In FDTD Simulation, Using the Analytical Field-Propagation Technique to Implement Arbitrary Plane Waves as Incident Source Conditions for the Lossy Half-Space

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APPLICATION

- Plane-wave oblique incident into a lossy half-space technique can be used at long-distance remote sensing simulation

  - Underground object detection
  - Industrial non-destructive detection
  - Human chest tumors detection
  - RF-human body interaction
  - Radar Cross-Section (RCS).

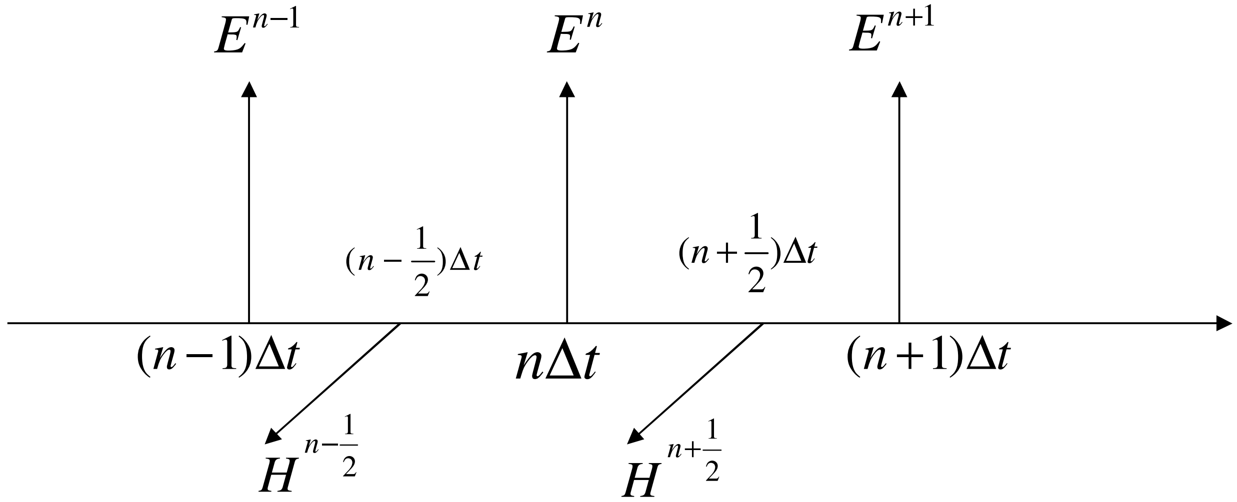
OUTLINE

- FDTD
- The update at the junction of total field and scattered field
- Lossy medium
- Arbitrary plane wave as the incident source
- The method of reflect waves in lossy medium
- The method of transmit waves in the lossy medium
- Simulation result
- Conclusion

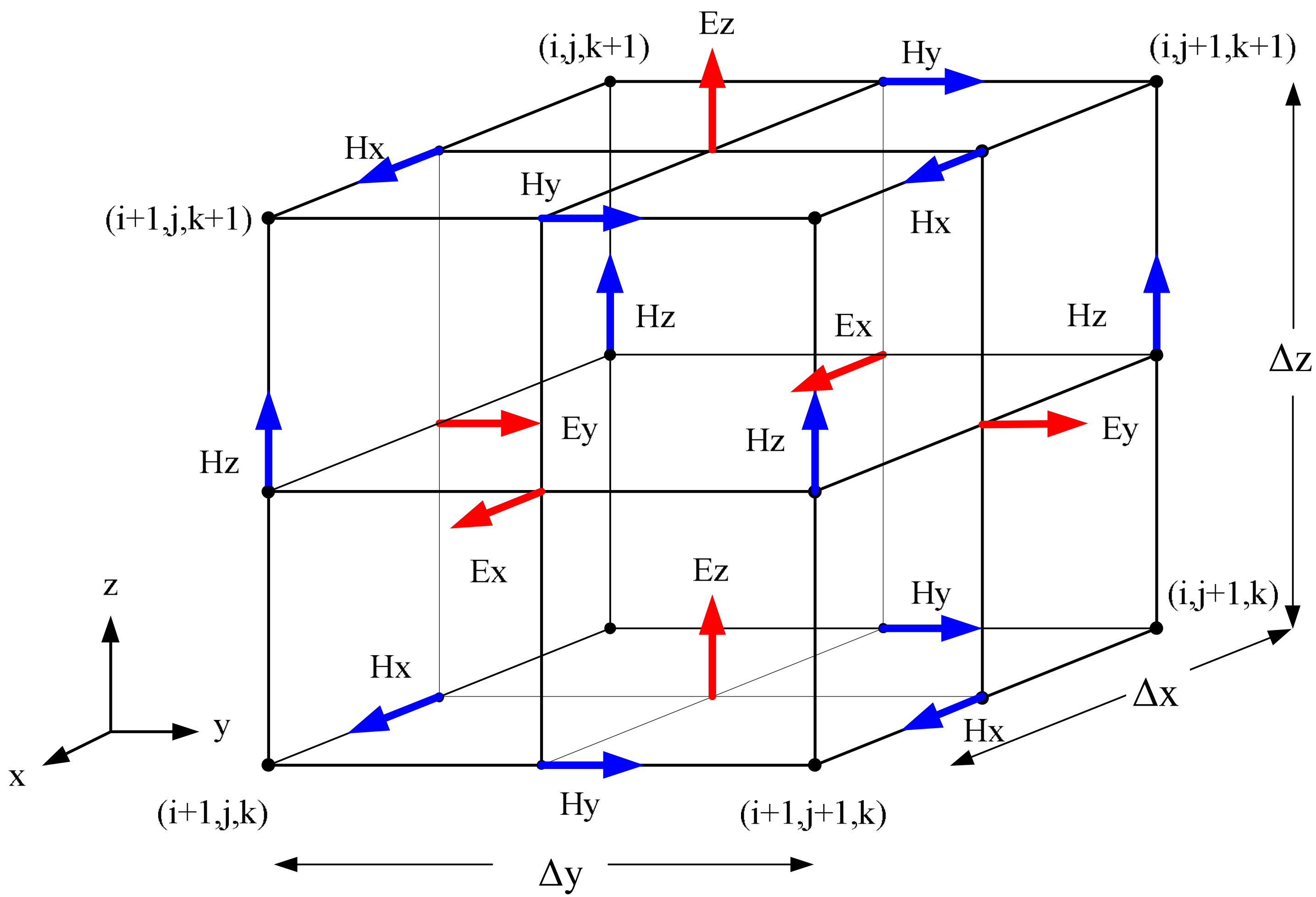
FDTD (Finite Difference Time Domain)

$$\nabla \times \vec{H} = \frac{\partial}{\partial t}\vec{D} + \vec{J} \Rightarrow \frac{E^{n} - E^{n-1}}{\Delta t} = -\frac{\sigma}{\varepsilon}E^{n-\frac{1}{2}} + \frac{1}{\varepsilon}\nabla \times \vec{H}^{n-\frac{1}{2}}$$

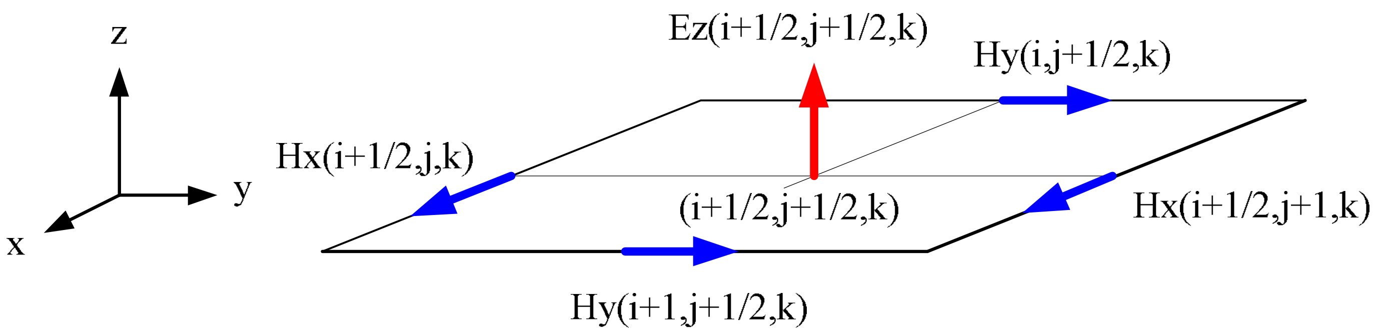
$$E^{n} = \frac{1 - \frac{\sigma \Delta t}{2\varepsilon}}{1 + \frac{\sigma \Delta t}{2\varepsilon}} E^{n-1} + \frac{\Delta t / \varepsilon}{1 + \frac{\sigma \Delta t}{2\varepsilon}} \nabla \times \vec{H}^{n-\frac{1}{2}}$$



ELECTRIC AND MAGNETIC FIELD CONFIGURATION IN THE UNIT GRID



FDTD E_Z



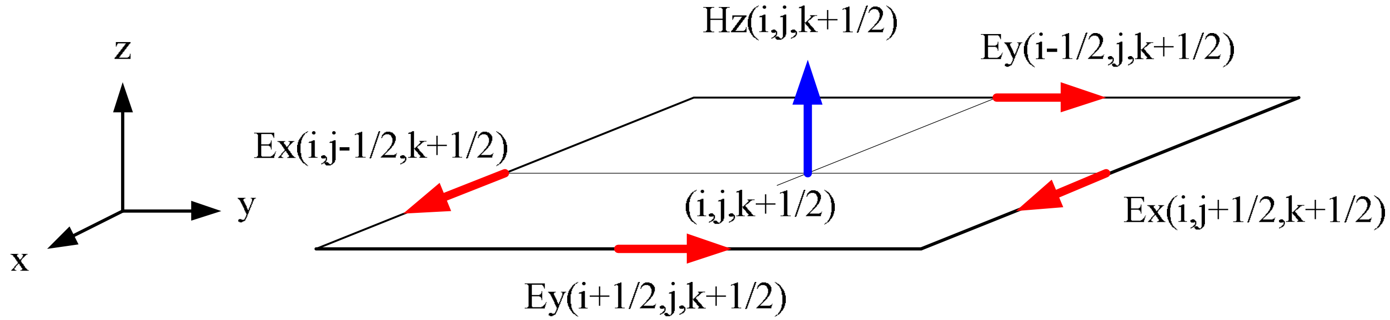
$$E_z^{n+\frac{1}{2}}(i+\tfrac{1}{2},j+\tfrac{1}{2},k)=A(m)\cdot E_z^{n+\frac{1}{2}}(i+\tfrac{1}{2},j+\tfrac{1}{2},k)$$

$$+B(m)\cdot[\frac{H_x^{n+\frac{1}{2}}(i+\tfrac{1}{2},j+1,k)-H_x^{n+\frac{1}{2}}(i+\tfrac{1}{2},j,k)}{\Delta y}$$

$$-\frac{H_y^{n+\frac{1}{2}}(i+1,j+\tfrac{1}{2},k)-H_y^{n+\frac{1}{2}}(i,j+\tfrac{1}{2},k)}{\Delta x}]$$

$$m=(i+\tfrac{1}{2},j+\tfrac{1}{2},k)$$

FDTD H_Z



$$H_z^{n+\frac{1}{2}}(i,j,k+\tfrac{1}{2}) = C(m)\cdot H_z^{n-\frac{1}{2}}(i,j,k+\tfrac{1}{2})$$

$$-D(m)\cdot\left[\frac{E_y^n(i+\frac{1}{2},j,k+\frac{1}{2})-E_y^n(i-\frac{1}{2},j,k+\frac{1}{2})}{\Delta x}\right.$$

$$\left.-\frac{E_x^n(i,j+\frac{1}{2},k+\frac{1}{2})-E_x^n(i,j-\frac{1}{2},k+\frac{1}{2})}{\Delta y}\right]$$

