Refraction & Snell's Law

Experiment One

Physics 192
Michigan State University

Before Lab

- **Due at the beginning of class (10 pt)**: answer the pre-lab theory questions
- Carefully read the entire lab guide

Experiment Overview

This is a two-week lab on the geometric perspective of optics. You will observe the behavior of light as it passes between media. Using a laser, you will characterize the optical properties of an acrylic lens.

- Determine the index of refraction and critical angle of a lens for a single wavelength of light
- Construct and test an optical micrometer
- **Produce a professionally formatted lab report**: communicate the details of your experiment to a general scientific reader, introduce each section to provide context for your work, include captions describing and interpreting plots, and show all relevant calculations

Light

Over 150 years ago, James Clerk Maxwell published his mathematical framework of electromagnetism, unifying phenomena which had previously been described independently: electricity, magnetism, and light. Maxwell's equations show that light is an oscillation in an underlying electromagnetic field, behaving as a wave. Radio waves, microwaves, infrared, visible light, ultraviolet, X-rays, and gamma-rays are all different manifestations of **electromagnetic radiation**. Figure 1 depicts the full electromagnetic spectrum, with a narrow band of color between 400–700 nm in wavelength. This region corresponds to **light**, the range of electromagnetic radiation visible to human eyes[1].

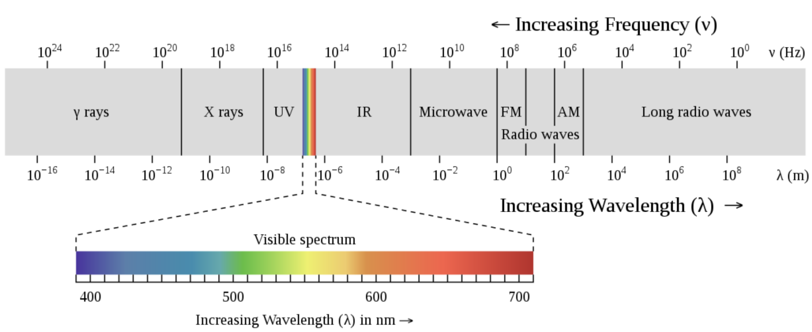


Figure 1: The electromagnetic spectrum, ranging from gamma-rays with wavelengths smaller than a nucleus to radio waves which can be longer than a continent.

[1]In physics, it is common to refer to *all* electromagnetic radiation as light.

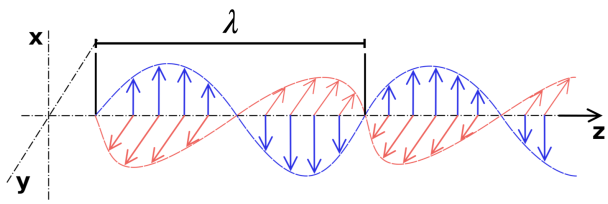


Figure 2

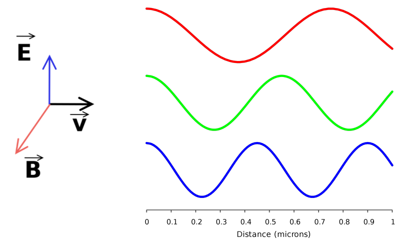


Figure 3

The **wavelength** is the *spatial* period of a wave, denoted by λ (lowercase Greek lambda) in Figure 2. The human visual system interprets different wavelengths as **colors**, with red corresponding to long and blue to short wavelengths (Figure 3). Waves also oscillate in time with a *temporal* period related to the **frequency** by $T = 1/f$. The speed of the wave is dependent upon these two quantities:

$$v = \lambda f. \tag{1}$$

The speed of light in vacuum is a universal constant, with a defined value of about $3 \cdot 10^8$ m/s. When light passes from vacuum into some medium, such as a lens or water, it experiences a reduction in speed. This is a consequence of interactions within the medium.

Figure 4 depicts a wave moving from left to right as it passes from vacuum into some material. **Wavefronts** are the bright lines which can be thought of as sequential peaks[2]. As the wave propagates to the right, the wavefront must remain continuous at the interface (pink dots). Consequently, the wave *must* change its direction to accommodate the change in speed.

