Part One

Section 3.9(pg. 107)

1)Show that if six integers are chosen at random, then at least two of them will have the same remainder when divided by 5.

- If you choose the first 10 numbers in the set Z^+ there is only 5 distinct remainders when dividing them by 10, these remainders equal = {1,2,3,4,0,1,2,3,4,0}
- Thus because there is only 5 distinct remainders, if you choose 6 random integers by pigeonhole principle there is at least two of them that will have the same remainder
- 6/5 = 1.2 with a remainder of one

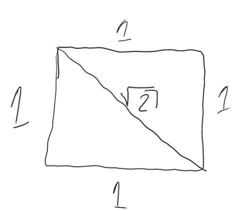
2)You deal a pile of cards, face down, from a standard 52-card deck. What is the least number of cards the pile must have before you can be assured that it contains at least five cards of the same suit?

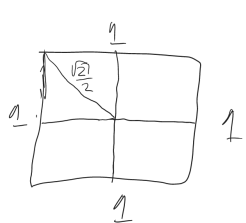
- You will need 17 cards to make sure that you will get 5 cards of the same suit
- 4 cards deal each time * deal 4 times = 16 total cards where all the suits can be the same within each deal of 4
- Thus adding one will assure that there is at least 5 cards of the same suit
- (4*4) + 1 = 17 cards total

3)What is the fewest number of times you must roll a six-sided dice before you can be assured that 10 or more of the rolls resulted in the same number?

- To assure that 10 or more rolls result in the same number, first you need 9 rolls to result in the same number
- Thus 9(amount of rolls containing the same number) * 6(amount of numbers) = 54
- If you add one to the total of 54 rolls then at least 10 of the points have the same value
- Thus there needs to be a total of 55 rolls

4)Select any five points on a square whose side-length is one unit. Show that at least two of these points are within $\frac{\sqrt{2}}{2}$ units of each other

- 

- 

- In the second square if you put four points one in each sub square then they are possibly farther then $\frac{\sqrt{2}}{2}$ from each other but that is only using four points
- If there is a 5th point introduced it has to go into or around one of the subsquares thus that point is less than $\frac{\sqrt{2}}{2}$ from one of the other four points
- Thus true

5)Prove that any set of seven distinct natural numbers contains a pair of numbers whose sum or difference is a multiple of 10.

- Take the first 7 digit from the set Z^+
- Assume (n + 1), (n + 2), (n + 3), (n + 4), (n + 5), (n + 6), (n + 7) where $n \in Z^+$
- If n = 0, then all the numbers are consecutive from one to seven
- 7 + 6 + 5 + 4 + 3 + 2 + 1 = 32
- Because the sum of all the consecutive numbers gives a minimum answer of 32 it is greater than 10 then there is at least one pair of numbers that when added or subtracted is multiple of 10
- Ex {(7,3), (6,4)} considering (n + 1), (n + 2), (n + 3), (n + 4), (n + 5), (n + 6), (n + 7) where n = 0
- Thus by pigeon hole principle there will always be at least one pair of numbers within a set of 7 distinct natural numbers, whose sum or difference is multiple by 10

Part Two

Section 11.3(pg. 214)

1) Let A = {1,2,3,4,5,6} , and consider the following equivalence relation on A: R = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(2,3),(3,2),(4,5),(5,4),(4,6),(6,4),(5,6),(6,5)} . List the equivalence classes of R.

- One only relates to one
  - [1] = {1}
- Two relates to two and three
  - [2] = {2,3}
- Three relates to three, two
  - [3] = {2,3}
- Four relates to four, five, six
  - [4] = {4,5,6}
- Five relates to five, four, six
  - [5] = {4,5,6}
- Six relates to six, four, five
  - [6] = {4,5,6}

5) There are two different equivalence relations on the set A = {a,b} . Describe them. Diagrams will suffice.

- R = {(a,a), (b,b)} (reflexive)
- R = {(a,a), (b,b), (a,b), (b,a)} (symmetric and transitive)

6) There are five different equivalence relations on the set A = {a,b, c }. Describe them all. Diagrams will suffice.

- R = {(a,a), (b,b), (c,c)}
- R = {(a,a), (b,b), (c,c), (b,c), (c,b)}
- R = {(a,a), (b,b), (c,c), (a,c), (c,a)}
- R = {(a,a), (b,b), (c,c), (a,b), (b,a)}
- R = {(a,a), (b,b), (c,c), (a,c), (c,a), (a,b), (b,a), (b,c), (c,b)}

9)Define a relation R on Z as xRy if and only if 4|(x+3y). Prove R is an equivalence relation. Describe its equivalence classes.

