R Code for Lecture 7

```
library(latex2exp)
library(TSA)
library(readr)
library(dplyr)
library(ggplot2)
```

Part 1 Fourth Moments of X_t

Consider the following ARCH(1) model:

$$X_t = \sigma_t \epsilon_t, \epsilon_t \sim \text{IID } N(0,1)$$

$$\sigma_t^2 = 0.1 + 0.9 X_{t-1}^2$$

```
Xt <- garch.sim(alpha = c(0.1, 0.9), n = 1000)
```

1. Fitting $\{X_t^2\}$ based on its ARMA representation:

$$X_t^2 = 0.1 + 0.9 X_{t-1}^2 + \eta_t, \eta_t \sim \text{WN}$$

```
arch1.arma <- arima(Xt^2, order = c(1,0,0), method = c("CSS"))
arch1.arma
```

```
##
## Call:
## arima(x = Xt^2, order = c(1, 0, 0), method = c("CSS"))
##
## Coefficients:
##          ar1  intercept
##       0.2255     0.5342
## s.e.  0.0308     0.1498
##
## sigma^2 estimated as 13.46:  log likelihood = -2718.72,  aic = NA
```

2. Fitting $\{X_t\}$ using conditional maximum likelihood method

```
arch1.mle <- garch(Xt, order = c(0,1), trace = F)
summary(arch1.mle)
```

```
##
## Call:
## garch(x = Xt, order = c(0, 1), trace = F)
##
## Model:
## GARCH(0,1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.61152 -0.64271  0.03336  0.71700  3.82660
```

```
##
## Coefficient(s):
##     Estimate  Std. Error  t value Pr(>|t|)
## a0  0.111051    0.008136    13.65   <2e-16 ***
## a1  0.764737    0.064528    11.85   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Diagnostic Tests:
##  Jarque Bera Test
##
## data:  Residuals
## X-squared = 1.4975, df = 2, p-value = 0.473
##
##
##  Box-Ljung test
##
## data:  Squared.Residuals
## X-squared = 0.15577, df = 1, p-value = 0.6931
```

Part 2 Monte Carlo Experiment for GARCH(1,1)

1. Estimating GARCH(1,1) model:

$$X_t = \sigma_t \epsilon_t, \epsilon_t \sim \text{IID } N(0,1)$$
$$\sigma_t^2 = 0.1 + 0.25 X_{t-1}^2 + 0.25 \sigma_{t-1}^2$$

```
data <- garch.sim(alpha = c(0.1, 0.25), beta = 0.25, n = 2500)
garch11 <- garch(data, order = c(1,1), trace = F)
summary(garch11)
```

```
##
## Call:
## garch(x = data, order = c(1, 1), trace = F)
##
## Model:
## GARCH(1,1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.93866 -0.66417 -0.01316  0.66059  3.66183
##
## Coefficient(s):
##     Estimate  Std. Error  t value Pr(>|t|)
## a0   0.12354     0.01800    6.862 6.77e-12 ***
## a1   0.22314     0.03214    6.944 3.82e-12 ***
## b1   0.10283     0.10195    1.009    0.313
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Diagnostic Tests:
##  Jarque Bera Test
```

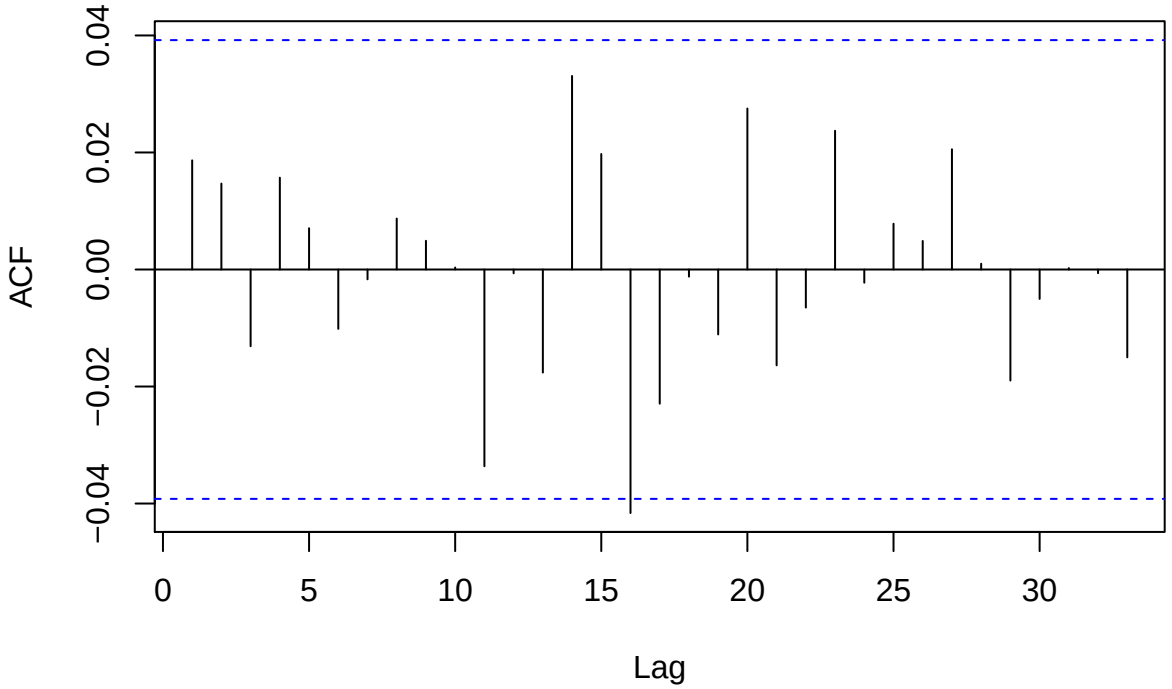
```
##
## data:  Residuals
## X-squared = 5.1328, df = 2, p-value = 0.07681
##
##
##  Box-Ljung test
##
## data:  Squared.Residuals
## X-squared = 0.008622, df = 1, p-value = 0.926
```

