6. Forwards and Futures

Exercises

1. The term structure of risk-free interest rates is as follows:

t	r
3m	0.025
6m	0.027
9m	0.03
1y	0.0285
15m	0.0295
18m	0.03
21m	0.0305
2y	0.031
3y	0.04
4y	0.0465
5y	0.05

All rates are expressed with continuous compounding. Determine the forward price in 6 months of the following assets:

(a) A zero-coupon bond with maturity nine months, per 100 face.
(b) A bond paying a 5% coupon semiannually with maturity 1.25 years, per 100 face.
(c) An equity with spot price 100 paying a 1.2% dividend continuously.
(d) An equity with spot price 300 that is expected to split 3 for 1 in 2 months. (Express the forward as a price per share.)
(e) A foreign currency with spot exchange rate 1.2—expressed as number of units foreign currency per unit domestic currency—in which the 6-month risk-free rate is 2.2%, expressed with continuous compounding. (Express your answer as number of units foreign currency per unit domestic currency.)

2. An equity's spot price S_0 is 100 at the moment. Using other traded market instruments, you have determined that the term structure of zero rates and equity forward prices is as follows:

t	r (continuous)	Ft
1m	0.02	100.0625195
2m	0.022	100.2628448
3m	0.0225	100.3882518
4m	0.0245	100.5934205
5m	0.025	100.8200105
6m	0.026	101.0732181

(a) What is the term structure of dividend rates in each month implied by these values? (Express your answer as the continuously compounded dividend rate from now to 1m, from 1m to 2m, 2m to 3m, and so on.)
(b) Suppose that the equity has an established record of paying its dividend, if any, at the end of each month, relative to the date for which you extracted this data. Under this assumption, what the is amount of the lump-sum cash dividend at the end of each month implied by this data? (Assume that the forward price as expressed here is *after* the payment of the fixed dividend.)

Applications
1. *The risk of a correlated hedge*
In the chapter, it is shown how one may hedge the risk of an asset with random price X using a hedge in a related asset with price Y by forming the portfolio short β units of Y per unit of X, where β is the minimum-variance hedge ratio:

$$\beta = \rho_{XY}\frac{\sigma_X}{\sigma_Y}$$

…with σ_X and σ_Y the daily standard deviations in the prices of their respective assets, and ρ_{XY} the correlation between the prices. The goal of this problem is to assess the risk of such a hedge. We assume that each asset's closing price tomorrow is normally distributed, with the mean equal to today's closing price and standard deviations as above.

(a) Take Z_1 and Z_2 to be independent standard normal random variables, and define $Z_3 = k_1Z_1 + k_2Z_2$. Show that Z_3 is a normal random variable with mean zero, and express its variance in terms of k_1 and k_2.
(b) Derive expressions for k_1 and k_2 that cause Z_3 to be a standard normal random variable with correlation ρ to Z_1.
(c) If X_0 and Y_0 are the prices of each asset today, then let:

$$X = X_0 + \sigma_X Z_1$$

$$Y = Y_0 + \sigma_Y\left(k_1Z_1 + k_2Z_2\right)$$

$$V_\Pi = X - \beta Y$$

…represent tomorrow's closing prices, with k_1 and k_2 chosen using your expression from (b) above. Show that V_Π is normally distributed, and find expressions for its mean and variance.
(d) Suppose $X_0 = 100$, $Y_0 = 30$, $\sigma_X = 2.5$, $\sigma_Y = 1.1$, and $\rho_{XY} = 0.85$. Use your results from the previous steps to calculate the 1-day loss on the hedged portfolio that will be exceeded with probability 5%.