Fundamental Concepts of Quantum Mechanics

Aug. 18, 2016

* Roll Call
* Syllabus & HW Guideline

Stern - Gerlach Expt. (Otto Stern & Walther Gerlach)

O. Stern in 1921 at Frankfurt

O. Stern & W. Gerlach in 1922

$\boxed{\text{Most quantum mechanical, least classical system}}$

1) Use <u>charge zero</u> particle because you don't want the Lorentz force to deflect the trajectory.

2) Atom as a whole is <u>very heavy</u> so that the classical trajectory can be legitimately applied.

<u>Silver Atom</u> $\quad {}^{47}Ag$ 107.87

47 protons
61 neutrons

in 1 amu ≈ mass of a single nucleon (proton or neutron)

5s

$$46\left\{\begin{array}{l|llll} 18 & 4s & 4p & 4d & 4f \\ 18 & 3s & 3p & 3d & \\ 8 & 2s & 2p & & \\ 2 & 1s & & & \end{array}\right.$$

$\mu_{Ag} \approx \mu_{e^-}$ since the atom is <u>filled up to 4d and only one electron left at <u>5S</u> level.</u>

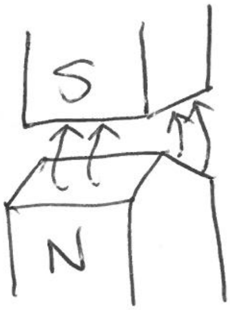
electron's intrinsic spin magnetic moment

$\frac{\mu_B}{\hbar}$ cf. Bohr magneton in CGS $\mu_B = \frac{e\hbar}{2m_e c}$

$$\mu_z = \frac{e}{m_e c} S_z \left(e<0 \text{ for electron}\right) = g\left(\frac{e}{2m_e c}\right) S_z \quad \left(g=2 \text{ for electron}\right)$$

3) Make <u>inhomogeneous magnetic field</u> to split different magnetic moments.

$$V(\vec{r}) = -\vec{\mu}\cdot\vec{B}(\vec{r}) \Rightarrow \vec{F} = -\vec{\nabla} V(\vec{r})$$

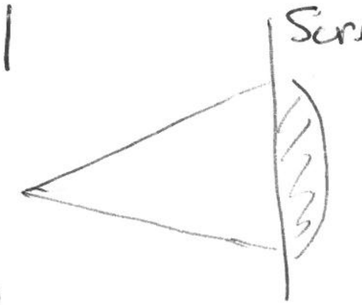


$$F_z = \frac{\partial}{\partial z}(\vec{\mu}\cdot\vec{B}) = \mu_z \frac{\partial B_z}{\partial z}$$

$$\frac{\partial B_z}{\partial z} < 0, \quad e < 0 \Rightarrow \underbrace{F_z \propto S_z}$$

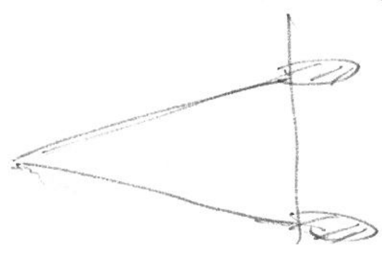
Spin aligns to the direction of force

If the spin is a classical object,
then $-|\vec{\mu}| < \mu_z < |\vec{\mu}|$



Continuous distribution expected from <u>classical physics</u>.

However, expts show "space quantization" !!