2. Model diagnostics via graphical plots.

```
e <- garch11$residuals
esq <- e^2
```

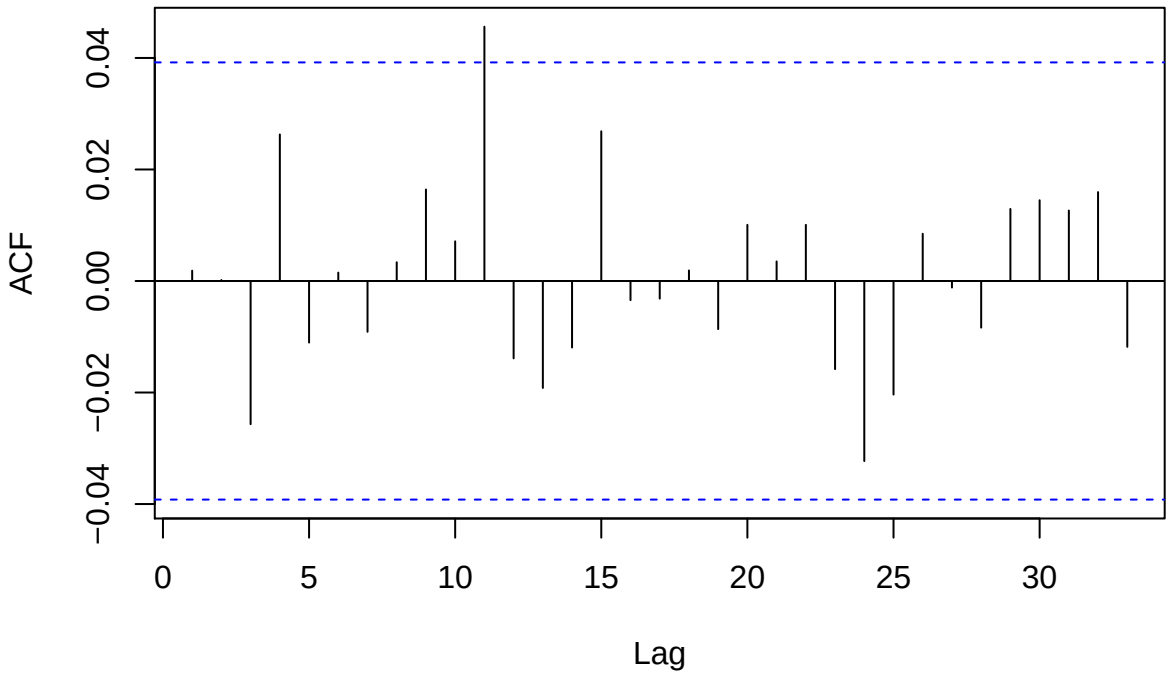
ACF of $\widehat{\epsilon}_t$

```
acf(e, na.action = na.pass, main = "")
```



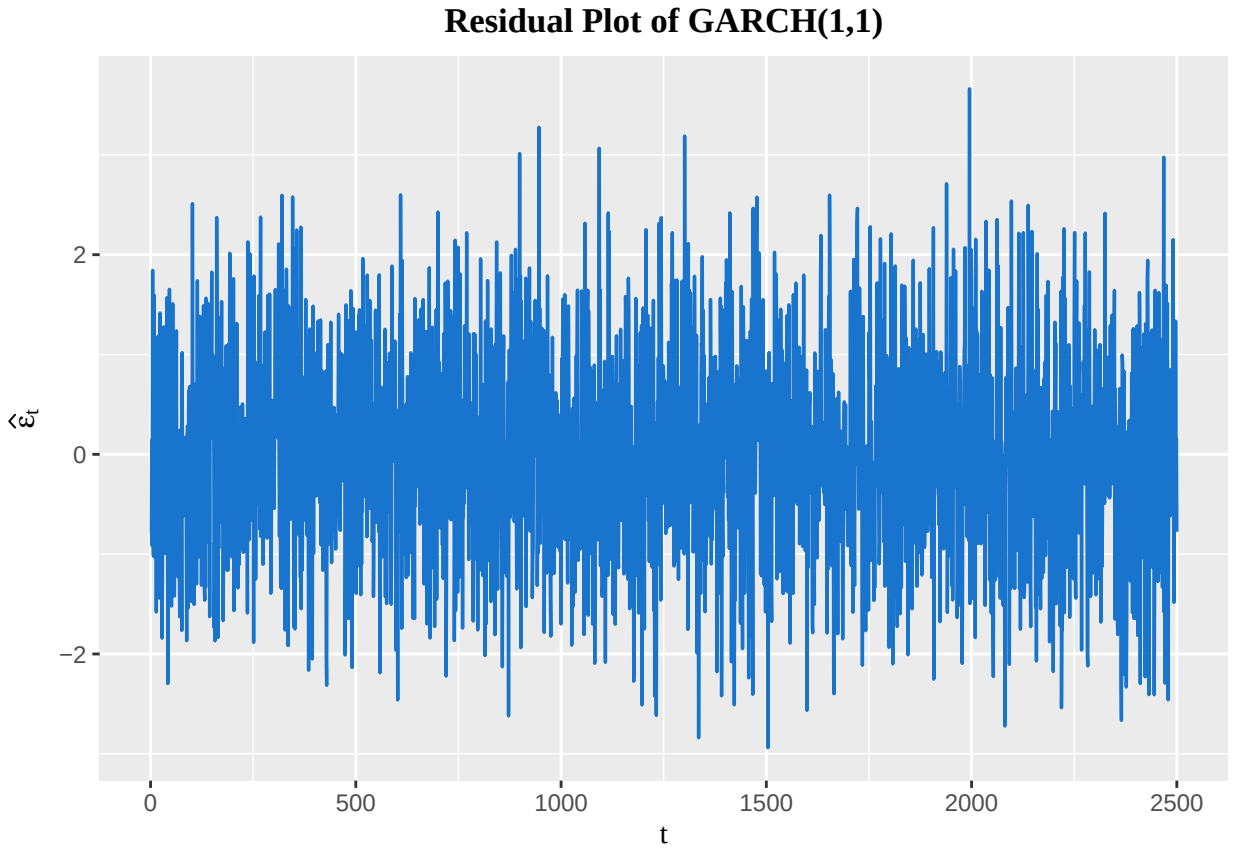
ACF of $\widehat{\epsilon}_t^2$

```
acf(esq, na.action = na.pass, main = "")
```



