Assignment 09

Task 9.1

$n = 2.600$: amount of potential partners

$r \in [1, 2600]$: ranks

$r = 1$: absolute best applicant when we watch all 2600.

If the woman sees $n=5$ persons: $[3, 2, 5, 1, 4]$

After the first 3 persons: $[2, 1, 3]$, Person 2 is the best so far.

The basic idea I have is that she should let pass a specific number of applicants. After that amount she picks whoever is better than applicant A. (My call would be $\sim \frac{1}{3}$)

x: amount of people strategically rejected

A: Applicant who was best until x

The calculation follows the idea that we want to find the probability that the $x+1$th applicant is beter than A.

(For calculation purposes (switch it up: $x \to x-1$, $x+1 \to x$)

$$P_x = \sum_{i=x}^{n} P(\text{best applicant}_i \cap \text{selected applicant}_i)$$

This is what we want to solve. I found a bit of help on the website www.cut-the-knot.org.

We can split that formular with Bayes rule

$$= \sum_{i=x}^{n} P(\text{best applicant}_i) \cdot P(\text{selected app}_x \mid \text{best app}_i)$$

Now we can pull out $\frac{1}{n}$ and just look at he first $x-1$ cases

$$= \sum_{i=x}^{n} \frac{1}{n} P(\text{best of first } i-1 \text{ applicants before } x)$$

$$= \sum_{i=x}^{n} \frac{1}{n} \cdot \frac{x-1}{i-1} = \frac{x-1}{n} \sum_{i=x}^{n} \frac{1}{i-1}.$$

Back to searching for the threshold x that maximizes P_x.

~~Therefore~~ we need to satisfy $1 \geq \sum_{i=x+1}^{n} \frac{1}{i-1} = f(x) \approx \left(\frac{n}{x}\right)$.

$\to x \approx e$, so P_x is $\approx \frac{1}{e}$. We need to reject the first 37 applicants.