STATISTICAL INFERENCES (2cr)
Chapter 10 One- and Two-Sample Tests of Hypotheses

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Math, IUSB

Outline

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1. 10.10 One- and Two-Sample Tests Concerning Variances
   - Hypothesis Test for one Variance
     - The Two-Tailed Test
     - One-Tailed Test
   - Hypothesis Testing About σ_1^2/σ_2^2

The Chi-Square Distribution

The chi-square distribution has only one parameter called the degrees of freedom.

The shape of a chi-squared distribution curve is skewed to the right for small df and becomes symmetric for large df.

The entire chi-square distribution curve lies to the right of the vertical axis.

The chi-square distribution assumes nonnegative values only, and these are denoted by the symbol χ^2 (read as "chi-square").

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The Shape of The Chi-Square Distribution

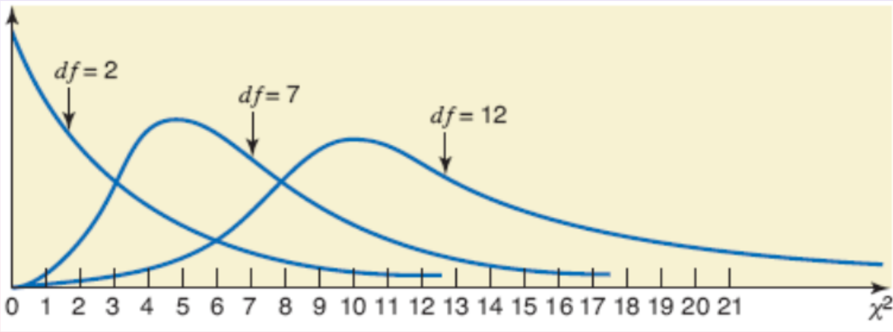


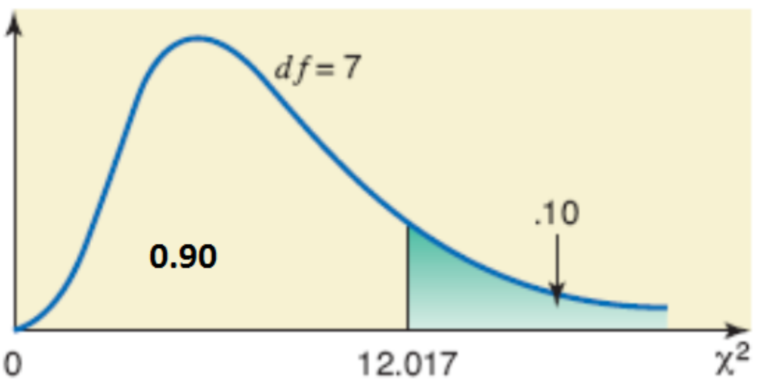
Table 5 Chi-Square Distribution

For df = 7 and .10 Area in the Right Tail

	Area in the Right Tail Under the Chi-Square Distribution Curve				
df	.995	...	.100	...	.005
1	0.000	...	2.706	...	7.879
2	0.010	...	4.605	...	10.597
.	...	...	...	...	...
.	...	...	...	...	...
.	...	...	...	...	...
7	0.989	...	12.017	...	20.278
.	...	...	...	...	...
.	...	...	...	...	...
.	...	...	...	...	...
100	67.328	...	118.498	...	140.169

Required value of χ^2

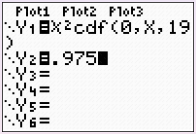
For df = 7 and .10 Area in the Right Tail



Use TI-83/84 to Find Critical χ^2 Value

For example, $n = 20$, df $= n - 1$=19, to find $\chi^2_{.025}$:

- Enter equation for χ^2 cdf : $\boxed{\text{Y=}}$ -> Y1= χ^2cdf(0, X, 19)
- Enter equation: Y2=1-.025=0.975.



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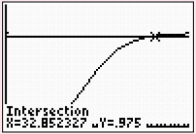
- Press $\boxed{\text{2ND}}$ and $\boxed{\text{CALC}}$ to find intersection of the two graphs. We can obtain $\chi^2_{.025} = 32.85$.



