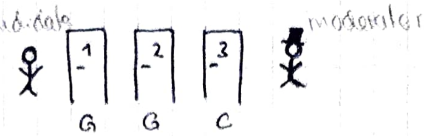
Task 3.4 - The Goat Problem



This problem divides at first glance in 2 cases:

Case 1: The candidate chooses door 3, such that the moderator opens a door with a goat random.

Case 2: The candidate chooses door 1, such sthat the moderator opens door 2 for sure.

Afterwards we must add the assumption that the candidate knows the moderators bias for Case 1. For example could it be that the moderator always takes the highest number possible (that is not the car). In fact, the bias is there for both cases.

where the prize can be

probability that moderator picks between two goat-doors mit $i \in \{1,2,3\}$

We define $\Omega = \{1,2,3\}$ and $P(M_i) = \frac{1}{2}$

initially: $P(Car_i) = \frac{1}{3}$ mit $i \in \{1,2,3\}$

Case 1: We assume the moderator is unbiased and picks doors with equal probability $\frac{1}{2}$.

$P(M_2 \mid Car_3) = \frac{1}{2}$ prob. that mod. opens 2 when car is in 3

$P(M_2 \mid Car_2) = 0$ prob. that mod. opens 2 when car is in 2

$P(M_2 \mid Car_1) = 1$ prob. that mod. opens 2 when car is in 1

$$P(Car_1 \mid M_2) = \frac{P(M_2 \mid Car_1) \cdot P(Car_1)}{P(\sum_{i=1}^{3} P(M_2 \mid Car_i) \cdot P(Car_i))} = \frac{1 \cdot \frac{1}{3}}{1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3}} = \frac{2}{3}$$

The candidate should change from door 3 to door 1. This way he/she doubles his/her winning probability from $\frac{1}{3}$ to $\frac{2}{3}$.

If we pick the door with a goat the probabilities change (we assume picking door 1)

$P(M_2 \mid Car_3) = 1$

$P(M_2 \mid Car_2) = 0$ this is just larger than 0 if the moderator can open the door that was picked.

$P(M_2 \mid Car_1) = 0$

The problem is far more complex than this small case. But we are not interested in game theory or biased moderators.