MTH9855 Homework Five

Likun Ouyang

March 20, 2018 Spring

5.1

(a) Calculate the historical (regressed, no intercept) beta, for each of these assets as of Dec 31, 2014. In each case, calculate the appropriate t-statistic on the coefficient to test the null hypothesis $\beta = 0$ and state whether you reject the null hypothesis.

Ans:

All betas for 3 assets are significant to reject the null hypothesis $\beta = 0$ regarding t-statistic

$$\beta_{IBM} = 0.907$$
$$\beta_{AAPL} = 1.136$$
$$\beta_{TSLA} = 1.310$$

In [1]:

```
import pandas as pd
import numpy as np
import statsmodels.api as sm
from scipy import stats
import matplotlib.pyplot as plt
```

```
C:\Users\Carter\Anaconda3\lib\site-packages\statsmodels\compat\pandas.py:56: FutureW
arning: The pandas.core.datetools module is deprecated and will be removed in a futu
re version. Please use the pandas.tseries module instead.
  from pandas.core import datetools
```

In [2]:

```
data = pd.read_csv("BetaExample.txt",sep="|")
data["DATE"] = pd.to_datetime(data["DATE"])
```

In [3]:

```
data_a = data[data["DATE"]<="2014-12-31"]
```

In [4]:

```
IBM = data_a[data_a.TICKER == "IBM"]
AAPL = data_a[data_a.TICKER == "AAPL"]
TSLA = data_a[data_a.TICKER == "TSLA"]
#drop those NaN rows
TSLA = TSLA[-TSLA.R.isnull()]
```

In [5]:

```
IMB_fitted = sm.OLS(IBM.R.values.reshape(-1, 1),IBM.RM.values.reshape(-1, 1)).fit()
IMB_fitted.summary()
```

Out[5]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.332
Model:	OLS	**Adj. R-squared:**	0.332
Method:	Least Squares	**F-statistic:**	3133.
Date:	Tue, 20 Mar 2018	**Prob (F-statistic):**	0.00
Time:	14:39:34	**Log-Likelihood:**	17657.
No. Observations:	6301	**AIC:**	-3.531e+04
Df Residuals:	6300	**BIC:**	-3.531e+04
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
x1	0.9067	0.016	55.970	0.000	0.875	0.938

Omnibus:	1227.530	**Durbin-Watson:**	1.964
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	34867.449
Skew:	0.179	**Prob(JB):**	0.00
Kurtosis:	14.519	**Cond. No.**	1.00

In [6]:

```
AAPL_fitted = sm.OLS(AAPL.R.values.reshape(-1, 1),AAPL.RM.values.reshape(-1, 1)).fit()
AAPL_fitted.summary()
```

Out[6]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.192
Model:	OLS	**Adj. R-squared:**	0.192
Method:	Least Squares	**F-statistic:**	1500.
Date:	Tue, 20 Mar 2018	**Prob (F-statistic):**	1.61e-294
Time:	14:39:34	**Log-Likelihood:**	13915.
No. Observations:	6301	**AIC:**	-2.783e+04
Df Residuals:	6300	**BIC:**	-2.782e+04
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
x1	1.1363	0.029	38.728	0.000	1.079	1.194

Omnibus:	1964.903	**Durbin-Watson:**	1.980
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	232261.428
Skew:	-0.417	**Prob(JB):**	0.00
Kurtosis:	32.732	**Cond. No.**	1.00

In [7]:

```
TSLA_fitted = sm.OLS(TSLA.R.values.reshape(-1, 1),TSLA.RM.values.reshape(-1, 1)).fit()
TSLA_fitted.summary()
```

Out[7]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.122
Model:	OLS	**Adj. R-squared:**	0.121
Method:	Least Squares	**F-statistic:**	157.4
Date:	Tue, 20 Mar 2018	**Prob (F-statistic):**	6.51e-34
Time:	14:39:34	**Log-Likelihood:**	2219.2
No. Observations:	1135	**AIC:**	-4436.
Df Residuals:	1134	**BIC:**	-4431.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
x1	1.3101	0.104	12.548	0.000	1.105	1.515

Omnibus:	262.175	**Durbin-Watson:**	2.005
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2947.095
Skew:	0.726	**Prob(JB):**	0.00
Kurtosis:	10.760	**Cond. No.**	1.00

(b) Compute the holdings vector $h \in \mathbb{R}^3$ for the unique portfolio which is dollarneutral (ie. self-financing) and which has unit exposure to AAPL and zero exposure to beta as of Dec 31, 2014.

