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Regime Shifts: Implications for Dynamic Strategies

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Regime shifts present significant challenges for investors because they cause performance to depart significantly from the ranges implied by long-term averages of means and covariances. But regime shifts also present opportunities for gain. The authors show how to apply Markov-switching models to forecast regimes in market turbulence, inflation, and economic growth. They found that a dynamic process outperformed static asset allocation in backtests, especially for investors who seek to avoid large losses.

Investors have long recognized that economic conditions frequently undergo regime shifts. The economy often oscillates between a steady, low-volatility state characterized by economic growth and a panic-driven, high-volatility state characterized by economic contraction. Evidence of such regimes has been documented in market turbulence,[1] inflation,[2] and GDP or GNP.[3]

Regime shifts present significant challenges for risk management and portfolio construction. For example, Ang and Bekaert (2002) demonstrated that regimes in financial assets lead to high downside correlations. Indeed, prior studies have shown that exposure to different countries' equity markets offers less diversification in down markets than in up markets.[4] The same is true for global industry returns (Ferreira and Gama 2004), individual stock returns (Ang and Chen 2002), hedge fund returns (Van Royen 2002), and international bond market returns (Cappiello, Engle, and Sheppard 2003). Recently, Chua, Kritzman, and Page (2009) conducted an extensive empirical study of conditional correlations and concluded diversification based on unconditional covariances is largely a myth.

Given this widespread evidence of nonstationarity, it may be preferable to manage risk and construct portfolios on the basis of regime-specific estimates of the relevant inputs. The goal of our research was to build regime-dependent investment strategies and backtest their performance out of sample. Many researchers have used Markov-switching models to "fit" a dataset and uncover evidence of regimes in sample, but far fewer have attempted out-of-sample forecasting. Nevertheless, as suggested by Harvey and Dahlquist (2001), if economic conditions are persistent and strongly linked to asset performance, then a dynamic asset allocation process should add value over static weights.

We built a simple regime-switching model for the following variables (described in detail in the "Defining Economic Regime Variables" section):

- Market turbulence
- Inflation
- Economic growth

For each of these variables, we measured the performance of a variety of assets and risk premiums under each regime.

Next, we turned to out-of-sample forecasting and backtesting. First, we tested the performance of the regime-switching approach for tactical asset allocation. We used regime forecasts to scale exposure to specific risk premiums over time and found that a dynamic process outperformed constant exposures. We then applied the same methodology to dynamic asset allocation across stocks, bonds, and cash. Again, we found that a dynamic process outperformed static asset allocation, especially for investors who seek to avoid large losses.

The Impact of Regimes on Investment Strategies

A significant body of research has sought to manage and even exploit the regime dependence of asset

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classes and investment strategies. Kritzman and Li (2010) presented a static solution to nonstationarity by designing event-sensitive portfolios. These portfolios are resilient to specific economic environments—such as financial turbulence—yet perform reasonably well on balance across all market conditions. Clarke and de Silva (1998) showed that in a world with more than one economic regime, an expanded opportunity set exists for investors who use regime-specific expected return and risk compared with those who maintain a static asset mix, but they did not prescribe a way for investors to anticipate these regimes in practice. Nevertheless, some evidence suggests that regimes may be predictable using Markov-switching models, as introduced by Hamilton (1989). Ang and Bekaert (2004) proposed a regime-switching model for country allocation based on modeling changes in the systematic risk of each country. They found that using a two-state Markov-switching model to estimate returns and covariances significantly improved the risk-adjusted return of optimized international equity portfolios. Guidolin and Timmermann (2006) used a four-state Markov-switching model to explain the joint returns of stocks and bonds and concluded that these regimes have an intuitive interpretation. They found predictive capacity in using a vector autoregressive forecasting model based on prior returns and dividend yields.

Our approach differs from these studies in that we did not rely on a specific asset-pricing model nor did we model regimes in returns directly. Rather, we extended the Kritzman and Li (2010) approach by using Markov-switching models to reallocate dynamically across event-sensitive portfolios. To avoid overfitting, we did not model regimes in asset returns, which is notoriously difficult and backward looking. Instead, we developed a forward-looking methodology that marries economic forecasting with portfolio construction. We forecasted regimes in the important drivers of asset returns (market turbulence and economic conditions) and then reallocated assets accordingly. In doing so, we built portfolios based on current economic conditions, as opposed to past returns.

Our research extends the existing literature on regime-dependent asset allocation in three ways. First, we provide an intuitive description of regime-switching models and compare the approach with a simple alternative of using thresholds to identify regimes. We also include a practitioner-friendly overview of the algorithm used to estimate Markov-switching models (in Appendix A) and a sample MATLAB code (in Appendix B), both of which allow for the technique to be used in a transparent fashion. Second, we use economic variables to estimate regimes for use in asset allocation, as opposed to modeling regimes directly on historical returns. Third, we analyze out-of-sample performance for dynamic strategies comprising a wide variety of risk factors and asset classes.

A Simple Introduction to Markov-Switching Models

We will begin by describing a basic Markov-switching model and why such a model is often superior to a naive approach to identifying regimes. Let us start with an example from outside of finance. Imagine that someone wears a heart rate monitor and you receive this person's heart rate data at one-minute intervals. While the person is sleeping, you observe a low average heart rate with low volatility. Then you notice a sudden rise in the average level of the heart rate and its volatility. The person is awake. Without seeing the person, you can reasonably conclude which "state" the person is in: asleep or awake. The heart rate data follow a Markov process; at any point in time, a "state" or "regime" (asleep or awake) generates observations (heart rates) from a specific distribution. The regimes change over time—hence the term "Markov-switching" process.[5]

Laverty, Miket, and Kelly (2002) provided a simple illustration of a Markov-switching process via simulation. The initial probability of being in regime i is given by

$$\Pr\left(X_1 = i\right) = p_i,$$

where X_1 is the first regime in the Markov chain. The transition probability matrix is denoted $\mathbf{\Gamma}$. Within the transition probability matrix, the elements γ_{ij} denote the probability of a transition into regime j from regime i, as follows:

$$\mathbf{\Gamma} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix},$$

and

$$\gamma_{ij} = \Pr\left(X_t = j \middle| X_{t-1} = i\right),$$

where t denotes time. Therefore, over time, the Markov chain is in either regime $X_t = 1$ or $X_t = 2$. Each regime generates observations Y_t that are consistent with a given distribution π_i. For a discrete distribution (with a finite set of possible values for Y_t), the equation

$$\pi_i = \Pr\left(Y_t = s \middle| X_t = i\right)$$

indicates that the current regime, X_t, dictates the probability that Y_t will take a specific value s. The distribution, π_i, may take many forms. For example, we could assume that Y_t is normally distributed with regime-specific mean μ_i and standard deviation σ_i and recast the preceding equation as

$$Y_t \sim N(\mu_i, \sigma_i),$$

where $N(\cdot)$ represents the normal distribution probability density function.