<u>Totally different from C.M.</u>

S_z + up; $S_z = +\frac{\hbar}{2}$

S_z − down; $S_z = -\frac{\hbar}{2}$

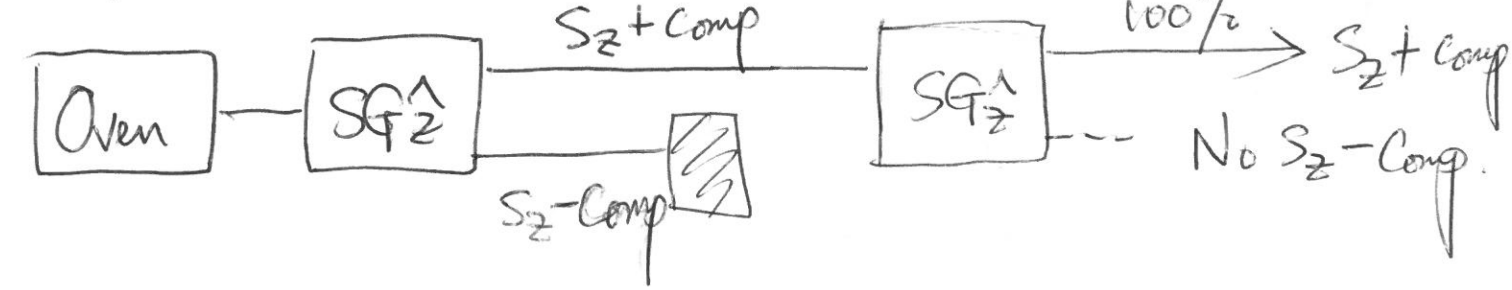
Planck's constant

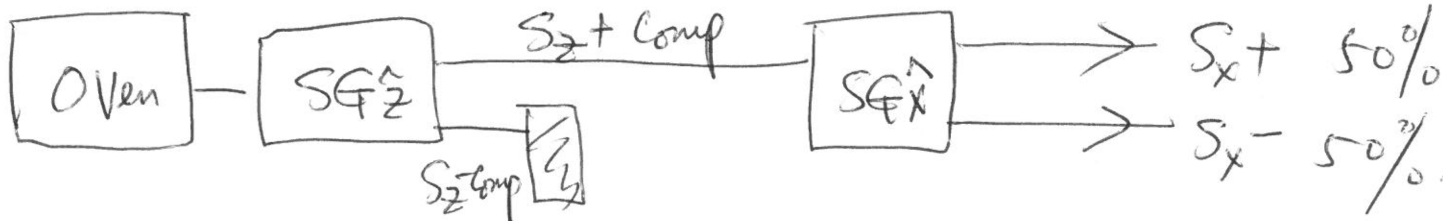
$\hbar = 1.0546 \times 10^{-27}$ erg-s

$= 6.5822 \times 10^{-16}$ eV-s

$\boxed{\text{Quantization of electron spin}}$

Expts





Does it mean that 50% of the atoms in the S_z+ beam coming out of the first apparatus ($SG\hat{z}$) are made up of atoms characterized by both S_z+ and S_x+, while the remaining 50% have both S_z+ and S_x-?

In classical picture, the spinning top with

$$\vec{L} = \overleftrightarrow{I} \cdot \vec{\omega}$$

has no problem in determining L_z and L_x simultaneously. In fact, no problem in classical mechanics to determine $\cancel{L_x}$, L_y and L_z once $\overleftrightarrow{I}$ and $\vec{\omega}$ are known. $\overleftrightarrow{I}$ is computable if we know the mass density and the geometric shape of the spinning top. Thus, there is no difficulty in specifying all the components of angular momentum simultaneously.

Now, let's do another expt. to pin down the reality in Q.M.

The model in which the atoms entering the third apparatus are visualized to have both S_z+ and S_x+ is clearly <u>unsatisfactory</u>.

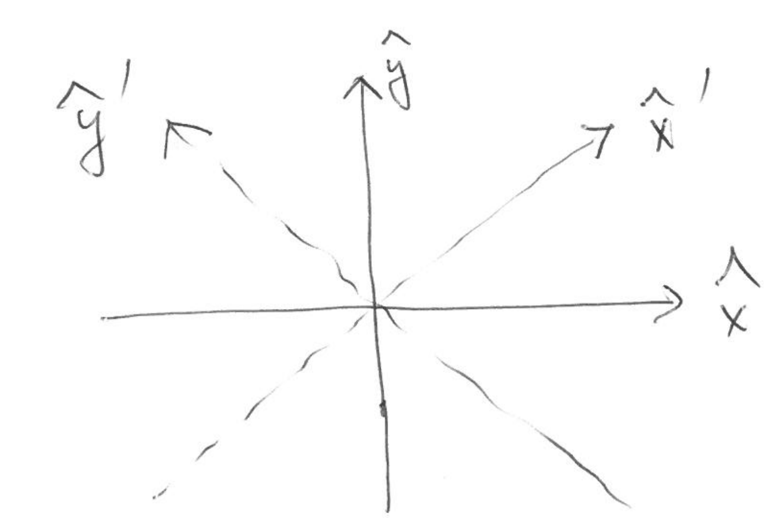
S_z and S_x <u>cannot</u> be determined <u>simultaneously</u>.

S_x+ beam from the second apparatus $SG\hat{x}$ completely <u>destroys</u> any previous information about S_z.
(wipes out)

$\boxed{\text{Analogy}}$ with polarization of light.

