SPAB assignment 09

task 9.3

- given
  global average sequencing depth λ
  local expected " λ,
  at any point in genome has gamma distr. with mean μ, variance $\sigma^2 = \mu/\beta$
  for a fixed value of λ at a specific point, the actual seq depth follows a Poisson distr. with mean λ

- tasks
  - compute param. k, β of gamma distr.
    (rate β, scale $1/\beta$, shape k)

mean $\mu = k\theta = k/\beta$ variance $\sigma^2 = k\theta^2 = k/\beta^2$

$\Leftrightarrow k = \mu\beta$ $\Leftrightarrow k = \sigma^2\beta^2$

$\Rightarrow \mu \cdot \beta = \sigma^2\beta^2$

$\Leftrightarrow \beta = \sigma^2/\mu$

$\Rightarrow k = \mu\beta = \mu \frac{\sigma^2}{\mu} = \sigma^2$

  - specify the resulting gamma-Poisson mixture distr.

$$\mathbb{P}[X=d] = \int_{\lambda=0}^{\infty} \underbrace{f_{k,\beta}}_{\text{Gamma}}(\lambda) \cdot \underbrace{\mathbb{P}_\lambda}_{\text{Poisson}}[X=d]\, d\lambda$$

$$\overset{k=\sigma^2,\ \beta=\sigma^2/\mu}{=} \int_{\lambda=0}^{\infty} \underbrace{f_{\sigma^2,\sigma^2/\mu}}_{\text{Gamma}}(\lambda)\ \underbrace{\mathbb{P}_\lambda}_{\text{Poisson}}[X=d]\, d\lambda$$