SPAB aso 10

①

$f(x) = \beta \exp(-\beta x) \quad , \quad x \geq 0$

1) method-of-moments estimator for β using mean

mean $\mu = \delta(\beta) = \frac{1}{\beta} \quad , \quad \mu = \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$

$\rightarrow \hat{\beta} = \frac{1}{\bar{x}}$

2) compute median, median-based estimator

median: $0{,}5 = \int_{-\infty}^{m} f(x)\,dx \quad , \quad F(x) = -\exp(-\beta x) \quad (+c,\ c \in \mathbb{R})$

$\Leftrightarrow 0{,}5 = F(m) - F(0)$, because $x \geq 0$

$\Leftrightarrow 0{,}5 = -\exp(-\beta m) + 1$

$\Leftrightarrow -0{,}5 = -\exp(-\beta m)$

$\Leftrightarrow \ln(1/2) = -\beta m$

$\Leftrightarrow m = -\frac{\ln(1/2)}{\beta} = \frac{-(\ln(1) - \ln(2))}{\beta} = \frac{\ln(2)}{\beta}$

cdf $= 1 - e^{-\beta x} \quad , \quad x \geq 0$

3) ML estimator for β

$\mathbb{P}_{\beta}[x] = \prod_{i=1}^{n} \mathbb{P}(X_i = x_i; \beta) = \prod_{i=1}^{n} (\beta \cdot e^{-\beta x})$

$\mathcal{L}_x(\beta) = \sum_{i=1}^{n} \log(\beta \cdot e^{-\beta x_i})$

$= \sum_{i=1}^{n} \log(\beta) + \log(e^{-\beta x_i})$

$= \sum_{i=1}^{n} \log(\beta) + (-\beta x_i)$

$= \log(\beta) \cdot (-\beta) \sum_{i=1}^{n} x_i$

$\hat{\theta}_{ML} = -\beta \log(\beta) \sum_{i=1}^{n} x_i$