$S_{z+} \Leftrightarrow$ x-polarized
$S_{z-} \Leftrightarrow$ y-polarized

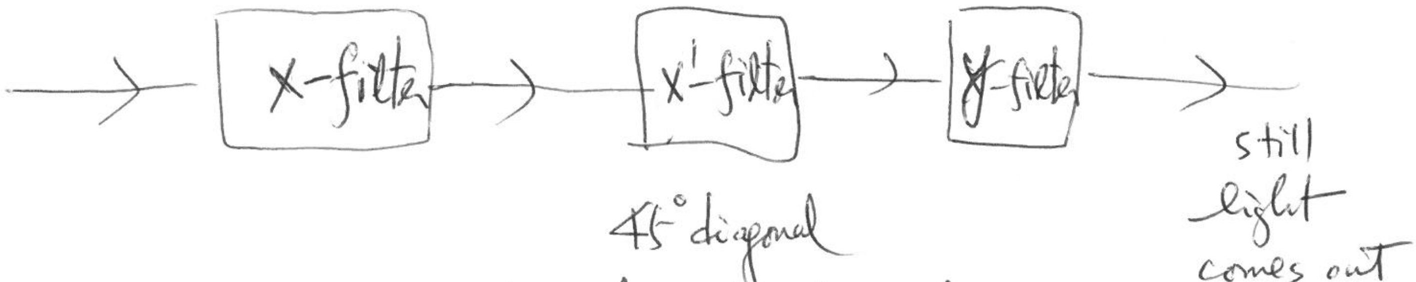
$$\vec{E} = \begin{cases} E_0 \hat{x} \cos(kz-\omega t) & \text{x-polarized light} \\ E_0 \hat{y} \cos(kz-\omega t) & \text{y-polarized light.} \end{cases}$$



$$\hat{x}' = \frac{1}{\sqrt{2}}(\hat{x} + \hat{y})$$

$$\hat{y}' = \frac{1}{\sqrt{2}}(-\hat{x} + \hat{y})$$

$$\boxed{\begin{array}{lcl} S_x+ & \longleftrightarrow & \hat{x}'\text{-polarized} \\ S_x- & \longleftrightarrow & \hat{y}'\text{-polarized} \end{array}}$$



The selection of the x'-polarized beam by the second polaroid <u>destroys</u> (wipes out) any previous information on light polarization

We may thus identify

$$|S_x+\rangle = \frac{1}{\sqrt{2}}|S_z+\rangle + \frac{1}{\sqrt{2}}|S_z-\rangle$$

$$|S_x-\rangle = -\frac{1}{\sqrt{2}}|S_x+\rangle + \frac{1}{\sqrt{2}}|S_z-\rangle$$

How about $|S_y+\rangle = ?$ and $|S_y-\rangle = ?$

⑥

$$\vec{E} = E_0\left[\frac{1}{\sqrt{2}}\hat{x}\cos(kz-\omega t) + \frac{1}{\sqrt{2}}\hat{y}\cos(kz-\omega t+\tfrac{\pi}{2})\right]$$

circularly polarized

$$\boxed{\frac{\vec{E}}{E_0} = Re(\vec{\epsilon})}$$

Complex Representation

$$\vec{\epsilon} = \left[\frac{1}{\sqrt{2}}\hat{x}\, e^{i(kz-\omega t)} + \underbrace{\frac{1}{\sqrt{2}}\hat{y}\, e^{i(kz-\omega t+\frac{\pi}{2})}}_{\frac{i}{\sqrt{2}}\hat{y}\, e^{i(kz-\omega t)}}\right]$$

