Research on Intraday Higher Moments and Stock Return

Based on Black-Litterman Model

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Abstract

We aim to incorporate realized higher moments into Markowitz Portfolio theory by Black-Litterman model. First, we use 5-minute returns of components of Dow Jones Index to obtain a daily measure of higher moments and analyze the relationship between intraday price movement and future stock returns. Second, we incorporate realized volatility and skewness as our investor’ s view into portfolio construction by using Black-Litterman Model. Our portfolio outperforms traditional Markowitz Portfolio.

1 Introduction

In this project, we analyze the relationship between higher moments (volatility, skewness and kurtosis) computed from intraday stock returns and the future stock returns, then, incorporate those moments as trading signals in Markowitz optimization by Black-Litterman model to improve the performance.

The relationship between higher moments and stock returns have attracted researchers' attention since 1976 (Kraus [1]). Recent papers con firm that higher moments of the underlying stock return distribution are related to future returns (Boyer and Mitton(2010)[2], Xing and Zhang(2010) [3]).

Though scholars hold various view on wether the moments positively or negatively affect the future returns under different settings, strong relationship between intraday realized higher moments and next week s stock returns have been testified. We test the effect and efficiency of higher moments on latest market data and create our trading strategy.

Thus, we divide our project into two parts.

Preliminary Test the effect of realized higher moments on predicting future stock returns. Construct Moment-Based Portfolios.

Trading Strategy and Back test Incorporate higher moments to improve Markowitz Optimization by Black-Litterman model.

2 Data Processing

We first describe the data collection and cleaning. We then show how the realized higher moments are computed.

2.1 Data

We analyze all 30 component stocks of Dow Jones industrial index (DJI) from March 5^{th}, 2013 to February 28^{th}, 2018. To compute the daily higher moments, we resort to high frequency price data.

We collect prices every 5 minutes in each trading day from Bloomberg terminals, and construct fi ve-minute log-returns for a total of 1255 trading days.

Missing Data

The advantage of analyzing components of DJI is the completeness of data. Although several tick price data might be missing, it cast no darkness on computing intraday higher moments. Only if the every tick data of a stock are missing in a trading day, we will drop the day out of our consideration.

Training Set: From March 5^{th}, 2013 to February 28^{th}, 2017.

Test Set: From March 1^{st}, 2017 to February 28^{th}, 2018.

2.2 Realized Higher Moments

Firstly, to compute the statistics of a stock, we defi ne the **i-th intraday log returns** for each company.

$$r_{t,i} = \log\left(P_{t-1+\frac{i}{N}}\right) - \log\left(P_{t-1+\frac{i-1}{N}}\right)$$

where P_k is the price observed at time k and N is the number of return observations in a trading day. In a trading day, there 6.5 trading hours, namely, $N = 6.5 \times 12 = 78$ observations per day.

Second, we define the daily realized higher moments (Andersen and Bollerslev (1998)[4]).

2.2.1 realized daily variance (RDVar)

1.Definition: The summation of squares of intraday high-frequency returns.

$$RDVar_t = \sum_{i=1}^{N} r_{t,i}^2$$

2.Description:

- It is a model free volatility measure. As sampling size increases, RDVar converges to sample variance.
- Note that RDVar represents the intraday volatility which is not the same as the variance used in Markowitz Model, which relates to the price movement in a longer term. In some way, it may be favorable.

2.2.2 realized Daily skewness (RDSkew)

1.Definition:

$$RDSkew_t = \frac{\sqrt{N}\sum_{i=1}^{N} r_{t,i}^3}{RDVar_t^{3/2}}$$

2.Description:

- It measures the degree of asymmetry of the daily return distribution.
- Negative values indicate that the stock s return distribution has a fatter left tail, vice versa.

2.2.3 realized daily kurtosis (RDKurt)

1.Definition:

$$RDKurt_t = \frac{N \sum_{i=1}^{N} r_{t,i}^4}{RDVar_t^2}$$

2.Description:

- It quantify the degree of tailedness of the daily return distribution.
- Large values indicate that the stock s return distribution has fatter tails and higher peak.

2.2.4 Adjust to weekly Realized higher moments

Third, adjust the moments above to weekly and monthly frequency, which will facilitate the Morkowitz part. The formula for weekly adapted higher moments are listed below. Note that, as is standard, we have annualized the realized volatility measure to facilitate the interpretation of results (See the first equation below).

-
$$RVol_t = \left(\frac{250}{5} \sum_{i=0}^{4} RDVar_{t-i} \right)^{1/2}$$

-
$$RSkew_t = \frac{1}{5} \sum_{i=0}^{4} RDSkew_{t-i}$$

-
$$Rkurt_t = \frac{1}{5} \sum_{i=0}^{4} RDKurt_{t-i}$$

Add visualization and interpretation

3 Efficiency Test Of Higher Moments

We test the efficiency by traditional information coefficients and cross-section linear regression. Note that only sketchy descriptions can be provided in this section, which ignites the intuition of creating trading strategies. In next section, we will discuss the characteristics of the higher moments in depth.

3.1 Statistical Characteristics

First, we compute the spearman correlation coefficient of moments in current week (or month) and return of next week (or month). In each week (or month), the average of the

Table 1: weekly statistics

	Rank IC	IC_IR	Positive significant rate	Negative significant rate
$RDVar$	0.0073	0.0291	0.0650	0.0850
$RDSkew$	-0.0172	-0.0778	0.0400	0.0600
$RDKurt$	-0.0111	-0.0578	0.0500	0.0850

Table 2: monthly statistics

	Rank IC	IC_IR	Positive significant rate	Negative significant rate
$RDVar$	0.0911	0.3426	**0.1633**	**0.0204**
$RDSkew$	-0.0057	-0.0283	0.0204	0.0612
$RDKurt$	0.0399	0.1994	0.1020	0.0612

correlation coeff is named as **Rank IC**, we report the average the weekly (monthly) Rank IC in Table 1 and 2, while the average scaled by its standard deviation is called **IC_IR**. And, we also compute the ratio of (positive and negative) significant IC.

