The Binomial Probability Distribution

A binomial experiment consists of n identical trials with probability of success p on each trial.

The probability of k successes in n trials is

$$P(x = k) = C_k^n \, p^k \, q^{n-k} = \frac{n!}{k!(n-k)!} \, p^k q^{n-k}$$

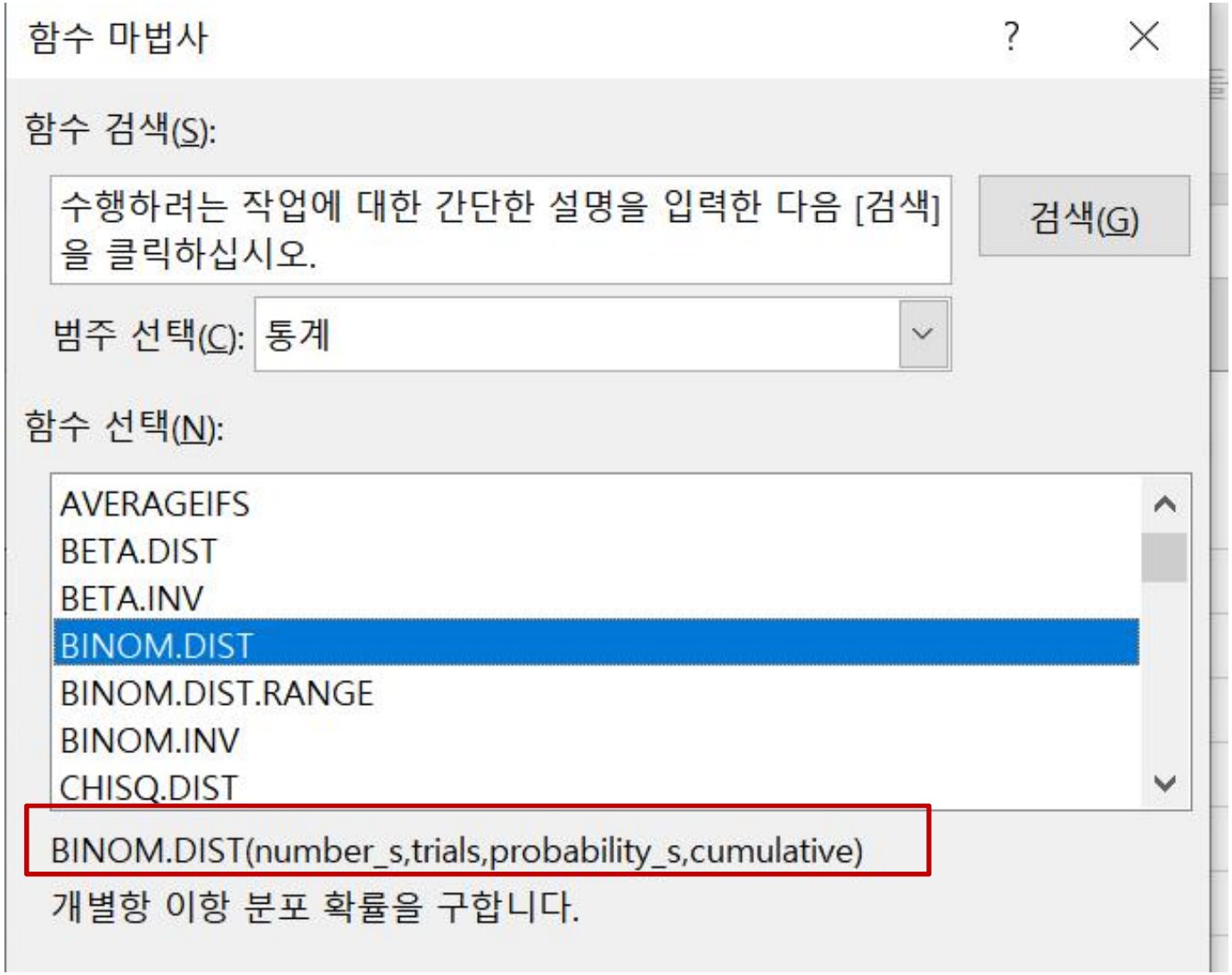
for values of $k = 0, 1, 2, \ldots, n$.

