STATISTICAL INFERENCES (2cr)
Chapter 9 One- and Two-Sample Estimation Problems

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Math, IUSB

Outline

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Point Estimate

A **point estimate** of some population parameter θ is a single value $\hat{\theta}$ of a statistic $\widehat{\Theta}$ calculated based on observed values $x_1, \ldots, x_n$ of sample $X_1, \ldots, X_n$.

Examples:

- Sample mean $\bar{x}$ is observed value of statistic $\overline{X}$ and point estimate of population mean μ.
- Sample proportion $\hat{p} = x/n$ is a point estimate of population proportion p.
- Sample variance s^2 is a point estimate of population variance σ^2.

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Unbiased Estimator

Definition 1. A statistic $\widehat{\Theta}$ is said to be an **unbiased** estimator of the parameter θ if the expected value of $\widehat{\Theta}$ equals θ, that is

$$\mu_{\widehat{\Theta}} = E(\widehat{\Theta}) = \theta.$$

Example 1. Sample mean $\overline{X}$ is an unbiased estimator of population mean μ.

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Unbiased Estimator

If $\hat{\Theta}$ is an unbiased estimator of θ, then $a\hat{\Theta} + b$ is an unbiased estimator of $a\theta + b$. But $\hat{\Theta}^r$ is a biased estimator of θ^r, $r \neq 1$.

Example 2. Sample variance S^2 is an unbiased estimator of population variance σ^2. However, sample standard deviation S is not an unbiased estimator of population standard deviation σ. In fact, if sample is drawn from normal population, then

$$E(S) = \frac{\sqrt{2}\,\Gamma(n/2)}{\sqrt{n-1}\,\Gamma[(n-1)/2]}\sigma < \sigma.$$

So

$$\hat{\sigma} = \frac{\sqrt{n-1}\,\Gamma[(n-1)/2]}{\sqrt{2}\,\Gamma(n/2)}S$$

is an unbiased estimator of σ.

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Variance of an Unbiased Estimator

The **most efficient unbiased estimator** of θ is such an unbiased estimator, $\widehat{\Theta}$, of θ that has the smallest variance among all possible unbiased estimators of θ, that is,

$$\mathrm{Var}(\widehat{\Theta}) \leqslant \mathrm{Var}(\widetilde{\Theta}), \quad \text{for any unbiased estimator } \widetilde{\Theta} \text{ of } \theta.$$

Outline

An Example

Example 1 Let x equal the weight in grams of 52-gram snack pack of candies. Assume that the distribution of X is normal with $\sigma = 2$ but an unknown μ. If the quality inspector decides to select a random sample of size $n = 10$, then he will get 10 observations $X_1, X_2, \ldots, X_{10}$ and use $\bar{X}$ as a point estimate of μ. Can he get an interval for μ?

$$\bar{X}, \quad Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}, \quad P(-1.65 < Z < 1.65) = 0.90$$

where $\sigma_{\bar{X}} = \sigma/\sqrt{10}$.

$$-1.65 < \frac{\bar{X} - \mu}{\sigma/\sqrt{10}} < 1.65$$

is equivalent to

$$\bar{X} - 1.65\frac{2}{\sqrt{10}} < \mu < \bar{X} + 1.65\frac{2}{\sqrt{10}}$$

So

$$P\left(\bar{X} - 1.65\frac{2}{\sqrt{10}} < \mu < \bar{X} + 1.65\frac{2}{\sqrt{10}}\right) = 0.90$$

