Contents

0.1 How can I print off and use this document?

Frankly, in just about any way thats useful to you. I am going to try something here, where I will try to make more or less the entirety of the notes associated with the Winter 2019 semester of BIOMEDE 211, Circuits, Systems, and Signals in Biomedical Engineering, to you, dear reader.
Please dont plagiarize this. If you were raised right, you ought to know what that is. If youd like my judgment on any sort of action, my opinions can be laid bare.
The first assignment I am giving you (worth 4% of your grade and which must be completed by the end of the semester) is to figure out where this document is located online, download it, print it off, sign your name to it, and get it to me. If you know who I am, I would expect a competent engineer to find that without much to-do about it. Start with Google, go from there. Further, for those in the class, BIOMEDE 211, Winter 2019, you must join Github and make at least four substantive contributions to this repository. The term all you engineers (and lawyers) cant wait to parse is substantive to which I will always enter a judgment which I deem final in this class, and I am ever in favor of beneficence over stricture. So, just help out the class in a way you think is helpful and watch those around you do the same. Failure to contribute to this living document by the end of the semester for those in this class will result in a loss of up to 4% of ones total grade outright.

0.2 On the first day, we will introduce ourselves.

Part I

Circuits

Chapter 1

I. Potential, current, energy, conservation

01/10/2019

Contents

1.1 What is electricity?

1.
2.
3.

1.2 Charge

1. Charge is the property of matter that causes it to experience a force when placed in an electromagnetic field; measured in coulombs (C)
2.
3. **How many electrons are needed to form one coulomb?** (What is the weight of all those electrons?)
4. One byte is eight bits. Bits are essentially a single electron stored in a transistor. **If we were to take all the electrons from one terabyte of well distributed information (equal number of ones and zeros), how many coulombs would we have?**

1.3 Current

1. The time rate of charge (charged particles) in motion; measured in amperes (A); defined mathematically as

$$i := dq/dt \tag{1.1}$$

where i is current, q is charge, and t is time

2. Conversely, the total charge transferred over time can be expressed as

$$Q := \int_{t_0}^{t} idt \tag{1.2}$$

3. 1 ampere is equal to 1 coulomb/second
4. Direct current, "DC"
5. Alternative current, "AC"

1.3.1 The directionality of current

Ultimately, the direction in which we say ”current” flows is largely arbitrary. As arbitrary as choosing one type of charge and calling it “positive” and another “negative”. The reason it doesn’t matter is that the only consequence of having chosen a “wrong direction” for the current in a given analysis is that we have to switch the sign of the value. Thus, 3 amps in one direction is *the exact same thing* as -3 in the opposite direction.

1. Thanks to Benjamin Franklin we say that current is

   i. **Positive in the direction in which positively charged particles flow** and

   ii. **Negative in the direction in which negatively charged particles**

   iii. We also now know that current results primarily from the movement of negatively charged particles (electrons) and therefore our convention is wrong in one sense, though convenient and entrenched enough that were not liable to change it in our life time (besides, the math comes out the same, and the actual flow of electrons will only matter to us in a few special circumstances, diodes)

1.3.2 The at times deadly serious nature of current

Much of the point of learning this material here is its eventual application by our hands or by the hands of those we work with. Before we put any of this stuff in our hands, we should probably know what is and is not safe.

1. 1 mA

2. 10 mA

3. 100 mA

4. 1000 mA

1.3.3 The “speed” of current

1.4 Potential (difference)

1. The amount of work needed to move a unit of (positive) charge from a reference point to another point [without producing an acceleration]).

2. Potential is measured in "volts" and is often called "voltage". In this class we will endeavor to avoid such a term as it can be very confusing to talk about potential as if there were such a *thing* as voltage.

3. Defined as
$$v := \frac{dw}{dq} \tag{1.3}$$

4. Potential describes the *potential* to do something. Increasing the potential is akin to increasing the height of a cliff. The height does not *do* anything other than increase what can be done on the drop. If potential is the cliff's height, charge would be pebbles you'd drop off the side, and current describes how fast those pebble fall.

5. In this class, and for the vast vast majority of electrical engineering work, we care about the *difference* in potential. One element held at 100 billion volts and another held at 100 billion + 1 volts has a *potential difference* of 1 V, which is less than a single AA battery.

6. Some typical voltages to be aware of

   Consumer level batteries (AA, AAA):

   Car batteries:

   The "mains" (levels provided by power companies to consumers):

   Power transmission lines:

1.5 Power

1. The time rate of expending or absorbing energy.

2. Quantifies the rate of energy transfer.

3. Mathematically:

4. Measured in watts: 1 W =

5. **Passive sign convention**:

1.6 Energy

1.

