STATISTICAL INFERENCES (2cr)
Chapter 10 One- and Two-Sample Tests of Hypotheses

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Math, IUSB

Outline

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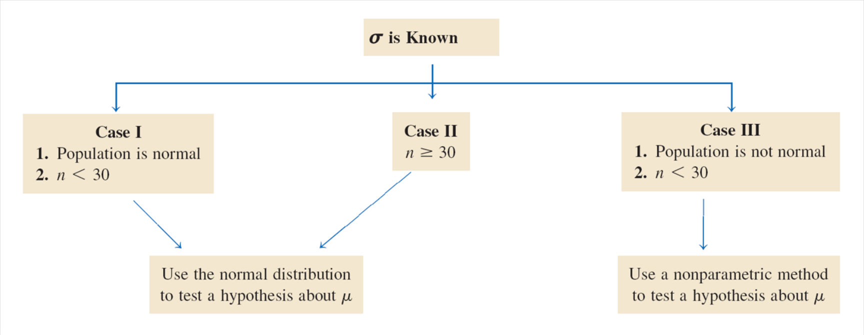
1. 10.4 Single Sample: Tests Concerning a Single Mean
   - Tests Concerning a Single Mean (Variance Known)
     - Cases I and II: The z Test
   - Tests Concerning a Single Mean (Variance Unknown)
     - Cases I and II: The t Test

Three Cases


σ is Known
Case I
1. Population is normal
2. $n < 30$
Case II
$n \geq 30$
Case III
1. Population is not normal
2. $n < 30$
Use the normal distribution to test a hypothesis about μ
Use a nonparametric method to test a hypothesis about μ

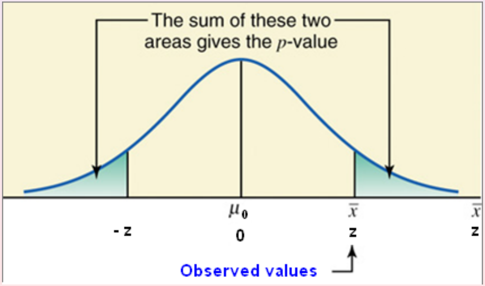
The Two-Tailed z-Test for $H_0 : \mu = \mu_0$

Step 1. State the hypotheses $H_0 : \mu = \mu_0, \quad H_1 : \mu \neq \mu_0$

Step 2. Choose the distribution: If σ is known and either population is normal or $n \geqslant 30$, then use normal distribution;

Step 3. Calculate p-value or find critical value. The test statistic is $z = \sqrt{n}(\bar{x} - \mu_0)/\sigma$

Find the p-value:



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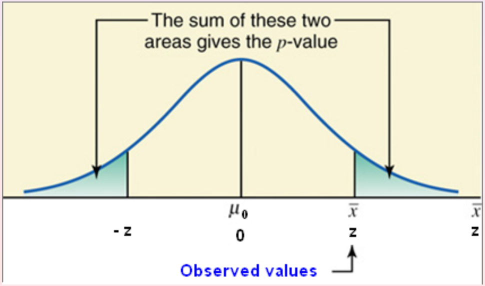

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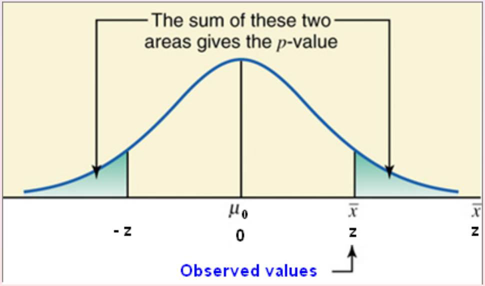

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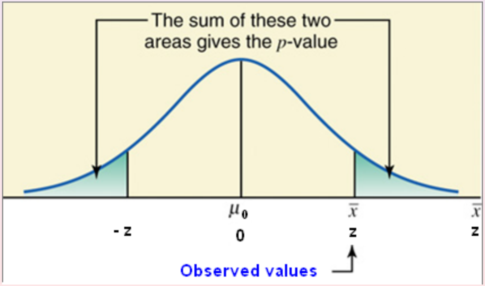

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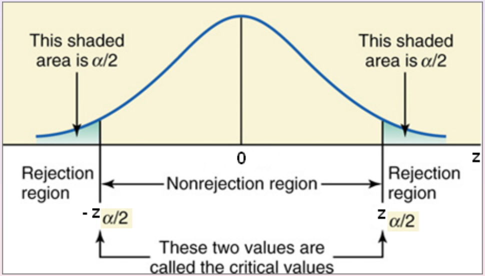
Find the critical value $z_{\alpha/2}$ for z:



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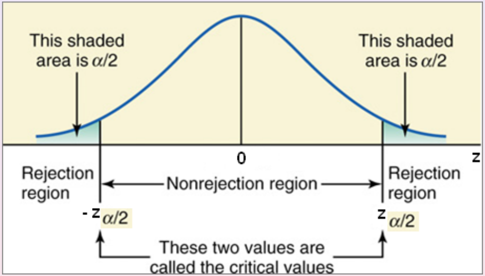

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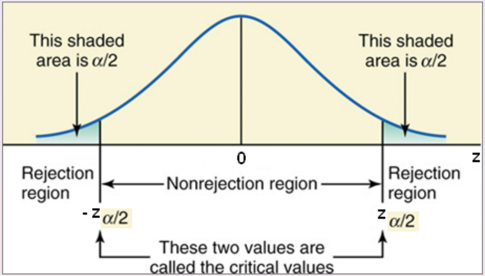

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Example

Example 1. Assume that the the thickness of spearmint gum manufactured for vending machines has a normal distribution with mean μ and $\sigma = 0.1$. Test the null hypothesis $H_0 : \mu = 7.5$ hundredths of an inch against $H_1 : \mu \neq 7.5$ based on the 10 observations

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7.65, 7.60, 7.65, 7.70, 7.55
7.55, 7.40, 7.40, 7.50, 7.50
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Solution:

Step 1. Hypotheses: $H_0 : \mu = 7.5 \quad H_1 : \mu \neq 7.5$

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Step 3.

$$\bar{x} = 7.55, \quad \alpha = 0.05,$$

$$z = (\bar{x} - \mu_0)/(\sigma/\sqrt{n}) = 1.581$$

The p-value Approach:

$$p\text{-value} = 2P(\bar{x} > 7.55) = 2P(z > 1.581) = 0.1139$$

The Critical Value Approach: $z_{\alpha/2} = 1.96$.

