R Code for Lecture 9

```r
library(WindR)        # load WindR package
library(TSA)
library(MTS)
library(vars)
library(dplyr)
```

1. Daily Data of S&P 500, S&P 400 (MidCap) and S&P 600 (SmallCap)

We download all data from "Wind Financial Terminal" (WTF) via its "Data Feed Services" (WDFS). To use WDFS in R/RSrtudio, one should first install the `WindR` package, then load it by `library(WindR)`[1], and launch it using `w.start()`.

```r
w.start()             # launch WindR DataFeed

index <- w.wsd("000001.SH,399106.SZ,HSCI.HI,SP500.SPI,SP400.SPI,SP600.SPI","close",
               "2011-01-01","2014-12-31","TradingCalendar=NYSE")$Data %>%
  rename(SH = `000001.SH`, SZ = `399106.SZ`, SP500 = `SP500.SPI`,
         SP400 = `SP400.SPI`, SP600 = `SP600.SPI`, date = `DATETIME`) %>%
  mutate(R.SH = (SH-lag(SH, n = 1L))/lag(SH, n = 1L),
         R.SZ = (SZ-lag(SZ, n = 1L))/lag(SZ, n = 1L),
         R.SP500 = (SP500-lag(SP500, n = 1L))/lag(SP500, n = 1L),
         R.SP400 = (SP400-lag(SP400, n = 1L))/lag(SP400, n = 1L),
         R.SP600 = (SP600-lag(SP600, n = 1L))/lag(SP600, n = 1L))
```

---

2. LSE of Vector Autoregressive Model using S&P 500/400/600 Data

```r
data <- filter(select(index, R.SP500, R.SP400, R.SP600), !is.na(R.SP500))
```

2.1 Fitting a VAR(p) model with $p = 1$ selected by BIC

Note: To use the `VAR()` function below, one need first install the `vars` package.

```r
SPvar.bic <- VAR(data, lag.max = 3, ic = "SC")
summary(SPvar.bic)

##
## VAR Estimation Results:
## =========================
## Endogenous variables: R.SP500, R.SP400, R.SP600
## Deterministic variables: const
## Sample size: 1004
## Log Likelihood: 11758.616
## Roots of the characteristic polynomial:
## 0.1698 0.02231 0.02231
## Call:
```

