Ch9. Large-Sample Tests of Hypotheses

In this chapter, the concept of a statistical test of hypothesis is formally introduced. The sampling distributions of statistics presented in Chapters 7 and 8 are used to construct large sample tests concerning the values of population parameters of interest to the experimenter.

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Introduction

◆ Suppose that a pharmaceutical company is concerned that the mean potency μ of an antibiotic meet the minimum government potency standards. They need to decide between two possibilities:

- The mean potency μ does not exceed the mean allowable potency.
- The mean potency μ exceeds the mean allowable potency.

◆ Similar to a courtroom trial. In trying a person for a crime, the jury needs to decide between one of two possibilities:

- The person is guilty.
- The person is innocent.

(1) To begin with, the person is assumed innocent.

(2) The prosecutor presents evidence, trying to convince the jury to reject the original assumption of innocence, and conclude that the person is guilty.

✓ This is an example of a **test of hypothesis.**

5 Parts of a Statistical test of hypothesis

1. The null hypothesis, H_0:

- Assumed to be true until we can prove otherwise.

2. The alternative hypothesis, H_a:

- generally the hypothesis that the researcher wishes to support

Court trial:	**Pharmaceuticals:**
H_0: innocent	**H_0: μ does not exceeds allowed amount**
H_a: guilty	**H_a: μ exceeds allowed amount**

3. The test statistic and its *p*-value:

- Test statistic: A single statistic calculated from the sample data
- p-value: a probability calculated using test statistic.

✓ *Help to deciding whether to reject or accept H_0*

5 Parts of a Statistical test of hypothesis

4. The rejection region:

- Rejection region: consisting of values that support the alternative hypothesis and lead to rejecting H_0
- Acceptance region: consisting of values that support the null hypothesis

5. Conclusion:

- Either "Reject H_0" or "Do not reject H_0", along with a statement about the reliability of your conclusion.

❖ How do you decide when to reject H_0?

- Depends on the **significance level, α,** the maximum tolerable risk you want to have of making a mistake, if you decide to reject H_0.

$$\alpha = P(\textit{falsely rejecting } H_0) = P(\textit{rejecting } H_0 \text{ when it is true})$$

- Usually, the significance level is $\alpha = .01$ or $\alpha = .05$.

Example

- You wish to show that the average hourly wage of carpenters in the state of California is different from $19, the national average.
- A random sample of 100 California carpenters provide a sample mean $\bar{x} = \$20$ with standard deviation $s = \$2$ for average hourly wage

Sol)

(1) Hypothesis

$$H_0: \mu = 19 \text{ versus } H_a: \mu \neq 19$$

(2) Test statistic

two tailed test of hypothesis

- Since the sample size is large, the sampling distribution of $\bar{x}$ is approximately normal with mean $\mu = 19$ and standard error $\frac{\sigma}{\sqrt{n}}$, estimated as $\frac{s}{\sqrt{n}} = \frac{2}{\sqrt{100}} = .2$

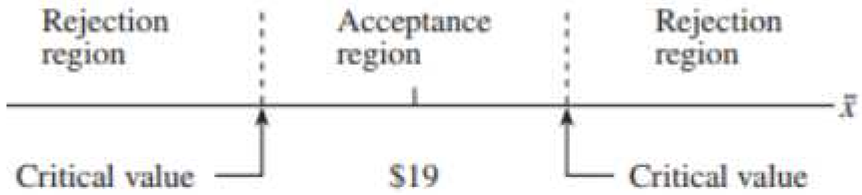
$\Rightarrow$ **the test statistic:** $z \approx \frac{20-19}{.2} = 5$

- $p-value = P(z > 5) + P(z < -5) \approx 0$ for two tailed test

✓ *The large value of the test statistic and the small $p-value$ mean that you have observed a very unlikely event, if indeed H_0 is true($\mu = 19$).*

(3) Rejection region

- California's average hourly wage was different from \$19 if the sample mean is either much less than \$19 or much greater than \$19.
- The two-tailed rejection region consists of very small and very large values of $\bar{x}$



(4) Conclusion

- There is no strong evidence to support $\boldsymbol{H_0}$ under significance level α =0.01 or 0.05

Large Sample Test of a population Mean, μ

- Take a random sample of size $n \geq 30$ from a population with mean μ and standard deviation s.
- We assume that either
  1. **σ is known or**
  2. ***s* $\approx \sigma$ since *n* is large**
- The hypothesis to be tested is
  - two tailed test $H_0{:}\mu = \mu_0$ versus $H_a{:}\ \mu \neq \mu_0$
  - one tailed test $H_0{:}\mu = \mu_0$ versus $H_a{:}\ \mu > \mu_0$ or $\mu < \mu_0$

