Parcial 1 Señales y sistemas - Juan Manuel Leyton Gómez

① $x(t) = 20 \sin(7t - \pi/2) - 3\cos(5t) + 2\cos(10t)$

Entrada $= -3.3 \to 5\,(v)$

Bits $= 5$ bits

$-20\cos(7t) - 3\cos(5t) + 2\cos(10t)$

$W_1 = 7$ $\quad T_1 = \frac{2\pi}{7} \to 10\pi$ $\quad f_1 = \frac{7}{2\pi} = 1.114\ Hz$

$W_2 = 5$ $\quad T_2 = \frac{2\pi}{5} \to 14\pi$ $\quad f_2 = \frac{5}{2\pi} = 0.796\ Hz$

$W_3 = 10$ $\quad T_3 = \frac{2\pi}{10} \to 7\pi$ $\quad f_3 = \frac{10}{2\pi} = 1.592\ Hz$

MCM $= 35$

$y = m_x + t$

Amplitud $= 20 + 3 + 2 = 25$

Correspondiente $= \frac{5 - (-3.3)}{25 - (-25)} = \frac{8.3}{50}$

$-3.3 = \frac{8.3}{50}(-25) + b$

Intercepto: $\frac{17}{20} = b$

$0.166\,x(t) + 0.85 = V^c(t)$

$\frac{V^c(t) - (-3.3\,V)}{0.2594}$

$f_{max} = 2F$ más alta

$f_{max} = 2\left(\frac{10}{2\pi}\right) = \frac{10}{\pi}$

$= 3.184\ Hz$

Dado que $f_{muestreo} > f_{max}$

$\therefore f_{muestreo} = 10\ Hz$

Frecuencia de filtro $= f_c = 2.5\ Hz$

$= \frac{1}{4} f_{muestreo}$

Niveles de cuantización $= 2^5 = 32$

$\frac{V}{nivel} = \frac{8.3}{32} = 0.2594$

2. Se procede a realizar el cambio de la variable del tiempo, dado la siguiente ecuación.

$$t = n/fs$$

Señal para discretizarla

- $x[n/fs] = 3\cos\left[1000\pi\left(\frac{n}{fs}\right)\right] + 5\,\text{sen}\left[2000\pi\left(\frac{n}{fs}\right)\right] + 10\cos\left[11000\pi\left(\frac{n}{fs}\right)\right]$
- $x[n/fs] = 3\cos\left[1000\pi\left(\frac{n}{5000}\right)\right] + 5\,\text{sen}\left[2000\pi\left(\frac{n}{5000}\right)\right] + 10\cos\left[11000\pi\left(\frac{n}{5000}\right)\right]$
- $x[n/fs] = 3\cos\left[\frac{\pi}{5}n\right] + 5\,\text{sen}\left[\frac{2\pi}{5}\right] + 10\cos\left[\frac{11\pi}{5}n\right]$

Para este último coseno su frecuencia no se halla en $[-\pi,\pi]$, lo que nos indica que es un alias, hace falta hallar la frecuencia original, entonces s resta 2π para dejarlo en el intervalo (Ω original $= \Omega$ copia $- 2\pi$)

$$\Omega_3 = \frac{11\pi}{5} \notin [-\pi,\pi]$$

$$\Omega_3 - 2\pi = \frac{11\pi}{5} - \frac{10\pi}{5} = \frac{\pi}{5}$$

Lo siguiente es volver a llevar esta frecuencia al coseno que consideremos, el cual quedaría con su frecuencia original de discretización.

$$x[n/fs] = 3\cos\left[\frac{\pi}{5}\right] + 5\,\text{sen}\left[\frac{2\pi}{5}\right] + 10\cos\left[\frac{\pi}{5}n\right]$$

Queda con la misma frecuencia angular que el primero, luego se procede a sumarlos.

