STATISTICAL INFERENCES (2cr)
Chapter 10 One- and Two-Sample Tests of Hypotheses

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Math, IUSB

Outline

Outline

1. 10.8 One Sample: Tests on a Single Proportion
   - The Two-Tailed z-Test
   - The Left-Tailed z-Test
   - The Right-Tailed z-Test

Two–Tailed z-Test for p

Step 1. State the hypotheses

$$H_0 : p = p_0, \quad vs \quad H_1 : p \neq p_0$$

Step 2. Choose the distribution: If

$$np_0 > 5 \quad \text{and} \quad nq_0 > 5 \quad q_0 = 1 - p_0$$

then use

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}}$$

as the test statistic and choose normal distribution.

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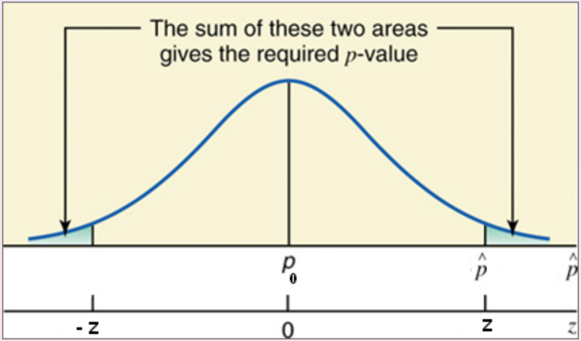
Step 3. Calculate p-value or find critical value.

Find the p-value:



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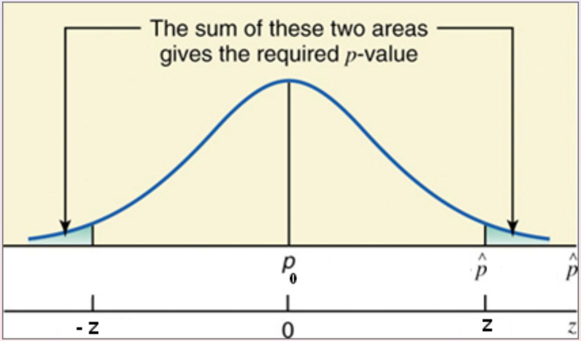

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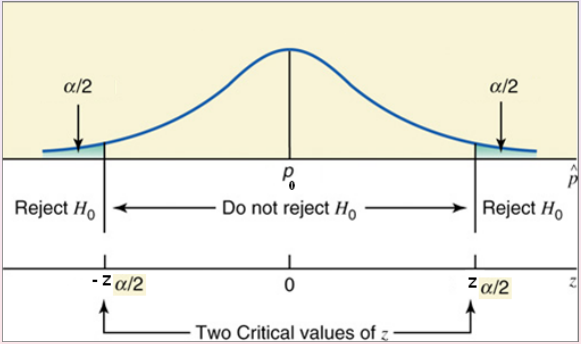
Step 3. Calculate p-value or find critical value.
Find the critical value $z_{\alpha/2}$ for z:



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Step 4. Make decision:
If p-value $< \alpha$, reject H_0, otherwise, do not reject H_0.
if $z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$, reject H_0, otherwise, do not reject H_0.

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Example

Example 1. In a *Time* magazine poll of adult Americans conducted by telephone March 15-17, 2005 by SRBI Public Affairs, 66% of the respondents said that there is too much violence on the television (*Time*, March 28, 2005). Assume that this result holds true for the 2005 population of all adults Americans. In a recent random sample of 1000 adult Americans, 70% said that there is too much violence on the television. Using the 2% significance level, can you conclude that the current percentage of adults Americans who think there is too much violence on the television is different from that for 2005?

Example

Solution:

Step 1. Hypotheses: $H_0 : p = 0.66 \quad H_1 : p \neq 0.66$

Step 2. $n = 1000$,

$$np_0 = 660 > 5, \quad nq_0 = 340 > 5$$

Use z-test and normal distribution.