Denote $E = 1.65 \frac{2}{\sqrt{10}} = 1.04$

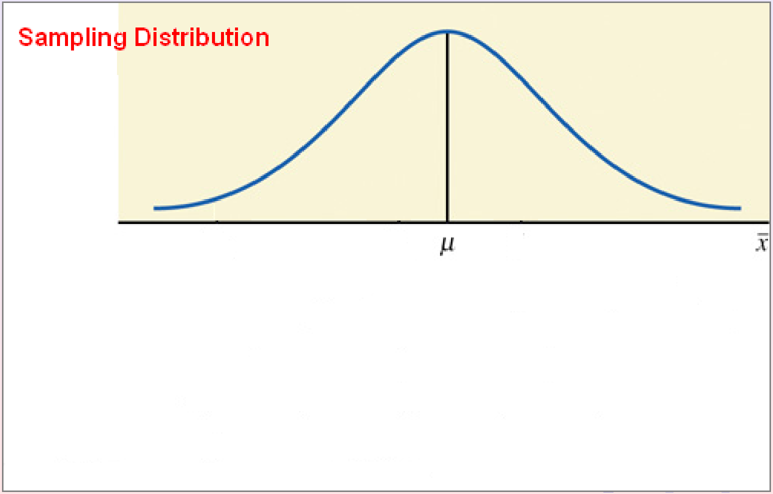
$$\hat{\mu}_L = \bar{X} - E, \quad \text{and} \quad \hat{\mu}_U = \bar{X} + E$$

Then

$$P(\hat{\mu}_L < \mu < \hat{\mu}_U) = 0.90$$

Interval $(\hat{\mu}_L, \hat{\mu}_U)$ is called a confidence interval for μ with 90% confidence level.

What does confidence level mean?



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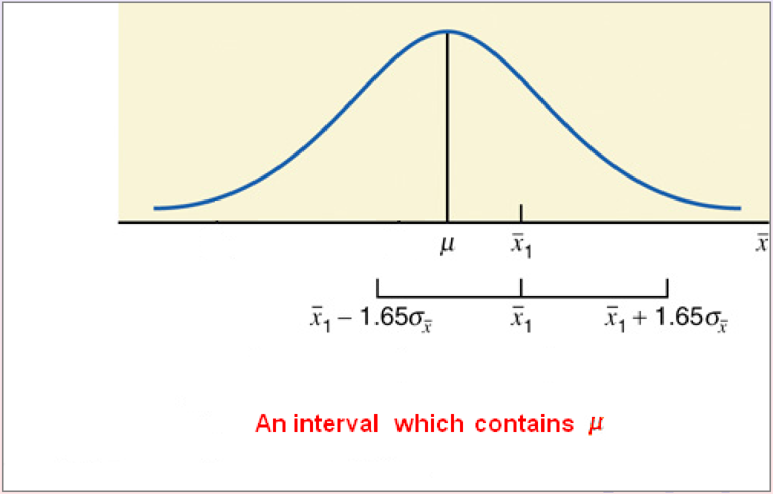

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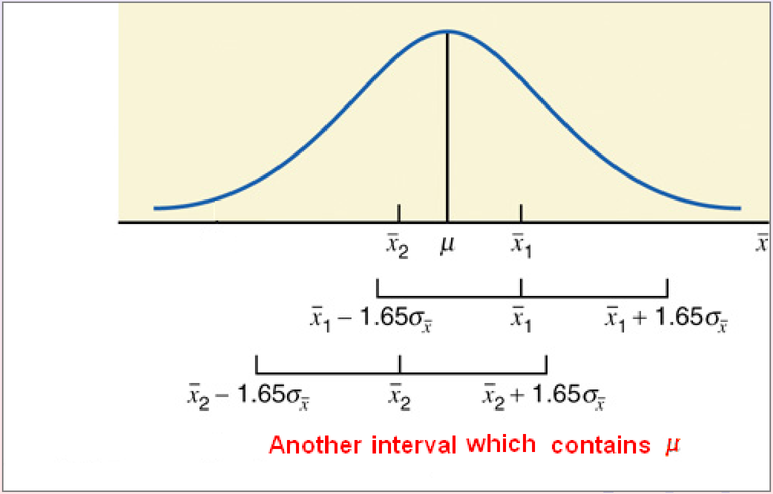

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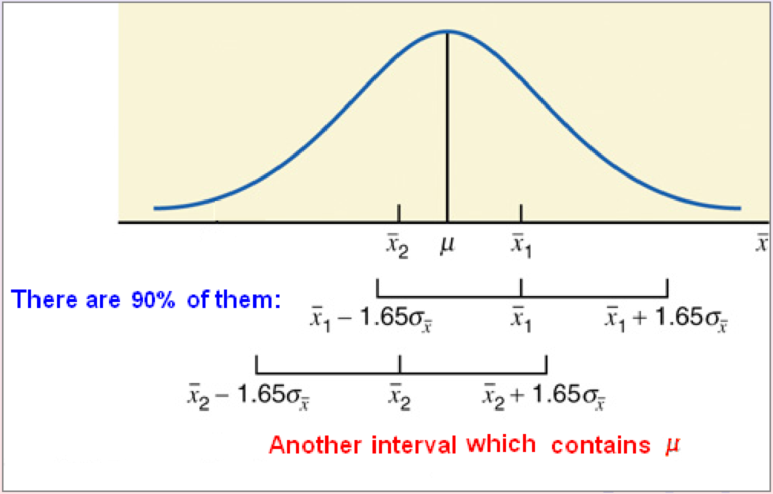

