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stats
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hw7: Sections 9.8-9.9 Exercise 9.40, 9.42, 9.43, 9.44.
Sections 9.10-9.11 Exercise 9.52, 9.53, 9.55, 9.60, 9.64, 9.66.

9.40 In a study conducted at Virginia Tech on the development of ectomycorrhizal, a symbiotic relationship between the roots of trees and a fungus, in which minerals are transferred from the fungus to the trees and sugars from the trees to the fungus, 20 northern red oak seedlings exposed to the fungus Pisolithus tinctorus were grown in a greenhouse. All seedlings were planted in the same type of soil and received the same amount of sunshine and water. Half received no nitrogen at planting time, to serve as a control, and the other half received 368 ppm of nitrogen in the form $NaNO_3$. The stem weights, in grams, at the end of 140 days were recorded as follows:

No Nitrogen 0.32 0.53 0.28 0.37 0.47 0.43 0.36 0.42 0.38 0.43

Nitrogen 0.26 0.43 0.47 0.49 0.52 0.75 0.79 0.86 0.62 0.46

Construct a 95% confidence interval for the difference in the mean stem weight between seedlings that receive no nitrogen and those that receive 368 ppm of nitrogen. Assume the populations to be normally distributed with equal variances.

$\bar{x}_{\bar{n}} = \sum(x_i)/n = 0.3989$
$s_{\bar{n}}^2 = (\sum(x_i - \bar{x}_{\bar{n}})^2/(n-1)) = 0.0728$

$\bar{x}_n = \sum(x_i)/n = 0.5650$
$s_n^2 = (\sum(x_i - \bar{x}_n)^2/(n-1)) = 0.1867$

$S_{both} = (\frac{(n-1)s_{\bar{n}}^2+(n-1)s_n^2}{n+n-2})^{1/2} = (0.0201)^{1/2} = 0.1417$

9.42 An experiment reported in Popular Science compared fuel economies for two types of similarly equipped diesel mini-trucks. Let us suppose that 12 Volkswagen and 10 Toyota trucks were tested in 90-kilometer-per-hour steady-paced trials. If the 12 Volkswagen trucks averaged 16 kilometers per liter with a standard deviation of 1.0 kilometer per liter and the 10 Toyota trucks averaged 11 kilometers per liter with a standard deviation of 0.8 kilometer per liter, construct a 90% confidence interval for the difference between the average kilometers per liter for these two mini-trucks. Assume that the distances per liter for the truck models are approximately normally distributed with equal variances.

$n_1 = 12$
$\bar{x}_1 = 16$
$s_1 = 1$

$n_2 = 10$
$\bar{x}_2 = 11$
$s_2 = 0.8$

$S = (\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2})^{1/2} = 0.9154$

$\alpha = 0.1$
$v = 20$
$t_{0.05,20} = 1.725$
$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,v} * S * (1/n_1 + 1/n_2)^{1/2} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,v} * S * (1/n_1 + 1/n_2)^{1/2}$
$4.324 < \mu_1 - \mu_2 < 5.676$

9.43 A taxi company is trying to decide whether to purchase brand A or brand B tires for its fleet of taxis. To estimate the difference in the two brands, an experiment is conducted using 12 of each brand. The tires are run until they wear out. The results are

Brand A:
$\bar{x}_1 = 36300$ kilometers,
$s_1 = 5000$ kilometers.
Brand B:
$\bar{x}_2 = 38100$ kilometers,
$s_2 = 6100$ kilometers.

Compute a 95% confidence interval for $\mu_A - \mu_B$ assuming the populations to be approximately normally distributed. You may not assume that the variances are equal.

$n_1 = 12$
$\bar{x}_1 = 36300$
$s_1 = 5000$

$n_2 = 12$
$\bar{x}_2 = 38100$
$s_2 = 6100$

$df = \lceil (s_1^2/n_1 + s_2^2/n_2)^2/((s_1^2/n_1)^2/(n_1-1) + (s_2^2/n_2)^2/(n_2-1)) \rceil = \lceil 20.3678 \rceil = 21$

$(\bar{x_1} - \bar{x_2}) \pm t_{\alpha/2,v}(s_1^2/n_1 + s_2^2/n_2)^{1/2} = -1800 \pm 4735.903$

$-6535.903 < \mu_A - \mu_B < 2935.903$

9.44 Referring to Exercise 9.43, find a 99% confidence interval for $\mu_1 - \mu_2$ if tires of the two brands are assigned at random to the left and right rear wheels of 8 taxis and the following distances, in kilometers, are recorded:

Taxi
Brand A B
1 34,400 36,700
2 45,500 46,800
3 36,700 37,700
4 32,000 31,100
5 48,400 47,800
6 32,800 36,400
7 38,100 38,900
8 30,100 31,500

Assume that the differences of the distances are approximately normally distributed.

$\bar{x}_1 = \sum x_1/n = 37250$
$s_1^2 = \sum(x_i - \bar{x}_1)^2/(n_1 - 1) = 42859935.5625$

$\bar{x}_2 = \sum x_2/n = 38362.5$
$s_2^2 = \sum(x_i - \bar{x}_2)^2/(n_2 - 1) = 38205502.7236$

$df = \lceil(s_1^2/n_1 + s_2^2/n_2)^2/((s_1^2/n_1)^2/(n_1-1) + (s_2^2/n_2)^2/(n_2-1))\rceil = \lceil 12.9342 \rceil = 13$

$t_{\alpha/2} \approx 3.0123$

$(\bar{x_1} - \bar{x_2}) \pm t_{\alpha/2,v}(s_1^2/n_1 + s_2^2/n_2)^{1/2} = -1112.5 \pm 9588.9513$

$(-10701.5, 8476.451)$

9.52 Compute 95% confidence intervals, using both methods on page 297, for the proportion of defective items in a process when it is found that a sample of size 100 yields 8 defectives.