Example

Solution:

Step 1. Hypotheses: $H_0 : p = 0.66 \quad H_1 : p \neq 0.66$

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Step 3.

$$\hat{p} = \frac{104}{590} = 0.1763, \quad \alpha = 0.02,$$

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}} = \frac{0.7 - 0.66}{\sqrt{0.66(0.34)/1000}} = 2.67$$

The p-value Approach: The p-value is

$$p\text{-value} = 2P(\hat{p} > 0.7) = 2P(z > 2.67) = 0.0076$$

The Critical Value Approach: The critical value is $z_{\alpha/2} = 2.33$.

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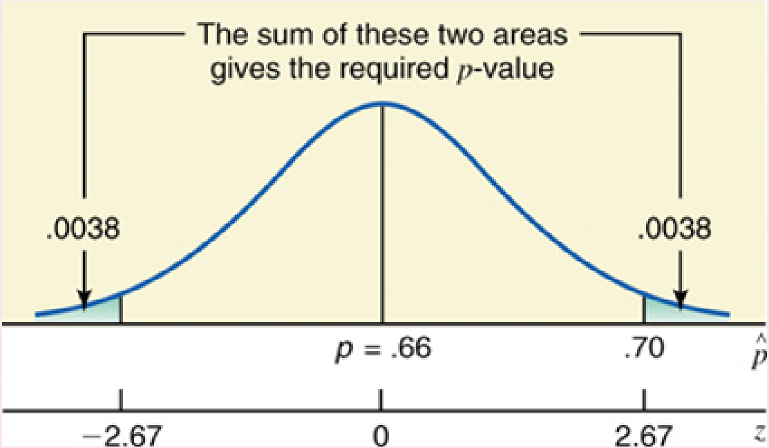

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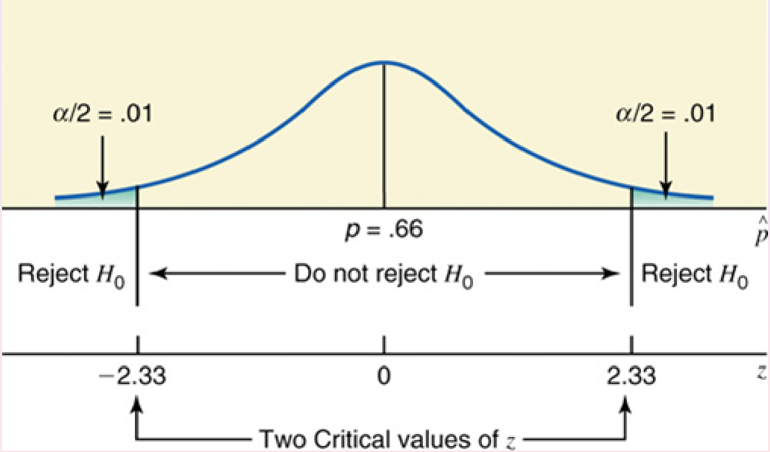

Outline

The Left-Tailed z-Test for p

Step 1. State the null and alternative hypotheses
$H_0 : p = p_0,$ or $H_0 : p \geqslant p_0,$ $H_1 : p < p_0$

Step 2. Choose the distribution;

Step 3. Find p-value or critical value; $z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}}$,

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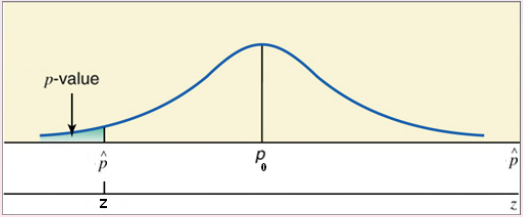

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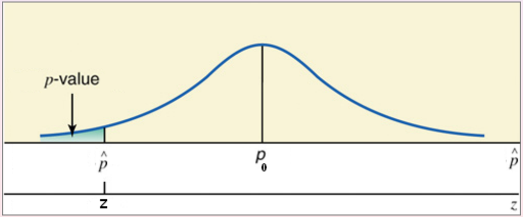

