SYSTEMATIC

~~ALGORITHMIC~~ TRADING. MTH9894

Lecture 4
Quantitative Investment Framework: Part 3

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COURSE STRUCTURE

SCOPE: OVERVIEW OF POPULAR THEMES IN QUANT TRADING

Lecture 1 (March 23th) Statistical Arbitrage

- Review of Avellaneda's and Meucci frameworks
- Course projects assignment

Lecture 2-3-4 (March 30th, April 6th, April 27th) Fundamental Quant Models

- Part 1: Common market anomalies,
- Part 2: More anomalies. Portfolio construction.
- Part 3: The role of factor decay. Cost of strategy implementation. Performance evaluation.

Lecture 5 (May 4th) High Frequency Trading Pt.1 (by Jarrod Yuster – CEO, Pico Trading)

- Raison d'être, infrastructure design and requirements
- Current state of research & market regulations.

Lecture 6 (May 11th) High Frequency Trading Pt.2 – Impact on Markets. Final project review

- Current market ecosystem. Who are liquidity takers and liquidity providers?
- Interaction of market participants. Another look at how trading impact investment performance.

Lecture 7 (May 18th) Modern Agency Algo Strategies

- A bit of history. Modern algos. SORs
- Performance evaluation and all kinds of TCA

Lecture 8 (May 25th) Course project presentation

Final Projects

- Research, replicate and (hopefully) improve existing investment strategy
- Working in groups of 2 or 3
- You may use any framework (R, Matlab, Python, Java …). The code will be reviewed
- **May 4th:** Working groups have to be finalized. Strategies are clearly stated.
- **May 11th:** Progress review. Groups are given 3-4 min to update/ask questions.
- **May 25th:** Final presentations. Each group will have 15 min to present the results.

Course Projects: Literature

Course Projects: Study one of these. Implement. Suggest Improvements

- The Little Book That Still Beats the Market by Joel Greenblatt
- The Handbook of Equity Market Anomalies, by Leonard Zacks (editor)
- Quantitative Equity Portfolio Management, by E. Qian, R. Hua, E. Sorensen

- Momentum: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=299107
  By Narasimhan Jegadeesh, Sheridan Titman
- Value and momentum everywhere by AQR (Asness, Moskowitz, and Pedersen) (posted on Forum)
- What is Dividend Premium: Laura Liu (posted on Forum)
- Generating Excess Returns through Global Industry Rotation: John Okunev
  http://papers.ssrn.com/sol3/papers.cfm?abstract_id=904106
- Multifactor Evaluation of Style Rotation: Kevin Q. Wang
  http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1339671

Course Projects: Suggestions

1. **Data sources**
   A. Quandl: https://www.quandl.com/
   B. Quantopian: https://www.quantopian.com/
   C. CRSP (Center for Research in Security Prices): http://www.crsp.com/
   D. Kenneth French site: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
   E. Yahoo and Google finance sites
2. **Improve StatArb model … Just a suggestion: calculate implied correlation:**
   A. For each stock in the universe for each day (or week) compute matrix *A(t)* of sign correlations of residual returns (e.g. after removing industry or sector returns) with residual returns of all other stocks over some trailing period (6-12 months). Retain top & bottom10% of highly correlated names (or set some minimum threshold). For a few stocks check most correlated names. Do they make sense?
   B. Optional: replace *A(t)* with PCA(*A(t)*) only keeping top few eigenvectors (check spectrum & apply random matrix theory). You can also try A(t) squared instead to overweight highly-correlated names.
   C. Your raw score for stock *I* (raw forward return) will be *A(t)* times stock returns over previous day minus the return of *I* (equivalent of replacing *A(t)* diagonal with -1). Smooth raw score by computing exponential moving average of this signal with 5-10 day half-life. Industry-neutralize and normalize raw score.
   D. Form portfolio (top/bottom deciles or just a factor-mimicking portfolio). Compute all stats (see next page)
3. **Fully replicate a momentum strategy (refer to chapter 8 or Zack's book)**
4. **Fully replicate "Dimensions of Popularity", by Ibbotson, Idzorek. Think what else you can use besides turnover to quantify stock's popularity.**

