SPAB · ass 11

③

$X \sim \text{Poisson}(\lambda)$, $Y \sim \text{Poisson}(\mu)$ independent, pmf $\frac{\lambda^k e^{-\lambda}}{k!}$

show: $Z := X+Y \sim \text{Poisson}(\lambda+\mu)$

considering the hint we start with

$$\mathbb{P}[X+Y=k] = \sum_{i=0}^{k} \mathbb{P}[X+Y=k, X=i]$$

$$= \sum_{i=0}^{k} \mathbb{P}[Y=k-i, X=i]$$

$$= \sum_{i=0}^{k} \mathbb{P}[Y=k-i]\, \mathbb{P}[X=i]$$

$$= \sum_{i=0}^{k} \frac{\mu^{k-i} e^{-\mu}}{(k-i)!} \frac{\lambda^i e^{-\lambda}}{i!}$$

$$= e^{-\mu-\lambda} \sum_{i=0}^{k} \frac{\mu^{k-i} \lambda^i}{(k-i)!\, i!} \cdot \frac{k!}{k!}$$

$$= e^{-(\mu+\lambda)} \frac{1}{k!} \sum_{i=0}^{k} \frac{k!}{(k-i)!\, i!} \mu^{k-i} \lambda^i$$

$$= e^{-(\lambda+\mu)} \frac{1}{k!} \sum_{i=0}^{k} \binom{k}{i} \mu^{k-i} \lambda^i$$

$$= e^{-(\lambda+\mu)} \frac{1}{k!} (\mu+\lambda)^k$$

$$= \frac{(\lambda+\mu)^k}{k!} e^{-(\lambda+\mu)} = \text{Poisson}(\lambda+\mu) \quad \blacksquare$$