---

[1]WDFS (WindR package) is not currently available for R/RStudio installed on Mac OS computers.

```
## VAR(y = data, lag.max = 3, ic = "SC")
##
##
## Estimation results for equation R.SP500:
## =======================================
## R.SP500 = R.SP500.l1 + R.SP400.l1 + R.SP600.l1 + const
##
##              Estimate Std. Error t value Pr(>|t|)
## R.SP500.l1 -0.2405222  0.1013294  -2.374   0.0178 *
## R.SP400.l1  0.2839293  0.1255829   2.261   0.0240 *
## R.SP600.l1 -0.1250348  0.0902428  -1.386   0.1662
## const       0.0005794  0.0003068   1.888   0.0593 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.009704 on 1000 degrees of freedom
## Multiple R-Squared: 0.01011, Adjusted R-squared: 0.007144
## F-statistic: 3.406 on 3 and 1000 DF,  p-value: 0.01717
##
##
## Estimation results for equation R.SP400:
## =======================================
## R.SP400 = R.SP500.l1 + R.SP400.l1 + R.SP600.l1 + const
##
##              Estimate Std. Error t value Pr(>|t|)
## R.SP500.l1 -0.2348136  0.1222116  -1.921   0.0550 .
## R.SP400.l1  0.2694693  0.1514633   1.779   0.0755 .
## R.SP600.l1 -0.1095613  0.1088403  -1.007   0.3144
## const       0.0005812  0.0003700   1.571   0.1166
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.0117 on 1000 degrees of freedom
## Multiple R-Squared: 0.005977,   Adjusted R-squared: 0.002995
## F-statistic: 2.004 on 3 and 1000 DF,  p-value: 0.1117
##
##
## Estimation results for equation R.SP600:
## =======================================
## R.SP600 = R.SP500.l1 + R.SP400.l1 + R.SP600.l1 + const
##
##              Estimate Std. Error t value Pr(>|t|)
## R.SP500.l1 -0.2425124  0.1352800  -1.793   0.0733 .
## R.SP400.l1  0.2959391  0.1676598   1.765   0.0778 .
## R.SP600.l1 -0.1872050  0.1204790  -1.554   0.1205
## const       0.0006774  0.0004096   1.654   0.0985 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01296 on 1000 degrees of freedom
## Multiple R-Squared: 0.01334, Adjusted R-squared: 0.01038
```

