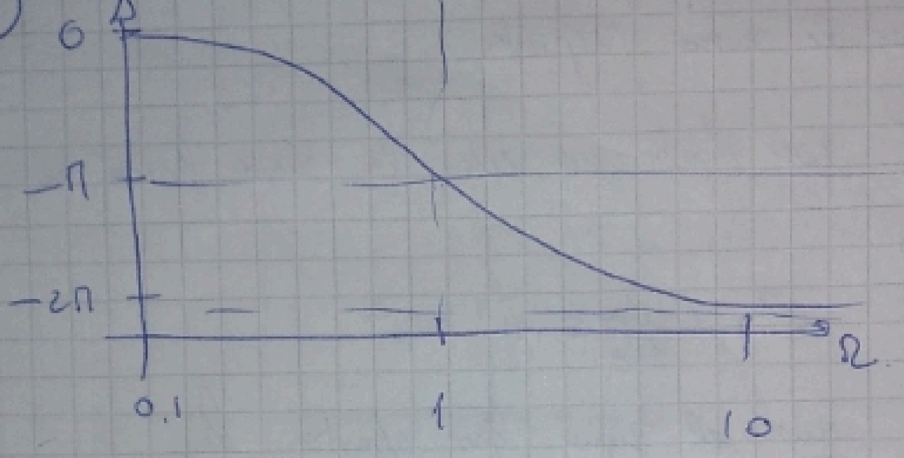
HOJA N°

FECHA

(2)



Sabemos que el 2° orden entonces es: $T(s) = \dfrac{s^2 + \frac{\omega_z}{Q_z}s + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$

Se tiene que $\theta_N(\omega) - \theta_D(\omega) = \theta(T(j\omega))$

En el caso del pasabajos $T(s) = \dfrac{1}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$

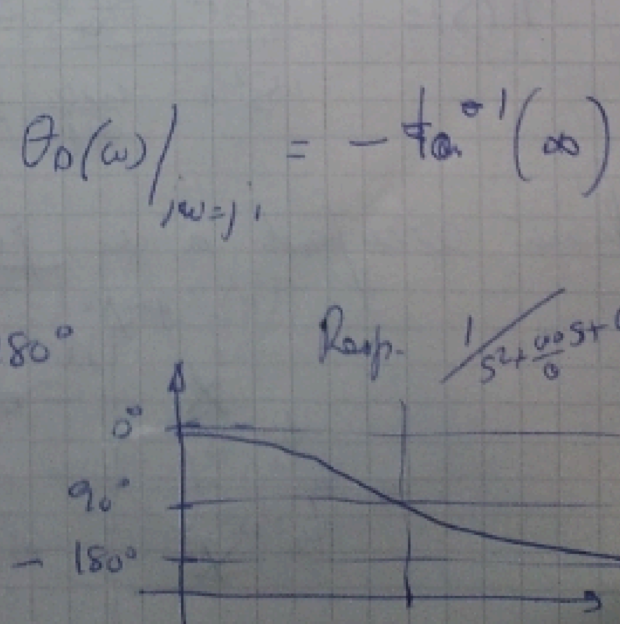
reemplazo $s = j\omega$ $\quad |T(j\omega)| = \dfrac{1}{\sqrt{\underbrace{(1-\omega^2)^2}_{\text{Parte real}} + \underbrace{\left(\frac{\omega}{Q}\right)^2}_{\text{Parte im.}}}}$

$$\theta_D(\omega) = -\tan^{-1}\left(\frac{\frac{\omega}{Q}}{1-\omega^2}\right)$$

$\theta_D(\omega)\Big|_{j\omega=j0} = 0°$ $\qquad \theta_D(\omega)\Big|_{j\omega=j1} = -\tan^{-1}(\infty) \cong -90°$

$\theta_D(\omega)\Big|_{j\omega=j\infty} \Rightarrow -180°$

Resp. $\dfrac{1}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$



NOTA

De acá deducimos que si agregamos $-180°$ en el numerador podríamos obtener la misma respuesta.