$\left(\because e^{\frac{i\pi}{2}} = i\right)$

relative phase factor

$S_y+ \longleftrightarrow$ right circularly polarized

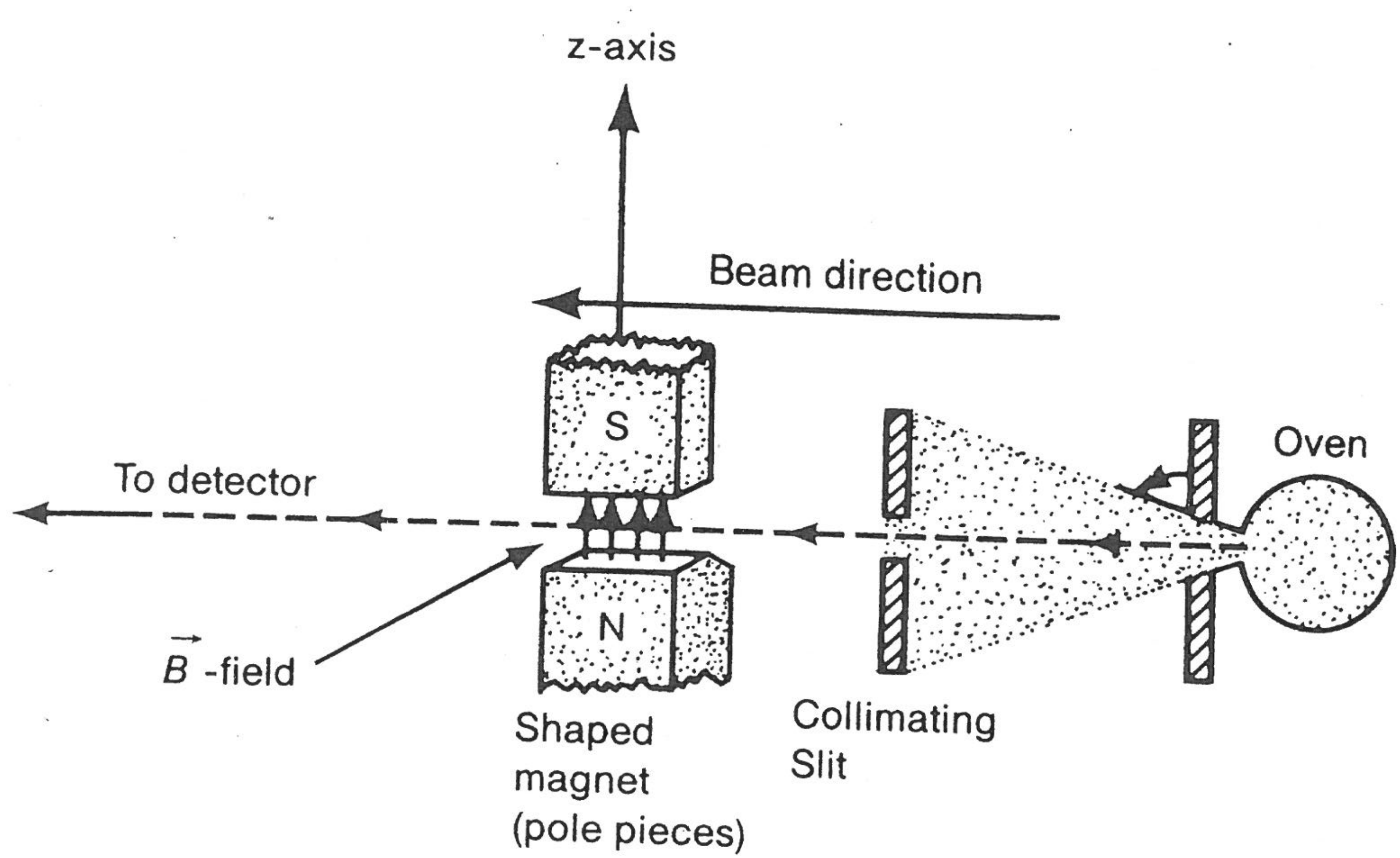
$S_y- \longleftrightarrow$ left "

or $$|S_y+\rangle = \frac{1}{\sqrt{2}}|S_z+\rangle + \frac{i}{\sqrt{2}}|S_z-\rangle$$