Step 4. Since the p-value $> \alpha$, we do not reject H_0.
Since $|z| < z_{\alpha/2}$, we do not reject H_0.

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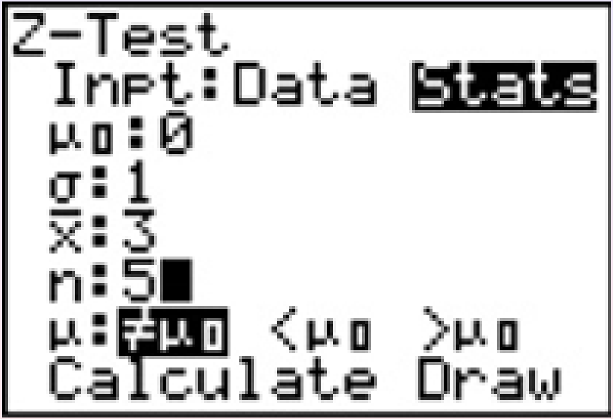
Using Excel

ZTEST | =2*ZTEST(B5:F6,7.5,0.1)

	A	B	C	D	E	F	G
1	Two-Tailed Z-Test						
2	Example 1:		H0: mu = mu0 vs H1: mu != mu0				
3	mu0=	7.5	sigma=	0.1			
4	alpha=	0.05	n=	10			
5	Data:	7.65	7.6	7.65	7.7	7.55	
6		7.55	7.4	7.4	7.5	7.5	
7	x-bar=	7.55	z=	1.5811	C. V. =	1.95996	
8	Using Z-test		P-val=	=2*ZTEST(B5:F6,7.5,0.1)			
9							
10							

ZTEST(array, x, [sigma])

Using TI-84



The two-tailed z-test can be done by CI

- Construct a $100(1-\alpha)\%$ CI.
- If the hypothesized μ_0 is not contained in the $100(1-\alpha)\%$ CI, we reject H_0 at the significant level α,
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Step 1. State the null and alternative hypotheses

$H_0 : \mu = \mu_0$, or $H_0 : \mu \geqslant \mu_0$, $H_1 : \mu < \mu_0$

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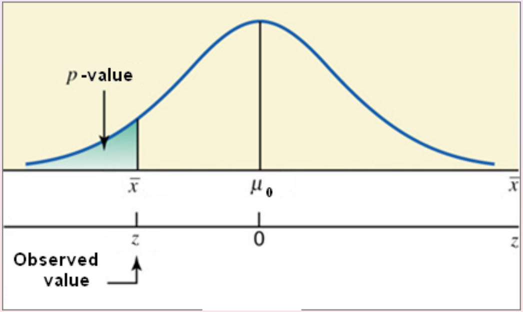

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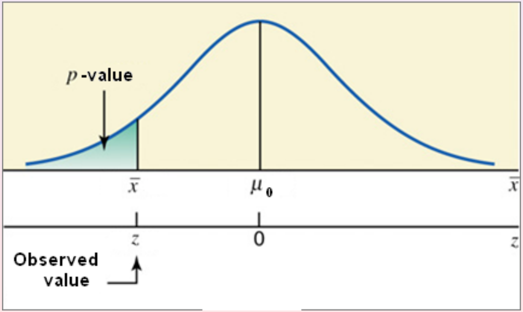

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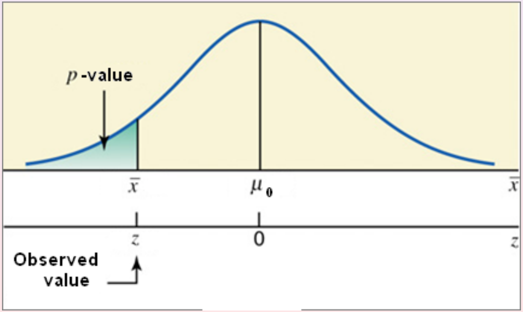

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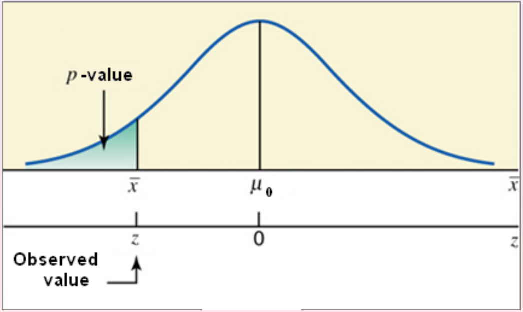

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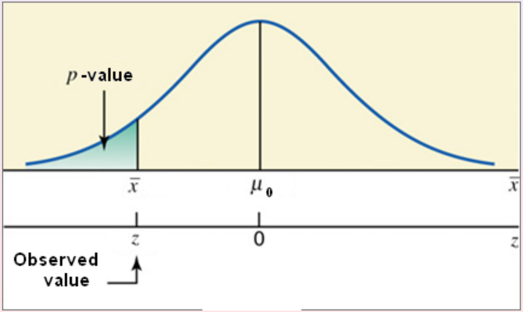