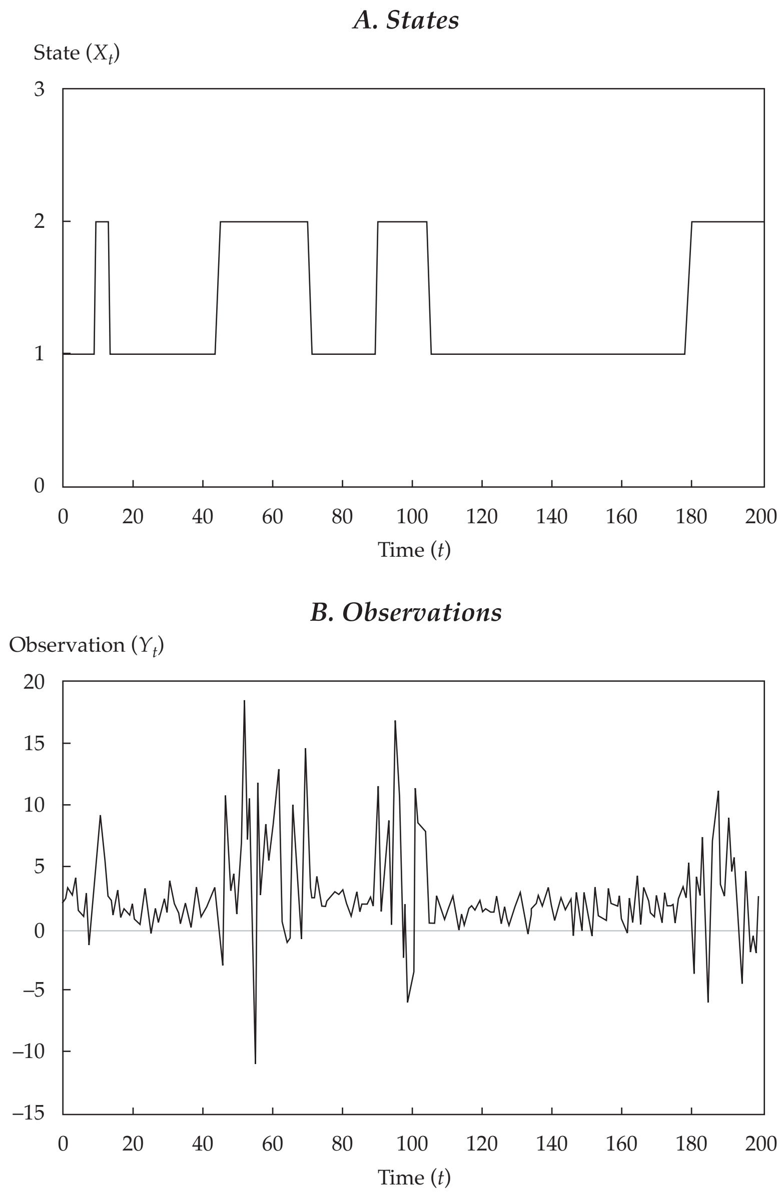
To show some of the advantages of Markov-switching models, we built a simple "toy" model with known parameters and simulated sample observations for Y_t. We made the following assumptions for this experiment:

1. In Regime 1, Y_t is normally distributed with a mean of 2 and a sigma of 1.
2. In Regime 2, Y_t is normally distributed with a mean of 4 and a sigma of 6.
3. The initial probability, p_i, of being in Regime 1 is 0.7.
4. Regime shifts are determined by the following transition probability matrix:

$$\mathbf{\Gamma} = \begin{pmatrix} 0.95 & 0.05 \\ 0.02 & 0.98 \end{pmatrix}.$$

Figure 1 shows the states (X_t) over time for 200 simulated observations, as well as the observations (Y_t) drawn from each state. We can easily visualize how State 2 generates more volatile observations with a higher mean than State 1.

Figure 1. Simulated Markov-Switching Process



In practice, we cannot observe the underlying states, X_t; we observe only the data, Y_t. Hence, as Laverty, Miket, and Kelly (2002) concluded,

> The real problem is the opposite of simulation, namely the observations are all that is known, and from there all the parameters defining the hidden Markov model must be inferred by mathematical and statistical reasoning. (p. 34)

We can think of Markov-switching models as regressions with hidden regime variables (these models are often referred to as hidden Markov models, or HMMs). In our empirical study, we used a standard method called "maximum likelihood estimation" to identify the regimes and parameters that best fit the data (see Appendix A for further details).

But why go through all the trouble? When dealing with regime shifts, we expect Markov-switching models to perform better than simple data partitions based on thresholds. For example, in Figure 1, if we had simply classified the observations that were in the highest quartile as being associated with Regime 2 (the high-mean regime), we would have misidentified the actual regime 40 times out of 200 observations. In contrast, a well-calibrated Markov-switching model would have misidentified the actual regime only three times. Arbitrary thresholds give false signals because they fail to capture the persistence in regimes as well as changing volatilities. For example, because Regime 2 has higher volatility, a large negative value is most likely to come from this regime, even though it has a higher mean. Likewise, if the previous observation were volatile, the current observation would be more likely associated with Regime 2, even though a simple threshold might classify it as part of the quiet regime. Markov-switching models are designed to capture these features of the data.

By modeling regime shifts, Markov-switching models also differ from traditional regressions. In fact, there are a number of ways in which Markov-switching models can complement standard econometric forecasting. For example, one could generate economic forecasts using regressions and use a Markov-switching model to partition historical data and thereby link the forecast to expected asset returns. Regime-switching models could also be used to improve upon standard econometric forecasting by allowing the regression coefficients themselves to take on different values in each regime. We deliberately kept the models in this article simple and did not integrate traditional regressions. Nevertheless, it is interesting to note how regime-switching models can be used in a broad range of applications.

Defining Economic Regime Variables

The regimes we identified in our empirical study are based on time series for financial market turbulence, inflation, and economic growth, which we define below. This list is clearly not meant to be exhaustive; however, each of these variables captures a different aspect of economic activity and provides a useful and concise dataset for our analysis.

Financial Market Turbulence. Following Chow, Jacquier, Kritzman, and Lowry (1999), we defined financial market turbulence as a condition in which asset prices behave in an uncharacteristic fashion given their historical pattern of behavior, including extreme price moves, decoupling of correlated assets, and convergence of uncorrelated assets. Specifically, we measured financial turbulence using the following multivariate distance measure (sometimes referred to as the squared Mahalanobis distance):

$$d_t = (\mathbf{y}_t - \boldsymbol{\mu})\boldsymbol{\Sigma}^{-1}(\mathbf{y}_t - \boldsymbol{\mu})',$$

where

$\mathbf{y}_t$ = vector of asset returns for period t

$\boldsymbol{\mu}$ = sample average vector of historical returns

$\boldsymbol{\Sigma}$ = sample covariance matrix of historical returns

Previous research has shown that this statistical characterization of financial turbulence is highly coincident with events in financial history widely regarded as turbulent (Kritzman and Li 2010). In our study, we computed a daily turbulence index for U.S. equities using returns for the 10 S&P 500 sector indices and a turbulence index for G–10 currencies using currency returns versus the U.S. dollar.[6] The components were equally weighted within each index. We estimated mean and covariance using equally weighted historical returns for the past 3 years for currencies and 10 years for equities (to capture longer equity market cycles). For our application, we averaged the daily turbulence scores within each month to create two monthly time series. The equity and currency turbulence time series begin in December 1975 and December 1977, respectively, and both end in December 2009.

Inflation. We measured inflation using monthly percentage changes in the seasonally adjusted U.S. Consumer Price Index for All Urban Consumers from February 1947 through December 2009 from the Federal Reserve Economic Data (FRED) website.[7]

Economic Growth. We measured economic growth using quarter-over-quarter percentage growth in the seasonally adjusted U.S. real gross national product from the second quarter of 1947 through the fourth quarter of 2009 from the FRED website.