Course Projects: Requirements

FACTOR DIAGNOSTICS

1. **Define a signal (for example 6 month return from 7 to 2 months ago)**
2. **Define a universe (for example S&P500, Russell 1000, Nikkei 225 etc.)**
   A. If you measure index return from T_1 to T_2 (typically you will work with closing prices), take index weights as of one day before T_1
   B. To evaluate forward performance of a signal from T_1 to T_2,include all stocks in your universe as of one day before T_1
3. **Calculate raw signal exposures for your universe. Neutralize signal within industries (U.S.) or sectors (all other counties) by grouping exposures, subtracting group means, and dividing by stdev (typically of the whole universe, sometime by stdev of a group, if they are drastically different from each other). Winsorize exposures from -3 to 3.**
4. **Define your rebalance strategy (daily/weekly/monthly). Backtest should be at least 5 years, 10 or more is better.**
5. **Calculate IC: rank correlation of your exposures to forward total industry-adjusted returns ("total" means adjusted for corporate actions: dividends, splits, etc. "Industry-adjusted" means your subtract equal weighed industry return)**
6. **Report average IC, moving-average IC, variance(IC), IC/stdev(IC), IC decay (e.g.) rank correlations of the exposures to the returns and to forward exposures over multiple forward periods. If satisfied with the result, move forward.**
7. **Further factor diagnostics: decile (A-C) and factor-mimicking portfolios (D-F)**
   A. For each period form 10 decile sub-portfolios. For each decile calculate total industry-adjusted forward equal weighted returns for multiple periods – days/weeks/months – a.k.a. excess return (ER). Compute hit ratio (HR) – % of stocks outperforming industry.
   B. Average across time for each forward period (e.g. 1,2,3,…N months). Report average ER, HR and t-stat: ER/stdev(ER).
   C. Split your entire testing period into 2 or more sub-periods and report ER/HR/t-stats for each. Are they different?

   Factor mimicking portfolios are most useful for multivariate case to evaluate the contribution of a new factor & also remove risk factors

   D. Compute factor-mimicking portfolios. When computing WLS, either use total industry-adjusted returns or (better) include industry dummies as factors. For true multivariate regression, include other risk factors (Fama-French, you can get exposures from their website)
   E. Assume your paper portfolio is a linear combination of your lagged factor-mimicking portfolios (how many – depends on the desired turnover of your strategy and/or optimal holding period from item 6 and 7B. Try a few. Lagged portfolios can be equal weighted or exponentially decayed.
   F. Compute cumulative excess return (over the benchmark index) of your paper portfolio, as well as information ratio, turnover, maximum drawdowns, skewness, kurtosis, and correlation of paper returns to benchmark returns.

Course Projects: Requirements

1. **Based on 1-7 decide on portfolio construction rules**
   A. Simple: invest 1/n of AUM in buying top & selling bottom decile. Continue buying/selling for n periods (n can be 1, it regulates turnover)
   B. At n+1 period sell/buy the portfolio you bought in the 1st period and buy/sell current top/bottom decides. Continue doing this
   C. Keep track of all trades. Fill your paper trades at close prices less some T-cost penalty. Try varying costs: 5bps, 10bps, etc.
2. **Do linear optimization. Maximize alpha while keeping maximum industry or sector exposures constrained. As a bonus, add T-Cost (as a linear function of size. Coefficient can be higher as stock capitalization goes down and/or stock volatility goes up). Will be happy to assist with providing more details on the T-Cost. For each optimization allocate some turnover (cumulative difference between current and optimized weights)**
3. **Actually do quadratic mean-variance optimization. Let me know if you have access to any Barra or Axioma models. I'll be happy to assist you and you get extra points for doing it.**
4. **If you did simple portfolio construction (as in item 1), report cumulative portfolio performance (ER/IR/hit ratios, max drawdown, skewness, kurtosis of returns <u>as a function of n and T-costs</u>. If you did steps 2 or3, report the same stats as a function of turnover, level of T-Cost and sector constraints. Try to evaluate portfolio capacity.**
5. **Calculate correlation of portfolio returns to market. It shouldn't be high… Does outperformance come from the long or short side of portfolio? If short side, you may need to add borrow costs (2-25 bps range per month of holding the position, make it higher as stock capitalization goes lower)**
6. **Bonus: you can regress portfolio returns on industry (sector) returns and check how much of the performance can be attributed to industry (sector) returns and how much to stock selection.**

Lecture Outline:

1. Return Factors (cont’d)
2. Factor Diagnostics
3. Multi-Factor return models
4. The curse of T-Cost (turnover, capacity)
5. Crowding

BUILDING AN ALPHA MODEL

FACTOR CONSTUCTION

1. **Compute raw factor (price to book)**
2. **Normalize factor exposure (winsorize, demean within county/sector/industry and then normalize)**
3. **Compute factor returns (mean(F) = Factor Premia, cov(F) = Factor Risk)**
4. **Typically (BARRA): GARCH(1,1) to estimate factor variances and EWMA to estimates factor correlations**
   A. https://www.msci.com/documents/10199/67b801e5-43ef-4200-8531-540851378835
   B. https://www.msci.com/resources/research/barra_risk_model_handbook.pdf
   C. http://www.northinfo.com/documents/8.pdf
5. **Compute residual (specific) risk**
6. **Compute view portfolios**
7. **Combine factor view portfolios into portfolio of factors**
8. **Compute implied alpha**
9. **Mean-variance optimization using alpha, risk and cost models**

How To Assess Factors

- Rank Correlations
- Sharpe Ratio and Information Ratio

$$Sharpe\ Ratio = \frac{R_{strategy} - R_{free}}{Volatiltiy_{strategy}}$$

$$Information\ Ratio = \frac{R_{strategy} - R_{benchmark}}{Tracking\ Error_{strategy}}$$