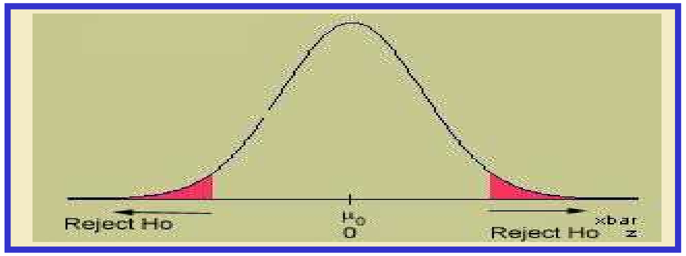
Test Statistic

- Assume to begin with that H_0 is true. The sample mean $\bar{x}$ is our best estimate of μ , and we use it in a standardized form as the **test statistic:**

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \approx \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

  since $\bar{x}$ has an approximate normal distribution with mean μ_0 and standard error $\sigma/\sqrt{n}$.

- If H_0 is true, the value of $\bar{x}$ should be close to μ_0, and z will be close to 0.
- If H_0 is false, $\bar{x}$ will be much larger or smaller than μ_0, and z will be much larger or smaller than 0, indicating that we should reject H_0.



Example

The average weekly earnings for female social workers is \$670. Do men in the same positions have average weekly earnings that are higher than those for women? A random sample of n=40 male social workers showed $\bar{x} = \$725$ and $s = \$102$. Test the appropriate hypothesis using $\alpha = .01$.

Sol)

(1) Null and alternative hypothesis:

$H_0: \mu = 670$ vs $H_a: \mu > 670$

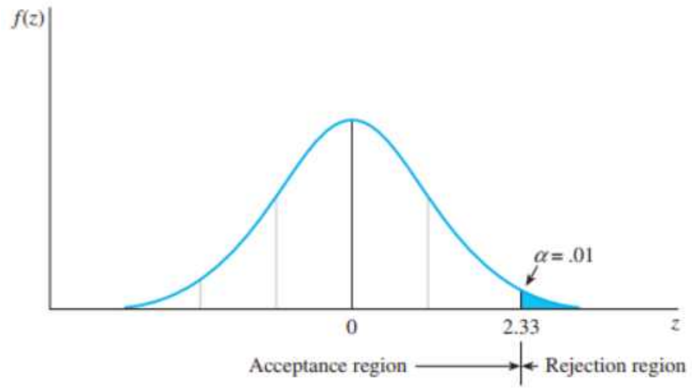
(2) Test Statistic

$$z \approx \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{725 - 670}{102/\sqrt{40}} = 3.41$$

(3) **Rejection Region**

For $\alpha = .01$, $z_{.01} = 2.33$

The null hypothesis will be rejected if the observed value of the test statistic, z, is greater than 2.33.



(4) **Conclusion**

Since test statistic $z = 3.41 > z_{.01} = 2.33$, you can reject H_0 and conclude that the average weekly earnings for male social workers are higher than the average for female social workers.

Example

- The daily yield for a local chemical plant has averaged 880 tons for the last several years. The quality control manager would like to know whether this average has changed in recent months.
- She randomly selects 50 days from the computer database and computes the average and standard deviation of the n=50 yields as $\bar{x} = 871$ tons and $s = 21$ tons, respectively. Test the appropriate hypothesis using $\alpha = .05$.

Sol)

(1) Null and alternative hypothesis:

$H_0: \mu = 880$ vs $H_a: \mu \neq 880$

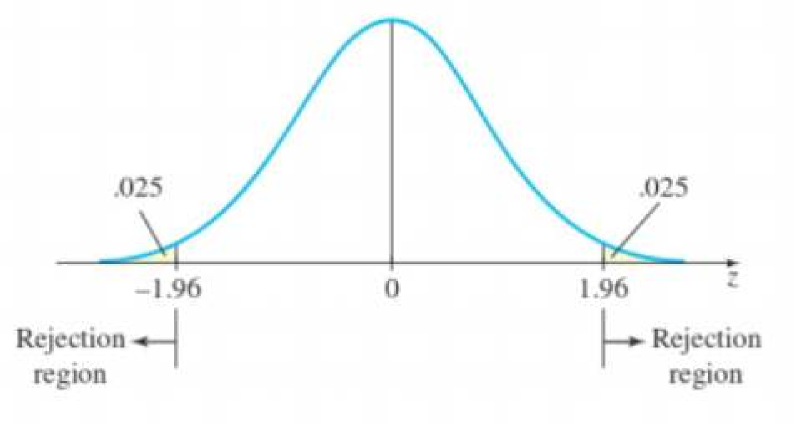
(2) Test Statistic: $z \approx \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{871 - 880}{21/\sqrt{50}} = -3.03$

(3) Rejection Region

For two tailed test, use both right and left tails for rejection region.

