Task 10.2

$$\delta(X) := \frac{1}{\bar{x}} \cdot \frac{n}{\sum_{i=1}^{n} X_i}$$

unbiasedness: $\mathbb{E}_\theta[\delta(X)] = g(\theta)$

$$b(\theta) := \mathbb{E}_\theta[\delta(X)] - g(\theta) \stackrel{!}{=} 0$$

$$= \mathbb{E}_\theta[\delta(X) - g(\theta)]$$

$$= \mathbb{E}\left[\frac{1}{\bar{x}} - \bar{x}\right]$$

$$= \frac{\frac{1}{\bar{x}}}{n-1}$$

In the end, $\bar{x}$ is an unbiased estimator for β. I used the common calculation here (also on Wikipedia) instead of an integral.

"This means your estimator is biased if you want to correct it, you have to calculate

$$\mathbb{E}[\delta(X)] - b(\theta)$$