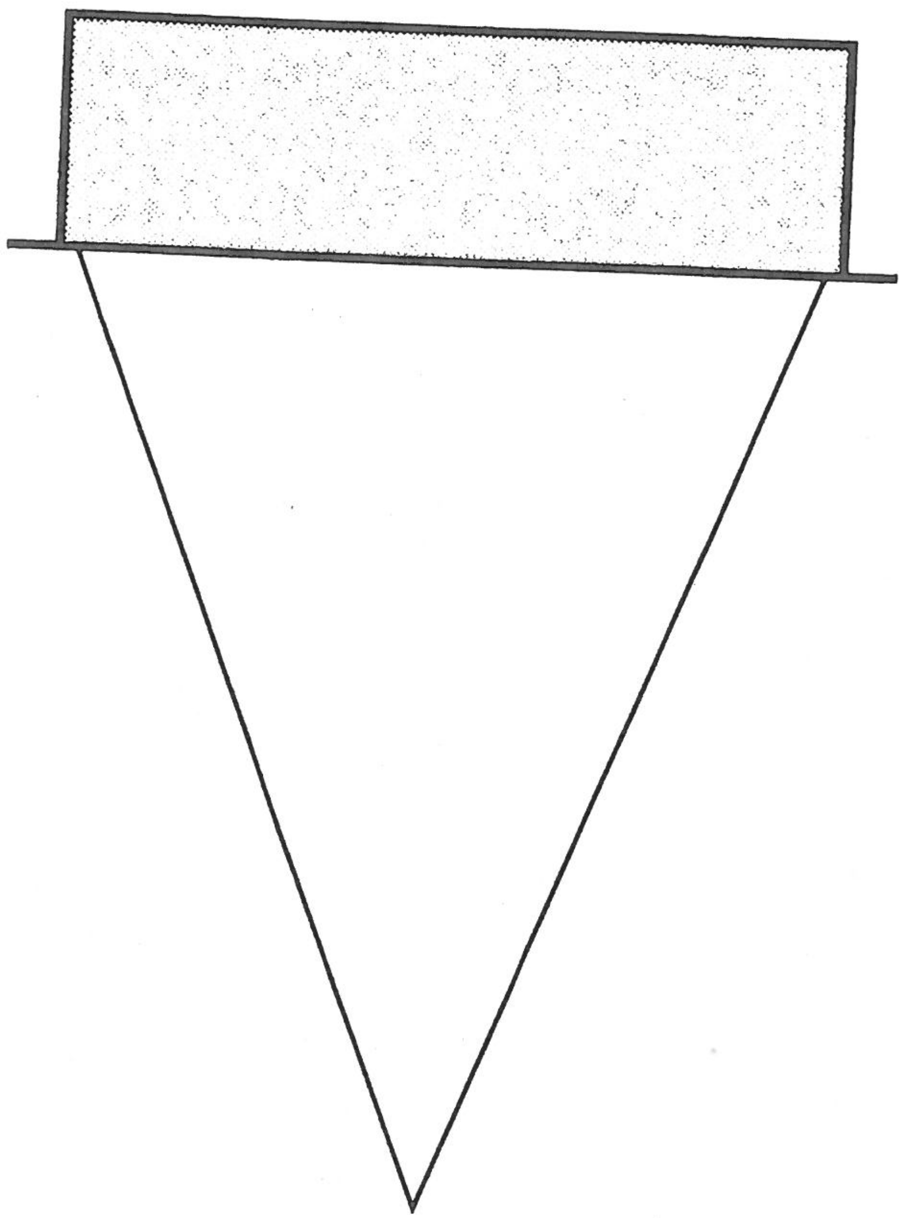
$$|S_y-\rangle = \frac{1}{\sqrt{2}}|S_z+\rangle - \frac{i}{\sqrt{2}}|S_z-\rangle$$

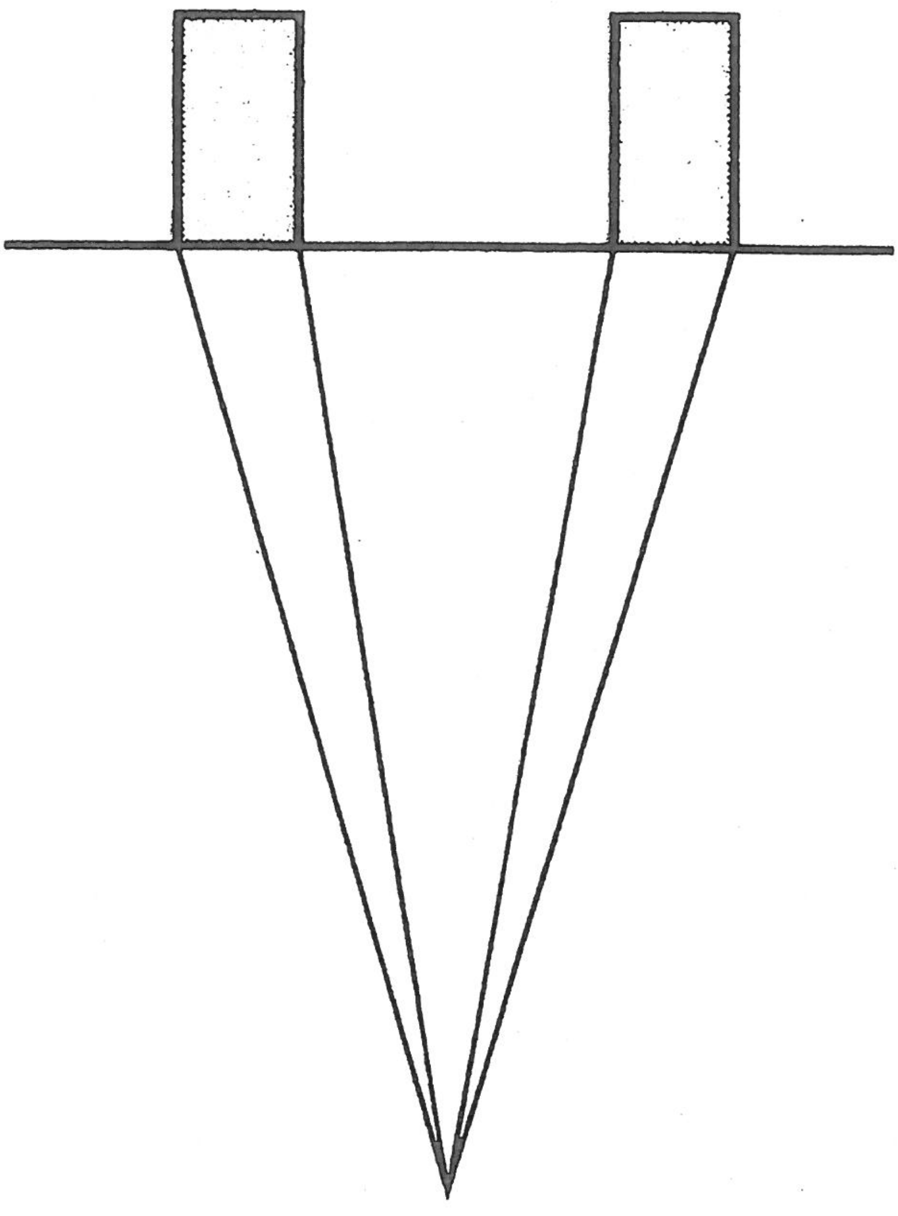
Complex Vector space $\Rightarrow$ Hilbert Space