$\alpha = .05 \Rightarrow z_{.025} = 1.96$

- Reject if $z > 1.96$ *or* $z < -1.96$



(4) Conclusion

Since test statistic z=- 3.03, falls in the rejection region, the manager can reject the null hypothesis that $\mu = 880$ tons and conclude that it has changed.

Large Sample Test of a population Mean, μ

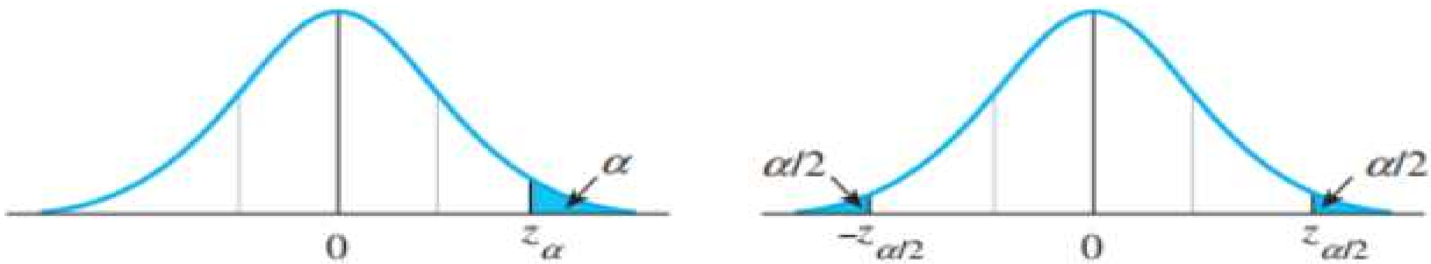
1. Null hypothesis: $H_0: \mu = \mu_0$
2. Alternative hypothesis:

One-Tailed Test	Two-Tailed Test
$H_a: \mu > \mu_0$ (or, $H_a: \mu < \mu_0$)	$H_a: \mu \neq \mu_0$

3. Test statistic: $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ estimated as $z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

4. Rejection region: Reject H_0 when

One-Tailed Test	Two-Tailed Test
$z > z_\alpha$ (or $z < -z_\alpha$ when the alternative hypothesis is $H_a: \mu < \mu_0$)	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$



p-value

- Once you've calculated the observed value of the test statistic, calculate its ***p*-value**:

- ***p*-value**(observed significance level of a statistical test)
  - the smallest value of α for which H_0 can be rejected.
  - If H_0 is rejected, this is the actual probability that we have made an incorrect decision.
  - ✓ *p-Value = Tail area (one or two tails) "beyond" the observed value of the test statistic*

- If this probability is very small, less than some **preassigned significance level, α,** H_0 can be rejected.
  - If *p*-value < α, H_0 is rejected.
  - If *p*-value > α, H_0 is not rejected.

Example

Previous example : The daily yield for a local chemical plant

Find p-value

Sol)

Hypothesis: $H_0: \mu = 880$ vs $H_a: \mu \neq 880$

Test statistic: z=-3.03

$\Rightarrow$ the smallest rejection region is $|z| > 3.03$ since two tailed test

$$p - value = P(|z| > 3.03) = P(z > 3.03) + P(z < -3.03)$$
$$= (1 - .9988) + .0012 = .0024$$

If p-value=.0024 < α , reject H_0

For this test, we can reject H_0 at either $\alpha = .01$ or $\alpha = .05$

Statistical Significance

- If the p-value is less than .01, H_0 is rejected. The results are highly significant.
- If the p-value is between .01 and .05, H_0 is rejected. The results are statistically significant.
- If the p-value is between .05 and .10, H_0 is usually not rejected. The results are only tending toward statistical significance.
- If the p-value is greater than .10, H_0 is not rejected. The results are not statistically significant.

✓ *The p-value approach is often preferred because*

- You can evaluate the test results at any significance level you choose
- Computer printouts usually calculate p-values

Example

- Standards set by government agencies indicate that Americans should not exceed an average daily sodium intake of 3300 milligrams (mg).
- To find out whether Americans are exceeding this limit, a sample of 100 Americans is selected, and the mean and standard deviation of daily sodium intake are found to be 3400 mg and 1100 mg, respectively.
- Use $\alpha = .05$ to conduct a test of hypothesis.

