Financial Econometrics

Lecture 1

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Outline

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One Period Returns

The *return* for an asset (e.g., stock, bond, etc) is a percentage defined as the change of the asset price as a fraction of its initial price. Returns exhibit more attractive statistical properties than prices. Financial econometrics typically models and analyzes return data rather than price series.

There are various definitions for the asset returns: Let P_t denote the price of an asset at time t.

- From time $t-1$ to t, assuming no dividend paid, the *one-period simple return* is defined as
$$R_t = (P_t - P_{t-1})/P_{t-1}$$
- The *one-period gross return*: $P_t/P_{t-1} = R_t + 1$.
- The *one-period log return*: $r_t = \log(P_t/P_{t-1}) = \log(R_t + 1)$. Note that when R_t is small, $r_t \approx R_t$, though theoretically $r_t \leq R_t$.

Multiperiod Returns

Consider multiperiod returns.

- the *k-period simple return* from time $t-k$ to t can be similarly defined as

$$R_t\left(k\right) = \left(P_t - P_{t-k}\right)/P_{t-k}$$

- The *k-period gross return*: $P_t/P_{t-k} = R_t(k) + 1$.
- The *k-period log return*: $r_t(k) = \log(P_t/P_{t-k}) = \log(R_t(k) + 1)$.
- $r_t(k)$ is the sum of the k one-period log returns

$$\begin{aligned} r_t\left(k\right) &= \log(R_t\left(k\right) + 1) \\ &= \log[(R_t + 1)(R_{t-1} + 1)\cdots(R_{t-k+1} + 1)] \\ &= \log(R_t + 1) + \log(R_{t-1} + 1) + \cdots + \log(R_{t-k+1} + 1) \\ &= r_t + r_{t-1} + \cdots + r_{t-k+1} \end{aligned}$$

In this class, returns refer to log returns unless specified otherwise.

Adjustment for Dividends

Let D_t denote the dividend payment between time $t-1$ and t. Assume that no dividends are re-invested in the asset.

$$R_t\left(k\right)=\frac{P_t+D_t+\cdots+D_{t-k+1}}{P_{t-k}}-1$$

$$r_t\left(k\right)=\sum_{j=0}^{k-1}\log\left(\frac{P_{t-j}+D_{t-j}}{P_{t-j-1}}\right)=r_t+\cdots+r_{t-k+1}$$

Continuously Compounded Returns and Bond Yields

r_t is also called *continuously compounded return*: If a simple annual interest rate r for a bank deposit is fixed and earnings are equally paid m times per year. Then the gross return at the end of one year is

$$(1 + r/m)^m \to e^r \text{ as } m \to \infty$$

i.e., for continuously compounded interest, r = the annual log return.

Bonds are quoted in annualized yields. Consider a zero-coupon bond with the face value 1, the current yield is r_t and the remaining duration is D. Its current price B_t satisfies $B_t \exp(Dr_t) = 1$ and so the annualized log return of holding the bond (ignoring that the remaining duration for B_{t+1} is $D - 1$) is

$$\log(B_{t+1}/B_t) = D(r_t - r_{t+1})$$

Outline

Behavior of Financial Return Data: Stationarity and Volatility Clustering

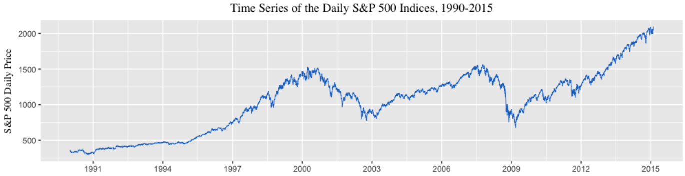
Stationarity

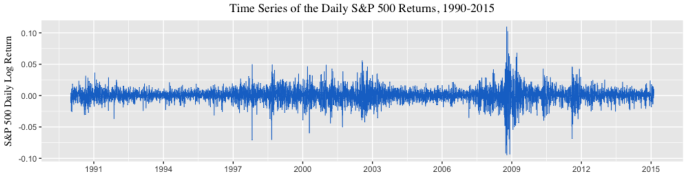
- Asset prices are often not stationary over time (e.g., financial crisis, economic growth/recession, technology improvement, etc).
- Most return sequences show certain level of stationarity (i.e., time invariant first two moments).
- The term *stationarity* will be formally defined in next lecture.

Volatility Clustering

- The volatility of asset returns is time varying.
- Large price changes (i.e., absolute returns) often occur in clusters.

Behavior of Financial Return Data: Stationarity and Volatility Clustering





Behavior of Financial Return Data: Heavy tails

Heavy tails

- The distribution of r_t often exhibits "heavier-than-normal" tails.
- A random variable X has "heavier tails" than Z if for sufficiently large $M \in \mathbb{R}_+$, $P(|X| > M) > P(|Z| > M)$.
- Example: t-distribution (with degree of freedom ν) whose PDF is

$$f_v(x) \propto (1 + x^2/\nu)^{-(\nu+1)/2}$$

with tails of the order $|x|^{-(\nu+1)}(\asymp |x|^{-(\nu+1)})$. $N(0,1)$ has tails of order $exp(-x^2/2)$. It is not hard to see that $\lim_{x\to\infty} \exp(x^2/2)/|x|^{\nu+1} = \infty$.

- Tail behavior is closely related to the existence of $E(|r_t|^p)$. For $X \sim N(0, \sigma^2)$, $E(|X|^p) < \infty$ for all $p \in \mathbb{R}_+$, but if $X \sim t_\nu$, $E(|X|^p) < \infty$ only for $p \in (0, \nu)$. r_t is typically assumed to have at least $E(r_t^2) < \infty$.