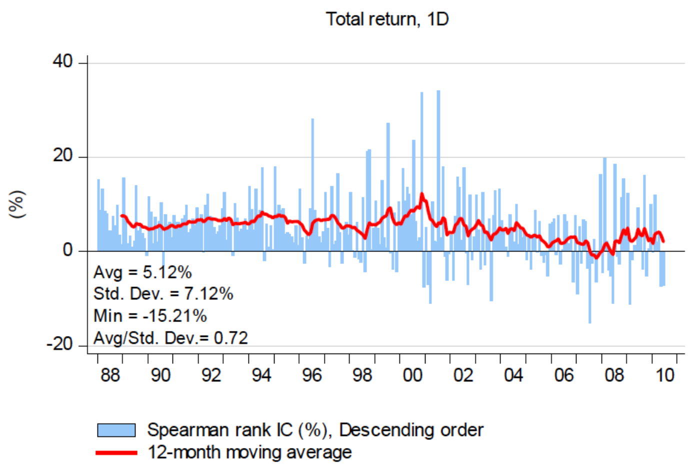
- Turnover (short-horizon strategies have higher turnover)

$$\text{Turnover} = \frac{\sum |\Delta W|}{\sum |W|}$$

- Maximum Drawdowns
- Skewness / Kurtosis: 3rd and 4th moments of return distribution
- Correlations with the market, other factors, co-linearity

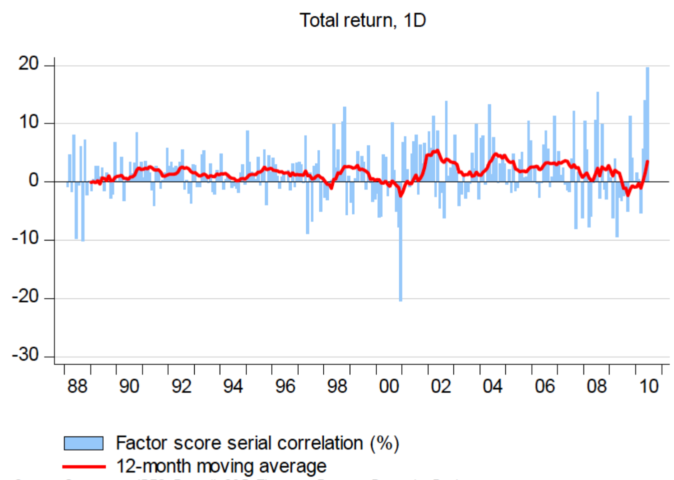
How To Assess Factors

Figure 16: Short-term price reversal



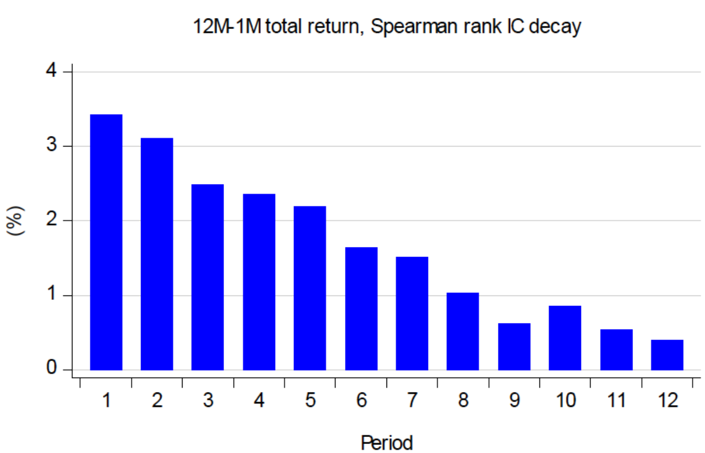
Source: Compustat, IBES, Russell, S&P, Thomson Reuters, Deutsche Bank

Figure 18: Signal serial correlation – reversal



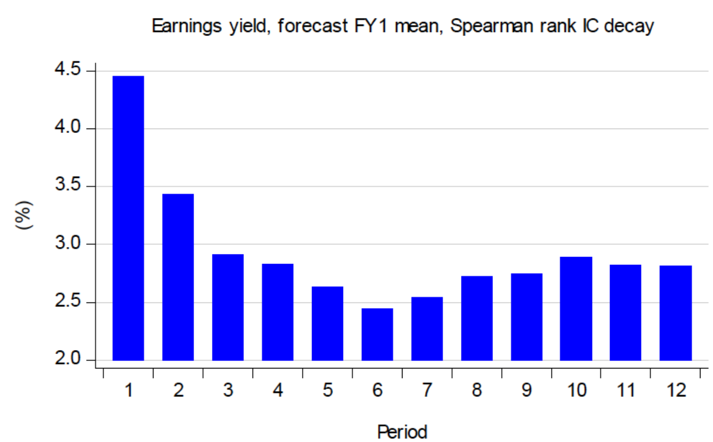
Source: Compustat, IBES, Russell, S&P, Thomson Reuters, Deutsche Bank