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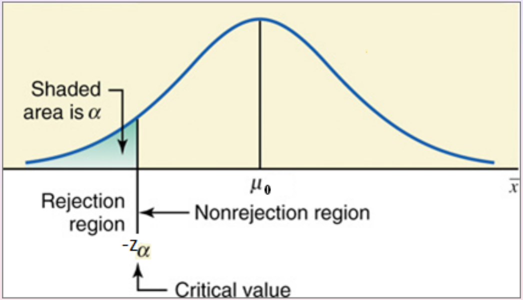

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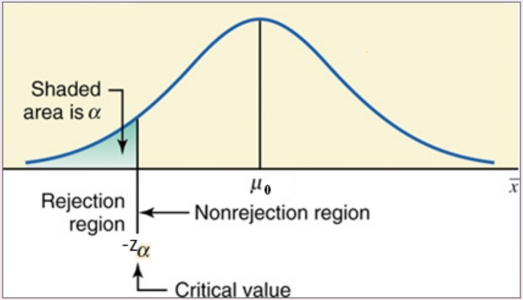

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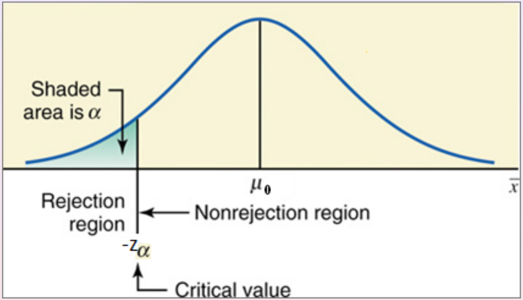

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If $\alpha = 0.01$, then $-z_\alpha = -2.34$.
If $\alpha = 0.05$, then $-z_\alpha = -1.64$.

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But p-value < 0.05, we reject H_0 at the significant level 0.05.
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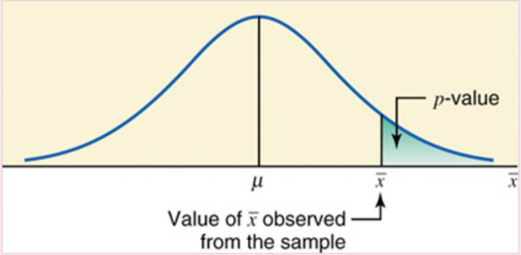

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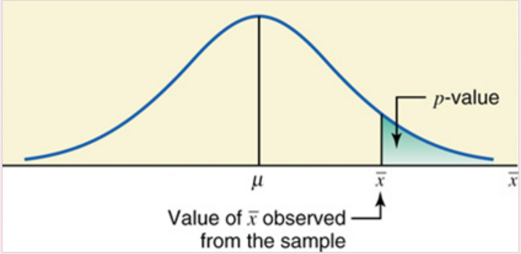
Step 3. Find p-value or critical value; $z = \sqrt{n}(\bar{x} - \mu_0)/\sigma$
Find the p-value:



The Right-Tailed z-Test

Step 1. State the null and alternative hypotheses
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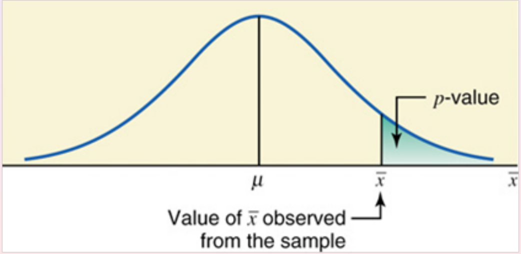

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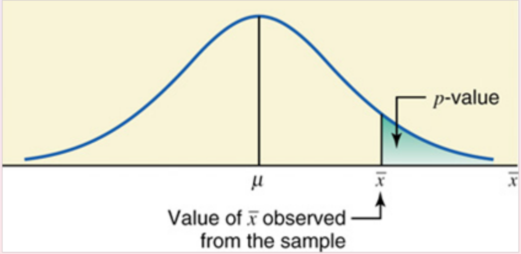

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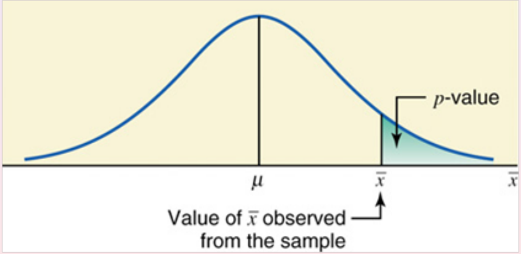

The Right-Tailed z-Test

Step 3. Find p-value or critical value.
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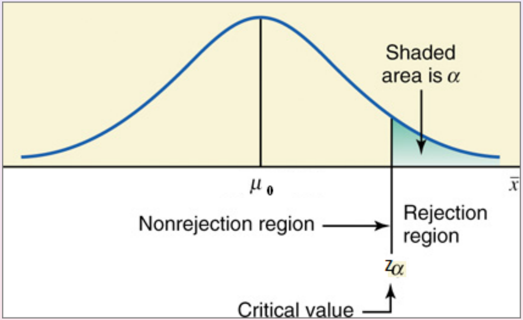

Step 4. Make decision:
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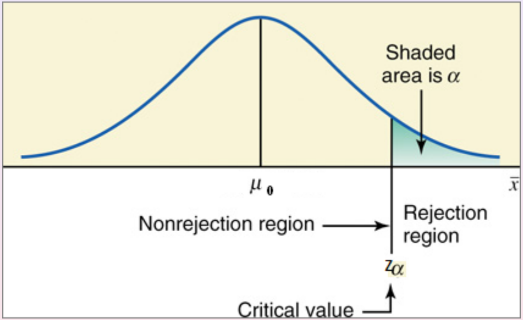