Summary:

- From IC, we know that the higher the realized volatility the higher the mean stock return. The monthly statistics shows a clearer pattern.
- The RDSkew affects future return negatively. The higher Realized skewness, the lower the future returns. It seems that the market favors the negative skewness.
- The RDKurt show an ambiguous pattern in weekly and monthly frequency. It is not a stable factor.

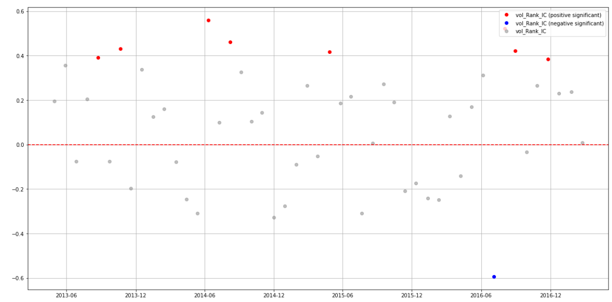

Fig 1: IC of Monthly Realized Volatility

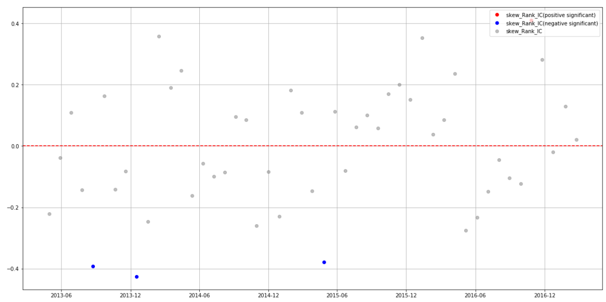

Fig 2: IC of Monthly Realized skewness

3.2 Cross-sectional Regressions

To further assess the relationship between future returns and realized volatility, realized skewness, and realized kurtosis, we carry out various cross-sectional regressions using the method proposed in Fama and MacBeth (1973)[5]. For each week t, we compute the realized moments for the $i-th$ firm and estimate the following cross-sectional regression on the week

$t+1$ returns.

$$r_{i,t+1} = \beta_{0,t} + \beta_{1,t} RVol_{i,t} + \beta_{2,t} RSkew_{i,t} + \beta_{3,t} RKurt_{i,t} + \varepsilon_{i,t+1} \quad (1)$$

Parameters

- $\mathbf{r_{i,t+1}}$, is the weekly (or monthly) return of the stock i for week (or month) $t+1$, as response variable.
- $\mathbf{RVol_{i,t}}$, $\mathbf{RSkew_{i,t}}$, $\mathbf{RKurt_{i,t}}$ are weekly (or monthly) higher moments as predictors.

In Table 3 and Table 4, We report the average of the coefficient estimates for the weekly regressions along with the p-value (in parentheses).

Equation 1: $r_{i,t+1} = \beta_{0,t} + \beta_{1,t} RVol_{i,t} + \varepsilon_{i,t+1}$

Equation 2: $r_{i,t+1} = \beta_{0,t} + \beta_{2,t} RSkew_{i,t} + \varepsilon_{i,t+1}$

Equation 3: $r_{i,t+1} = \beta_{0,t} + \beta_{3,t} RKurt_{i,t} + \varepsilon_{i,t+1}$

Equation 4: $r_{i,t+1} = \beta_{0,t} + \beta_{1,t} RVol_{i,t} + \beta_{2,t} RSkew_{i,t} + \beta_{3,t} RKurt_{i,t} + \varepsilon_{i,t+1}$

Table 3: Cross-Sectional Regressions (weekly)

	1	2	3	4
intercept	0.000855	0.002484	0.003865	0.001258
RVol	0.011892 (0.3775)			0.016367 (0.3940)
RSkew		-0.0014 (0.4467)		-0.00113 (0.4377)
RKurt			-0.00027 (0.4578)	-0.00025 (0.4871)
Adjusted R^2	0.062604	0.049874	0.040744	0.047793
counts of significant weeks	32/200	20/200	16/200	

Table 4: Cross-Sectional Regressions (monthly)

	1	2	3	4
intercept	-0.01192	0.010803	-0.00154	-0.02169
RVol	0.158607 (0.3179)			0.142732 (0.3498)
RSkew		-0.00132 (0.4692)		0.000014 (0.4802)
RKurt			0.002377 (0.4751)	0.002434 (0.4381)
Adjusted R^2	0.070748	0.035209	0.041129	0.047667
counts of significant months	10/49	3/49	4/49	

The first to three column report the results of the regression of the stock return on lagged realized volatility, realized skewness and realized kurtosis. The results present that p values are around 0.4, which means they do not show statistical significance. Especially, coefficient of Realized kurtosis shifts from negative to positive when the regression is conducted by monthly data. Hence, it confirm that Realized kurtosis is not a satisfying factor. We will exile it from our optimization model, the second part of our project.

However, the first column presents relationship between stock return and lagged realized volatility. The coefficient is 0.011892 weekly and 0.158607 monthly, with the strongest explanatory power in all of the linear regressions. This implies that realized volatility affects the return **positively.**

4 Realized Moments and the Cross-Section of Stock Returns

We now further examine the interaction between the effects of realized skewness and realized volatility on returns. Here, we construct two kinds of portfolio to analyze the relationships.

Note that we rebalance every month.
Type I Portfolios using a single sort on realized volatility and skewness respectively.
Type II Portfolios using a **double sort** on realized skewness and realized volatility and then examine subsequent stock returns.

4.1 Portfolio of Type I

First, we form three tertiles portfolios with different levels of realized volatility and skewness. We sort the 30 DJI components according to realized volatility (skewness) in ascending order, then, cluster them into 3 groups. We assume that, in each group, stocks share similar realized volatility.

4.1.1 Sort by Realized Volatility

We show the monthly average return and the total return across the entire test set and relative statistics below.