Figure 34: IC decay, price momentum



Source: Compustat, IBES, Russell, S&P, Thomson Reuters, Deutsche Bank

Figure 35: IC decay, value



Source: Compustat, IBES, Russell, S&P, Thomson Reuters, Deutsche Bank

Time Horizon – Another Form of Diversification

- Investment style is mostly defined by time horizon
  - Toolsets and performance systems
  - Analysts "style" and focus
  - Turnover – good proxy for horizon
- Blending and varying different time horizons can improve performance
  - Works when traditional approaches fail
- Time horizon diversification has worked well in every crisis
  - Lengthen Time Horizon in your strategies when risk subsides
  - Shorten your time horizon when risk aversion rises and investors aren't concerned with fundamentals

Correlation of Strategies Based on Time Horizon[1]
1977-2010

	Time Horizon			
	Long	**Medium**	**Short**	**Ultrashort**
Long	100%	(58%)	(36%)	7%
Medium		100%	42%	(4%)
Short			100%	0%
Ultrashort				100%

Combining Strategies

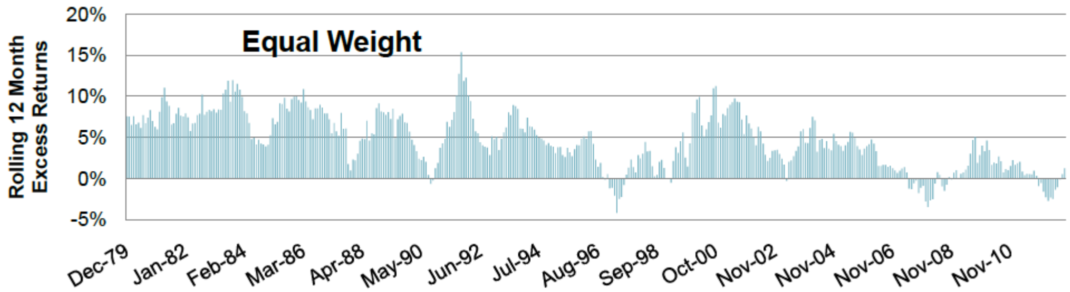
Global Portfolio Optimization
Black, Fischer; Litterman, Robert
Financial Analysts Journal; Sep/Oct 1992; 48, 5; ABI/INFORM Global
pg. 28

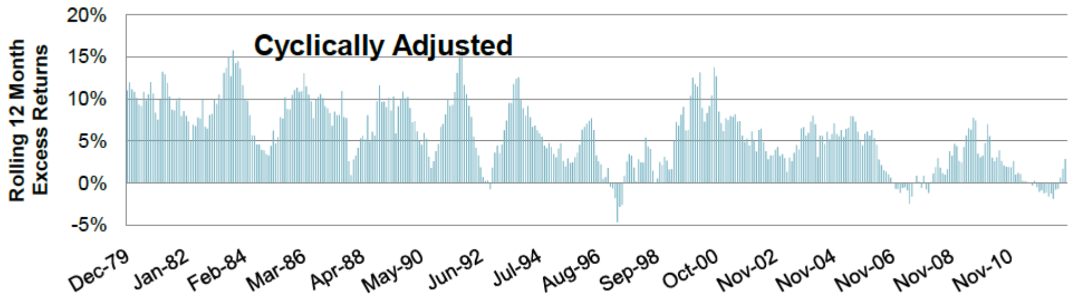
- How to maximize a bag full of tricks?
  - Equal weight: no opinion on individual strategy efficacy
  - No opinion is actually an opinion on the risk/reward trade-offs
  - Results aren’t so bad!
  - A standard to beat
- Cyclically adjusted:
  - Strategies have varying degrees of risk/reward profile depending on their relationships with risk
  - For example, in recession, we favor profitability and momentum, but caution is to the upside risk
- Opinion Pooling:
  - Black-Litterman Framework: fundamentally mean-variance trade-off between factors;
  - Copula-based: similar to BL framework with more esoteric statistics, performs better in some rare cases
- Black-Litterman Framework
  - (http://papers.ssrn.com/sol3/papers.cfm?abstract_id=334304)
  - https://www3.nd.edu/~zda/BL_JOI.pdf
  - Fundamentally no different from a fundamental manager hedging his/her bet between growth and value;
  - Risk, Reward, as well as Correlation among strategies are considered simultaneously

Combining Strategies

- Centrality of risk allocation across strategies, not the expected returns
- Expected returns, or alphas, can be derived as $\alpha=\lambda \times Inv(COV) \times W$, where λ is the risk aversion parameter, *COV* is the covariance matrix of the strategies, W is allocation across strategies
- Expected returns being a single number disguise the underlying economics behind the investment decision