$$m=(i,j,k+\tfrac{1}{2})$$

COURANT STABLE CONDITION

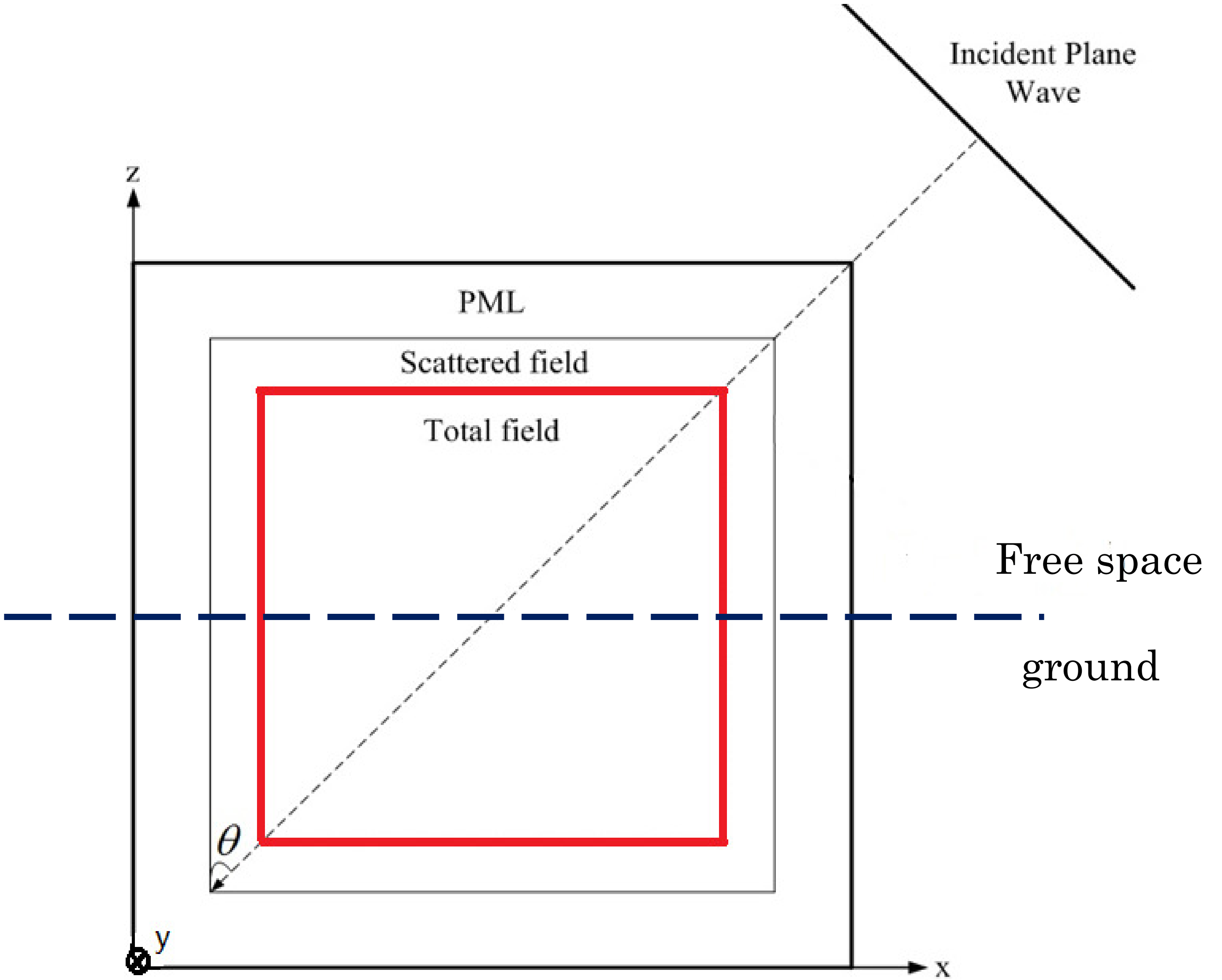
When using the FDTD method to calculate, in order to keep the calculation result stable, the selected time-discrete interval and space-discrete interval must meet the Courant stability condition.

$$c\Delta t \le \frac{1}{\sqrt{\left[\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}\right]}}$$

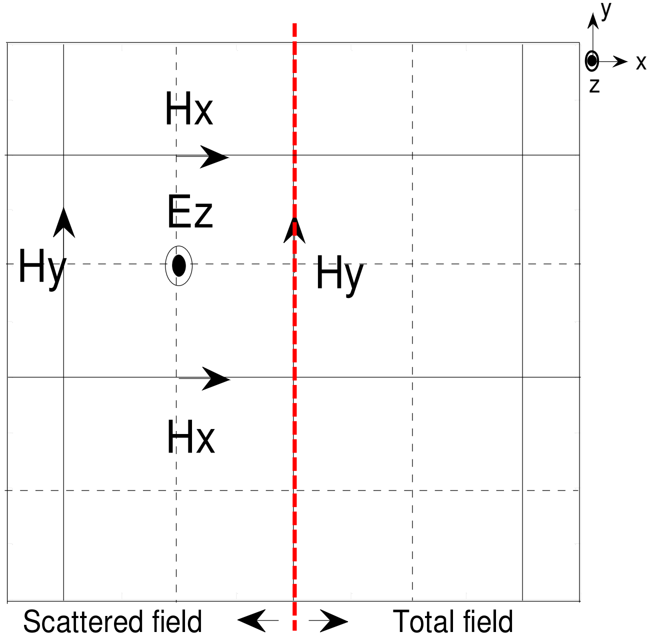
To avoid the error of numerical calculation, the dispersion of the wave must be satisfied

$$\delta \le \frac{\lambda_{\min}}{10}$$

THE UPDATE AT THE JUNCTION OF TOTAL FIELD AND SCATTERED FIELD



Schematic diagram of the junction of the total field and the scattered field at E_z

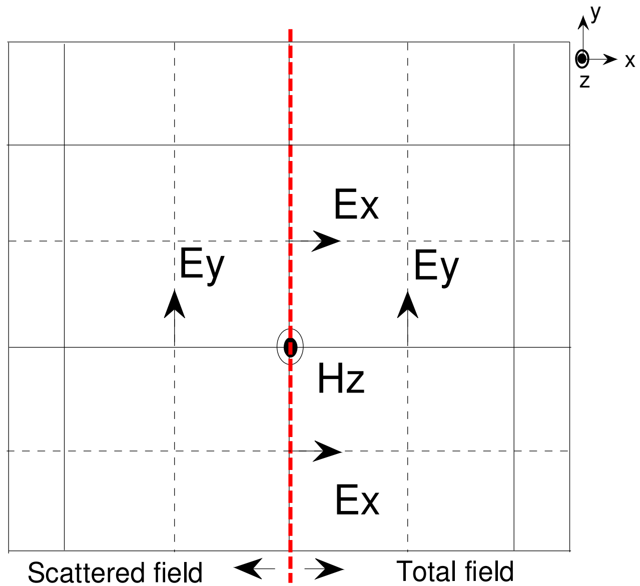


$$E_{zs}^{n+1}(i+\frac{1}{2}, j+\frac{1}{2}, k) = A(m)\cdot E_{zs}^{n}(i+\frac{1}{2}, j+\frac{1}{2}, k)$$

$$+B(m)\cdot[\frac{(H_{y}^{n+\frac{1}{2}}(i+1, j+\frac{1}{2}, k) - \boxed{H_{yi}^{n+\frac{1}{2}}(i+1, j+\frac{1}{2}, k)}) - H_{ys}^{n+\frac{1}{2}}(i, j+\frac{1}{2}, k)}{\Delta x}$$

$$-\frac{H_{xs}^{n+\frac{1}{2}}(i+\frac{1}{2}, j+1, k) - H_{xs}^{n+\frac{1}{2}}(i+\frac{1}{2}, j, k)}{\Delta y}]$$

SCHEMATIC DIAGRAM OF THE JUNCTION OF THE TOTAL FIELD AND THE SCATTERED FIELD AT H_z



$$H_z^{n+\frac{1}{2}}(i,j,k+\frac{1}{2}) = H_z^{n-\frac{1}{2}}(i,j,k+\frac{1}{2})$$

$$+C(m)\cdot[\frac{E_x^n(i,j+\frac{1}{2},k+\frac{1}{2}) - E_x^n(i,j-\frac{1}{2},k+\frac{1}{2})}{\Delta y}$$

$$-\frac{E_y^n(i+\frac{1}{2},j,k+\frac{1}{2}) - (E_{ys}^n(i-\frac{1}{2},j,k+\frac{1}{2}) + \boxed{E_{yi}^n(i-\frac{1}{2},j,k+\frac{1}{2})})}{\Delta x}]$$

FDTD Program structure flow

Electric field incident field update

Total field and scatter field electric field update

Magnetic field incident field update

Total field and scatter field magnetic field update

LOSSY MEDIUM

- When the medium is lossy, there will be a problem of dispersion. The speed of each frequency is different, and the transmission angle θ_t of each frequency is also different. At this time, the plane wave cannot be analyzed in the time domain.

- $n_1 \sin(\theta_i) = \mathrm{n}_2 \sin(\theta_t)$ $\qquad \beta_1 \sin(\theta_i) = \beta_2 \sin(\theta_t)$

- The dielectric constant of the conductor: $\varepsilon_c = \varepsilon_0 (\varepsilon_{r2} + \dfrac{\sigma_2}{j\omega\varepsilon_0})$

- Wave number in free space: $k_1 = \omega\sqrt{\mu\varepsilon} = \beta_1$

- Wave number of dielectric layer conductor: $k_2 = \omega\sqrt{\mu\varepsilon_c}$

PLANE WAVE ： DIFFERENTIAL GAUSSIAN PULSE

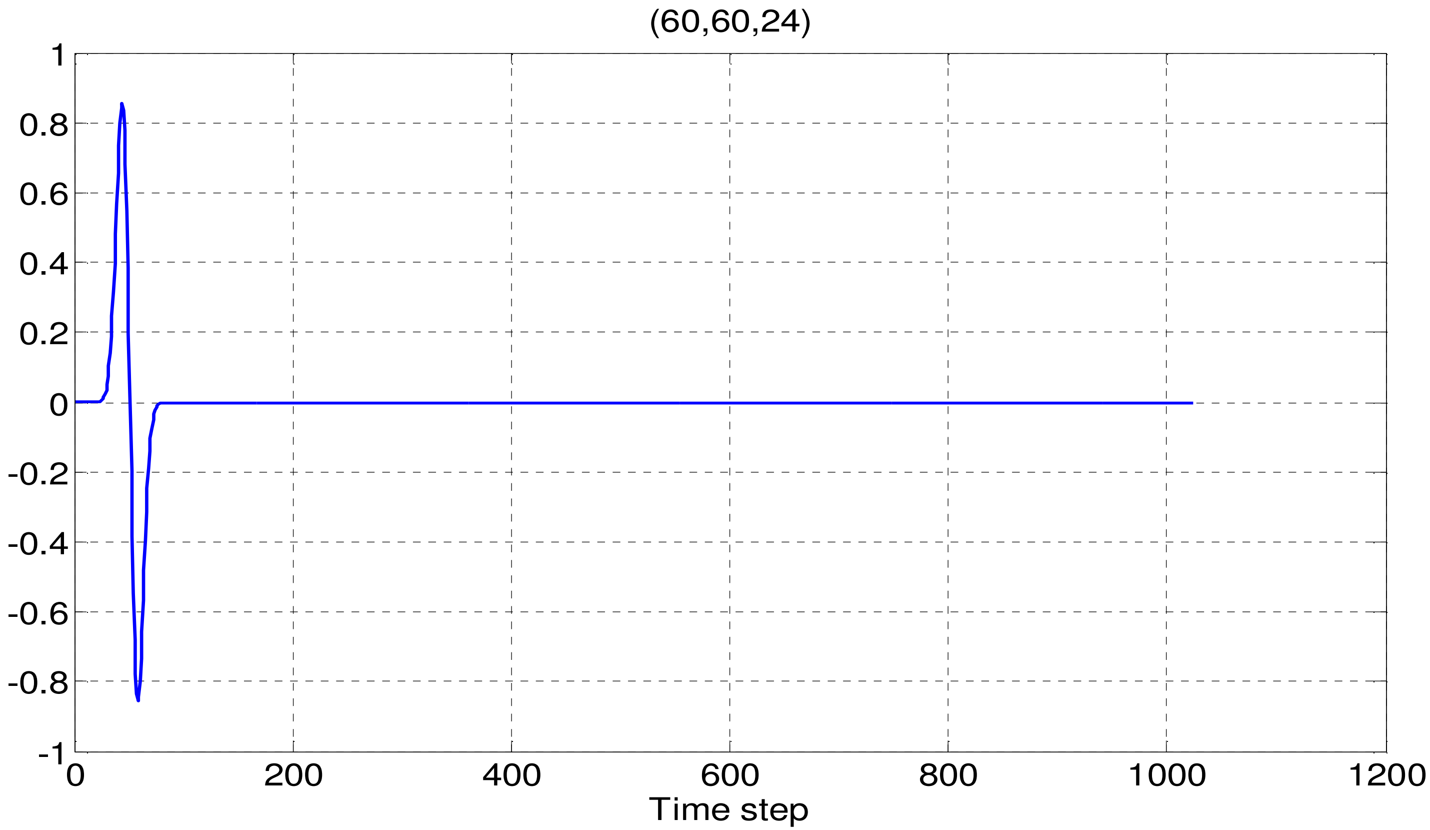
$$E_y^{inc} = (-2)\cdot\left(\frac{t+\frac{r}{c}-t_0}{T}\right)\cdot\exp\left[-\left(\frac{t+\frac{r}{c}-t_0}{T}\right)^2\right]$$

$$H_x^{inc} = \frac{1}{\eta_0}\cdot(-2)\cdot\left(\frac{t+\frac{r}{c}-t_0}{T}\right)\cdot\exp\left[-\left(\frac{t+\frac{r}{c}-t_0}{T}\right)^2\right]$$