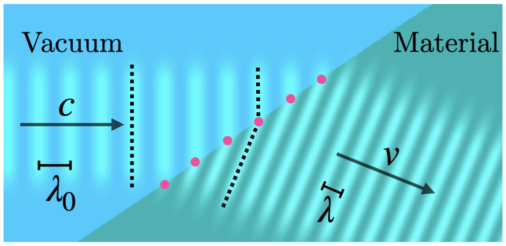


Figure 4: A plane wave in vacuum moving at speed c experiences a reduction in speed and wavelength as it passes into a material.

The change in direction as a wave passes between media is called **refraction**. This mechanism is responsible for rainbows and enables the properties of lenses, fiber optics, and many other technologies. The refractive behavior of a material is characterized by its **index of refraction (IOR)**, defined as

$$n = \frac{c}{v} \tag{2}$$

where c is the speed of light in vacuum and v is the speed within the material. Light passing into a material with a larger value of n experiences a greater change in speed, resulting in a larger change in direction. The reduction in speed comes at the expense of the wavelength, as shown in Figure 4. The frequency of a light wave of a single color is determined by the source; it remains fixed regardless of transitions between media. Only the wavelength is subject to change.

Geometric Optics

The wave model of light elucidates the mechanism of refraction. **Geometric optics** is a problem solving method which treats light as rays that move in straight lines. A light ray can only change direction upon reaching a material boundary[3] indicated by a dark blue line in Figure 5. The *incident* ray reaches the boundary and splits into two components:

- The *reflected* ray does not pass into the material
- The *refracted* ray is transmitted into the material

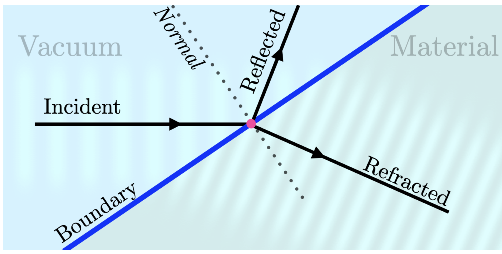


Figure 5: Light rays from the previous example, including a ray for light *reflected* at the boundary and a dotted reference line labeled *normal*.

[2] The wavefront of a wave is the set of all points with the same phase.

[3] A change in direction is also possible when the index of refraction varies within a medium. This process is responsible for mirages over hot roads and the twinkling of stars.

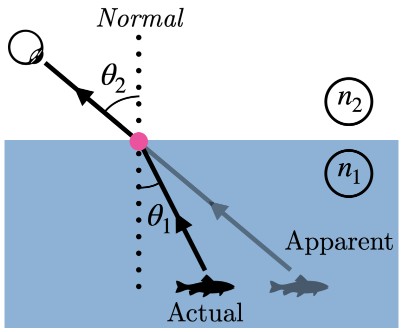


Figure 6

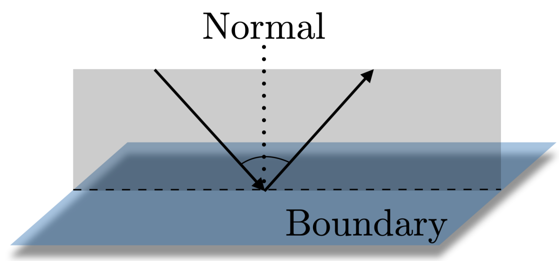


Figure 7

Imagine looking at a fish in a pond. The light from the fish that reaches your eye refracts at the boundary between water and air, making the fish appear to be in a different location as shown in Figure 6. Geometric optics can be used to determine the actual angular location of the fish. The only coordinate necessary to describe a ray is its angle relative to the interface. The incident, reflected, and refracted rays all lie in a single plane as shown in Figure 7. The **surface normal** is a line extending *perpendicular* to the surface, from which all angles are measured.

Figure 8 shows a general example in which an incident ray moves through medium 1 with IOR n_1. At the boundary, part of the ray is reflected and part is transmitted, refracting into medium 2 with IOR n_2. The **law of reflection** states that the incident ray reflects symmetrically about the surface normal:

$$\theta_1 = \theta_1'. \tag{3}$$

The refracted ray is described by the **law of refraction**,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{4}$$

where the subscripts refer to the first (before boundary) and second (after boundary) medium. Also known as **Snell's law**, the equation relates the indices of refraction to the incident and refracted angles.

