SPAB assignment II

①

$X = (X_1, ..., X_n)$ ECDT $\widehat{F_n(X)} : \mathbb{R} \rightarrow [0,1]$

1) distribution of $\widehat{F_n(X)}(\omega)$ for each $x \in \mathbb{R}$?

ECDT is an estimator for the CDF of X so it converges to the underlying distribution of X

2) mean & variance

mean $E(X) = \frac{1}{n} \sum_{i=1}^{n} X_i$

var $Var(X) = E[(X - E[X])^2]$
$= E[(x - \bar{x})^2]$
$= \frac{1}{n}(\sum_{i=1}^{n}(x_i - \bar{x})^2)$

3) $\widehat{F_n(X)}(x)$ unbiased?

$\widehat{F_n(X)}(\omega)$ would be unbiased, if the long time average of the estimator equals to the mean / median of the underlying distribution. Since its mean already equals the mean of X, $\widehat{F_n(X)}(\omega)$ is mean-unbiased.