

# Financial Econometrics

## Lecture 1

Fu Ouyang

School of Finance  
Nankai University

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# Outline

Introduction to This Course

Asset Returns

Stylized Features of Financial Returns

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Stylized Features of Financial Returns

# One Period Returns

The *return* for an asset (e.g., stock, bond, etc) is a percentage defined as the change of the asset price as a fraction of its initial price. Returns exhibit more attractive statistical properties than prices. Financial econometrics typically models and analyzes return data rather than price series.

There are various definitions for the asset returns: Let  $P_t$  denote the price of an asset at time  $t$ .

- From time  $t - 1$  to  $t$ , assuming no dividend paid, the *one-period simple return* is defined as

$$R_t = (P_t - P_{t-1})/P_{t-1}$$

- The *one-period gross return*:  $P_t/P_{t-1} = R_t + 1$ .
- The *one-period log return*:  $r_t = \log(P_t/P_{t-1}) = \log(R_t + 1)$ . Note that when  $R_t$  is small,  $r_t \approx R_t$ , though theoretically  $r_t \leq R_t$ .

# Multiperiod Returns

Consider multiperiod returns.

- the  $k$ -period *simple return* from time  $t - k$  to  $t$  can be similarly defined as

$$R_t(k) = (P_t - P_{t-k})/P_{t-k}$$

- The  $k$ -period *gross return*:  $P_t/P_{t-k} = R_t(k) + 1$ .
- The  $k$ -period *log return*:  $r_t(k) = \log(P_t/P_{t-k}) = \log(R_t(k) + 1)$ .
- $r_t(k)$  is the sum of the  $k$  one-period log returns

$$\begin{aligned}r_t(k) &= \log(R_t(k) + 1) \\&= \log[(R_t + 1)(R_{t-1} + 1) \cdots (R_{t-k+1} + 1)] \\&= \log(R_t + 1) + \log(R_{t-1} + 1) + \cdots + \log(R_{t-k+1} + 1) \\&= r_t + r_{t-1} + \cdots + r_{t-k+1}\end{aligned}$$

In this class, returns refer to log returns unless specified otherwise.

# Adjustment for Dividends

Let  $D_t$  denote the dividend payment between time  $t - 1$  and  $t$ . Assume that no dividends are re-invested in the asset.

$$R_t(k) = \frac{P_t + D_t + \cdots + D_{t-k+1}}{P_{t-k}} - 1$$

$$r_t(k) = \sum_{j=0}^{k-1} \log \left( \frac{P_{t-j} + D_{t-j}}{P_{t-j-1}} \right) = r_t + \cdots + r_{t-k+1}$$

# Continuously Compounded Returns and Bond Yields

$r_t$  is also called *continuously compounded return*: If a simple annual interest rate  $r$  for a bank deposit is fixed and earnings are equally paid  $m$  times per year. Then the gross return at the end of one year is

$$(1 + r/m)^m \rightarrow e^r \text{ as } m \rightarrow \infty$$

i.e., for continuously compounded interest,  $r$  = the annual log return.

Bonds are quoted in annualized yields. Consider a zero-coupon bond with the face value 1, the current yield is  $r_t$  and the remaining duration is  $D$ . Its current price  $B_t$  satisfies  $B_t \exp(Dr_t) = 1$  and so the annualized log return of holding the bond (ignoring that the remaining duration for  $B_{t+1}$  is  $D - 1$ ) is

$$\log(B_{t+1}/B_t) = D(r_t - r_{t+1})$$

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Asset Returns

Stylized Features of Financial Returns



# Behavior of Financial Return Data: Stationarity and Volatility Clustering

## Stationarity

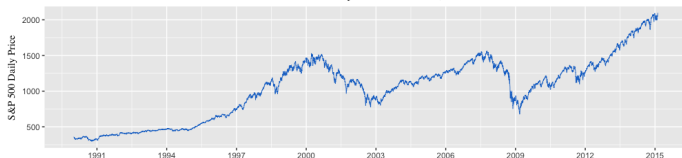
- Asset prices are often not stationary over time (e.g., financial crisis, economic growth/recession, technology improvement, etc).
- Most return sequences show certain level of stationarity (i.e., time invariant first two moments).
- The term *stationarity* will be formally defined in next lecture.