where $C_k^n = \frac{n!}{k!(n-k)!}$ Or use notation ${}_nC_x$

$$n! = n(n-1)(n-2)\ldots(2)(1) \quad and \quad 0! = 1$$

Excel 활용

수식 > 함수 > 통계



BINOM.DIST 함수

BINOM.DIST(number_s, trials, probability_s, cumulative)

Binomial(n, p)

- $P(x = k)$: BINOM.DIST(k, n, p, FALSE) 개별확률
- $P(x \leq k)$: BINOM.DIST(k, n, p, TRUE) 누적확률
- $P(x \leq k) = .7$
  BINOM.INV(n, p, 0.7)

The Poisson Probability Distribution

- Let x represent the number of events that occur in a period of time or space during which an average of μ such events can be expected to occur.
  - Assumption: events occur randomly and independently of one another.

⇒ the random variable x can be modeled by the **Poisson random variable:**

- Let μ be the average number of times that an event occurs in a certain period of time or space. The probability of k occurrences of this event is

$$P(x = k) = \frac{\mu^k \, e^{-\mu}}{k!}$$

for values of $k = 0, 1, 2, 3, , \ldots$ $e = 2.71828$

Poisson 함수

POISSON.DIST(x, mean, cumulative)

- **x** : 사건의 수 필수 요소입니다. 사건의 수입니다.
- **mean** : 평균 필수 요소입니다. 기대값입니다.
- **cumulative** : 누적이면 True, 확률이면 False

- $P(x = k)$: POISSON.DIST(k, mean, False) 개별확률
- $P(x \leq k)$: POISSON.DIST(k, mean, True) 누적확률

THE HYPERGEOMETRIC PROBABILITY DISTRIBUTION

- A population contains M successes and N M failures. The probability of exactly k successes in a random sample of size *n* is

$$P(x = k) = \frac{C_k^M C_{n-k}^{N-M}}{C_n^N}$$

for values of *k* that depend on *N*, *M*, and *n* with $C_n^N = \frac{N!}{n!(N-n)!}$

- **HYPGEOM.DIST**
  (sample_s, number_sample, population_s, number_pop, cumulative)

  - **sample_s :** 표본의 성공 도수
  - **number_sample** : 표본 크기
  - **population_s** : 모집단의 성공 도수
  - **number_pop** : 모집단 크기

$P(x = k)$: HYPGEOM.DIST(k, n, M, N, false)
$P(x \leq k)$: HYPGEOM.DIST(k, n, M, N, true)

Example)

N=12, n=4, M=3, and (N-M)=9.

$$p(x) = \frac{C_x^M C_{n-x}^{N-M}}{C_n^N} = \frac{C_x^3 C_{4-x}^9}{C_4^{12}}$$

$$p(0) = \frac{C_0^3 C_4^9}{C_4^{12}} = \frac{1(126)}{495} = .25,\ p(1) = .51, p(2) = .22, p(3) = .02$$

x = k	n	M	N	P(x=k)	p(x <= k)
0	4	3	12	=HYPGEOM.DIST(A63,B63,C63,D63,FALSE)	=HYPGEOM.DIST(D63,E63,F63,G63,TRUE)
1	4	3	12	=HYPGEOM.DIST(A64,B64,C64,D64,FALSE)	=HYPGEOM.DIST(D64,E64,F64,G64,TRUE)
2	4	3	12	=HYPGEOM.DIST(A65,B65,C65,D65,FALSE)	=HYPGEOM.DIST(D65,E65,F65,G65,TRUE)
3	4	3	12	=HYPGEOM.DIST(A66,B66,C66,D66,FALSE)	=HYPGEOM.DIST(D66,E66,F66,G66,TRUE)

x = k	n	M	N	p(x=k)	p(x <= k)
0	4	3	12	0.254545455	0.254545455
1	4	3	12	0.509090909	0.763636364
2	4	3	12	0.218181818	0.981818182
3	4	3	12	0.018181818	1