```
## F-statistic: 4.507 on 3 and 1000 DF,  p-value: 0.003783
##
##
##
## Covariance matrix of residuals:
##           R.SP500    R.SP400    R.SP600
## R.SP500 9.417e-05 0.0001080 0.0001159
## R.SP400 1.080e-04 0.0001370 0.0001466
## R.SP600 1.159e-04 0.0001466 0.0001679
##
## Correlation matrix of residuals:
##         R.SP500 R.SP400 R.SP600
## R.SP500  1.0000  0.9506  0.9218
## R.SP400  0.9506  1.0000  0.9667
## R.SP600  0.9218  0.9667  1.0000
```

```
## Refitting the model by leaving out insignificant coefficients
restrict(SPvar.bic, thresh = 0.1)
```

```
##
## VAR Estimation Results:
## =======================
##
## Estimated coefficients for equation R.SP500:
## ============================================
## Call:
## R.SP500 = R.SP500.l1 + R.SP400.l1 + R.SP600.l1 + const
##
##    R.SP500.l1    R.SP400.l1    R.SP600.l1          const
## -0.2405221625  0.2839293483 -0.1250347791  0.0005794004
##
##
## Estimated coefficients for equation R.SP400:
## ============================================
## Call:
## R.SP400 = R.SP500.l1 + R.SP400.l1 + R.SP600.l1 + const
##
##    R.SP500.l1    R.SP400.l1    R.SP600.l1          const
## -0.2348135696  0.2694693388 -0.1095613489  0.0005812373
##
##
## Estimated coefficients for equation R.SP600:
## ============================================
## Call:
## R.SP600 = R.SP500.l1 + R.SP400.l1 + R.SP600.l1 + const
##
##    R.SP500.l1    R.SP400.l1    R.SP600.l1          const
## -0.2425124106  0.2959390856 -0.1872049676  0.0006773561
```

```
## ACF of residuals
ACF.SPvar.bic <- acf(residuals(SPvar.bic), na.action = na.pass, plot = F)
plot(ACF.SPvar.bic)
```
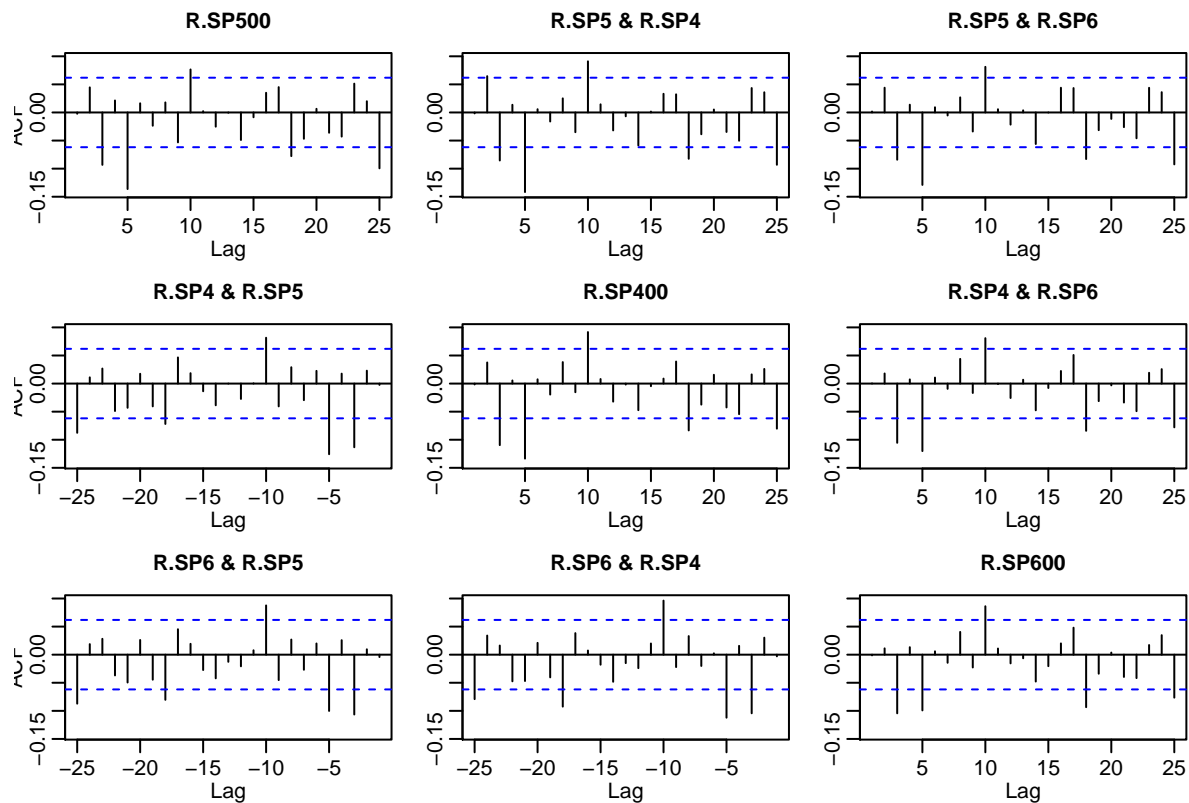
```
## 
## Portmanteau tests
serial.test(SPvar.bic)
```

```
## 
##   Portmanteau Test (asymptotic)
## 
## data:  Residuals of VAR object SPvar.bic
## Chi-squared = 194.74, df = 135, p-value = 0.000587
```