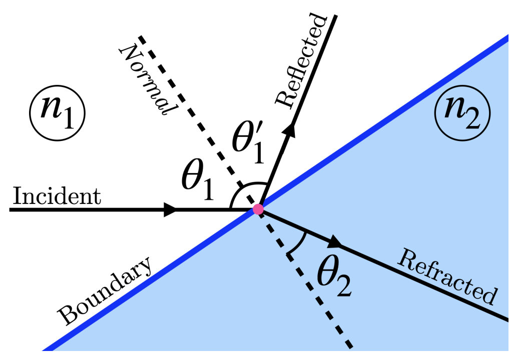


Figure 8

Key Concepts

- Refraction is the change in direction of a wave as it passes a material boundary
- The index of refraction, defined as $n = c/v$, characterizes the amount by which a ray is bent when passing into a material. Optically dense materials have a larger value of n, resulting in more bending and lower speed
- All angles are measured relative to a line perpendicular to material boundary called the surface normal
- If the first medium has a lower IOR than the second, the ray is bent towards the normal. If the first medium has a higher IOR, the ray is bent away
- Refraction is described by Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, where
  - n_1 is the IOR of the first medium
  - θ_1 is the angle of the incoming ray
  - n_2 is the IOR of the second medium
  - θ_2 is the angle of the outgoing ray

Total Internal Reflection

In most situations, a ray of light that reaches a material boundary will both reflect and refract. Under certain circumstances, the ray of light can be *entirely* reflected. Figure 9 is a picture looking at the water-air boundary from within a fish tank. The boundary acts as a mirror, with no light transmitted from the air side. This is an example of **total internal reflection (TIR)**, the phenomenon by which waves are entirely reflected at a boundary.

Figure 9

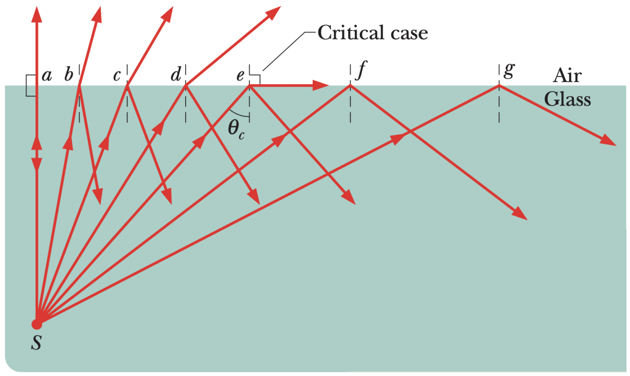


Figure 10

TIR is only possible for specific incident angles under the condition that the second medium has a smaller IOR: $n_2 < n_1$. In the fish tank, the second IOR is that of air. You would never observe this effect looking from air into water. The second condition is illustrated in Figure 10. Rays from the point labeled S are shown reaching the surface at different incident angles. Moving from left to right, the refracted angle tips towards the interface. At the angle labeled e, the refracted ray is parallel to the surface: $\theta_2 = 90°$. The **critical angle** is that incident angle resulting in a refracted angle of exactly 90°, labeled as θ_c.

The critical angle can be obtained mathematically from Snell's law if the indices of refraction are known. When $\theta_2 = 90°$,

$$n_1 \sin\theta_c = n_2 \sin 90° = n_2 \quad \Rightarrow \quad \sin\theta_c = \frac{n_2}{n_1}.$$

The critical angle can be obtained explicitly using the inverse sine function,

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right). \tag{5}$$

For all angles larger than θ_c, such as those at f and g in Figure 10, the entire ray is reflected. To observers at those locations, the surface would appear as a mirror, having no light pass from the opposite side.

Optical Micrometer

The properties of refraction can be exploited to displace a beam of light by a small amount. Figure 11 shows the configuration in which a light ray passes through an ideal rectangular lens (the faces are parallel and perfectly flat). Along its path, the beam encounters *two* boundaries, indicated by dark blue lines.

