Ch10. Inference from Small Samples

The basic concepts of large-sample statistical estimation and hypothesis testing for practical situations involving population means and proportions were introduced in Chapters 8 and 9.
This chapter presents hypothesis tests and confidence intervals for population means and variances when the sample sizes are small. Unlike the large-sample techniques, these small sample methods require the sampled populations to be normal, or approximately so.

Introduction

- When the sample size is small, the estimation and testing procedures of Chapter 8 and 9 are not appropriate.
- There are equivalent small sample test and estimation procedures for
  - ✓ μ, single population mean
  - ✓ $\mu_1 - \mu_2$, difference between two population means
  - ✓ σ^2, single population variance
  - ✓ σ_1^2 and σ_1^2, comparison of two population variances,

The Sampling Distribution of the Sample Mean

- When we take a sample from a normal population, the sample mean $\bar{x}$ has a normal distribution for any sample size *n*, and

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

$\frac{\bar{x}-\mu}{s/\sqrt{n}}$ is not normal!

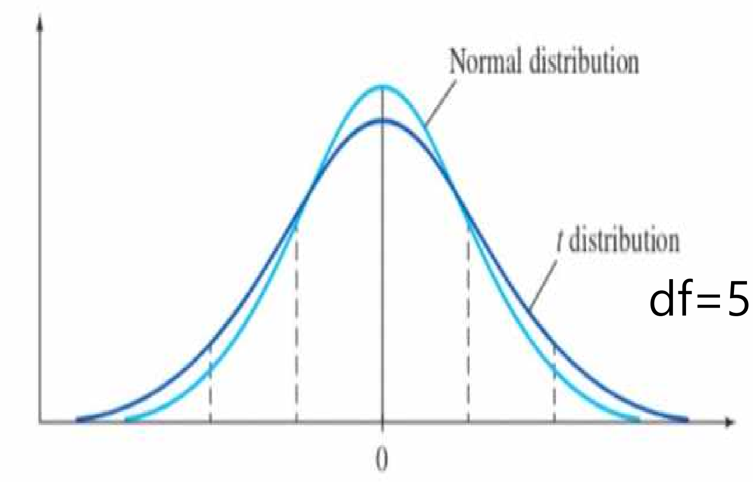
has a standard normal distribution.

- But if σ is unknown, and we must use *s* to estimate it, the resulting statistic **is not normal**

Student's t Distribution

- Fortunately, this statistic does have a sampling distribution that is well known to statisticians, called the **Student's t distribution,** with ***n*-1 degrees of freedom.**

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

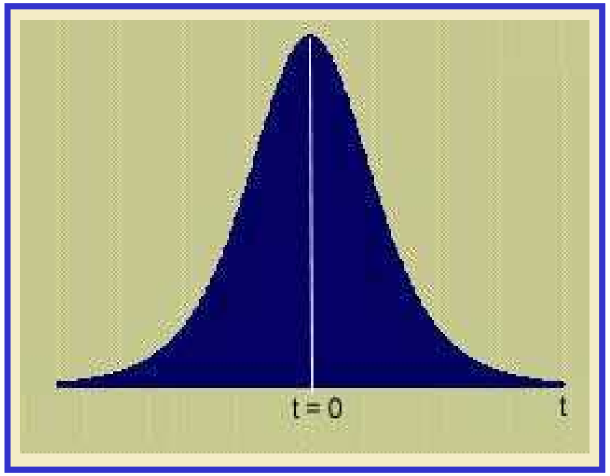


✓ Assumptions for one-sample t:
- Random Sample from Normal distribution

- We can use this distribution to create estimation testing procedures for the population mean μ.

Properties of Student's *t*

- Mound-shaped and symmetric about 0.
- More variable than *z*, with "heavier tails"

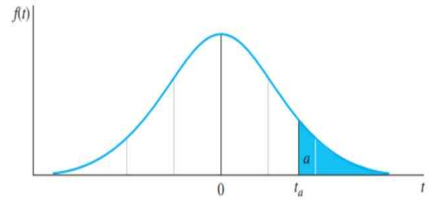


- Shape depends on the sample size *n* or the **degrees of freedom, *n*-1.**
- As *n* increases the shapes of the *t* and *z* distributions become almost identical.

Using the *t*-Table

- Table 4 gives the values of *t* that cut off certain critical values in the tail of the *t* distribution.
- Index ***df*** and the appropriate tail area ***a*** to find $\boldsymbol{t_a}$, the value of *t* with area ***a*** to its right.

df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$
1	3.078	6.314	12.706	31.821
2	1.886	2.920	4.303	6.965
3	1.638	2.353	3.182	4.541
4	1.533	2.132	2.776	3.747
5	1.476	2.015	2.571	3.365
6	1.440	1.943	2.447	3.143
7	1.415	1.895	2.365	2.998
8	1.397	1.860	2.306	2.896
9	1.383	1.833	2.262	2.821
10	1.372	1.812	2.228	2.764
11	1.363	1.796	2.201	2.718
12	1.356	1.782	2.179	2.681
13	1.350	1.771	2.160	2.650
14	1.345	1.761	2.145	2.624
15	1.341	1.753	2.131	2.602



For a random sample of size $n = 10$, find a value of *t* that cuts off .025 in the right tail.