- It is reflexive because if we consider 4|(x+3x) to be true then there is a pair (x, x)
- It is symmetric because

  $4 | (x+3y)$

  $3(x+3y = 4a)$

  $3x+9y = 12a$

  $3x+y = 12a-8y$

  - $3x+y = 4(3a-2y)$
- It is transitivity because ({xRy ^ yRz} implies xRz)

  $4 | (x+3y)$

  $4 | (y+3z)$

  $4a = x+3y$

  $4b = y+3z$

  prove $(4c = x+3z)$

  - • $4a+4b = x+4y+3z$
    • $4a+4b-4y = x+3z$
    • $4(\underbrace{a+b-y}_{\text{some Integer}}) = x+3z$
    • $4(c) = x+3z$
    • thus $x+3z$ is divisable by 4 thus true

- [1] = 4 | (x + 3)
  - {...-7, -3, 1, 5, 9…}
- [2] = 4 | (x +6)
  - {...-10, -2, 2, 6, 10}
- [3] = 4 | (x + 9)
  - {...-5, -1, 3, 7, 11...}

11) Prove or disprove: If R is an equivalence relation on an infinite set A, then R has infinitely many equivalence classes.

- False, mod 4 only has 4 equivalence classes
- One for remainder zero, one, two and three

Section 12.1(pg. 228)

1)Suppose A = { 0,1,2,3,4 } , B = { 2,3,4,5 } and f = { (0,3),(1,3),(2,4),(3,2),(4,2)} . State the domain and range of f . Find f (2) and f (1).

- Domain = {0,1,2,3,4}
- Range = {2,3,4}
- f(2) = 4
- f(1) = 3

2) Suppose A = { a,b, c,d } , B = { 2,3,4,5,6 } and f = { (a,2),(b,3),(c,4),(d,5)} . State the domain and range of f . Find f (b) and f (d).

- Domain = {a,b,c,d}
- Range = {2,3,4,5}
- f(b) = 3
- f(d) = 5

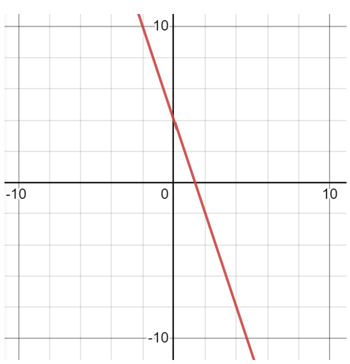
3) There are four different functions f : { a,b } → { 0,1 } . List them. Diagrams suffice.

- {(a,0), (b,0)}
- {(a,1), (b,0)}
- {(a,0), (b,1)}
- {(a,1), (b,1)}

5) Give an example of a relation from { a,b, c,d } to { d, e} that is not a function

- {(a,d), (b,d), (b,e), (c,e), (c,d), (d,d)}

7) Consider the set f = { (x, y) ∈ Z × Z : 3x + y = 4 } . Is this a function from Z to Z? Explain.

- Yes it is a function because every input has a distinct output
- 3x + y = 4
- y = 4 - 3x(line thus its a function)
- Plotted on desmos
- 

Section 12.2(pg. 232)

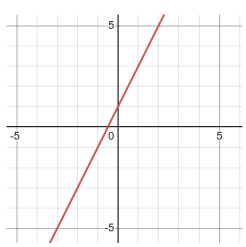
1)Let A = {1,2,3,4} and B = { a,b, c } . Give an example of a function f : A → B that is neither injective(one to one) nor surjective(onto).

- {(b,3),(b,4),(b,2)}

3)Consider the cosine function cos : R → R. Decide whether this function is injective(one to one) and whether it is surjective(onto). What if it had been defined as cos : R → [−1,1]?

- f(x) = cos is neither injective or surjective without restriction to a interval like [-1,1]
    - injective(one to one) a multiple of inputs(2pi and 4pi both equal 1) give the same output thus not one to one
    - surjective (onto) has no real numbers when you go beyond the interval [-1,1] for example there is no cos(x) = 2
- The function is surjective when it's in the interval [-1,1] because every input has a real output, it is not injective because cos(0) = cos(pi/2)

5)A function f : Z → Z is defined as f (n) = 2n +1. Verify whether this function is injective(one to one) and whether it is surjective(onto).

- The function is injective(one to one) because it is a line where every input has a unique output
    - 

- The function is not surjective(onto), because for every input has a output that is odd(2n+1), so not every output has a input it corresponds to

15) This question concerns functions f : { A,B,C,D,E,F,G } → { 1,2,3,4,5,6,7 } . How many such functions are there? How many of these functions are injective(one to one)? How many are surjective(onto)? How many are bijective(both one to one and onto)?

- The total number of functions is equal to 7^7 = 823543
- 7! = 5040 functions are both surjective and injective because both of the sets are of the same length

16) This question concerns functions f : { A,B,C,D,E } → { 1,2,3,4,5,6,7 } . How many such functions are there? How many of these functions are injective(one to one)? How many are surjective(onto)? How many are bijective(both one to one and onto)?

- 7^5 = 16,807 total number of functions
- There is no surjective functions since both sets are not the same length thus it cant be bijective

- 7!/2! = 7*6*5*4*3 because you can only choose 5 from the second set and there is two you can't choose
- 2520 = total number of injective functions