Table 5: Weekly and monthly Realized Volatility Portfolios

weekly	feature	avg weekly return	total return	win rate
group 1	low vol	0.002382	0.4764	0.3050
group 2	mid vol	0.002720	0.5439	0.3000
group 3	high vol	0.002798	0.5595	0.3950
	feature	avg monthly return	total return	win rate
group 1	low vol	0.006738	0.3301	0.2041
group 2	mid vol	0.008601	0.4215	0.3265
group 3	high vol	0.015186	0.7441	0.4694

From Table 5, we know that no matter we rebalance every week or month, long stocks with high realized volatility will generate higher return. Such portfolios outperform the others with a probability of 39.5% and 46.94%. It implies that intraday volatility predicts higher future return.

To display the performance clearly, we plot the trajectory of each portfolio in Fig 5 and 6. We regard the average portfolio as the benchmark plotted in bold red line. For details, Fig 8 and 9 in appendix can be referred.

It is obvious that, weekly portfolio fluctuates greatly than monthly portfolio. Excessively frequent rebalance could be detrimental to total return.

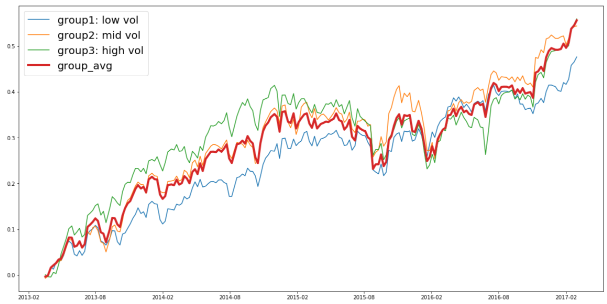

Fig 3: Weekly Realized Volatility Portfolios

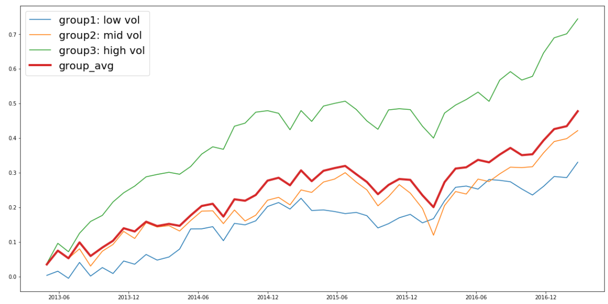

Fig 4: Monthly Realized Volatility Portfolios

4.1.2 Sort by Realized Skewness

We show the monthly average return and the total return across the entire test set and relative statistics below.

Table 6: Weekly and monthly Realized skewness Portfolios

weekly	feature	avg weekly return	total return	win rate
group 1	low skewness	0.002527	0.5055	0.3050
group 2	mid skewness	0.002773	0.5546	0.3750
group 3	high skewness	0.002112	0.4223	0.3200
monthly	feature	avg monthly return	total return	win rate
group 1	low skewness	0.011586	0.5677	0.3878
group 2	mid skewness	0.007633	0.3740	0.3469
group 3	high skewness	0.010751	0.5268	0.2653

From Table 6, in we know that lower and middle skewness (close to normal distribution) generates higher return. If we rebalnce the portfolio weekly, group 2 outperforms the others with a probability of 37.5%, while the monthly portfolio with a winning probability only 34.69%. If we rebalnce the portfolio monthly, group 1 outperforms the others with a probability of 38.78%, which means that lower skewness (left skewed distribution).

It implies that intraday volatility predicts higher future return.

To display the performance clearly, we plot the trajectory of each portfolio in Fig 5 and 6. We regard the average portfolio as the benchmark plotted in bold red line. For details, Fig 10 and 11 in appendix can be referred.

It is obvious that, weekly portfolio fluctuates greatly than monthly portfolio. Excessively frequent rebalance could be detrimental to total return.

Fig 5: Weekly Realized Skewness Portfolios

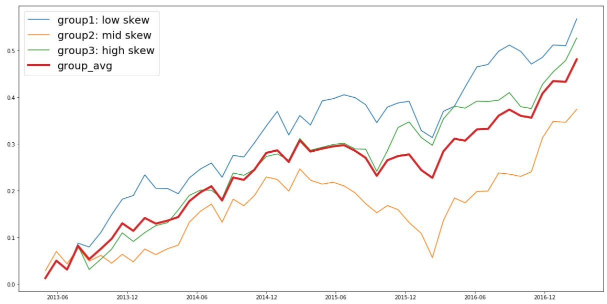

Fig 6: Monthly Realized Skewness Portfolios

4.2 Portfolio of Type II

We construct portfolios using a double sort on realized volatility and realized skewness, then examine subsequent stock returns. First, we form **three tertiles** portfolios with different levels of realized volatility monthly. Within each of these portfolios, we also create three portfolios that have different levels of realized skewness.

Each week, stocks are first sorted by realized volatile into three tertiles and then, within each tertile, stocks are sorted once again into three tertiles by realized skewness ascending. We present the monthly return in Table 7. The total returns are in parentheses.

Table 7: Double Sorting on Realized Volatility and Realized Skewness

		skewness		
		low	mid	high
volatility	low	0.004035898	0.008989452	0.006437246
		(0.1978)	(0.4405)	(0.3154)
	mid	0.009226201	0.006629761	0.010604401
		(0.4521)	(0.3249)	(0.5196)
	high	0.012576143	0.020146075	0.011181704
		(0.6162)	(**0.9872**)	(0.5479)

We observe that in higher realized volatility and mid skewness portfolios translate into higher returns.

5 Markowitz Optimization And Black-Litterman Model

5.1 Background

Markowitz Portfolio Theory (MPT) introduces mean-variance trade-off in investment, assembling a portfolio of assets such that the expected return μ is maximized for a given level of risk, defined as variance Σ. Its key insight is that an asset's risk and return should not be assessed by itself, but by how it contributes to a portfolio's overall risk and return.