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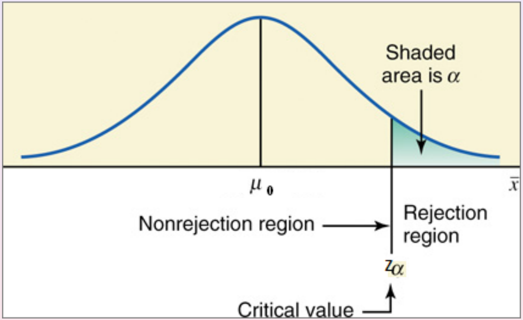

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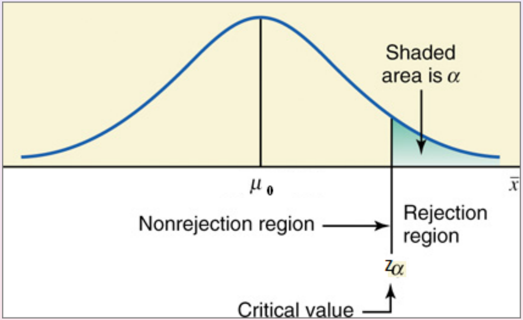

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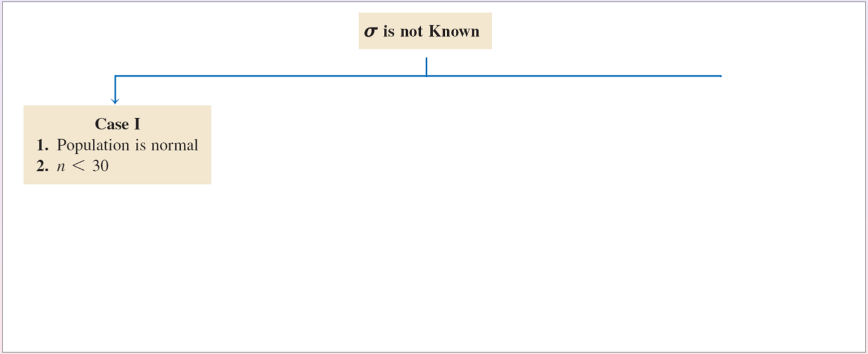
Outline

Three Cases

σ is not Known

Three Cases



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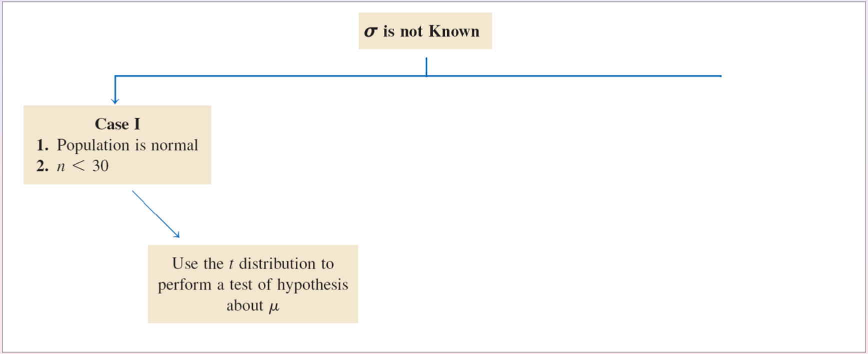
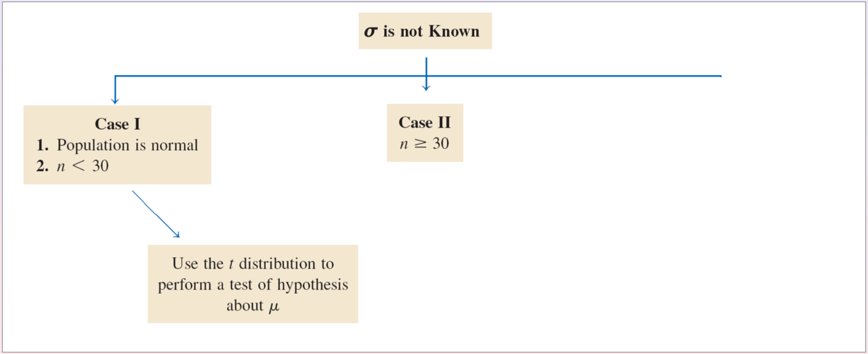

σ is not Known
Case I
1. Population is normal
2. $n < 30$
Use the t distribution to perform a test of hypothesis about μ

Three Cases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n ≥ 30
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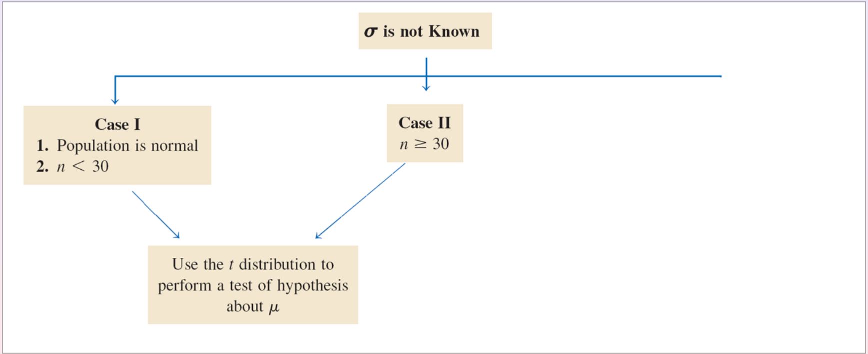
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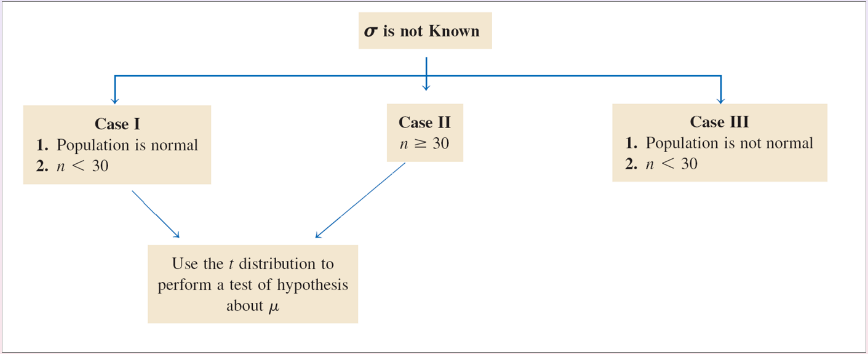
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$n \geq 30$
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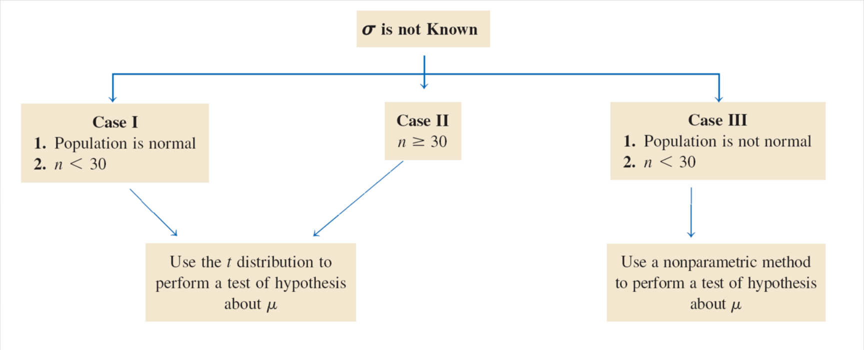
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Use a nonparametric method to perform a test of hypothesis about μ

