Notation

To ensure that everyone is up to speed on notation, let's review

- the notation (ML_Notation.ipynb) that we used in the "Classical Machine Learning" part of the intro course.
- additional notation (Intro_to_Neural_Networks.ipynb) used in the "Deep Learning" part of the intro course

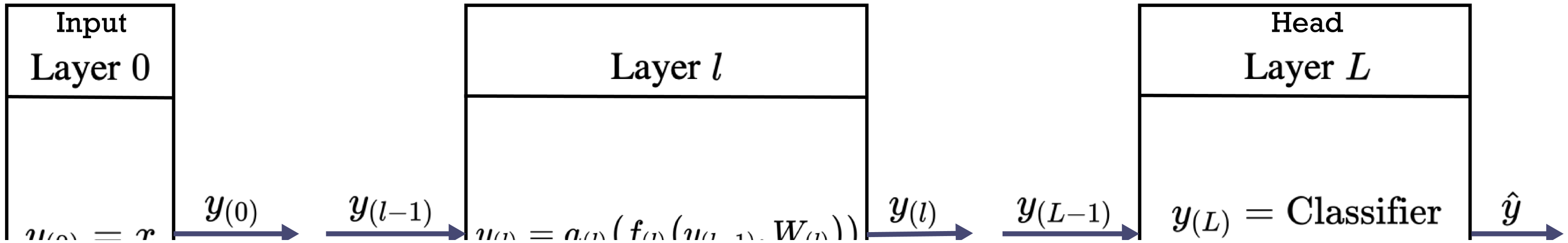
Representations

A path through a Neural Network can be viewed as a sequence of representation transformations

- transforming *raw features* $\mathbf{y}_{(0)} = \mathbf{x}$
- into *synthetic features* $\mathbf{y}_{(l)}$
  - varying with layer $1 \le l \le (L-1)$
- of increasing abstraction

Thus, the output anywhere along the path is an *alternate representation* of the input

Path through a Neural Network



Input
Layer 0
$y_{(0)}$
$y_{(l-1)}$
Layer l
$y_{(l)}$
$y_{(L-1)}$
Head
Layer L
$y_{(L)} = \text{Classifier}$
$\hat{y}$

Shallow features are less abstract: "syntax", "surface"

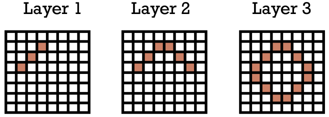
Deeper features are more abstract: "semantics", "concepts"

- We may even interpret the features as "pattern matching" regions or concepts in the raw feature space.

For example, in a CNN

- shallow features are primitive shapes
- deeper features seem to recognize combinations of shallower features

Input features detected by layer


Layer 1
Layer 2
Layer 3

Saliency Maps and Corresponding Patches
Single Layer 5 Feature Map
On 9 Maximally Activating Input images

Layer 5 ? Feature Map (Row 11, col 1).

Attribution: [https://arxiv.org/abs/1311.2901 (https://arxiv.org/abs/1311.2901)](https://arxiv.org/abs/1311.2901)

In the simple architectures of the Intro course, we mostly ignored the intermediate representations

$$\mathbf{y}_{(l)} \; : \; 1 \le l \le (L-1)$$

The layers were referred to as "hidden" for a reason !

We will discover uses for intermediate representations and show how to build a "feature extractor" to obtain them from a given architecture.

Recurrent Neural Networks

With a sequence $\mathbf{x^{(i)}}$ as input, and a sequence $\mathbf{y}$ as a potential output, the questions arises:

- How does an RNN produce $\mathbf{y}_{(t)}$, the t^{th} output ?

Some choices

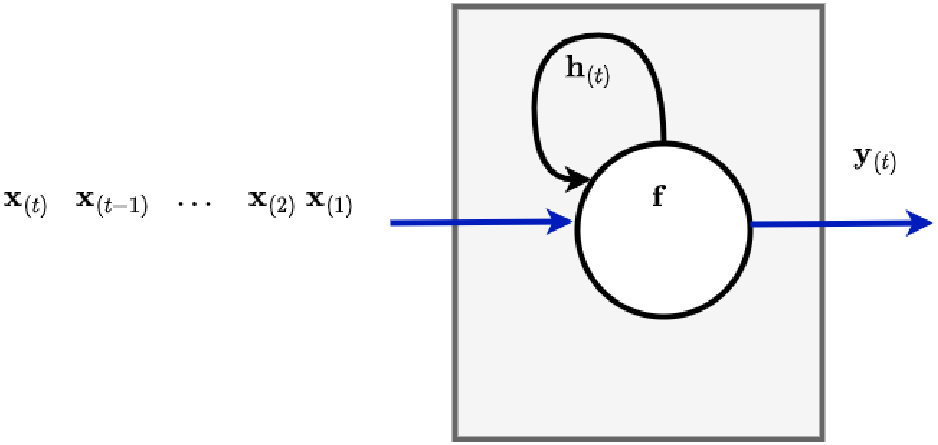
- Predict $\mathbf{y}_{(t)}$ as a direct function of the prefix of $\mathbf{x}$ of length t:
$$p(\mathbf{y}_{(t)} | \mathbf{x}_{(1)} \ldots \mathbf{x}_{(t)})$$

