R Code for Lecture 6

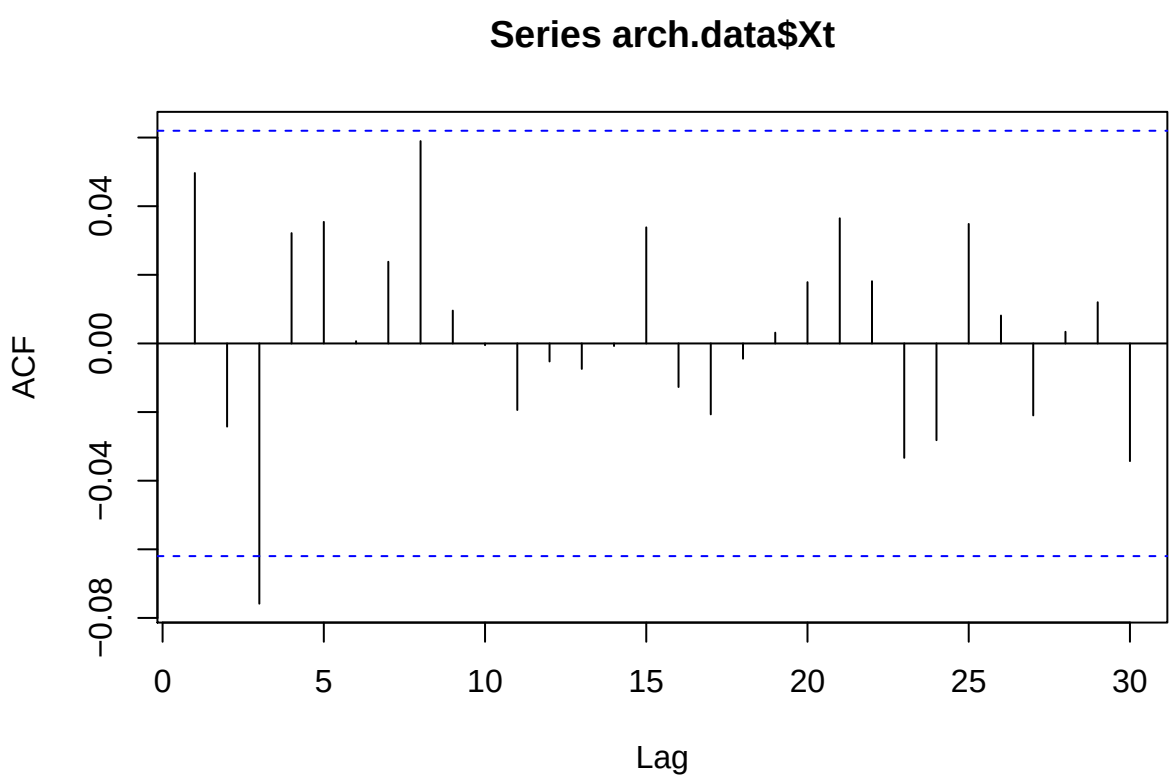
```
library(TSA)
library(ggplot2)
library(latex2exp)
```

ARCH(1) Model

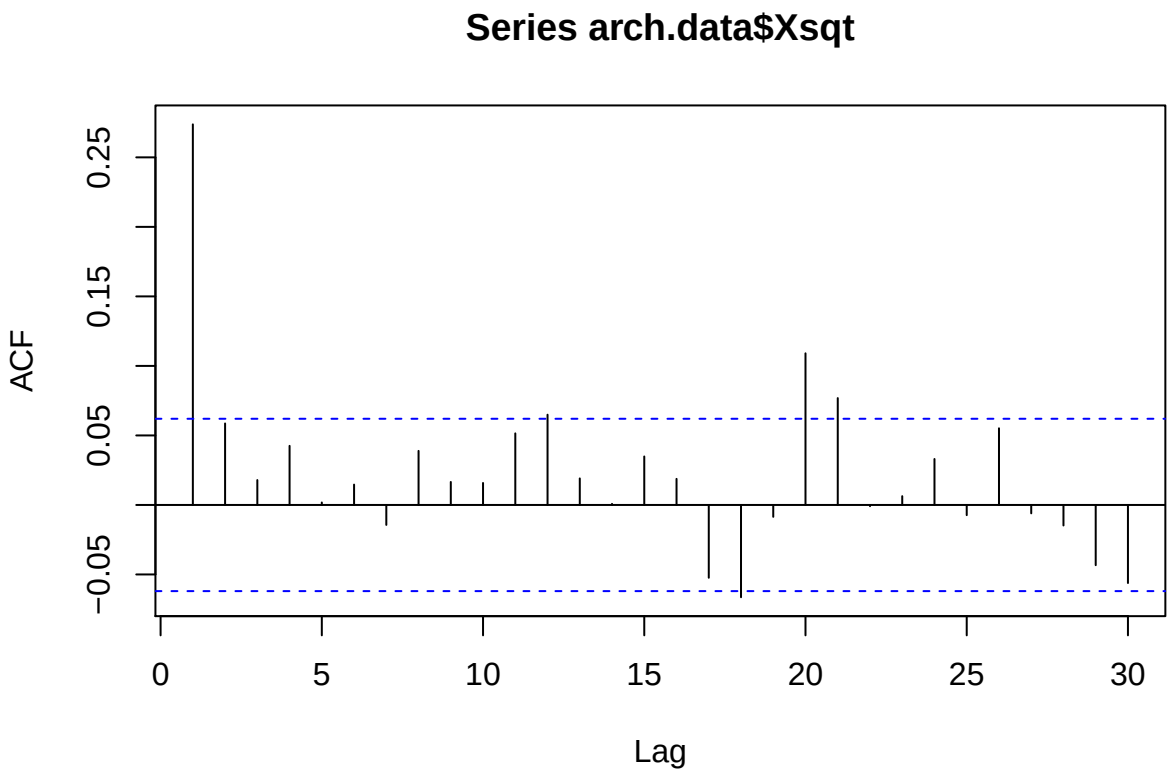
$$X_t = \sigma_t \epsilon_t, \epsilon_t \sim N(0,1)$$
$$\sigma_t^2 = 0.6 + 0.4 X_{t-1}^2$$

```
X <- garch.sim(alpha = c(0.6, 0.4), n = 1000)
Xsq <- X^2
sig2 <- 0.6 + 0.4*lag(Xsq)
arch.data <- data.frame(time = c(1:length(X)), Xt = X, Xsqt = Xsq, sig2t = sig2)
```