2.

3.

4.

1.7 Conservation

Here, as elsewhere, things will be conserved. In electrical circuits there are two laws of conservation that will matter most for us:

1. **The Conservation of Mass.**

2. **The Conservation of Energy.**

In evaluating circuits, the main focus of the first third of this class, it will be the application of these two conservative laws that will enable us to "solve" them. That is, by understanding (1) how energy is generated and used and (2) how charges move around in closed loops ("circuits") we will be able to predict the behavior of the myriad electrical systems which may cross our paths.

1.8 Worksheet

1.8.1 Problem 1, constant charge through a cross-section

How much charge passes through a cross-section of a conductor in 60 seconds if a DC current value is measured at 0.1 mA? **Solution**

1.8.2 Problem 2, arbitrary charge through a cross-section

Determine the total charge entering a terminal between $t = 0$ seconds and $t = 10$ seconds if the current (in amps) passing through is

$$i(t) = \frac{1}{\sqrt{5t+2}}. \tag{1.4}$$

Solution

1.8.3 Problem 3, a "tera"ble puzzle

Approximately how much current is necessary to transmit one terabyte of information in an hour? **Solution**

1.8.4 Problem 4, power necessary to run a pacemaker

A cardiac pacemaker will provide approximately 5,000 J of energy over 5 years. Determine the capacity of a 5 V lithium battery necessary to drive this pacing such that only 40% of its energy is spent over that time. **Solution**

1.8.5 Problem 5, energy needed to excite a neuron

A colleague of yours has been in their lab ginning up new neurons. You, as their resident electrical expert, are tasked with determining the energy consumed by the cell. If the current and voltage variations are found to be functions of time ($t \geq 0$)

$$i(t) = 3t \tag{1.5}$$
$$v(t) = 10e^{6t} \tag{1.6}$$

determine the energy consumed between 0 and 2 ms. **Solution**

1.8.6 Problem 6, a thump to the chest

(a) A typical defibrillator delivers 200-1000 V in less than 10 ms. How much current is needed to deliver 120, 240, and 360 Joules?
(b) A human heart ways about 300 grams. From approximately how high of a cliff would one have to drop a heart such that the impact was equivalent to the energy delivered to someone's chest from a defibrillator? **Solution**

Chapter 2

II. Circuit elements

01/15/2019

2.1 Active v. passive

2.2 Ohm's Law and what it means

2.3 Sources

2.4 Resistors

2.4.1 Resistance, R

2.4.2 Resistivity, ρ

2.4.3 Conductance

2.5 Capacitors

2.5.1 Its time varying behavior

2.5.2 Charge accumulation

2.5.3 A simple example

2.6 Inductors

2.7 Impedance

2.7.1 A quick note on "imaginary" numbers

2.8 Equivalent impedance

2.8.1 Impedances in general

2.8.2 Resistors

2.8.3 Capacitors

2.8.4 Delta-Wye (Δ-Y) transformations

2.8.5 A few examples

2.9 Grounds

2.10 Conductors

2.11 Operational amplifiers

2.12 Diodes

2.13 Switches

2.14 Transistors

2.15 Transformers

2.16 Worksheet

2.16.1 Problem 1, expressing power in ohms

2.16.2 Problem 2, a couple toaster based problems

2.16.3 Problem 3, currently conducting power

2.16.4 Problem 4, conductance of a sodium channel

2.16.5 Problem 5, resistance of a simple tissue

Chapter 3

III. Operational amplifiers

01/17/2019

3.1 Some details

3.2 Some rules

3.3 Some conveniences

3.4 Some examples

3.4.1 Inverting amplifier

3.4.2 Non-inverting amplifier

3.4.3 Voltage follower

3.4.4 Summing amplifier

3.4.5 Differential amplifier (as homework)

Chapter 4

Circuit analysis: I. Nodal analysis

01/22/2019

4.1 Nodes and branches

4.2 Kirchhoff's Laws

4.2.1 Kirchhoff's Current Law

4.2.2 Kirchhoff's Voltage Law

4.3 Nodal analysis

4.4 Solving simultaneous equations

4.4.1 Cramer's Rule

Chapter 5

Circuit analysis: II. Mesh analysis; Homework I

01/24/2019

5.1 Mesh analysis

5.2 Steps of mesh analysis

5.3 Writing mesh equations directly in matrix form

Chapter 6

Circuit analysis: III. Supernodes and supermeshes

01/29/2019 Lecture 6.