Row = $df = n - 1 = 9$

Column subscript = $a = .025$

$\boldsymbol{t_{.025} = 2.262}$

Small Sample Inference for a Population Mean μ

- The basic procedures are the same as those used for large samples.

◆ **Test of hypothesis**

- Test $\mathrm{H_0}: \mu = \mu_0$ *versus* $\mathrm{H_a}$: one of three
- Test statistic: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
- Rejection region based on a t-distribution with $df = n - 1$ or using p-values

◆ For a **100(1–α)% confidence interval** for the population mean μ:

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

Where $t_{\alpha/2}$ is the value of t that cuts off area $\alpha/2$ in the tail of a t-distribution with $df = n - 1$

Example

A new process for producing synthetic diamonds can be operated at a profitable level only if the average weight of the diamonds is greater than .5 karat.

To evaluate the profitability of the process, six diamonds are generated, with recorded weights .46, .61, .52, .48, .57, and .54 karat. Do the six measurements present sufficient evidence to indicate that the average weight of the diamonds produced by the process is in excess of .5 karat? Use $\alpha = 0.05$.

Sol)

Data : .46, .61, .52, .48, .57, .54

$$\bar{x} = \frac{.46 + .61 + \cdots + .54}{6} = .53$$

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{1.701 - \frac{(3.18)^2}{6}}{5}} = 0.0559$$

Example

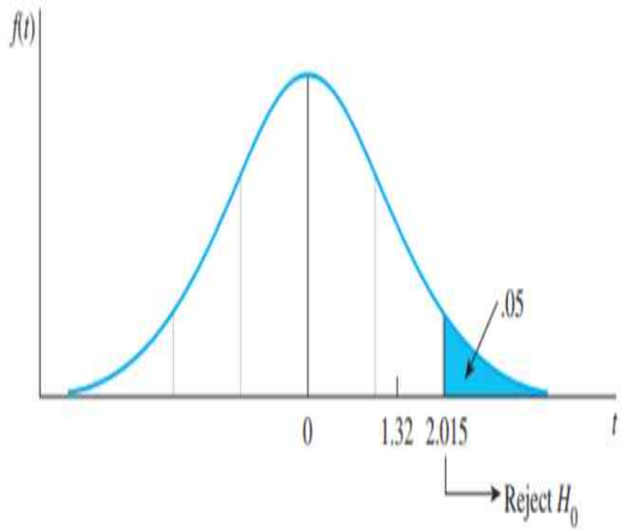
Sol)

(1−2)Hypotheses : $H_0: \mu = .5$ versus $H_a: \mu > .5$

(3) Test statistic: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{.53 - .5}{.0559 / \sqrt{6}} = 1.32$

(4) Rejection region:

for $\alpha = 0.05$, reject H_0 if T.S. $> t_\alpha(n-1=5) = 2.015$



(5) Conclusion

Since T.S $=1.32 < 2.015$,

Do not reject H_0

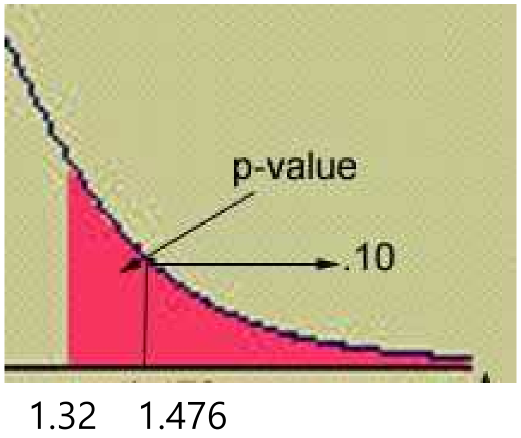
The data do not present sufficient evidence to indicate that the mean diamond weight exceeds .5 karat.

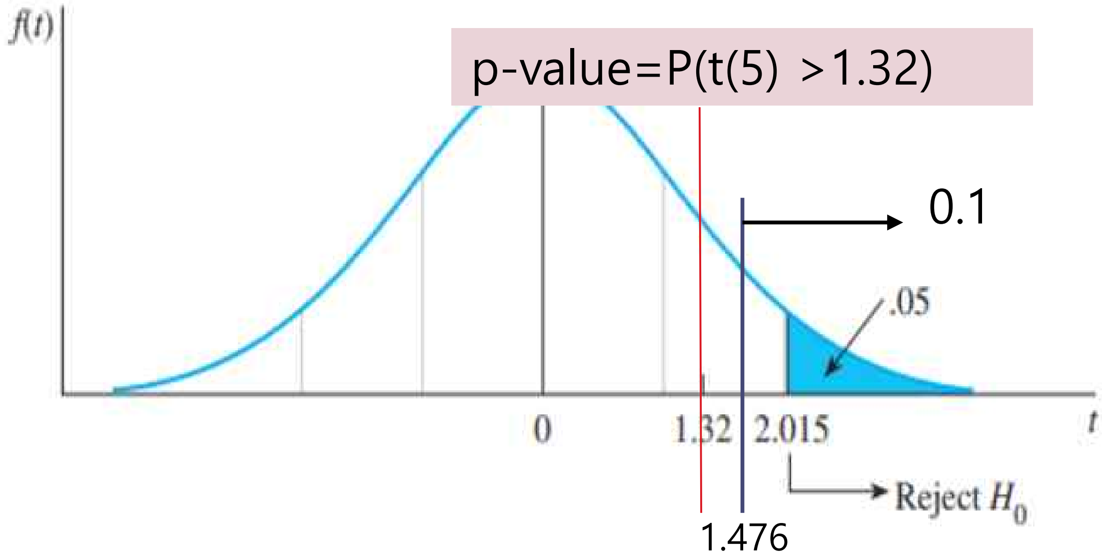
- **Approximating the p-value**
  - approximate the p-value for the test using Table 4