In-Sample Regimes in Turbulence, Inflation, and Economic Growth

We calibrated a two-regime Markov-switching model for each of our four variables individually: equity turbulence, currency turbulence, inflation, and economic growth. For each variable, we assumed that observations from Regime 1 were normally distributed with a given mean and standard deviation and that observations from Regime 2 were normally distributed with a different mean and standard deviation. Several more advanced regime-switching models have been suggested (see, for example, Gray 1996), including AR, ARMA, and GARCH models, but we used a simple model to avoid overfitting. We assumed the transition matrix is a 2×2 matrix, as in the example described earlier. We solved for the parameters that best explain the data for a given variable. **Table 1** shows the persistence, the mean, and the standard deviation of the two regimes for each variable.[8]

Our results revealed the presence of a "normal" regime and an "event" regime in each series. For example, the event regime for economic growth—the recession regime—has an average GNP growth of –0.14% and a standard deviation of 0.96%, compared with an average GNP growth of 1.09% and a standard deviation of 0.84% for the normal regime. For all four variables, the event regime is characterized not only by more challenging investment conditions on average ($\boldsymbol{\mu}$) but also by greater volatility in those conditions ($\boldsymbol{\Sigma}$). These results are interesting because we did not constrain the model to align in this way; these are simply the characteristics that best explain the data. It is also interesting to note that all the persistence numbers are larger than 50% and that the persistence of each event regime is lower than that of the nonevent regime. This difference is intuitive; the natural reaction to turbulence is to reduce portfolio risk and the natural reaction to recession is to stimulate the economy, both of which should eventually make conditions more stable. However, high-inflation regimes are highly persistent, perhaps because of the self-reinforcing cycle of higher wages and higher prices. Table 1 also shows the standard error associated with each parameter estimate. The 95% confidence interval for a given parameter can be derived by adding and subtracting 1.96 times the standard error from the point estimate. At a 95% confidence level, six of the eight regimes exhibit persistence that is statistically greater than 50%. The standard errors of each estimate for mu and sigma indicate that for each variable, the differences in mean and standard deviation across regimes are also statistically meaningful.

Figure 2 shows each variable's historical probability of being in the event regime over time. Regimes across variables are somewhat correlated, but these correlations are not unusually high. During the recent global financial crisis, all variables were in the event regime. Inflation was classified as being in the event regime because of uncertainty about the inflation rate, not because of observed rising inflation—yet another example of the difference between Markov-switching models and simple threshold-based regimes.

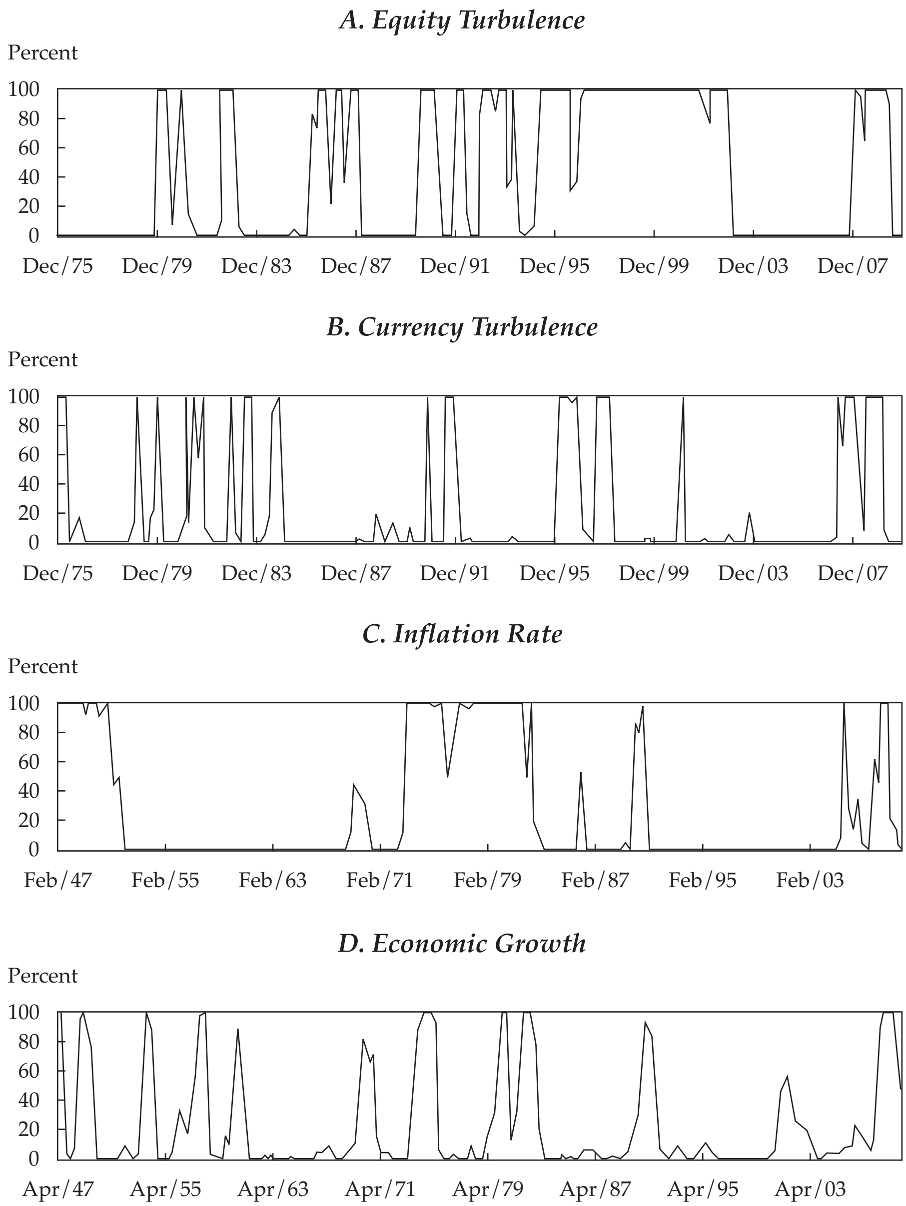
Next, we compiled a large dataset on various risk premiums. In a recent study, Bender, Briand, Nielsen, and Stefek (2010) suggested that diversification across risk premiums is more effective than diversification across asset classes. For tactical asset allocation (TAA) managers and hedge fund

Table 1. Markov-Switching Model: In-Sample Estimation Results

	Regime 1 (normal)			Regime 2 (event)		
	Persistence	Mu	Sigma	Persistence	Mu	Sigma
Equity turbulence	92.33%	0.65	0.28	90.47%	1.89	1.13
Standard error	8.40	0.00	0.01	6.14	0.00	0.00
Currency turbulence	91.50%	0.88	0.33	68.46%	2.14	1.22
Standard error	5.11	0.00	0.00	15.43	0.01	0.02
Inflation rate	98.44%	2.62%	0.70%	95.23%	6.66%	1.81%
Standard error	4.95	0.12	0.02	8.29	0.11	0.07
Economic growth	90.74%	1.09%	0.84%	67.80%	–0.14%	0.96%
Standard error	8.56	0.04	0.02	11.32	0.07	0.04

Note: Persistence is defined as the estimated transition probability of staying in the current regime.

Figure 2. Probability of the Event Regime



investors who can easily access risk premiums, this approach should generate better risk-adjusted performance than asset class diversification provides.