Direct function

- Loop
    - Uses a "latent state" that is updated with each element of the sequence, then predict the output

$$
\begin{array}{ll}
p(\mathbf{h}_{(t)} | \mathbf{x}_{(t)}, \mathbf{h}_{(t-1)}) & \text{latent variable } \mathbf{h}_{(t)} \text{encodes } [\mathbf{x}_{(1)} \ldots \mathbf{x}_{(t)}] \\
p(\mathbf{y}_{(t)} | \mathbf{h}_{(t)}) & \text{prediction contingent on latent variable}
\end{array}
$$

Loop with latent state



Latent state

The *latent state* $\mathbf{h}_{(t)}$ is a kind of memory that acts as a *summary* of the prefix of sequence $\mathbf{x}$ through time step $\tt%:

$$\mathbf{h}_{(t)} = \text{summary}(\mathbf{x}_{([1:t])})$$

Note that $\mathbf{h}_{(t)}$ is a *vector* of fixed length.

Thus, it is a *fixed length* representation of the key aspects of a sequence $\mathbf{x}$ of potentially *unbounded* length.

Example

Let's use an RNN to compute the sum of a sequence numbers

- the latent state $\mathbf{h}_{(t)}$ can be maintained as

$$\mathbf{h}_{(t)} = \text{summary}(\mathbf{x}_{([1:t])}) = \sum_{t'=1}^{t} \mathbf{x}_{(t')}$$

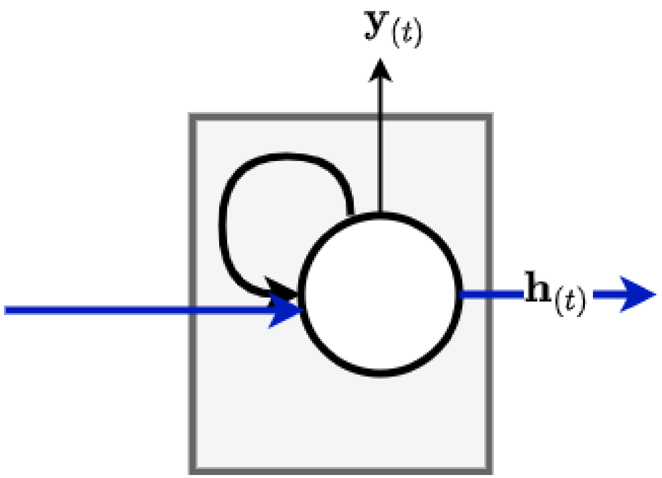
- by updating $\mathbf{h}_{(t)}$ in the loop

$$\mathbf{h}_{(t)} = \mathbf{h}_{(t-1)} + \mathbf{x}_{(t)}$$

Let's make this concrete with an example: a sequence of words

RNN

Machine Learning is easy not hard



$\mathbf{h}_{(t)}$ is a **fixed length** vector that "summarizes" the prefix of sequence $\mathbf{x}$ up to element t.

The sequence is processed element by element, so order matters.

$$
\begin{aligned}
\mathbf{h}_{(0)} &= \text{summary}([\text{Machine}]) \\
\mathbf{h}_{(1)} &= \text{summary}([\text{Machine, Learning}]) \\
&\vdots \\
\mathbf{h}_{(t)} &= \text{summary}([\mathbf{x}_{(0)}, \ldots \mathbf{x}_{(t)}]) \\
&\vdots \\
\mathbf{h}_{(5)} &= \text{summary}([\text{Machine, Learning, is, easy, not, hard}])
\end{aligned}
$$

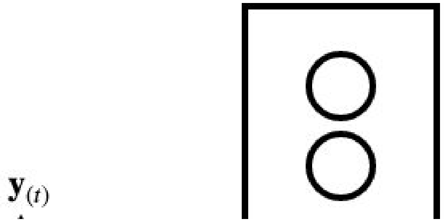
The importance of $\mathbf{h}_{(t)}$ being *fixed length*

- can be used as input to other types of Neural Network layers
- which *don't* process sequences.