Se concluye que la señal obtenida en tiempo discreto por el conversor análogo digital es:

$$x[n/fs] = 13\cos\left[\left(\frac{\pi}{5}\right)n\right] + 5\,\text{sen}\left[\left(\frac{2\pi}{5}\right)n\right]$$

③. Los intervalos de $x_2(t)$, se tiene que dividir la integral en 3 partes.

$$\bar{P}_{x1-x2} = \lim_{T\to\infty} \frac{1}{T} \left[\int_0^{T/4} (A\cos(w_0 t) - 1)^2 dt \right.$$

$$\left. + \int_{T/4}^{3T/4} (A\cos(w_0 t) + 1)^2 dt + \int_{3T/4}^{T} (A\cos(w_0 t) - 1)^2 dt \right]$$

• Utilizamos factorización y los términos de cada uno

$$\bar{P}_{x1-x2} = \lim_{T\to\infty} \frac{1}{T} \left[\int_0^{T/4} (A^2\cos^2(w_0 t) - 2A\cos(w_0 t) + 1)\, dt \right.$$

$$\left. + \int_{T/4}^{3T/4} (A^2\cos^2(w_0 t) + 2A\cos(w_0 t) + 1)\, dt + \int_{3T/4}^{T} (A^2\cos^2(w_0 t) - 2A\cos(w_0 t))\, dt \right]$$

• Resolveremos primero para $0 \leq t < \frac{T}{4}$

$$\int_0^{T/4} (A^2\cos^2(w_0 t) - 2A\cos(w_0 t)) + 1)\, dt \to \cos^2(w_0 t)$$

$$= \frac{1 + \cos(2w_0 t)}{2}$$

$$A^2\cos^2(w_0 t) = \frac{A^2}{2}(1 + \cos(2w_0 t))$$

$$\frac{A^2}{2} \int_0^{T/4} 1\, dt + \frac{A^2}{A} \int_0^{T/4} \cos(2w_0 t)\, dt - 2A$$

$$\int_0^{T/4} \cos(w_0 t)\, dt + \int_0^{T/4} 1\, dt$$

• Separamos la integral

$$\int_0^{T/4} 1\, dt = \frac{T}{4} \; ; \quad \cdot \int_0^{T/4} \cos(2w_0 t)\, dt = \left[\frac{\operatorname{sen}(2w_0 t)}{2w_0} \right]_0^{T/4}$$

$$= \frac{1}{2w_0} \left(\operatorname{sen}\left(\frac{T}{2} \underbrace{w_0}_{\frac{2\pi}{T}}\right) - \operatorname{sen}(0) \right)$$

$$= \frac{1}{2w_0}\left(\left(\operatorname{sen}\left(\frac{\cancel{T}}{2}\frac{2\pi}{\cancel{T}}\right)^{0} - \operatorname{sen}(0)\right)\right) \boxed{=0}$$

$$* \int_0^{T/4} \cos(w_0 t)\,dt = \left[\frac{\operatorname{sen}(w_0 t)}{w_0}\right]_0^{T/4} = \frac{1}{w_0}\left(\operatorname{sen}\left(w_0 \frac{T}{4}\right) - \operatorname{sen}(0)\right)$$

$$= \frac{1}{w_0}\left(\operatorname{sen}\left(\frac{2\pi}{\cancel{T}} \cdot \frac{\cancel{T}}{4}\right)\right)$$

$$= \frac{1}{w_0}\left(\cancel{\operatorname{sen}}^{1}\left(\frac{\pi}{2}\right)\right) = \frac{1}{w_0} \rightarrow \boxed{\frac{T}{2\pi}}$$

$$* \int_0^{T/4} 1\,dt = \frac{T}{4}$$

- Sustituimos todos los términos

$$\frac{A^2}{2}\cdot\frac{T}{4} + \frac{A^2}{2}\cdot 0 - 2A\cdot\frac{T}{2\pi} + \frac{T}{4} \boxed{= \frac{A^2 T}{8} - \frac{AT}{\pi} + \frac{T}{4}}$$

Luego para $T/4 \leq t \leq 3T/4$

$$\int_{T/4}^{3T/4} \left(A^2\cos^2(w_0 t) + 2A\cos(w_0 t) + 1\right) dt$$

$$A^2\cos^2(w_0 t) = \frac{A^2}{2}\left(1 + \cos(2w_0 t)\right)$$

- Separamos la integral

$$\frac{A^2}{2}\int_{T/4}^{3T/4} 1\,dt + \frac{A^2}{2}\int_{T/4}^{3T/4}\cos(2w_0 t)\,dt + 2A\int_{T/4}^{3T/4}\cos(w_0 t)\,dt$$

$$+ \int_{T/4}^{3T/4} 1\,dt$$

- Evaluamos cada integral

$$\int_{T/4}^{3T/4} 1\,dt = \frac{3T}{4} - \frac{T}{4} = \boxed{\frac{T}{2}} \; ; \quad \cdot \int_{T/4}^{3T/4} \cos(w_0 t)\,dt = \left.\frac{\operatorname{sen}(2w_0 t)}{2w_0}\right]_{T/4}^{3T/4}$$