Exhibit 1 shows our data sources. We classified risk premiums on the basis of how they should relate to regimes in turbulence, inflation, and economic growth. We expected turbulence to affect almost all risk premiums. For inflation, we focused on Treasury Inflation-Protected Securities (TIPS), gold, and the yield curve. For economic growth, the relevant risk premiums were stocks versus bonds and cyclical stocks versus noncyclical stocks.

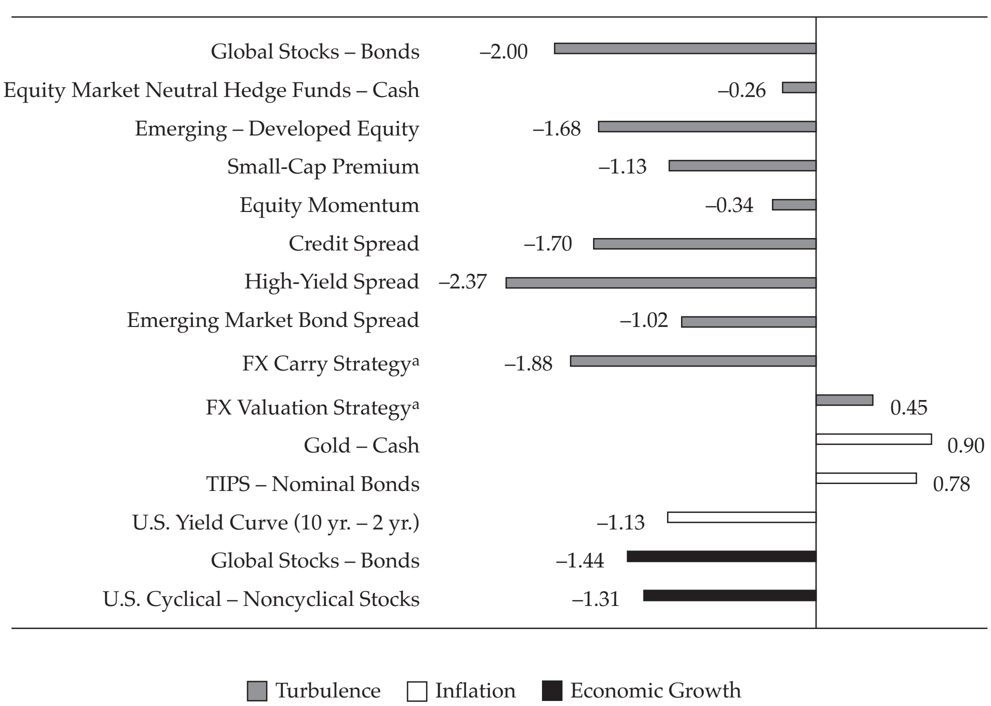
Figure 3 shows the performance of various risk premiums during the event regime compared with during the normal regime, normalized by standard deviation. For example, the performance of the "Global Stocks – Bonds" risk premium is 2.00 standard deviations lower during the event regime than during the normal regime. As expected, foreign exchange (FX) valuation (a defensive premium) outperformed during the currency turbulence event regime, and "Gold – Cash" as well as "TIPS – Nominal Bonds" outperformed during the high-inflation regime. Overall, we found that regimes in turbulence, inflation, and economic growth have a significant impact on the performance of risk premiums.

Exhibit 1. Risk Premium Data

Risk Premium	Start Date	Data Proxy
Turbulence		
Global stocks – Bonds	Feb. 1973	MSCI World Equities minus Barclays U.S. Treasury
Equity market neutral hedge funds – Cash	Jan. 1996	HFRI Equity Market Neutral Composite minus JPMorgan U.S. 1 Month Cash
Emerging – Developed equity	Jan. 1988	MSCI Emerging minus MSCI Developed Equities
Small-cap premium	Jan. 1979	Russell 2000 minus Russell 1000
Equity momentum	Jul. 1926	Kenneth French momentum-sorted portfolios, top decile minus bottom decile
Credit spread	Feb. 1973	Barclays U.S. Credit Intermediate minus U.S. Treasury Intermediate
High-yield spread	Aug. 1983	Barclays U.S. High-Yield Intermediate minus U.S. Treasury Intermediate
Emerging market bond spread	Jan. 1994	JPMorgan Emerging Market Bond Index minus Barclays 10-Year U.S. Treasury Index
FX carry strategy	Jan. 1975	RBS / ABN naive carry
FX valuation strategy	Jan. 1975	RBS / ABN naive valuation
Inflation		
Gold – Cash	Jan. 1978	S&P / GSCI Gold Index minus JPMorgan U.S. 1 Month Cash
TIPS – Nominal bonds	Feb. 1973	Barclays U.S. TIPS[a] minus Barclays U.S. Treasury
U.S. yield curve (10 yr. – 2 yr.)	Mar. 1976	Merrill Lynch U.S. 7–10 Year Treasury minus Merrill Lynch 1–3 Year Treasury
Economic growth		
Global stocks – Bonds	Feb. 1973	Same as above
U.S. cyclical – Noncyclical stocks	Jul. 1926	Based on Kenneth French's 10 industry returns with our own definition of cyclical industries

[a]We used a regression model to backfill TIPS historical returns prior to February 1999.

Figure 3. Risk Premium Performance: In Sample (Event Mean – Nonevent Mean/Full Sample Volatility)



[a]Based on currency turbulence.

Out-of-Sample Analysis: Tactical Asset Allocation

Our in-sample results showed that the Markov-switching models succeeded in partitioning historical data in a meaningful way, but they did not reveal whether a dynamic process would outperform a static process out of sample. To test out-of-sample performance, we designed the following experiment. Each month, we implemented these steps:

1. We calibrated our Markov-switching model using a growing window of data from inception up to that point in time.
2. When the model indicated an upcoming event regime, we implemented defensive tilts, as shown in **Table 2**.[9]
3. We compared the performance of the dynamic portfolio of risk premiums and constant exposures and rolled the experiment forward.

When the event regime was simultaneously predicted for more than one variable, we added the relevant tilts. For example, if turbulence and recession were both predicted to occur at the same time, the resulting exposure to "stocks versus bonds" would be 10% – 5% – 5% = 0%. The positions we used in Table 2 were arbitrary; obviously, we could have constructed and tilted the portfolio in multiple ways. Hence, this example is meant to be illustrative; it is not a proposed investment strategy.

Table 3 compares the performance, before costs, of the constant exposure portfolio and the dynamic process. The dynamic process increased risk-adjusted performance by 41% compared with constant exposure. Moreover, by anticipating crises, the dynamic process significantly reduced downside risk, as evidenced by its lower skewness, lower value at risk (VaR), and lower maximum drawdown. The dynamic tilts resulted in approximately 150% portfolio turnover per year. Rather than make a transaction cost assumption, we report the break-even transaction cost that would reduce the dynamic strategy's information ratio to that of the static strategy. As long as transaction costs are less than 133 bps per two-way transaction, the dynamic strategy will result in outperformance net of costs. This outperformance is likely driven by the model's ability to identify and anticipate persistent regimes in which returns to risk are subject to large losses. Directional returns are admittedly very difficult to predict, but volatility may be easier. Indeed, much of the improvement in information ratio is attributable to the lower volatility (and downside risk) of the dynamic strategy.