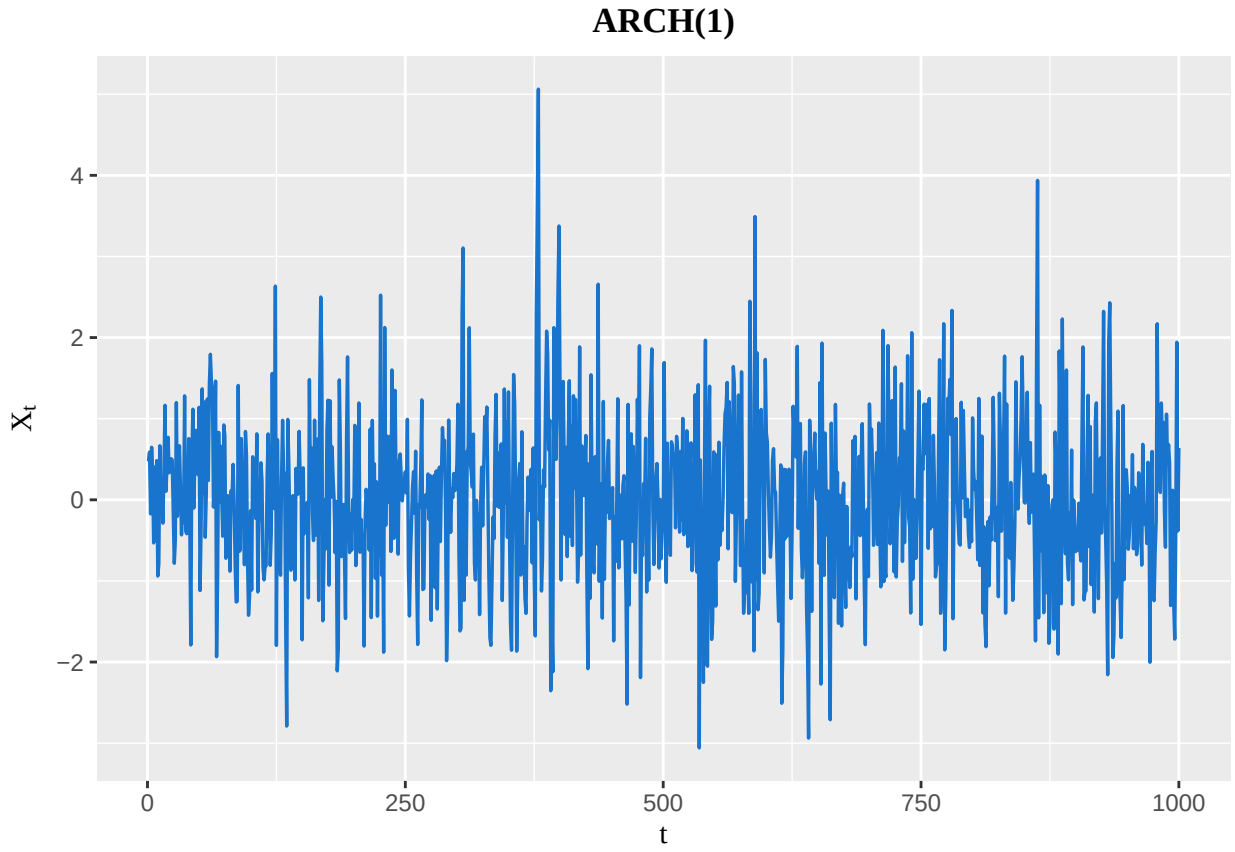
```
acf(arch.data$Xt, na.action = na.pass)
```