Una deducción casi directa es usar la misma función en el numerador.

$$s^2 + \frac{\omega_z}{Q_z}s + \omega_z^2$$

Pero queremos que el signo sea opuesto. Luego

$$\theta_N(j\infty) = -180°$$
$$\theta_N(j1) = -90°$$
$$\theta_N(j0) = 0°$$

si $\theta_N = \left(\dfrac{\frac{\omega}{Q_z}}{1-\omega^2}\right)$

$\Downarrow$

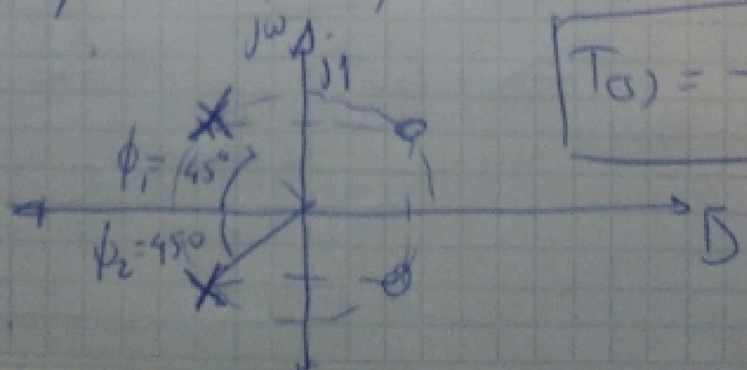
Necesitamos invertir el signo del numerador $\Rightarrow$

$$s^2 - \frac{\omega_z}{Q_z}s + \omega_z^2$$

luego

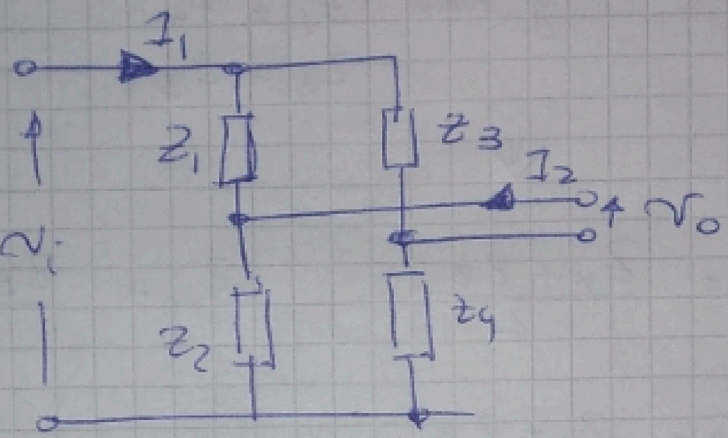
$$T(s) = \frac{s^2 - \frac{\omega_z}{Q_z}s + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Esta función corresponde a un pasatodo.



$$\boxed{T(s) = \frac{s^2 - \sqrt{2}s + 1}{s^2 + \sqrt{2}s + 1}}$$

Una posible implementación pasiva es mediante estructura tipo lattice



Podemos analizar este circuito mediante matriz de impedancias

$$\begin{cases} v_i = Z_{11} I_1 + Z_{12} I_2 \\ v_0 = Z_{21} I_1 + Z_{22} I_2 \end{cases}$$

$$Z_{11} = \left.\frac{v_i}{I_1}\right|_{I_2=0} = (Z_1 + Z_2) \,//\, (Z_3 + Z_4)$$

$$Z_{12} = \left.\frac{v_i}{I_2}\right|_{I_1=0} = (Z_1 + Z_3) \,//\, (Z_2 + Z_4)$$

$$Z_{21} = \left.\frac{v_0}{I_1}\right|_{I_2=0} = (Z_1 + Z_3) \,//\, (Z_2 + Z_4)$$

$$Z_{22} = \left.\frac{v_0}{I_2}\right|_{I_1=0} = (Z_1 + Z_2) \,//\, (Z_2 + Z_4)$$