$w_0 = \frac{2\pi}{T}$, $\frac{1}{2w_0}\left[sen\left(\frac{4\pi}{T}\cdot\frac{3T}{\pi}\right) - sen\left(\frac{4\pi}{T}\cdot\frac{T}{4}\right)\right] = 0$

- $\int_{T/4}^{3T/4} \cos(w_0 t)\,dt = \left[\frac{sen\, w_0 t}{w_0}\right]_{T/4}^{3T/4} = \frac{1}{\frac{2\pi}{T}}$

$\left[sen\left(\frac{2\pi}{T}\cdot\frac{3T}{4}\right) - sen\left(\frac{2\pi}{T}\cdot\frac{T}{4}\right)\right]$

$= \frac{1}{\frac{2\pi}{T}}\left[-1-1\right] = \frac{-2T}{2\pi} = \frac{-T}{\pi}$

- Sustituimos todos los términos

$\frac{A^2}{2}\cdot\frac{T}{2} + 0 + 2A\cdot\left(-\frac{T}{\pi}\right) + \frac{T}{2} = \frac{A^2 T}{4} - \frac{2AT}{\pi} + \frac{T}{2}$

- Para $3T/4 \le t \le T$

$\int_{3T/4}^{T} (A^2\cos^2(w_0 t) + 1)\,dt \rightarrow A^2\cos^2(w_0 t) = \frac{A^2}{2}(1+\cos(2w_0 t))$

- Separamos la integral

$\frac{A^2}{2}\int_{3T/4}^{T} 1\,dt + \frac{A^2}{2}\int_{3T/4}^{T} \cos(2w_0 t)\,dt - 2A\int_{3T/4}^{T} 1\,dt$

- Se evalúa cada integral

$\int_{3T/4}^{T} 1\,dt = T - \frac{3T}{4} = \frac{T}{4}$; $\int_{3T/4}^{T} \cos(2w_0 t)\,dt = \left[\frac{sen(2w_0 t)}{2w_0}\right]_{3T/4}^{T}$

$= \frac{sen(2w_0 T) - sen\left(\frac{3T}{2} w_0\right)}{2w_0}$, $w_0 = \frac{2\pi}{T} \rightarrow 2w_0 T = 4\pi$

$2w_0\cdot\frac{3T}{4} = 3\pi$

$sen(4\pi) - sen(3\pi) = 0$

- $\int_{3T/4}^{T} \cos(w_0 t)\,dt = \left[\frac{\operatorname{sen}(w_0 t)}{w_0}\right]_{3T/4}^{T} = \frac{\operatorname{sen}(w_0 T) - \operatorname{sen}(w_0 \cdot \frac{3T}{4})}{w_0}$

$w_0 = \frac{2\pi}{T} \to wT = 2\pi$

$w \cdot \frac{3T}{4} = \frac{3\pi}{2}$; $\frac{1}{\frac{2\pi}{T}}\left[\operatorname{sen}(2\pi) - \operatorname{sen}\left(\frac{3\pi}{2}\right)\right]$

$= \frac{T}{2\pi} + (-1) = \frac{T}{2\pi}$

$\int_{3T/4}^{T} 1\,dt = T - \frac{3T}{4} = \frac{T}{4}$

- Se sustituyen todos los términos

$\frac{A^2}{2} \cdot \frac{T}{4} + 0 - 2A \cdot \frac{T}{2\pi} + \frac{T}{4} = \frac{A^2 T}{8} - \frac{AT}{\pi} + \frac{T}{4}$

- Se suman los 3 resultado y luego se calcula la potencia media para llegar al siguiente límite:

$\bar{P}_{x1-x2} = \lim_{T\to\infty} \frac{1}{T}\left[\left(\frac{A^2T}{8} - \frac{AT}{\pi} + \frac{1}{4}\right) + \left(\frac{A^2T}{4} - \frac{2AT}{\pi} + \frac{T}{2}\right) + \left(\frac{A^2T}{4} + \frac{AT}{\pi} + \frac{T}{4}\right)\right]$

$\lim_{T\to\infty} \frac{1}{\cancel{T}}\left[\frac{A^2\cancel{T}}{2} - \frac{4\cancel{T}}{\pi} + \cancel{T}\right] = \boxed{\frac{A^2 T}{2} - \frac{4T}{\pi} + T}$

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$C_n = \frac{1}{T}\int_{t_i}^{t_f} x(t) e^{-jnw_0 t}\,dt \to x(t) = \sum_n C_n e^{jnw_0 t}$

$x'(t) = \frac{d}{dt}x(t) = \frac{d}{dt}\left\{\sum_n C_n e^{jnw_0 t}\right\} = \sum_n C_n e^{jnw_0 t} jnw_0$

$x''(t) = \frac{d}{dt}\left\{\sum_n C_n e^{jnw_0 t}(jnw_0)\right\} = \sum_n C_n e^{jnw_0 t}(jnw_0)^2$

$\tilde{C}_n = \frac{(x''(1), e^{jnw_0 t})}{\|e^{jnw_0 t}\|^2} = \int_{t_i}^{t_f} \frac{x''(1) e^{-jnw_0 t}\,dt}{T}$; $T = t_f - t_i$

$$\tilde{C_n} = C_n(jnw_o)^2 = \int_{ti}^{tf} \frac{x''(t)e^{-jnw_ot}dt}{T}$$

$$C_n = \frac{1}{(tf-ti)(jnw_o)^2} \int_{ti}^{tf} x''(t)e^{-jnw_ot}dt = \frac{1}{(ti-tf)n^2w_o^2} \int_{ti}^{tf} x''(t)e^{-jnw_ot}dt$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos(nw_ot) + b_n\operatorname{sen}(nw_ot)$$

$$x'(t) = \sum_{n=1}^{N} a_n(-nw_o)\operatorname{sen}(nw_ot) + b_n(nw_o)\cos(nw_ot)$$

$$x''(t) = \sum_{n=1}^{N} a_n(-nw_o)(nw_o)\cos(nw_ot) + b_n(nw_o)(-nw_o)\operatorname{sen}(nw_ot)$$

$$\tilde{a_n} = \frac{2}{T}\int_{ti}^{TF} x''(t)\cos(nw_ot)dt \; ; \; \tilde{b_n} \frac{2}{T}\int_{Ti}^{tf} x''(t)\operatorname{sen}(nw_ot)dt$$

$$a_n(-n^2w_o^2) = \frac{2}{T}\int_{ti}^{tf} x''(t)\cos(nw_ot)dt$$

$$a_n = \frac{2}{-Tn^2w_o^2}\int_{ti}^{tf} x''(t)\cos(nw_ot)dt$$

$$b_n(-n^2w_o^2) = \frac{2}{T}\int_{ti}^{tf} x''(t)\operatorname{sen}(nw_ot)dt$$

$$C_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)dt = \frac{1}{t}\int_{-d_2}^{-d_1} \frac{A}{d_2-d_1}(t+d_2)dt + \frac{1}{T}\int_{-d_1}^{d_1} A\,dt + \frac{1}{T}\int_{d_1}^{d_2} -\frac{A}{d_2-d_1}(t-d_2)dt$$

$$= \frac{1}{T}\left[\frac{A}{d_2-d_1}\left(\frac{t^2}{2}+d_2t\right)\Big|_{-d_2}^{-d_1} + At\Big|_{-d_1}^{d_1} - \frac{A}{d_2-d_1}\left(\frac{t^2}{2}-d_2t\right)\Big|_{d_1}^{d_2}\right] = \frac{1}{T}\left[\frac{A}{d_2-d_1}\left(\frac{d_1^2}{2}-d_1d_2-\frac{d_2^2}{2}+d_2d_2\right)\right.$$

$$\left.+A(d_1+d_1) - \frac{A}{d_2-d_1}\left(\frac{d_2^2}{2}-d_2d_2-\frac{d_1^2}{2}+d_1d_2\right)\right]$$