Behavior of Financial Return Data: Asymmetry and Aggregational Gaussianity

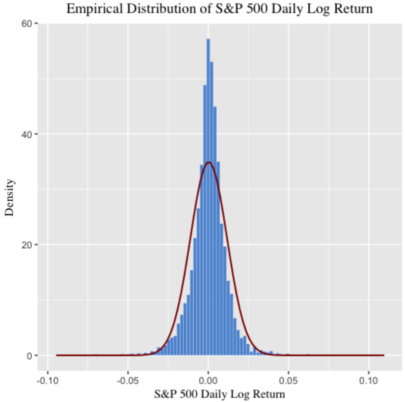
Asymmetry

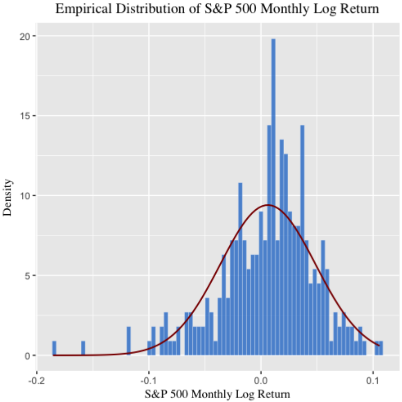
- The distribution of r_t is often negatively skewed (long left tails).
- Risk aversion: the market tends to react more strongly to negative shocks than positive ones.

Aggregational Gaussianity

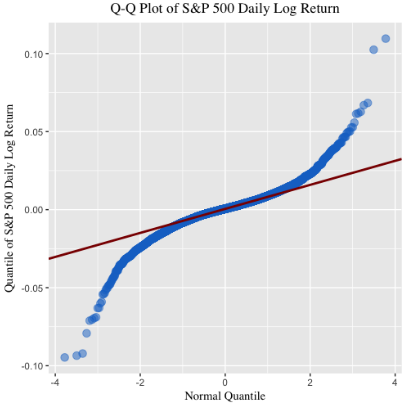
- Recall $r_t(k) = r_t + r_{t-1} + \cdots + r_{t-k+1}$.
- As k increases (long time horizon), the distribution of $r_t(k)$ tends to a normal distribution (think the central limit theorem).

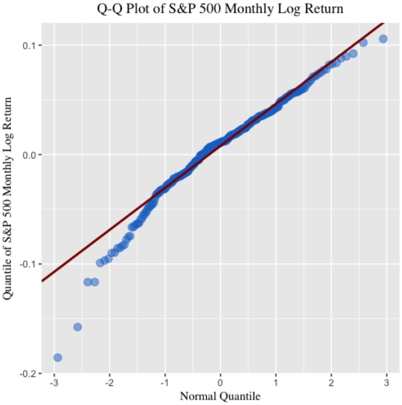
Behavior of Financial Return Data: Heavy tails, Asymmetry and Aggregational Gaussianity





Behavior of Financial Return Data: Heavy tails, Asymmetry and Aggregational Gaussiananity





Behavior of Financial Return Data: Long Range Dependence

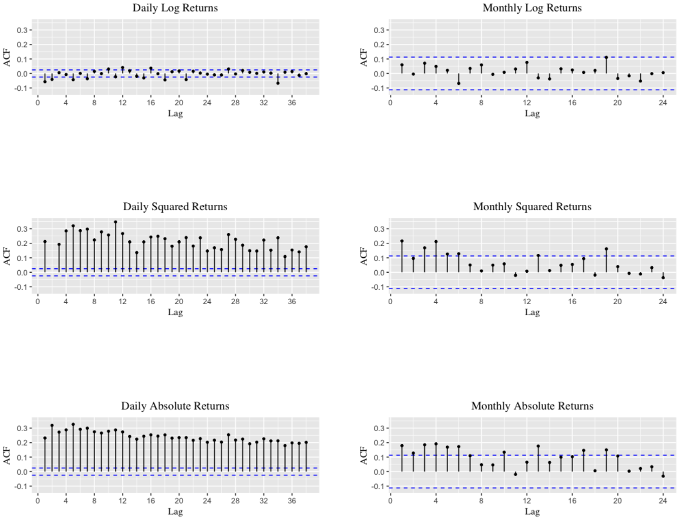
Long Range Dependence

- The returns are almost serially uncorrelated, but not independent!
- Use the sample *autocorrelation function* (ACF) $\widehat{\rho}_k$ to measure the serial correlation of a return series $\{r_t\}_{t=1}^{T}$. Define $\widehat{\rho}_k = \widehat{\gamma}_k / \widehat{\gamma}_0$ where

$$\widehat{\gamma}_k = \frac{1}{T} \sum_{t=1}^{T-k} (r_t - \bar{r})(r_{t+k} - \bar{r})$$

  is the sample *autocovariance* at lag k and $\bar{r} = \sum_{t=1}^{T} r_t / T$.
- $\{r_t^2\}_{t=1}^{T}$ and $\{|r_t|\}_{t=1}^{T}$ (*long memory*) exhibit significant (95% confidence interval for $H_0 : \rho_k = 0$) ACF $\Rightarrow$ nonlinear dependencies over time.
- ρ_k's become weaker and less persistent for longer sampling intervals.

Behavior of Financial Return Data: Long Range Dependence



Behavior of Financial Return Data: Leverage Effect

Leverage Effect

- Assets returns are often negatively correlated with the changes of their volatilities.
- Prices down $\Rightarrow$ firms more leveraged (riskier) $\Rightarrow$ more volatile prices.
- Volatility up $\Rightarrow$ investors demand risk premiums $\Rightarrow$ prices down.
- Risk aversion: volatilities caused by price decline are typically larger than the appreciations due to declined volatilities.

Behavior of Financial Return Data: Leverage Effect