Ans:

We have the following equation to solve in order to compute the holding vector that satisfy all conditions.

$$\begin{aligned} h_1 + h_2 + h_3 &= 0 \\ h_2 &= 1 \\ 0.907h_1 + 1.136h_2 + 1.310h_3 &= 0 \end{aligned}$$

The solution is $h = [-0.432, 1, -0.568]$, which means we sell 0.432 dollar IBM, sell 0.568 dollar TSLA, buy 1 dollar AAPL

In [8]:

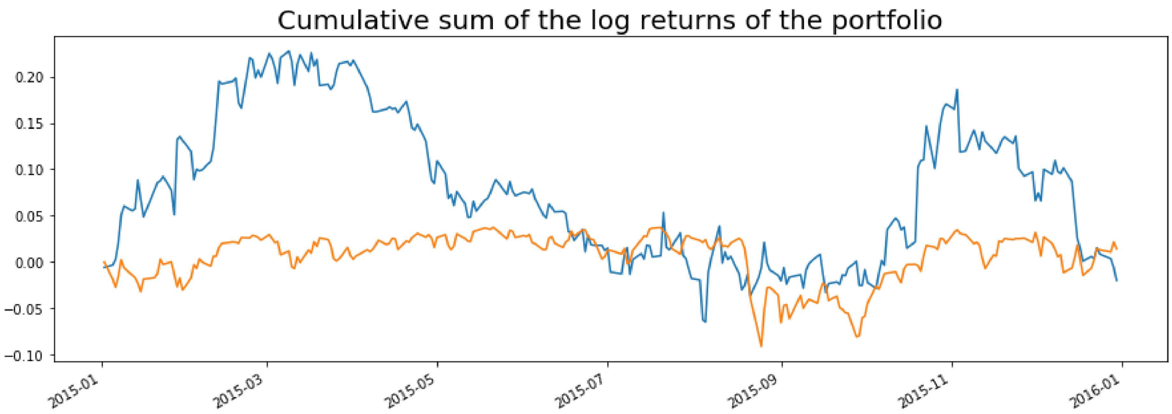
```
A = np.array([[1,1,1],[0,1,0],[0.907,1.136,1.31]])
b = np.array([0,1,0])
h=np.linalg.solve(A,b)
tickers=data['TICKER'].unique()
for a,holdings in zip(tickers,h):
    print(a,':',holdings)
```

```
IBM : -0.4317617866
AAPL : 1.0
TSLA : -0.5682382134
```

(c) Compute the daily returns of the portfolio from (b) over the period Jan 1, 2015 to Dec 31, 2015. Assume that each day, the portfolio is rebalanced back to the initial holdings vector $h \in \mathbb{R}^3$. Plot the cumulative sum of the log returns.

In [9]:

```
start_date = "2015-01-01"
end_date = "2015-12-31"
mask = (data['DATE'] > start_date) & (data['DATE'] <= end_date)
data_c = data[mask]
IBM_C = data_c[data_c.TICKER=='IBM'].R.values.reshape(-1,1)
AAPL_C = data_c[data_c.TICKER=='AAPL'].R.values.reshape(-1,1)
TSLA_C = data_c[data_c.TICKER=='TSLA'].R.values.reshape(-1,1)
R = np.concatenate((IBM_C,AAPL_C,TSLA_C), axis=1)
daily_return = np.dot(R,h)
market_return = data_c[data_c.TICKER=='TSLA'].RM.values.reshape(-1,1).flatten()
fig, ax = plt.subplots(figsize=(15,5))
fig.autofmt_xdate()
plt.plot(data_c[data_c.TICKER=='IBM'].DATE,np.cumsum(daily_return),data_c[data_c.TICKER=='IBM'].DAT
plt.title("Cumulative sum of the log returns of the portfolio",fontsize=20)
plt.show()
```



(d) Compute the realized correlation of the returns in part (c) to the market's return. Construct a statistical test of the null hypothesis that the correlation is zero. Is the realized correlation significantly different from zero at the 95% level?