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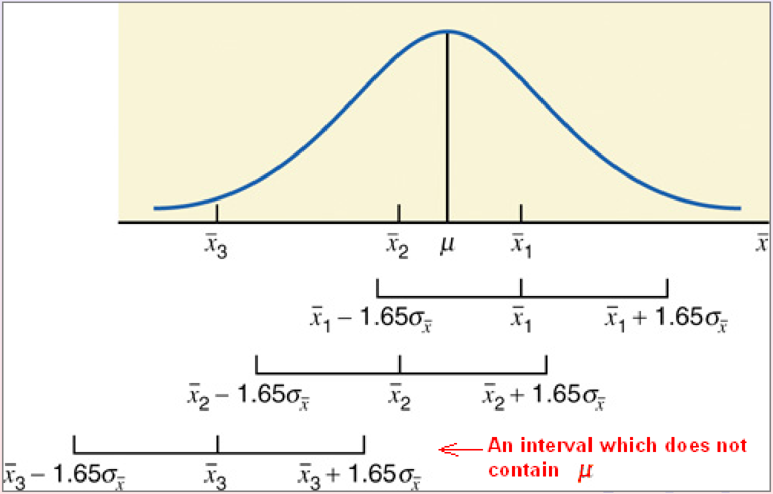

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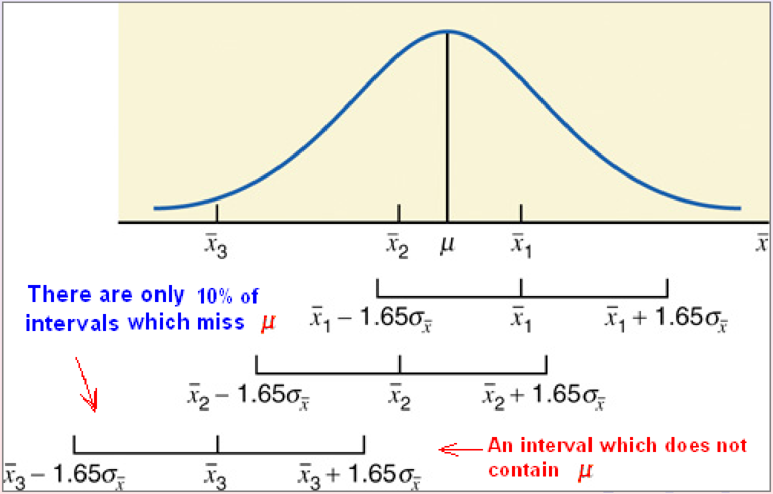


Confidence Interval for θ

Definition. Let θ be a population parameter and $\hat{\Theta}_L$ and $\hat{\Theta}_U$ be two statistics and $0 < \alpha < 1$. If

$$P(\hat{\Theta}_L < \theta < \hat{\Theta}_U) = 1 - \alpha$$

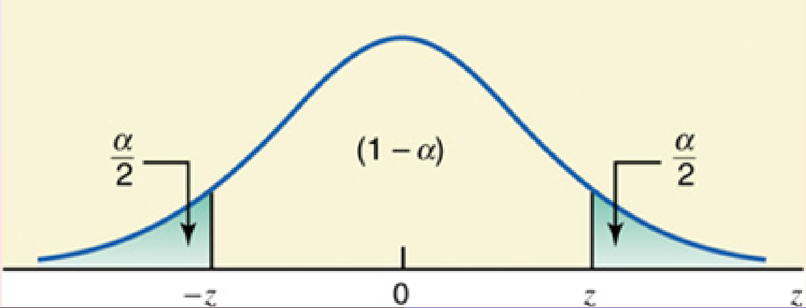
then interval $(\hat{\Theta}_L, \hat{\Theta}_U)$ is called a $100(1-\alpha)\%$ confidence interval for θ.