General form of Markowitz Optimization Problem is equation 2, and equation 3 with risk-free asset x_0.

$$\begin{aligned} \max_{x} \quad & \mu^T x \\ s.t \quad & x^T V x \leq \sigma^2 \\ & e^T x = 1 \end{aligned} \tag{2}$$

$$\begin{aligned} \max_{x,x_0} \quad & \mu_0 x_0 + \mu^T x \\ s.t \quad & x^T V x \leq \sigma^2 \\ & x_0 + e^T x = 1 \\ & x_0 \leq 1 \end{aligned} \tag{3}$$

Though Markowitz Portfolio Theory is theoretical important, it fails to match the real world in some ways. One we want to mention is that the risk, return, and correlation measures are based on expected values. In traditional MPT, predictions are substituted based on historical measurements of asset return and volatility. It is prevalent to use average historical return of stock i, $\frac{1}{T}\sum_{l=1}^{T} r_i^{(t-l)}$, as the estimator of μ_i, and historical covariance matrix $\Big(Cov(r_i, r_j)\Big)_{i,j}$ to substitute V.

Very often such expected values fail to reflect new circumstances that did not exist when the historical data were generated.

The Black-Litterman Model seeks to overcome the problems and provides an approach for choosing the expected return μ and covariance V to be used in a portfolio model. We can merge the market' s view and an investors' s view, and thereby provide a model reflecting the investors' s edge.

5.2 Construct Portfolio Using Black-Litterman Model

We now walk through and illustrate the usage of the Black-Litterman model in three simple steps.

5.2.1 Step 1: Starting Point

Basic assumption: underlying the Black-Litterman model is that the expected return of a security should be consistent with market equilibrium unless the investor has a specific view on the security (Fischer Black and Robert Litterman, 1990 [6]).

Since Black-Litterman Model combines market historical information (Markowitz's view) and investors' perspective to optimize the investment, average historical return $\vec{R}$ and covariance Σ are still useful components to estimate μ and V.

Here, we assume that traditional MPT above serves as a reasonable estimate of the true expected returns in the sense that

$$\vec{R} = \mu + \varepsilon_{\vec{R}}, \quad \varepsilon_{\vec{R}} \sim N(\vec{0}, \tau\Sigma) \tag{4}$$

for some small parameter $\tau > 0$. We can think about $\tau\Sigma$ as our confidence in how well we can estimate the expected returns. With large τ (higher deviation), R is deemed to be greatly volatile around the true value of μ. In other words, a $\tau < 1$ implies a high confidence in historical information and vice versa.

5.2.2 Step 2: Expressing an Investor's Views

If there are K vies in the Black-Litterman model are expressed as a K-dimensional vector $\bar{b}$ with

$$\bar{b} = A\mu + \varepsilon_{\bar{b}}, \quad \varepsilon_{\bar{b}} \sim N(0, \Omega) \tag{5}$$

where A is a $K \times N$ matrix (explained in the following example) and Ω is a $K \times K$ matrix expressing the confidence in the views. For explicit details, readers can refer to Page 16 on Slides.

5.2.3 Step 3: Combining an Investor's Views with Markowitz Model

Having specified the relationship between markowitz and an investor' s views separately, we are now ready to combine the two together. Now, we recall equation 4 and 5, and stack these two equations together in the form of

$$\begin{cases} \vec{R} = \mu + \varepsilon_{\vec{R}}, & \varepsilon_{\vec{R}} \sim N(\vec{0}, \tau\Sigma) \\ \bar{b} = A\mu + \varepsilon_{\bar{b}}, & \varepsilon_{\bar{b}} \sim N(0, \Omega) \end{cases} \Rightarrow y = X\mu + \varepsilon, \varepsilon \sim N(0, V) \tag{6}$$

where

$$y = \begin{pmatrix} R \\ \bar{b} \end{pmatrix}, \quad X = \begin{pmatrix} I \\ A \end{pmatrix}, \quad V = \begin{pmatrix} \tau\Sigma & 0 \\ 0 & \Omega \end{pmatrix} \tag{7}$$

It is intuitive to estimate μ by **Generalized Least Squares** (GLS) estimator in equation 8 as well as its variance in equation 10.

$$\mu_{BL} = (X^T V^{-1} X)^{-1} X^T V^{-1} y \tag{8}$$

$$= \left[(\tau\Sigma)^{-1} + A^T \Omega^{-1} A\right]^{-1} \left[(\tau\Sigma)^{-1} R + A^T \Omega^{-1} \bar{b}\right] \tag{9}$$

$$\begin{aligned} Var(\mu_{BL}) &= (X^T V^{-1} X)^{-1} \\ &= \left[(\tau\Sigma)^{-1} + A^T \Omega^{-1} A\right]^{-1} \end{aligned} \tag{10}$$

Define $W := Var(\mu_{BL})$

5.2.4 Step 4: Crystalize an Investor's Views

Now, we apply the results in Section 4, and incorporate realized volatility and realized skewness into Black-Litterman model.

In Section 4, We sort the 30 DJI components according to realized volatility (skewness) in ascending order, then, cluster them into 3 groups. We find that stocks with high intraday realized volatility and medium realized skewness outperforms the others. We can express this result by setting the matrix A and Ω in equation 7. In order to introduce our method, let us take an example.

- For instance, at time t, we consider realized volatility(RVol) in previous n_s[1] days (i.e. from day $t - n\_s$ to day $t - 1$).
- The 30 component stocks in Dow Jones industrial index serve as our asset pool. We compute the sample variance of all $RVol_k^{(i)}$, $i = 1, \cdots, 30$, $k = t - n\_s, \cdots, t - 1$, and denote it as ω_{RVol_t}.
- We compute the average RVol of each stock, record them in vector $\overline{RVol}_t = (\overline{RVol}_t^{(1)}, \cdots, \overline{RVol}_t^{(30)})^T$.
- We sort the stocks ascending according to $\overline{RVol}_t^{(i)}$, and divide them into 3 groups. Group 1 is of lowest RVol, while Group 3 with the highest.
- According to results in section 4.2, we choose Group 3. Then, we can write down $A_{\mathrm{RVol}_t} = (a_t^{(1)}, \cdots, a_t^{(30)})$, while

$$a_t^{(i)} = \begin{cases} 1 & \text{if stock i is in group 3} \\ 0 & o.w. \end{cases} \tag{11}$$