Boundary 1: the beam reaches the first interface at incident angle θ_1. It refracts from air *into* the lens at an angle θ_2

Boundary 2: the beam reaches the second boundary at an angle θ_2. It refracts from the lens *into* air at an angle θ_1

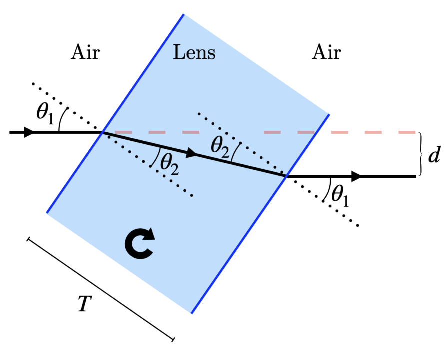


Figure 11: A beam of light passing into a rectangular lens of thickness T. The dashed red line represents the undeviated beam.

The example has a very nice symmetry, resulting in an outgoing beam that is exactly parallel to the incident beam, but displaced by a small distance, d. Rotating the lens has the effect of increasing or decreasing the beam displacement by very small amounts. An **optical micrometer** is a device that uses this lens shape with a protractor and laser to measure small lateral displacements over very large distances. The displacement is not very sensitive to the rotation angle, allowing for very fine adjustment. The equation for displacement can be derived from geometry shown in Figure 11, resulting in

$$d = \frac{T}{\cos\theta_2}\sin(\theta_1 - \theta_2) \tag{6}$$

where T is the thickness of the lens, θ_1 is the incident angle (typically set by the experimenter), and θ_2 is the refracted angle *inside* the lens.

Unfortunately, real lenses cannot be made with such precision that any two faces are exactly parallel. A real instrument will have a slight angular deviation between the incident and outgoing beams resulting from misaligned faces, as shown in Figure 12. The dashed red lines indicate the ideal displacement from the incident ray. Misalignment causes the outgoing ray to deviate by an angle θ_{dev}.

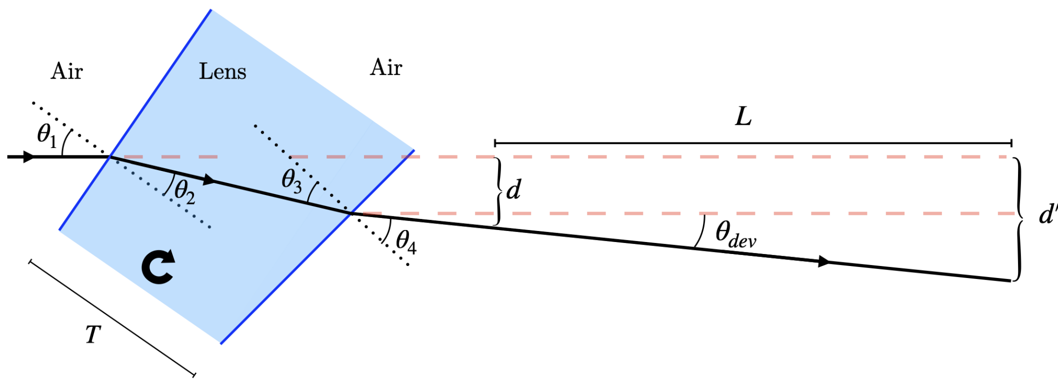


Figure 12: The lateral deviation grows from d to d' as the distance L from the lens increases.

The deviation angle is a measure of the error associated with the device for measurements at large distances. The geometry shown in Figure 12 can be used to approximate the deviation angle as

$$\theta_{dev} = \frac{d' - d}{L}. \tag{7}$$

Lasers and Safety

Laser is an acronym for "Light Amplification by Stimulated Emission of Radiation." Exploiting the properties of discrete energies of atomic orbitals, lasers produce a narrow beam of **monochromatic** light, or light with a single wavelength (color). There are two lasers available for this experiment.

- A red laser with a wavelength of 636 nm
- A green laser with a wavelengths of 532 nm

The lasers used in this lab have a lower power output, but the beam is still very intense. Always exercise caution when in use.