6.1 Nodal analysis with an independent current source

6.2 Nodal analysis with voltage sources, Supernodes

6.3 Nodal analysis with controlled sources

6.4 Mesh analysis with current sources

6.5 Mesh analysis with controlled sources, Supermeshes

Chapter 7

Circuit analysis: IV. Circuit theorems

01/31/2019 Lecture 7.

7.1 Circuit theorems

7.2 Linearity

7.3 Superposition

7.4 Source transformation

7.5 Thevenin equivalents

7.6 Norton equivalents

7.7 Equivalents with dependents

Chapter 8

Circuit analysis: V. When to choose between analyses

02/05/2019 Lecture 8.

Chapter 9

A review of the material thus far; Homework II

02/07/2019 Lecture 9.

9.1 How to measure voltage and current

Exam I

02/12/2019

Part II

Systems

Chapter 10

The Laplace Transform: I. What it is and why it is important

02/14/2019 Lecture 10.

10.1 How do we know our world looks like this?

10.2 Euler's identity / Euler's formula

10.3 The Laplace transform

10.4 The Laplace transform of 1

10.5 The s-plane

10.6 The linearity of the Laplace transform

10.7 The Laplace transform of e^{at}

10.8 The Laplace transform of dx/dt

10.9 The Laplace transform in RLC circuits

10.9.1 Resistors

10.9.2 Inductors

10.9.3 Capacitors

10.9.4 RLC

10.10 Two important places, zeros and poles

Chapter 11

The Laplace Transform: II. How to use it

02/19/2019 Lecture 11.

11.1 The inverse Laplace transform

11.2 The Laplace transform of sin

11.3 The Laplace transform of t^n

11.4 Some applicability

Chapter 12

Circuits as ODEs: I. First-order

02/21/2019 Lecture 12.

12.1 Source-free RC circuits

12.1.1 One resistor, one capacitor

12.1.2 Two or more resistors and/or capacitors

12.2 Source-free "active" circuits

12.3 First-order systems with sources

12.4 Several singular functions

12.4.1 Unit step function, $u(t - t_0) = 1, t > t_0$

The Laplace transform of the unit step function

12.4.2 Unit impulse function, $\delta(t) = du(t)/dt$

Its "sifting" abilities

The Laplace transform of the unit impulse function

12.4.3 Unit ramp function, $r(t) = \int u(t)dt$

The Laplace transform of the unit impulse function

Chapter 13

Circuits as ODEs: II. Second-order

02/26/2019 Lecture 13.

13.1 A series RLC circuit

Chapter 14

System response: I. Convolution; Homework III

02/28/2019 Lecture 14.

14.1 An introduction to thinking in systems

Viewing everything as a "system".

14.1.1 Domains of interest, of command

14.1.2 The time-domain, or: our typical realm

14.1.3 The frequency-domain, or: our new realm

14.1.4 The s-domain, or: our magical realm

14.2 Inputs and outputs

14.3 Somewhere in the between

14.4 Convolution in the time-domain

14.5 Multiplication in the frequency- and s-domain

Chapter 15

System response: II. Stability

03/12/2019 Lecture 15.

15.1 An introduction

15.1.1 What do we mean by stability?

15.2 Undamped, $\zeta = 0$

15.3 Underdamped, $0 < \zeta < 1$

15.4 Overdamped, $\zeta > 1$

Part III

& Signals

Chapter 16

System response: III. The frequency domain

03/14/2019 Lecture 16.

Chapter 17

System response: IV. Filters

03/19/2019 Lecture 17.

Chapter 18

System response: V. Feedback; Homework IV

03/21/2019 Lecture 18.

Exam II

03/26/2019

Part IV

in Biomedical Engineering

Chapter 19

Bioelectricity: I. Passive properties

03/28/2019 Lecture 19.

19.1 Modeling biological material with a simple circuit, $R_1 + (R_2||C)$

19.2 Resistance-Reactance Plane

19.3 What can we do with this information?

Chapter 20

Bioelectricity: II. Active properties

04/02/2019 Lecture 20.

Chapter 21

Bioelectricity: III. Measurement

04/04/2019 Lecture 21.

Chapter 22

Digital circuits: I. Discretization

04/09/2019 Lecture 22.

Chapter 23

Digital circuits: II. Logic; Homework V

04/11/2019 Lecture 23.

Chapter 24

Happenstance: A few BME specific situations

04/16/2019 Lecture 24.

Chapter 25

Circumstance: A few BME specific standards

04/18/2019 Lecture 25.

Chapter 26

A philosophy of circuits, systems, and signals; Homework VI

04/23/2019 Lecture 26.

Exam III

04/26/2019