- Similarly, we can adapt it to process realized skewness signal, and get $\omega_{\mathrm{RSkew}_t}$ and A_{RSkew_t}. But now, group 2 is favorable.
- We instill our view into the Black-Litterman model by setting

$$A = \begin{pmatrix} A_{\mathrm{RVol}_t} \\ A_{\mathrm{RSkew}_t} \end{pmatrix}, \quad \Omega = \begin{pmatrix} \omega_{\mathrm{RVol}_t} & 0 \\ 0 & \omega_{\mathrm{RSkew}_t} \end{pmatrix} \tag{12}$$

6 Realization

6.1 Parameters

- `initial wealth=10000`
- `horizon = 20` # monthly rebalance
- `start = 100` # the day on which you are first given a portfolio to rebalance
- `number_of_samples = 60` # how many samples are to be used in computing return averages and covariances
- `sample_frequency = 1`
- `tau=2` # confidence parameter

[1] n_s is short for "number of samples"

Know that we are confident on our view of market, we set the confidence parameter $\tau = 2$.

6.2 Optimization

Here, we just write down the essential part of the Black-Litterman Optimization. For details, please resort to `BACKTEST_FOR_BOTH_MODELS_MULTISIGNAL.py`.

```
BL_mu = W*(A.T*np.linalg.inv(Omega)*b+np.linalg.inv(V)*mu)
U = np.linalg.cholesky(W)

x0 = Variable(1)
x = Variable(n)
y = Variable(n)
total_trans_cost = Variable(1)

objective = Maximize(mu0*x0+BL_mu.T*x)
constraints = [norm(U*x)<=sigma,
                   x0+sum_entries(x)+total_trans_cost==1,
                   x==xx+y,
                   trans_cost*sum_entries(abs(y))<=total_trans_cost,
                   x0>=0,
                   max_entries(abs(x-1.0/(n+1)))<=0.05
                   ]
prob = Problem(objective, constraints)
result = prob.solve()
prob.solve()
```

6.3 Results

Here, we use equal-weighted portfolio as benchmark. And, the Black-litterman Model is also compared with corresponding Markowitz model. It is obvious that, our Black-litterman portfolio outperforms the traditional Markowitz almost at any time. However, generally equal-weighted portfolio catch the market trend better and gain better returns.

Table 8: Performance

	Initial wealth	Final Value	Return per annum
Markowitz_wealth	10000	16036.92299	0.120327914
BlackLitterman_wealth	10000	16372.28044	**0.125919643**
Benchmark	10000	17012.87827	0.136363562

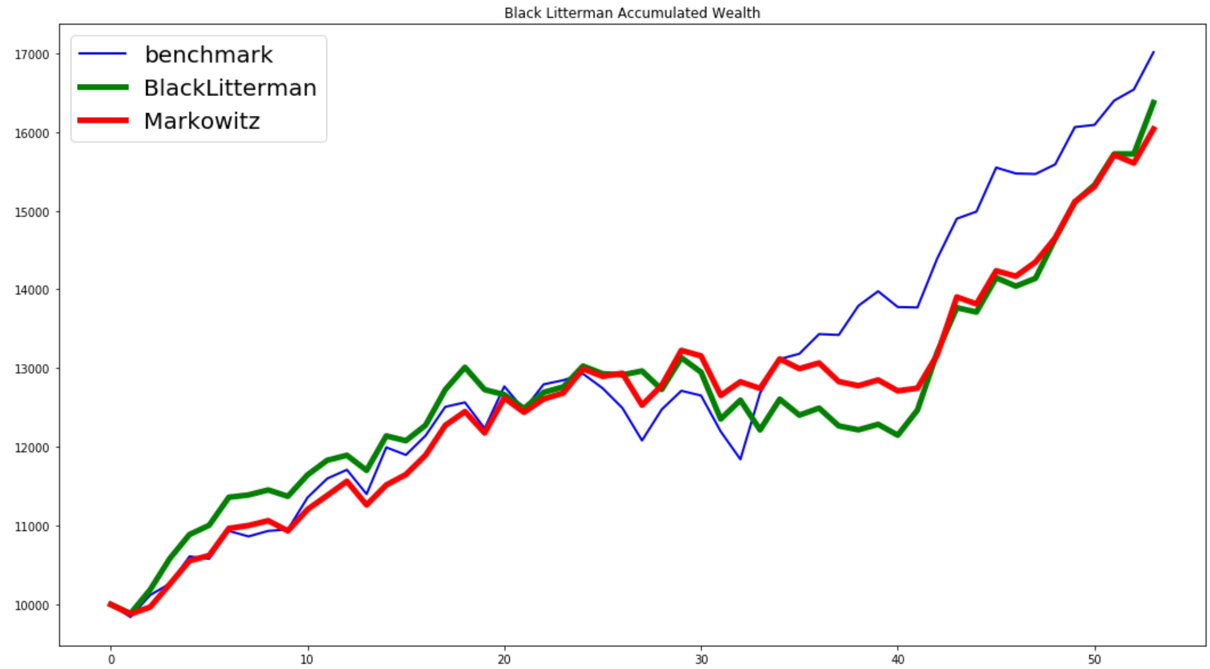

Fig 7: Black Litterman Accumulated Wealth

7 Conclusions

In this project, we introduce model-free estimates of intraday Realized volatility, skewness and kurtosis. we use 5-minute returns of components of Dow Jones Index to obtain a daily measure of higher moments and analyze the relationship between intraday price movement and future stock returns. We find that realized skewness and realized volatility predict next week s stock returns in the cross-section better than realized kurtosis, in the cross-section regressions. Kurtosis shows an ambiguous pattern when forecasting stock return.

By constructing single sorting portfolio based on realized volatility and skewness respectively (section 4), we find that realized volatility is positively related to future stock returns, while the realized skewness is slightly negatively. A portfolio that buys stocks with high realized volatility generates average monthly return of 1.52%, and 1.16% when buying stocks with low skewness. Double sorting portfolio (section 4.2) confirms that stocks with high realized volatility and low skewness simultaneously will generates favorable profits.