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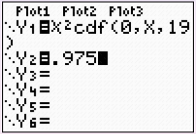

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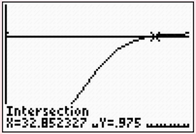

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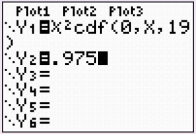

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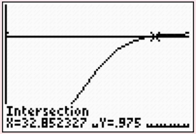

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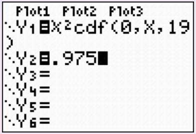

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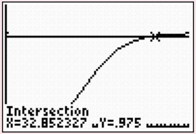

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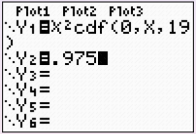

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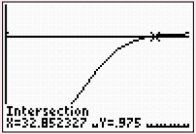

Two–Tailed Test for σ^2

Let s^2 be the sample variance of $x_1, \ldots, x_n$ and σ^2 be the population variance.

Step 1. State the hypotheses

$$H_0 : \sigma^2 = \sigma_0^2, \quad vs \quad H_1 : \sigma^2 \neq \sigma_0^2$$

Step 2. Choose the distribution: If population is normal then use

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

which has χ^2-distribution with df = $n-1$ if H_0 is true.

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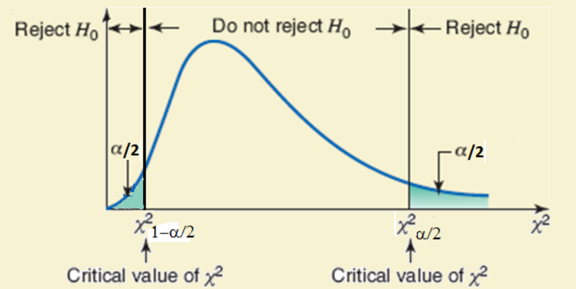
Two–Tailed Test for σ^2

Step 3. Calculate p-value or find critical value.

Find the p-value: Twice the smaller tail area at χ^2.
Using TI83/4, p-value is the smaller one of 2*χ^2cdf(0, χ^2, $n-1$) and 2*χ^2cdf(χ^2, E99, $n-1$). See example below.
The critical values for χ^2 are $\chi^2_{\alpha/2}$ and $\chi^2_{1-\alpha/2}$:



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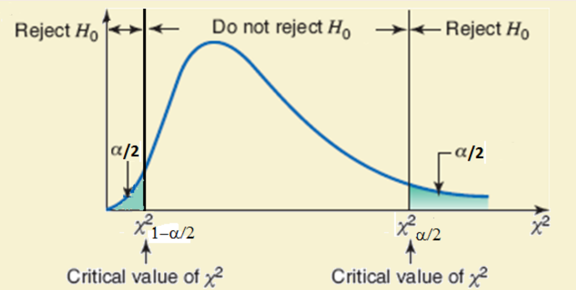

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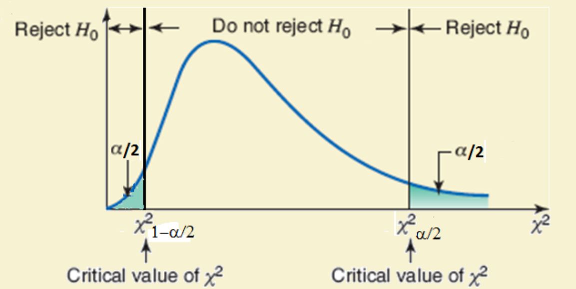

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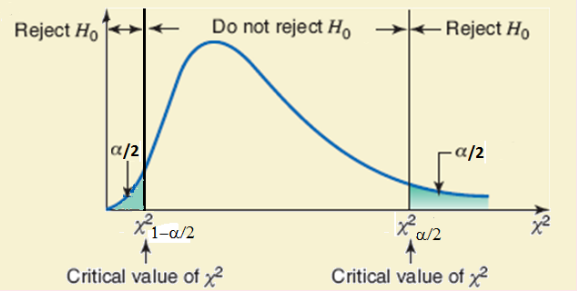

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Two–Tailed Test for σ^2

Step 4. Make decision:
If p-value $< \alpha$, reject H_0, otherwise, do not reject H_0.
if $\chi^2 < \chi^2_{1-\alpha/2}$ or $\chi^2 > \chi^2_{\alpha/2}$, reject H_0, otherwise, do not reject H_0.

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The variance of scores on a standardized mathematics test for all high school seniors was 150 in 2009. A sample of scores for 20 high school seniors who took this test this year gave a variance of 170. Test at the 5% significance level if the variance of current scores of all high school seniors on this test is different from 150. Assume that the scores of all high school seniors on this test are (approximately) normally distributed.