df	$t_{.100}$	$t_{.050}$
1	3.078	6.314
2	1.886	2.920
3	1.638	2.353
4	1.533	2.132
5	1.476	2.015
6	1.440	1.943

- Since the observed value of t=1.32 is smaller than $t_{.10} = 1.476$,

  p-value > .10





Testing the Difference between Two Means

As in Chapter 9,
independent random samples of size n_1 and n_2 are drawn from populations 1 and 2 with means μ_1 and μ_2
and variances σ_1^2 and σ_2^2.

✓ Since the sample sizes are small, the two populations must be **normal**

- To test:

  $H_0: \mu_1 - \mu_2 = D_0$ versus H_a: one of three

  where D_0 is some hypothesized difference, usually 0.

Testing the Difference between Two Means

- The test statistic used in Chapter 9

$$z \approx \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$z \approx \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$: normal

- does not have either a *z* or a *t* distribution,
- and cannot be used for small-sample inference.

- We need to make one more assumption, that the population variances, although unknown, are equal. i.e. $\sigma_1^2 = \sigma_2^2 = \sigma^2$

  $\Rightarrow$ the standard error of the difference in the two sample mean is

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

- Estimate the common variance with

$$\boldsymbol{s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

- The resulting test statistic,

$$t = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

~ t distribution with $n_1 + n_2 - 2$ degrees of freedom

Estimating the Difference between Two Means

- Create a 100(1-α)% confidence interval for μ_1-μ_2.

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

$$\text{with } s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

Remember the three assumptions:

1. **Original populations normal**
2. **Samples random and independent**
3. **Equal population variances.**

Example

A course can be taken for credit either by attending lecture sessions at fixed times and days, or by doing online sessions that can be done at the student's own pace and at those times the student chooses.
The course coordinator wants to determine if these two ways of taking the course resulted in a significant difference in achievement as measured by the final exam for the course. Table below gives the scores on an examination with 45 possible points for one group of $n_1 = 9$ students who took the course online, and a second group of $n_2 = 9$ students who took the course with conventional lectures.

Do these data present sufficient evidence to indicate <u>that the average grade for students who take the course online is significantly higher than for those who attend a conventional class?</u>

Online	Classroom
32	35
37	31
35	29
28	25
41	34
44	40
35	27
31	32
34	31

Example (continued~)

Sol)
Let μ_1 and μ_2 be the mean scores for the online group and the classroom group, respectively.

(1) H_0: $\mu_1 - \mu_2 = 0$ (or H_0: $\mu_1 = \mu_2$) versus
(2) H_a: $\mu_1 - \mu_2 > 0$ (or H_0: $\mu_1 > \mu_2$)

(3) Test statistic

✓ *To conduct the t-test for these two independent samples, assume that the sampled populations are both normal and have the same variance σ^2*

from the data, s_1^2 =4.9441 and s_2^2 =4.4752 *: not different enough*

- The pooled estimate of the common variance as

$$s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{8(4.9941)^2 + 8(4.4752)^2}{9+9-2} = 22.2361$$

- T. S. $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \dfrac{35.22 - 31.56}{\sqrt{22.2361\left(\frac{1}{9}+\frac{1}{9}\right)}} = 1.65$

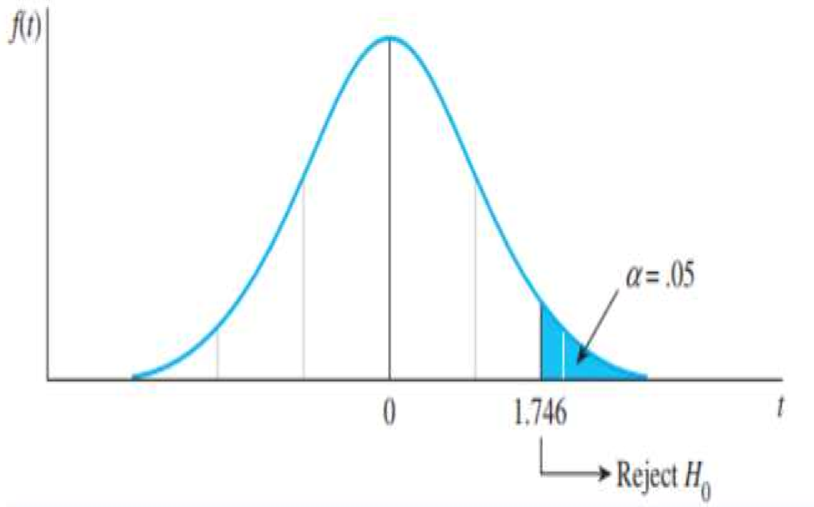
Example (continued~)

(4) Rejection Region

Reject H_0 if T.S. > t_α distribution with $n_1 + n_2 - 2 = 16$
For $\alpha = .05$, $t_{.05} = 1.746$

(5) Since T.S. $1.65 < t_{.05} = 1.746$
Do not reject H_0

There is insufficient evidence to indicate that the average online course grade is higher than the average conventional course grade at the 5% level of significance.