Figure 4 shows when each of the event regimes was predicted to occur.[10] The forecasts themselves exhibit substantial persistence, which prevented excessive turnover in the backtest. One notable exception is the period between January 1990 and December 1996 for equity turbulence, in which some event regime forecasts lasted only one or two months. One obvious question is, were these short-lasting signals useful? It turns out that they were important in avoiding losses associated with some

Table 2. Risk Premiums and Recommended TAA Tilts

		Event Regime Tilts		
	Default Exposure	Turbulence	Recession	Inflation
Risk premiums				
Global stocks – Bonds	10%	–5%	–5%	
Small-cap premium	10	–5		
Equity momentum	10	–5		
Equity market neutral hedge funds – Cash	10	–5		
Emerging – Developed equity	10	–5		
Credit spread	10	–5		
High-yield spread	10	–5		
U.S. yield curve (10 yr. – 2 yr.)	10			–5%
Emerging market bond spread	10	–5		
FX carry strategy[a]	10	–5		
Defensive trades				
Gold – Cash	0%			10%
TIPS – Nominal bonds	0			10
U.S. cyclical – Noncyclical stocks	0		10%	
FX valuation strategy[a]	0	10%		
Total notional exposure	100%	55%	15%	25%

[a]Based on currency turbulence.

Table 3. Static and Dynamic TAA Model Performance

	Static	With Dynamic Tilts
Annualized excess return	5.99%	6.89%
Annualized volatility	8.37%	6.83%
Information ratio	0.72	1.01
Skewness	–1.56	–1.01
5% VaR	–3.39%	–2.72%
Maximum drawdown	–41.48%	–32.69%
Break-even transaction cost (two-way)	na	1.33%

na = not applicable.

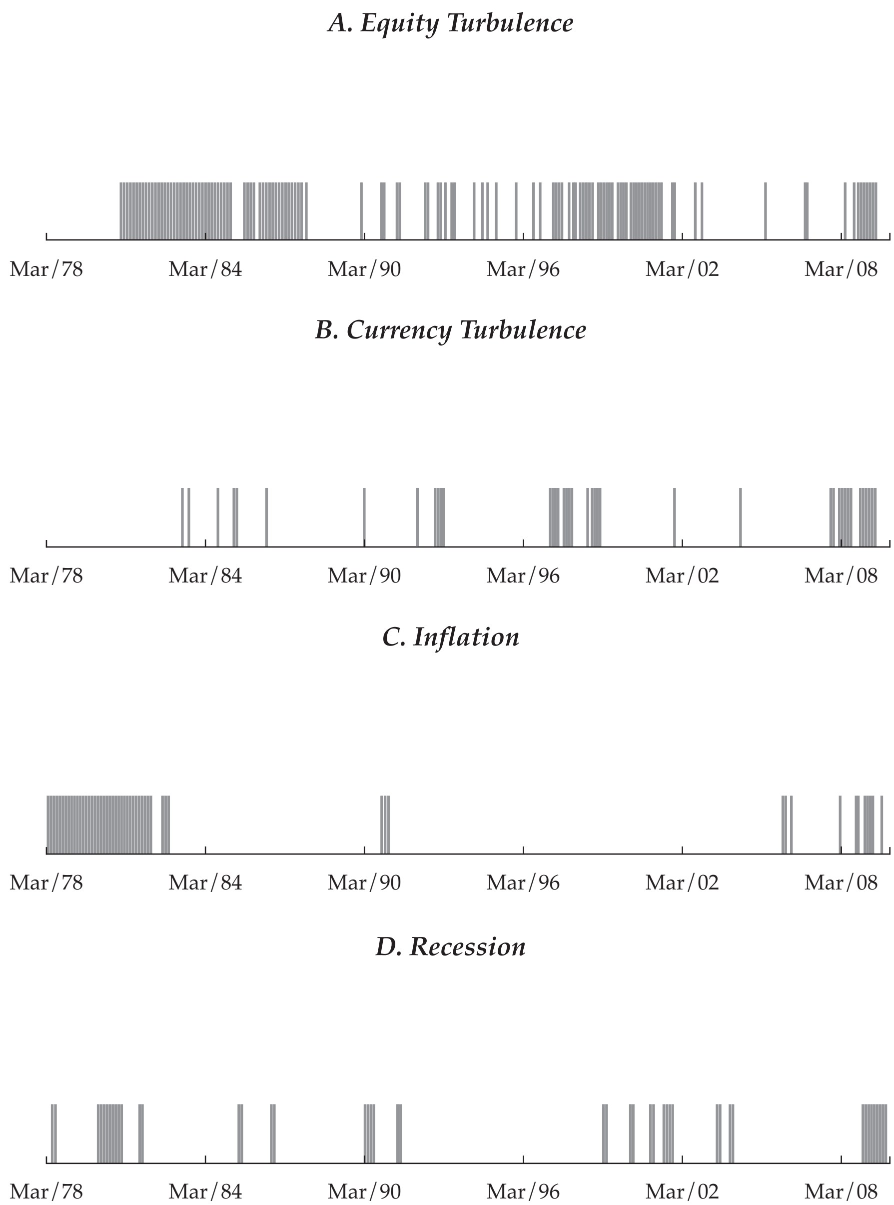
Note: The dynamic strategy turns over the portfolio approximately 1.5 times per year.

risk premiums. In the month following these turbulence forecasts, the annualized return of the small-cap premium was –8.4%, compared with 2.0% in the other months in this period. The same was true for equity momentum, which returned an annualized –4.8% in the forecasted turbulent months, compared with 6.5% in the other months. As long as overall turnover was not excessive, these occasional short-lasting signals added value, so we did not find it necessary to introduce further smoothing.

Dynamic Asset Allocation

Although TAA with risk premiums provided an obvious application for regime-switching models, unfortunately, many investors cannot access risk

Figure 4. Out-of-Sample Regime Forecasts: Event Regimes



premiums directly. Moreover, strategic investors, such as pension plans, must invest with a fairly long time horizon. Nonetheless, as evidenced by the recent global financial crisis—during which to "do nothing" became a significant active bet—even strategic investors should react to significant regime shifts. To address this concern, we applied the same methodology as in the TAA example to allocation across asset classes.

We began with a simple static portfolio allocated 60% to stocks and 40% to bonds.[11] The allocation was further subdivided into U.S. equity versus foreign equity and government bonds versus corporate bonds. **Exhibit 2** shows the data we used in this experiment. Using the same forward-looking event regime probabilities as shown in Figure 4, we implemented the defensive tilts shown in **Table 4** in anticipation of turbulent, recessionary, or inflationary periods. In light of the asset allocation constraints faced by many institutional investors, we limited the size of the tilts to 5% or 10%. We decided to tilt away from U.S. stocks but not global stocks during a recession because we used U.S. GNP data. These tilts are meant for illustrative purposes only.

Exhibit 2. Asset Allocation Data

Asset Class	Start Date	Data Proxy
U.S. equity	Feb. 1973	S&P 500 Index
Foreign equity	Feb. 1973	MSCI EAFE Index
U.S. government bonds	Feb. 1973	Barclays U.S. Government
U.S. corporate bonds	Feb. 1973	Barclays U.S. Corporate
Cash	Feb. 1973	JPMorgan 1 Month Cash

Table 5 shows the performance, before costs, of the baseline allocation with the dynamic asset allocation strategy. Although the tilts are small, the dynamic strategy still had a meaningful impact in reducing downside risk. The dynamic strategy improved the ratio of return to value at risk (return / VaR) from 0.90 to 1.13. The magnitude and duration of large losses were also reduced, as evidenced by the drawdown analysis in **Table 6**. This model had very little turnover—only 34% per year; therefore, transaction costs had little impact on the results. A two-way cost of 5.53% or larger would have reduced the dynamic strategy's return / VaR to less than that of the static allocation.