Sol)

Hypothesis: $H_0 : \mu = 3300$ vs $H_a : \mu > 3300$

Test statistic: $z \approx \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{3400 - 3300}{1100 / \sqrt{100}} = .91$

(1) critical value approach

rejection region : z > 1.645 for $\alpha = .05$

Since z = .91 is not greater than the critical value 1.645,

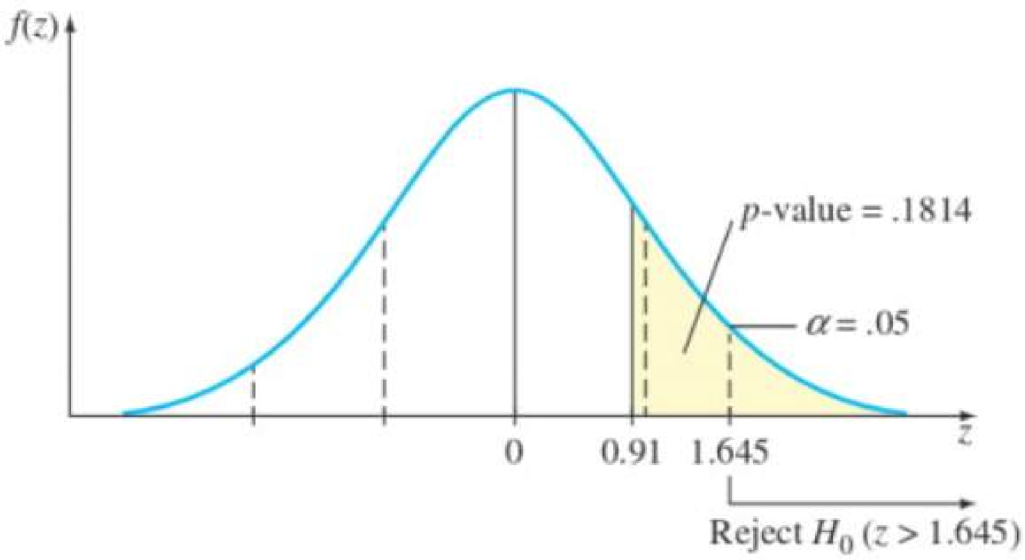
H_0 is not rejected

(2) $p - value$ approach

$p - value = P(z > .91) = 1 - .8186 = .1814$

Since $p - value = .1814 > \alpha = .05$, H_0 is not rejected and the results are not statistically significant

Conclusion: There is not enough evidence to indicate that the average daily sodium intake exceeds 3300 mg.



Two types of Errors

There are **two types of errors** which can occur in a statistical test.

(a) Courtroom Trial		
	Actual Fact	
Decision	**Innocent**	**Guilty**
Guilty	Error 1	Correct
Not Guilty	Correct	Error 2

(b) Statistical Test of Hypothesis		
	Null Hypothesis	
Decision	**True**	**False**
Reject H_0	Type I Error	Correct
Accept H_0	Correct	Type II Error

Define:

α = P(Type I error) = P(reject H_0 when H_0 is true)

β = P(Type II error) = P(accept H_0 when H_0 is false)

$\Rightarrow 1-\beta =$ P(reject H_0 when H_a is true) : power of test

Two types of Errors

We want to keep the probabilities of error as small as possible.

- The value of α is the significance level, and is controlled by the experimenter.
- The value of β is difficult to calculate.

  - when H_0 is false and H_a is true, it may not be able to specify an exact value for μ , but only a range of values.

✓ *Without a measure of reliability, it is not wise to conclude that H_0 is true.*

✓ *Rather than risk an incorrect decision, it is better to conclude that you do not have enough evidence to reject H_0. Instead of accepting H_0, you should "not reject" or "fail to reject" H_0.*

We write: There is insufficient evidence to reject H_0.