	Equal Weight	CyclicallyAdjusted	Black-Litterman
Excess Return	4.5%	5.6%	13.3%
Tracking Error	2.5%	2.8%	2.5%
Information Ratio	1.8	2.0	5.3
Max (excess)	2.8%	2.8%	3.8%
Min (excess)	-1.9%	-2.1%	-2.7%
Hit Rate (excess)	69%	69%	96%





Portfolio Construction

- Accounting for the ingredients for constructing a portfolio
  - Client objective: tracking error, geographical preferences etc.
  - Benchmark selection: cap-weighted indices or custom benchmark or short-term instruments?
  - Rebalance frequencies / Turnover
  - T-Cost estimation (commissions, clearing charges, impact, cost of borrow)
- Optimizer to glue them together
  - Objective $\max(W'\alpha - \lambda W'\Omega \mathrm{W} - Impact - Short\ Cost \cdots)$
  - Generic optimizers like Matlab, financial apps from Barra, Axioma, Northfield
- Where does "alpha modeling" stop and "portfolio construction" begin?
  - The more flexible the mandate, the more blurred the line between alpha model and portfolio construction
  - Flexibility means significant room to time strategies

Performance Attributions – Risk Analysis

Date	19-Apr-2013
Managed	Global Long/Short
Benchmark	USD Cash
Market	MSCI World
Base Currency	USD
Long Volatility	31.2%
Short Volatility	31.7%
Tracking Error	5.6%

Portfolio Relative Contribution to Tracking Error (RCTE)

	Long	Short	Total
Factor	99%	99%	34%
THEMES	1%	2%	24%
CONTROL	-3%	1%	9%
INDUSTRY	90%	86%	-1%
COUNTRY	1%	1%	2%
CURRENCY	10%	10%	0%
Residual	1%	1%	66%
Total	100%	100%	100%
Beta	2.55	2.60	-0.04
Dispersion Measure	81%	80%	

Factor Relative Contribution to Tracking Error (RCTE)

	Long		Short		Total	
Theme > Factor	**Exposure**	**RCTE**	**Exposure**	**RCTE**	**Exposure**	**RCTE**
CAPITAL USE	1.7	-0.3%	0.8	-0.3%	0.8	-0.3%
CURRENT VALUE	1.7	0.7%	0.3	0.4%	1.5	0.4%
DEEP VALUE	-1.3	-0.4%	-2.7	-0.5%	1.4	0.9%
MOMENTUM	1.2	-2.0%	-0.6	-0.8%	1.8	16.4%
PROFITABILITY	2.5	-0.6%	0.6	-0.1%	1.9	6.7%
QUALITY	0.8	0.1%	0.3	-0.1%	0.5	0.1%
TECHNICAL	1.6	3.2%	1.4	3.0%	0.2	0.1%
CONTROL						
COMMODITY	0.0	0.0%	0.0	0.0%	0.0	0.1%
LEVERAGE	0.6	0.4%	0.6	0.2%	0.0	0.2%
LIQUIDITY	0.7	0.8%	1.1	1.2%	-0.4	0.5%
MARKET	-0.2	-3.5%	0.1	-0.2%	-0.3	4.7%
MOMENTUM	0.2	-0.1%	0.2	-0.1%	0.0	0.2%
RISK	-2.2	-4.6%	-1.8	-4.1%	-0.3	2.2%
SIZE	-4.8	3.7%	-4.7	3.6%	-0.1	0.7%

Performance Attributions – Factor Attributions

Portfolio Return Contribution

	Long	Short	Total
Risk Free	0.13%	0.12%	0.01%
Factor	8.96%	5.95%	3.01%
THEMES	-1.08%	-2.21%	1.13%
CONTROL	1.47%	1.71%	-0.24%
INDUSTRY	8.58%	6.45%	2.12%
Residual	5.55%	1.01%	4.54%
Total	**14.64%**	**7.08%**	**7.56%**

