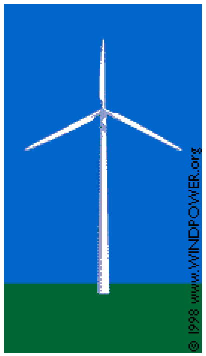
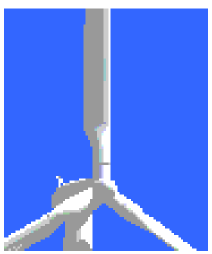


Photo courtesy of Vattenfall.

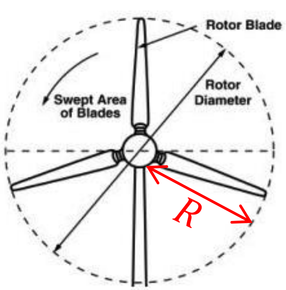
Example: Data-driven wind farm power maximization



<www.windpowr.org>



<www.windpowr.org>

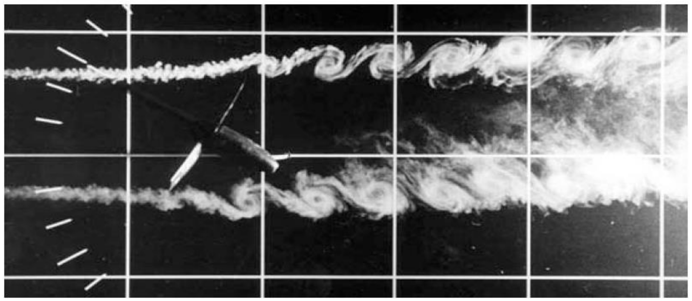


Yaw angle

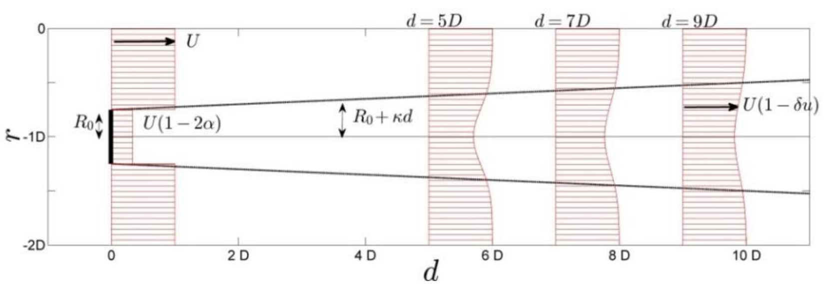
Blade pitch angle

Tip speed ratio

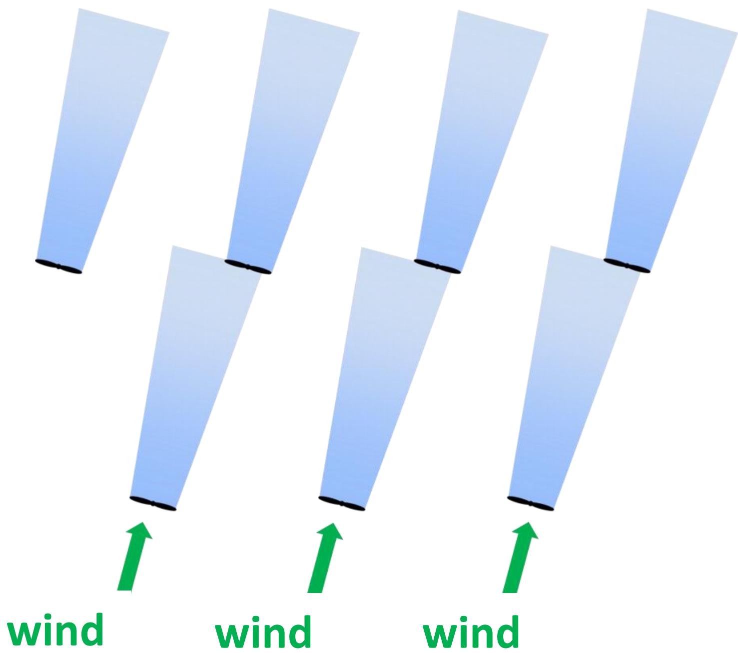
Control Inputs

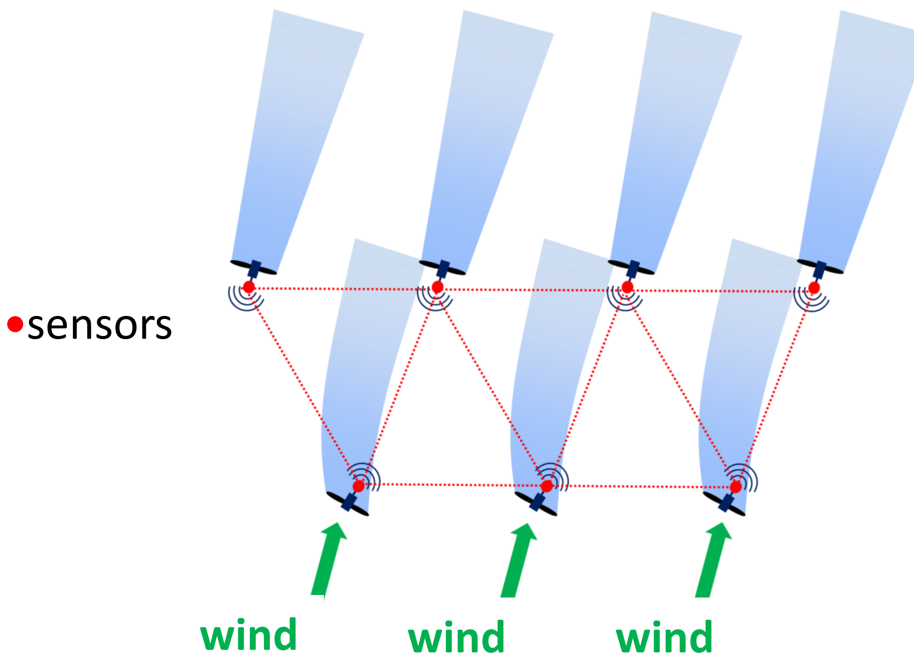


Flow visualization of wake deflection. Conducted at the Royal institute of Technology at 1987, Dahlberg (2003)



