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Homework 7

Question 1.

(a) The variables in the program represent the binary decision of whether or not to assign a detection to a particular track. That is, we can let the binary variable x_{ij} represent whether the detection i is assigned to track j. Here we have that $i \in \{1, 2, ..., M\}$ and $j \in \{1, 2, ..., N\}$. Therefore there are $M \times N$ binary variables in total.

(b) We can define the objective function of this linear program to maximize the sum (total) of similarity scores from each track or detection pair that is assigned to each other. In other words, we want to find the assignment of detection to tracks that maximize the total similarity between assigned pairs. Mathematically the objective function is:

$$\max_{i,j} \sum_i \sum_j s_{ij} x_{ij} \tag{1}$$

Where s_{ij} denotes the similarity score between detection i and track j as given by the evaluator

(c)

$$\begin{aligned}
&\max_{i,j} \sum_i \sum_j s_{ij} x_{ij} \\
\text{subject to} &\sum_i x_{ij} \leq 1 \forall j \in \{1, 2, ..., M\} \\
&\sum_i x_{ij} \leq 1 \forall j \in \{1, 2, ..., N\} \\
&x_{ij} \leq \sum_k^{i-1} x_{kj} \text{ for } i \in \{2, 3..., M\} \text{ and } j \in \{2, 3..., N\} \\
&x_{ij} \leq \sum_k^{j-1} x_{kj} \text{ for } i \in \{2, 3..., M\} \text{ and } j \in \{2, 3..., N\} \\
&x_{ij} \in \{0, 1\}
\end{aligned} \tag{2}$$

Going by the constraints:

(i) $\sum_i x_{ij} \leq 1$: each detection can only be assigned to one track;
(ii) $\sum_i x_{ij} \leq 1$: each track can only be assigned to one detection;
(iii) $x_{ij} \leq \sum_k^{i-1} x_{kj}$: each track can only be assigned to detection if and only if it has been assigned to at least a previous detection;
(iv) $x_{ij} \leq \sum_k^{j-1} x_{kj}$: the detection can only be assigned to an active track;
(v) $x_{ij} \in \{0, 1\}$: all the variables are binary.