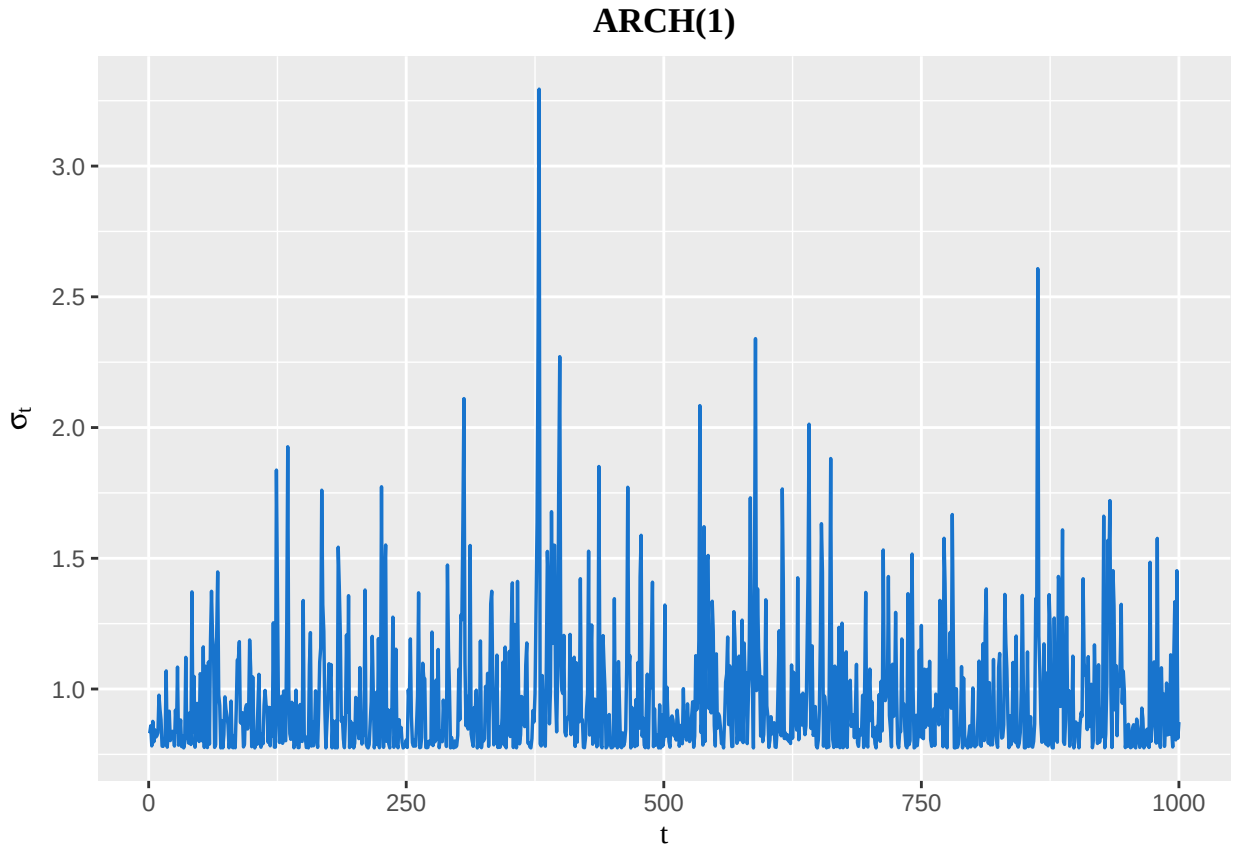

```
acf(arch.data$Xsqt, na.action = na.pass)
```



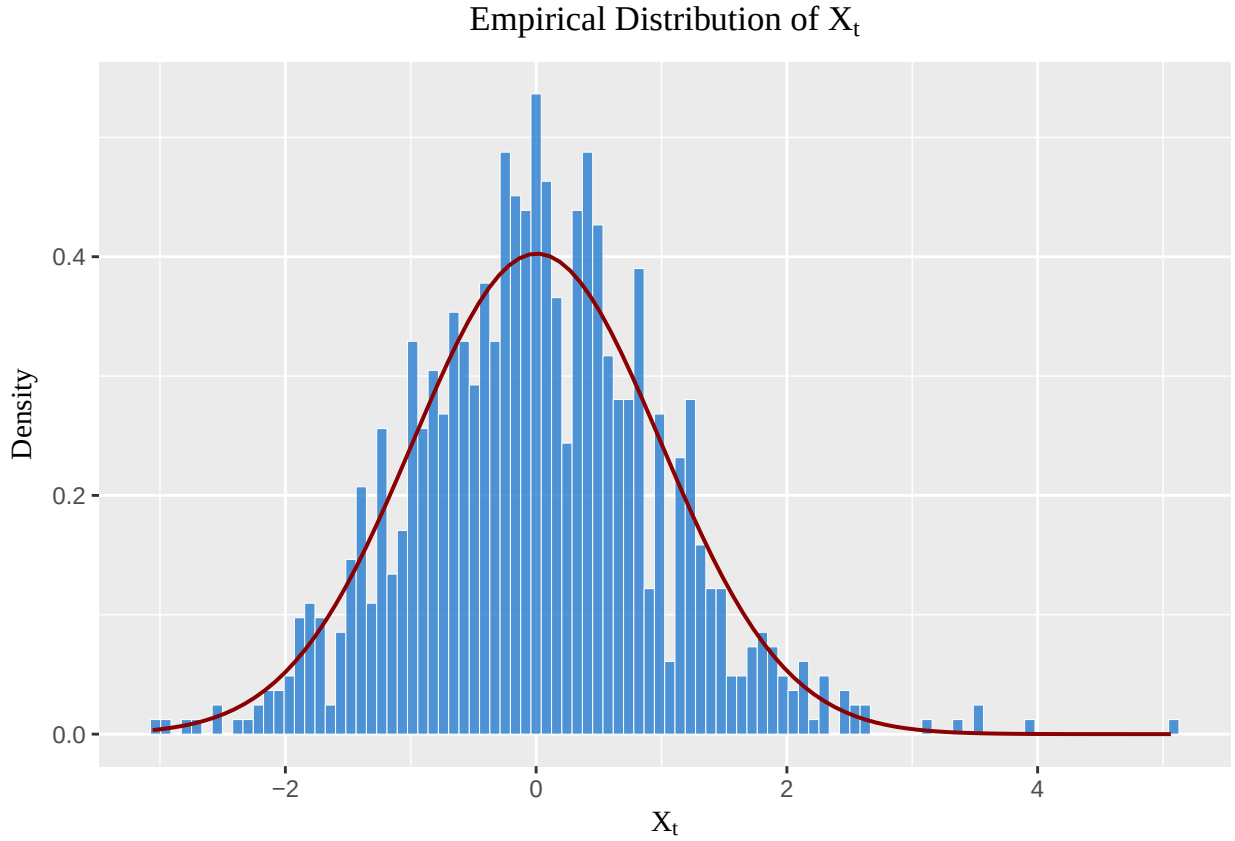

```
fig11 <- ggplot(arch.data, aes(time)) +
  geom_line(aes(y = Xt), size = 0.6, color = "dodgerblue3") +
  labs(title = "ARCH(1)",
       x = TeX("$t$"), y = TeX("$X_t$")) +
  theme(axis.title = element_text(family = "serif"),
        plot.title = element_text(hjust = 0.5, family = "serif", face = "bold"))
fig11
```




```
fig12 <- ggplot(arch.data, aes(time)) +
  geom_line(aes(y = sqrt(sig2t)), size = 0.6, color = "dodgerblue3") +
  labs(title = "ARCH(1)",
       x = TeX("$t$"), y = TeX("$\\sigma_t$")) +
  theme(axis.title = element_text(family = "serif"),
        plot.title = element_text(hjust = 0.5, family = "serif", face = "bold"))
fig12
```