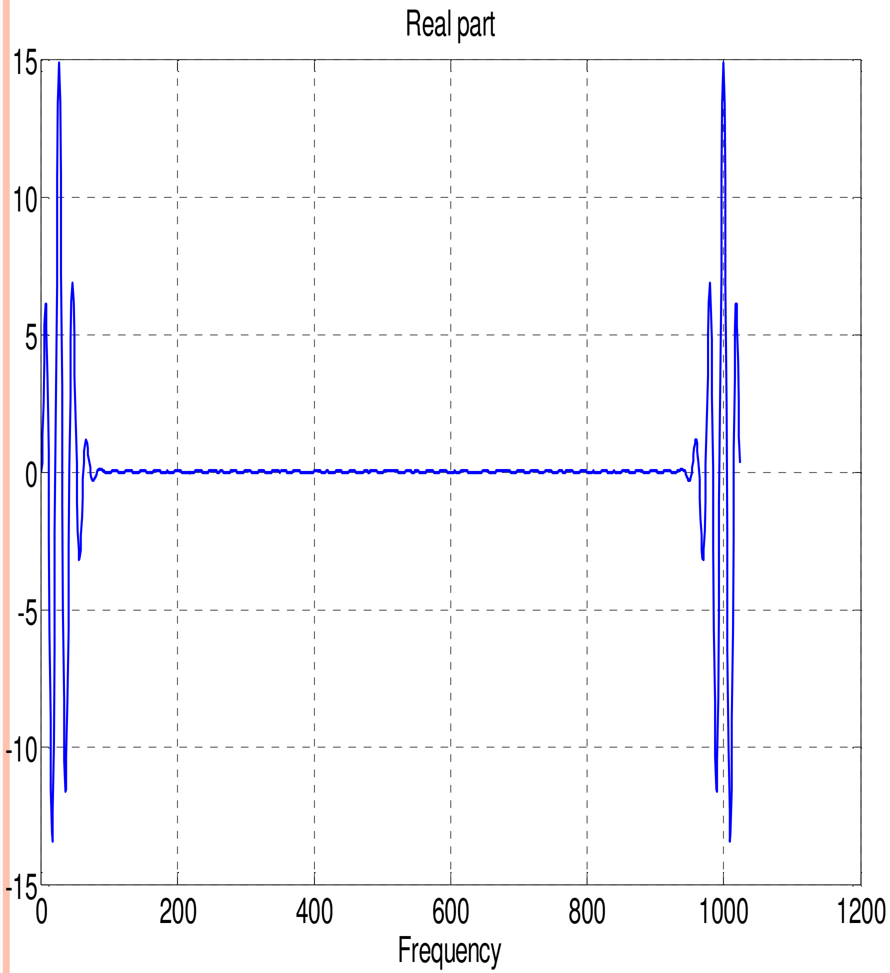
$$r = (k+0.5-k_0)*\Delta z \quad \rightarrow \quad \begin{aligned} \mathrm{r} = {} & (\mathrm{i}+0.5-\mathrm{i}_{r0})\cdot\sin(\theta)\cdot\cos(\phi)\cdot\Delta\mathrm{x} \\ & +(\mathrm{j}+0.5-\mathrm{j}_{r0})\cdot\sin(\theta)\cdot\sin(\phi)\cdot\Delta\mathrm{y} \\ & +(\mathrm{k}+0.5-\mathrm{k}_{r0})\cdot\cos(\theta)\cdot\Delta\mathrm{z} \end{aligned}$$

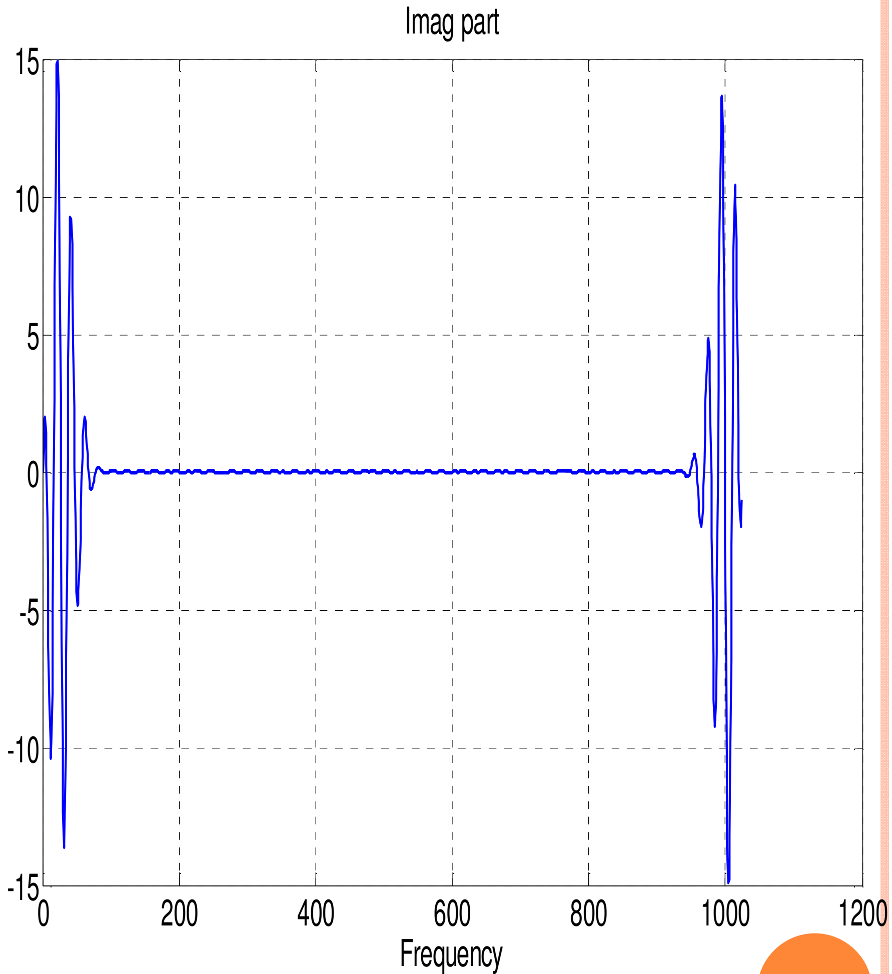
ARBITRARY PLANE WAVE AS THE INCIDENT SOURCE

- Simulation space size x=136、y=136、 z=50， Incident angle $\theta=30$ 、$\phi=60$
- time step=1024、z_ground=18



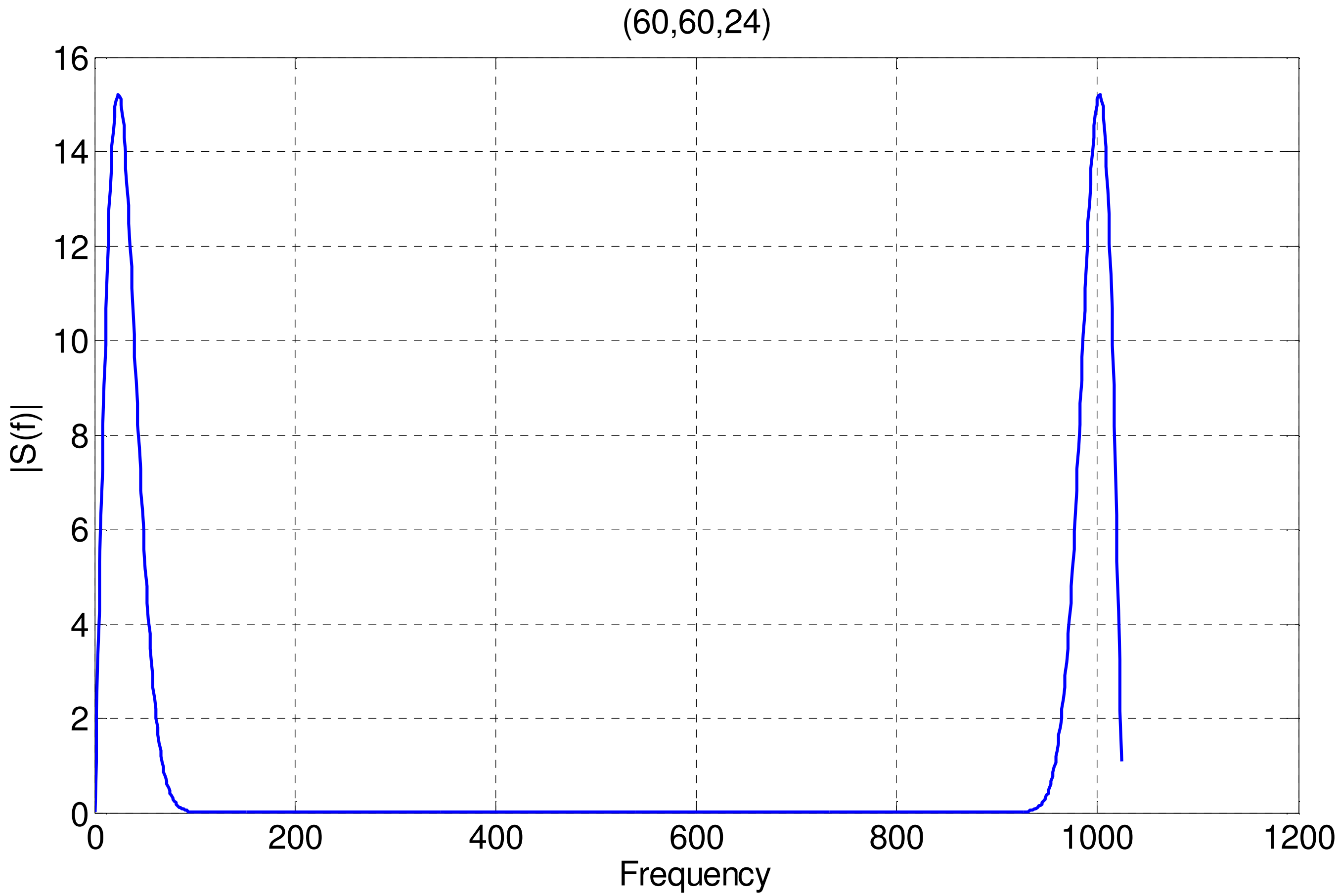
GAUSSIAN PULSE IS CONVERTED BY FFT





THE AMPLITUDE AFTER FFT CONVERSION

$$|S(f)| = \sqrt{(Real)^2 + (Imag)^2}$$



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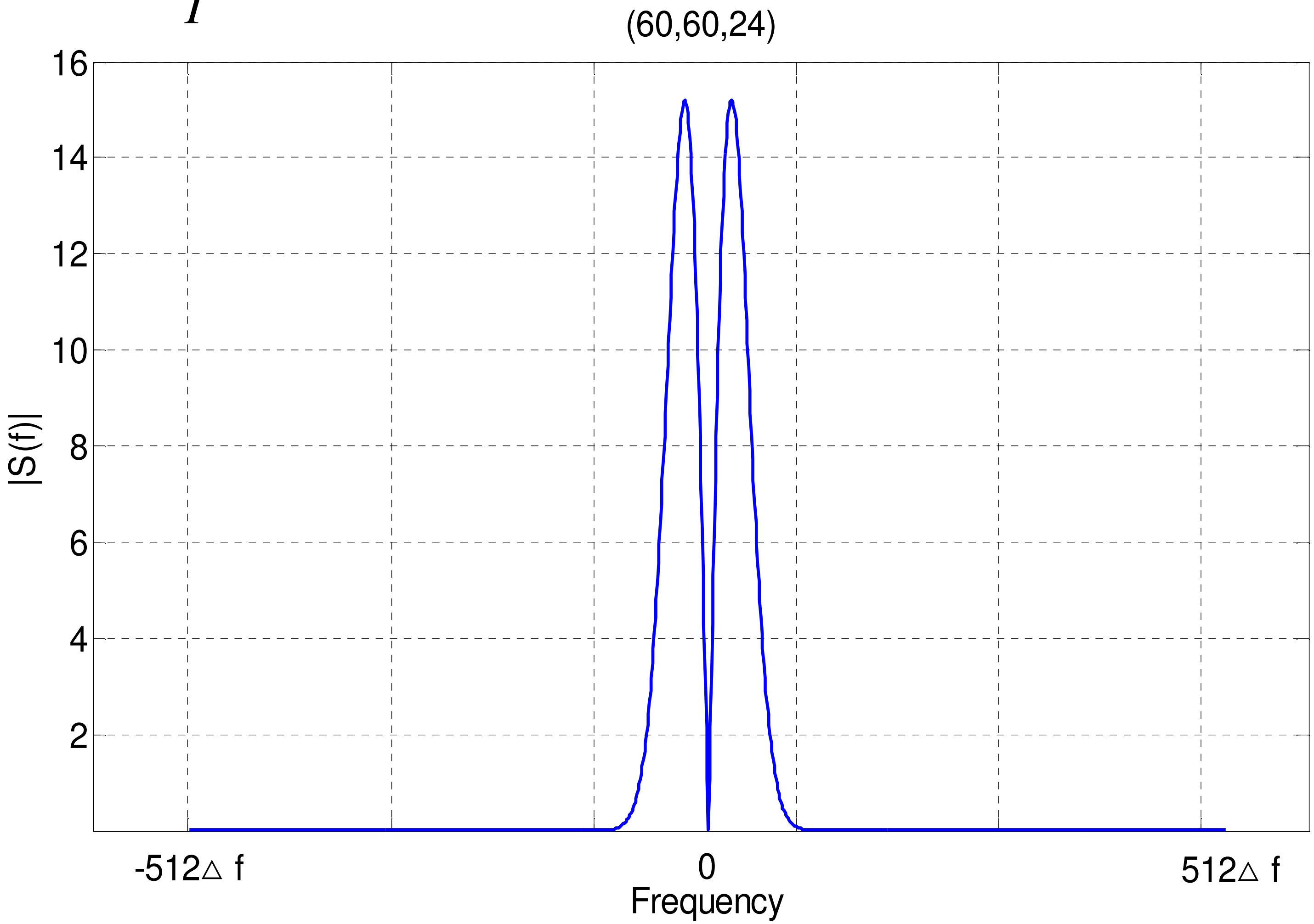
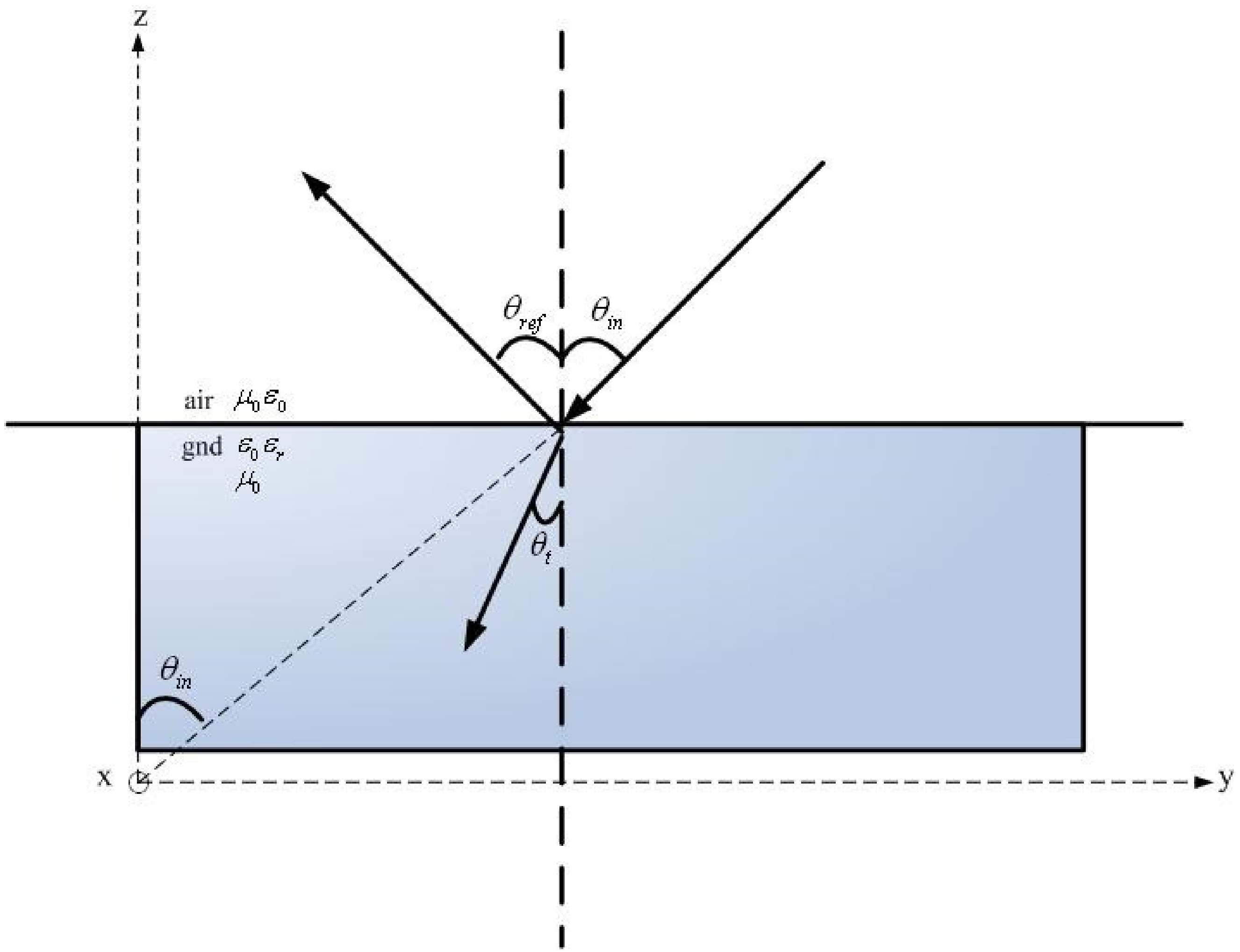
DIFFERENTIAL GAUSSIAN PULSE SPECTRUM