✓ p-value =P(t > 1.65)

From table4, .05 < p-value < .10
✓ $t_{.1}$=1.337 and $t_{.05}$ =1.746

⇒ Because the p-value is greater than .05, most researchers would report the results as not significant.

Example (continued~)

- Use a lower 95% confidence bound to estimate the difference $\mu_1 - \mu_2$ for the above Example
- Does the lower confidence bound indicate that the online average is significantly higher than the classroom average?

Sol) $$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

$$\Rightarrow \ (35.22 - 31.56) \pm 1.746\sqrt{22.2361\left(\frac{1}{9}+\frac{1}{9}\right)}$$

⇒ lower bound 3.66 - 3.88 = -.22

Since the value ($\mu_1 - \mu_2$)=0 0 is included in the confidence interval, it is possible that the two means are equal.

There is insufficient evidence to indicate that the online average is higher than the classroom average.

Testing the Difference between Two Means

- How can you tell if the equal variance assumption is reasonable?

◆ Rule of Thumb:

- If the ratio, $\dfrac{larger\ \ s^2}{smaller\ \ s^2} \leq 3,$
  the equal variance assumption is reasonable.

- If the ratio, $\dfrac{larger\ \ s^2}{smaller\ \ s^2} > 3,$
  use an alternative test statistic.

Testing the Difference between Two Means

- If the population variances cannot be assumed equal, the test statistic

$$t \approx \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} \qquad df \approx \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}}$$

has an approximate *t* distribution with degrees of freedom given above. This is most easily done by computer.

The Paired-Difference Test

- Sometimes the assumption of independent samples is intentionally violated, resulting in a **matched-pairs** or **paired-difference test**.
- By designing the experiment in this way, we can eliminate unwanted variability in the experiment by analyzing only the differences,

$$d_i = x_{1i} - x_{2i}$$

to see if there is a difference in the two population means, $\mu_1 - \mu_2$.

Example

One Type A and one Type B tire are randomly assigned to each of the rear wheels of five cars. Compare the average tire wear for types A and B using a test of hypothesis.

Car	1	2	3	4	5
Type A	10.6	9.8	12.3	9.7	8.8
Type B	10.2	9.4	11.8	9.1	8.3

$$\mathrm{H}_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 \neq 0$$

✓ But the samples are not independent. The pairs of responses are linked because measurements are taken on the same car.

The Paired-Difference Test

- To test $H_0: \mu_1 - \mu_2 = 0$, we test $H_0: \mu_d = 0$ using the test statistics

$$t = \frac{\bar{d} - 0}{s_d / \sqrt{n}}$$

Where n=number of pairs, $\bar{d}$ *and* s_d are the mean and standard deviation of the differences, d_i.

- Use the p-value or a rejection region based on a t-distribution with df = n-1.

Example

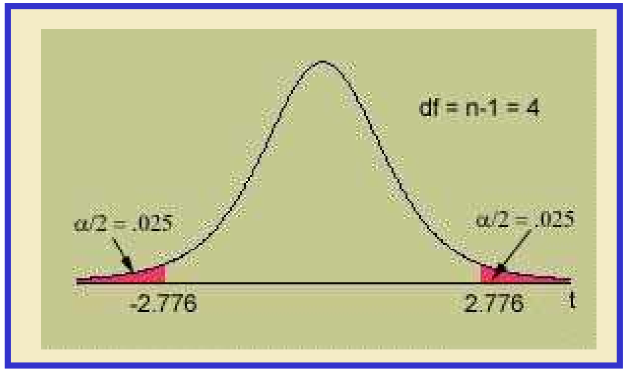
$$H_0 : \mu_d = 0$$
$$H_a : \mu_d \neq 0$$

Car	1	2	3	4	5
Type A	10.6	9.8	12.3	9.7	8.8
Type B	10.2	9.4	11.8	9.1	8.3
Difference	.4	.4	.5	.6	.5

Calculate $\bar{d} = \frac{\sum d_i}{n} = .48 \quad s_d = \sqrt{\frac{\sum d_i^2 - \frac{\left(\sum d_i\right)^2}{n}}{n-1}} = .0837$

- **Test statistic**: $t = \frac{\bar{d}-0}{s_d/\sqrt{n}} = \frac{.48-0}{.0837/\sqrt{5}} = 12.8$

- **Rejection region**:
Reject H_0 if $t > 2.776$ *or* $t < -2.776$ *for* $\alpha = .05$



- **Conclusion**: Since t=12.8 >2.776, reject H_0
  There is a difference in the average tire wear for the two types of tires

◆ Some notes

- You can construct a 100(1-α)% confidence interval for a paired experiment using

$$\bar{d} \pm t_{\alpha/2} \frac{s_d}{\sqrt{n}}$$

- Once you have designed the experiment by pairing, you MUST analyze it as a paired experiment.
- If the experiment is not designed as a paired experiment in advance, do not use this procedure.