INDUSTRY Return Contribution

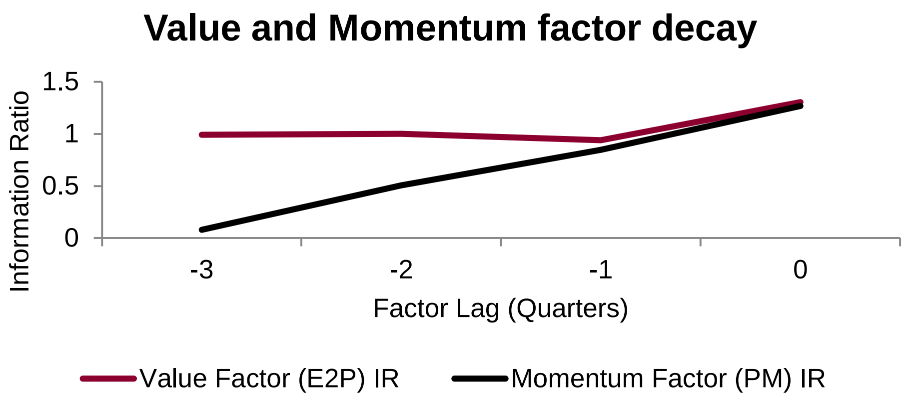
	Long		Short		Total	
Group	**Avg. Weight**	**Return**	**Avg. Weight**	**Return**	**Avg. Weight**	**Return**
Telecommunicatio	3.6%	0.6%	0.6%	0.2%	3.0%	0.4%
Utilities	0.0%	0.0%	0.4%	0.0%	-0.4%	0.0%
Information Techn	8.2%	0.8%	2.2%	0.2%	6.0%	0.6%
Financials	9.1%	0.9%	10.9%	1.1%	-1.8%	-0.2%
Health Care	3.8%	0.5%	0.7%	0.1%	3.1%	0.4%
Consumer Staples	12.1%	1.1%	10.4%	1.2%	1.7%	0.0%
Consumer Discret	28.7%	2.5%	23.1%	2.0%	5.6%	0.6%
Industrials	18.6%	2.6%	21.4%	3.1%	-2.9%	-0.5%
Energy	11.9%	0.3%	9.8%	-0.3%	2.1%	0.6%
Materials	3.9%	-0.9%	8.2%	-1.2%	-4.3%	0.2%

Factor Return Contribution

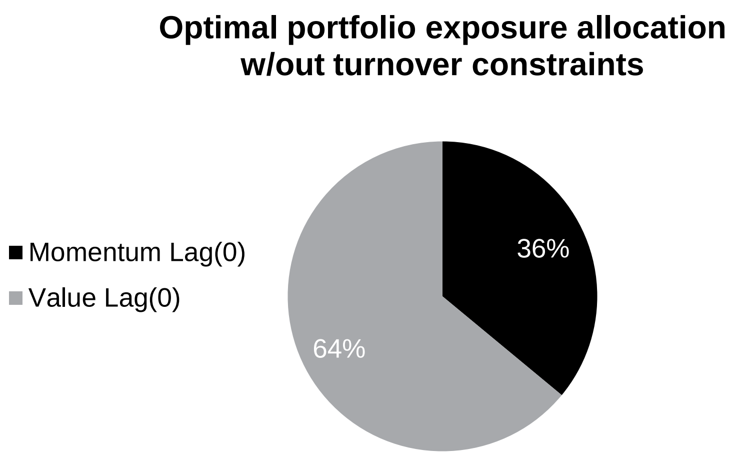
	Long		Short		Total	
Theme > Factor	**Avg. Exp.**	**Return**	**Avg. Exp.**	**Return**	**Avg. Exp.**	**Return**
CAPITAL USE	0.4	-0.7%	0.2	-0.3%	0.2	-0.3%
CURRENT VALUE	0.3	-0.7%	0.0	-0.1%	0.3	-0.7%
DEEP VALUE	-0.4	-0.7%	-0.5	-0.5%	0.1	-0.2%
MOMENTUM	0.5	1.5%	-0.3	-1.3%	0.8	2.8%
PROFITABILITY	1.3	0.0%	0.2	0.2%	1.1	-0.3%
QUALITY	0.1	0.0%	0.0	-0.1%	0.0	0.1%
TECHNICAL	-0.3	-0.5%	-0.4	-0.1%	0.1	-0.4%
CONTROL						
CRUDE	0.2	0.3%	0.5	0.5%	-0.3	-0.2%
MARKET	0.5	-1.9%	0.7	-2.3%	-0.1	0.4%
LEVERAGE	0.3	-0.2%	0.3	0.1%	0.0	-0.2%
LIQUIDITY	0.2	0.4%	0.5	0.7%	-0.2	-0.3%
SIZE	-1.3	-0.4%	-1.4	-0.6%	0.1	0.2%
RISK	-0.7	3.4%	-0.7	3.4%	0.0	-0.1%

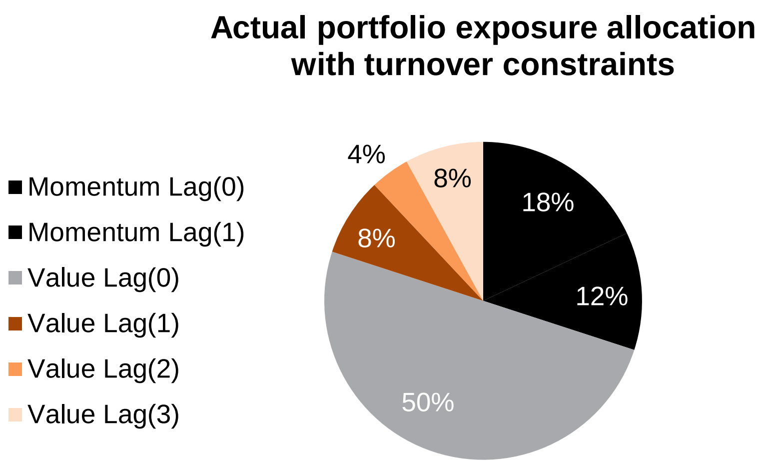
Fast Factors & the Curse of Portfolio Turnover