Conclusion

Investors typically construct portfolios with inputs estimated from the average of a variety of disparate regimes. We showed how to apply a Markov-switching model to partition history into meaningful regimes based on measures of financial turbulence, inflation, and economic growth. We extended this approach to tilt portfolios dynamically in accordance with the relative likelihood of a particular regime. Finally, we presented evidence showing that regime-switching asset allocation significantly improves performance compared with an unconditional static alternative.

This article qualifies for 1 CE credit.

Appendix A. Estimating Regimes in Practice: The Baum–Welch Algorithm

The statistical procedure for estimating the parameters of a Markov-switching model (specifically, a hidden Markov model in which regimes are not observable) is well established. However, many research papers describing the technique are either too technical for some practitioners or specific to a very narrow model specification. We offer here a simple description of the technique for a two-state model governed by simple discrete distributions. The same principles can be extended to estimate more complex regime models. Instead of including a thorough mathematical treatment,[12] we provide a simple numerical example along with intuition. Appendix B contains a version of the MATLAB code we used to estimate regimes for a more general two-state model governed by normal distributions.

Table 4. Asset Classes and Recommended Tilts

		Event Regime Tilts		
	Static Allocation	Turbulence	Recession	Inflation
U.S. equity	30%	–5%	–10%	
Foreign equity	30	–5		
U.S. government bonds	20	5	10	–5%
U.S. corporate bonds	20	5		–5
Cash	0			10
Sum of absolute values	100%	20%	20%	20%

Table 5. Static and Dynamic Asset Allocation Performance

	Static	With Dynamic Tilts
Annualized return	9.45%	9.43%
5% annual VaR	–10.44%	–8.34%
Return / VaR	0.90	1.13
Annualized volatility	9.88%	8.98%
Skewness	–0.36	–0.34
Worst year	–34.93%	–29.51%
Break-even transaction cost (two-way)	na	5.53%

na = not applicable.

Note: The dynamic strategy has an average annual turnover of 34%.

Maximum likelihood estimation (MLE) is typically used to "fit" a Markov-switching model to a dataset. The goal is to solve for the entire set of parameters—for example, an initial state probability, two means, two standard deviations, and a transition matrix—that is most likely to have produced the data we observe. Unfortunately, there is no closed-form solution to this problem because the states underlying the model are unobservable. We must resort to using an algorithm to effectively "learn" the best set of parameters by taking an initial estimate for the transition matrix and improving this estimate in each successive iteration. The Baum–Welch algorithm, which is a special case of an expectation-maximization (EM) algorithm, provides an efficient way to solve this problem. To illustrate the algorithm, we present the following example. For ease of understanding, this step-by-step example is based on a discrete distribution that can take on only two values. However, the same intuition can be extended to solve more complex hidden Markov models, such as those with normal distributions.

Imagine that we are presented with six data observations for an economic variable Y_t that can take on only two values: 5% and –3%. We assume there are two regimes for this variable and that the probability p^1 of the variable registering as 5% in Regime 1 might differ from the probability of a 5% value in Regime 2, p^2 (we use superscripts to denote regime dependence and subscripts to denote time periods). We would like to determine the full set of parameters—the transition probability matrix ($\boldsymbol{\Gamma}$) governing regime shifts, the probability distributions (p^1 and p^2) for Y_t in each regime, and the initial probability (z) of Regime 1—that is most likely to have produced the six data points we observe. In mathematical notation, we want to solve for the set of parameters that maximizes the probability of observing the data given the parameters:

$$\left(\boldsymbol{\Gamma}^*, p^{1*}, p^{2*}, z^*\right) = \max_{\Theta}\left\{P\left[\left(Y_1\, Y_2 \ldots Y_6\right) \mid \left(\boldsymbol{\Gamma}, p^1, p^2, z\right)\right]\right\},$$

where Θ is the set of all possible choices of parameters $(\boldsymbol{\Gamma}, p^1, p^2, z)$ and $P[(Y_1\, Y_2 \ldots Y_6) \mid (\boldsymbol{\Gamma}, p^1, p^2, z)]$ represents the probability of observing the six data points we were provided, conditional on a given set of parameters.

Following the Baum–Welch algorithm, we begin by picking arbitrary values for the transition matrix, $\boldsymbol{\Gamma}$, as shown in Panel A of **Table A1**. Assuming for the moment that these are indeed the true transition probabilities, we could compute the precise probability of each sequence of unobservable states ($q_1\, q_2 \ldots q_6$). Because we have assumed some persistence (a 70% chance of remaining in the current regime), a regime sequence with some persistence, such as (1 1 2 2 1 1), is more likely than one that oscillates erratically, such as (1 2 1 2 1 2). The overall likelihood of observing the data is, therefore, the probability-weighted sum—or "expectation"—of the data likelihood for each possible regime sequence. We can then solve for the two distributions and a new set of transition probabilities that jointly maximize this "expected" likelihood (hence the name "expectation-maximization algorithm"). We feed these new transition probabilities back into the algorithm and repeat the process, refining our estimates of the transition matrix until the algorithm converges and the likelihood has been maximized. Each iteration is guaranteed to produce a new transition matrix that contributes at least as much likelihood as the previous estimate.[13]

Table 6. Drawdown Analysis for Static and Dynamic Asset Allocation

Static Allocation		With Dynamic Tilts	
Maximum Loss	Length[a]	Maximum Loss	Length[a]
–35.2%	Ongoing	–30.1%	Ongoing
–27.7	34	–25.7	34
–21.6	45	–17.1	39
–12.8	14	–12.0	14
–11.9	14	–10.8	10

[a]Months from inception to recovery.

Table A1. One Iteration of the Baum–Welch Algorithm (Simple Example)

Panel A. Initial Estimates Used to Evaluate Paths		Prob. of Transitioning to Regime 1	Prob. of Transitioning to Regime 2	Prob. of Stopping	Prob. That y = 5%	Prob. That y = –3%
Init. prob. of Regime 1	When in Regime 1:	0.70	0.20	0.10	0.60	0.40
0.15	When in Regime 2:	0.20	0.70	0.10	0.30	0.70

Panel B.		Working Forward		Working Backward		Accounting for All Information	
Time	Obs. Data for y_t	Prob. of Being in Regime 1 and Seeing y's	Prob. of Being in Regime 2 and Seeing y's	Prob. of Being in Regime 1 and Seeing y's	Prob. of Being in Regime 2 and Seeing y's	Prob. of Being in Regime 1 and Seeing All y's	Prob. of Being in Regime 2 and Seeing All y's
t = 1	5%	0.090	0.255	0.001	0.002	0.00013	0.00050
t = 2	–3	0.046	0.138	0.004	0.003	0.00016	0.00047
t = 3	–3	0.024	0.074	0.010	0.005	0.00024	0.00039
t = 4	5	0.019	0.017	0.022	0.013	0.00042	0.00021
t = 5	5	0.010	0.005	0.048	0.033	0.00048	0.00015
t = 6	5	0.005	0.002	0.100	0.100	0.00047	0.00016