Confidence Interval for μ with Known σ

Confidence Interval for μ The $(1-\alpha)100\%$ *confidence interval* for μ is

$$\bar{x} \pm z_{\alpha/2}\sigma_{\hat{\mu}}$$

where $E = z_{\alpha/2}\sigma_{\hat{\mu}}$ is the margin of error $\sigma_{\hat{\mu}} = \frac{\sigma}{\sqrt{n}}$ and $z_{\alpha/2}$ satisfies

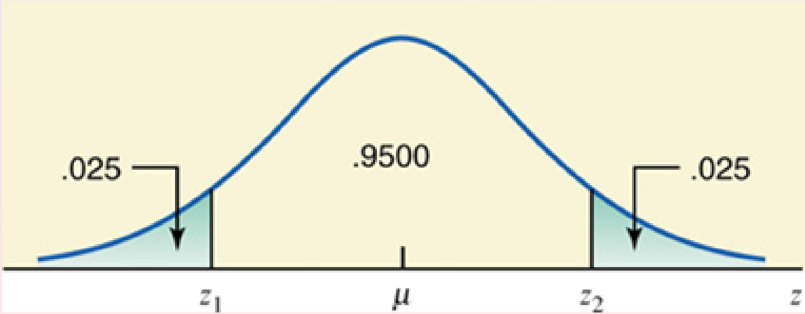


z value with $\alpha = 0.05$

z value in the confidence interval for μ with $\alpha = 0.05$

$$1 - \alpha = 1 - 0.05 = 0.95, \quad \frac{\alpha}{2} = 0.025$$

$$z_{\alpha/2} = z_{0.025} = 1.96$$

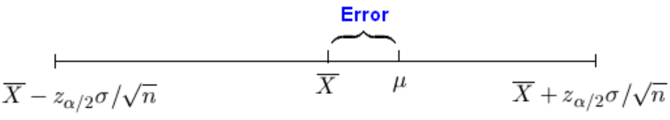


Commonly used z values

Table 8.1 z Values for Commonly Used Confidence Levels

Confidence Level	Areas to Look for in Table IV	z Value
90%	.0500 and .9500	1.64 or 1.65
95%	.0250 and .9750	1.96
96%	.0200 and .9800	2.05
97%	.0150 and .9850	2.17
98%	.0100 and .9900	2.33
99%	.0050 and .9950	2.57 or 2.58

Error in Estimating μ by $\overline{X}$



Error in estimating μ by $\overline{X}$

The $100(1-\alpha)\%$ confidence interval

$$\bar{x} \pm z_{\alpha/2}\sigma/\sqrt{n}$$

also means that when we use $\bar{x}$ to estimate μ, we can be $100(1-\alpha)\%$ confident that the error will not exceed the margin of error $E = z_{\alpha/2}\sigma/\sqrt{n}$

Example

Example 1(cont.) Find the 99% confidence interval for μ based on the data of Example 1:

```
55.95, 56.54, 57.58, 55.13, 57.48
56.06, 59.93, 58.30, 52.57, 58.46
```

Solution

$$\bar{x} = 56.8, \quad \alpha = 0.01,$$

$$z_{\alpha/2} = z_{0.005} = 2.57$$

The 99% confidence interval for μ is

$$\left[\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right] = [55.17, 58.43]$$