Equating T-Cost and Turnover leads to underutilization of faster factors and loss of alpha



- "Slow" & "Fast" factors – similar IR if we use most recent values (no lag)
- Factors have negative correlation
- T-cost assumptions impose an artificial limit on exposure to the momentum factor and force the use of lagged factors
- In the end, despite equally compelling levels given current information...we are forced to allocate more to Value because of execution inefficiency





Trading Large Caps vs. Small Caps

Market makers supply average liquidity in LC names, but nearly non-exists in SC

Supply

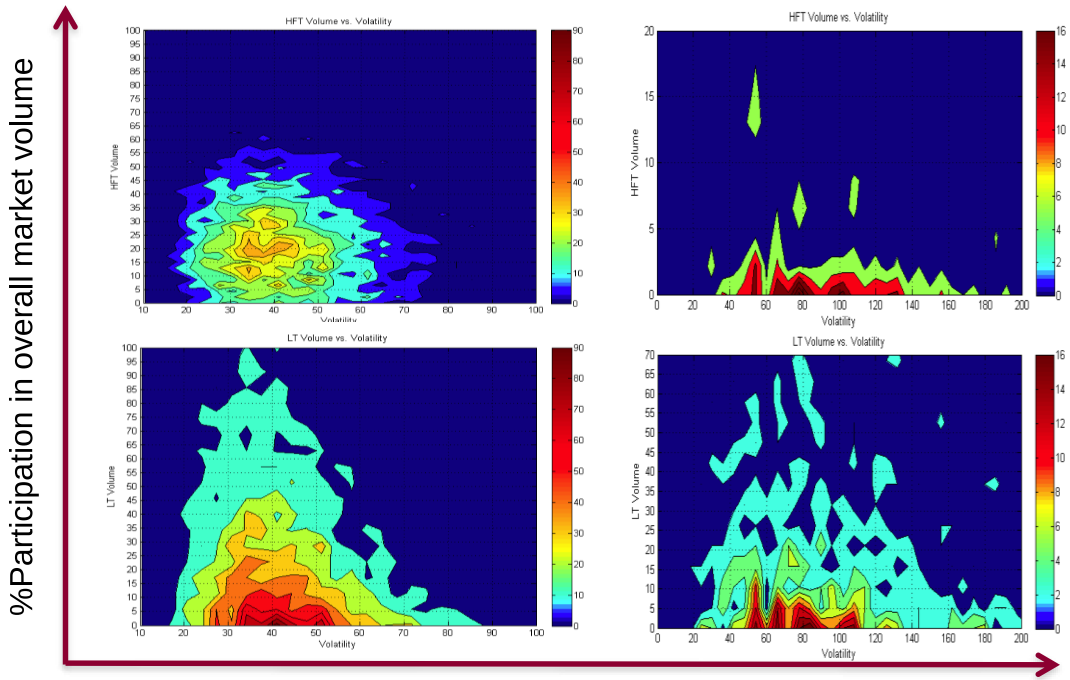
<u>Market Makers</u>:
In top 75 names provide liquidity within "normal" volatility regime and non beyond that mark

Demand

<u>Institutional Investors:</u>
Trade top and bottom names identically without realizing that liquidity supply is much different.

Frequently originate small trades across wide range of stocks: cash flows, rebalances

Top 75 Nikkei stocks | **Bottom 75 Nikkei stocks**



%Volatility over 30-min trading period, annualized

Color: Event probability: Frequent Event Rare Event

Outside of the most liquid names and "normal" volatility, investors need to (a) monitor fragmented market & patiently wait for rare natural liquidity or (b) dislocate the stock price to attract liquidity

Factor Crowding

Typical measures

- Active bets – number of institutional managers holding the stock as one their top 10/20/50 relative overweights (from FactSet's ownership database)
- Trade persistence – several consecutive quarters of net purchases by aggregated institutional investors (13F)
- Long-term price momentum – strong outperformance over the trailing two years
- Sell-Side analyst "Buy" rating – analyst sentiment. (contrarian indicator). When consensus recommendations are unanimous, there is essentially no further room to upgrade/downgrade, so one can expect reversion to the mean.
- High expectations – increase in short- and long-term earnings estimates vs. 2 years ago, combined with high forward P/E multiples
- Based on securities lending (Data Explorers) – only looks at the positions of long-short managers.
- Strong pairwise intraday correlations of residual returns– (Cahan, Luo – DB)
    - http://www.bfjlaward.com/pdf/25907/14-23_cahan_jpm_0716.pdf
    - http://finpko.faculty.ku.edu/myssi/FIN938/Crowding_DBQS_Portfolios_Under_Construction_Feb-1-2012.pdf
- Cross-stock factor based on co-movement of unrelated securities held by institutional investors.
    - https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2817059

BARRA-type Single Factor Model

Consider a single factor model in the form of a cross-sectional regression at time t

$$\underset{(N\times 1)}{\mathbf{R}_t} = \underset{(N\times 1)}{\boldsymbol{\beta}}\underset{(1\times 1)}{f_t} + \underset{(N\times 1)}{\varepsilon_t}, t = 1, \ldots, T$$

- $\boldsymbol{\beta}$ is an $N \times 1$ vector of observed values of an asset specific attribute (e.g., market capitalization, industry classification, style classification)

- f_t is an unobserved factor realization.