Inference Concerning a Population Variance

- Sometimes the primary parameter of interest is not the population mean μ but rather the population variance σ^2. We choose a random sample of size *n* from a normal distribution.

- The sample variance s^2 can be used in its standardized form

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

which has a Chi-Square distribution with *n* - 1 degrees of freedom.

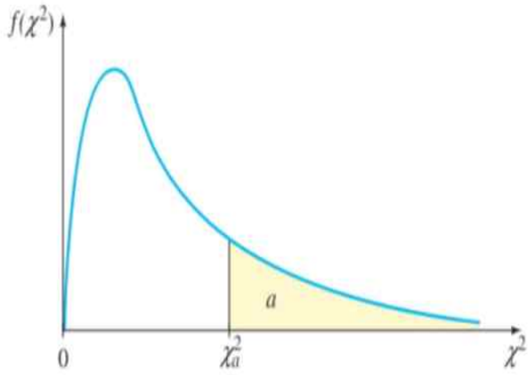
Inference Concerning a Population Variance

Table 5 gives both upper and lower critical values of the chi-square statistic for a given *df.*

TABLE 5
(continued)

$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$	*df*
2.70554	3.84146	5.02389	6.63490	7.87944	1
4.60517	5.99147	7.37776	9.21034	10.5966	2
6.25139	7.81473	9.34840	11.3449	12.8381	3
7.77944	9.48773	11.1433	13.2767	14.8602	4
9.23635	11.0705	12.8325	15.0863	16.7496	5
10.6446	12.5916	14.4494	16.8119	18.5476	6
12.0170	14.0671	16.0128	18.4753	20.2777	7
13.3616	15.5073	17.5346	20.0902	21.9550	8

For example, the value of chi-square that cuts off .05 in the upper tail of the distribution with *df* = 5 is $\chi^2_{.05} = 11.07.$



Format of the Chi-Square Table from Table 5 in Appendix I

df	$\chi^2_{.995}$	. . .	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	. . .	$\chi^2_{.005}$	*df*
1	.0000393		.0039321	.0157908	2.70554	3.84146		7.87944	1
2	.0100251		.102587	.210720	4.60517	5.99147		10.5966	2
3	.0717212		.351846	.584375	6.25139	7.81473		12.8381	3
4	.206990		.710721	1.063623	7.77944	9.48773		14.8602	4
5	.411740		1.145476	1.610310	9.23635	11.0705		16.7496	5
6	.0675727		1.63539	2.204130	10.6446	12.5916		18.5476	6
.	.		.	.	.	.		.	.
.	.		.	.	.	.		.	.
.	.		.	.	.	.		.	.
15	4.60094		7.26094	8.54675	22.3072	24.9958		32.8013	15
16	5.14224		7.96164	9.31223	23.5418	26.2962		34.2672	16
17	5.69724		8.67176	10.0852	24.7690	27.5871		35.7185	17
18	6.26481		9.39046	10.8649	25.9894	28.8693		37.1564	18
19	6.84398		10.1170	11.6509	27.2036	30.1435		38.5822	19
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Inference Concerning a Population Variance

- Test H_0: $\sigma^2 = \sigma_0^2$ versus H_a: *one or two tailed*

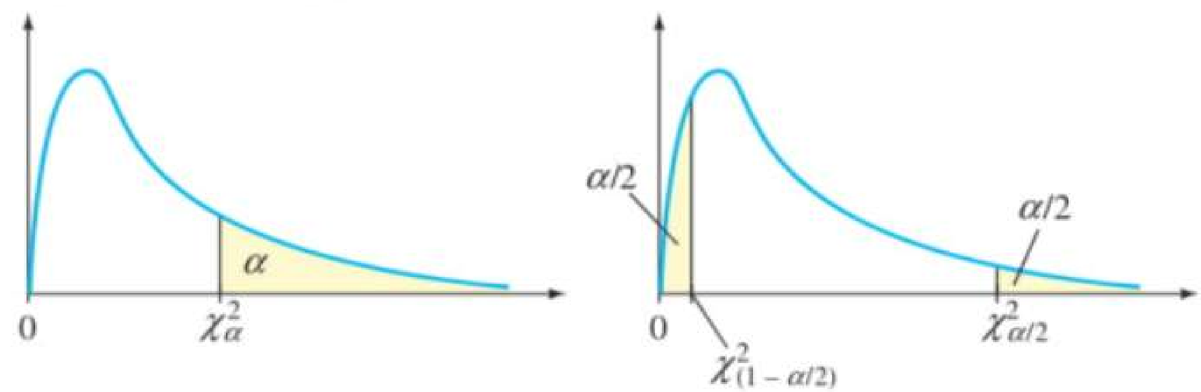
$$\sigma^2 > (<)\, \sigma_0^2 \quad \text{or} \quad \sigma^2 \neq \sigma_0^2$$

- Test statistic: $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$

  with a rejection region based on a chi-square distribution with df = n-1.