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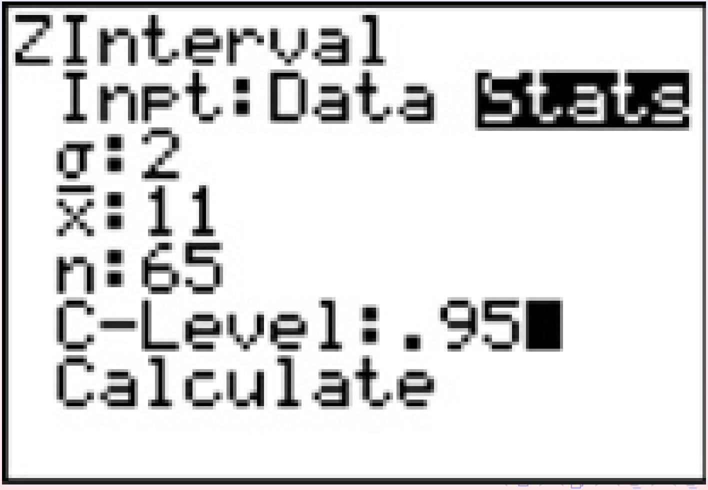
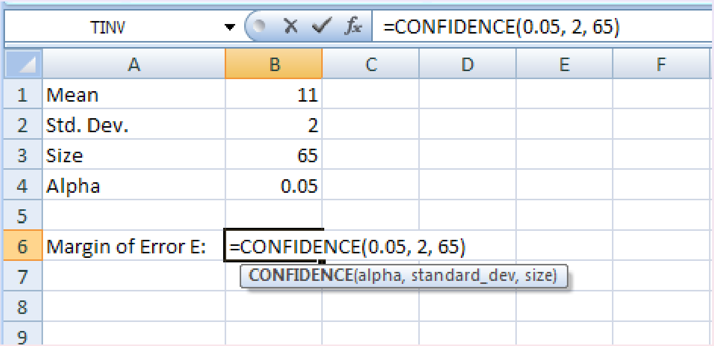
Using TI-8x to Obtain CI: ZInterval



Using Excel to Obtain CI: CONFIDENCE



Using Excel to Obtain CI: CONFIDENCE

C10 f_x

	A	B	C	D	E	F	G
1	Mean	11					
2	Std. Dev.	2					
3	Size	65					
4	Alpha	0.05					
5							
6	Margin of Error E:	0.486207					
7							
8							

Outline

Sample size, confidence level and width of confidence interval

The CI $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ is centered at $\bar{x}$ with width $w_n(\alpha) = 2E = 2z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$.

(a) when α is fixed, as $n \nearrow \infty$, $w_n(\alpha) \searrow 0$.

(b) when n is fixed, there is a **trade-off** between confidence level and width of confidence interval. As $1-\alpha \nearrow 1$ $(\alpha \searrow 0)$, then $z_{\alpha/2} \nearrow \infty$, so $w_n(\alpha) \nearrow \infty$.

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Given margin of error, determine sample size

Because

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \longrightarrow E^2 = z_{\alpha/2}^2 \frac{\sigma^2}{n}$$

In order to obtain an CI with a given margin of error E, the sample size should be

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = z_{\alpha/2}^2 \frac{\sigma^2}{E^2}$$

What if σ is unknown?
1. Take preliminary sample of any size(better be 30 or greater). Use sample standard deviation s to estimate σ.
2. Estimate σ based on other study done earlier.
3. $\sigma \approx$range/4.

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Example

Example 2 Let x be the excess weight of soap in a 1000-gram bottle. Assume that the distribution of x is normal with μ and standard deviation of $\sigma = 13$. What sample size is required so that we have 95% CI for μ with margin of error 1.5?

Solution $\sigma^2 = 169$. $E = 1.5$. $\alpha = 0.05$, $z_{\alpha/2} = 1.96$.

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \frac{1.96^2(169)}{1.5^2} = 288.5468.$$

So we should choose $n = 289$.

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Example

Example 3: Price of Textbook If we know that the maximum price is \$170 and the cheapest textbook is \$10. We want to estimate the mean(average) price μ of the college textbooks. Confidence level is 99% with margin of error $E =$\$5. What is the sample size?

Solution Since we know that the maximum price is \$170 and the cheapest textbook is \$10, we can estimate σ by

$$\sigma = \frac{\text{range}}{4} = (170 - 10)/4 = 40.$$

$\alpha = 0.99$, $z_{\alpha/2} = 2.575$, $E = 5$,

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = (2.575(40)/5)^2 = 424.36 \approx 425.$$

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Example

Example 4: A light bulb manufacturer sells a light bulb that has a mean life of 1450 hours with standard deviation of 33.7 hours. A new manufacturing process is being tested and there is interest in knowing the mean life μ of the new bulbs. How large a sample is required so that $\bar{x} \pm 5$ is a 95% CI for μ? You may assume that the standard deviation is still the same.