- **NEVER look directly into the beam or its reflection!**
- **Always pay attention to reflections. Do not let reflections catch your classmates.**
- **Avoid positioning your head in the plane of the laser.**

1 Theory Questions

The University of Colorado Boulder has a very nice simulation demonstrating the principles of reflection and refraction at https://phet.colorado.edu/sims/html/bending-light/latest/bending-light_en.html. You are highly encouraged to play with the simulation to visualize refraction. Open the "More Tools" simulation; change between waves and rays; change the IORs, incident angle, and color. Observe the effects.

1. (2 pt) In your own words, describe refraction in 3-4 sentences.

2. (1 pt) What are the units of index of refraction? Is it possible for a material to have $n < 1$?

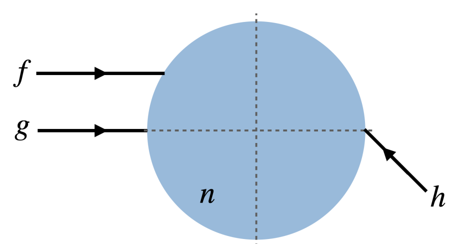
3. (1 pt) The picture in Figure 9 shows how the interface between water and air can act as a mirror, with no light passing from above the surface. If you could stand at the bottom of the fish tank looking straight up, would it be possible to see what is above the water?

4. (1 pt) Many materials have an index of refraction that depends on the wavelength as

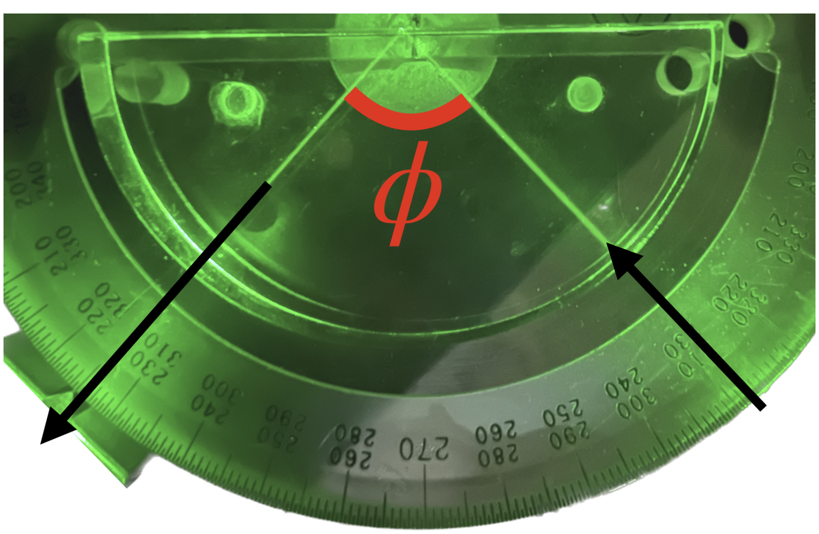
$$n(\lambda) = A + \frac{B}{\lambda^2},$$

   where A and B are empirically determined constants. Given that $\lambda_{red} > \lambda_{blue}$, should the IOR be larger for red or green light?

5. (1 pt) A wave traveling perpendicular to a boundary will not change its direction. The figure below shows three different rays labeled f, g, and h incident on a glass disk. The disk has IOR n and is centered on the dashed lines. Which rays, if any, will change direction when passing into the disk?



6. (1 pt) The picture below shows a green laser passing from air into a D-shaped lens mounted to a protractor. The beam internally reflects at the flat surface. Estimate the angle ϕ, indicated by the red arc. Give an uncertainty $\delta\phi$ based on your confidence in your measurement.



7. This question is about linearizing data. In this experiment, you will direct a laser beam into a lens. You will control the incident angle θ_1 and measure the refracted angle, θ_2. For this question, the laser refracts from air into the lens as shown in Figure 13. The IOR of air is $n_{air} \approx 1$. The IOR of the lens is labeled as n.

