STATISTICAL INFERENCES (2cr)
Chapter 9 One- and Two-Sample Estimation Problems

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Math, IUSB

Outline

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1. 9.10 Estimating a Proportion
   - Estimating Single Proportion
   - Sample Size for Estimating p
2. 9.11 Difference of Two Proportions

Do you know how to read the polls' results?

- If the the opinion poll says candidate A leads B 3%: 45% vs 42%, what does it mean?
- The poll gives sample proportions not the population proportions.
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- Y of the n voters will vote for A.
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Normal Approximation of Sampling Distribution of $\hat{p}$:

- By the C.L.T., if n is large, then

$$P\left[-z_{\alpha/2} \leqslant \frac{\overline{X}-p}{\sqrt{p(1-p)/n}} \leqslant z_{\alpha/2}\right] \approx 1-\alpha.$$

- Since $p \approx \hat{p}$,

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Confidence Interval for Population Proportion p:

- If sample size n is large, i.e. $n\hat{p} > 5$ and $n\hat{q} = n(1 - \hat{p}) > 5$,
- the $100(1 - \alpha)\%$ confidence interval for the population proportion p is

$$\overline{X} \pm E, \quad E = z_{\alpha/2}\sqrt{\hat{p}(1 - \hat{p})/n}.$$

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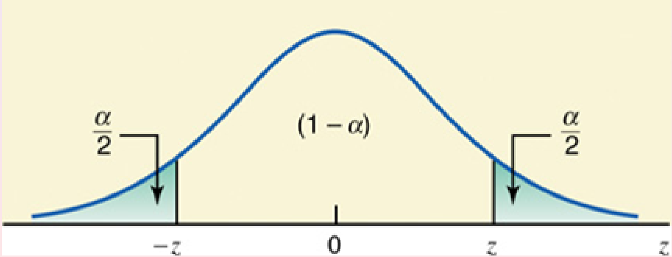

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- Let $a = 1 + \frac{z^2_{\alpha/2}}{n}$, $b = -2\overline{X} - \frac{z^2_{\alpha/2}}{n}$, $c = \overline{X}^2$. $H(p) = ap^2 + bp + c$.
- The solutions of $H(p) = 0$ are

$$\hat{p}_L = \frac{-b-\sqrt{b^2-4ac}}{2a}, \quad \hat{p}_U = \frac{-b+\sqrt{b^2-4ac}}{2a}.$$

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Example 2:

A machine shop manufactures toggle levers. A lever is flawed if a standard nut cannot be screwed onto the threads. Let p equal the proportion of flawed toggle levers that the shop manufactures. If there were 24 flawed levers out of a sample of 642 that were selected randomly from the production line.

(a) Give a point estimate of p.

(b) Find a 95% confidence interval for p.

Solution of Example 2

Solution

(a) The point estimate is $\hat{p} = 24/642 = 0.037$.

(b) $\alpha = 0.05$, $z = 1.96$, $s_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/n} = 0.0075$.

$E = 0.0147$ and 95% CI for p is $[0.0223, 0.0517]$.

If the machine shop wants to be 99% confident with a margin of error as small as 0.001, how large a sample should be selected?

What is the most conservative sample size?

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Determine Sample Size by Margin of Error

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$$E^2 = z^2_{\frac{\alpha}{2}} \frac{\hat{p}(1-\hat{p})}{n}$$

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- If $\hat{p}$ is not available, use the *most conservative* estimate

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In §6.7 Example 2, have found a point estimate of p: $\hat{p} = 0.037$ and the margin of error $E = 0.0147$ with 95% confidence.

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Solution of Example 3

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(a) $\alpha = 0.01$, $z_{\frac{\alpha}{2}} = 2.575$

$$n = \left(\frac{z}{E}\right)^2 \hat{p}(1-\hat{p}) = \left(\frac{2.575}{0.001}\right)^2 (0.037)(1-0.037) = 236,256$$

(b) The most conservative sample size is

$$n = \left(\frac{z_{\frac{\alpha}{2}}}{2E}\right)^2 = \left(\frac{2.575}{2(0.001)}\right)^2 = 1,657,657$$

Example 4:

- Consider two groups of women:

  Group 1: women who spend less than \$500 annually on clothes.

  Group 2: women who spend over \$1000 annually on clothes.

- Let p_1 and p_2 be the proportions of women in the two groups who believe that the clothes are too expensive, respectively.
- If $Y_1 = 1009$ out of $n_1 = 1230$ women from Group 1 and $Y_2 = 207$ out of $n_2 = 340$ from Group 2 believe that the clothes are too expensive.

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$$\hat{p}_1 = \frac{Y_1}{n_1} = 0.82, \quad \hat{p}_2 = \frac{Y_2}{n_2} = 0.61$$

The point estimate of $p_1 - p_2$ is $\hat{p}_1 - \hat{p}_2 = 0.21$.

(b) Since Y_1 and Y_2 are independent r.v.'s. $Y_1 \sim b(n_1, p_1)$ and $Y_2 \sim b(n_2, p_2)$,by the C.L.T.,

$$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_1(1-\hat{p}_1)/n_1 + \hat{p}_2(1-\hat{p}_2)/n_2}} \sim N(0,1)$$

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Thus $\hat{p}_1 - \hat{p}_2 \pm E = [0.154, 0.266]$.

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(b) Since Y_1 and Y_2 are independent r.v.'s. $Y_1 \sim b(n_1, p_1)$ and $Y_2 \sim b(n_2, p_2)$,by the C.L.T.,

$$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_1(1-\hat{p}_1)/n_1 + \hat{p}_2(1-\hat{p}_2)/n_2}} \sim N(0,1)$$

From this we an approximate $100(1-\alpha)\%$ CI for $p_1 - p_2$ is $\hat{p}_1 - \hat{p}_2 \pm E$, where

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = 0.056.$$

Thus $\hat{p}_1 - \hat{p}_2 \pm E = [0.154, 0.266]$.