- $\Delta f = \dfrac{1}{T}$ 、 $T = N \times \Delta t$ 、 $\Delta t = 3.6 \times 10^{-12} s$



SCHEMATIC DIAGRAM OF OBLIQUE INCIDENT HALF-SPACE WITH LOSSLESS MEDIUM



Reflected wave and transmitted wave

- Reflected wave:

$$E_r = \Gamma \cdot E_{ro} \cdot (-2) \cdot \left(\frac{t + \frac{r_r}{c} - t_0}{T}\right) \cdot \exp\left[-\left(\frac{t + \frac{r_r}{c} - t_0}{T}\right)^2\right]$$

$$H_r = \frac{\Gamma}{\eta_0} \cdot H_{r0} \cdot (-2) \cdot \left(\frac{t + \frac{r_r}{c} - t_0}{T}\right) \cdot \exp\left[-\left(\frac{t + \frac{r_r}{c} - t_0}{T}\right)^2\right]$$

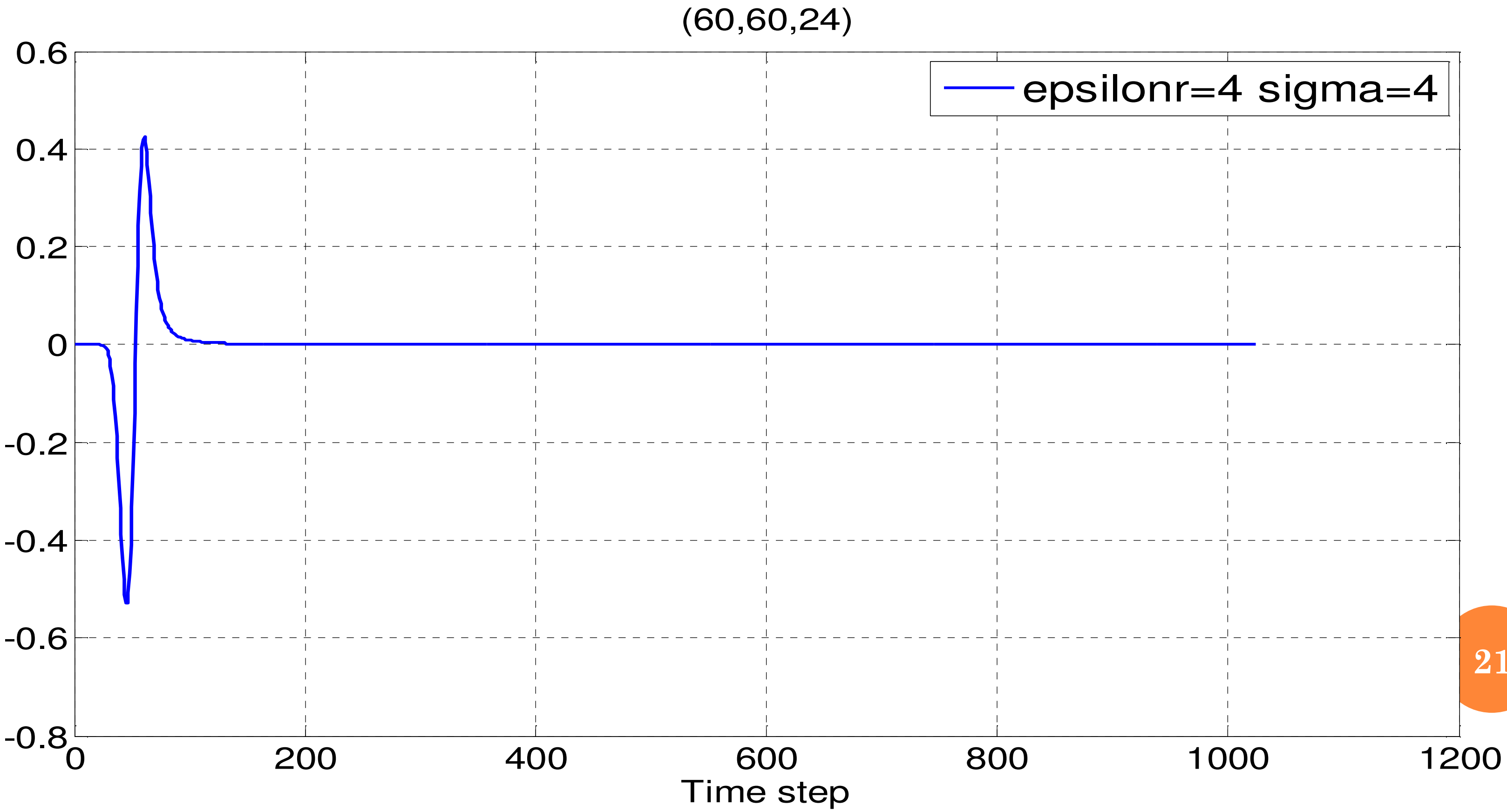
- Transmitted wave:

$$E_t = \tau \cdot E_{to} \cdot (-2) \cdot \left(\frac{t + \frac{r_t}{c} - t_0}{T}\right) \cdot \exp\left[-\left(\frac{t + \frac{r_t}{c} - t_0}{T}\right)^2\right]$$

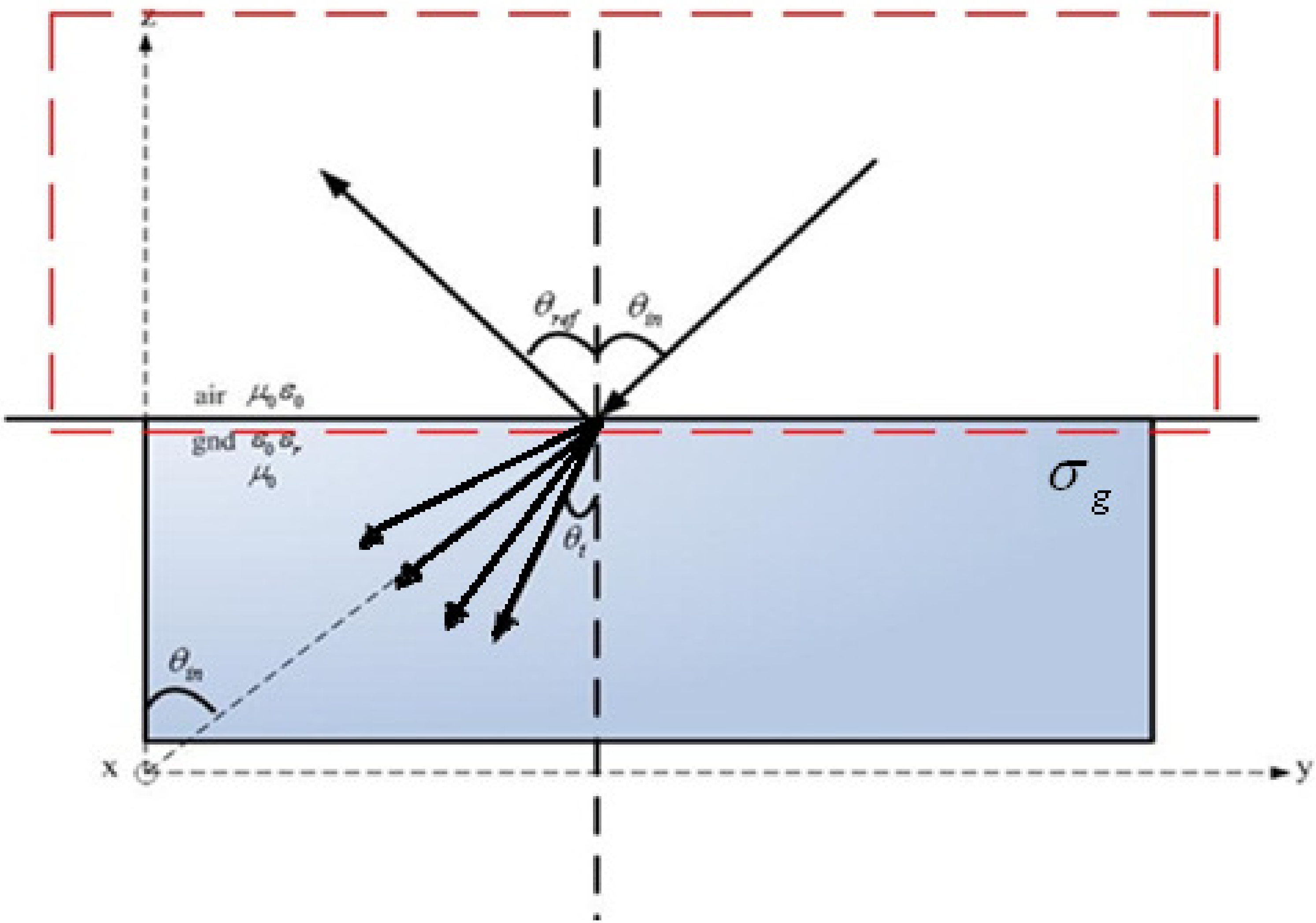
$$H_t = \frac{\tau}{\eta_0} \cdot H_{to} \cdot (-2) \cdot \left(\frac{t + \frac{r_t}{c} - t_0}{T}\right) \cdot \exp\left[-\left(\frac{t + \frac{r_t}{c} - t_0}{T}\right)^2\right]$$