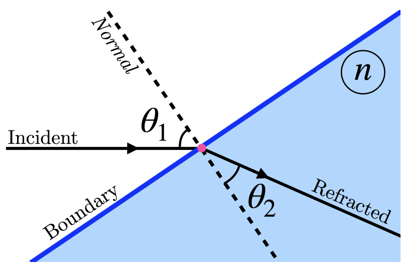


Figure 13

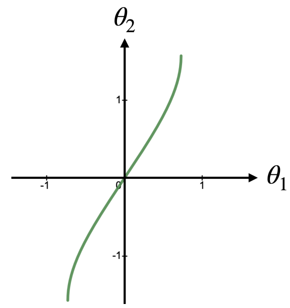


Figure 14

(a) (1 pt) Write the equation for Snell's law when $n_1 = n_{air} = 1$ and $n_2 = n$.

(b) (1 pt) Figure 14 is a graph of θ_2 vs. θ_1. What function represents the green curve?

(c) (1 pt) Extracting the value of n from a plot of θ_2 vs. θ_1 would be unnecessarily complicated. Instead, you can calculate the sines of your angle measurements, organized as shown below. Substitute X and Y into your equation from part (a) and sketch a plot of Y vs. X. What is the slope of the line?

θ_1	θ_2	$Y = \sin\theta_1$	$X = \sin\theta_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

2 Experiment Setup

Placed on your bench is a large optical rail which holds various components in place. The rail is designed to help with the alignment of each piece in your system. Every component mounts to the rail on a post that can be rotated, raised, and lowered. You must take care to align the components whenever you make changes to your system. You have two lenses to be used for different parts of your experiment:

1. D-shaped lens for IOR characterization
2. Rectangular lens for the optical micrometer

Fixed at opposite ends of the optical rail are a laser and screen (graph paper). Your lens will be positioned on a rotating stage fitted with a 360° protractor. Rotating the stage changes the incident angle of the laser beam. The protractor will be used to measure incident and refracted angles. For each part of the experiment, you need to align the beam with the face of the lens:

- Place the lens in the desired orientation on the rotating stage.
- Point the laser at the lens and pay attention to the reflected part of the beam.
- Align the beam such that the reflected part returns to where the beam leaves the laser. All optical components have lock screws to keep pieces held in place. Loosen the screw to make an adjustment, then tighten to lock the position. None of your components should wobble after alignment.
- The beam can be tilted using adjustment screws on the back of the laser. Do not overtighten the screws. Alignment is usually an iterative process.
- Once alignment is achieved, lock all components. The rotating stage should be the only moving part when experimenting.

3 D-shaped Lens

For this part of the experiment, you will measure the index of refraction of the D-shaped lens. You will do so using two different configurations as shown below. For all measurements, the IOR of air can be approximated as $n_{air} = 1$.

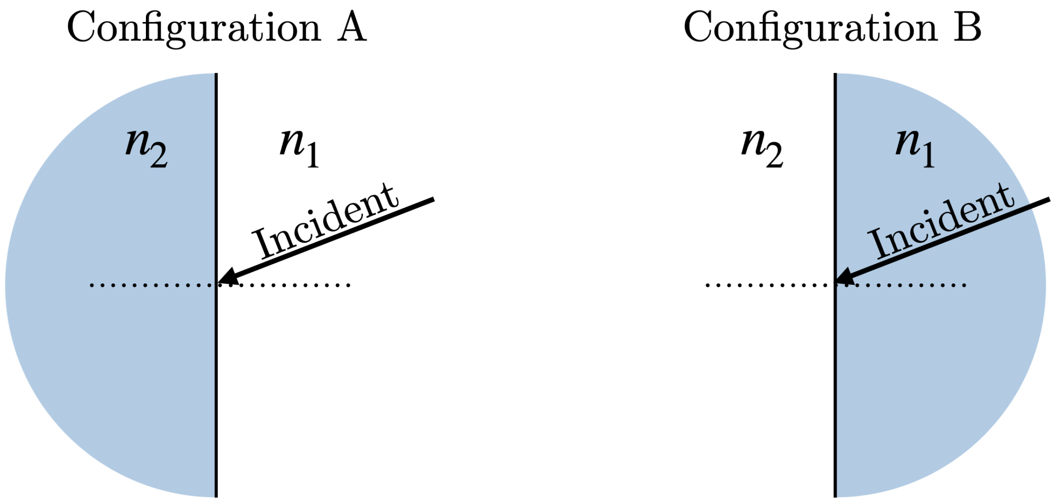


Figure 15

1. (2 pt) Write a short introduction to this part of the experiment. Include the color and wavelength of your laser. Include the relevant equations, stating the dependent and independent variables. You should return to this question *after* doing your experiment.