- Rejection region: Reject H_0 when

One-tailed test	Two-tailed test
$\chi^2 > \chi^2_{\alpha}$ (or $\chi^2 < \chi^2_{(1-\alpha)}$)	$\chi^2 > \chi^2_{\alpha/2}$ or $\chi^2 < \chi^2_{(1-\alpha/2)}$



- **(1- α)100% Confidence Interval for σ^2**

$$\frac{(n-1)s^2}{\chi^2_{(\alpha/2)}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}}$$

where $\chi^2_{(\alpha/2)}$ and $\chi^2_{(1-\alpha/2)}$ are the upper and lower χ^2 -values, which locate one-half of a in each tail of the chi-square distribution.

Assumption: The sample is randomly selected from a normal population.

Example

A cement manufacturer claims that his cement has a compressive strength with a standard deviation of 10 kg/cm^2 or less.
A sample of n = 10 measurements produced a mean and standard deviation of 312 and 13.96, respectively.

Do these data present sufficient evidence to reject the manufacturer's claim?

Sol)

(1-2) $\mathbf{H_0}$: $\boldsymbol{\sigma^2 = 10^2 = 100}$ versus $\mathbf{H_a}$: $\boldsymbol{\sigma^2 > 100}$

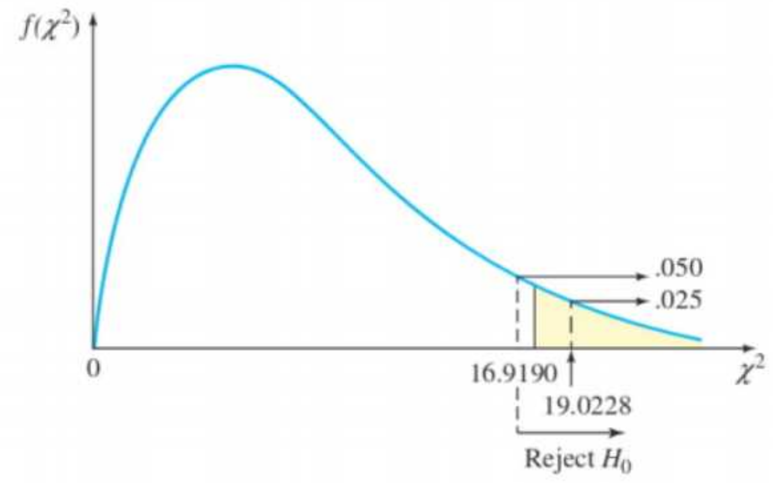
(3) Test statistic $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(10-1)(13.96)^2}{100} = 17.55$

(4) Reject $\mathbf{H_0}$ for $\alpha = .05$

if T.S. > $\chi^2_{.05}\ (n-1=9) = 16.91$

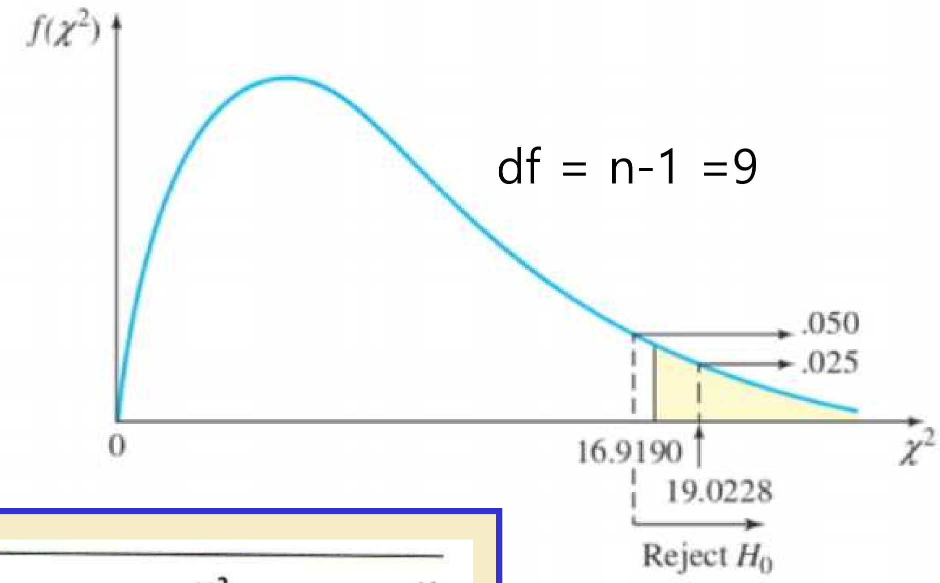
(5) Since 17.55 > 16.91, reject $\mathbf{H_0}$

The standard deviation of the cement strengths is more than 10



p-value = $P(\chi^2(9) > 17.55)$

$\Rightarrow$ .025 < p-value < .05



$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$	df
2.70554	3.84146	5.02389	6.63490	7.87944	1
4.60517	5.99147	7.37776	9.21034	10.5966	2
6.25139	7.81473	9.34840	11.3449	12.8381	3
7.77944	9.48773	11.1433	13.2767	14.8602	4
9.23635	11.0705	12.8325	15.0863	16.7496	5
10.6446	12.5916	14.4494	16.8119	18.5476	6
12.0170	14.0671	16.0128	18.4753	20.2777	7
13.3616	15.5073	17.5346	20.0902	21.9550	8
14.6837	16.9190	19.0228	21.6660	23.5893	9

