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stats
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hw5: Sections 9.3-9.4 Exercise 9.2, 9.5, 9.6, 9.8, 9.11. Study the textbook Examples in Sections 9.1-9.4.

9.2 An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

$n = 30$
$\bar{x} = 780$
$\sigma = 40$
$P = 0.96$

$$\begin{aligned}\bar{x} \pm z_{0.02}(\sigma/n^{1/2}) &= 780 \pm 2.0537(40/30^{1/2}) \\ &= 780 \pm 14.9981 \\ &\to (765.0019, 794.9981)\end{aligned}$$

9.5 A random sample of 100 automobile owners in the state of Virginia shows that an automobile is driven on average 23,500 kilometers per year with a standard deviation of 3900 kilometers. Assume the distribution of measurements to be approximately normal.

(a) Construct a 99% confidence interval for the average number of kilometers an automobile is driven annually in Virginia.

$n = 100$
$\bar{x} = 23500$
$s = 3900$

$$\begin{aligned}\bar{x} \pm z_{0.005} \cdot \frac{s}{n^{1/2}} &= 23500 \pm 2.5758(3900/10) \\ &\to (22495.438, 24504.562)\end{aligned}$$

(b) What can we assert with 99% confidence about the possible size of our error if we estimate the average number of kilometers driven by car owners in Virginia to be 23,500 kilometers per year?

$$\begin{aligned}error &\leq z_{\alpha/2} \cdot \frac{s}{n^{1/2}} \\ &= 2.5758(390) \\ &= 1004.562\end{aligned}$$

It is at most 1004.562.

9.6 How large a sample is needed in Exercise 9.2 if we wish to be 96% confident that our sample mean will be within 10 hours of the true mean?

$n = 30$
$\bar{x} = 780$
$\sigma = 40$
$err = 10$
$P = .96$

needed $n = \lceil(\frac{z_{\alpha/2}\sigma}{err})^2\rceil = \lceil(\frac{2.055 \cdot 40}{10})^2\rceil = 68$

9.8 An efficiency expert wishes to determine the average time that it takes to drill three holes in a certain metal clamp. How large a sample will she need to be 95% confident that her sample mean will be within 15 seconds of the true mean? Assume that it is known from previous studies that $\sigma = 40$ seconds.

$\sigma = 40$
$err = 15$
$P = .95$

needed $n = \lceil(\frac{z_{\alpha/2}\sigma}{err})^2\rceil = \lceil(1.9599 \cdot 40/15)^2\rceil = 28$

9.11 A machine produces metal pieces that are cylindrical in shape. A sample of pieces is taken, and the diameters are found to be 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, and 1.03 centimeters. Find a 99% confidence interval for the mean diameter of pieces from this machine, assuming an approximately normal distribution.

$\bar{x} = \sum x_i/n \approx 1.0055$

$s = (\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1})^{1/2} = 0.0246$

$$
\begin{aligned}
ci &= \bar{x} \pm t_{\alpha/2,n-1} \cdot s/n^{1/2} \\
&= 1.0056 \pm 0.0275 \\
&= (0.9781, 1.0331)
\end{aligned}
$$