Example

Solution:

Step 1. Hypotheses: $H_0 : \sigma^2 = 150 \quad H_A : \sigma^2 \neq 150$

Step 2. The population is (approximately) normal. We use the chi-square distribution to test a hypothesis about σ^2.

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Example

Step 3. $\alpha = 0.05$,

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(20-1)(170)}{150} = 21.533$$

The p-value Approach: The p-value is

$$p\text{-value} = 2\min[\chi^2\mathrm{cdf}(0, 21.533, 19), \chi^2\mathrm{cdf}(21.533, \mathrm{E}99, 19)]$$

$$= 0.6162207$$

The Critical Value Approach:
The critical values are $\chi^2_{.975} = 8.907$ and $\chi^2_{.025} = 32.852$.

Step 4. Since the p-value $> \alpha$, we do not reject H_0.
Since $\chi^2_{1-\alpha/2} < \chi^2 < \chi^2_{\alpha/2}$, we do not reject H_0.

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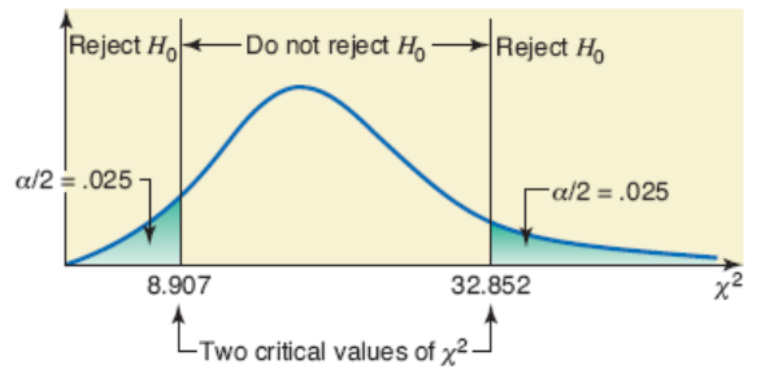
Critical Value of Two-Tailed



Confidence Interval for σ^2

The $100(1-\alpha)\%$ confidence interval for σ^2 is

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$H_0 : \sigma^2 = \sigma_0^2$, normal population, $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$

Alternative Hypothesis	Rejection Region	p-value
$H_1 : \sigma^2 < \sigma_0^2$	$\chi^2 < \chi^2_{1-\alpha}(n-1)$	The area to the left of χ^2
$H_1 : \sigma^2 > \sigma_0^2$	$\chi^2 > \chi^2_{\alpha}(n-1)$	The area to the right of χ^2

Outline

Sampling distribution of s_1^2/s_2^2

- Let $x_{11}, \ldots, x_{1n_1}$ be a sample from normal population with variance σ_1^2,
  and $x_{21}, \ldots, x_{2n_2}$ be a sample from normal population with variance σ_2^2.
- Assume the two samples are independent and have sample variances s_1^2 and s_2^2, respectively.
- The distribution of $\frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2}$ is F-distribution with numerator degrees of freedom $n_1 - 1$ and denominator degrees of freedom $n_2 - 1$.

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The *F* Critical Value (Table 6)

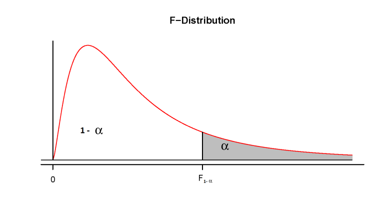

Table 7 gives F_α for $\alpha = .100, .050, .025, .010$, and $.005$.
For example, $\nu_1 = 5$, $\nu_2 = 7$, $F_{.025} = 5.29$ by Table 6.

