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stats
Mon Mar 18 10:26:41 EDT 2019
Homework 8: Sections 9.12-9.13 Exercise 9.72, 9.74, 9.78, 9.80.
Study the textbook Examples in Sections 9.12-9.13.

9.72 A random sample of 20 students yielded a mean of $\bar{x} = 72$ and a variance of $s^2 = 16$ for scores on a college placement test in mathematics. Assuming the scores to be normally distributed, construct a 98% confidence interval for σ^2 .

$n = 20$
$\bar{x} = 72$
$s^2 = 16$
$a = 0.02$

$(n-1)S^2/\chi^2_{\alpha/2} < \sigma^2 < (n-1)S^2/\chi^2_{1-\alpha/2}$

$(20-1) * 16/36.191 < \sigma^2 < (20-1) * 16/7.633$

$8.3999 < \sigma^2 < 39.8271$

9.74 Construct a 99% confidence interval for σ^2 in Exercise 9.11 on page 283.

$\alpha = 0.01$
$n = 9$
$s = (\sum_{i=1}^{n}(x_i - \bar{x})^2/(n-1))^{1/2} = 0.0245$

summon formula: $(n-1)S^2/\chi^2_{\alpha/2} < \sigma^2 < (n-1)S^2/\chi^2_{1-\alpha/2}$

compute: $0.0002 < \sigma^2 < 0.0036$

9.78 Construct a 90% confidence interval for σ_1^2/σ_2^2 in Exercise 9.43 on page 295. Were we justified in assuming that $\sigma_1^2 = \sigma_2^2$ when we constructed the confidence interval for $\mu_1 - \mu_2$?

$n_1 = 12$
$n_2 = 12$
$\bar{x_1} = 36300$
$\bar{x_2} = 38100$
$s_1 = 5000$
$s_2 = 6100$

$\frac{s_1^2}{s_2^2 * f_{\alpha/2}(v_1,v_2)} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2 * f_{\alpha/2}(v_1,v_2)}{s_2^2}$

$\frac{5000^2}{6100^2 * 2.82} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{5000^2 * 2.82}{6100^2}$

$0.2382 < \frac{\sigma_1^2}{\sigma_2^2} < 1.8946$

So it is possible that $\sigma_1^2 = \sigma_2^2$.

9.80 Construct a 95% confidence interval for σ_A^2/σ_B^2 in Exercise 9.49 on page 295. Should the equal-variance assumption be used?

$n_1 = 15$
$\bar{x_A} = \sum(x_A)/n_A = 57.3/15 = 3.82$

$s_A^2 = \frac{\sum(x_A - \bar{x_A})^2}{n_A - 1} = 0.6074$

$\bar{x_B} = \sum(x_B)/n_B = 4.94$
$s_B^2 = \frac{\sum(x_B - \bar{x_B})^2}{n_B - 1} = 0.5682$

$\frac{s_A^2}{s_B^2 * f_{\alpha/2}(v_A, v_B)} < \frac{\sigma_A^2}{\sigma_B^2} < \frac{s_A^2 * f_{\alpha/2}(v_A, v_B)}{s_B^2}$

$1.0688/2.98 < \frac{\sigma_A^2}{\sigma_B^2} < 1.0688 * 2.98$

$0.3586 < \frac{\sigma_A^2}{\sigma_B^2} < 3.1850$

So it is possible that $\sigma_A^2 = \sigma_B^2$.