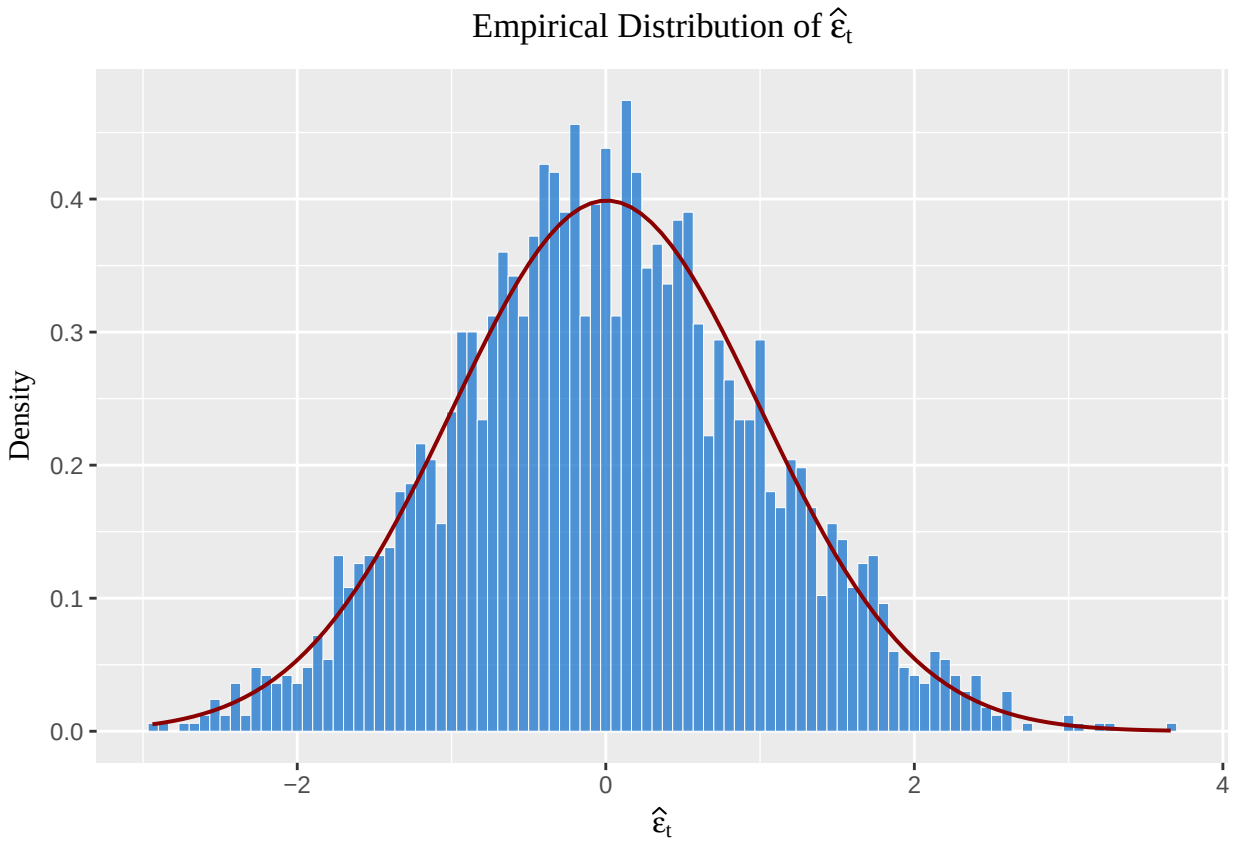
Time series of $\widehat{\epsilon}_t$

```
res <- data.frame(time = c(1:length(e)), et = e, et2 = esq)
fig11 <- ggplot(res, aes(time)) +
  geom_line(aes(y = et), size = 0.6, color = "dodgerblue3") +
  labs(title = "Residual Plot of GARCH(1,1)",
       x = TeX("$t$"), y = TeX("$\\widehat{\\epsilon}_t$")) +
  theme(axis.title = element_text(family = "serif"),
        plot.title = element_text(hjust = 0.5, family = "serif", face = "bold"))
fig11
```