Inference Concerning Two Population Variances

- We can make inferences about the ratio of two population variances in the form a ratio.
- We choose two independent random samples of size n_1 and n_2 from normal distributions.
- If the two population variances are equal, the statistic

$$F = \frac{s_1^2}{s_2^2}$$

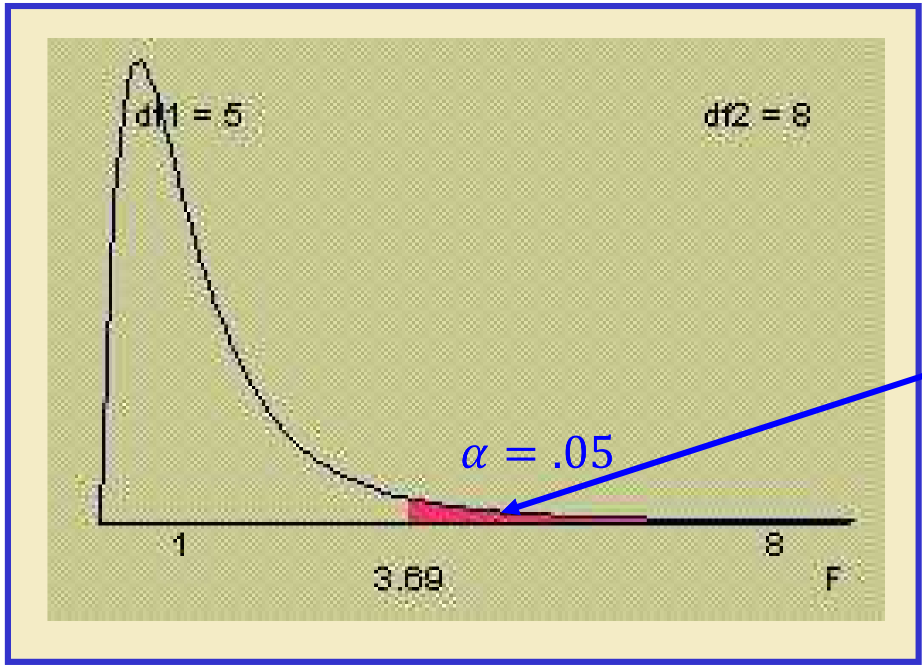
has an F distribution with $df_1 = n_1 - 1$ and $df_2 = n_2 - 1$ degrees of freedom.

✓ **Assumptions for s_1^2 / s_2^2 to have an F distribution**

- Random and independent samples are drawn from each of two normal populations.
- The variability of the measurements in the two populations is the same and can be measured by a common variance, $\boldsymbol{\sigma^2}$; that is , $\sigma^2 = \sigma_1^2 = \sigma_2^2$

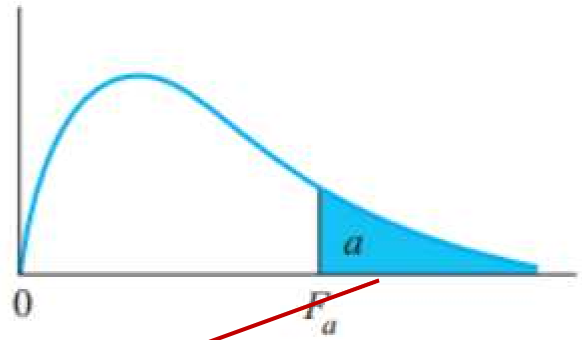
Inference Concerning Two Population Variances

- Table 6 gives only upper critical values of the F statistic for a given pair of df_1 and df_2.



For example, the value of F that cuts off .05 in the upper tail of the distribution with $df_1 = 5$ and $df_2 = 8$ is F =3.69.

F table(table 6)



Format of the *F* Table from Table 6 in Appendix I

		df_1					
df_2	a	1	2	3	4	5	6
1	.100	39.86	49.50	53.59	55.83	57.24	58.20
	.050	161.4	199.5	215.7	224.6	230.2	234.0
	.025	647.8	799.5	864.2	899.6	921.8	937.1
	.010	4052	4999.5	5403	5625	5764	5859
	.005	16211	20000	21615	22500	23056	23437
2	.100	8.53	9.00	9.16	9.24	9.29	9.33
	.050	18.51	19.00	19.16	19.25	19.30	19.33
	.025	38.51	39.00	39.17	39.25	39.30	39.33
	.010	98.50	99.00	99.17	99.25	99.30	99.33
	.005	198.5	199.0	199.2	199.2	199.3	199.3
3	.100	5.54	5.46	5.39	5.34	5.31	5.28
	.050	10.13	9.55	9.28	9.12	9.01	8.94
	.025	17.44	16.04	15.44	15.10	14.88	14.73
	.010	34.12	30.82	29.46	28.71	28.24	27.91
	.005	55.55	49.80	47.47	46.19	45.39	44.84