## Volatility Clustering

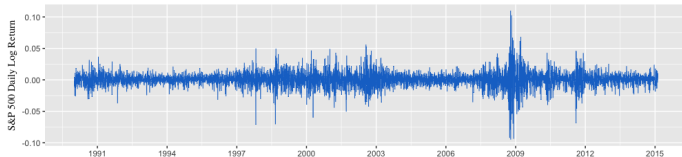
- The volatility of asset returns is time varying.
- Large price changes (i.e., absolute returns) often occur in clusters.

# Behavior of Financial Return Data: Stationarity and Volatility Clustering

Time Series of the Daily S&P 500 Indices, 1990-2015



Time Series of the Daily S&P 500 Returns, 1990-2015



# Behavior of Financial Return Data: Heavy tails

## Heavy tails

- The distribution of  $r_t$  often exhibits “heavier-than-normal” tails.
- A random variable  $X$  has “heavier tails” than  $Z$  if for sufficiently large  $M \in \mathbb{R}_+$ ,  $P(|X| > M) > P(|Z| > M)$ .
- Example:  $t$ -distribution (with degree of freedom  $\nu$ ) whose PDF is

$$f_\nu(x) \propto (1 + x^2/\nu)^{-(\nu+1)/2}$$

with tails of the order  $|x|^{-(\nu+1)} (\asymp |x|^{-(\nu+1)})$ .  $N(0, 1)$  has tails of order  $\exp(-x^2/2)$ . It is not hard to see that  $\lim_{x \rightarrow \infty} \exp(x^2/2)/|x|^{\nu+1} = \infty$ .

- Tail behavior is closely related to the existence of  $E(|r_t|^p)$ . For  $X \sim N(0, \sigma^2)$ ,  $E(|X|^p) < \infty$  for all  $p \in \mathbb{R}_+$ , but if  $X \sim t_\nu$ ,  $E(|X|^p) < \infty$  only for  $p \in (0, \nu)$ .  $r_t$  is typically assumed to have at least  $E(r_t^2) < \infty$ .

# Behavior of Financial Return Data: Asymmetry and Aggregational Gaussianity

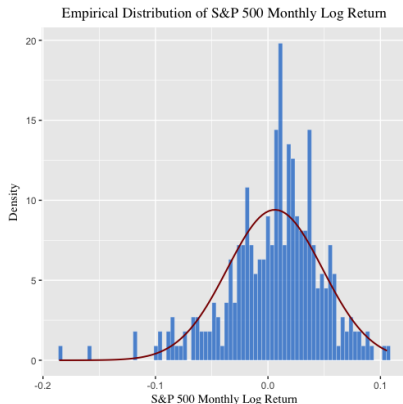
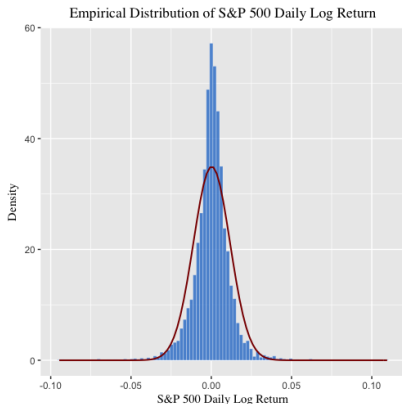
## Asymmetry

- The distribution of  $r_t$  is often negatively skewed (long left tails).
- Risk aversion: the market tends to react more strongly to negative shocks than positive ones.