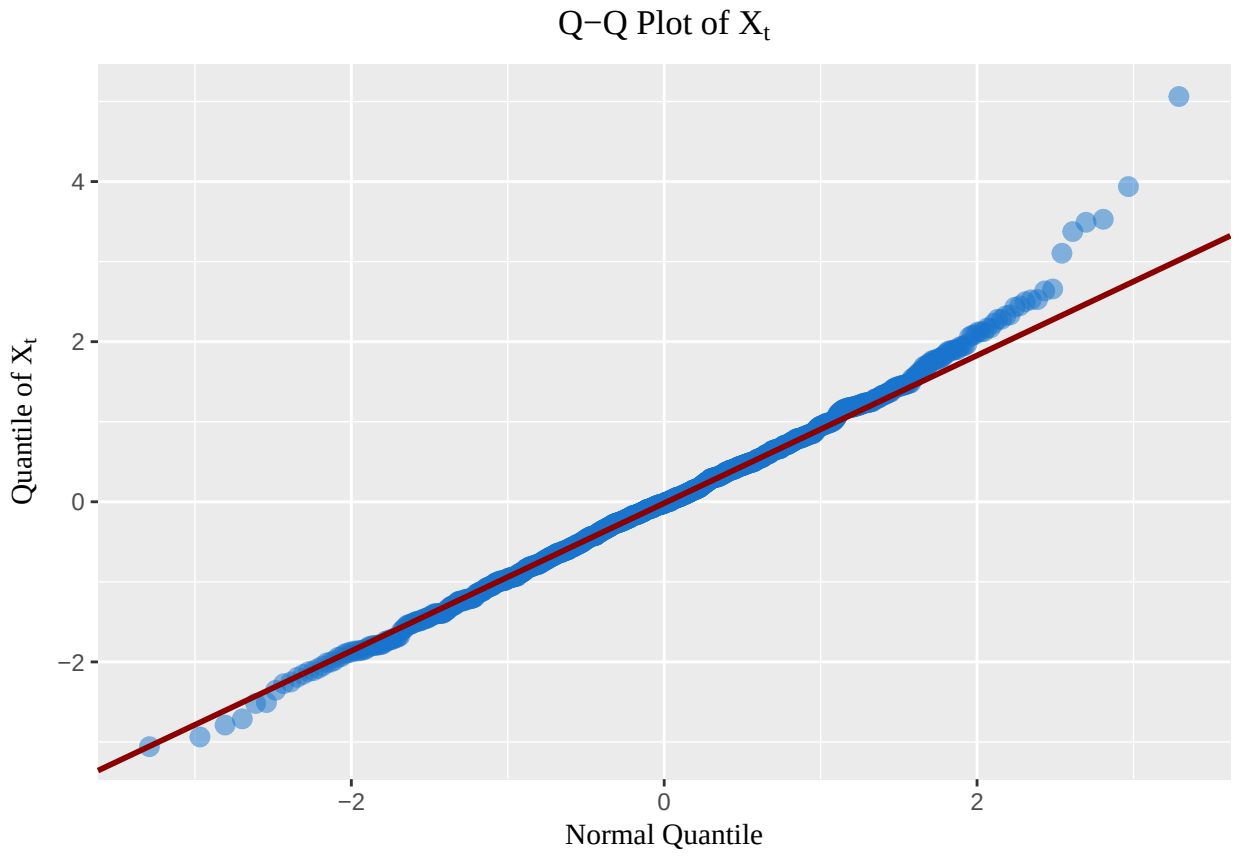



```
fig13 <- ggplot(arch.data, aes(Xt)) +
  geom_histogram(aes(y = ..density..), bins = 100, color="white",
                 fill="dodgerblue3",
                 alpha = 0.75, size = 0.1) +
  stat_function(fun = dnorm, color = "darkred", size = 0.7,
                args = list(mean = mean(arch.data$Xt,
                                        na.rm = T),
                            sd = sd(arch.data$Xt,
                                    na.rm = T))) +
  labs(title = TeX("Empirical Distribution of $X_t$"),
       x = TeX("$X_t$"), y = "Density") +
  theme(axis.title = element_text(family = "serif"),
        plot.title = element_text(hjust = 0.5, family = "serif"))
fig13
```




```
y <- quantile(arch.data$Xt, c(0.25, 0.75), na.rm = T)
x <- qnorm(c(0.25, 0.75))
slope <- diff(y)/diff(x)
int   <- y[1] - slope*x[1]

fig14 <- ggplot(arch.data, aes(sample = Xt)) +
  geom_qq(alpha = 0.5, size = 3, color = "dodgerblue3") +
  geom_abline(intercept=int, slope=slope, color = "darkred", size = 1) +
  labs(title = TeX("Q-Q Plot of $X_t$"),
       x = "Normal Quantile", y = TeX("Quantile of $X_t$")) +
  theme(axis.title = element_text(family = "serif"),
        plot.title = element_text(hjust = 0.5, family = "serif"))
fig14
```