Empirical distribution of $\widehat{\epsilon}_t$

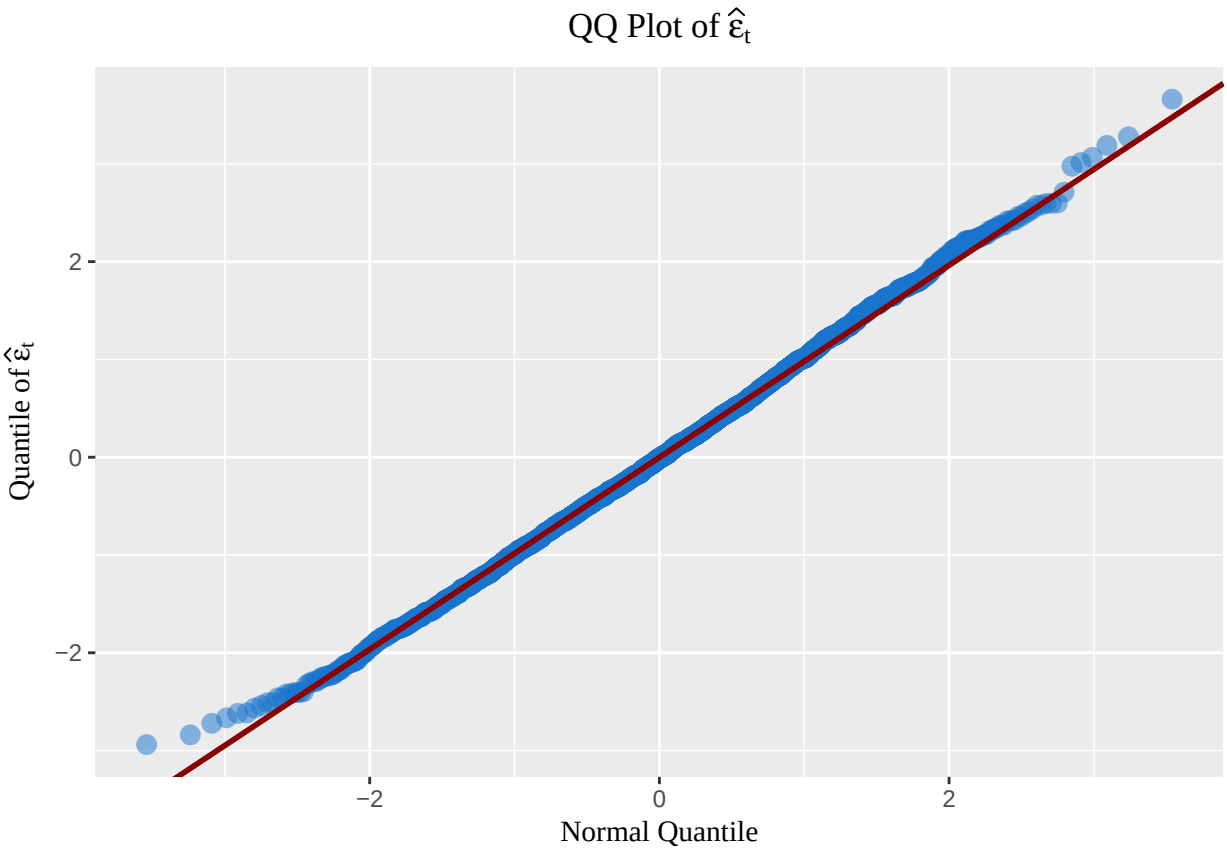
```
fig12 <- ggplot(res, aes(et)) +
  geom_histogram(aes(y = ..density..), bins = 100, color="white",
                 fill="dodgerblue3",
                 alpha = 0.75, size = 0.1) +
  stat_function(fun = dnorm, color = "darkred", size = 0.7,
                args = list(mean = mean(res$et,
                                        na.rm = T),
                            sd = sd(res$et,
                                    na.rm = T))) +
  labs(title = TeX("Empirical Distribution of $\\widehat{\\epsilon}_t$"),
       x = TeX("$\\widehat{\\epsilon}_t$"), y = "Density") +
  theme(axis.title = element_text(family = "serif"),
        plot.title = element_text(hjust = 0.5, family = "serif"))
fig12
```



QQ-plot of $\widehat{\epsilon}_t$ against $N(0,1)$

```
y <- quantile(res$et, c(0.25, 0.75), na.rm = T)
x <- qnorm(c(0.25, 0.75))
slope <- diff(y)/diff(x)
int   <- y[1] - slope*x[1]

fig13 <- ggplot(res, aes(sample = et)) +
  geom_qq(alpha = 0.5, size = 3, color = "dodgerblue3") +
  geom_abline(intercept=int, slope=slope, color = "darkred", size = 1) +
  labs(title = TeX("QQ Plot of $\\widehat{\\epsilon}_t$"),
       x = "Normal Quantile", y = TeX("Quantile of $\\widehat{\\epsilon}_t$")) +
  theme(axis.title = element_text(family = "serif"),
        plot.title = element_text(hjust = 0.5, family = "serif"))
fig13
```