Lo hacemos simétrico de tal manera que:

$$Z_{12} = Z_{21}$$

y hace $Z_1 = Z_4$
$Z_2 = Z_3$

$$\Downarrow$$

$$Z_{11} = \frac{Z_A + Z_B}{2} \qquad Z_{12} = \frac{Z_B - Z_A}{2} = Z_{21}$$

$$Z_{22} = \frac{Z_A + Z_B}{2}$$

La matriz transferencia se calcula por definición:

$$\begin{cases} V_i = A V_0 - I_0 B \\ I_1 = C V_0 - I_0 D \end{cases} \Rightarrow \quad A = \left.\frac{V_i}{V_0}\right|_{I_0=0} \quad B = \left.\frac{V_i}{I_0}\right|_{V_0=0}$$

$$C = \left.\frac{I_1}{V_0}\right|_{I_2=0} \quad D = \left.\frac{I_1}{-I_0}\right|_{V_0=0}$$

Deduciendo:

$$A = \frac{V_i}{V_0} = \frac{Z_{11}\cancel{I_i}}{Z_{21}\cancel{I_i}} \Rightarrow A = \frac{Z_{11}}{Z_{21}} \Rightarrow \frac{1}{A} = \frac{v_o}{v_i} = T(s)$$

luego ya sabemos que $\boxed{T(s) = \frac{Z_{12}}{Z_{11}}}$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

$$C = \frac{1}{Z_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}}$$

Ahora igualamos $T(S)$ a nuestro modelo.

$$T(s) = \frac{s^2 - \sqrt{2}\,s + 1}{s^2 + \sqrt{2}\,s + 1} = \frac{\dfrac{Z_B - Z_A}{Z}}{\dfrac{Z_A + Z_B}{Z}} = \frac{Z_B - Z_A}{Z_B + Z_A}$$

$$N(s) = Z_B - Z_A = s^2 - \sqrt{2}\,s + 1$$

$$D(S) = Z_B + Z_A = s^2 + \sqrt{2}\,s + 1$$

$D(S)$ es fácil podemos hacer con $Z_B = LC$ y $Z_A = R$:

$$\frac{1}{sC} + sL + R = \frac{1 + s^2LC + sCR}{sC}$$

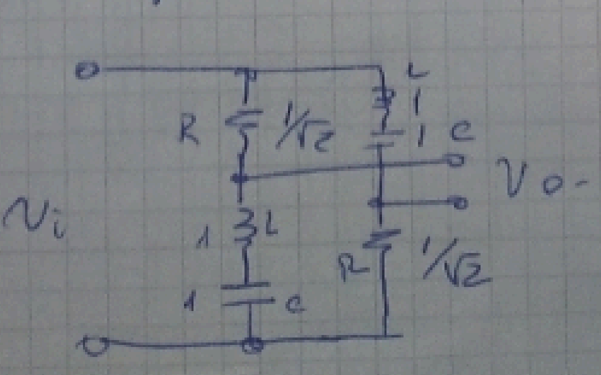
y $N(S)$: $\dfrac{1}{sC} + sL - R \Rightarrow \dfrac{1 + s^2LC - sRC}{sC}$

$$\frac{N(S)}{D(S)} = \frac{1 + s^2LC - sRC}{1 + s^2LC + sRC} \Rightarrow T(s) = \frac{s^2 - \frac{1}{RC}s + 1}{s^2 + \frac{1}{RC}s + 1}$$

Con lo cual $\dfrac{1}{RC} = \sqrt{2}$ $\quad \dfrac{1}{LC} = 1$

Si $\boxed{C = 1} \Rightarrow \boxed{R = \frac{1}{\sqrt{2}}} \rightarrow \boxed{L = 1}$

El circuito sintetizado sería:



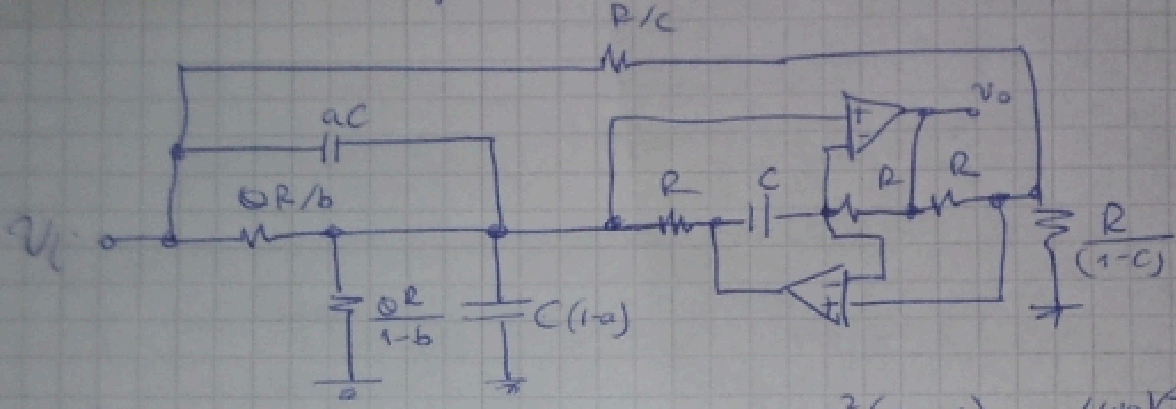
Calculo de la Matriz Z y T_{ABCD}

$$Z = \begin{pmatrix} \dfrac{1+sRC+s^2LC}{2sC} & \dfrac{1-sRC+s^2LC}{2sC} \\[2ex] \dfrac{1-sRC+s^2LC}{2sC} & \dfrac{1+sRC+s^2LC}{2sC} \end{pmatrix}$$

$$\boxed{Z = \begin{pmatrix} \dfrac{1+s\sqrt{2}+s^2}{2s} & \dfrac{1-s\sqrt{2}+s^2}{2s} \\[2ex] \dfrac{1-s\sqrt{2}+s^2}{2s} & \dfrac{1+s\sqrt{2}+s^2}{2s} \end{pmatrix}}$$

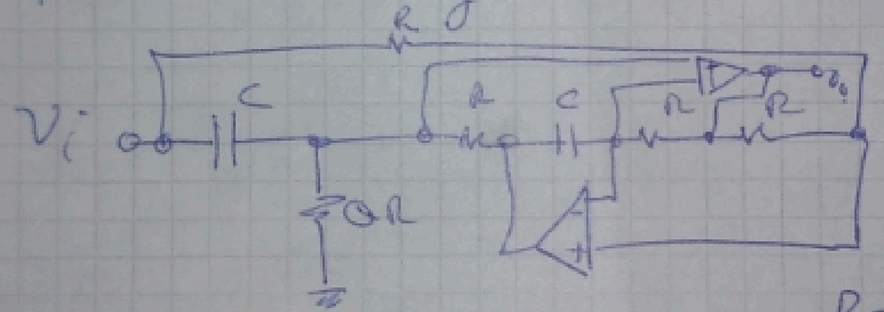
$$\boxed{T_{ABCD} = \begin{pmatrix} \dfrac{s^2+\sqrt{2}s+1}{s^2+\sqrt{2}s+1} & \dfrac{\dfrac{(1+\sqrt{2}s+s^2)^2-(1-\sqrt{2}s+s^2)^2}{2s}}{1-s\sqrt{2}+s^2} \\[3ex] \dfrac{2s}{1-s\sqrt{2}+s^2} & \dfrac{1+\sqrt{2}s+s^2}{1-\sqrt{2}s+s^2} \end{pmatrix}}$$

En la estructura bicuadrática GIC..



Para un All-pass $T(s) = \dfrac{s^2(2a-c) + s\left(\frac{\omega_0}{Q}\right)(2b-c) + c\omega_0^2}{s^2 + \frac{\omega_0}{Q} + \omega_0^2}$

necesitamos $a = c = 1$ y $b = 0 \Rightarrow$



$R = \dfrac{1}{\omega_0 C}.$

luego si $\omega_0^2 = 1 \Rightarrow R = C = 1$ y

$Q = \dfrac{1}{\sqrt{2}} \Rightarrow QR = \dfrac{1}{\sqrt{2}}$