The Two-Tailed t-Test for $H_0 : \mu = \mu_0$

Step 1. State the hypotheses $H_0 : \mu = \mu_0, \quad H_1 : \mu \neq \mu_0$

Step 2. Choose the distribution: If σ is unknown and either population is normal or $n \geqslant 30$, then use t distribution;

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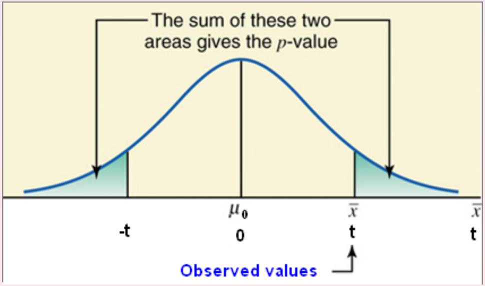

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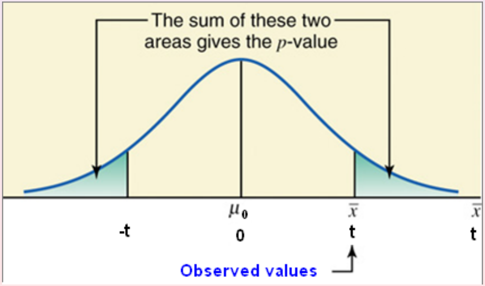

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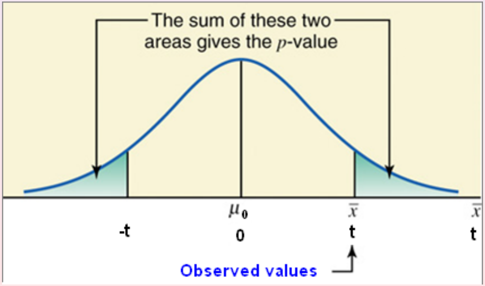

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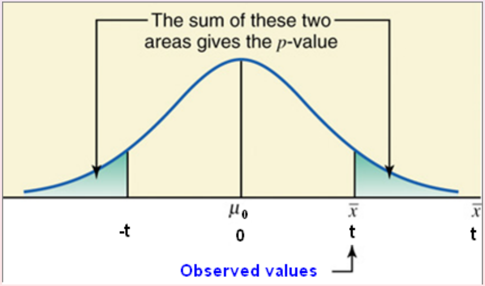
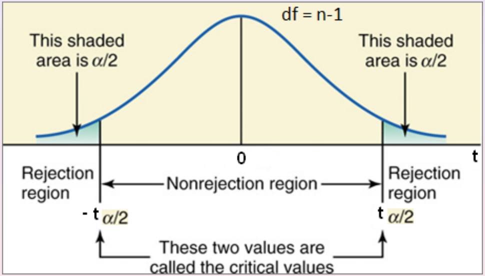


The Two-Tailed t-Test for $H_0 : \mu = \mu_0$

Step 3. Find p-value or critical value.
Find the critical value $t_{\alpha/2}$ for t:



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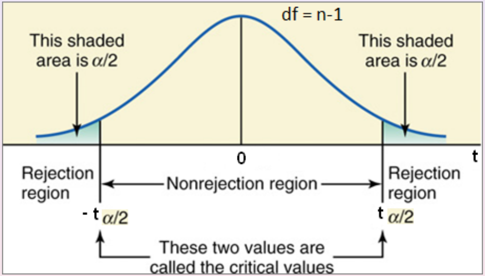

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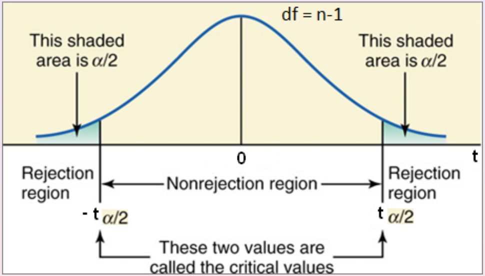

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Example

Example 1. A company that manufactures brackets for an auto maker selected 15 brackets from the production line and performs a torque test. The goal is for mean torque to equal 125. Let the toque have a normal distribution. The 15 observations are

```
128, 149, 136, 114, 126, 142, 124,
122, 118, 122, 129, 118, 122, 129, 136
```

Test $H_0 : \mu = 125$ against a two-tailed alternative hypothesis ($\alpha = 0.05$).