Ans:

The realized correlation of the returns in part (c) to the market return is 0.0245.

Its statistical test's p-value is 0.699, which is not significantly different from zero at 95% level.

In [10]:

```
from scipy.stats.stats import pearsonr
pearsonr(market_return, daily_return)
```

Out[10]:

```
(0.029277171987615556, 0.64435287983432687)
```

5.2

(a) Calculate $\mathbb{E}[h'r]$ and $\mathbb{V}[h'r]$. Note that $\mathbb{V}[h'r]$ can be expressed as $\mathbb{V}[h'r] = f(\beta, \sigma_M^2) + g(\sigma_1^2, \ldots, \sigma_n^2)$; find functions f() and g() explicitly.

Ans:

$$\mathbb{E}[h'r] = \sum \frac{1}{n}\mathbb{E}[r_i] = \sum \frac{1}{n}(\beta\mathbb{E}[r_M] + \mathbb{E}[\epsilon_i]) = \beta\mathbb{E}[r_M] + \frac{1}{n}\sum \mathbb{E}[\epsilon_i]$$

$$\mathbb{V}[h'r] = \mathbb{V}[\beta r_M + \frac{1}{n}\sum \epsilon_i] = \beta^2\sigma_M^2 + \frac{1}{n^2}\sum \sigma_i^2$$

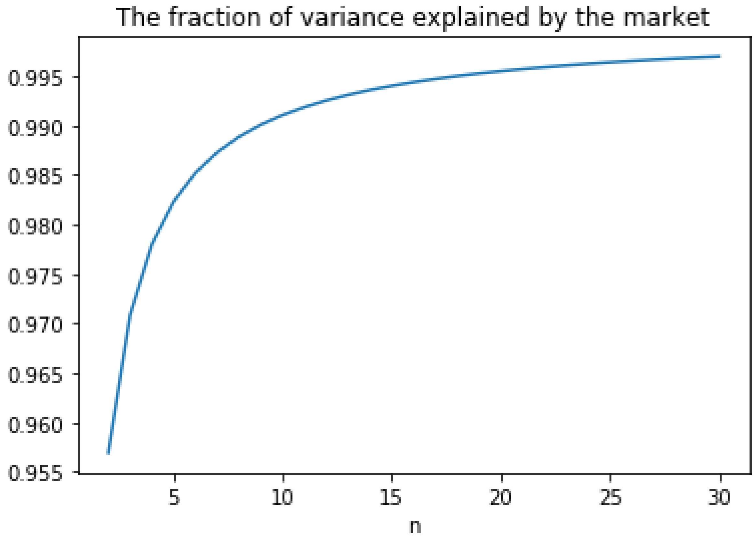
where

$$f(\beta, \sigma_M^2) = \beta^2\sigma_M^2$$
$$g(\sigma_1^2, \ldots, \sigma_n^2) = \frac{1}{n^2}\sum_{i=1}^{n} \sigma_i^2$$

(b) Take $\beta = 0.5$ and $\sigma_M = 0.2$. Assume that each constituent fund has an annualized volatility target of 10% and all $\sigma_i \approx 0.03$. The “fraction of variance explained by the market” for the fund-of-funds is defined to be f /(f + g). Numerically compute and plot this fraction as a function of n for n = 2 . . . 30.