(Bra&Ket space)



The Stern-Gerlach experiment.





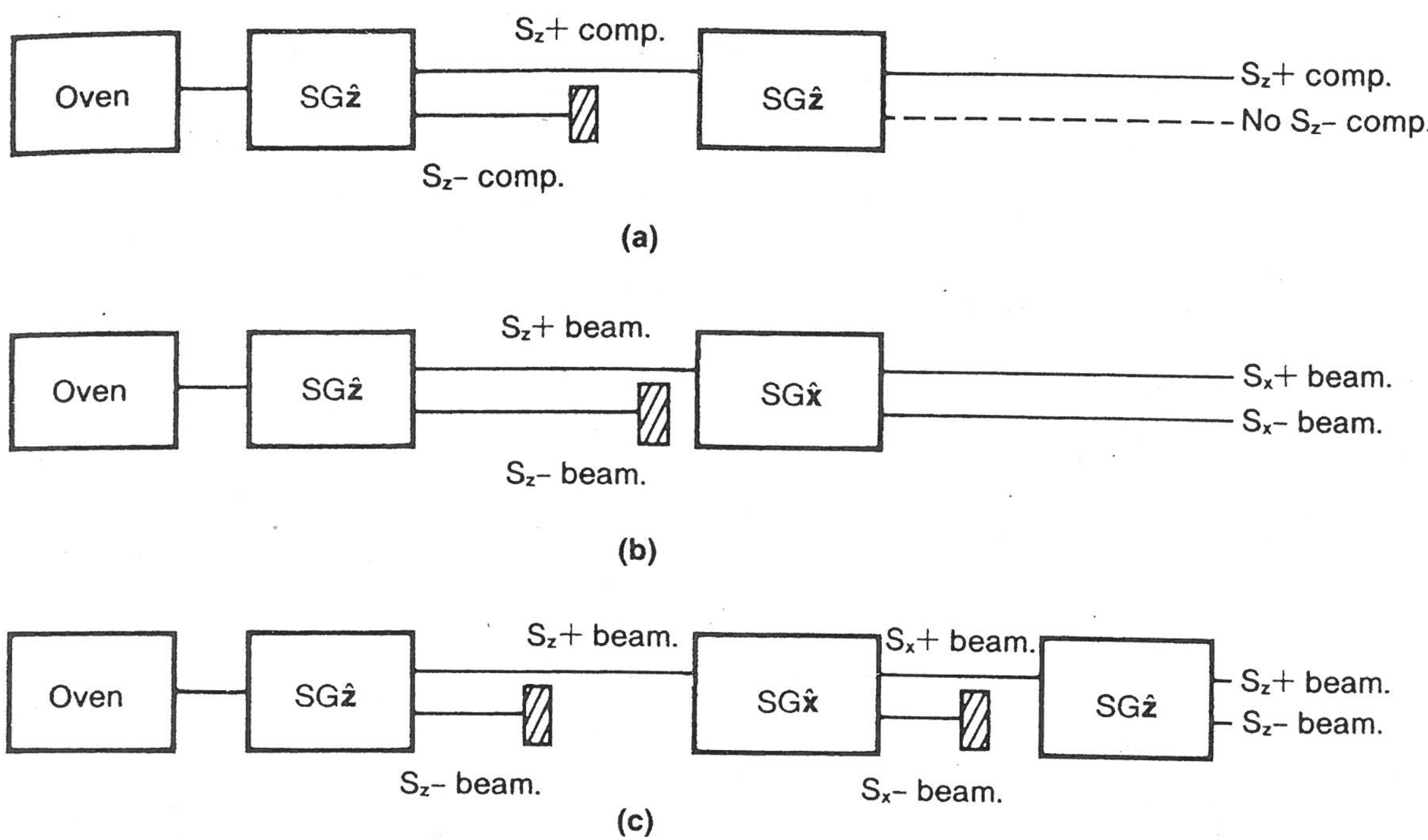


FIGURE 1.3. Sequential