Solution:

Step 1. Hypotheses: $H_0 : \mu = 125 \quad H_1 : \mu \neq 125$

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Example

Step 3.

$$t = (\bar{x} - \mu_0)/(s/\sqrt{n}) = 1.076$$

The p-value Approach:

$$p\text{-value} = 2P(t > 1.076) = 0.3001$$

The Critical Value Approach: $\alpha = 0.05$, $t_{\alpha/2} = 2.145$.

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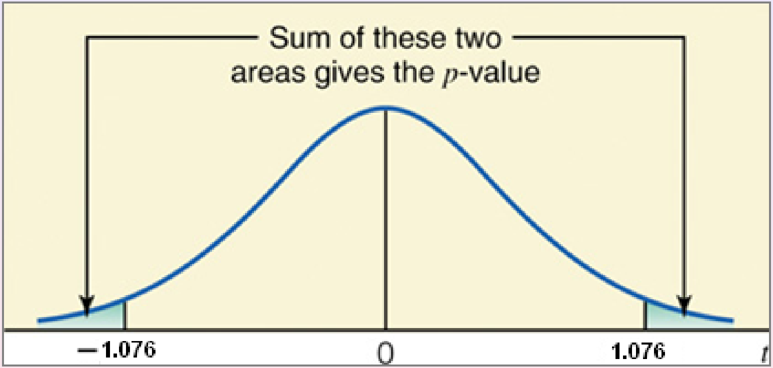
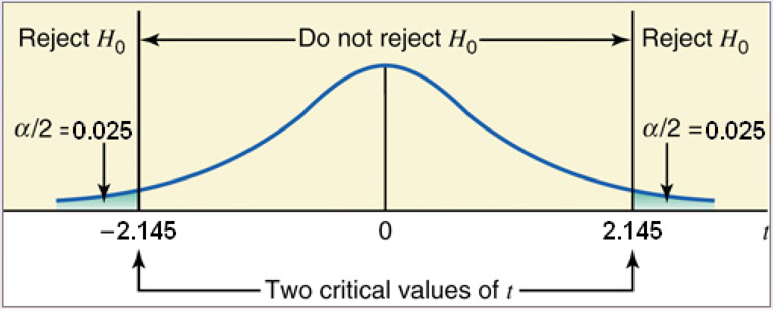
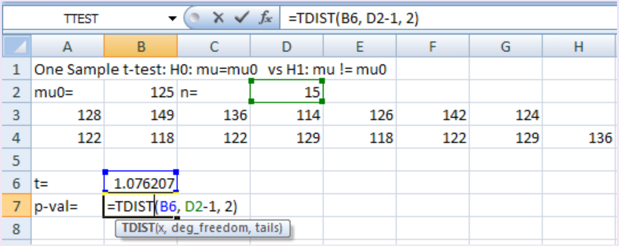
P-value Value of t–Two-Tailed



Critical Value of t–Two-Tailed

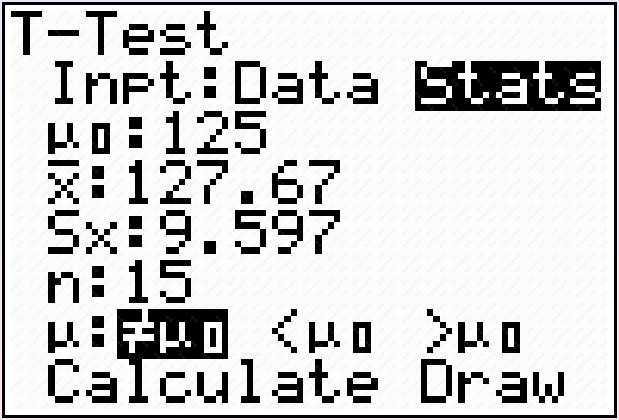


Using Excel


TTEST =TDISTT(B6, D2-1, 2)

	A	B	C	D	E	F	G	H
1	One Sample t-test: H0: mu=mu0 vs H1: mu != mu0							
2	mu0=	125	n=	15				
3	128	149	136	114	126	142	124	
4	122	118	122	129	118	122	129	136
5								
6	t=	1.076207						
7	p-val=	=TDIST(B6, D2-1, 2)						
8		TDIST(x, deg_freedom, tails)						

Using TI-84


T-Test
Inpt:Data Stats
μ0:125
x̄:127.67
Sx:9.597
n:15
μ:≠μ0 <μ0 >μ0
Calculate Draw

Using TI-84

```
T-Test
 μ≠125
 t=1.077510215
 p=.2994692797
 x̄=127.67
 Sx=9.597
 n=15
```

The two-tailed t-test can be done by CI

- Construct a $100(1-\alpha)\%$ CI using t distribution.
- If the hypothesized μ_0 is not contained in the $100(1-\alpha)\%$ CI, we reject H_0 at the significant level α,
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The Left-Tailed t-Test

Step 1. State the null and alternative hypotheses
$H_0 : \mu = \mu_0$, or $H_0 : \mu \geqslant \mu_0$, $H_1 : \mu < \mu_0$

Step 2. Choose the distribution;

Step 3. Find p-value or critical value; $t = \sqrt{n}(\bar{x} - \mu_0)/s$, $df = n - 1$,
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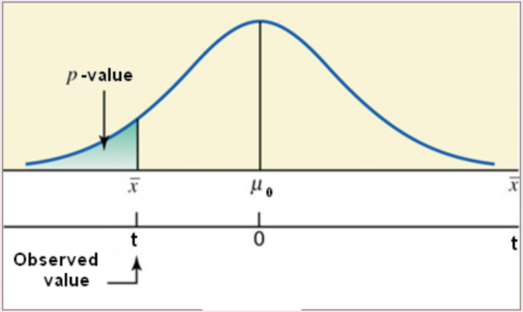