In the second part (section 5), we incorporate realized volatility and skewness as our investor's view into portfolio construction by using Black-Litterman Model. We combine the investor's view and historical market price information together to improve the estimation of expected return μ. Then, we use it to create a portfolio, obtaining an annualized return of 12.59%, which surpasses the traditional Markowitz Portfolio.

8 Future Work

Insignificant Coefficients

In Section 3.2, we conduct cross-sectional regression on higher moments and future stock returns. However, the coefficients are not statistically significant. The possible reasons may be,

- Realized skewness is regarded as a kind of anomalies. DJI consists of mature companies with high capitalization, so they may not be susceptible to anomalies.
- Realized higher moments may consist of assortment of other famous factors, such as momentum factors or size factors. To unveil the truth of this story, we need to remove the impurity hidden inside the realized higher moments.

In the future, we can using S&P 500 components instead of DJI, and focus on small and immature companies.

Improve Black-Litterman

In section 5.2.4, we incorporate realized higher moments by setting Matrices A and Ω by the form of eq (12). Here, we assume that the factor realized volatility and skewness are uncorrelated, because we set Ω to be a **diagonal matrix**. We need to test if this assumption holds. Or, we have to put the covariance of these two signals on non-diagonal entries.

Reference

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Appendix A Portfolio: Sort by higher moments

