

**IE801 & IE481 Game Theory with Multi-Agent Reinforcement Learning**

**Homework 1**

**Due: 2019. 10. 7**

**Submit the hard copy in the class (before the lecture starts)**

**(15pts) Problem 1 (Sharing game)**

Two people are trying to share a 100\$. They each simultaneously announce a real number from 0 to 100\$. If the sum of the numbers is less than or equal to 100\$, each person receives the number he announced. If the sum is greater than 100\$, then the player who announced the smaller number, say  $x$ , receives  $x$  while the other person receives  $100 - x$ . If the sum is greater than 100 and both numbers are the same, then each receives 50\$.

(1) Formulate this situation a strategic game (i.e., define players, actions, payoff functions)

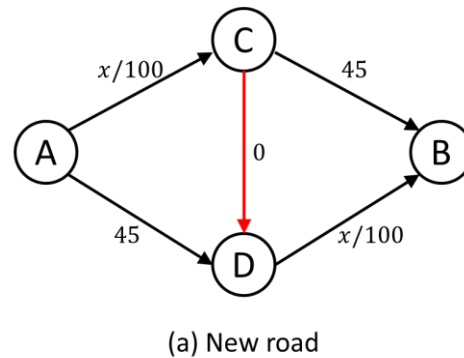
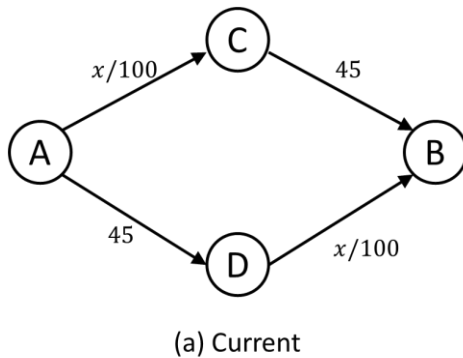
(2) Show that (50, 50) is a Nash equilibrium

(3) Can you find more Nash equilibria? Prove your answer

**(15pts) Problem 2 (Nash equilibrium route for reaching a destination)**

Consider the following traffic network. You need to reach destination B starting from A. The number on the edge represents the travel time required to move between two nodes. For example, the travel time from node C to B is 45 (minutes).

- The travel time for the routes A-C and D-B depends on the number of car  $x$  using that route.
- The travel time for the routes A-D and C-B are fixed as 45.



Now, assume that 4000 cars want to move from A to B as part of the morning commute. The two possible routes that each car can choose are: the upper route through C and the lower route through D.

(1) Formulate this problem as a game by 4000 players

(2) What is a Nash equilibrium? Describe the conditions where each car is not willing to deviate from. (express this condition using  $x$ , the number of cars moving upper route A-C-B)

(3) What will happen if the city constructs a new really fast highway between C and D. Assume the travel time between C and D is 0. How will this new road affect the commuting time?

**(15Pts) Problem 3 (Normal form game with three-players)**

Consider the 3-player normal form game bellows:

	L	R
T	(5, 5, 5)	(2, 6, 2)
B	(6, 2, 2)	(3, 3, -1)
N		

(when player 3 choose to play N)

	L	R
T	(2, 2, 6)	(-1, 3, 3)
B	(3, -1, 3)	(0, 0, 0)
F		

(when player 3 choose to play F)

Here, each player has two strategies:

- (T, B) for player 1,
- (L, R) for player 2
- (N, F) for player 3.

In this notation, player 3 gets to select left or right table, player 2 selects the column, and player 1 selects the row. For example, if they play (T, L, F) they each get (2,2,6) respectively.

(1) List all of the pure strategy Nash equilibrium profiles of this game.

(2) Compute the maxmin value (or security level) for player 1.

(3) Suppose it is common knowledge that player 3 will play N with probability 0.8 and F with probability 0.2. This induces a game just between player 1 and player 2. Show the Normal Form of this induced game (include payoffs for all three players).

(4) What are the pure strategy Nash Equilibria in the induced game?

**(15pts) Problem 4 (Extension of Cournot Duopoly game to multiple firms)**

In Cournot Duopoly game, assume there are  $n$  firms.

- Each firm can choose the production quantity  $q_i \geq 0$  for  $i = 1, \dots, n$
- The market price decrease as the firms' total production  $Q = \sum_{i=1}^n q_i$  increases as:

$$P(Q) = \begin{cases} a - Q & \text{if } Q \leq a \\ 0 & \text{if } Q > a \end{cases}$$

- The cost function of each firm  $i$  is  $C_i(q_i) = cq_i$  with  $c < a$
- The firm  $i$ 's profit, equal to its revenue minus its cost, is

$$u_i(q_1, \dots, q_n) = q_i P(Q) - C_i(q_i)$$

(1) Find the best response function of each firm and set up the conditions for  $(q_1^*, \dots, q_n^*)$  to be a Nash equilibrium

(2) How the price  $P(Q)$  changes as the number of firms  $n$  increases

**(20pts) Problem 5 (Mixed Nash strategy for animals)**

Two animals are fighting over some prey. Each can be passive or aggressive. Each prefers to be aggressive if its opponent is passive, and passive if its opponent is aggressive; given its own stance, it prefers the outcome when its opponent is passive to that in which its opponent is aggressive. The following are the conditions you can use to formulate the strategic behaviors of the animal

- take each player's payoff to (Aggressive, Aggressive) to be 0
- take each player's payoff to (Passive, Aggressive) to be 1
- Each player is indifferent between the outcome (Passive, Passive) and the lottery that assigns probability 0.5 to (Aggressive, Aggressive) and probability 0.5 to (Aggressive, Passive)
- Each player is indifferent between the outcome (Passive, Aggressive) and the lottery that assigns probability  $\frac{2}{3}$  to the outcome (Aggressive, Aggressive) and probability  $\frac{1}{3}$  to the outcome (Passive, Passive).

(1) Represent this game as a normal form game

(2) Plot the best response curves for the two animals

(3) Find all Nash equilibria (including pure and mixed strategies)

**(20pts) Problem 6 (Nash and Correlated equilibria for a two player game)**

Consider the following two player game:

	L	R
T	6, 6	2, 8
B	8, 2	0, 0

(1) Find all pure strategy Nash equilibria and the payoff vector for each one

(2) Find all mixed strategy Nash equilibria and the payoff vector for each one

(3) Identify all correlated equilibria by specifying the set of inequalities that they need to satisfy

(4) Among all the correlated equilibria, provide one with the largest sum of payoffs