Example

Previous example : The daily yield for a local chemical plant

Calculate β and the power of the test (1-β) when μ is actually equal to 870 tons

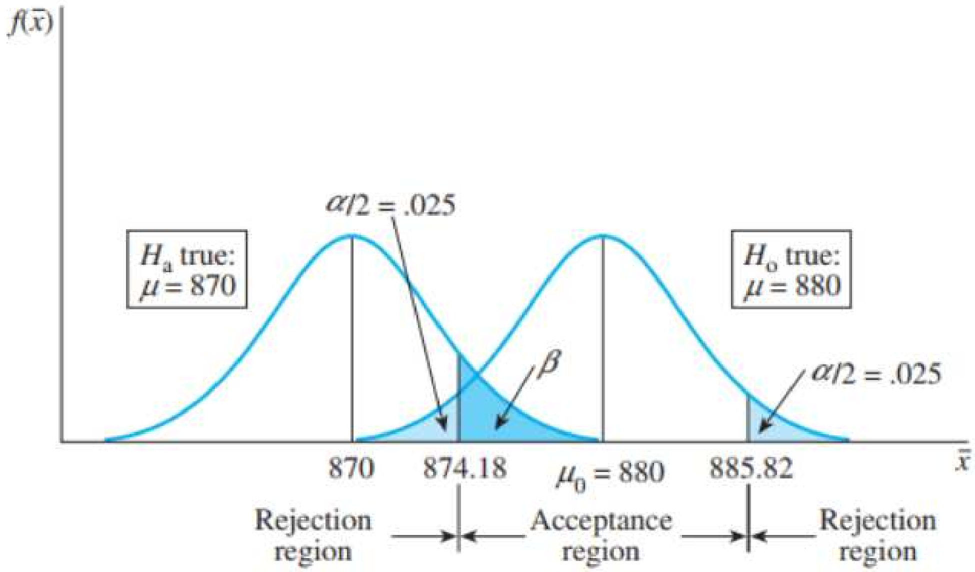
Sol)

Hypothesis: $H_0: \mu = 880$ vs $H_a: \mu \neq 880$

For $\alpha = .05$, acceptance region: $-1.96 < \frac{\bar{x}-880}{s/\sqrt{n}} < 1.96$

$\Rightarrow 874.18 < \bar{x} < 885.82$

β, the probability of accepting H_0 when $\mu = 870$



$$\beta = P(accept \quad H_0 \text{ when } \mu = 870)$$
$$= P(874.18 < \bar{x} < 885.82 \ when\ \mu = 870)$$
$$= P\left(\frac{874.18 - 870}{21/\sqrt{50}} < \frac{\bar{x} - 870}{s/\sqrt{n}} < \frac{885.82 - 870}{21/\sqrt{50}} \right)$$
$$= P(1.41 < z < 5.33) = 1 - .9207 = .0793$$

$$1 - \beta = 1 - .0793 = .9207$$

- The probability of correctly rejecting H_0, given that μ is really equal to 870, is .9207, or approximately 92 chances in 100.

The Power of the test

- Values of $(1-\beta)$ can be calculated for various values of μ_a different from $\mu_0 = 880$ to measure the power of the test.

- For example, if $\mu_a = 885$

$$\beta = P(accept \quad H_0 \text{ when } \mu = 885)$$

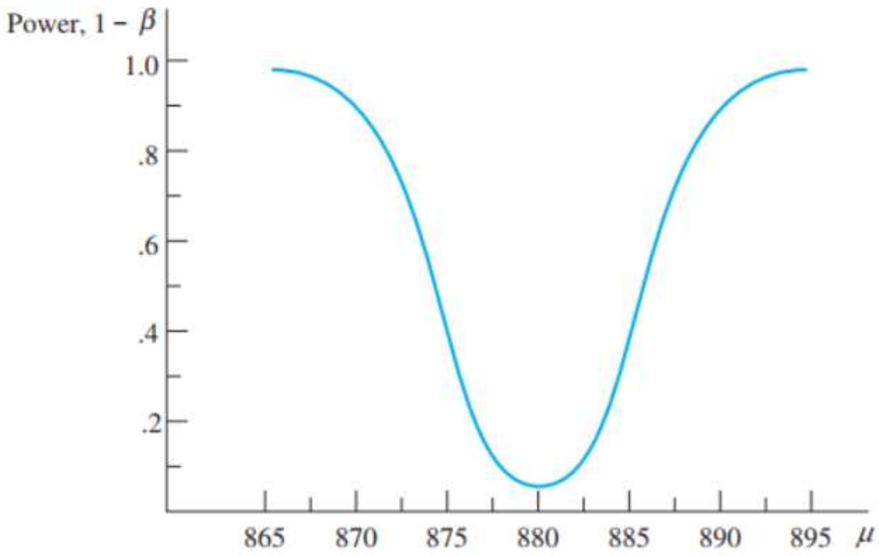
$$= P(874.18 < \bar{x} < 885.82 \ when \ \mu = 885)$$

$$= P\left(\frac{874.18-885}{21/\sqrt{50}} < \frac{\bar{x}-870}{s/\sqrt{n}} < \frac{885.82-885}{21/\sqrt{50}}\right)$$

$$= P(-3.64 < z < .28) = .6103 - 0 = .6103$$

$$1 - \beta = 1 - .6103 = .3897$$

μ_a	$(1-\beta)$	μ_a	$(1-\beta)$
865	.9990	883	.1726
870	.9207	885	.3897
872	.7673	888	.7673
875	.3897	890	.9207
877	.1726	895	.9990
880	.0500		



Testing the Difference between Two Means

- A random sample of size n_1 drawn from population 1

  with mean μ_1 *and* variance σ_1^2.
- A random sample of size n_2 drawn from population 2

  with mean μ_2 *and* variance σ_2^2.