THE METHOD OF REFLECT WAVES IN LOSSY MEDIUM

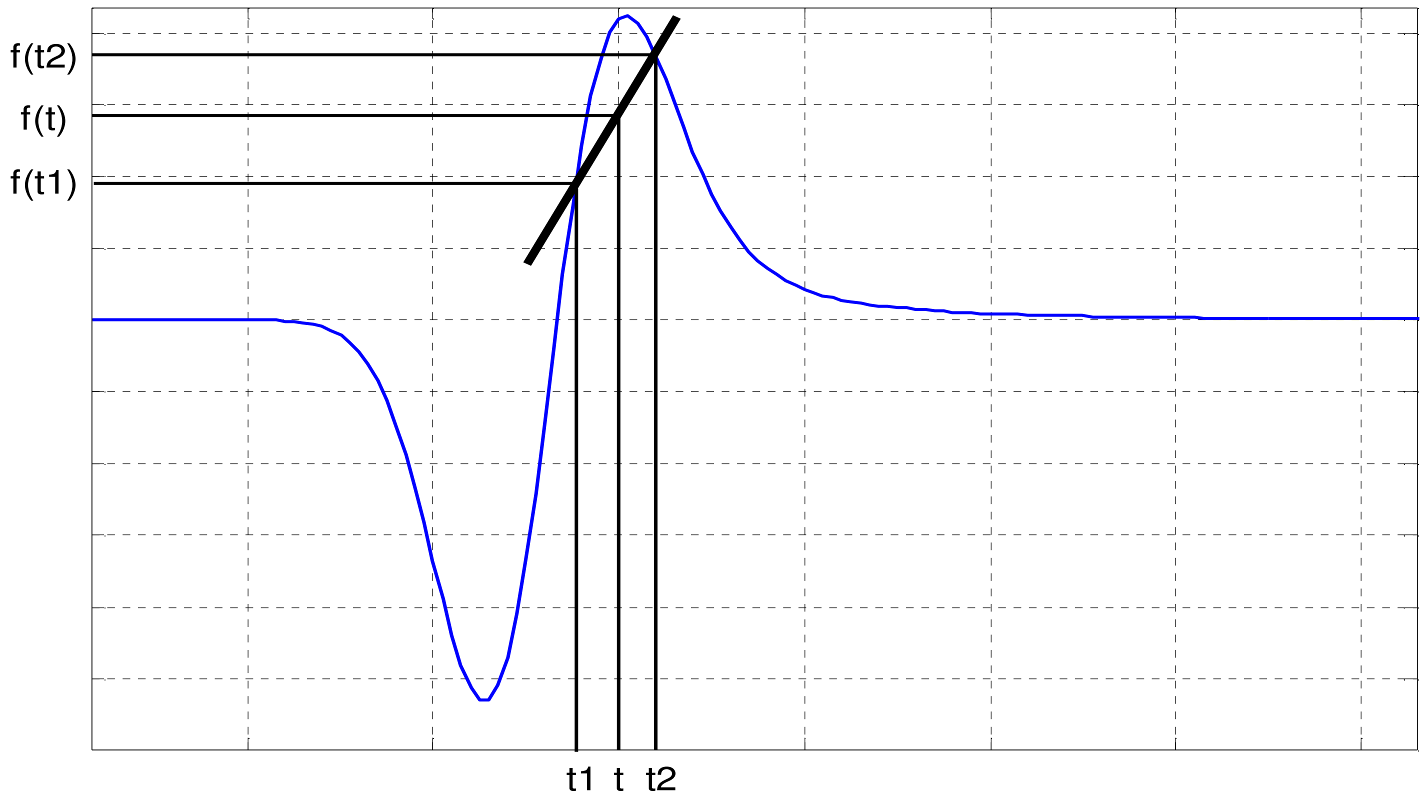
- (1) $S(f)\Gamma(f)$
- (2) $f(t) = F^{-1}\{S(f)\Gamma(f)\}$



SCHEMATIC DIAGRAM OF REFLECTION WAVE WITH LOSSY MEDIUM



Schematic diagram of Reflected wave Linear Interpolation



$$y = f(t) = \frac{f(t_2) - f(t_1)}{t_2 - t_1}(t - t_1) + f(t_1)$$

LOSSLESS MEDIUM HALF SPACE FORMULA

$$E_1 = E_0 \cdot (-2) \cdot \left(\frac{t + \frac{z - z_0}{c} - t_D}{T} \right) \cdot \exp\left[- \left(\frac{t + \frac{z - z_0}{c} - t_D}{T} \right)^2 \right]$$

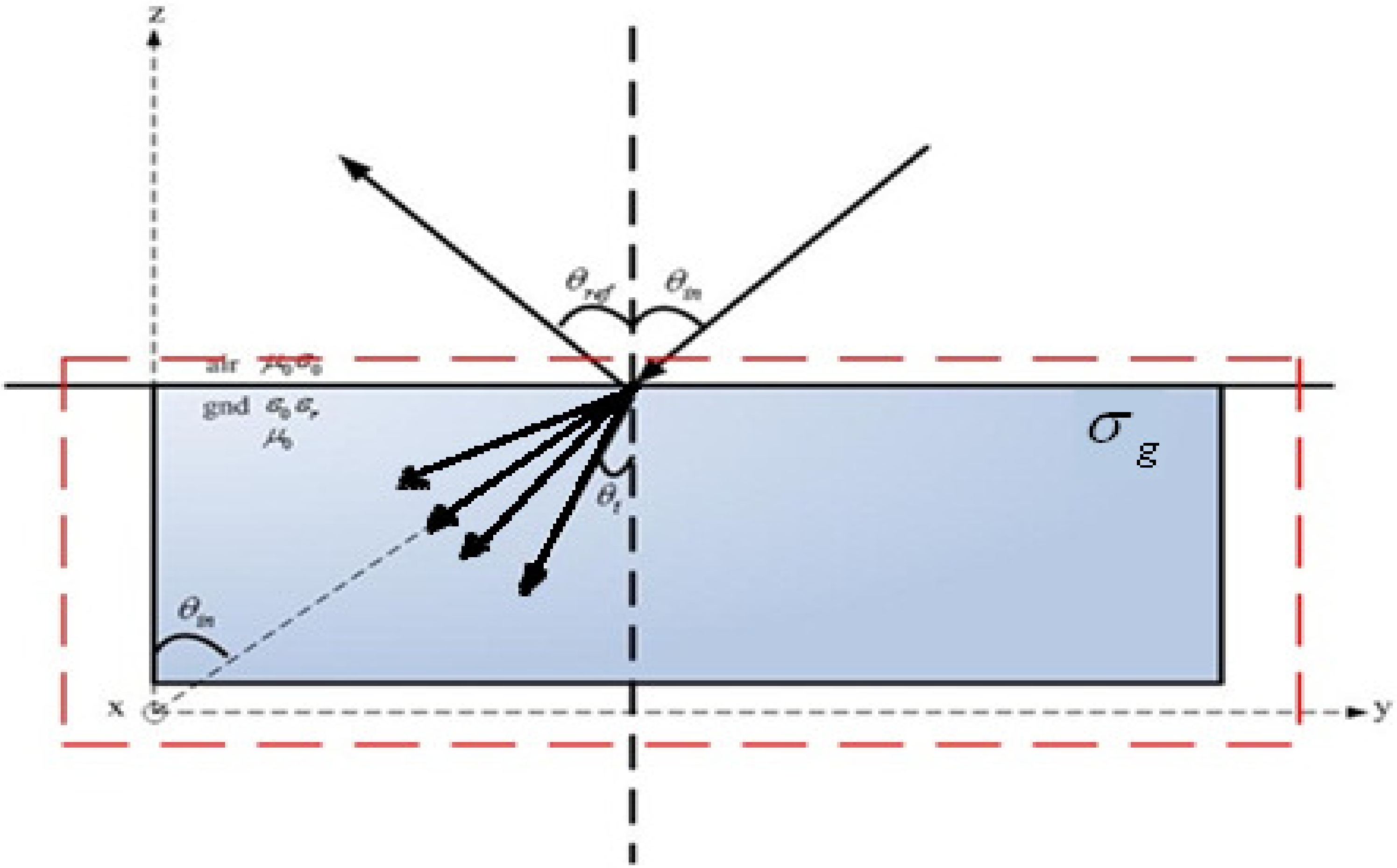
$$\boxed{+\Gamma \cdot \mathrm{E}_{r0} \cdot (-2) \cdot \left(\frac{t + \frac{(-1)\cdot|z_G - z_0|}{\cos(\theta_i)} \cdot \frac{1}{c} - \frac{\hat{k}_r \cdot (\bar{r} - \bar{r}_G)}{c} - t_D}{T} \right) \cdot \exp\left[- \left(\frac{t + \frac{(-1)\cdot|z_G - z_0|}{\cos(\theta_i)} \cdot \frac{1}{c} - \frac{\hat{k}_r \cdot (\bar{r} - \bar{r}_G)}{c} - t_D}{T} \right)^2 \right]}$$

$$for \quad z \geq z_G$$

$$E_2 = \tau \cdot E_{to} \cdot (-2) \cdot \left(\frac{t + \frac{(-1)\cdot|z_G - z_0|}{\cos(\theta_i)} \cdot \frac{1}{c} - \frac{\hat{k}_t \cdot (\bar{r} - \bar{r}_G)}{c_g} - t_0}{T} \right) \cdot \exp\left[- \left(\frac{t + \frac{(-1)\cdot|z_G - z_0|}{\cos(\theta_i)} \cdot \frac{1}{c} - \frac{\hat{k}_t \cdot (\bar{r} - \bar{r}_G)}{c_g} - t_0}{T} \right)^2 \right]$$

$$for \quad z < z_G$$

SCHEMATIC DIAGRAM OF TRANSMISSION WAVE IN THE LOSSY MEDIUM



THE METHOD OF TRANSMIT WAVES IN THE LOSSY MEDIUM

- Method: $\mathrm{S}(\mathrm{f})\tau(\mathrm{f})\,\mathrm{e}^{-j\beta_k \widehat{\theta_k}\cdot\bar{r}}\mathrm{e}^{-j\omega t_{DG}}\;e^{-\alpha_k \widehat{\theta_k}\cdot\bar{r}}$

- Adopt Method(1):

$$\mathrm{f(t)} = F^{-1}\{\mathrm{S(f)}\tau\mathrm{(f)}\,\mathrm{e}^{-j\beta_k \widehat{\theta_k}\cdot\bar{r}}\mathrm{e}^{-j\omega t_{DG}}\;e^{-\alpha_k \widehat{\theta_k}\cdot\bar{r}}\}$$

- Adopt Method(2):

DFT spectrum accumulation to the half method

DFT

$$x(t_n) = \frac{1}{N}\sum_{k=1}^{N} x(k) e^{-j(\beta_k - j\alpha_k)\widehat{\theta_k}\cdot\bar{r}} e^{j\omega_k t_n} e^{-j\omega_k t_{DG}}$$

$\omega_k = 2\pi(k-1)\Delta f$ 、 $t_n = (n-1)\Delta t$ 、 $\Delta f = \dfrac{1}{T} = \dfrac{1}{N\Delta t}$

$x(k)$ Is the transmitted coefficient in the frequency domain multiplied by the differential Gaussian pulse in the frequency domain

$t_{DG} = \dfrac{|z_G - z_0|}{\cos(\theta_i)} \cdot \dfrac{1}{c}$ Is the delay time from the free space to the contact with the dielectric layer

DFT SPECTRUM ACCUMULATION TO THE HALF METHOD

$$x(t_n) = \boxed{\frac{1}{N}}\{x(1)e^{-j\beta_1\widehat{\theta_1}\cdot\bar{r}}e^{-\alpha_1\widehat{\theta_1}\cdot\bar{r}}e^{j\omega_1 t_n}e^{-j\omega_1 t_{DG}}$$

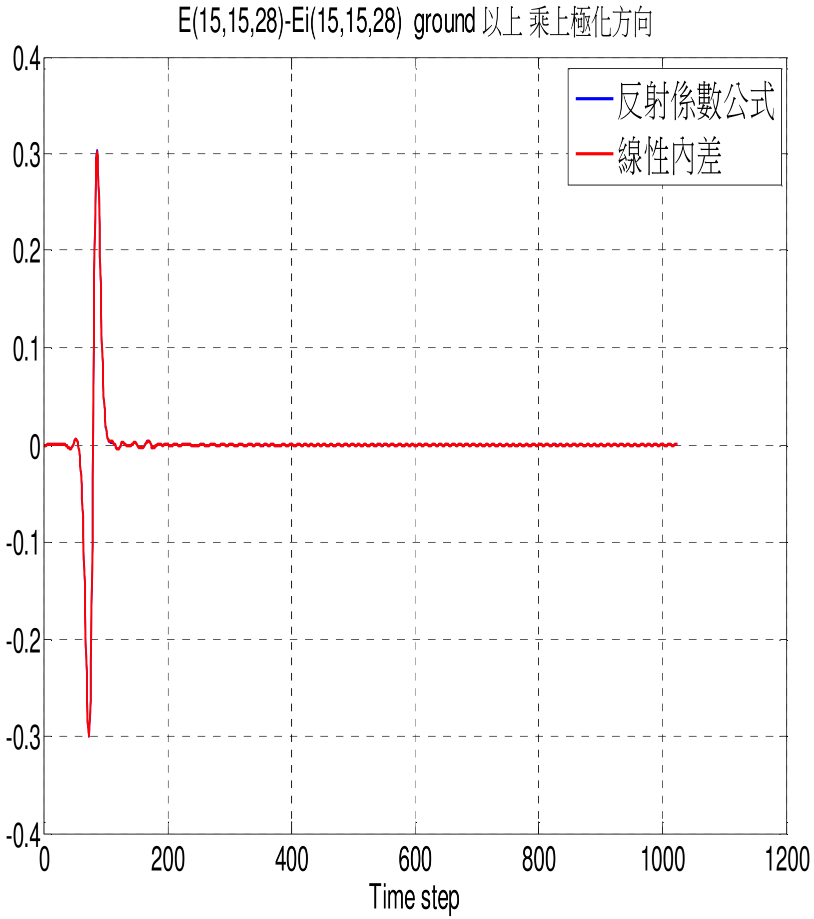
$$\boxed{+\sum_{k=2}^{N/2} 2\left|x(k)\right|\cos\left[\omega_k t_n - \beta_k\widehat{\theta_k}\cdot\bar{r} + \phi_k - \omega_k t_{DG}\right]e^{-\alpha_k\widehat{\theta_k}\cdot\bar{r}}}$$