Note: Using AIC (set `ic = "AIC"`) and HQIC result in a VAR(p) model with $p = 3$ and $p = 1$, respectively.

2.2 Fitting a VAR(p) model with $p = 2$

```
SPvar2 <- VAR(data, p = 2)
summary(SPvar2)
```

```
## 
## VAR Estimation Results:
## =========================
## Endogenous variables: R.SP500, R.SP400, R.SP600
## Deterministic variables: const
## Sample size: 1003
## Log Likelihood: 11758.272
## Roots of the characteristic polynomial:
## 0.2843 0.2843 0.2815 0.2778 0.2289 0.2289
```

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```
## Call:
## VAR(y = data, p = 2)
##
##
## Estimation results for equation R.SP500:
## ========================================
## R.SP500 = R.SP500.l1 + R.SP400.l1 + R.SP600.l1 + R.SP500.l2 + R.SP400.l2 + R.SP600.l2 + const
##
##             Estimate Std. Error t value Pr(>|t|)
## R.SP500.l1 -0.2494751  0.1016901  -2.453   0.0143 *
## R.SP400.l1  0.2493178  0.1276772   1.953   0.0511 .
## R.SP600.l1 -0.0873447  0.0923683  -0.946   0.3446
## R.SP500.l2 -0.1438598  0.1013881  -1.419   0.1562
## R.SP400.l2  0.3234439  0.1256101   2.575   0.0102 *
## R.SP600.l2 -0.1392876  0.0903271  -1.542   0.1234
## const       0.0005611  0.0003067   1.829   0.0676 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.009675 on 996 degrees of freedom
## Multiple R-Squared: 0.01986, Adjusted R-squared: 0.01395
## F-statistic: 3.363 on 6 and 996 DF,  p-value: 0.002751
##
##
## Estimation results for equation R.SP400:
## ========================================
## R.SP400 = R.SP500.l1 + R.SP400.l1 + R.SP600.l1 + R.SP500.l2 + R.SP400.l2 + R.SP600.l2 + const
##
##             Estimate Std. Error t value Pr(>|t|)
## R.SP500.l1 -0.2466229  0.1229353  -2.006   0.0451 *
## R.SP400.l1  0.2495521  0.1543517   1.617   0.1062
## R.SP600.l1 -0.0837235  0.1116659  -0.750   0.4536
## R.SP500.l2 -0.1232594  0.1225702  -1.006   0.3148
## R.SP400.l2  0.3080651  0.1518527   2.029   0.0428 *
## R.SP600.l2 -0.1546628  0.1091983  -1.416   0.1570
## const       0.0005690  0.0003708   1.534   0.1252
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.0117 on 996 degrees of freedom
## Multiple R-Squared: 0.01114, Adjusted R-squared: 0.005181
## F-statistic:  1.87 on 6 and 996 DF,  p-value: 0.08302
##
##
## Estimation results for equation R.SP600:
## ========================================
## R.SP600 = R.SP500.l1 + R.SP400.l1 + R.SP600.l1 + R.SP500.l2 + R.SP400.l2 + R.SP600.l2 + const
##
##             Estimate Std. Error t value Pr(>|t|)
## R.SP500.l1 -0.2632594  0.1359985  -1.936   0.0532 .
## R.SP400.l1  0.2824000  0.1707532   1.654   0.0985 .
## R.SP600.l1 -0.1622995  0.1235317  -1.314   0.1892
```

```
## R.SP500.l2 -0.2267199  0.1355946  -1.672    0.0948 .
## R.SP400.l2  0.3951433  0.1679887   2.352    0.0189 *
## R.SP600.l2 -0.1630499  0.1208019  -1.350    0.1774
## const       0.0006788  0.0004102   1.655    0.0983 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01294 on 996 degrees of freedom
## Multiple R-Squared: 0.01941, Adjusted R-squared: 0.0135
## F-statistic: 3.286 on 6 and 996 DF,  p-value: 0.003312
##
##
##
## Covariance matrix of residuals:
##           R.SP500    R.SP400    R.SP600
## R.SP500 0.0000936 0.0001076 0.0001154
## R.SP400 0.0001076 0.0001368 0.0001463
## R.SP600 0.0001154 0.0001463 0.0001674
##
## Correlation matrix of residuals:
##         R.SP500 R.SP400 R.SP600
## R.SP500  1.0000  0.9507  0.9218
## R.SP400  0.9507  1.0000  0.9667
## R.SP600  0.9218  0.9667  1.0000
##
## Refitting the model by leaving out insignificant coefficients
restrict(SPvar2, thresh = 0.05)

##
## VAR Estimation Results:
## =======================
##
## Estimated coefficients for equation R.SP500:
## ============================================
## Call:
## R.SP500 = R.SP500.l1 + R.SP400.l1 + R.SP600.l1 + R.SP500.l2 + R.SP400.l2 + R.SP600.l2 + const
##
##    R.SP500.l1    R.SP400.l1    R.SP600.l1    R.SP500.l2    R.SP400.l2
## -0.2494750631  0.2493177528 -0.0873446981 -0.1438597770  0.3234439354
##    R.SP600.l2         const
## -0.1392876104  0.0005611218
##
##
## Estimated coefficients for equation R.SP400:
## ============================================
## Call:
## R.SP400 = R.SP500.l1 + R.SP400.l1 + R.SP600.l1 + R.SP500.l2 + R.SP400.l2 + R.SP600.l2 + const
##
##    R.SP500.l1    R.SP400.l1    R.SP600.l1    R.SP500.l2    R.SP400.l2
## -0.2466229374  0.2495520825 -0.0837235418 -0.1232594306  0.3080650674
##    R.SP600.l2         const
## -0.1546628399  0.0005689542
##
```