- The hypothesis of interest involves the difference, $\mu_1 - \mu_2$, in the form:

  H_0: $\mu_1 - \mu_2 = D_0$ versus H_a: one of three alternatives

  where D_0 is some hypothesized difference, usually 0.

The Sampling Distribution of $\bar{x}_1 - \bar{x}_2$

1. The mean of $\bar{x}_1 - \bar{x}_2$ is $\mu_1 - \mu_2$, the difference in the population means

2. The standard error of $\bar{x}_1 - \bar{x}_2$ is $\mathrm{SE} = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

3. If the sample sizes are large, the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is approximately normal, and SE can be estimated

   as $\mathrm{SE} = \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

✓ Assumption: The samples are randomly and independently selected from the two populations and $n_1 \geq 30$ and $n_2 \geq 30$.

Testing the Difference between Two Means

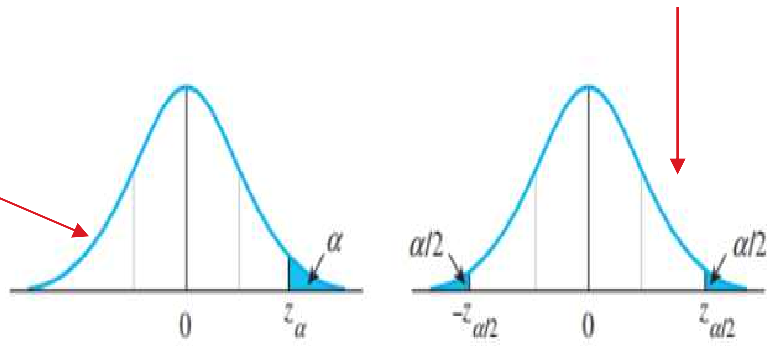
- **H_0: $\mu_1 - \mu_2 = D_0$ versus H_a: one of three alternatives**

- Test statistic: $z \approx \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

  with rejection regions and/or p − *values* based on the standard normal z distribution

- **Rejection region : Reject H_0 when**

One-Tailed Test	**Two-Tailed Test**
$z > z_\alpha$ [or $z < -z_\alpha$ when the alternative hypothesis is $H_a : (\mu_1 - \mu_2) < D_0$] or when p-value $< \alpha$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$



Example

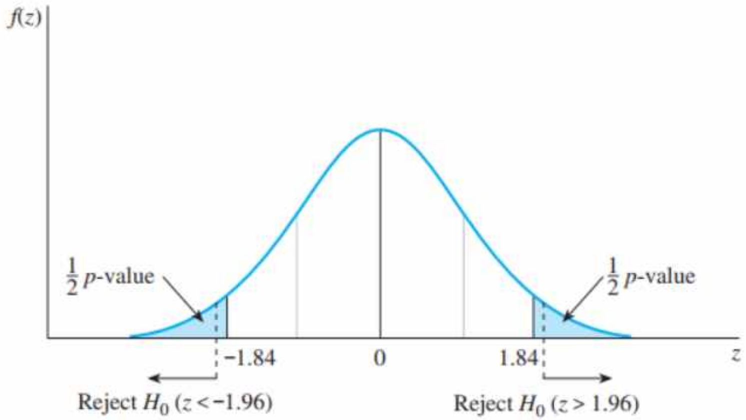
- To determine whether car ownership affects a student's academic achievement, two random samples of 100 male students were each drawn from the student body.
- The grade point average for the $n_1 = 100$ nonowners of cars had an average and variance equal to $\bar{x}_1 = 2.70$ and $s_1^2 = .36$ and $\bar{x}_2 = 2.54$ and $s_1^2 = .40$ for the $n_2 = 100$ car owners.
- Do the data present sufficient evidence to indicate a difference in the mean achievements between car owners and nonowners of cars? Test using $\alpha = .05$.

Sol) (1) $\mathbf{H_0: \mu_1 - \mu_2 = D_0 = 0}$ **versus** $\mathbf{H_a: \mu_1 - \mu_2 \neq 0}$

(2) T.S. $z \approx \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \dfrac{(2.70 - 2.54) - 0}{\sqrt{\frac{.36}{100} + \frac{.40}{100}}} = 1.84$

(3) rejection region: for $\alpha = .05$
since |z|=1.84 < 1.96, do not reject $\mathbf{H_0}$

or p-value=$P(z > 1.84) + P(z < -1.84)$
=(1-.9671) +.0329=.0658
$> \alpha = .05$
$\Rightarrow$ do not reject $\mathbf{H_0}$



Confidence Interval for $\mu_1 - \mu_2$

Example

- Construct a 95% confidence interval for the difference in average academic achievements between car owners and nonowners.