## Aggregational Gaussianity

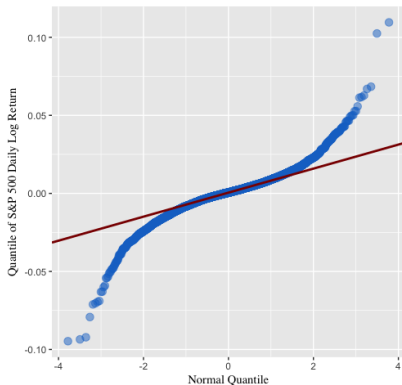
- Recall  $r_t(k) = r_t + r_{t-1} + \cdots + r_{t-k+1}$ .
- As  $k$  increases (long time horizon), the distribution of  $r_t(k)$  tends to a normal distribution (think the central limit theorem).

# Behavior of Financial Return Data: Heavy tails, Asymmetry and Aggregational Gaussianity

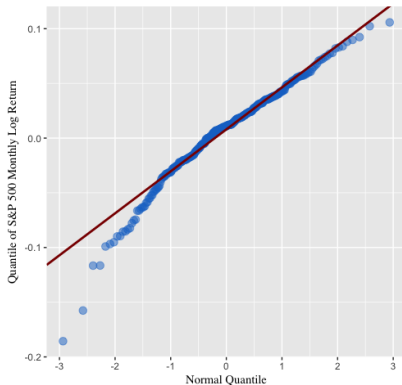


# Behavior of Financial Return Data: Heavy tails, Asymmetry and Aggregational Gaussianity

Q-Q Plot of S&amp;P 500 Daily Log Return



Q-Q Plot of S&amp;P 500 Monthly Log Return



# Behavior of Financial Return Data: Long Range Dependence

## Long Range Dependence

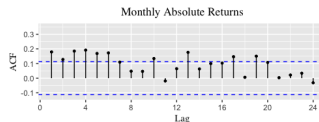
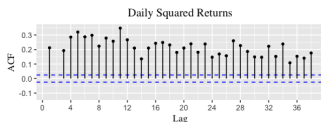
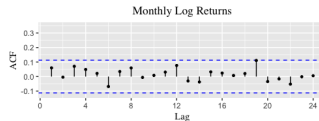
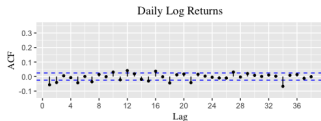
- The returns are almost serially uncorrelated, but not independent!
- Use the sample *autocorrelation function* (ACF)  $\hat{\rho}_k$  to measure the serial correlation of a return series  $\{r_t\}_{t=1}^T$ . Define  $\hat{\rho}_k = \hat{\gamma}_k / \hat{\gamma}_0$  where

$$\hat{\gamma}_k = \frac{1}{T} \sum_{t=1}^{T-k} (r_t - \bar{r})(r_{t+k} - \bar{r})$$

is the sample *autocovariance* at lag  $k$  and  $\bar{r} = \sum_{t=1}^T r_t / T$ .

- $\{r_t^2\}_{t=1}^T$  and  $\{|r_t|\}_{t=1}^T$  (*long memory*) exhibit significant (95% confidence interval for  $H_0 : \rho_k = 0$ ) ACF  $\Rightarrow$  nonlinear dependencies over time.
- $\rho_k$ 's become weaker and less persistent for longer sampling intervals.

# Behavior of Financial Return Data: Long Range Dependence





# Behavior of Financial Return Data: Leverage Effect

## Leverage Effect

- Assets returns are often negatively correlated with the changes of their volatilities.
- Prices down  $\Rightarrow$  firms more leveraged (riskier)  $\Rightarrow$  more volatile prices.
- Volatility up  $\Rightarrow$  investors demand risk premiums  $\Rightarrow$  prices down.
- Risk aversion: volatilities caused by price decline are typically larger than the appreciations due to declined volatilities.

# Behavior of Financial Return Data: Leverage Effect

