Task 1

$\mathbb{P}[X=2^n] = 2^{-n} \quad \forall n \geq 1 \quad \wedge \quad \mathbb{P}[X=k] = 0 \quad \forall$ other k

- verify $\sum_{n=1}^{\infty} 2^{-n} = 1$

$\sum_{n=0}^{\infty} q^k = \frac{1}{1-q}$, $|q|<1$

$$\sum_{n=1}^{\infty} 2^{-n} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \left(\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n\right) - \left(\frac{1}{2}\right)^0 = \frac{1}{1-\frac{1}{2}} - 1 = 2 - 1 = 1 \quad \checkmark$$

- $$E[X] = \sum_{n=1}^{\infty} x_i \, \mathbb{P}[X=x_i] = 1 \cdot 0 + 2 \cdot \frac{1}{2} + 3 \cdot 0 + 4 \cdot \frac{1}{4} + 5 \cdot 0 + 6 \cdot 0 + \dots$$

$$= \sum_{n=1}^{\infty} 2^n \cdot 2^{-n} = \sum_{n=1}^{\infty} 1$$

- $$E[X^2] = \sum_{n=1}^{\infty} x_i^2 \, \mathbb{P}[X=x_i] = \sum_{n=1}^{\infty} (2^n)^2 \cdot 2^{-n} = \sum_{n=1}^{\infty} 2^{2n-n}$$

- $$E[\sqrt{X}] = \sum_{n=1}^{\infty} \sqrt{x_i} \, \mathbb{P}[X=x_i] = \sum_{n=1}^{\infty} \sqrt{2^n} \cdot 2^{-n} = \sum_{n=1}^{\infty} 2^{n/2 - n}$$

- $$E[\log_2 X] = \sum_{n=1}^{\infty} \log_2(x_i) \, \mathbb{P}[X=x_i] = \sum_{n=1}^{\infty} \log_2(2^n) \cdot 2^{-n} = \sum_{n=1}^{\infty} \frac{n}{2^n}$$