Panel C.		Normalizing Probabilities		Probability of Each Transition Having Occurred			
Time	Obs. Data for y_t	Prob. of Being in Regime 1	Prob. of Being in Regime 2	Prob. of Transitioning from Regime 1 to 1	Prob. of Transitioning from Regime 1 to 2	Prob. of Transitioning from Regime 2 to 1	Prob. of Transitioning from Regime 2 to 2
t = 1	5%	0.210	0.790	na	na	na	na
t = 2	–3	0.257	0.743	0.142	0.068	0.115	0.675
t = 3	–3	0.379	0.621	0.203	0.054	0.175	0.568
t = 4	5	0.660	0.340	0.350	0.029	0.310	0.311
t = 5	5	0.755	0.245	0.601	0.059	0.154	0.186
t = 6	5	0.750	0.250	0.661	0.094	0.089	0.156
Total		3.011[a]	2.989[b]	1.957[c]	0.304[d]	0.844[e]	1.895[f]

(continued)

Table A1. One Iteration of the Baum–Welch Algorithm (Simple Example) (continued)

Panel D.		Joint Probability of Regime and Data Observations			
Time	Obs. Data for y_t	Prob. of Being in Regime 1 and y_t = 5%	Prob. of Being in Regime 2 and y_t = 5%	Prob. of Being in Regime 1 and y_t = –3%	Prob. of Being in Regime 2 and y_t = –3%
t = 1	5%	0.210	0.790	0.000	0.000
t = 2	–3	0.000	0.000	0.257	0.743
t = 3	–3	0.000	0.000	0.379	0.621
t = 4	5	0.660	0.340	0.000	0.000
t = 5	5	0.755	0.245	0.000	0.000
t = 6	5	0.750	0.250	0.000	0.000
Total		2.376[g]	1.624[h]	0.636[i]	1.364[j]

Panel E. New (Improved) Estimates		Prob. of Transitioning to Regime 1	Prob. of Transitioning to Regime 2	Prob. of Stopping	Prob. That y = 5%	Prob. That y = –3%
Init. Prob. of Regime 1	When in Regime 1:	0.65	0.10	0.25	0.79	0.21
0.21	When in Regime 2:	0.28	0.63	0.08	0.54	0.46

na = not applicable.

[a]Sum = Expected number of days in Regime 1.
[b]Sum = Expected number of days in Regime 2.
[c]Sum = Expected number of transitions from Regime 1 to 1.
[d]Sum = Expected number of transitions from Regime 1 to 2.
[e]Sum = Expected number of transitions from Regime 2 to 1.
[f]Sum = Expected number of transitions from Regime 2 to 2.
[g]Expected number of days in Regime 1 where y_t = 5%.
[h]Expected number of days in Regime 1 where y_t = –3%.
[i]Expected number of days in Regime 2 where y_t = 5%.
[j]Expected number of days in Regime 2 where y_t = –3%.

Table A1 shows the first iteration of the algorithm in detail for this simple example. The steps are as follows:

1. Compute "forward probabilities" (Table A1, Panel A, left). For example, for t = 1, compute the probability of being in Regime 1 and observing y = 5%: $f_1^1 = zp^1 = 0.15 \times 0.6 = 0.09$. For t = 2, calculate the probability of having observed the t = 1 data and then transitioning to Regime 1 and observing the new data point, y = –3%. This calculation requires summing the associated probabilities for the path that began in Regime 1 and the path that began in Regime 2: $f_2^1 = \left(zp^1\right)\left(\gamma^{11}\right)\left(1-p^1\right)+\left(zp^2\right)\left(\gamma^{21}\right)\left(1-p^1\right)$ = (0.09 × 0.7 × 0.4) + (0.255 × 0.2 × 0.4) = 0.046. We proceed in this fashion until we reach the final data point.
2. Compute "backward probabilities" (Table A1, Panel B, middle). Starting at the end, for t = 6, calculate the probability of the regime sequence stopping, $b_6^1 = \left(1-\gamma^{11}-\gamma^{12}\right)$, given that Regime 1 prevails. This probability is simply 0.1, based on our initial guess for the transition probabilities. Note that while the probability of the regime sequence stopping is not part of the general model specification, we have included it in this simple example so that the calculations are precise.[14] Now, work backward step by step to t = 1. If we knew only the future but not the past at time t = 5, the probability of being in Regime 1 and observing the t = 6 data point, y = 5%, would be equal to $b_5^1 = \left(1-\gamma^{11}-\gamma^{12}\right)\left(\gamma^{11}\right)\left(p^1\right)+\left(1-\gamma^{22}-\gamma^{21}\right)\left(\gamma^{12}\right)\left(p^2\right) = (0.1 \times 0.7 \times 0.6) + (0.1 \times 0.2 \times 0.3) = 0.048$. As with the forward probabilities from Step 1, this calculation involves summing the probabilities associated with a path through Regime 1 and a path through Regime 2 in t = 6. We proceed backward through time in this fashion until we reach the first data point.
3. Combine information from the forward and backward probabilities. These are called "smoothed probabilities" (Table A1, Panel B, right) because they incorporate information from the full data sample. In other words, with a full understanding of what happened throughout the six time periods, what was the probability of being in Regime 1 at, say, t = 3 and observing the entire data series? It is equal to the product of the forward and backward probabilities: $s_3^1 = f_3^1 b_3^1 = 0.024 \times 0.01 = 0.00024$.

   Note that this estimate for t = 3 assumes knowledge of the future because it is based in part on data from subsequent periods. Therefore, when conducting such backtests as those in this paper, it is important to calibrate the Markov-switching model using only data that would have been available at a given point in time (in this example, using only Periods 1–3) to avoid look-ahead bias. Nevertheless, for descriptive analysis, analyzing the full dataset can be very informative.
4. The probability of having been in Regime 1 at time t = 3 is the smoothed probability normalized by the sum of all smoothed probabilities for t = 3—namely, $s_3^1 / \left(s_3^1 + s_3^2\right) = 0.00024/(0.00024 + 0.00039) = 0.379$, or about 38%. These values are shown in Table A1 (Panel C, left).
5. Next, we calculate the probability of experiencing a given transition (1 to 1, 1 to 2, 2 to 1, and 2 to 2). For example, the probability of having switched from Regime 1 to 2 between t = 1 and t = 2 is $\left(f_1^1\right)\left(\gamma^{12}\right)\left(b_2^2\right)\left(1-p^2\right)/\left(s_2^1+s_2^2\right)$ = (0.09 × 0.2 × 0.003 × 0.7) / (0.00016 + 0.00047) = 0.068. These values are shown in Table A1 (Panel C, right).
6. Finally, as shown in Panel D of Table A1, we break out the joint probability of being in a given regime and seeing a given value for y. For t = 1, the probability of being in Regime 1 and observing y = 5% is f_1^1 = 0.09, and because we know with certainty that the observation for y was 5%, the probability that we observe –3% is simply zero.

From these metrics we can calculate a complete set of new parameters (Table A1, Panel E). The probability of transitioning from Regime 1 to Regime 1 (at any given point in time) is equal to the total probability of transitioning from Regime 1 to Regime 1 divided by the total probability of being in Regime 1, or 1.957 / 3.011 = 0.65. The new probability distribution parameter—for example, the probability that y = 5% in Regime 1—is computed as the expected number of days in Regime 1 where y = 5% divided by the total expected number of days in Regime 1, or 2.376 / 3.011 = 0.79. The initial probability is simply the probability of being in Regime 1 at t = 1 (i.e., 0.21).

The data show that the algorithm decreased the estimates of regime persistence in the first iteration and dramatically increased the probability that Regime 1 will result in an observation of y = 5% (to 79%).