A typical example is a model for text classification (sentiment)

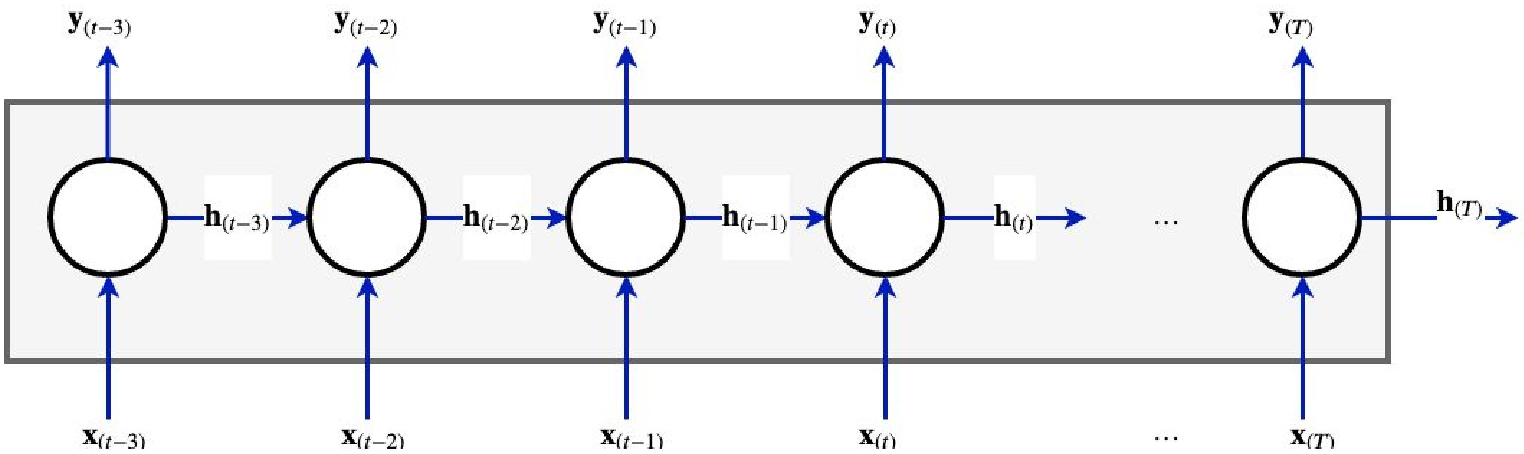
- Using an RNN to create a fixed length encoding of a variable length sequence
- A Head Layer that is a Binary Classifier

RNN Many to one; followed by classifier


$\mathbf{y}_{(t)}$

Unrolled RNN diagram

RNN many to many API



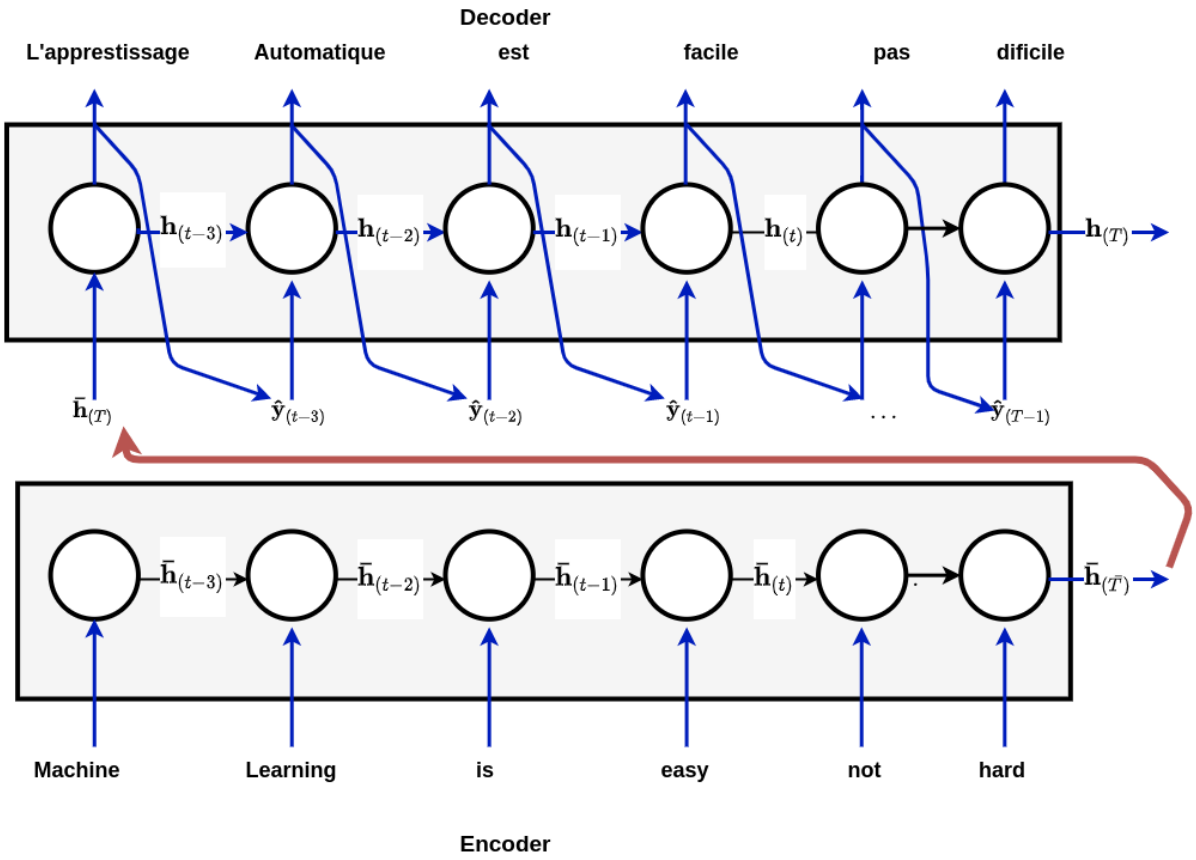
Encoder-Decoder architecture; Auto-regressive