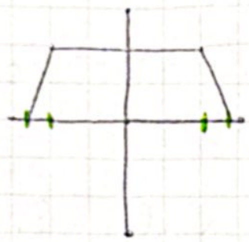
$$= \frac{1}{T}\left[\frac{2A}{d_2-d_1}\left(\frac{d_1^2}{2} - d_1 d_2 - \frac{d_2^2}{2} + d_2^2\right) + 2Ad_1\right]$$

si $A=1$ $d_1=1$ $d_2=2$ $T=2d_2=4$

$$C_n = -\frac{1}{\frac{2n^2\pi^2}{2}}\left(\cos\left(n\frac{2\pi}{4}\cdot 2\right) - \cos\left(n\frac{2\pi}{4}\cdot 1\right)\right)$$

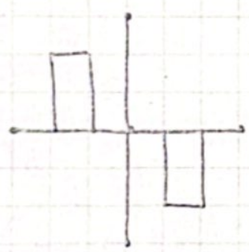
$$b_n = \frac{2}{-Tn^2 w_0^2}\int_{t_i}^{t_f} x''(t)\,\mathrm{sen}(n w_0 t)\,dt$$

*Encontramos el espectro de fourier



$$x''(t) = A\delta(t+d_2) - A\delta(t+d_1) - A\delta(t-d_1) + A\delta(t-d_2)$$

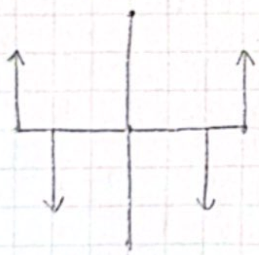
⇓



$$C_n = \frac{1}{-Tn^2 w_0^2}\int_{-T/2}^{T/2} x''(t)e^{-jnw_0 t}\,dt$$

$$C_n = -\frac{1}{Tn^2 w_0^2}\int_{-T/2}^{T/2} A[\delta(t+d_2) - \delta(t+d_1) - \delta(t-d_1) + \delta(t-d_1)]e^{-jnw_0 t}\,dt$$

⇓



$$C_n = \frac{-A}{Tn^2 w_0^2}\left(e^{-jnw_0(-d_2)} - e^{-jnw_0(-d_1)} - e^{-jnw_0 d_1} + e^{-jnw_0 d_2}\right)$$

$$C_n = \frac{-A}{Tn^2 w_0^2}\left(e^{jnw_0 d_2} + e^{-jnw_0 d_2} - (e^{jnw_0 d_1} + e^{-jnw_0 d_1})\right)$$

$$C_n = -\frac{A}{Tn^2 w_0^2}\left(2\cos(nw_0 d_2) - 2\cos(nw_0 d_1)\right)$$

$$= \frac{-2A}{Tn^2\frac{4\pi^2}{T^2}}$$

$$\left(\cos\left(n\frac{2\pi}{T}d_1\right) - \cos\left(n\frac{2\pi}{T}d_1\right)\right)$$

$$C_0 = \frac{1}{4}\left[\frac{2.1}{1}\left(\frac{1}{2}-2-2+4\right)+2.1.1\right] = \frac{1}{4}\left[2\left(\frac{1}{2}\right)+2\right]$$

$$= \frac{3}{4}$$

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} (x(t))^2 dt = \frac{2}{T}\int_{-T/2}^{0} (x(t))^2 dt = \frac{2}{t}\int_{-d_2}^{-d_1}\left(\frac{A}{d_2-d_1}\right)^2$$

$$(t+d_2)^2\,dt + \frac{2}{t}\int_{-d_1}^{0} A^2 dt \to P_x = \frac{2}{t}\left(\frac{A}{d_2 d_1}\right)^2$$

$$(t^2+2td_2+d_2^2)\Big|_{-d_2}^{-d_1} + \frac{2}{T}A^2 t\Big|_{-d_1}^{0}$$

$$P_x = \frac{2}{T}\left(\frac{A}{d_2-d_1}\right)^2 (d_1^2-2d_2d_1+d_2^2-d_2^2+2d_2^2-d_2^2)$$

$$+\frac{2}{T}A^2(0+d_1)$$

$$P_x = \frac{1}{6}+\frac{1}{2} = \frac{2}{3}$$