Example: Data-driven wind farm power maximization





Greedy strategy:

$$\sum_{i=1}^{N} \underset{x_i}{\text{maximize}}\, P_i(\boldsymbol{x}; U, \theta)$$

$$\leq$$

Cooperative strategy:

$$\underset{\boldsymbol{x}}{\text{maximize}} \sum_{i=1}^{N} P_i(\boldsymbol{x}; U, \theta)$$

$P_i(\boldsymbol{x}; U, \theta)$:Power of wind turbine i

$\boldsymbol{x} = (x_1, \ldots, x_N)$: Control inputs for N wind turbines

Example: Data-driven wind farm power maximization

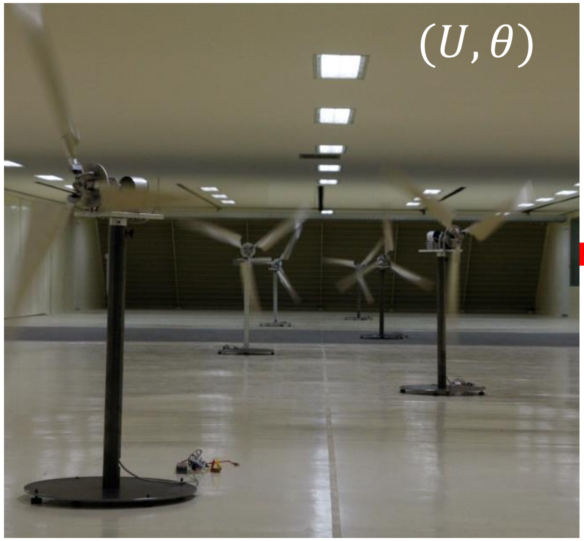
(U, θ) : **Fixed wind condition (Fixed context)**