Part 3 Application of GARCH Modeling

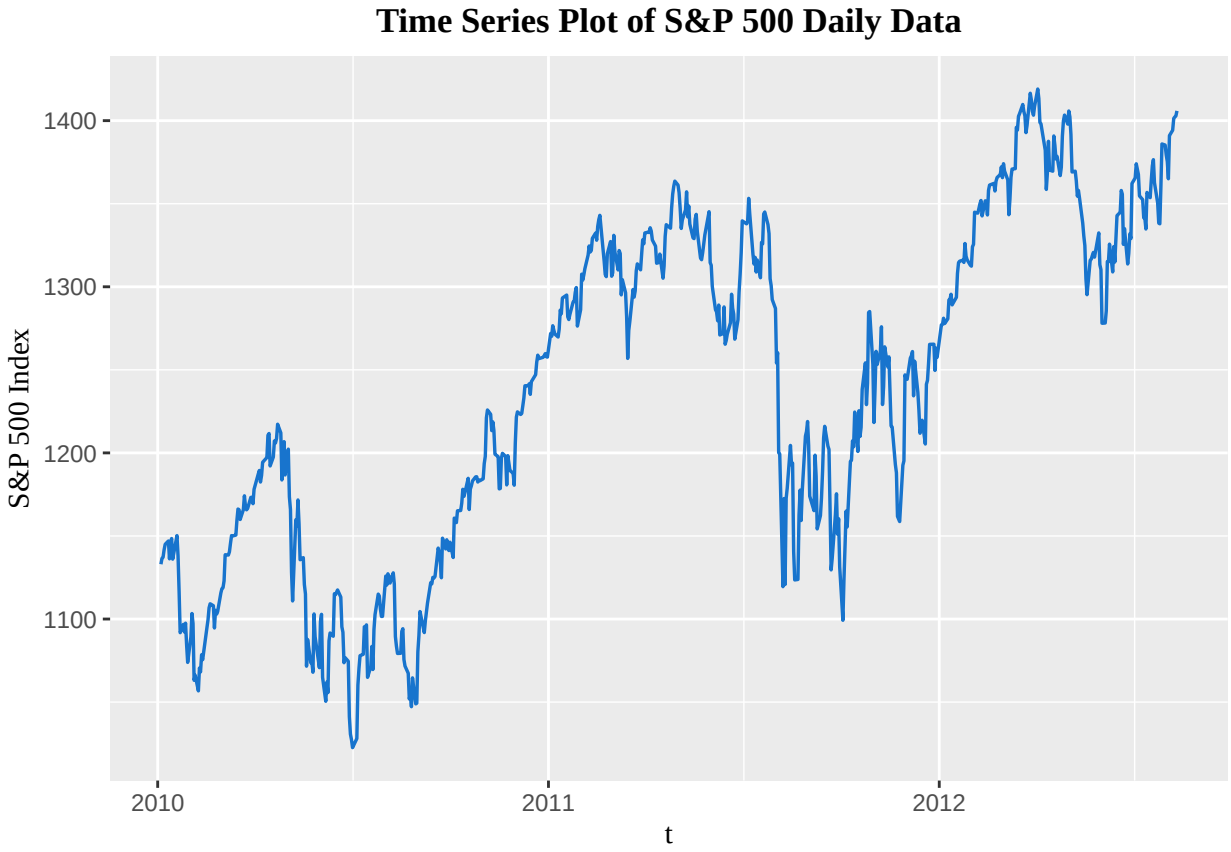
1. Loading S&P 500 data (daily return, Jan 4, 2010 - Aug 10, 2012)

```
setwd("/Users/ouyangfu/Dropbox/Teaching/financial econometrics/2018/data")
sp500d <- read_csv("sp500day.csv", col_names = T) %>%
  select(Date, `Adj Close`) %>%
  rename(index = `Adj Close`) %>%
  mutate(lindex = lag(index, n = 1L), ret = (index-lindex)/lindex)

sp500d$Date <- as.Date(strptime(as.character(sp500d$Date), "%m/%d/%Y"))
sp500d <- arrange(sp500d, Date)
sp500d <- filter(sp500d, (Date >= as.Date("2010-01-04")) &
                   (Date <= as.Date("2012-08-10")))
```

2. Time series plot

```
fig21 <- ggplot(sp500d, aes(Date)) +
  geom_line(aes(y = index), size = 0.6, color = "dodgerblue3") +
  labs(title = "Time Series Plot of S&P 500 Daily Data",
       x = TeX("$t$"), y = "S&P 500 Index") +
  theme(axis.title = element_text(family = "serif"),
        plot.title = element_text(hjust = 0.5, family = "serif", face = "bold"))
fig21
```



3. Fitting data with GARCH(1,1) model

```
garch11.sp500d <- garch(sp500d$ret, order = c(1,1), trace = F)
summary(garch11.sp500d)
```

```
##
## Call:
## garch(x = sp500d$ret, order = c(1, 1), trace = F)
##
## Model:
## GARCH(1,1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.72795 -0.60974 -0.06616  0.47359  4.10755
##
## Coefficient(s):
##     Estimate  Std. Error  t value Pr(>|t|)
## a0 3.238e-06   1.137e-06    2.847  0.00441 **
## a1 1.162e-01   1.777e-02    6.540 6.17e-11 ***
## b1 8.634e-01   2.174e-02   39.718  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Diagnostic Tests:
##  Jarque Bera Test
##
```

