29.5 ELECTRIC POTENTIAL ENERGY

Consider a system of charges. The charges of the system exert electric forces on each other. If the position of one or more charges is changed, work may be done by these electric forces. We define *change in electric potential energy* of the system as negative of the work done by the electric forces as the configuration of the system changes.

Consider a system of two charges q_1 and q_2. Suppose, the charge q_1 is fixed at a point A and the charge q_2 is taken from a point B to a point C along the line ABC (figure 29.7).

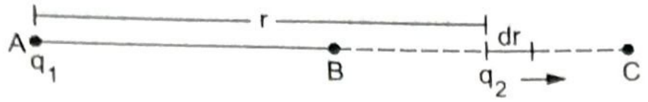

Figure 29.7

Let the distance $AB = r_1$ and the distance $AC = r_2$.

Consider a small displacement of the charge q_2 in which its distance from q_1 changes from r to $r + dr$. The electric force on the charge q_2 is

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \text{ towards } \overrightarrow{AB}.$$

The work done by this force in the small displacement dr is

$$dW = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} dr.$$

The total work done as the charge q_2 moves from B to C is

$$W = \int_{r_1}^{r_2} \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} dr = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

No work is done by the electric force on the charge q_1 as it is kept fixed. The change in potential energy $U(r_2) - U(r_1)$ is, therefore,

$$U(r_2) - U(r_1) = -W = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right). \quad \ldots \text{(29.5)}$$

We choose the potential energy of the two-charge system to be zero when they have infinite separation (that means when they are widely separated). This means $U(\infty) = 0$. The potential energy when the separation is r is

$$U(r) = U(r) - U(\infty)$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{q_1 q_2}{4\pi\varepsilon_0 r}. \quad \ldots \text{(29.6)}$$

The above equation is derived by assuming that one of the charges is fixed and the other is displaced. However, the potential energy depends essentially on the separation between the charges and is independent of the spatial location of the charged particles. Equations (29.5) and (29.6) are, therefore, general.

Equation (29.6) gives the electric potential energy of a pair of charges. If there are three charges q_1, q_2 and q_3, there are three pairs. Similarly for an N-particle system, the potential energy of the system is equal to the sum of the potential energies of all the pairs of the charged particles.

29.6 ELECTRIC POTENTIAL

The electric field in a region of space is described by assigning a vector quantity $\vec{E}$ at each point. The same field can also be described by assigning a scalar quantity V at each point. We now define this scalar quantity known as *electric potential*.

Suppose, a test charge q is moved in an electric field from a point A to a point B while all the other charges in question remain fixed. If the electric potential energy changes by $U_B - U_A$ due to this displacement, we define the *potential difference* between the point A and the point B as

$$V_B - V_A = \frac{U_B - U_A}{q}. \qquad \dots \ (29.7)$$

Conversely, if a charge q is taken through a potential difference $V_B - V_A$, the electric potential energy is increased by $U_B - U_A = q(V_B - V_A)$. This equation defines potential difference between any two points in an electric field. We can define absolute electric potential at any point by choosing a reference point P and saying that the potential at this point is zero. The electric potential at a point A is then given by (equation 29.7)

$$V_A = V_A - V_P = \frac{U_A - U_P}{q}. \qquad \dots \ (29.8)$$

So, *the potential at a point A is equal to the change in electric potential energy per unit test charge when it is moved from the reference point to the point A.*

Suppose, the test charge is moved in an electric field without changing its kinetic energy. The total work done on the charge should be zero from the work–energy theorem. If W_{ext} and W_{el} be the work done by the external agent and by the electric field as the charge moves, we have,

$$W_{ext} + W_{el} = 0$$

or, $$W_{ext} = -W_{el} = \Delta U,$$

where ΔU is the change in electric potential energy. Using this equation and equation (29.8), the potential at a point A may also be defined as follows:

The potential at a point A is equal to the work done per unit test charge by an external agent in moving the test charge from the reference point to the point A (without changing its kinetic energy).

The choice of reference point is purely ours. Generally, a point widely separated from all charges in question is taken as the reference point. Such a point is assumed to be at infinity.

As potential energy is a scalar quantity, potential is also a scalar quantity. Thus, if V_1 is the potential at a given point due to a charge q_1 and V_2 is the potential at the same point due to a charge q_2, the potential due to both the charges is $V_1 + V_2$.

29.7 ELECTRIC POTENTIAL DUE TO A POINT CHARGE

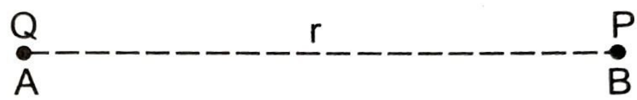

Figure 29.8

Consider a point charge Q placed at a point A (figure 29.8). We have to find the electric potential at a point P where $AP = r$. Let us take the reference point at $r = \infty$. Suppose, a test charge q is moved from $r = \infty$ to the point P. The change in electric potential energy of the system is, from equation (29.6),

$$U_P - U_\infty = \frac{Qq}{4\pi\varepsilon_0 r}.$$

The potential at P is, from equation (29.8),

$$V_P = \frac{U_P - U_\infty}{q} = \frac{Q}{4\pi\varepsilon_0 r}. \qquad \text{... (29.9)}$$

The electric potential due to a system of charges may be obtained by finding potentials due to the individual charges using equation (29.9) and then adding them. Thus,

$$V = \frac{1}{4\pi\varepsilon_0} \sum \frac{Q_i}{r_i}.$$