3.1 Configuration A

Begin by mounting the lens flat on the optical stage. The lens has extrusions allowing the it to be fixed in place. Follow the alignment procedure outlined in Section 2. Figure 16 shows how refracted angles can be measured using an index card. The dark blue line represents the surface normal, and the red line is the refracted beam. The cyan line is related to the incident angle, set by the user.

1. (2 pt) Carefully align your system. The beam must hit the center of the lens and reflect directly back to where it is emitted from the laser. Have you TA check the alignment for the first configuration. What happens if the beam does not hit the center of the lens? *Hint*: refer to theory Question 5.

2. (1 pt) Sketch a ray diagram with the lens in configuration A. Label the surface normal, the incident angle θ_1, and the refracted angle θ_2.

3. (1 pt) In this configuration, the beam refracts from air *into* the lens. Do you expect the incident angle to be larger or smaller than the refracted angle?

4. Construct a table to store your measurements. You should measure for the following incident angles: $\theta_1 = 10°, 20°, 30°, 40°, 50°$. To minimize errors, you should measure for rotations to the left and right. Use the table below as a starting point, and add columns as needed for calculations. In your lab report, the units must be indicated.

θ_0	θ_1	θ_{2L}	θ_{2R}	$\theta_{2,avg}$	$\delta\theta_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

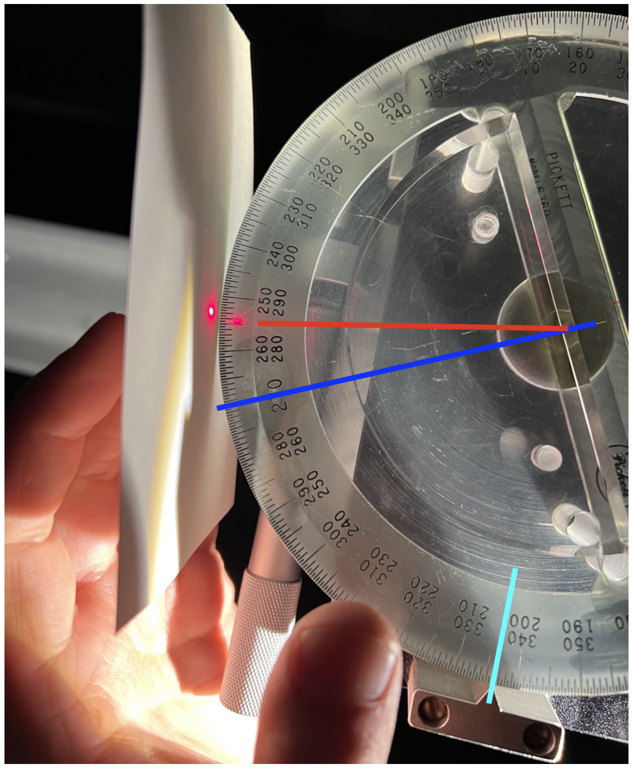


Figure 16

5. Record the angle θ_0. This corresponds to the angle indicated on the protractor when the beam is directly aligned with the surface normal (i.e. the incident angle is zero). All incident angles should be measured relative to this number.

6. (2 pt) Measure the refracted angles for each incident angle. Use an index card as shown in Figure 16. Do one direction first, then switch to the other. Determine your uncertainty in the refracted angles. Note: the standard error is the uncertainty in the average angle measured. If this number is zero, the uncertainty is the *minimum* uncertainty associated with the protractor.

7. (2 pt) As shown in theory Question 7, directly plotting the angles is not a good way to extract the IOR from your data. Instead, perform a substitution for the x and y variables that results in a linear relationship. *Hint*: define $X(\theta_2)$ and $Y(\theta_1)$ such that Snell's law can be written as $Y = nX$ where n is the IOR of the lens. Show your work.