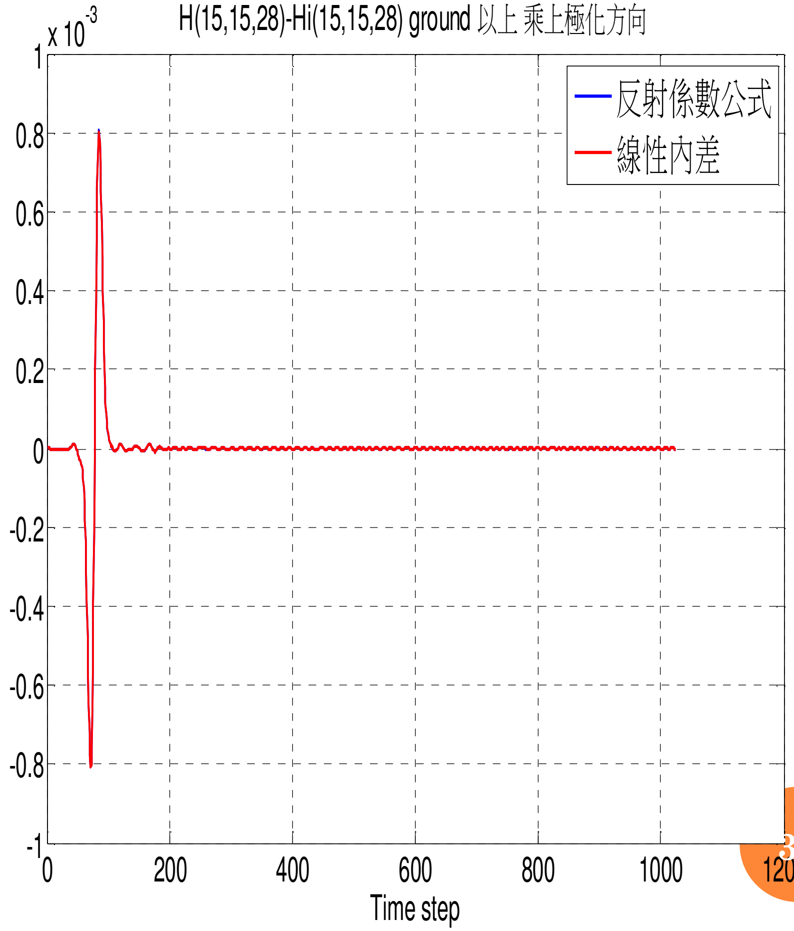
$$+x(\frac{N}{2}+1)e^{-j\beta_{-(\frac{N}{2}-1)}\widehat{\theta_{-(\frac{N}{2}-1)}}\cdot\bar{r}}e^{-\alpha_{-(\frac{N}{2}-1)}\widehat{\theta_{-(\frac{N}{2}-1)}}\cdot\bar{r}}e^{j2\pi\omega_{-(\frac{N}{2}-1)}t_n}e^{-j\omega_{-(\frac{N}{2}-1)}t_{DG}}\}$$

SIMULATION TIME COMPARISON

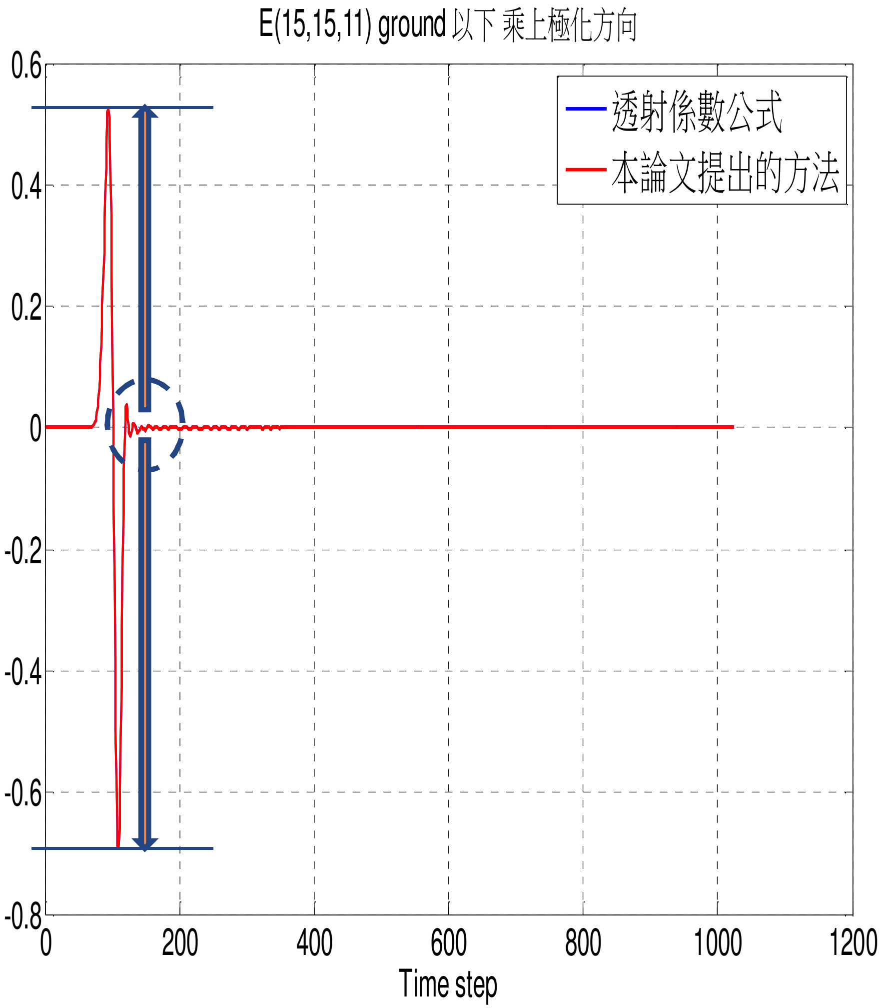
	$x=30\ \ y=30\ \ z=40$ $\varepsilon_r=4\quad\sigma_2=0.1$ The number of output files is the same	$x=30\ \ y=30\ \ z=40$ $\varepsilon_r=4\quad\sigma_2=0.1$ The number of output files is the same
Adopt method	iFFT	DFT spectrum accumulation to the half method
Calculation time	4945 seconds (約1.373 hrs)	1403 seconds (約23.3833 min)

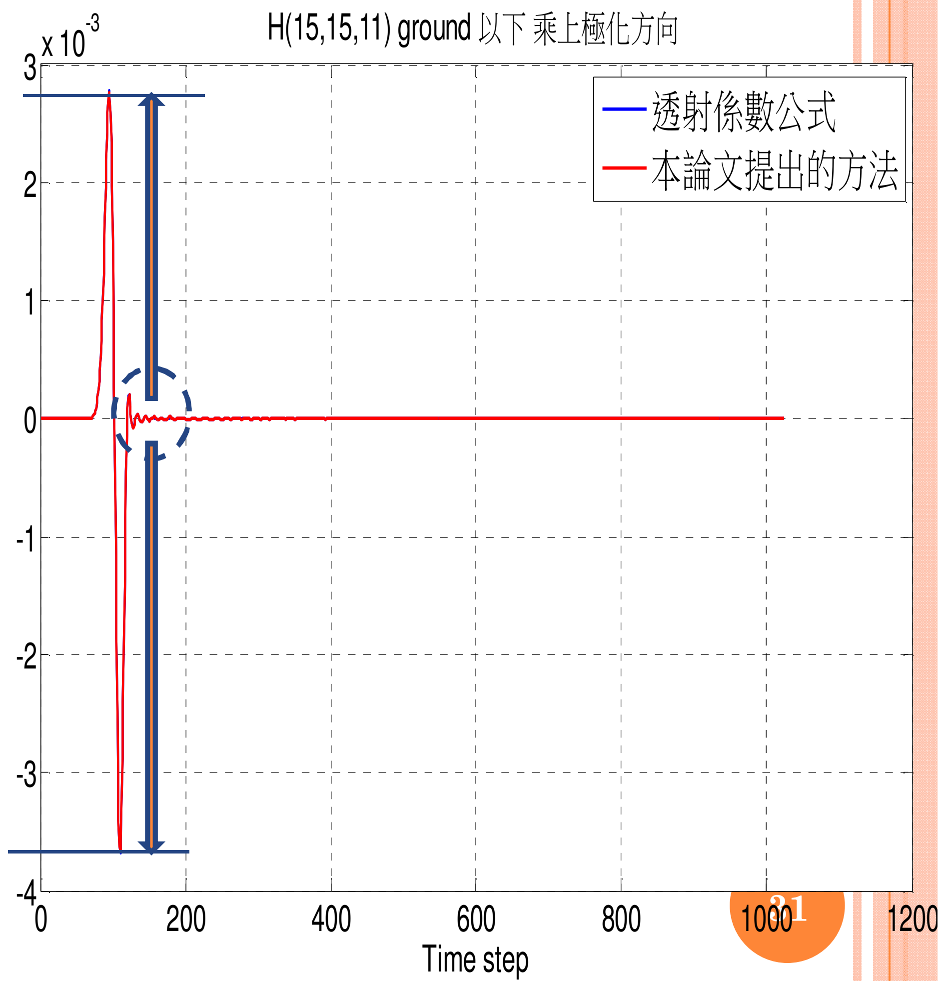
Simulation result
Normal incident electric and magnetic field reflected waves