Sol) Confidence interval for the difference in two population means

$$(\bar{x}_1 - \bar{x}_2) \pm 1.96\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$\Rightarrow \quad (2.7 - 2.54) \pm 1.96\sqrt{\frac{.36}{100} + \frac{.40}{100}}$$

$$\Rightarrow \quad .16 \pm .17$$

$$\Rightarrow \quad -.01 < \mu_1 - \mu_2 < .33$$

Since the hypothesized difference, $\mu_1 - \mu_2 = 0$, is contained in the confidence interval, you should not reject $\mathbf{H_0}$

Testing a Binomial Proportion p

A random sample of size n from a binomial population to test

- $H_0: p = p_0$ versus H_a : one of three alternatives
- Test statistic: $z \approx \frac{\hat{p}-p_0}{SE} = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}}$,

where $\hat{p} = \frac{x}{n}$, x : number of success in n binomial trials

with rejection regions and/or

$p-values$ based on the standard normal z distribution.

✓ Assumption: n is large enough so that the sampling distribution of $\hat{p}$ can be approximated by a normal distribution ($np_0 > 5$ *and* $nq_0 > 5$).

Example

- Regardless of age, about 20% of American adults participate in fitness activities at least twice a week. A random sample of 100 adults over 40 years old found 15 who exercised at least twice a week. Is this evidence of a decline in participation after age 40? Use α = .05.

Sol)

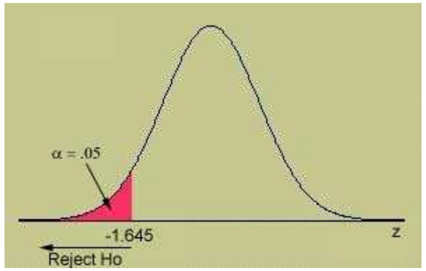
(1) Hypothesis : $H_0 : p = .2$ vs $H_a : p < .2$

(2) Test statistic :

$$z \approx \frac{\hat{p}-p_0}{SE} = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.15-.2}{\sqrt{\frac{.2(.8)}{100}}} = -1.25$$

(3) Since $z = -1.25 >$ -1.645

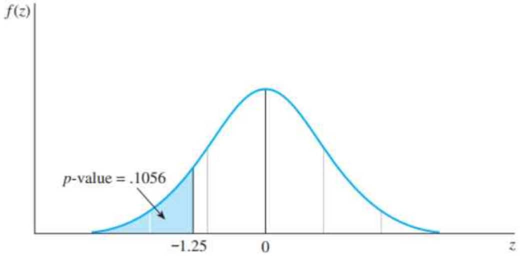
Do not reject $\mathbf{H_0}$



p-value= $P(z < -1.25)$=.1056 > $\alpha = .05$

⇒ do not reject $\mathbf{H_0}$

✓ There is not enough evidence to indicate that p is less than .2 for people over 40



Testing the Difference between Two proportions

- A random sample of size n_1 drawn from binomial population 1 with parameter p_1.
- A random sample of size n_2 drawn from binomial population 2 with parameter p_2.

- The hypothesis of interest involves the difference, $p_1 - p_2$, in the form:

H_0: $p_1 - p_2 = D_0$ versus H_a: one of three alternatives

where D_0 is some hypothesized difference, usually 0.

The Sampling Distribution of $\hat{p}_1 - \hat{p}_2$

1. The mean of $\hat{p}_1 - \hat{p}_2$ is $p_1 - p_2$, the difference in the population means

2. The standard error of $\hat{p}_1 - \hat{p}_2$ is $\text{SE} = \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$

3. If the sample sizes are large, the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is approximately normal.

4. The standard error is estimated differently, depending on the hypothesized difference, $\mathbf{D_0}$.

Testing the Difference between Two Proportions

(1) H_0: $p_1 - p_2 = 0$ versus

(2) H_a: one of three alternatives

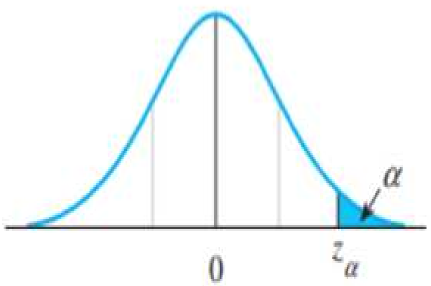
(3) Test statistics $z \approx \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\left(\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}\right)}} = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

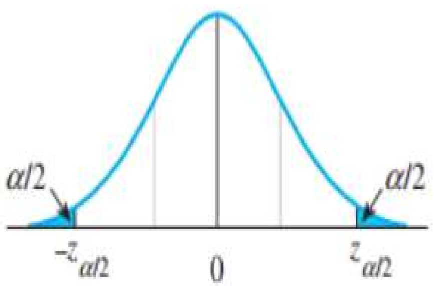
with $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$ to estimate the common value of $p_1 = p_2 = p$ and rejection regions or $p-values$ based on the standard normal z distribution.