Input (control actions)

$$\boldsymbol{x}^1 = (x_1^1, \dots, x_N^1)$$

$$\boldsymbol{x}^2 = (x_1^2, \dots, x_N^2)$$

$$\vdots$$



Output (power measurements)

$$y^1 = \sum_{i=1}^{N} P_i(\boldsymbol{x}^1; \theta, U) + \epsilon^1$$

$$y^2 = \sum_{i=1}^{N} P_i(\boldsymbol{x}^2; \theta, U) + \epsilon^2$$

$$\vdots$$

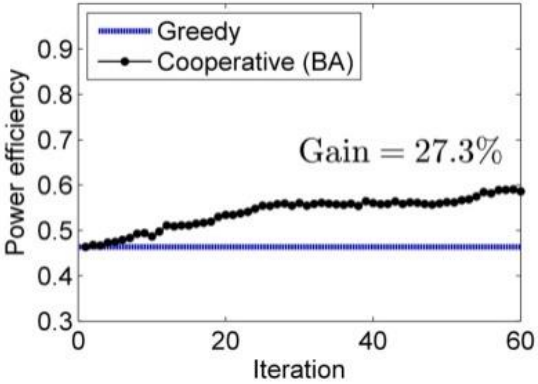
(y = total wind farm power)

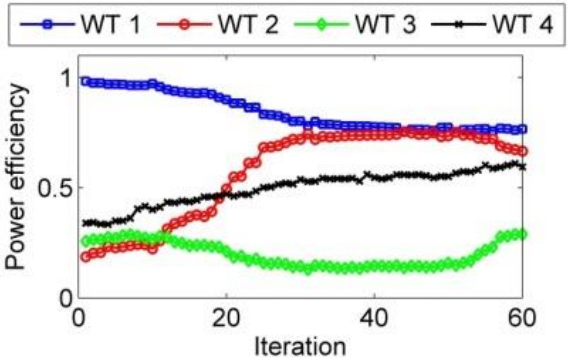
$$\underset{\boldsymbol{x}}{\text{maximize}} \;\; f(\boldsymbol{x}; U, \theta) = \sum_{i=1}^{N} P_i(\boldsymbol{x}; \theta, U)$$

$$\text{subject to} \;\; \boldsymbol{x}^l \le \boldsymbol{x} \le \boldsymbol{x}^u$$

