Optimization in Finance – Problem set 1

1.1 Find and classify all critical points of the following functions:

(a) $f(x) = \dfrac{x^2 - x - 2}{x - 3}$

(b) $f(x, y) = x^2 + y^2 - 3x + 12$

(c) $f(x, y) = x^2 - y^2 + 4x + 8y$

(d) $f(x, y) = x^3 + y^3 - 3xy$

(e) $f(x_1, x_2, x_3) = 2x_1^2 + x_1x_2 + 4x_2^2 + x_1x_3 + x_3^2 + 2$

(f) $f(x_1, x_2, x_3) = 25 - x_1^2 - x_2^2 - x_3^2$

(g) $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + 3x_3^2 - x_1x_2 + 2x_1x_3 + x_2x_3$

(h) $f(x, y, z) = x^2 + 2y^2 + 3z^2 + 2xy + 2xz$

1.2 Explain why the Second Derivative Test for n variables with $n = 1$, gives the same result as the Second Derivative Test in the case of one variable.
Work out the general case of the Second Derivative Test for two variables.

1.3 (Exam Summer 2014) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be defined by

$$f(x, y, z) = x^3 - z^3 + 3x^2 + xy^2 + y^2 + 3z.$$

(i) Find all critical points of f

(ii) State the second order derivative test and decide the nature of the critical point $(x, y, z) = (-1, \sqrt{3}, -1)$.

1.4 Find and classify the critical points of

$$f(x, y, z) = \frac{3x^4 - 6x^2 + 1}{z^2 + 1} - y^2.$$