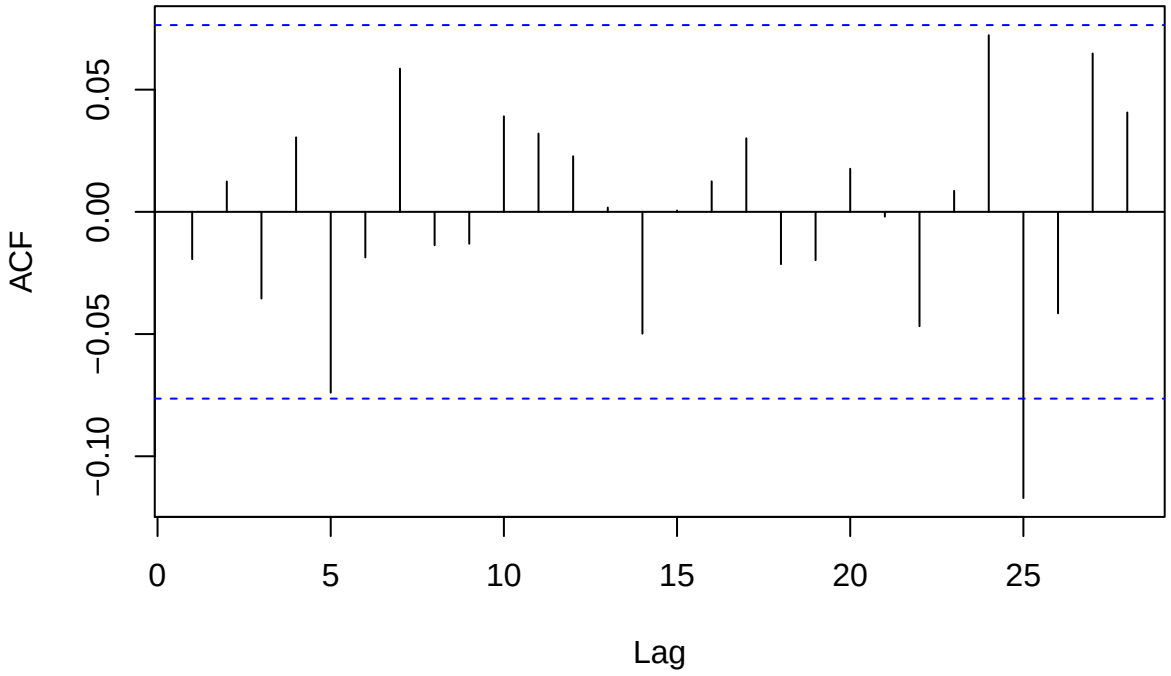
```
## data:  Residuals
## X-squared = 59.012, df = 2, p-value = 1.533e-13
##
##
##  Box-Ljung test
##
## data:  Squared.Residuals
## X-squared = 8.7513, df = 1, p-value = 0.003094
```

4. Model diagnostics

```
e <- garch11.sp500d$residuals
esq <- e^2
eabs <- abs(e)
```

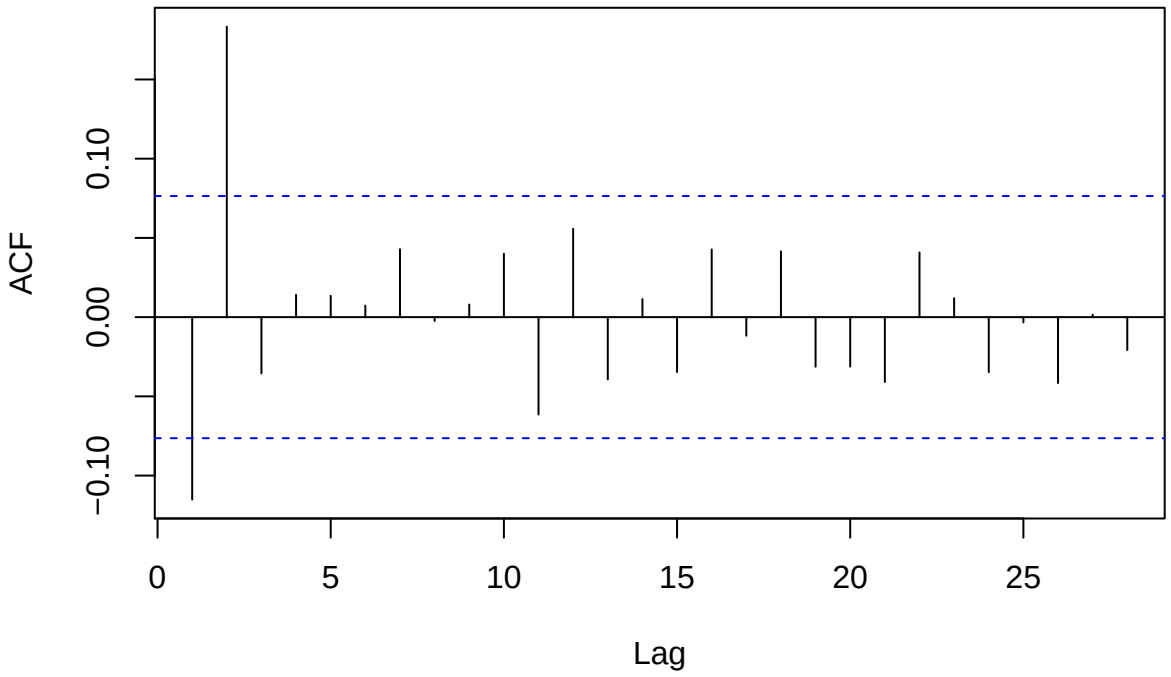
ACF of $\widehat{\epsilon}_t$

```
acf(e, na.action = na.pass, main = "")
```