Solution $\sigma = 33.7$. $E = 5$. $\alpha = 0.05$, $z_{\alpha/2} = 1.96$.

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \frac{1.96^2(33.7)^2}{5^2} = 174.5147.$$

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Confidence Bounds (Limits)

- Sometimes, we are only interested in an upper/lower limits for μ.
- A $100(1-\alpha)\%$ **lower confidence bound** for μ with known σ^2 is

$$\overline{X} - z_\alpha \frac{\sigma}{\sqrt{n}}$$

- A $100(1-\alpha)\%$ **upper confidence bound** for μ with known σ^2 is

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Example

Example 5: During the Friday night shift, $n = 28$ mints were selected at random from a production line and weighted. They had an average weight of $\bar{x} = 21.45$ grams. Assume that the weights of all the mints have a normal distribution with a unknown mean μ and a known standard deviation $\sigma = 0.30$. Give a 90% lower confidence bound for μ.

Solution $n = 28$, $\bar{x} = 21.45$, $\sigma = 0.30$, $\alpha = 0.1$. A $100(1-\alpha)\% = 90\%$ lower confidence bound for μ is

$$\bar{x} - z_\alpha \frac{\sigma}{\sqrt{n}} = 21.45 - 1.28\frac{0.30}{\sqrt{28}} = 21.45 - 0.073 = 21.38.$$

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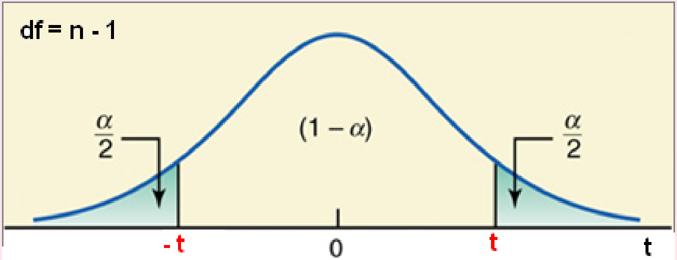
Confidence Interval for μ with Unknown σ

Replace σ and $z_{\alpha/2}$ with s and $t_{\alpha/2}$ respectively

If σ is not know, the $(1-\alpha)100\%$ *confidence interval* for μ is

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$$

where $E = t_{\alpha/2}\frac{s}{\sqrt{n}}$ is the margin of error and $t_{\alpha/2}$ satisfies



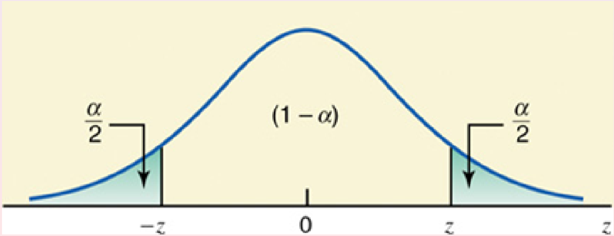
Large Sample Confidence Interval for μ w/ Unknown σ

If $n \geqslant 30$, replace σ with s

The $(1-\alpha)100\%$ *large sample confidence interval* for μ is

$$\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}$$

where $E = z_{\alpha/2}\frac{s}{\sqrt{n}}$ is the margin of error and $z_{\alpha/2}$ satisfies



Example

Example 6: As a clue to the amount of organic waste in Lake Macatawa, a count was made of the number of bacteria colonies in 100 milliliters of water. The number of colonies, in hundreds, for $n = 30$ samples of water from the east basin yield

93	140	8	120	3	120	33	70	91	61
7	100	19	98	110	23	14	94	57	9
66	53	28	76	58	9	73	49	37	92

Find a 90% exact and large sample confidence interval for the mean number μ_E of colonies in 100 milliliters of water in the east basin.

Solution $\bar{x} = 60.37$, $s = 39.62$, Choose $\alpha = 0.1$.
The exact 90% confidence interval is

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 60.37 \pm 1.70\frac{39.62}{\sqrt{30}} = 60.37 \pm 12.29$$

$$= [48.08, 72.66].$$

A 90% large sample confidence interval for μ_E is

$$\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}} = 60.37 \pm 1.65\frac{39.62}{\sqrt{30}} = 60.37 \pm 11.899$$

$$= [48.468, 72.265].$$