In [11]:

```
#(b) Numerically compute and plot the "fraction of variance explained by the market" for the fund-
beta = 0.5
sigma_m = 0.2
sigma_i = 0.03
f = beta**2*sigma_m**2
n = np.arange(2,31)
g = sigma_i**2/n
plt.plot(n,f/(f+g))
plt.title("The fraction of variance explained by the market")
plt.xlabel("n")
plt.show()
```



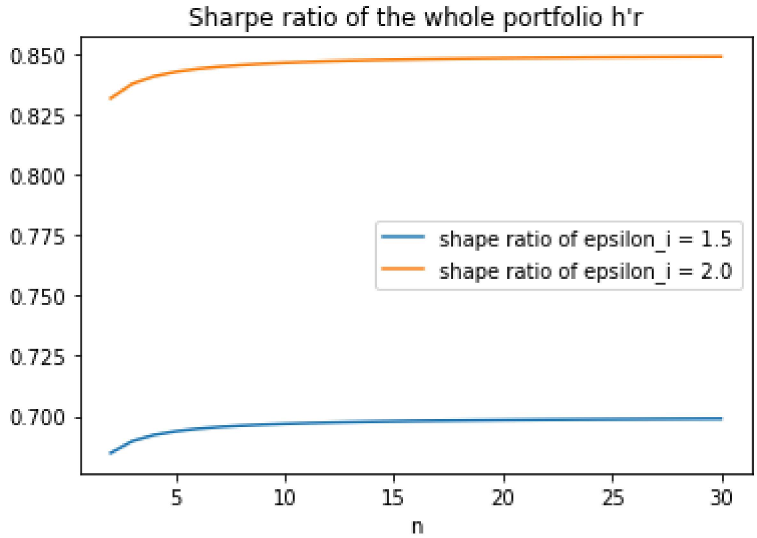
(c) Take the same assumptions as (b). Further assume that each ϵ_i has a Sharpe ratio of 1.5, so that $\mathbb{E}[\epsilon_i] = 1.5\sigma_i$, and the market's expected annual return is $\mathbb{E}[r_M] = 0.07$. The fund-of-funds charges a fee of 0.01 on capital. Numerically compute and plot the Sharpe ratio, $\mathbb{E}[h'r - 0.01]/\sqrt{\mathbb{V}[h'r]}$ as a function of n for n = 2 . . . 30. How does this change if the Sharpe ratio of ϵ_i is 2.0 rather than 1.5?

Ans:

If the Sharpe ratio of ϵ_i is 2.0 rather than 1.5, the Sharpe ratio of the whole portfolio increase as the figure below.

In [12]:

```
#(c) Numerically compute and plot the Sharpe ratio
E_i = 1.5* sigma_i
E_m = 0.07
E_portfolio = beta*E_m + E_i
plt.title("Sharpe ratio of the whole portfolio h'r")
plt.xlabel("n")
plt.plot(n, (E_portfolio-0.01)/np.sqrt(f+g),label="shape ratio of epsilon_i = 1.5")
E_i = 2.0* sigma_i
E_portfolio = beta*E_m + E_i
plt.plot(n, (E_portfolio-0.01)/np.sqrt(f+g),label="shape ratio of epsilon_i = 2.0")
leg = plt.legend(loc='best')
plt.show()
```



(d) If the fund-of-funds could simply invest in a single fund with the same properties as the others except that this fund has $\beta = 0$ and $\sigma_i = 0.1$, would that be better or worse, in terms of Sharpe ratio, than the above scenario?

Ans:

It would be better in terms of Sharpe ratio than the above secnario.

Given the same assumption as above, we have $\mathbb{E}[\epsilon_i] = 1.5\sigma_i$ and $\sigma_i = 0.1$.

Then the whole Sharpe ratio is

$$\mathbb{E}[h'r - 0.01]/\sqrt{\mathbb{V}[h'r]} = (\mathbb{E}[\epsilon_i] - 0.01)/\sigma_i) = 1.5 - \frac{1}{100\sigma_i} \approx 1.4 > 0.7$$

so It would be better in terms of Sharpe ratio to invest in a single fund with $\beta = 0$.