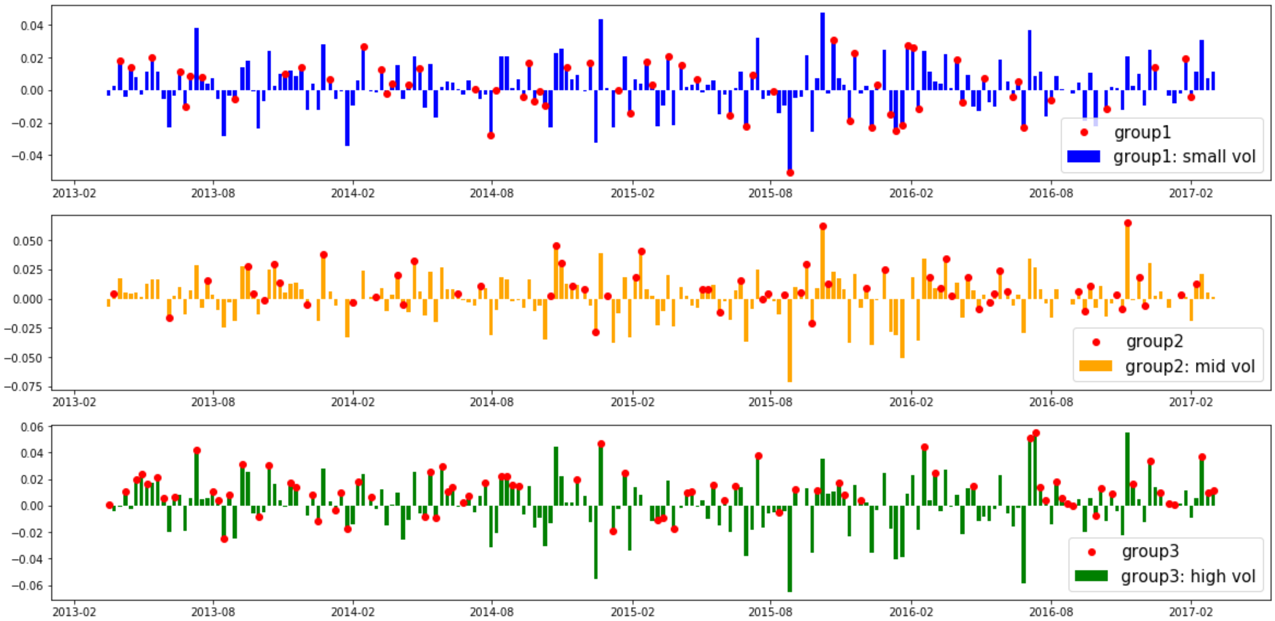

Fig 8: Details of Weekly Realized Volatility Portfolio

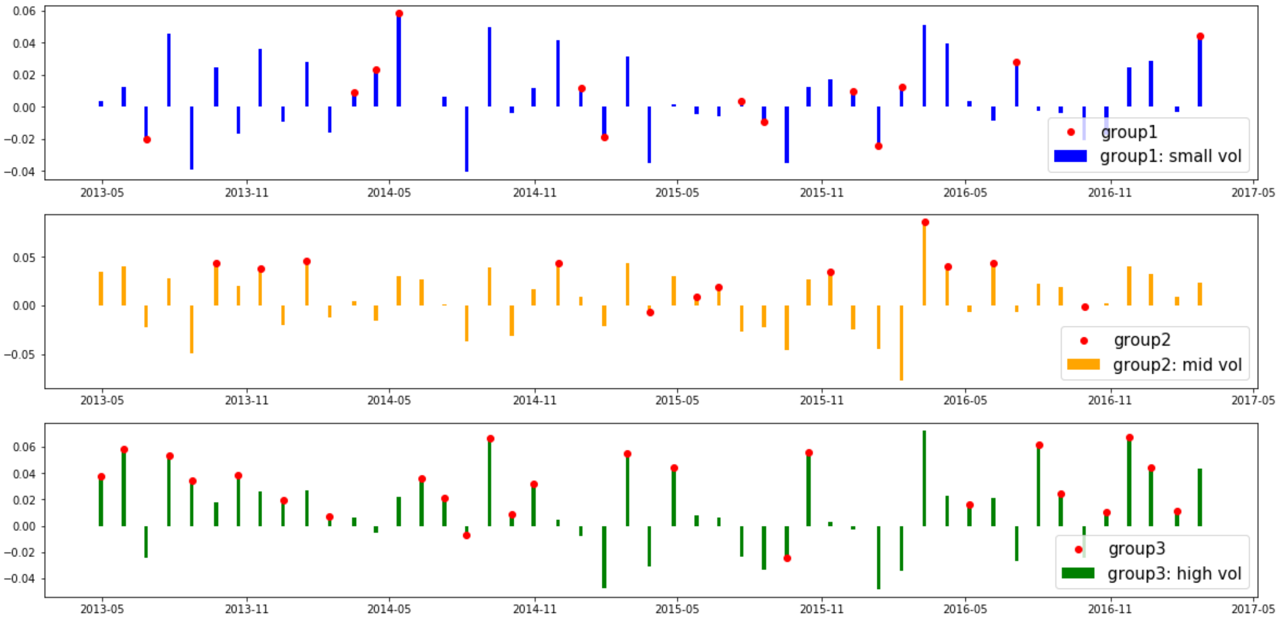

Fig 9: Details of monthly Realized Volatility Portfolio

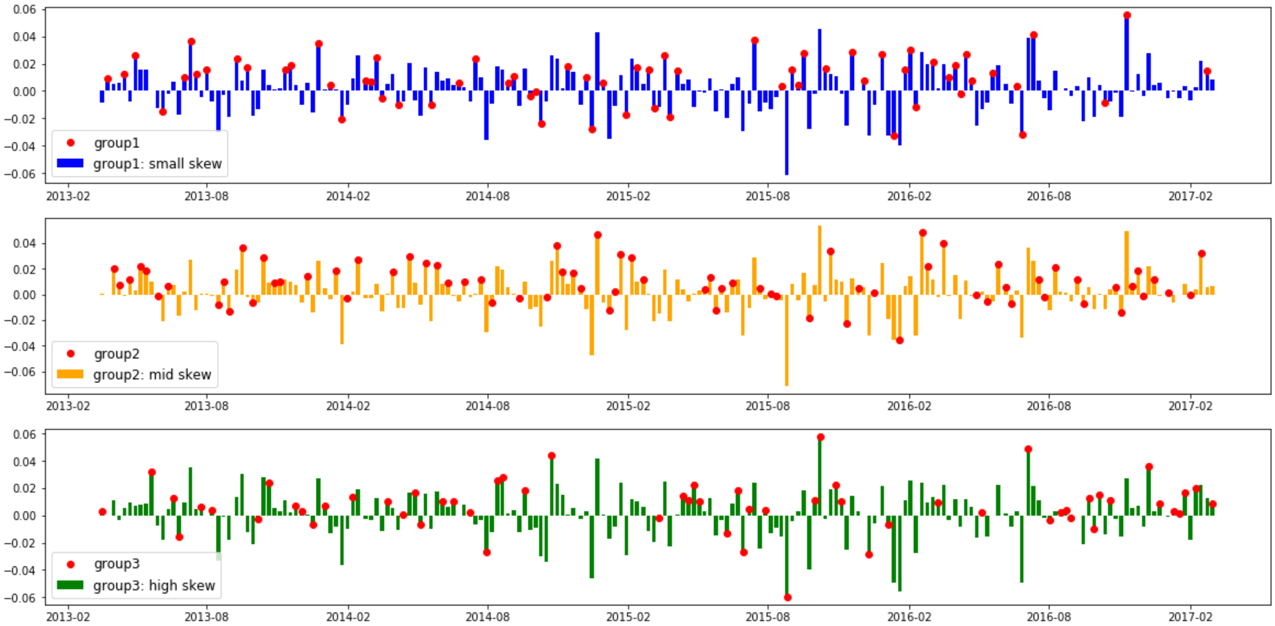

Fig 10: Details of Weekly Realized skewness Portfolio

