FX Problems

Exercises

1. The spot EURUSD exchange rate is 1.29 dollars per Euro. The volatility of the USDEUR rate is 8%. The 3M risk-free rate in USD is 0.3095%, and the 3M risk-free rate in EUR is 0.183%. Calculate the following, making it clear where appropriate in which currency your result is denominated:

(a) Long an option to pay \$1.3 million and receive €1 million in 3M.

(b) The delta hedge that would be used by a party short the option in part a. (This will consist of a certain quantity of the USD money market asset and a certain quantity of the EUR money market asset.)

(c) The value of a one-touch option that pays \$1 million if EUR touches the level 1.275 USD within the next 3M. The cash amount is paid at the time of the touch.

The general form of the pricing formula for one-touch options is shown below. Define the following quantities:

Q = the quantity of pricing currency paid in the event of touch

 $\eta = \text{upper} / \text{lower barrier indicator: } \eta = 1 \text{ for an upper barrier, } \eta = -1 \text{ for a lower barrier}$

 ω = payment time indicator: ω = 0 for pay-at-hit; ω = 1 for pay-at-expiry

 X_0 = the spot value of one unit of foreign in pricing currency

H = the barrier FX level, expressed as the price of one unit of foreign in pricing currency r = the pricing currency risk-free rate

 r_i = the foreign currency risk-free rate

T = time to expiry

 $\sigma = FX$ rate volatility

Then the value of the one-touch is given by:

$$V_{one-touch} = Qe^{-\omega rT} \left[\left(\frac{H}{X_0} \right)^{\frac{\theta+\theta}{\sigma}} N(\eta e_+) + \left(\frac{H}{X_0} \right)^{\frac{\theta-\theta}{\sigma}} N(-\eta e_-) \right]$$

$$\theta = \frac{r - r_f}{\sigma} - \frac{\sigma}{2}$$

$$\vartheta = \sqrt{\theta^2 + 2(1 - \omega)r}$$

$$e_{\pm} = \frac{1}{\sigma\sqrt{T}} \left(\pm \ln \frac{X_0}{H} - \sigma \vartheta T \right)$$

(d) The value of a no-touch option that pays €1 million if EUR does not touch the level 1.3 USD within the next 3M.