```
## 
## Estimated coefficients for equation R.SP600:
## ===========================================
## Call:
## R.SP600 = R.SP500.l1 + R.SP400.l1 + R.SP600.l1 + R.SP500.l2 + R.SP400.l2 + R.SP600.l2 + const
## 
##     R.SP500.l1     R.SP400.l1     R.SP600.l1     R.SP500.l2     R.SP400.l2
## -0.2632593696  0.2824000464 -0.1622995114 -0.2267199442  0.3951432540
##     R.SP600.l2          const
## -0.1630498826  0.0006788162
```

```
## 
## ACF of residuals
ACF.SPvar2 <- acf(residuals(SPvar2), na.action = na.pass, plot = F)
plot(ACF.SPvar2)
```
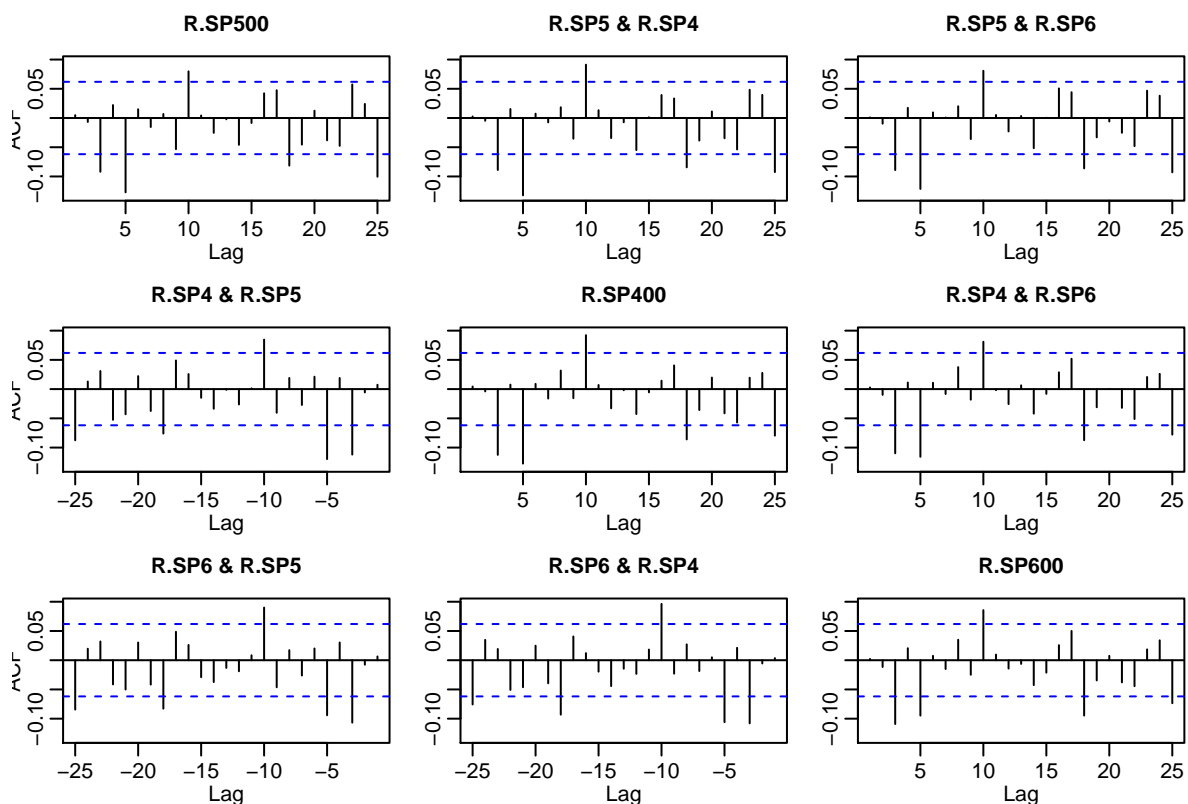


```
## 
## Portmanteau tests
serial.test(SPvar2)
```

```
## 
##  Portmanteau Test (asymptotic)
## 
## data:  Residuals of VAR object SPvar2
## Chi-squared = 166.02, df = 126, p-value = 0.009758
```

Note: The MLE of a VAR(p) model can be implemented by the `VARMA()` function in the `MTS` package.

3. LSE of Vector Autoregressive Model using S&P500, SH and SZ Data