Without knowing $f(\boldsymbol{x}; U, \theta)$

Example: Data-driven wind farm power maximization

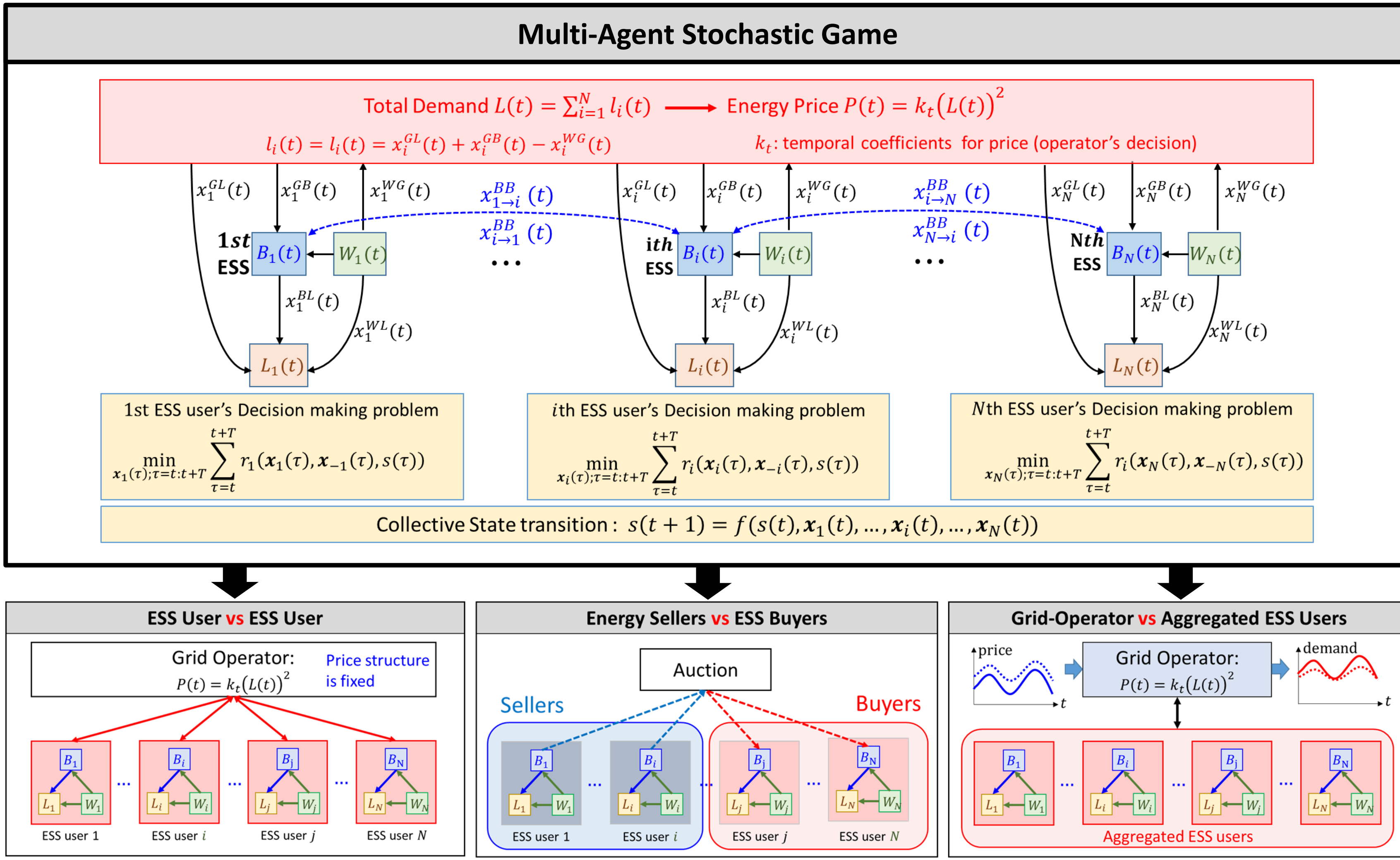




Greedy control (initial)

Cooperative control (after convergence)