Stern-Gerlach experiments.

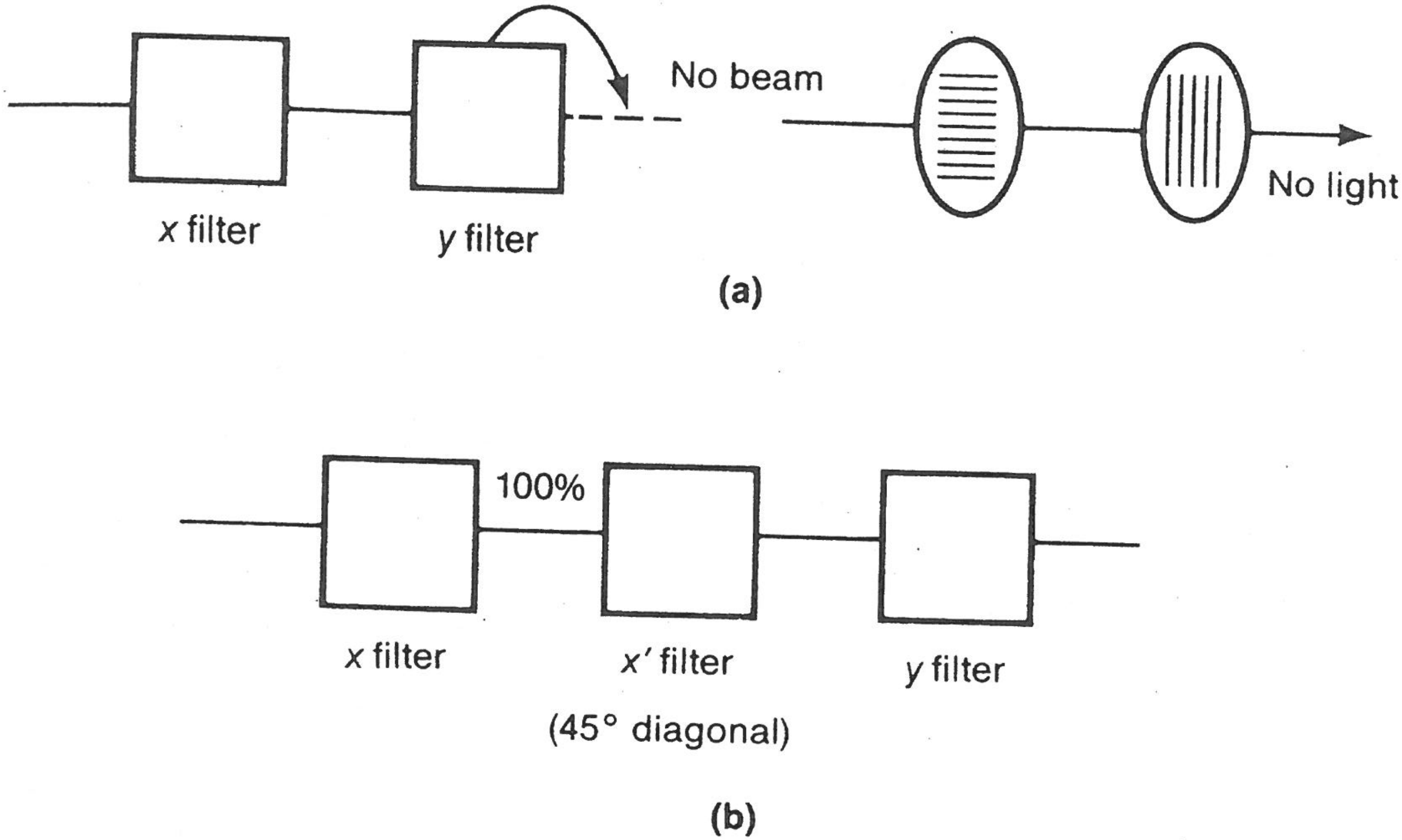


FIGURE 1.4. Light beams subjected to Polaroid filters.

