Jordan Winkler
stats
Thu Mar 21 19:50:57 EDT 2019
Homework 9: Sections 10.1-10.3 Exercise 10.1, 10.2, 10.4, 10.5, 10.6, 10.14. Study the textbook Examples in Sections 10.1-10.3.

10.1 Suppose that an allergist wishes to test the hypothesis that at least 30% of the public is allergic to some cheese products. Explain how the allergist could commit

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+-----+-----+-----+
|     |h0 t |h0 f |
+-----+-----+-----+
|-r h0| t   |err2 |
+-----+-----+-----+
|r h0 |err1 |  t  |
+-----+-----+-----+
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a) a type 1 error:

They refuted the null hypothesis and came to the conclusion that less than 30% of the public are allergic to some cheese products. But in reality 30% or more are allegic to cheese products.

b) a type 2 error:

It is the case that less than 30 percent of the public are allergic to cheese, but the null hypethesis was not rejected. So the allergist believes that 30 percent or more people are allergic to some types of cheese.

10.2 A sociologist is concerned about the effectiveness of a training course designed to get more drivers to use seat belts in automobiles.

a) What hypothesis is she testing if she commits a type I error by erroneously concluding that the training course is ineffective?

She is looking for evidence to accapt the hypothesis that the training course was effective, if she is at risk of a type 2 error.

b) What hypothesis is she testing if she commits a type II error by erroneously concluding that the training course is effective?

She is looking for evidence to reject the effectiveness of the training course, if she is at risk of a type 2 error.

10.4 A fabric manufacturer believes that the proportion of orders for raw material arriving late is p = 0.6. If a random sample of 10 orders shows that 3 or fewer arrived late, the hypothesis that p = 0.6 should be rejected in favor of the alternative p ¡ 0.6. Use the binomial distribution.

a) Find the probability of committing a type I error if the true proportion is p = 0.6.

P(type 1 error)= $\sum_{i=0}^{3}\binom{10}{i}0.6^i 0.4^{10-i}$= 0.0548

b) Find the probability of committing a type II error for the alternatives $p =$ $0.3, p = 0.4$, and p = 0.5.

p = 0.3 P(type 2 error)= $\sum_{i=4}^{10}\binom{10}{i}0.3^i 0.7^{10-i}$= 0.3504

p = 0.4
P(type 2 error)= $\sum_{i=4}^{10}\binom{10}{i}0.4^i 0.6^{10-i}$= 0.6177

p = 0.5
P(type 2 error)= $\sum_{i=4}^{10}\binom{10}{i}0.5^i 0.5^{10-i}$= 0.8281

10.5 Repeat Exercise 10.4 but assume that 50 orders are selected and the critical region is defined to be $x \leq 24$, where x is the number of orders in the sample that arrived late. Use the normal approximation.

a)
$\mu = np = 30$
$\sigma = (npq)^{1/2} = 3.4641$
$P(\text{type 1 error}) = P(\frac{x-30}{3.4641} < (24 + .5 - 30)/3.4641) = P(z < 1.59) = 0.0559$

b)
$\mu = 50 * .3 = 15$
$\sigma = 3.2403$
$P(\text{type 2 error}) = P((x - 15)/3.2403 > (24 + .5 - 15)/3.24) = P(z > 2.93) =$ 0.0071

$\mu = 50 * .4 = 20$
$\sigma = (npq)^{1/2} = 3.4641$
$P((x - 20)/3.4641 > (24 + .5 - 20)/3.4641) = P(z > 1.30) = 0.0968$

$\mu = 50 * 0.5 = 25$
$\sigma = (npq)^{1/2} = 3.5355$
$P((x - 25)/3.5355 > (24 + .5 - 25)/3.5355)$= $P(z > -0.14) = 0.5557$

10.6 The proportion of adults living in a small town who are college graduates is estimated to be p = 0.6. To test this hypothesis, a random sample of 15 adults is selected. If the number of college graduates in the sample is anywhere from 6 to 12, we shall not reject the null hypothesis that p = 0.6; otherwise, we shall conclude that $p \neq 0.6$.

a) Evaluate α assuming that p = 0.6. Use the binomial distribution.

$\alpha = P(\text{type 1 error}) = 1 - \sum_{x=6}^{12} \binom{15}{x} 0.6^x 0.4^{15-x} = 0.06$

b) Evaluate β for the alternatives p = 0.5 and p = 0.7.

$p = 0.5$
$\beta = P(\text{type 2 error}) = \sum_{x=6}^{12} bin(x, 15, 0.5) = 0.8454$

$p = 0.7$
$\beta = P(\text{type 2 error}) = \sum_{x=6}^{12} bin(x, 15, 0.7) = 0.8695$

c) Is this a good test procedure?

Nope. Error probability is high.

10.14 A manufacturer has developed a new fishing line, which the company claims has a mean breaking strength of 15 kilograms with a standard deviation of 0.5 kilogram. To test the hypothesis that $\mu = 15$ kilo- grams against the alternative that $\mu < 15$ kilograms, a random sample of 50 lines will be tested. The critical region is defined to be $\bar{x} < 14.9$.

a) Find the probability of committing a type I error when H_0 is true.

$P(\text{type 1 error}) = P((\bar{x} - 15)/0.0707 < (14.9 - 15)/(0.0707) = P(z < -1.414) = 0.0793$

b) Evaluate β for the alternatives $\mu = 14.8$ and $\mu = 14.9$ kilograms.

$\mu = 14.8$
$P(\text{type 2 error}) = P(\frac{\bar{x}-14.8}{0.0707} \geq \frac{14.9-14.8}{0.0707}) = P(z \geq 1.414) = 0.0793$

$\mu = 14.9$
$P(\text{type 2 error}) = P(\frac{\bar{x}-14.9}{0.0707} \geq \frac{14.9-14.9}{0.0707}) = P(z \geq 0) = 0.5$