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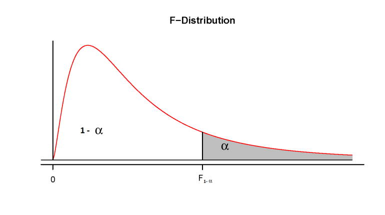

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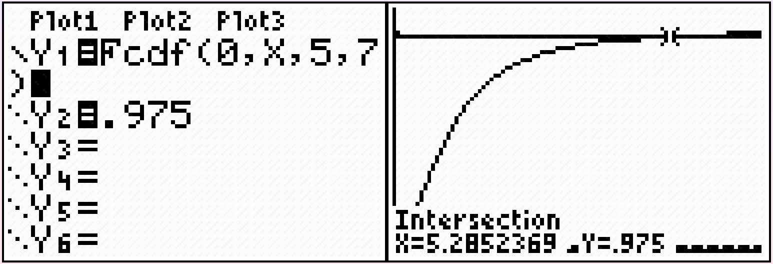
The F Critical Value using Technology

- In Excel: $F_{.025}$=F.INV.RT(0.025,5,7)=5.285236852, or =F.INV(1-0.025,5,7)=5.285236852.
- In TI-8x: Graph Y1=Fcdf(0, X, 5,7) and Y2=1-.025=0.975, the x-coordinate of the intersection is $F_{.025}$=5.2852369.



- In R: $F_{.975}$= "qf(.975,5,7)=5.285237".

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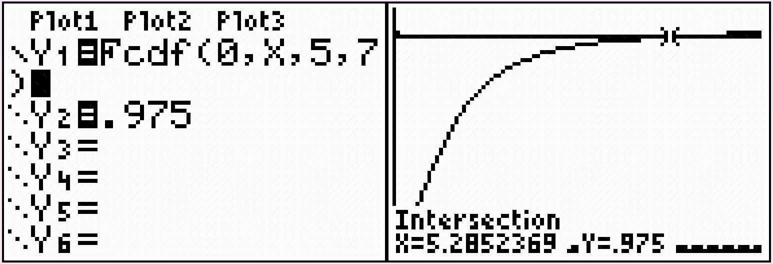

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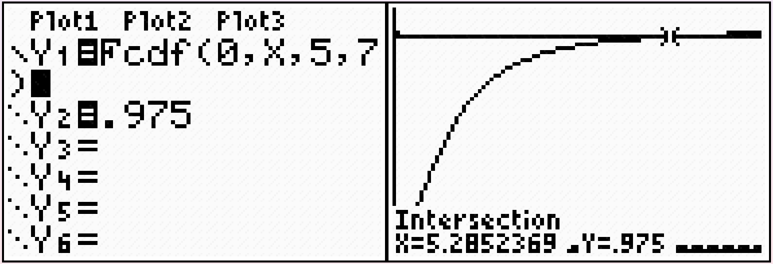

- In R: $F_{.975}$= “qf(.975,5,7)=5.285237”.

Hypothesis Testing About σ_1^2/σ_2^2

$H_0 : \sigma_1^2/\sigma_2^2 = 1$, normal populations, $F = \frac{s_1^2}{s_2^2}$.

Alternative Hypothesis	Rejection Region	p-value
$H_1 : \sigma_1^2/\sigma_2^2 \neq 1$	$F < F_{1-\alpha/2}(n_1 - 1, n_2 - 1)$ or $F > F_{\alpha/2}(n_1 - 1, n_2 - 1)$	Twice the smaller tail area at F
$H_1 : \sigma_1^2/\sigma_2^2 > 1$	$F > F_{\alpha}(n_1 - 1, n_2 - 1)$	The area to the left of F
$H_1 : \sigma_1^2/\sigma_2^2 < 1$	$F < F_{1-\alpha}(n_1 - 1, n_2 - 1)$	The area to the right of F

TI-83/84: $P(a < F(m,n) < b) =$Fcdf(a, b, m, n).

If $H_0 : \sigma_1^2/\sigma_2^2 = 1$ is accepted, then we can assume equal variance in the two-sample t-test for comparing two population means.

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Example

Pressures(mm Hg) under the Pelvis during Static Conditions for spinal cord injury(SCI) and healthy control groups.

CONTROL	131	115	124	131	122	117	88	114	150	169
SCI	60	150	130	180	163	130	121	119	130	148

Assume the populations are normal. Test $H_0 : \sigma_C^2 = \sigma_{SCI}^2$ vs $H_A : \sigma_C^2 \neq \sigma_{SCI}^2$ at level $\alpha = 0.05$.

Solution:

- $F = \frac{s_C^2}{s_{SCI}^2} = \frac{21.8^2}{32.2^2} = .458$, df.num =df.deno =10-1=9.
- p-value: tail areas at $F = .458$ are Fcdf(0, .458,9,9)=.1301 and Fcdf(.458, E99, 9,9)=.8699. Twice the smaller tail area = 2(.1301)=0.2602$> \alpha$.
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