(4) Rejection region: Reject H_0 when

One-Tailed Test	Two-Tailed Test
$z > z_\alpha$ [or $z < -z_\alpha$ when the alternative hypothesis is $H_a : (p_1 - p_2) < 0$]	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$

or when p-value $< \alpha$





Example

The records of a hospital show that 52 men in a sample of 1000 men versus 23 women in a sample of 1000 women were admitted because of heart disease.
Do these data present sufficient evidence to indicate a higher rate of heart disease among men admitted to the hospital? Use $\alpha = .05$.

Sol)
Assume that the number of patients admitted for heart disease
~ approximate binomial prob. dist for both men and women with parameters p_1 and p_2, respectively

(1) H_0: $p_1 - p_2 = 0$ versus

(2) H_a: $p_1 - p_2 > 0$

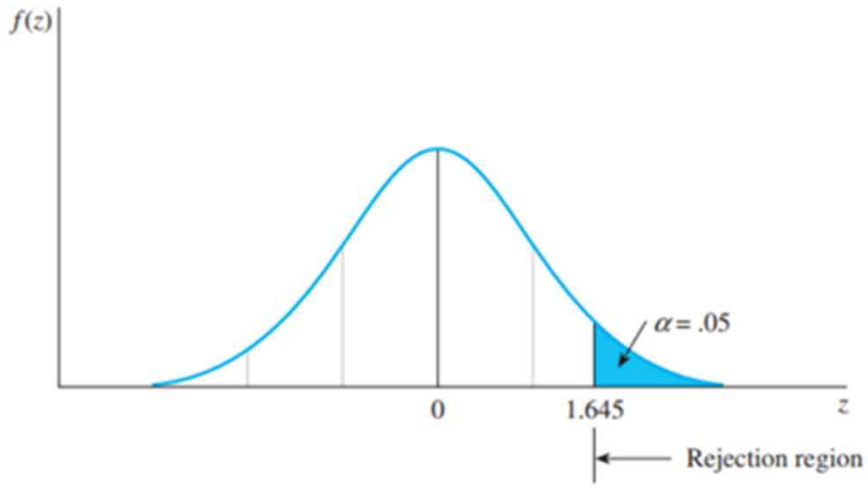
(3) Since $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{52 + 23}{1000 + 1000} = .0375$

$$\text{T. S. } z \approx \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{.052 - .023}{\sqrt{(.0375)(.9625)\left(\frac{1}{1000} + \frac{1}{1000}\right)}} = 3.41$$

(4) Since TS = 3.41 > 1.645 Reject H_0 for $\alpha = .05$

Or p-value = $P(z > 3.41)$=.0003 < $\alpha = .05$

$\Rightarrow$ Reject H_0 for for $\alpha = .05$



- The data present sufficient evidence to indicate that the percentage of men entering the hospital because of heart disease is higher than that of women for $\alpha = .05$.

Key Concepts

I. Parts of a Statistical Test

1. Null hypothesis: a contradiction of the alternative hypothesis
2. Alternative hypothesis: the hypothesis the researcher wants to support.
3. Test statistic and its *p*-value: sample evidence calculated from sample data.
4. Rejection region - critical values and significance levels: values that separate rejection and nonrejection of the null hypothesis
5. Conclusion: Reject or do not reject the null hypothesis, stating the practical significance of your conclusion.

Key Concepts

II. Errors and Statistical Significance

1. The significance level α is the probability if rejecting H_0 when it is in fact true.

2. The *p*-value is the probability of observing a test statistic as extreme as or more than the one observed; also, the smallest value of α for which H_0 can be rejected.

3. When the *p*-value is less than the significance level α, the null hypothesis is rejected. This happens when the test statistic exceeds the critical value.

4. In a Type II error, β is the probability of accepting H_0 when it is in fact false. The power of the test is $(1 - \beta)$, the probability of rejecting H_0 when it is false.

Key Concepts

III. Large-Sample Test Statistics Using the z Distribution

To test one of the four population parameters when the sample sizes are large, use the following test statistics:

Parameter	Test Statistic
μ	$z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
p	$z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}}$
$\mu_1 - \mu_2$	$z = \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$
$p_1 - p_2$	$z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$