

## Non-Vanilla Payoffs Problems

### Applications

#### 1. A dispersion trade

Dispersion trading seeks to capture gains relating to the correlation among individual constituents of an equity basket, such as an index. Because of diversification effects, the variance of the basket return should be less than the sum of the constituents' variances, with the degree of difference attributable to the correlations of the constituents to one another. The purpose of this problem is to construct a simplified dispersion trade and investigate its correlation behaviors.

Consider the two assets, which pay no dividends, and a basket of the two constructed of fixed quantities of each:

| asset  | S0  | basket quantity (shares) |
|--------|-----|--------------------------|
| A      | 125 | 0.4                      |
| B      | 40  | 1.25                     |
| basket | 100 | 1                        |

For 6-month European options on these underlyings, the Black-Scholes implied volatilities for at-the-money forward (ATMf) options are 0.325 for asset A, 0.45 for asset B, and 0.31 for the basket. Note that the implied volatility for the basket option is calculated as if the basket price followed a geometric Brownian motion in its own right.

The risk-free interest rate is 1% to all maturities, expressed with continuous compounding.

- Determine the price of ATMf straddles on 0.4 shares of asset A, 1.25 shares of asset B, and a single share of the basket.
- Using your preferred method (quasi-analytic or Monte Carlo), build a pricer for an option on the basket using the correlation as an input. With your pricer, determine the values of ATMf call and put options on one share of the basket under the following correlation scenarios: -1, 0, 1. (Note that a quasi-analytic pricer will likely fail with inputs at either extreme; use correlations very, very near the extreme values to answer the question if you have chosen this implementation. If you use a Monte Carlo pricer, in addition to your final prices provide 95% confidence intervals for the prices, and report how many trials were used to obtain these results.)
- Determine the correlation implied by the market price of the ATMf straddle.
- You purchase, in the same quantities as the basket, ATMf straddles on asset A and asset B individually. What quantity of ATMf straddles on the index is needed to finance your purchase? Is the resulting portfolio long or short the correlation between the assets?
- Use your pricer to determine the PL of the portfolio in part d under an increase of 0.01 in the market-implied asset correlation; do the same for a decrease of 0.01 in the market-implied correlation.
- Use your pricer to estimate the deltas of the portfolio in part d with respect to each of the underlying assets.