Table A2 shows the convergence of the algorithm for our example. The likelihood is maximized at 0.005224.[15] The algorithm concludes that Regime 1 will almost certainly generate y = 5% and that this regime is not highly persistent (58%). In contrast, Regime 2 is more persistent (73%) and is expected to generate a value of y = 5% less than half the time (45%). This framework can be extended to solve for two normal distributions by using values from a Gaussian probability density function (PDF) in place of probabilities for discrete values, such as 5% and –3%.

Table A2. Convergence of the Baum–Welch Algorithm (Simple Example)

Iteration	Likelihood	Change in Likelihood	Persistence in Regime 1	Persistence in Regime 2	Prob. That y = 5% in Regime 1	Prob. That y = 5% in Regime 2
1	0.0006	na	0.62	0.67	0.84	0.52
2	0.0021	0.0014	0.60	0.70	0.90	0.48
3	0.0028	0.0008	0.59	0.72	0.96	0.46
4	0.0039	0.0011	0.58	0.72	0.98	0.46
5	0.0048	0.0008	0.58	0.72	0.99	0.45
6	0.0051	0.0003	0.58	0.72	1.00	0.45
7	0.0052	0.0001	0.58	0.73	1.00	0.45
8	0.0052	0.0000	0.58	0.73	1.00	0.45

na = not applicable.

Appendix B. MATLAB Code for Estimating Regimes

The following MATLAB code applies the Baum–Welch algorithm to fit a two-regime hidden Markov model for any times series, y, under the assumption that y follows a normal distribution with a regime-switching mean and standard deviation. We make no guarantees about the accuracy of this code; however, it produced consistent and useful results in our research.

```
function [A,mu,sigma,p,smoothed] = fit_hmm(y)

T=length(y);
% Simple initial guesses for parameters - can be changed
mu=[mean(y),mean(y)]+randn(1,2)*std(y);
sigma=[std(y),std(y)];
A=[.8,.2;.2,.8];
p=.5;
iteration=2;
likelihood(1)=-999;
change_likelihood(1)=Inf;
tolerance=0.000001;

while change_likelihood(iteration-1) > tolerance

    for t=1:T % 0. probability of observing data, based on gaussian PDF
        B(t,1)=exp(-.5*((y(t)-mu(1))/sigma(1)).^2)/(sqrt(2*pi)*sigma(1));
        B(t,2)=exp(-.5*((y(t)-mu(2))/sigma(2)).^2)/(sqrt(2*pi)*sigma(2));
    end
    forward(1,:)=p.*B(1,:);
    scale(1,:)=sum(forward(1,:));
```



```
        forward(1,:)=forward(1,:)/sum(forward(1,:));
        for t=2:T % 1. probability of regimes given past data
            forward(t,:)=(forward(t-1,:)*A).*B(t,:);
            scale(t,:)=sum(forward(t,:));
            forward(t,:)=forward(t,:)/sum(forward(t,:));
        end
        backward(T,:)=B(T,:);
        backward(T,:)=backward(T,:)/sum(backward(T,:));
        for t=T-1:-1:1 % 2. probability of regime given future data
            backward(t,:)=(A*backward(t+1,:)')'.*B(t+1,:);
            backward(t,:)=backward(t,:)/sum(backward(t,:));
        end
        for t=1:T % 3-4. probability of regimes given all data
            smoothed(t,:)=forward(t,:).*backward(t,:);
            smoothed(t,:)=smoothed(t,:)/sum(smoothed(t,:));
        end
        for t=1:T-1 % 5. probability of each transition having occurred
            xi(:,:,t)=(A.*(forward(t,:)'*(backward(t+1,:).*B(t+1,:))));
            xi(:,:,t)=xi(:,:,t)/sum(sum(xi(:,:,t)));
        end
        p=smoothed(1,:);
        exp_num_transitions=sum(xi,3);
        A(1,:)=exp_num_transitions(1,:)/sum(sum(xi(1,:,:),2),3);
        A(2,:)=exp_num_transitions(2,:)/sum(sum(xi(2,:,:),2),3);
        mu(1)=(smoothed(:,1)'*y)'/sum(smoothed(:,1));
        mu(2)=(smoothed(:,2)'*y)'/sum(smoothed(:,2));
        sigma(1)=sqrt(sum(smoothed(:,1).*(y-mu(1)).^2)/sum(smoothed(:,1)));
        sigma(2)=sqrt(sum(smoothed(:,2).*(y-mu(2)).^2)/sum(smoothed(:,2)));
        likelihood(iteration+1)=sum(sum(log(scale)));
        change_likelihood(iteration)=abs(likelihood(iteration+1)-likelihood(iteration));
        iteration=iteration+1;

    end

end
```

Notes

1. See Chow, Jacquier, Kritzman, and Lowry (1999) and Kritzman, Lowry, and Van Royen (2001).
2. See Kim (1993) and Kumar and Okimoto (2007).
3. See Hamilton (1989), Goodwin (1993), Luginbuhl and de Vos (1999), and Lam (2004).
4. See, for example, Ang and Bekaert (2002); Kritzman, Lowry, and Van Royen (2001); and Baele (2003) regarding regime shifts and Longin and Solnik (2001) and Campbell, Koedijk, and Kofman (2002) regarding correlation asymmetries.
5. The Markov property implies that next period's regime depends only on the regime that prevailed last period—not on prior observations. This type of model is significantly easier to work with than a model that has a "long memory" of historical states.
6. The equity sectors are consumer discretionary, consumer staples, energy, financials, health care, industrials, information technology, materials, telecommunications, and utilities. Currencies, priced in U.S. dollars, include AUD, CAD, CHF, EUR, GBP, JPY, NOK, NZD, and SEK.
7. http://research.stlouisfed.org/fred2.
8. We took the square root of the turbulence index in order to obtain a variable that more closely approximates a normal distribution. Raw turbulence scores range from zero to arbitrarily large numbers and under the assumption of asset normality would be chi-squared distributed with degrees of freedom equal to the number of assets.
9. Note that risk premiums were rescaled to have similar risk levels. We accounted for publication lags in the GNP and CPI data. Turbulence is based on readily available market prices, so a publication lag is not necessary.
10. Our goal was to avoid large negative returns that may coincide with event regimes. Therefore, in this example we calibrated each signal to forecast an event regime approximately 20–30% of the time.
11. The 60/40 allocation is roughly equal to the asset breakdown of the top 1,000 pension funds in the United States, as reported by *Pensions & Investments* on 26 January 2009 (www.pionline.com/apps/pbcs.dll/article?AID=/20090126/CHART3/901229965/-1/PENSIONFUNDDIRECTORY).
12. See Hamilton (1989), Bilmes (1998), and Baum, Petrie, Soules, and Weiss (1970) for mathematical details.
13. Technically speaking, the algorithm is guaranteed to converge to a local maximum of the likelihood function. See Baum et al. (1970) for more information.
14. Markov-switching models typically assume that the data-generating process will continue forever. Actual data contradict this assumption, however, because the process inevitably ends. The probability of stopping can be important when the number of data points is very small, such as in our simple example. As the number of data points grows, the implicit likelihood that the process "stops" becomes extremely small relative to the other transition probabilities, and therefore, it is not critical to model this aspect in most real-world applications. We were able to ignore this technical aspect throughout our empirical analysis.
15. Likelihood is computed as the sum of the smoothed probabilities for both regimes, evaluated at any time period.

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