Inference Concerning Two Population Variances

- Test $H_0: \sigma_1^2 = \sigma_2^2$ versus H_a: *one or two tailed*

  $H_a: \sigma_1^2 > (<)\ \sigma_2^2$ or $\sigma_1^2 \neq \sigma_2^2$

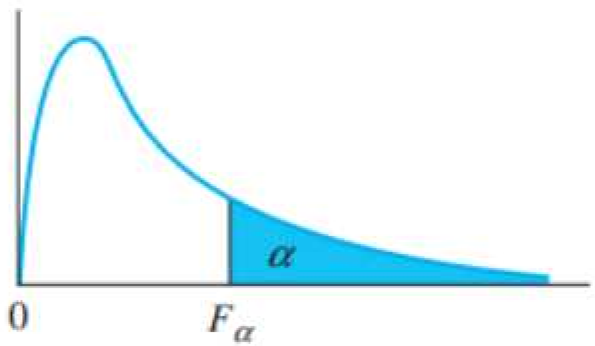
- Test statistic: $F = \dfrac{s_1^2}{s_2^2}$

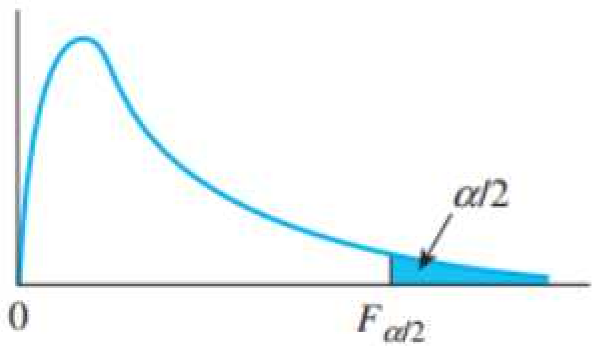
  where s_1^2 is the larger sample variance

- Rejection region: Reject H_0 when

One-tailed test	Two-tailed test
$F > F_\alpha$	$F > F_{\alpha/2}$ ~ $F(n_1 - 1, n_2 - 1)$

  Or when p-value $< \alpha$





- **Confidence Interval**

$$\frac{s_1^2}{s_2^2}\frac{1}{F_{df_1,df_2}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2}F_{df_2,df_1}$$

- where $df_1 = (n_1 - 1)$ and $df_2 = (n_2 - 1)$.
  F_{df_1,df_2} is the tabulated critical value of F corresponding to df_1 and df_2 degrees of freedom in the numerator and denominator of F, respectively, with area α /2 to its right.

- **Assumptions**: The samples are randomly and independently selected from normally distributed populations.

Example

- An experimenter has performed a lab experiment using two groups of rats. He wants to test H_0: $\mu_1 = \mu_2$, but first he wants to make sure that the population variances are equal.

	Standard (2)	**Experimental (1)**
Sample size	10	11
Sample mean	13.64	12.42
Sample Std Dev	2.3	5.8

Preliminary test :

$$\mathrm{H}_0 : \sigma_1^2 = \sigma_2^2 \text{ versus } \mathrm{H}_a : \sigma_1^2 \neq \sigma_2^2$$

Example (Continued)

- $\mathrm{H}_0: \sigma_1^2 = \sigma_2^2$ versus $\mathrm{H}_a: \sigma_1^2 \neq \sigma_2^2$

- Test statistic: $\mathrm{F} = \frac{s_1^2}{s_2^2} = \frac{5.8^2}{2.3^2} = 6.36$

✓ *We designate the sample with the larger standard deviation as sample 1, to force the test statistic into the upper tail of the F distribution.*

- The rejection region is two-tailed, with α = .05, but we only need to find the upper critical value, which has $\alpha/2$ = .025 to its right.
- From Table 6, with df_1=10 and df_2 = 9, we reject H_0 if F > 3.96.
- Conclusion: Reject H_0. There is sufficient evidence to indicate that the variances are unequal. Do not rely on the assumption of equal variances for your *t test!*

Key Concepts

I. Experimental Designs for Small Samples

1. Single random sample: The sampled population must be normal.

2. Two independent random samples: Both sampled populations must be normal.

    a. Populations have a common variance σ^2.

    b. Populations have different variances

3. Paired-difference or matched-pairs design: The samples are not independent.

Key Concepts

II. Statistical Tests of Significance

1. Based on the t, F, and χ^2 distributions
2. Use the same procedure as in Chapter 9
3. Rejection region—critical values and significance levels: based on the t, F, and χ^2 distributions with the appropriate degrees of freedom
4. Tests of population parameters: a single mean, the difference between two means, a single variance, and the ratio of two variances

III. Small Sample Test Statistics

To test one of the population parameters when the sample sizes are small, use the following test statistics:

Key Concepts

Parameter	Test Statistic	Degrees of Freedom
μ	$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$n - 1$
$\mu_1 - \mu_2$ (equal variances)	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$	$n_1 + n_2 - 2$
$\mu_1 - \mu_2$ (unequal variances)	$t \approx \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	Satterthwaite's approximation
$\mu_1 - \mu_2$ (paired samples)	$t = \dfrac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$	$n - 1$
σ^2	$\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$	$n - 1$
σ_1^2/σ_2^2	$F = s_1^2/s_2^2$	$n_1 - 1$ and $n_2 - 1$