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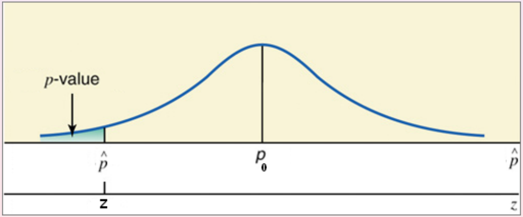

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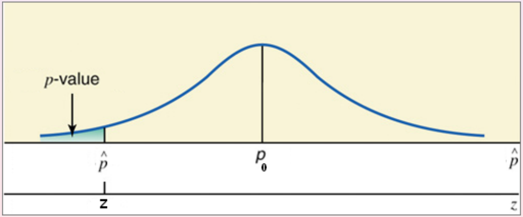

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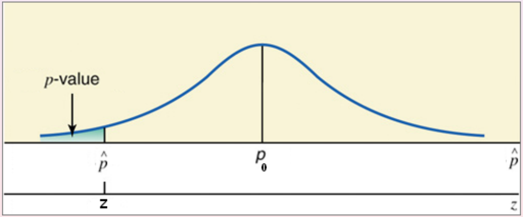

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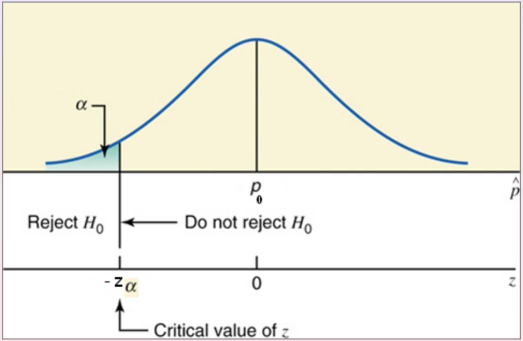

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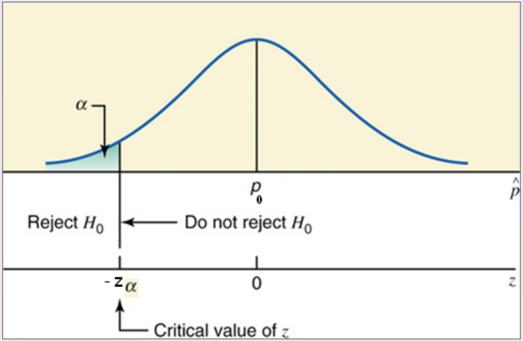

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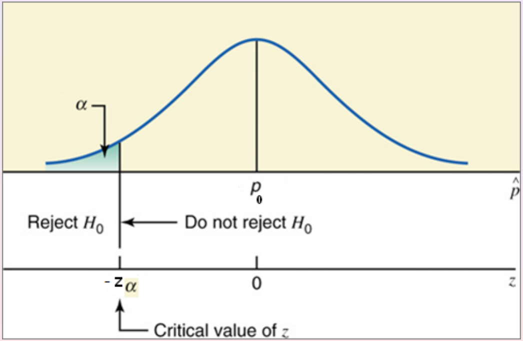

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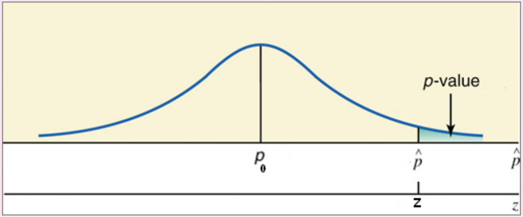

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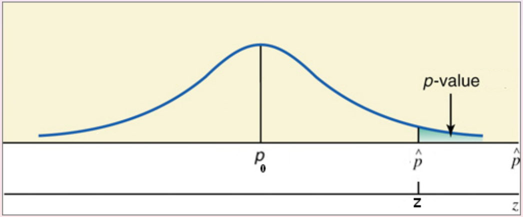

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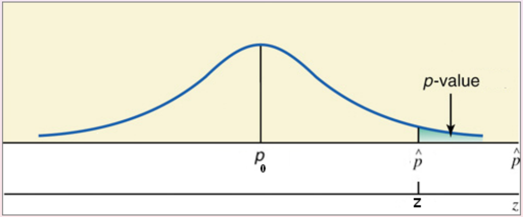