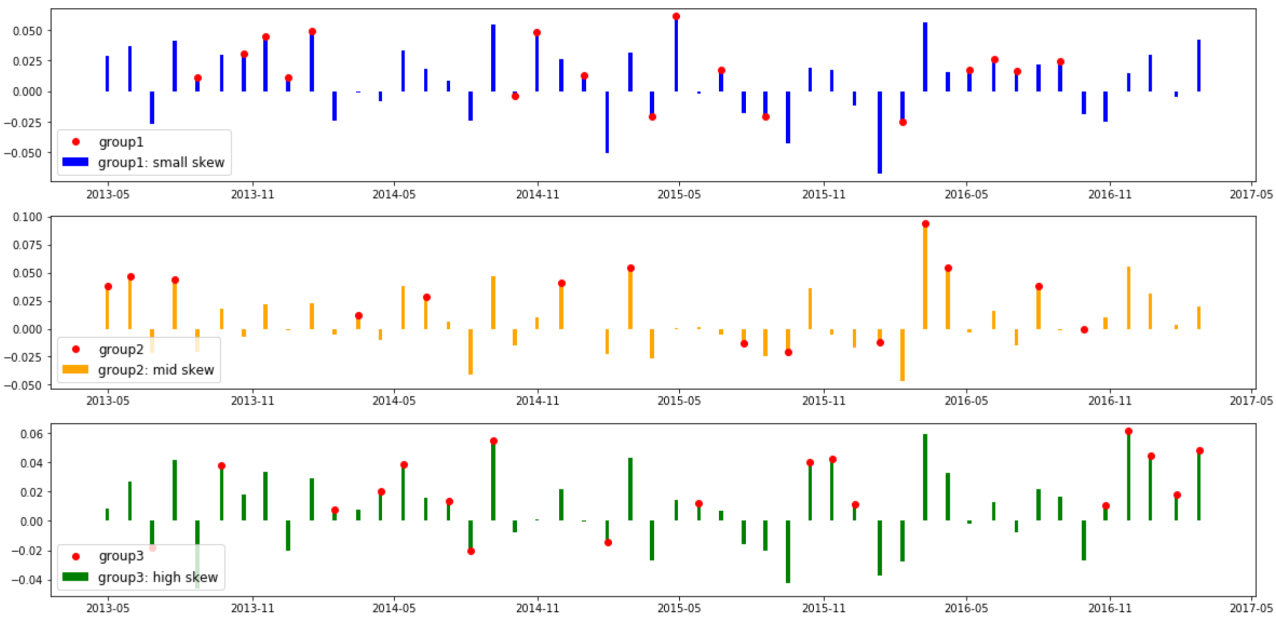

Fig 11: Details of monthly Realized skewness Portfolio

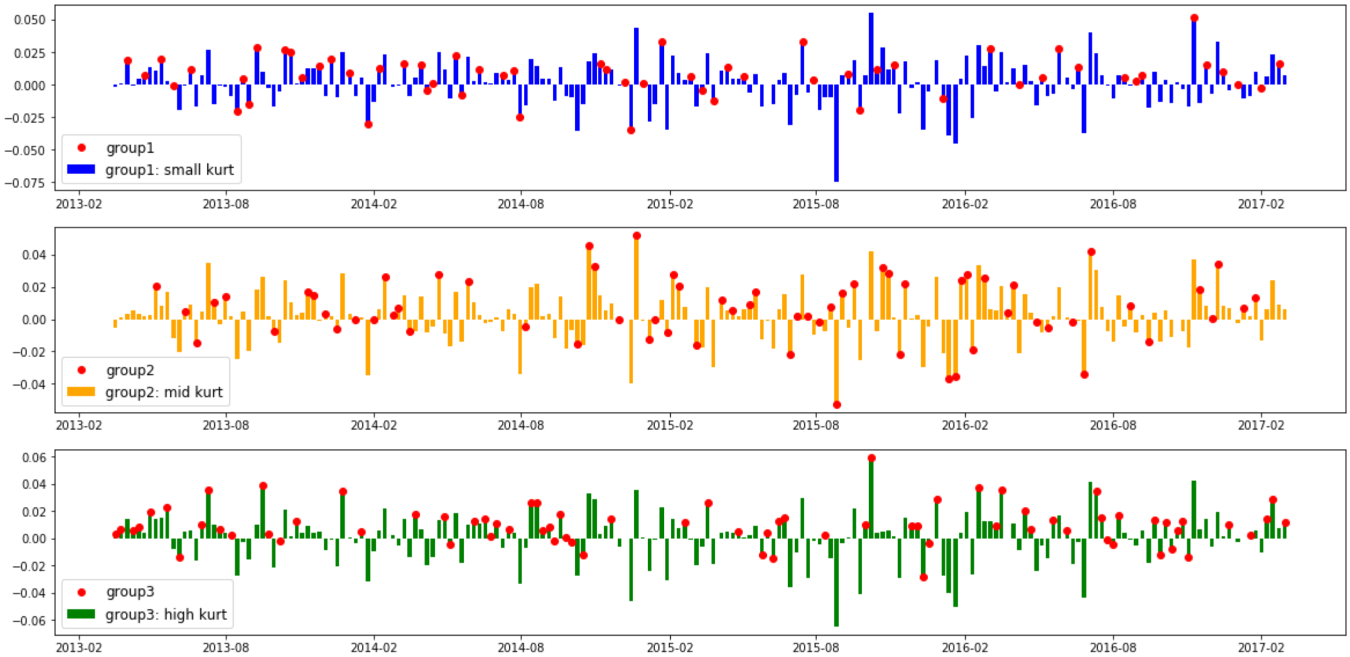

Fig 12: Details of Weekly Realized kurtosis Portfolio

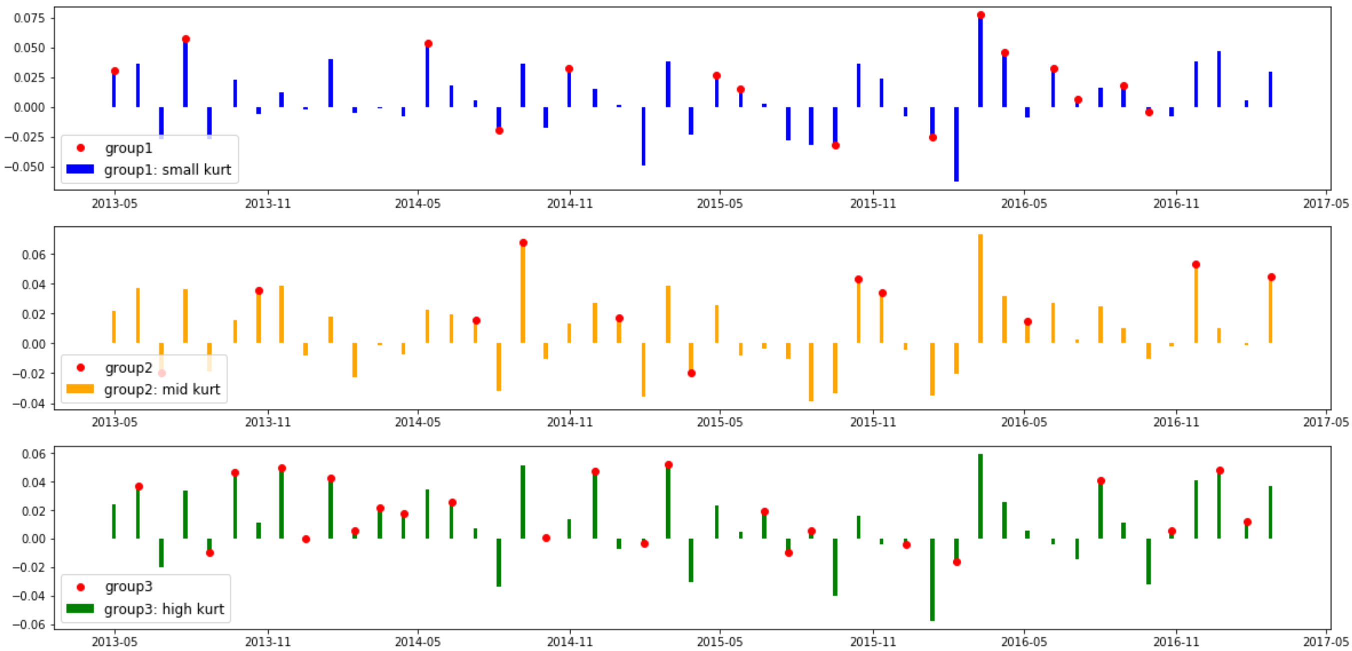

Fig 13: Details of monthly Realized kurtosis Portfolio