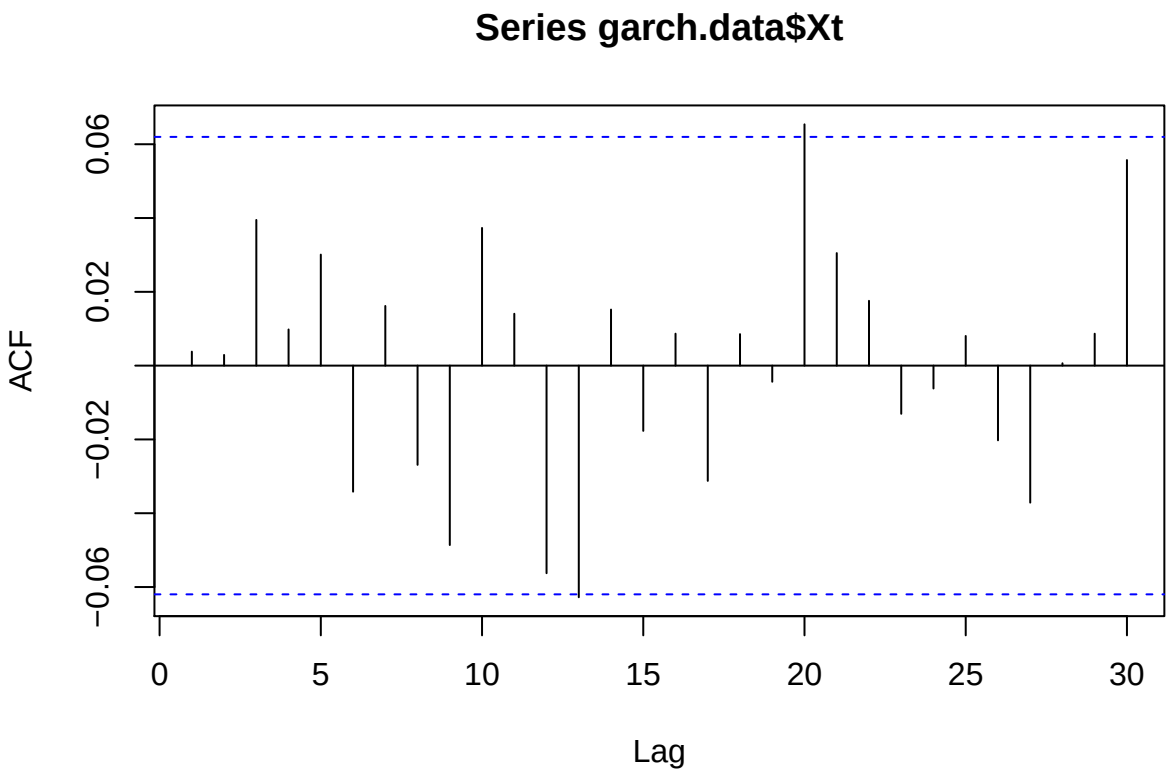


GARCH(1,1) Model

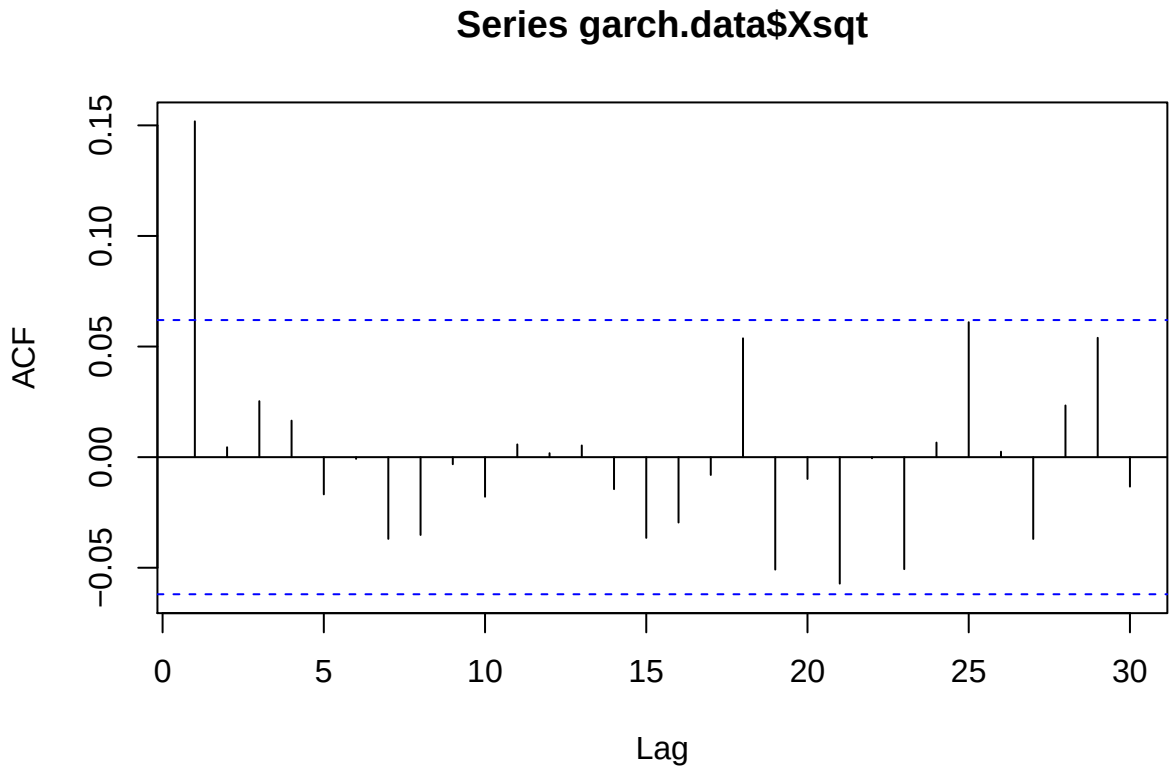
$$X_t = \sigma_t \epsilon_t, \epsilon_t \sim N(0,1)$$
$$\sigma_t^2 = 0.6 + 0.2X_{t-1}^2 + 0.2\sigma_{t-1}^2$$

```
X <- garch.sim(alpha = c(0.6, 0.2), beta = 0.2, n = 1000)
Xsq <- X^2
sig2 <- vector("numeric"); sig2[1] <- NA
for (t in 2:length(X)) {
  sig2[t] <- 0.6/(1-0.2) + 0.2*sum(Xsq[1:(t-1)]*0.2^c((t-2):0))
}
garch.data <- data.frame(time = c(1:length(X)), Xt = X, Xsqt = Xsq, sig2t = sig2)
```