- $var(f_t) = \sigma_f^2$; $cov(f_t, \varepsilon_{it}) = 0$, for all i, t; $var(\varepsilon_{it}) = \sigma_i^2, i = 1, \ldots, N$.

Estimation

For each time period $t = 1, \ldots T$, the vector of factor betas, $\boldsymbol{\beta}$, is treated as data and the factor realization f_t, is the parameter to be estimated. Since the error term ε_t is heteroskedastic, efficient estimation of f_t is done by weighted least squares (WLS) (assuming the asset specific variances σ_i^2 are known)

$$\begin{aligned} \hat{f}_{t,wls} &= (\boldsymbol{\beta}'\mathbf{D}^{-1}\boldsymbol{\beta})^{-1}\boldsymbol{\beta}'\mathbf{D}^{-1}\mathbf{R}_t,\ t = 1, \ldots, T \\ \mathbf{D} &= diag(\sigma_1^2, \ldots, \sigma_N^2) \end{aligned} \tag{8}$$

Note 1: σ_i^2 can be consistently estimated and a feasible WLS estimate can be computed *Use Gauss Jordan elimination instead of matrix inversion*

$$\begin{aligned} \hat{f}_{t,fwls} &= (\boldsymbol{\beta}'\hat{\mathbf{D}}^{-1}\boldsymbol{\beta})^{-1}\boldsymbol{\beta}'\hat{\mathbf{D}}^{-1}\mathbf{R}_t,\ t = 1, \ldots, T \\ \hat{\mathbf{D}} &= diag(\hat{\sigma}_1^2, \ldots, \hat{\sigma}_N^2) \end{aligned}$$

Note 2: Other weights besides $\hat{\sigma}_i^2$ could be used

Factor Mimicking Portfolio

The WLS estimate of f_t in (8) has an interesting interpretation as the return on a portfolio $\mathbf{h} = (h_1, \ldots, h_N)'$ that solves

$$\min_{\mathbf{h}} \frac{1}{2}\mathbf{h}'\mathbf{D}\mathbf{h} \text{ subject to } \mathbf{h}'\boldsymbol{\beta} = 1$$

The portfolio $\mathbf{h}$ minimizes asset return residual variance subject to having unit exposure to the attribute β and is given by

$$\mathbf{h}' = (\boldsymbol{\beta}'\mathbf{D}^{-1}\boldsymbol{\beta})^{-1}\boldsymbol{\beta}'\mathbf{D}^{-1}$$

The estimated factor realization is then the portfolio return

$$\hat{f}_{t,wls} = \mathbf{h}'\mathbf{R}_t$$

When the portfolio $\mathbf{h}$ is normalized such that $\sum_i^N h_i = 1$, it is referred to as a *factor mimicking portfolio.*

Combining with Risk Model

A. Using daily estimates for factor returns compute GARCH(1,1) factor volatilities

$$\sigma_t^2 = (1-\alpha-\beta)\sigma_0^2 + \alpha f_{t-1}^2 + \beta\sigma_{t-1}^2$$

A. Compute EWMA for factor correlations (typically 6-12mos half-life)

B. Consider shrinking off-diagonal elements

C. Calculate residual <u>stock</u> returns x and fit GARCH(1,1) to these processes.

D. Full risk model is $W = diag(x) + bF\,b'$, where covariance matrix F is estimated in steps A-C

E. Construct factor mimicking portfolios as

$$h'_{GLS} = (b'W^{1}b)^{-1}\,b'W^{1}$$

F. Using Woodbury matrix identity to avoid inversing W; inverse F instead.

G. Covariance matrix of factor mimicking portfolios is naturally

$$h'Wh = F + h'\,diag(x)\,h$$

Combining Factors

A. Optimal factor weights: $w = (h'\Omega h)^{-1} < F >$

B. Combining into portfolio of factors: $P = w'h' = ((h'\Omega h)^{-1} < F >)'h'$

C. Implied stock-level alphas: $\alpha = \Omega w'h' = ((h'\Omega h)^{-1} < F >)'h'$

D. Sometime alphas are rescaled such that portfolio of factors had certain level of IR consistent with historical observation

$$\alpha_{scale} = hist\_IR \, \alpha / [P'\alpha / sqrt(P'\Omega P)]$$