```
data <- filter(select(index, R.SP500, R.SH, R.SZ), !is.na(R.SP500*R.SH))
```

```
var.bic <- VAR(data, lag.max = 3, ic = "SC")
summary(var.bic)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: R.SP500, R.SH, R.SZ
## Deterministic variables: const
## Sample size: 1004
## Log Likelihood: 9894.493
## Roots of the characteristic polynomial:
## 0.1581 0.09534 0.002063
## Call:
## VAR(y = data, lag.max = 3, ic = "SC")
##
##
## Estimation results for equation R.SP500:
## =======================================
## R.SP500 = R.SP500.l1 + R.SH.l1 + R.SZ.l1 + const
##
##              Estimate Std. Error t value Pr(>|t|)
## R.SP500.l1 -0.0717810  0.0316558  -2.268   0.0236 *
## R.SH.l1     0.0542360  0.0530986   1.021   0.3073
## R.SZ.l1    -0.0455520  0.0432490  -1.053   0.2925
## const       0.0005654  0.0003074   1.839   0.0662 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.009724 on 1000 degrees of freedom
## Multiple R-Squared: 0.00602, Adjusted R-squared: 0.003038
## F-statistic: 2.019 on 3 and 1000 DF,  p-value: 0.1096
##
##
## Estimation results for equation R.SH:
## ====================================
## R.SH = R.SP500.l1 + R.SH.l1 + R.SZ.l1 + const
##
##              Estimate Std. Error t value Pr(>|t|)
## R.SP500.l1  1.814e-01  3.576e-02   5.072 4.69e-07 ***
## R.SH.l1    -5.301e-03  5.997e-02  -0.088    0.930
## R.SZ.l1    -2.276e-02  4.885e-02  -0.466    0.641
## const       9.421e-05  3.472e-04   0.271    0.786
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01098 on 1000 degrees of freedom
## Multiple R-Squared: 0.02585, Adjusted R-squared: 0.02293
## F-statistic: 8.845 on 3 and 1000 DF,  p-value: 8.652e-06
```

```
##
##
## Estimation results for equation R.SZ:
## ===================================
## R.SZ = R.SP500.l1 + R.SH.l1 + R.SZ.l1 + const
##
##              Estimate Std. Error t value Pr(>|t|)
## R.SP500.l1  0.1883520  0.0438446   4.296 1.91e-05 ***
## R.SH.l1    -0.1628486  0.0735437  -2.214   0.0270 *
## R.SZ.l1     0.1377281  0.0599016   2.299   0.0217 *
## const       0.0000731  0.0004258   0.172   0.8637
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01347 on 1000 degrees of freedom
## Multiple R-Squared: 0.02243, Adjusted R-squared: 0.0195
## F-statistic: 7.648 on 3 and 1000 DF,  p-value: 4.674e-05
##
##
##
## Covariance matrix of residuals:
##            R.SP500       R.SH       R.SZ
## R.SP500 9.456e-05 8.568e-06 6.313e-06
## R.SH    8.568e-06 1.206e-04 1.262e-04
## R.SZ    6.313e-06 1.262e-04 1.814e-04
##
## Correlation matrix of residuals:
##          R.SP500    R.SH   R.SZ
## R.SP500 1.00000 0.08022 0.0482
## R.SH    0.08022 1.00000 0.8529
## R.SZ    0.04820 0.85293 1.0000
##
## Refitting the model by leaving out insignificant coefficients
restrict(var.bic, thresh = 0.05)

##
## VAR Estimation Results:
## =======================
##
## Estimated coefficients for equation R.SP500:
## ============================================
## Call:
## R.SP500 = R.SP500.l1 + R.SH.l1 + R.SZ.l1 + const
##
##     R.SP500.l1        R.SH.l1        R.SZ.l1          const
## -0.0717810384  0.0542360381 -0.0455519652  0.0005653507
##
##
## Estimated coefficients for equation R.SH:
## =========================================
## Call:
## R.SH = R.SP500.l1 + R.SH.l1 + R.SZ.l1 + const
##
```