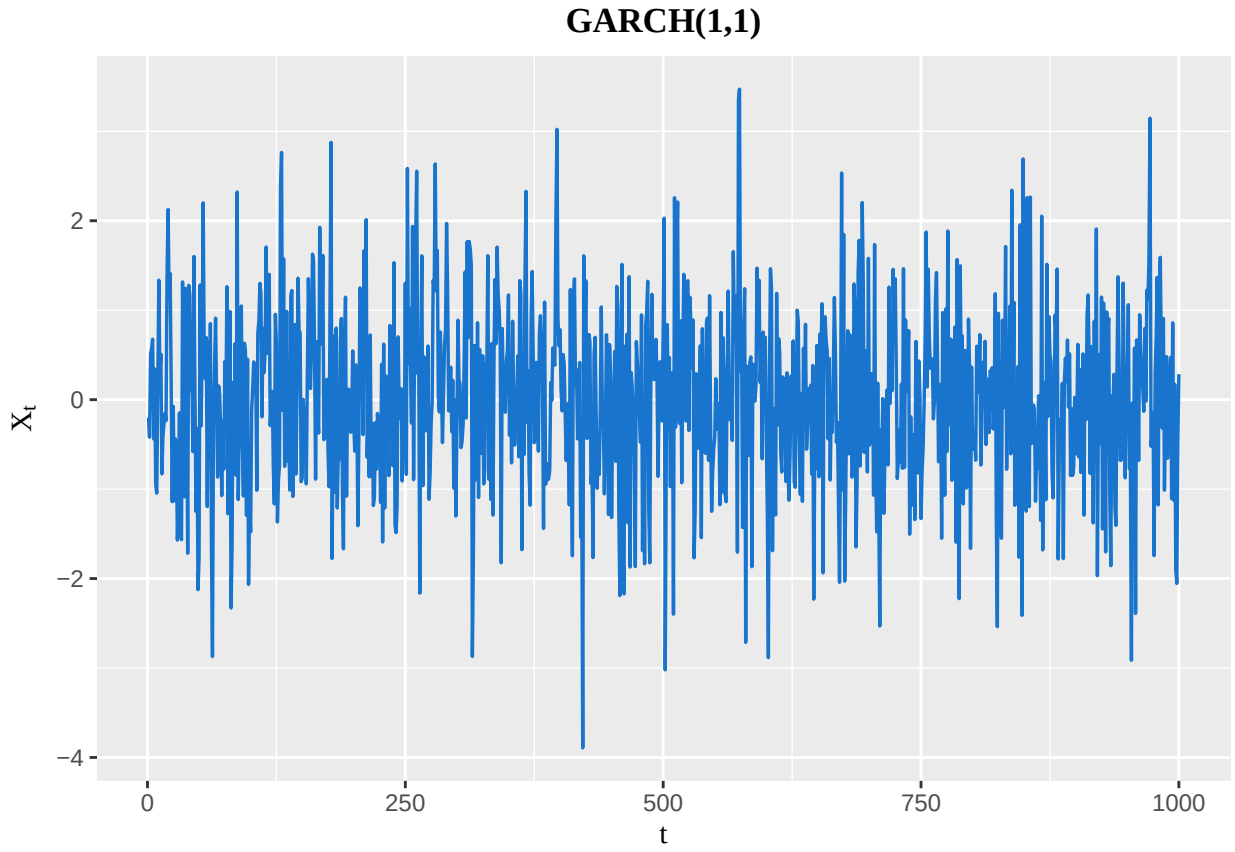
```
acf(garch.data$Xt, na.action = na.pass)
```




```
acf(garch.data$Xsqt, na.action = na.pass)
```