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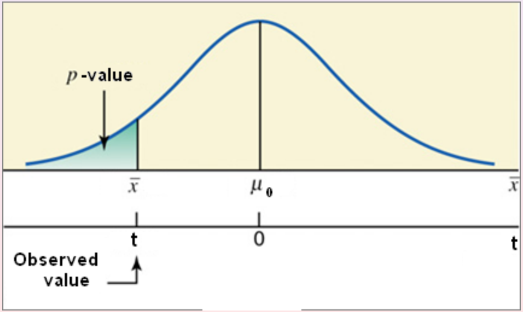

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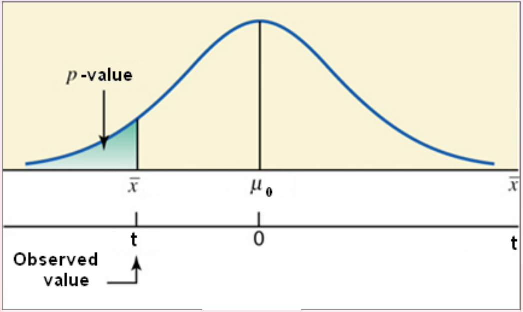

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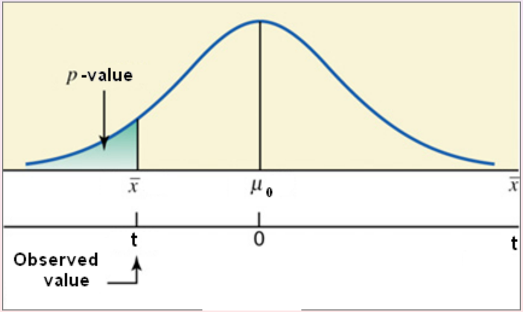

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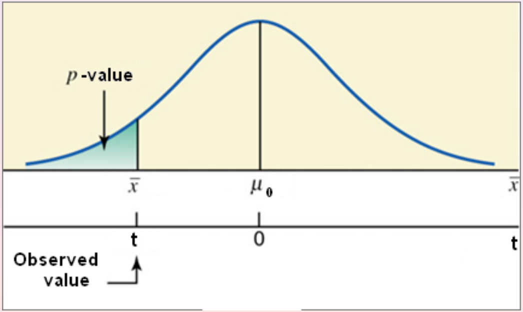

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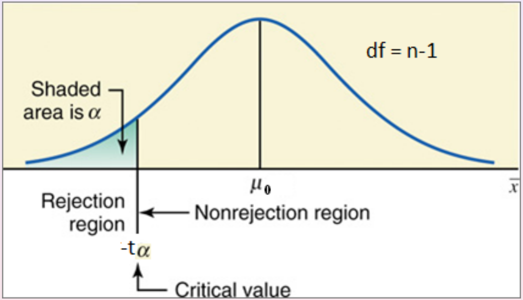

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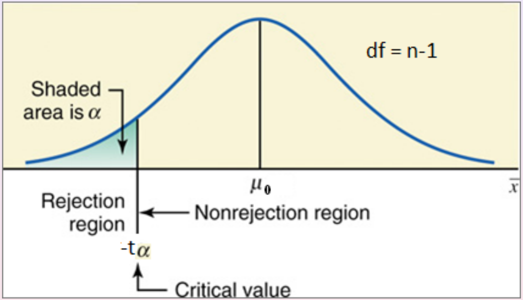

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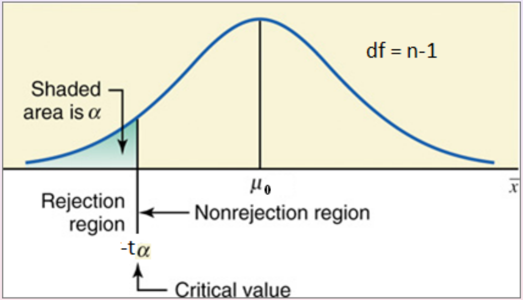

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The Right-Tailed t-Test

Step 1. State the null and alternative hypotheses
$H_0 : \mu = \mu_0$, or $H_0 : \mu \leqslant \mu_0$, $H_1 : \mu > \mu_0$

Step 2. Choose the distribution;

Step 3. Find p-value or critical value; $t = \sqrt{n}(\bar{x} - \mu_0)/s$, $df = n - 1$,
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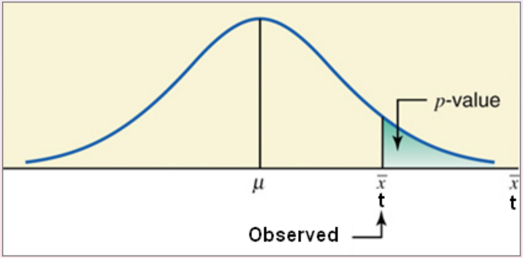

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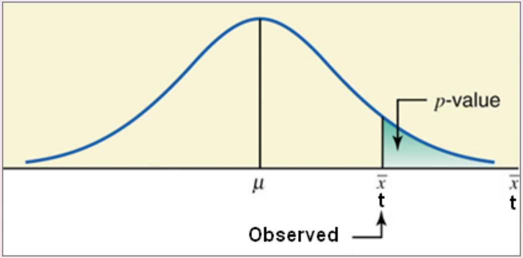

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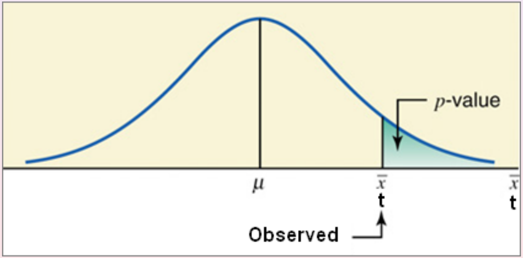