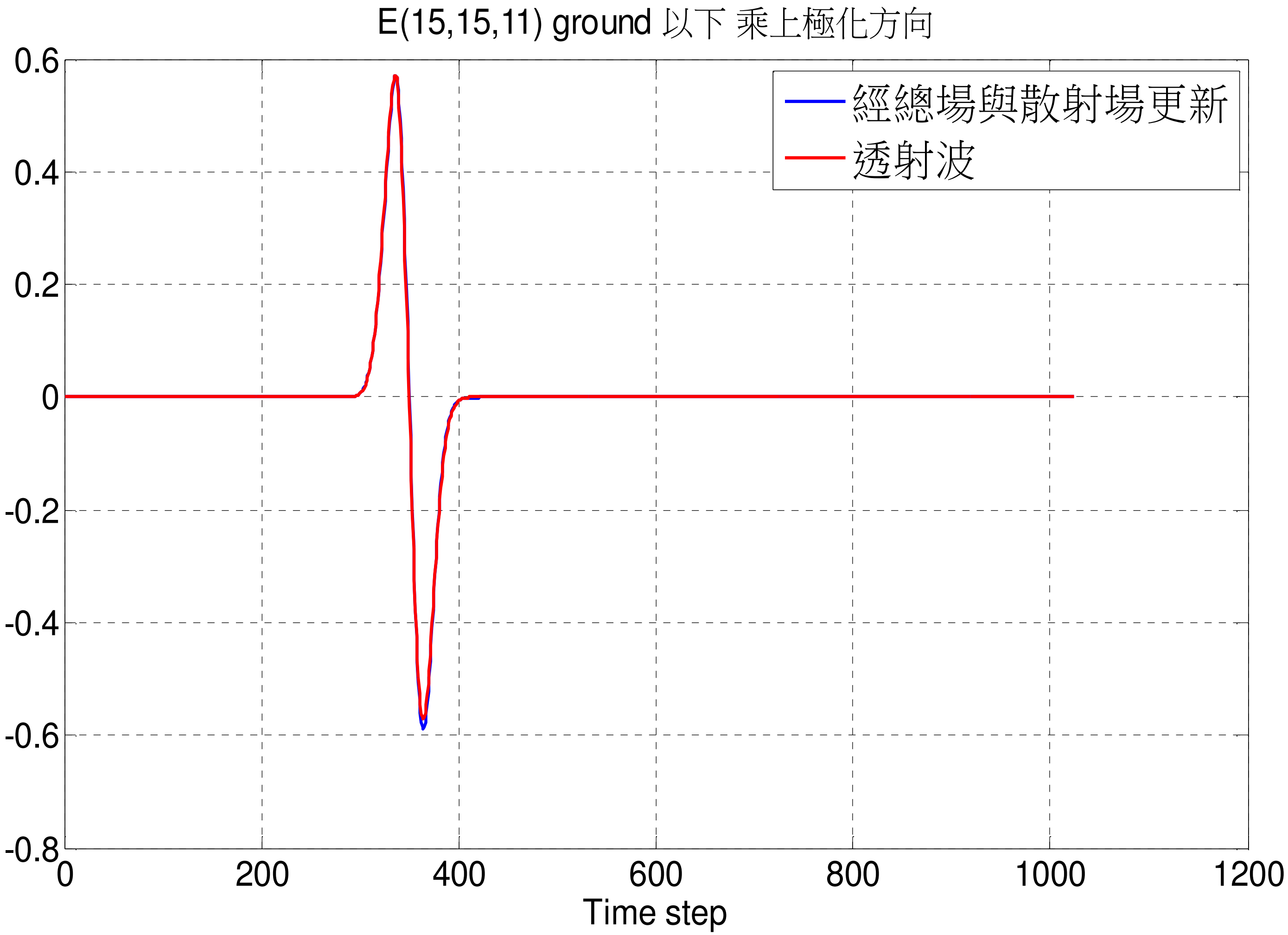


Normal incident electric and magnetic field Transmitted wave

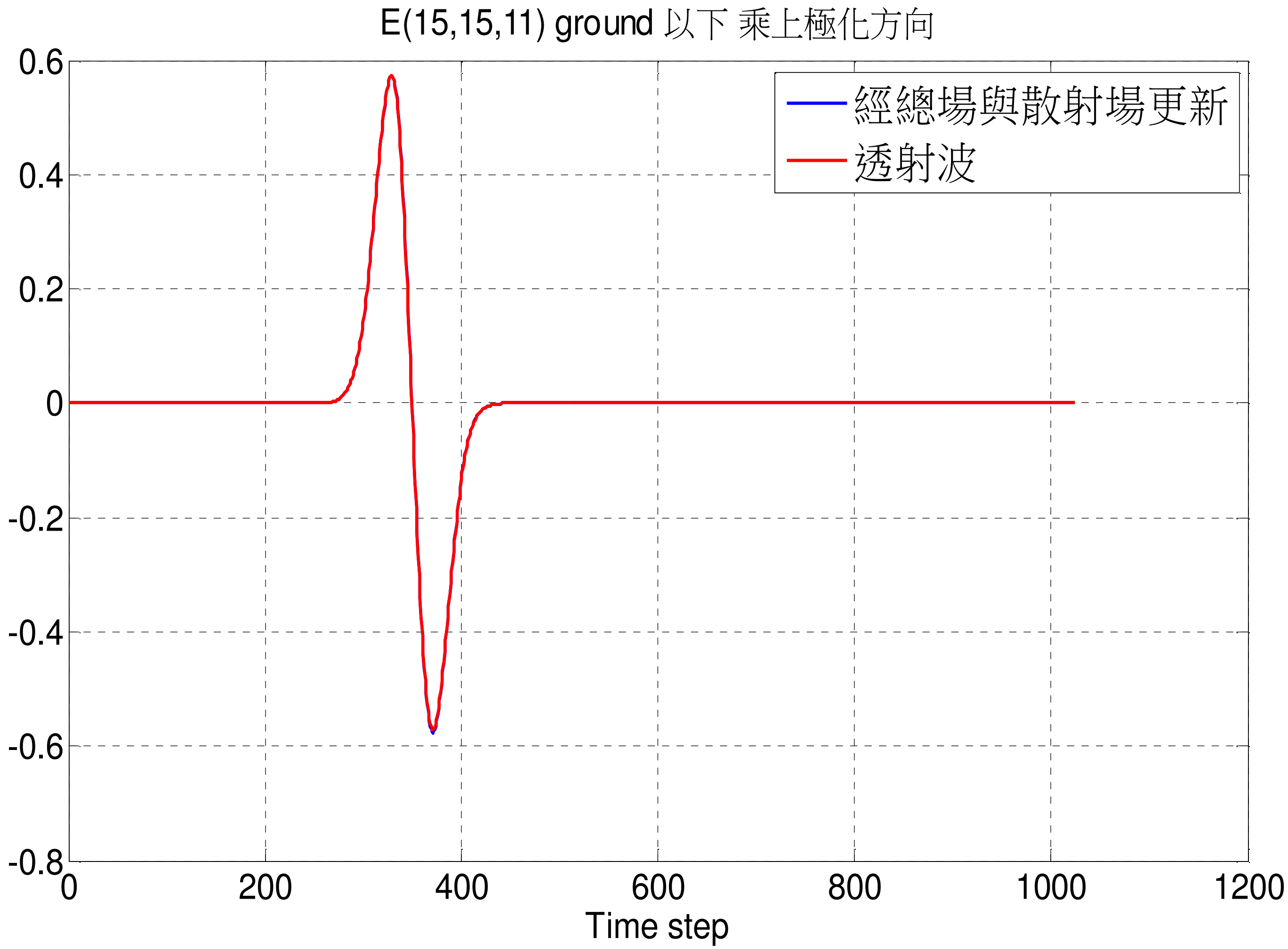




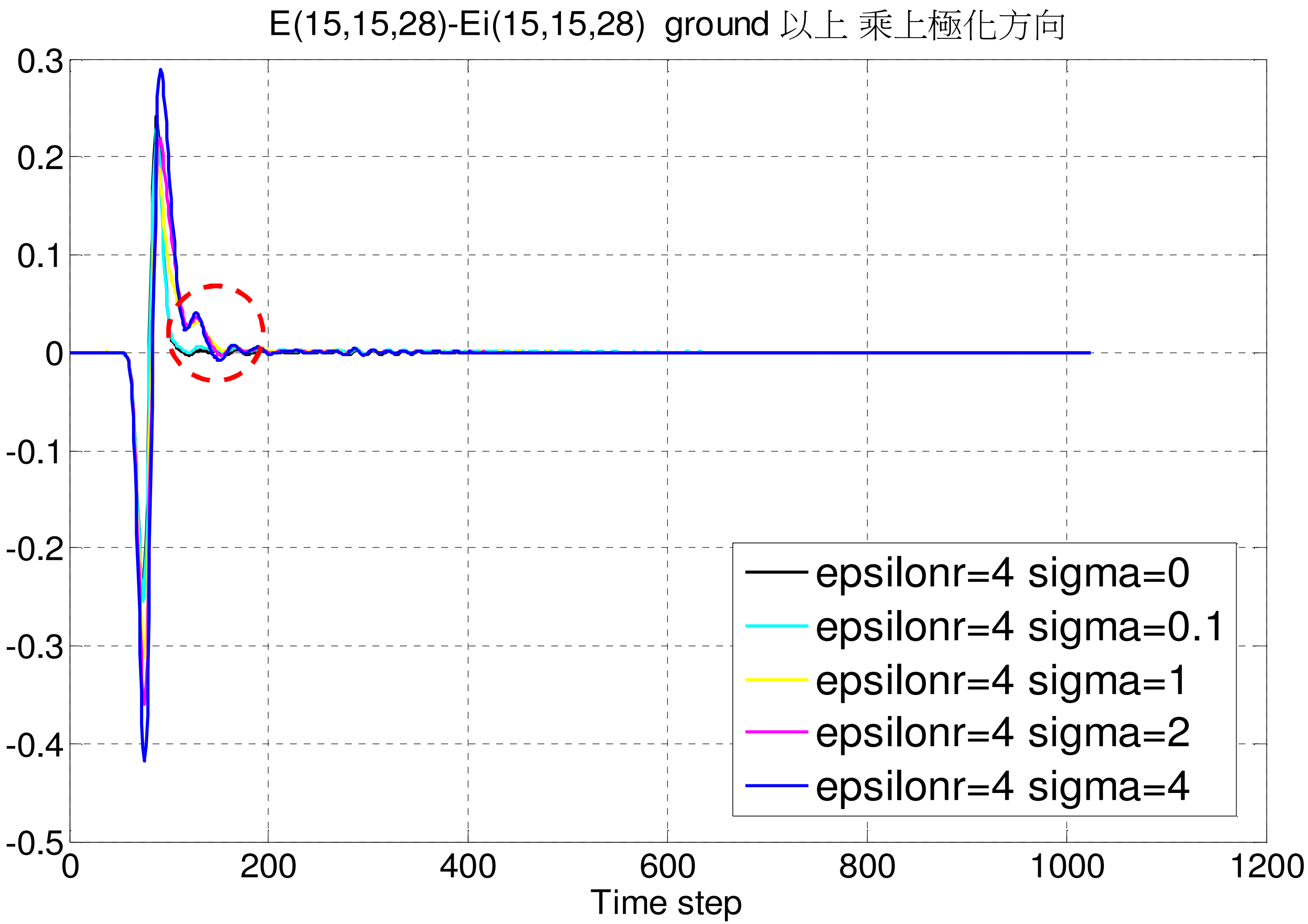
TRANSMITTED WAVE TWO TIMES THE PULSE WIDTH OF THE TOTAL FILED OF ELECTRIC FIELD



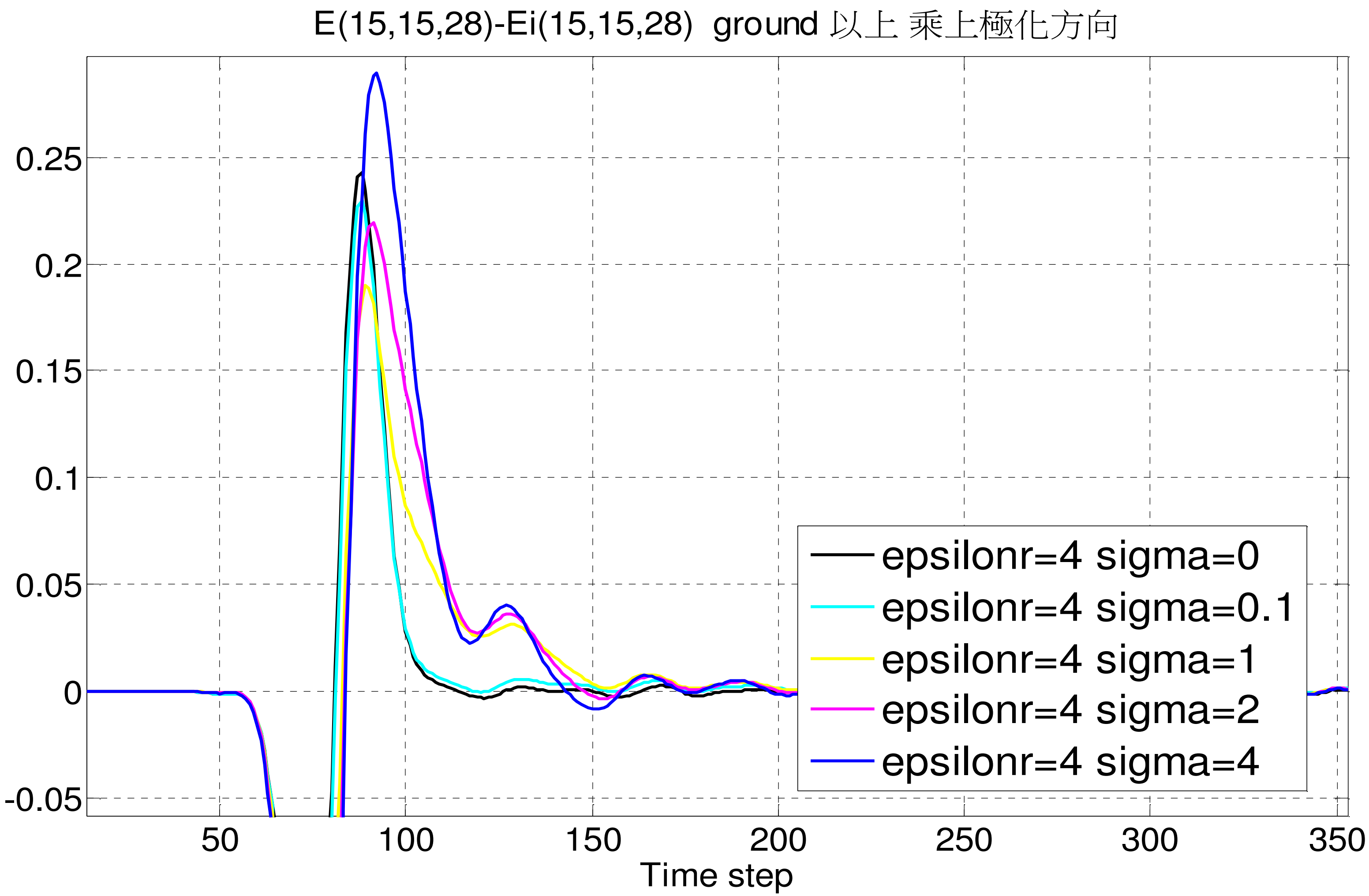
TRANSMITTED WAVE
THREE TIMES THE PULSE WIDTH OF THE TOTAL FILED OF ELECTRIC FIELD