8. (2 pt) Recall that when a quantity $x \pm \delta x$ is measured and used to calculate a function $f(x)$, the uncertainty in the calculation is $\delta f = \left|\frac{df}{dx}\right| \cdot \delta x$. Calculate the uncertainties δX and δY. **Important**: your answer should depend on your protractor uncertainties, $\delta\theta$. **These numbers must be in radians**.

9. (1 pt) Add four more columns to your spreadsheet for calculations and compute the following values (you will use these numbers for plotting)

X	Y	δX	δY
$\vdots$	$\vdots$	$\vdots$	$\vdots$

10. (2 pt) Use curve.fit to plot Y vs. X. Choose the appropriate fit type, add error bars for δX and δY using the numbers from the previous question, and determine $n \pm \delta n$ for the D-shaped lens.

3.2 Configuration B

In order to better estimate the IOR of the D-shaped lens, a second set of measurements can be made using the same equipment in a different orientation. Rotate the lens on the optical stage by 180° such that the beam from the laser passes into the lens as shown in Configuration B of Figure 15.

1. Carefully align your system in configuration B. The beam must hit the center of the lens and reflect directly back to where it is emitted at $\theta_1 = 0°$.

2. (2 pt) Sketch a ray diagram with the lens in configuration B. Label the surface normal, the incident angle θ_1, and the refracted angle θ_2. *Hint*: light does not refract when it passes a boundary directly along the normal.

3. (1 pt) In this configuration, the beam refracts from the lens *into* air ($n_1 > n_2$). Do you expect the incident angle to be larger or smaller than the refracted angle?

4. (2 pt) Determine the critical angle for the lens. Do so by rotating to the left and right, and averaging to get your best estimate. State as $\theta_c \pm \delta\theta_c$ where $\delta\theta_c$ is your estimated uncertainty. **Explain your procedure**. Calculate the IOR using Equation 5.

5. (9 pt) **Repeat questions 4–10** from the previous section for the lens in configuration B, but with incident angles $\theta_1 = 5°, 10°, 15°, 20°, 25°$. *Note*: when plotting your results, the substitutions should be functions $X(\theta_1)$ and $Y(\theta_2)$.

6. (1 pt) Use all previous measurements of the IOR of the lens to calculate your best estimate $\bar{n} \pm \delta\bar{n}$.

4 Optical Micrometer Experiment

In the previous section, you explored how characteristics of a material can be measured using a laser. In this experiment, you will apply the principles of geometric optics to build and test optical measurement tool. Using the rectangular lens, you will construct an optical micrometer as shown in the theory section. Your goal is to determine the deviation angle associated with your lens. For this experiment, you will make use of the screen. Make sure you have a new piece of graph paper.

1. (2 pt) Write a short introduction to this part of the experiment. Include the relevant equations, stating the dependent and independent variables. You should return to this question *after* doing your experiment.

2. (1 pt) Sketch a digram of your optical micrometer.

3. (1 pt) Measure the thickness T of your lens as precisely as possible.

4. (1 pt) Carefully align your system, ensuring that the beam reflects back to the laser at $\theta_1 = 0°$.

5. (1 pt) With the screen at the end of the optical rail, measure the distance L. Estimate the uncertainty and record $L \pm \delta L$. With $\theta_1 = 0°$, mark the position of the beam on the screen with a pen. As you rotate the lens, you will find that the beam is displaced from this location.

6. (1 pt) For an incident angle of $\theta_1 = 10°$, record the refraction angle θ_2 *inside* the lens using an index card. Mark the location of the transmitted beam on the screen with a pen. Continue the process for incident angles of $20°, 30°, 40°$, and $50°$. Repeat the measurements for rotations in the opposite direction.

7. (1 pt) Refer to Figure 12. The distance labeled L can be taken to be the distance from the lens to the screen. As L gets small, the true lateral displacement d' approaches the ideal displacement, d. Use your left-right averaged angles θ_2 to calculate the ideal displacements using Equation 6.

8. (2 pt) Calculate θ_{dev} for each incident angle using Equation 7. Determine $\theta_{dev} \pm \delta\theta_{dev}$ statistically.