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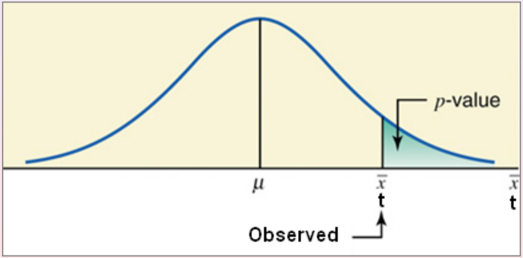

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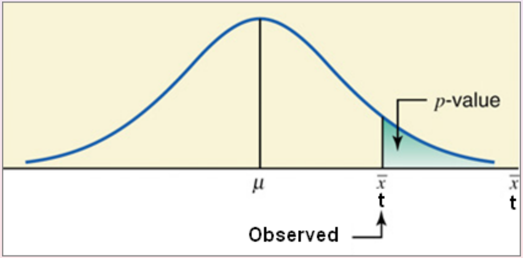

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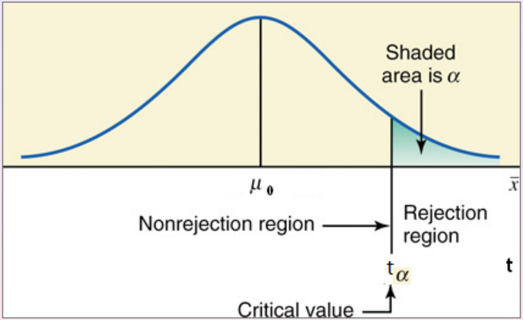

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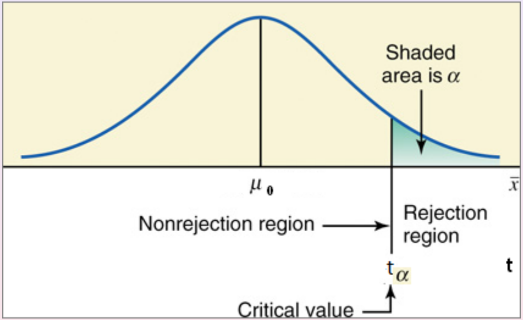

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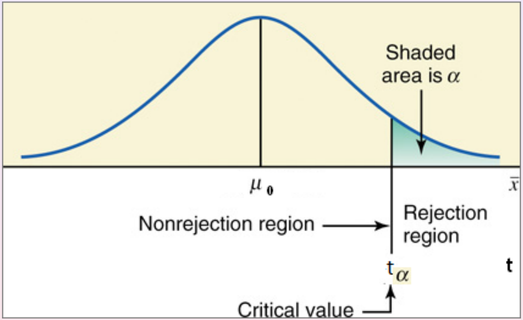

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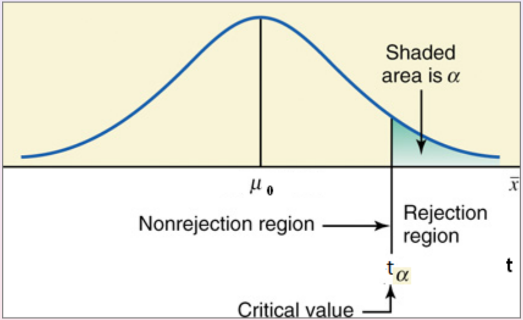

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Example

Example 2. Let x be the Brinell hardness measurement of ductile iron subcritically annealed. Assume that x has a normal distribution with mean μ. Test $H_0 : \mu = 170$ against the alternative hypothesis that the mean hardness is greater than 170 based on the following 25 observations(Use significance level $\alpha = 0.05$.)

```
170, 167, 174, 179, 179, 156, 163, 156, 187
156, 183, 179, 174, 179, 170, 156, 187,
179, 183, 174, 187, 167, 159, 170, 179
```

Solution:

Step 1. Hypotheses:

$$H_0 : \mu = 170 \quad H_1 : \mu > 170$$

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Example 2. Let x be the Brinell hardness measurement of ductile iron subcritically annealed. Assume that x has a normal distribution with mean μ. Test $H_0 : \mu = 170$ against the alternative hypothesis that the mean hardness is greater than 170 based on the following 25 observations(Use significance level $\alpha = 0.05$.)

```
170, 167, 174, 179, 179, 156, 163, 156, 187
156, 183, 179, 174, 179, 170, 156, 187,
179, 183, 174, 187, 167, 159, 170, 179
```

Solution:

Step 1. Hypotheses:

$$H_0 : \mu = 170 \quad H_1 : \mu > 170$$

Example

Step 2. This is Case I. $s = 10.31$, $n = 25$, σ is not known, use t distribution.

Step 3. $\bar{x} = 172.52$, $s = 10.31$, $t = (\bar{x} - \mu_0)/(s/\sqrt{n}) = 1.2218$, $df = 24$.

The p-value Approach:

$$p\text{-value} = P(t > 1.2218) = 0.1168$$

The Critical Value Approach:

$\alpha = 0.05$, $t_\alpha = 1.7109$.

Step 4. p-value > 0.05, we do not reject H_0 at the significant level 0.05.

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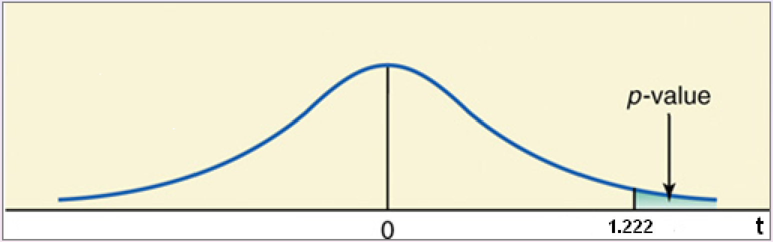
P-value of *t*–Right-Tailed



Critical value of t–Right-Tailed