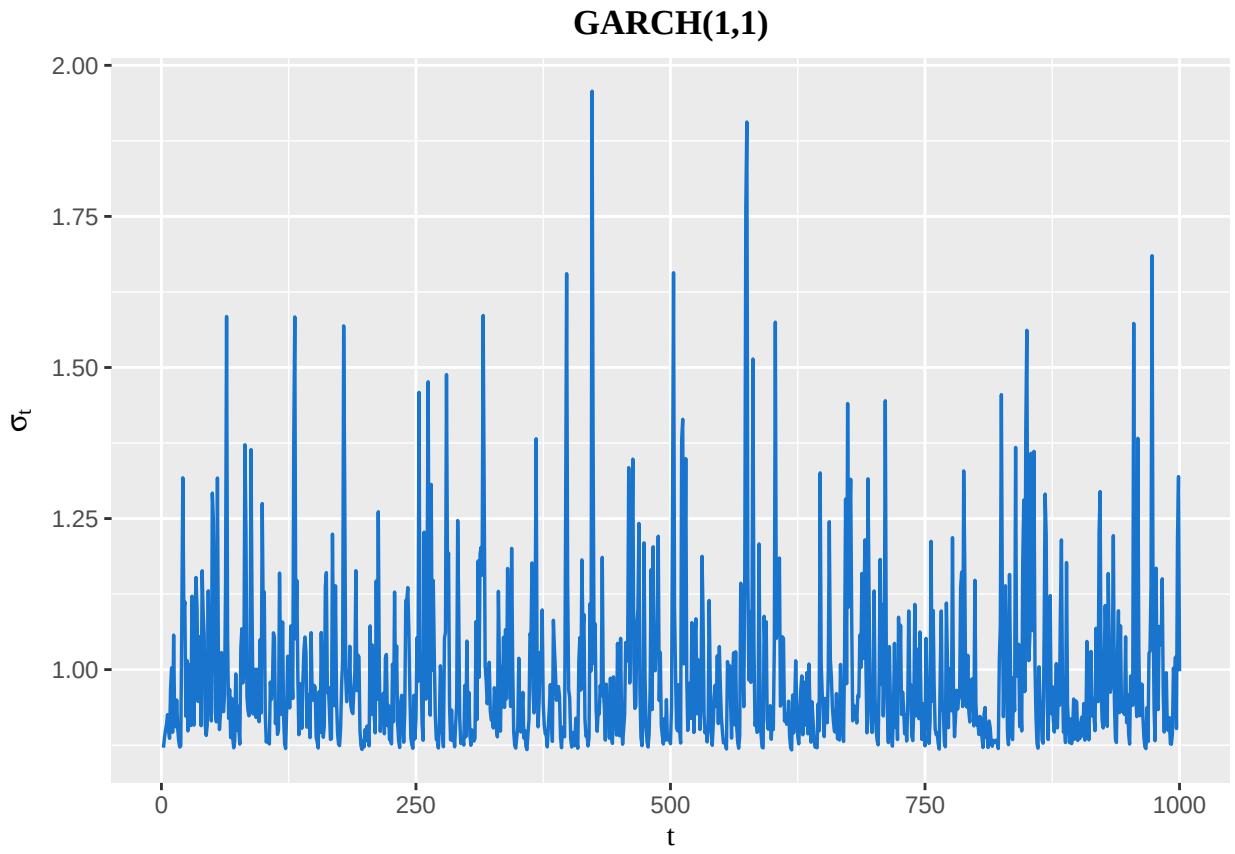



```
fig21 <- ggplot(garch.data, aes(time)) +
  geom_line(aes(y = Xt), size = 0.6, color = "dodgerblue3") +
  labs(title = "GARCH(1,1)",
       x = TeX("$t$"), y = TeX("$X_t$")) +
  theme(axis.title = element_text(family = "serif"),
        plot.title = element_text(hjust = 0.5, family = "serif", face = "bold"))
fig21
```

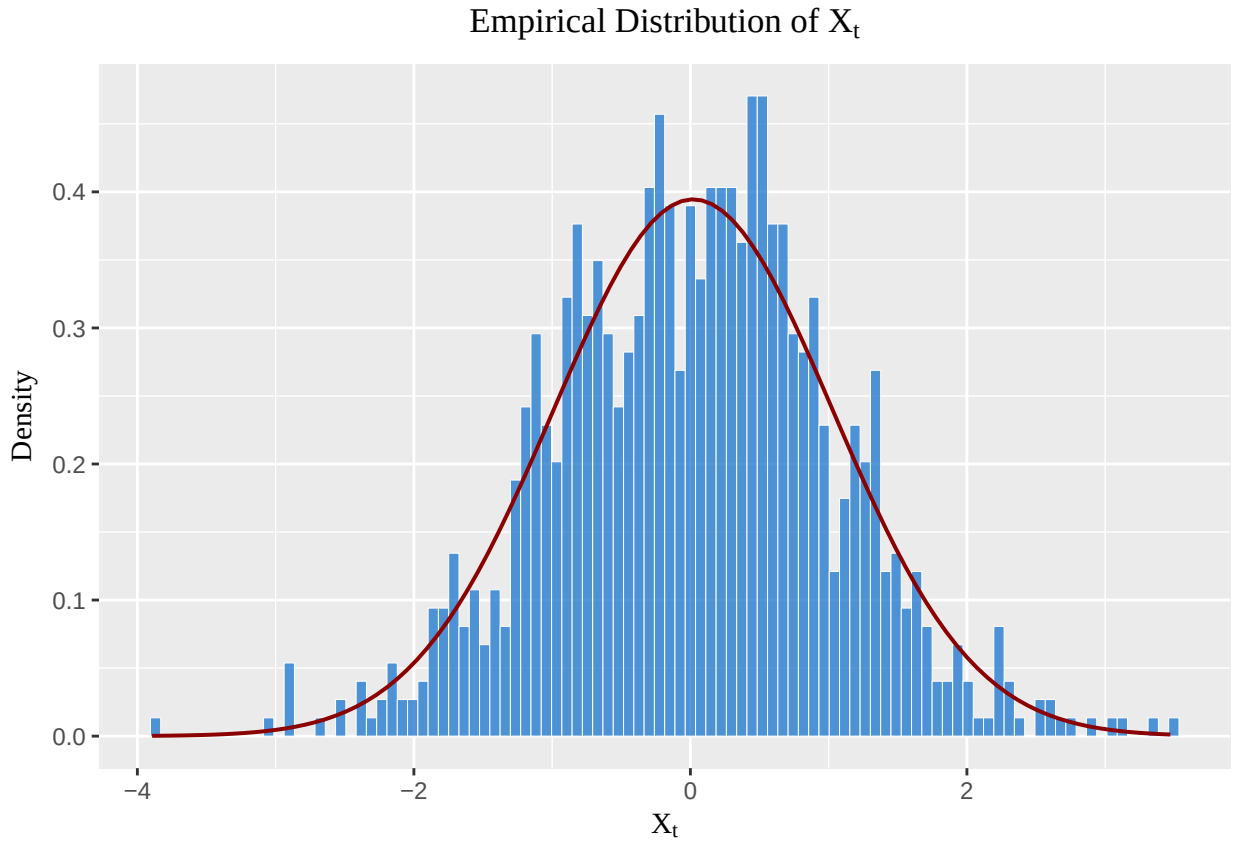



```
fig22 <- ggplot(garch.data, aes(time)) +
  geom_line(aes(y = sqrt(sig2t)), size = 0.6, color = "dodgerblue3") +
  labs(title = "GARCH(1,1)",
       x = TeX("$t$"), y = TeX("$\\sigma_t$")) +
  theme(axis.title = element_text(family = "serif"),
        plot.title = element_text(hjust = 0.5, family = "serif", face = "bold"))
fig22
```

```
## Warning: Removed 1 rows containing missing values (geom_path).
```