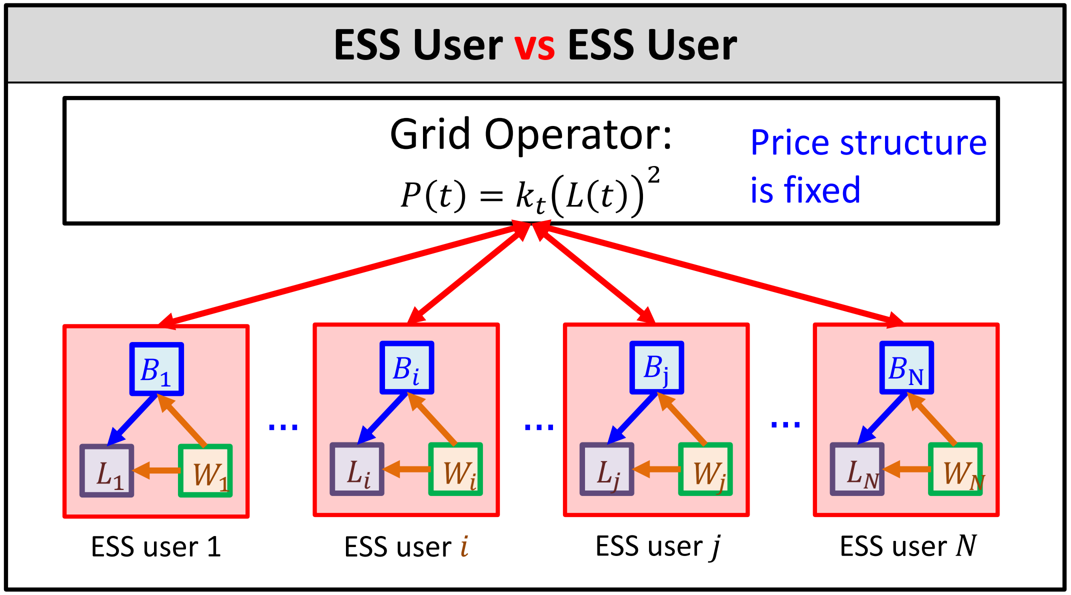
Smart Grid + AI



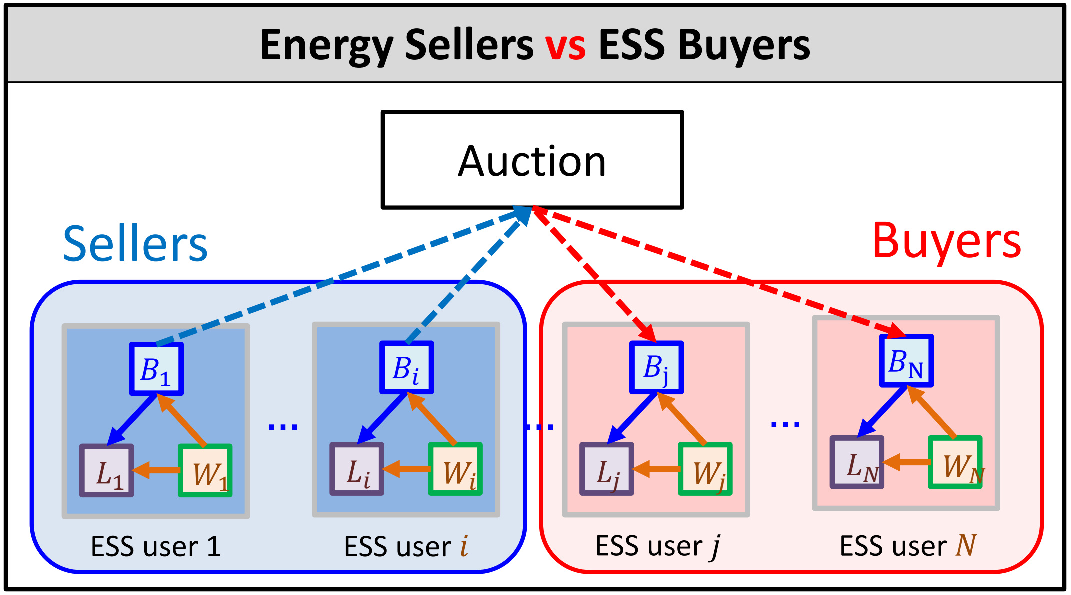
<Non-Cooperative Game among N ESS users>

<Energy Trading Game Modeled by Double Auction>

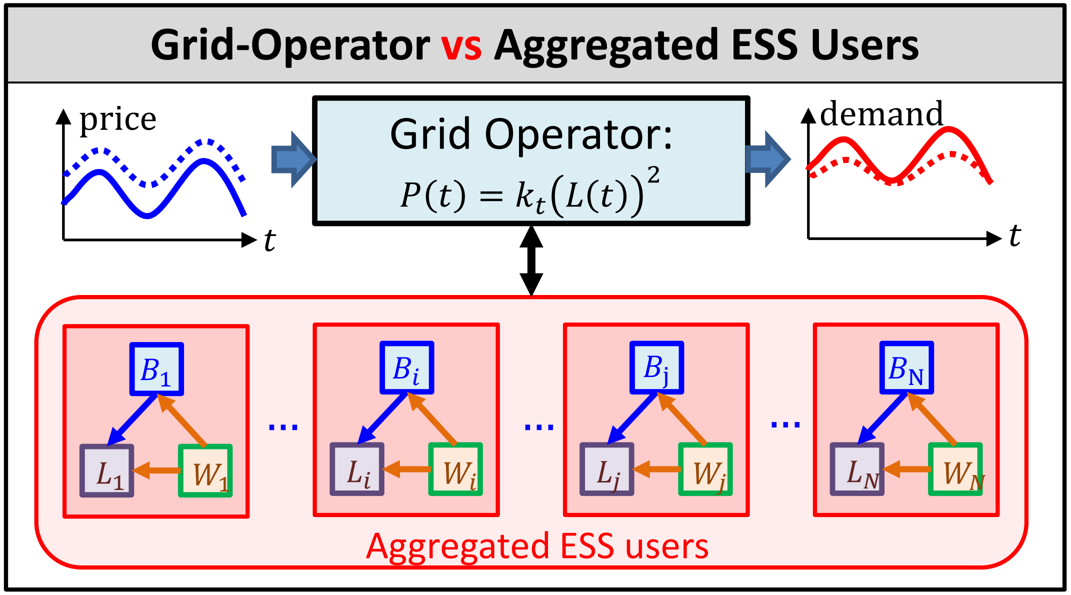
<Sequential Game Modeled by Stackelberg Game>



<Non-Cooperative Game among N ESS users>



<Energy Trading Game Modeled by Double Auction>



<Sequential Game Modeled by Stackelberg Game>