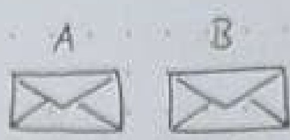
4.3



$X = x$ amount in first envelope

Y amount in other envelope

$\mathbb{P}[Y = 2x \mid X = x] = \frac{1}{2}$, $\mathbb{P}[Y = \frac{1}{2}x \mid X = x] = \frac{1}{2}$

$\mathbb{E}[Y \mid X = x] = \frac{1}{2}(2x) + \frac{1}{2} \cdot \frac{x}{2} = \frac{5}{4} > x$

At ~~the~~ first glance it is a completely symmetric problem and it seems that no matter which envelope you choose, you can not expect getting more money. But why the weird $\mathbb{E}[Y \mid X = x]$? This is because ~~we~~ of the calculation that prefers the bigger amount we could win. But in fact if we would take the other envelope, and we could reconsider again we would end in an endless loop of switching envelopes.

If we take this example:

If we choose A, with switching we earn 10€. ~~On~~ ~~the~~ So we gain x. If we would have chosen B in the first place, we would loose $\frac{x}{2}$ (50 10€). Because we determine Xs value with our choice. So we are earning 10€ or loosing 10€, so the correct way of calculating μ is

$$\mathbb{E} = \frac{1}{2}x + \frac{1}{2}(-x) = 0$$

So swapping makes no sense at all.