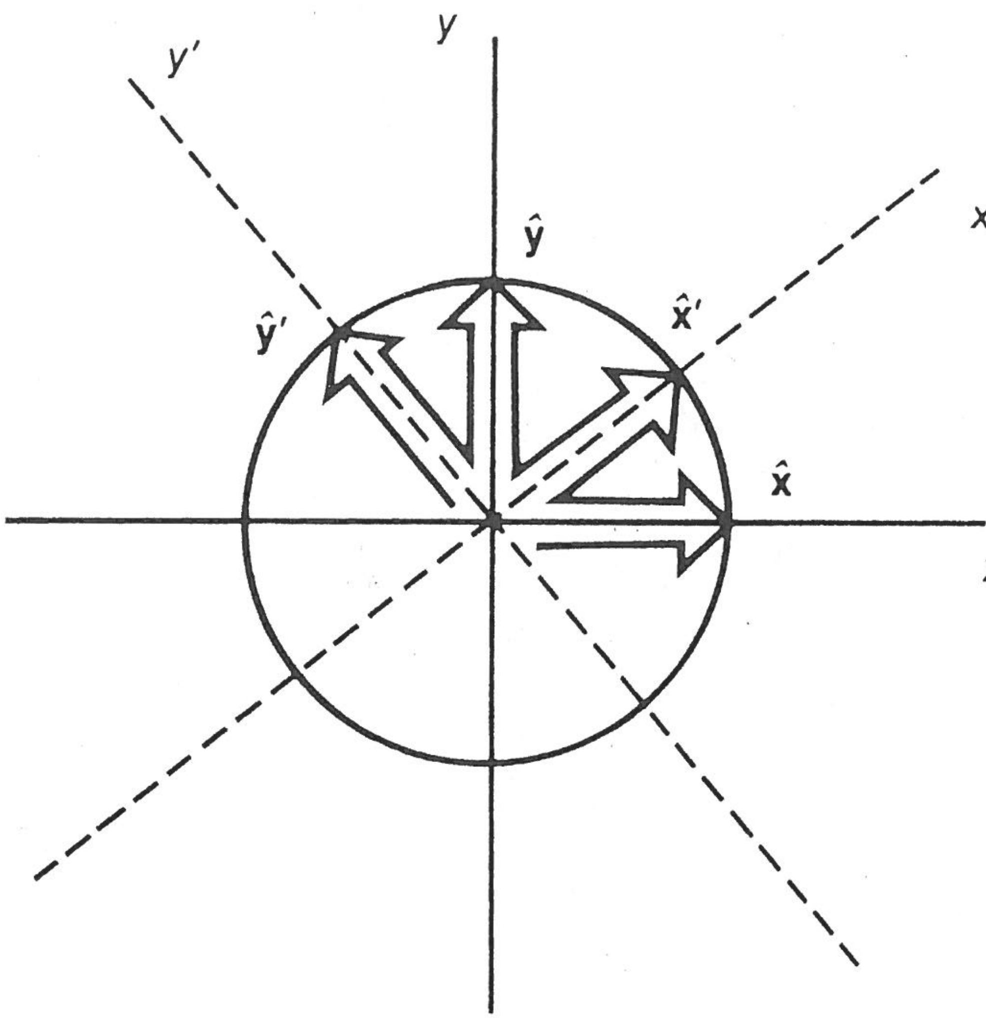


FIGURE 1.5. Orientations of the x'- and y'-axes.

assical electrodynamics. Using Figure 1.5 we obtain

$$E_0\hat{\mathbf{x}}'\cos(kz-\omega t) = E_0\left[\frac{1}{\sqrt{2}}\hat{\mathbf{x}}\cos(kz-\omega t)+\frac{1}{\sqrt{2}}\hat{\mathbf{y}}\cos(kz-\omega t)\right],$$

$$E_0\hat{\mathbf{y}}'\cos(kz-\omega t) = E_0\left[-\frac{1}{\sqrt{2}}\hat{\mathbf{x}}\cos(kz-\omega t)+\frac{1}{\sqrt{2}}\hat{\mathbf{y}}\cos(kz-\omega t)\right]. \tag{1.1.8}$$