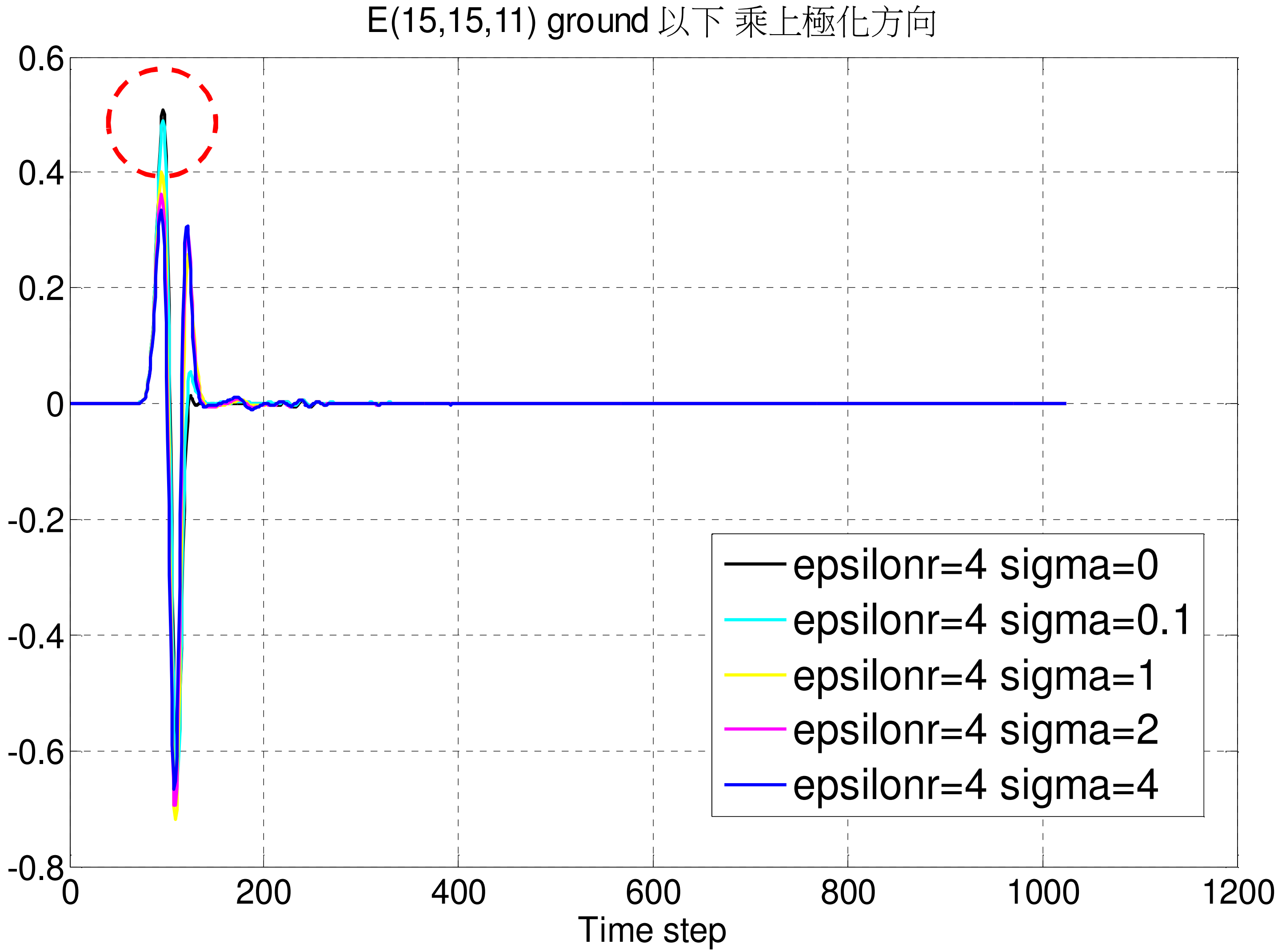
DIFFERENT CONDUCTIVITY COEFFICIENTS OF THE ELECTRIC FIELD REFLECTION WAVES



DIFFERENT CONDUCTIVITY COEFFICIENTS OF THE ELECTRIC FIELD REFLECTION WAVES ZOOM IN

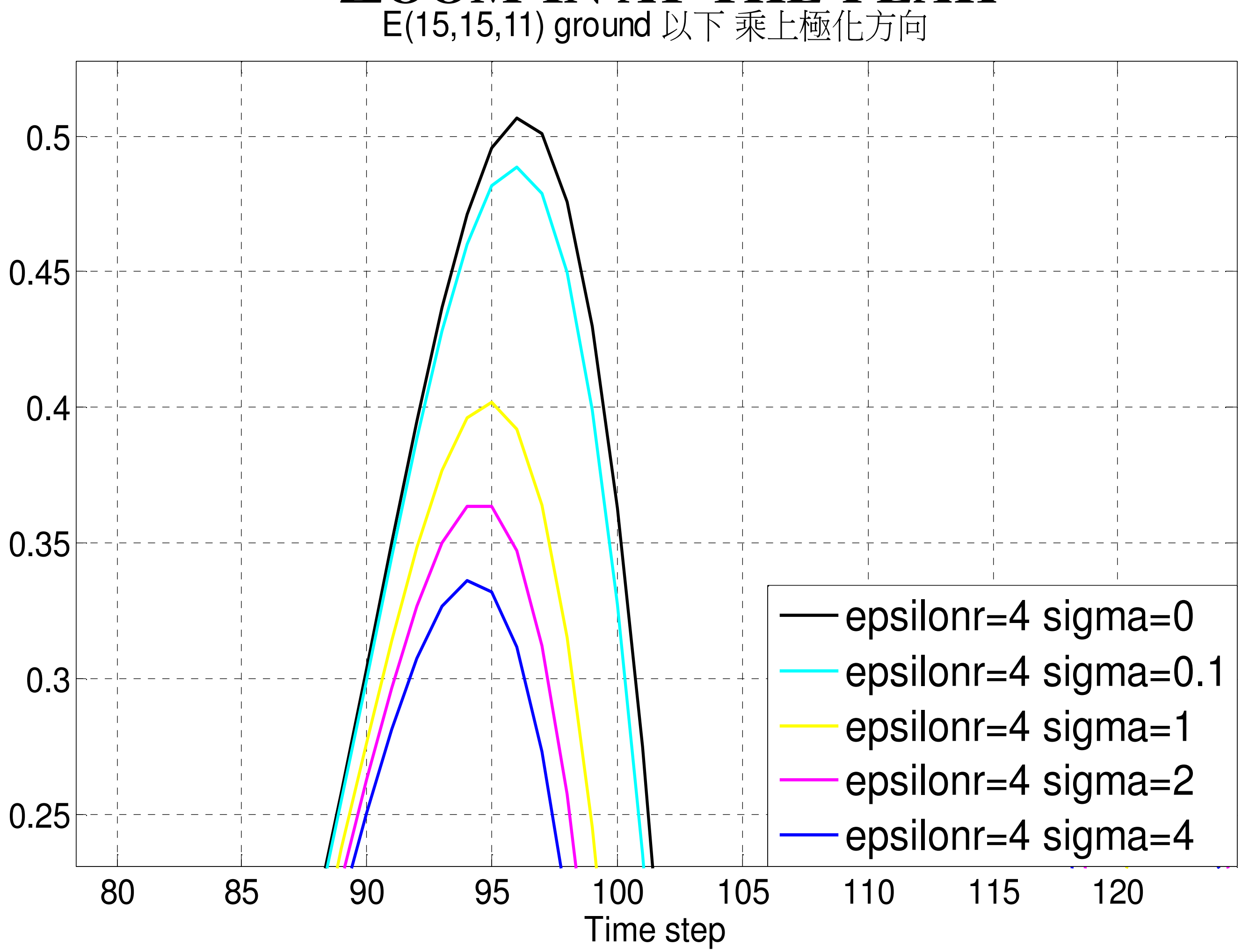


Different conductivity coefficients of the Electric field transmission waves Transmitted wave

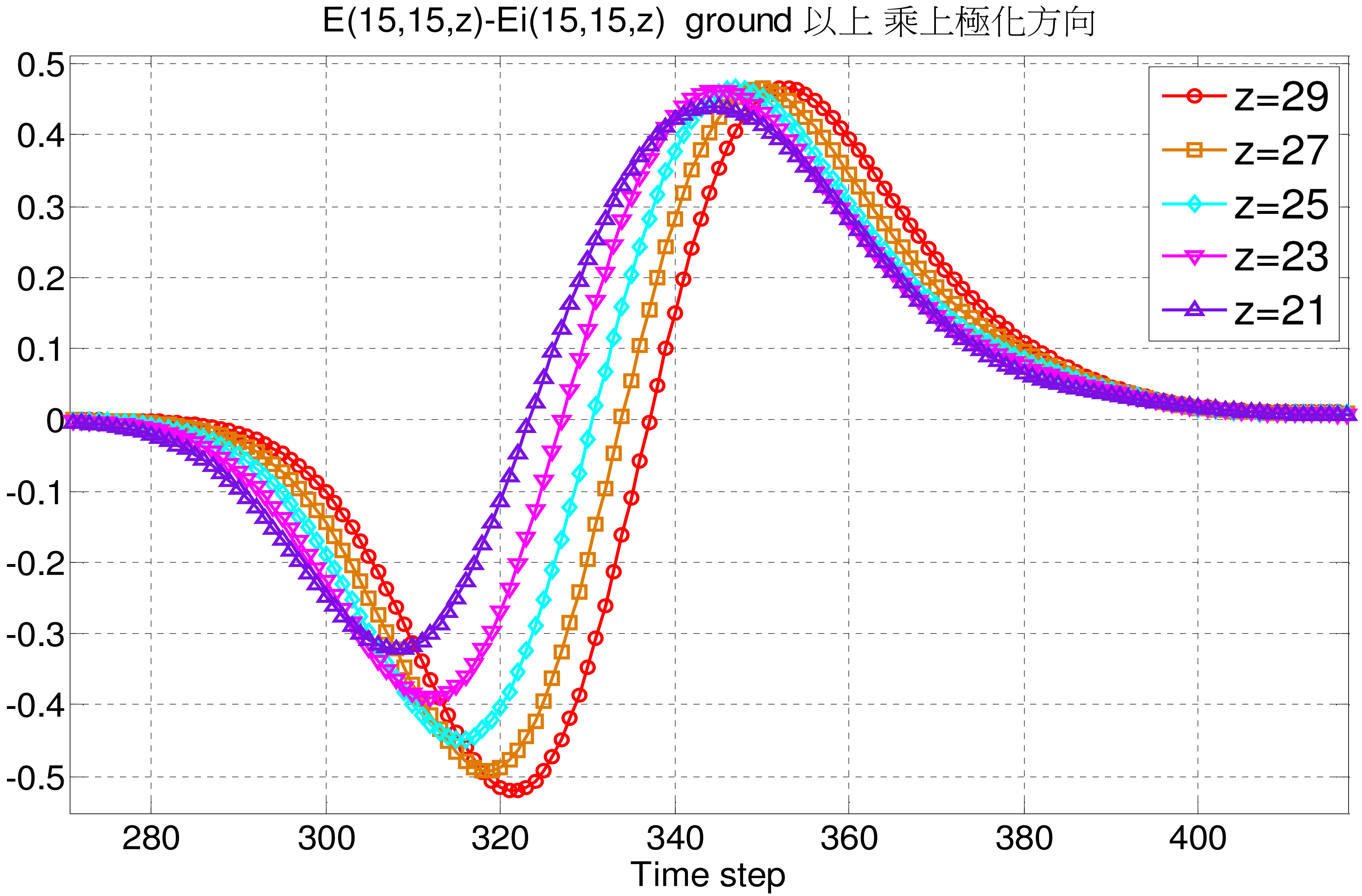


Different conductivity coefficients of the Electric field transmission waves Transmitted wave

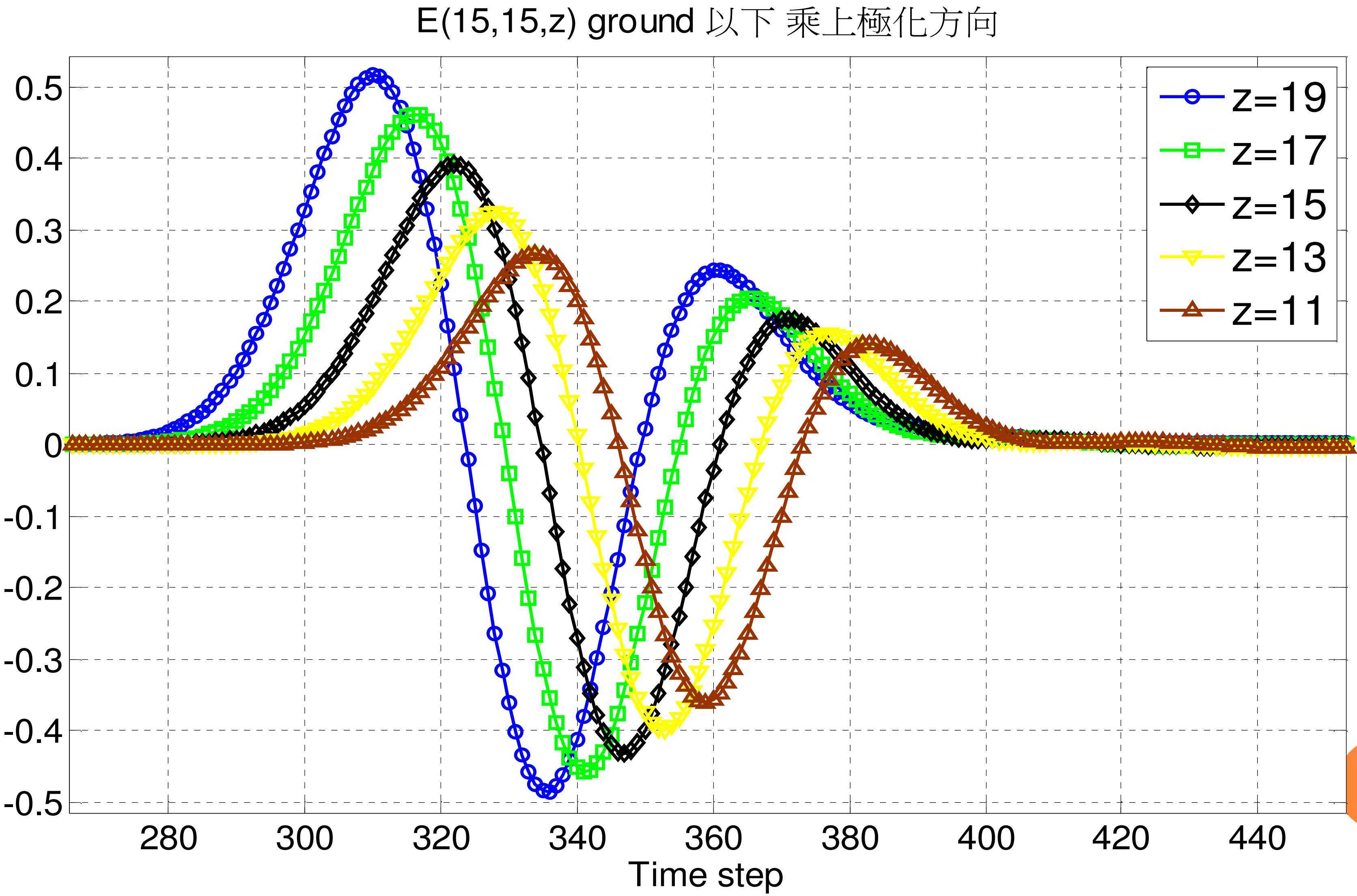
Zoom in at the peak



COMPARISON OF THE ELECTRIC FIELD REFLECTION WAVES AT DIFFERENT HEIGHTS (LOSSY MEDIUM)



Comparison of the electric field transmitted waves at different depths (Lossy Medium)



CONCLUSION

- The development of the plane wave obliquely incident on the lossy half-space can make the simulated environment closer to the actual situation. In terms of transmitted waves, some papers adopt the (SIBC) method, which is intended to replace the lossy dielectric layer. Its advantage is to reduce the solution space and generate Significant calculation saves time, but its disadvantage is that it is impossible to observe the change of the transmitted wave in the lossy medium, only the reflected wave above the lossy medium layer can be observed, however, our essay can do both.

- As far as we know, there are no papers on the development and implement of normal incident and oblique incident plane waves in the three-dimensional lossy half-space.

Thanks for your listening