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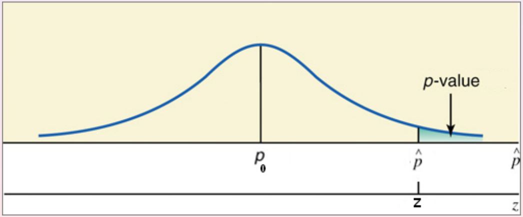

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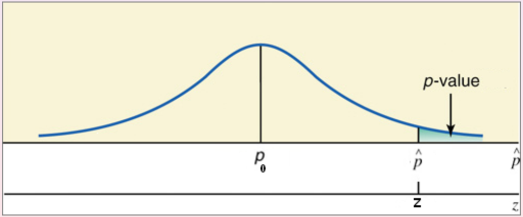

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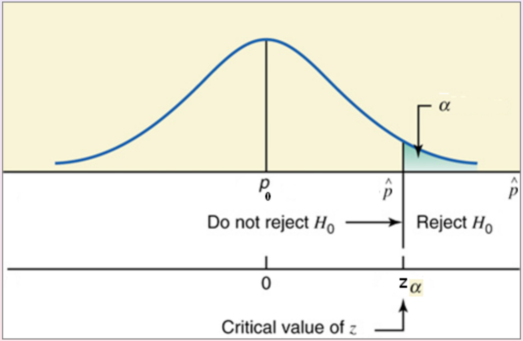

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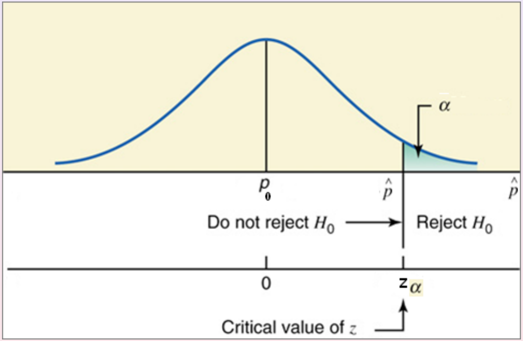

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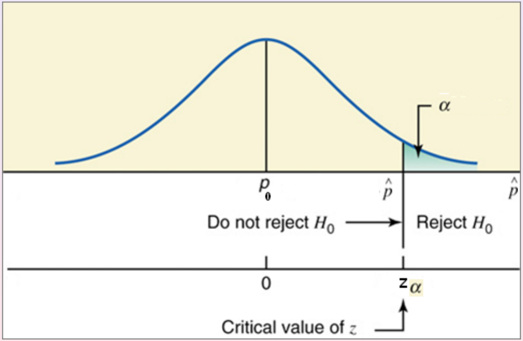

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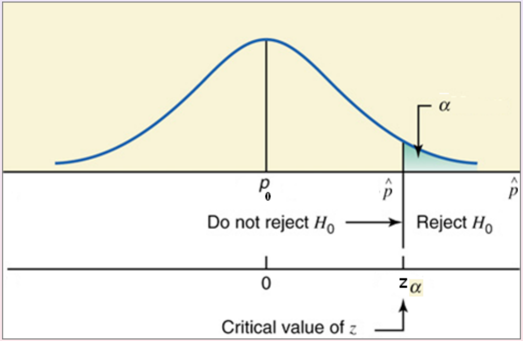

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Example 2. Let p be the proportion of drivers who use a seat belt in a state that does not have a amendatory seat belt law. It was claimed that $p = 14\%$. An advertising campaign was conducted to increase the proportion. Two months after the campaign, 104 out of a random sample of 590 drivers were wearing their seat belts. Was the campaign successful?($\alpha = 0.01$).

Solution:

Step 1. Hypotheses: $H_0 : p = 0.14 \quad H_1 : p > 0.14$

Step 2. $n = 590$,

$$np_0 = 590(0.14) = 82.6 > 5, \quad nq_0 = 590(1-0.14) = 507.4 > 5$$

Use z-test and normal distribution.

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$$p\text{-value} = P(\hat{p} > 0.1763) = P(z > 2.541) = 0.0055$$

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