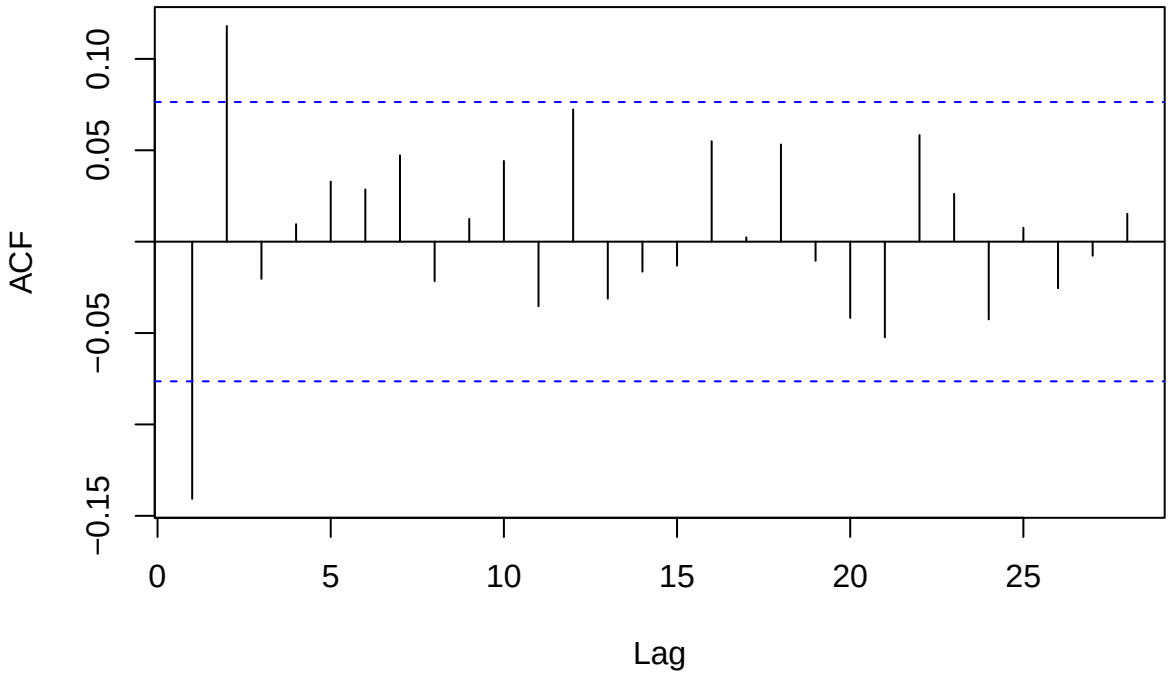
ACF of $\widehat{\epsilon}_t^2$

```
acf(esq, na.action = na.pass, main = "")
```



ACF of $|\widehat{\epsilon}_t|$

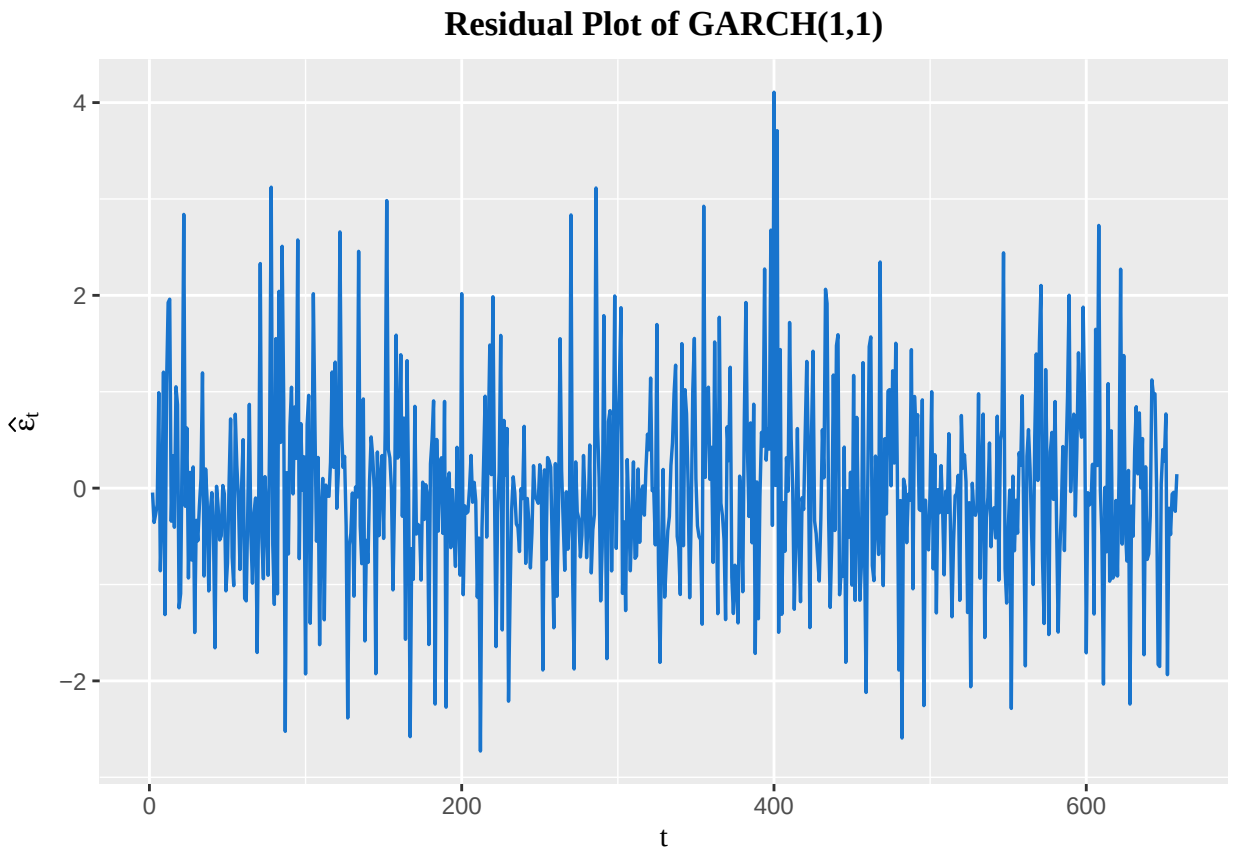
```
acf(eabs, na.action = na.pass, main = "")
```