$n = 100$
$x_{sample} = 8$
$\alpha = 0.05$

$\hat{p} = \bar{x}/n = 0.08$
$\hat{q} = 1 - \hat{p} = 0.92$
$z_{\alpha/2} = z_{0.025} = 1.96$

$\hat{p} - z_{\alpha/2}(\frac{\hat{p}\hat{q}}{n})^{1/2} < p < \hat{p} + z_{\alpha/2}(\frac{\hat{p}\hat{q}}{n})^{1/2}$

$0.0268 < p < 0.1332$ with 95 percent confidence.

using a big book formula
$\frac{\hat{p}+\frac{z^2_{\alpha/2}}{2n}}{1+\frac{z^2_{\alpha/2}}{n}} - \frac{z_{\alpha/2}}{1+\frac{z^2_{\alpha/2}}{n}}(\frac{\hat{p}\hat{q}}{n} + \frac{z^2_{\alpha/2}}{4n^2}) < p < \frac{\hat{p}+\frac{z^2_{\alpha/2}}{2n}}{1+\frac{z^2_{\alpha/2}}{n}} + \frac{z_{\alpha/2}}{1+\frac{z^2_{\alpha/2}}{n}}(\frac{\hat{p}\hat{q}}{n} + \frac{z^2_{\alpha/2}}{4n^2})$
Giving the inverval $(0.0411 < p < 0.1500)$ which is wider than the other method.

9.53
(a) A random sample of 200 voters in a town is selected, and 114 are found to support an annexation suit. Find the 96% confidence interval for the fraction of the voting population favoring the suit.

$n = 200$
$x_{sample} = 114$
$\alpha = 0.04$
$\hat{p} = x/n = 114/200 = 0.57$
$\hat{q} = 1 - \hat{p} = 0.43$

$z_{\alpha/2} = 2.055$

$\hat{p} - z_{\alpha/2}(\frac{\hat{p}\hat{q}}{n})^{1/2} < p < \hat{p} + z_{\alpha/2}(\frac{\hat{p}\hat{q}}{n})^{1/2}$

$0.4981 < p < 0.6419$

(b) What can we assert with 96% confidence about the possible size of our error if we estimate the fraction of voters favoring the annexation suit to be 0.57?

$\hat{p} = 0.57$
$error \leq z_{\alpha/2}(\hat{p}(1-\hat{p})/n)^{1/2}$= 0.0719

9.55 A new rocket-launching system is being considered for deployment of small, short-range rockets. The existing system has p = 0.8 as the probability of a successful launch. A sample of 40 experimental launches is made with the new system, and 34 are successful.

(a) Construct a 95% confidence interval for p.

$x = 34$
$n = 40$
$\alpha = 0.05$
$\hat{p} = x/n = 0.85$
$\hat{q} = 0.15$
$z_{\alpha/2} = 1.96$

$\hat{p} - z_{\alpha/2}(\frac{\hat{p}\hat{q}}{n})^{1/2} < p < \hat{p} + z_{\alpha/2}(\frac{\hat{p}\hat{q}}{n})^{1/2}$

$0.7393 < p < 0.9607$

(b) Would you conclude that the new system is better?

It is probably about the same since the old system was in the middle of the the confidence interval of the new system.
9.60 How large a sample is needed if we wish to be 99% confident that our sample proportion in Exercise 9.51 will be within 0.05 of the true proportion of homes in the city that are heated by oil?

$n = 1000$
$x = 228$
$\hat{p} = x/n = 0.228$
$\hat{q} = 0.772$

$z_{\alpha/2} = 2.575$

$n = \lceil z_{\alpha/2}^2 * \frac{\hat{p}(1-\hat{p})}{err^2} \rceil = \lceil 465.02 \rceil = 466$

9.64 A study is to be made to estimate the proportion of residents of a certain city and its suburbs who favor the construction of a nuclear power plant near the city. How large a sample is needed if one wishes to be at least 95% confident that the estimate is within 0.04 of the true proportion of residents who favor the construction of the nuclear power plant?

$err = 0.04$
$\alpha = 0.05$
$z_{\alpha/2} = 1.96$
$n = \lceil \frac{z_{\alpha/2}^2}{4err^2} \rceil = \lceil 600.25 \rceil = 601$

9.66 Ten engineering schools in the United States were surveyed. The sample contained 250 electrical engineers, 80 being women; 175 chemical engineers, 40 being women. Compute a 90% confidence interval for the difference between the proportions of women in these two fields of engineering. Is there a significant difference between the two proportions?

$n_1 = 250$
$x_1 = 80$
$n_2 = 175$
$x_2 = 40$

$\hat{p_1} = 0.3200$
$\hat{q_1} = 0.6799$
$\hat{p_2} = 0.2286$

$\hat{q}_2 = 0.7714$

$\alpha = 0.1$
$z_{\alpha/2} = 1.645$
$(\hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2}(\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}\hat{q}}{n_2})^{1/2}$
$(0.0201, 0.1627)$