A very common architecture pairs two RNN's

- an Encoder, which summarizes the input sequence $\mathbf{x}_{([1:\bar{T}])}$ via final latent state $\bar{\mathbf{h}}_{(\bar{T})}$
- a Decoder, which takes the input summary $\bar{\mathbf{h}}_{(\bar{T})}$ and outputs sequence $\hat{\mathbf{y}}_{([1:T])}$

It is used for *Sequence to Sequence* tasks where both the input and output are sequences.

Encoder-Decoder for language translation

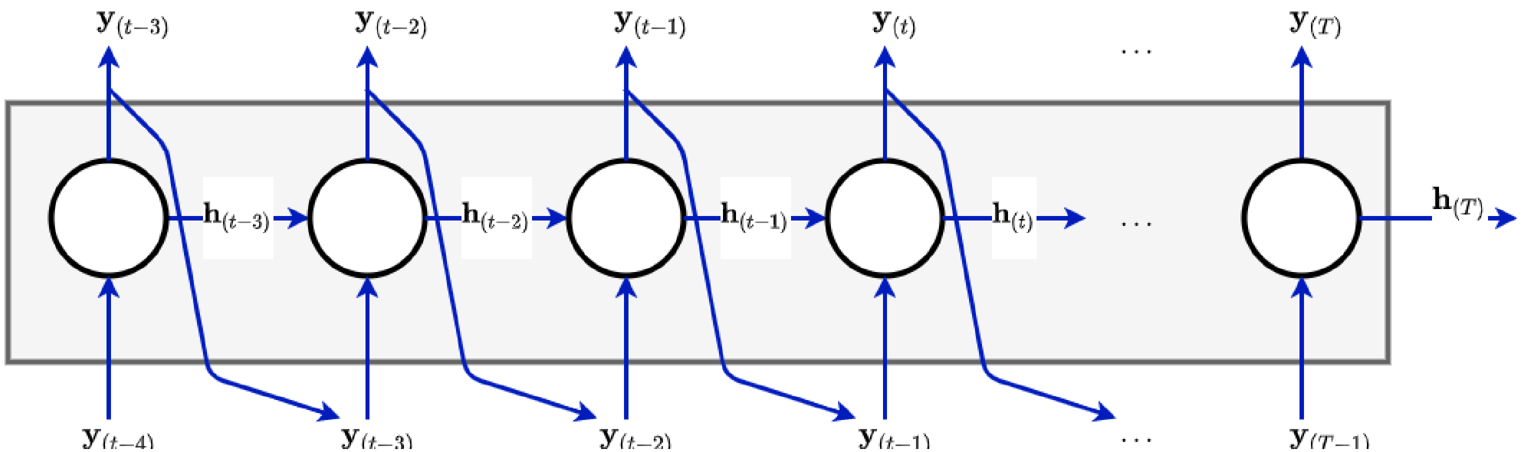


Notice that

- the output $\hat{\mathbf{y}}_{(t-1)}$ of the Decoder at position $(t-1)$
- is used as the *input* at position t

This is called *auto-regressive* behavior.

Test time: no forcing



In [2]:

```python
print("Done")
```

```
Done
```