Time series of $\widehat{\epsilon}_t$

```
res <- data.frame(time = c(1:length(e)), et = e, et2 = esq)
fig31 <- ggplot(res, aes(time)) +
  geom_line(aes(y = et), size = 0.6, color = "dodgerblue3") +
  labs(title = "Residual Plot of GARCH(1,1)",
       x = TeX("$t$"), y = TeX("$\\widehat{\\epsilon}_t$")) +
  theme(axis.title = element_text(family = "serif"),
```

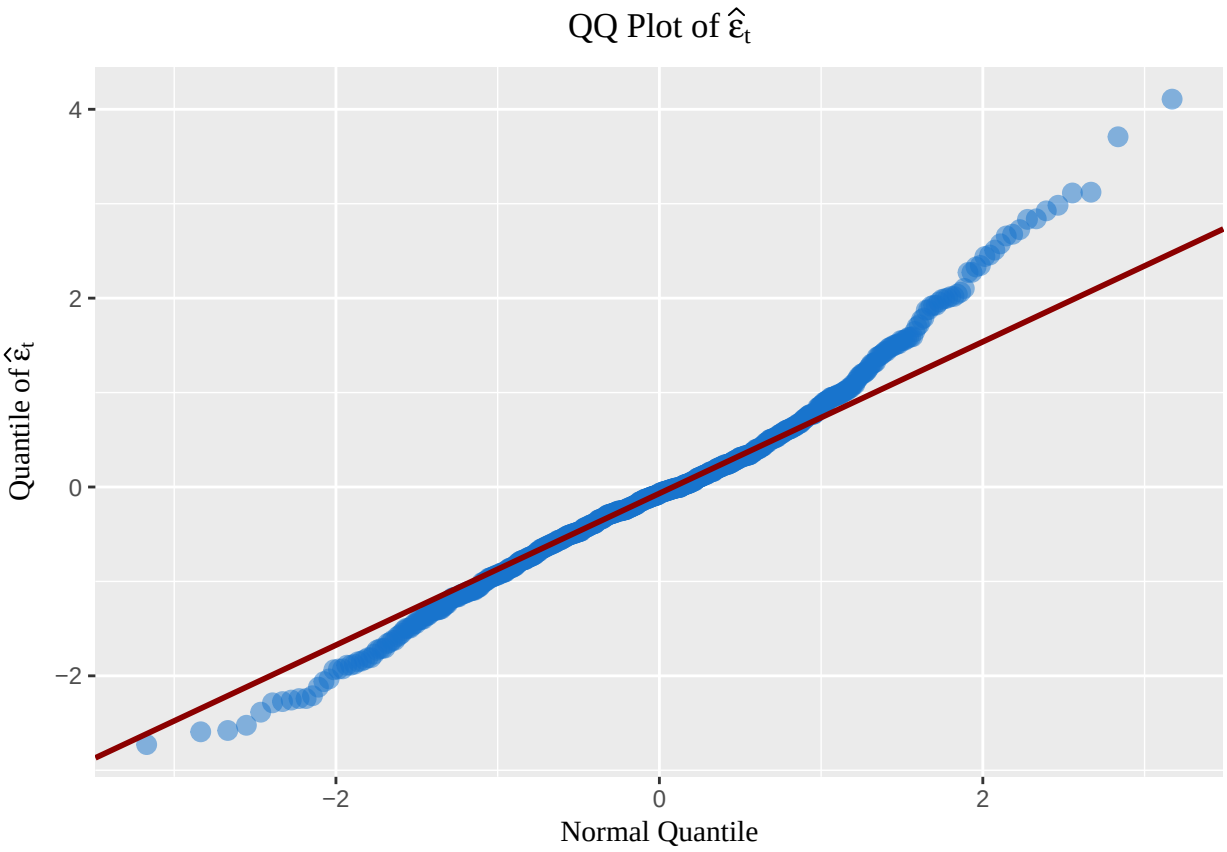
```
        plot.title = element_text(hjust = 0.5, family = "serif", face = "bold"))
fig31
```



QQ-plot of $\widehat{\epsilon}_t$ against $N(0,1)$

```
y <- quantile(res$et, c(0.25, 0.75), na.rm = T)
x <- qnorm(c(0.25, 0.75))
slope <- diff(y)/diff(x)
int   <- y[1] - slope*x[1]

fig32 <- ggplot(res, aes(sample = et)) +
  geom_qq(alpha = 0.5, size = 3, color = "dodgerblue3") +
  geom_abline(intercept=int, slope=slope, color = "darkred", size = 1) +
  labs(title = TeX("QQ Plot of $\\widehat{\\epsilon}_t$"),
       x = "Normal Quantile", y = TeX("Quantile of $\\widehat{\\epsilon}_t$")) +
  theme(axis.title = element_text(family = "serif"),
        plot.title = element_text(hjust = 0.5, family = "serif"))
fig32
```


QQ Plot of $\hat{\varepsilon}_t$
Quantile of $\hat{\varepsilon}_t$
Normal Quantile
4
2
0
−2
−2
0
2