```
##      R.SP500.l1        R.SH.l1        R.SZ.l1           const
##   0.1813551363 -0.0053007640 -0.0227577933  0.0000942059
##
##
## Estimated coefficients for equation R.SZ:
## =========================================
## Call:
## R.SZ = R.SP500.l1 + R.SH.l1 + R.SZ.l1 + const
##
##      R.SP500.l1        R.SH.l1        R.SZ.l1           const
##   1.883520e-01 -1.628486e-01  1.377281e-01  7.310059e-05
```

```
ACF.var.bic <- acf(residuals(var.bic), na.action = na.pass, plot = F)
plot(ACF.var.bic)
```
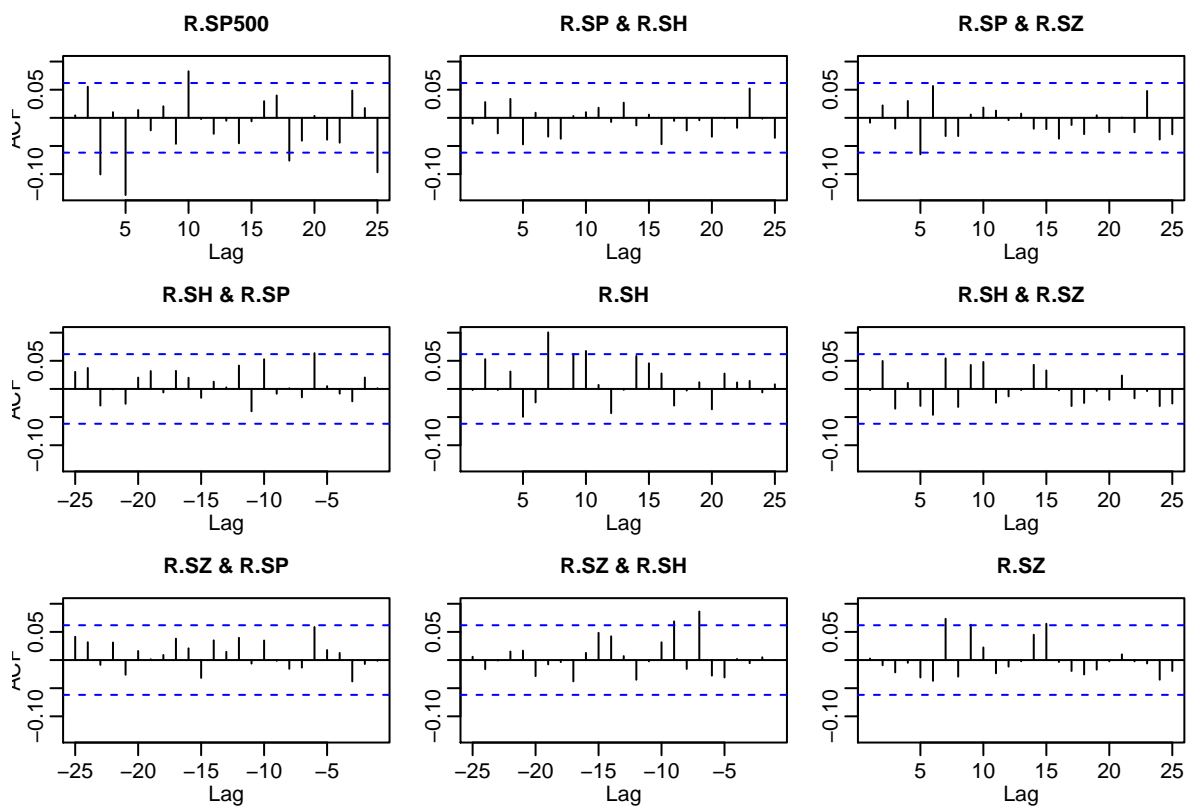


```
serial.test(var.bic)
```

```
##
##  Portmanteau Test (asymptotic)
##
## data:  Residuals of VAR object var.bic
## Chi-squared = 211.75, df = 135, p-value = 2.712e-05
```