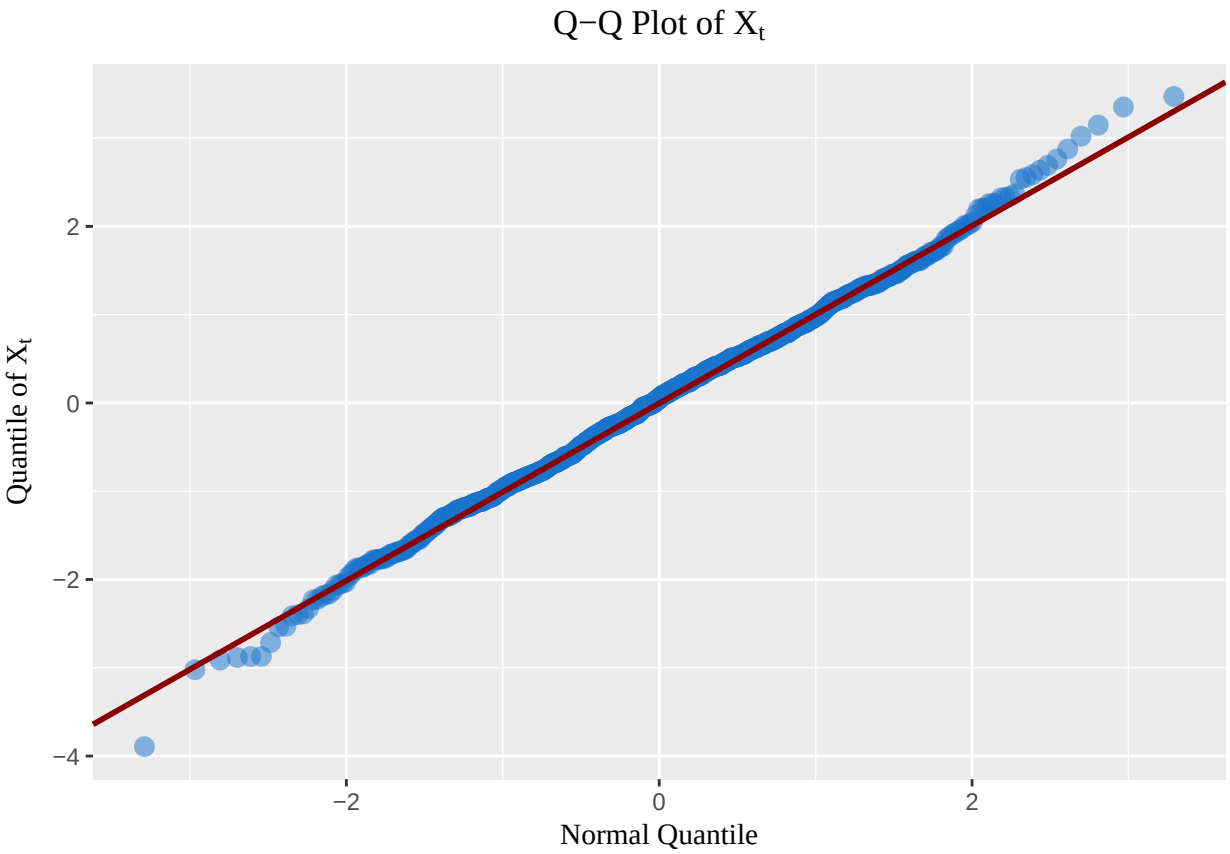



```
fig23 <- ggplot(garch.data, aes(Xt)) +
  geom_histogram(aes(y = ..density..), bins = 100, color="white",
                 fill="dodgerblue3",
                 alpha = 0.75, size = 0.1) +
  stat_function(fun = dnorm, color = "darkred", size = 0.7,
                args = list(mean = mean(garch.data$Xt,
                                        na.rm = T),
                            sd = sd(garch.data$Xt,
                                    na.rm = T))) +
  labs(title = TeX("Empirical Distribution of $X_t$"),
       x = TeX("$X_t$"), y = "Density") +
  theme(axis.title = element_text(family = "serif"),
        plot.title = element_text(hjust = 0.5, family = "serif"))
fig23
```




```
y <- quantile(garch.data$Xt, c(0.25, 0.75), na.rm = T)
x <- qnorm(c(0.25, 0.75))
slope <- diff(y)/diff(x)
int   <- y[1] - slope*x[1]

fig24 <- ggplot(garch.data, aes(sample = Xt)) +
  geom_qq(alpha = 0.5, size = 3, color = "dodgerblue3") +
  geom_abline(intercept=int, slope=slope, color = "darkred", size = 1) +
  labs(title = TeX("Q-Q Plot of $X_t$"),
       x = "Normal Quantile", y = TeX("Quantile of $X_t$")) +
  theme(axis.title = element_text(family = "serif"),
        plot.title = element_text(hjust = 0.5, family = "serif"))
fig24
```


Q−Q Plot of X_t
Quantile of X_t
Normal Quantile
2
0
−2
−4
−2
0
2