



CMOS CALCULATOR

Joshua Clark

Third Year Project Report for the Final
Honour School of Computer Science

May 2015

Abstract

The aim of this project was to build a system to simulate the interaction of CMOS transistors, with the following interactive elements:

- Simplification and CMOS implementation of arbitrary logical expressions
- Simulation to visualise the change in flow of potential as the state of inputs changed
- Simulation of delay in propagation of state for large networks.

There are many tools in existence to design CMOS circuits, but most focus on the physical implementation and the exact positioning of transistors, rather than a more abstract approach, as may be required when reasoning about the layout of circuits, this tool seeks to remedy that issue. It quickly became apparent that the approach that I was using had various similarities to a compiler, which will be a focus of this document. Additionally, this report will discuss alternative approaches not used in the final implementation to produce the final CMOS output.

Contents

1	Introduction	4
1.1	Typical Use	5
1.2	Document Conventions	5
2	Requirements and Design	6
2.1	Requirements	6
2.2	Design	6
3	Definitions	7
3.1	Logical Expressions	7
3.2	CMOS Transistors	8
4	Implementation	10
4.1	Model	10
4.1.1	Logical Expressions	10
4.1.2	Transistors and Wires	11
4.1.3	Helpers	15
4.2	View and Controller	19
5	Comparison to a Compiler	24

6	Testing	26
6.1	General Testing Strategy	26
6.2	Parsing	26
6.3	Quine-McCluskey Algorithm	26
6.4	Transistor Generation and Drawing	26
7	Conclusions and Further Work	30
7.1	Conclusions	30
7.2	Further Work	31
7.2.1	Model	31
7.2.2	View	33
7.2.3	Controller	33
8	Acknowledgements	34
A	Listings	36
A.1	Model	36
A.2	View	52
A.3	Controller	57
A.4	Helper	64
B	Test Results	73

List of Algorithms

1	Converting DNF expression to transistors	12
2	Minimising the number of transistors	14
3	Determining the optimal sequence of negations	15
4	Traversing transistors to determine drivenness	16
5	Helper methods to determine drivenness	17
6	Quine-McCluskey – Logical Expression to Boolean Function . .	18
7	Quine-McCluskey – Prime Implicants	19
8	Quine-McCluskey – Essential and covering implicants	20
9	Transistor Drawing Algorithm	22
10	Optimised Transistor Drawing Algorithm	23

List of Figures

1	Initial Design	7
2	N-transistor and P-transistor, respectively	9

3	Standard nor- and nand- gate implementations	11
4	Logical Expression Class Relations	11
5	Gate and Wire Class Relations	12
6	Compiler Structure	24
7	Tri-state buffer	32
8	$\neg a$	73
9	$(a \wedge b) \vee (\neg a \wedge \neg b \wedge c)$	73
10	$a \wedge b \wedge c$ with negation applied to the result	74
11	$a \vee b \vee c$ and $(a \vee b \vee c) \wedge (a \vee b \vee c)$ with negation applied to the result	74
12	$a \oplus b \oplus c$	74

1 Introduction

The idea of logical functions is familiar to many, and even a schoolchild can grasp the concept of a box receiving true/false inputs and producing a true/false output, and how lots of these boxes in interesting combinations can yield more interesting functions. Through introducing these concepts, and producing more and more intricate diagrams, they can then see how more complicated ideas, such as addition and subtraction are implemented, and maybe how these are used in the modern computers used today. However, what is rarely discussed is what is inside these boxes: they are usually presented as the lowest level inputs and treated as the basic building blocks of a computer's processor. The layer underneath this is built up of Complementary Metal-Oxide Semiconductors, henceforth referred to as CMOS. The treatment of this document mainly concerns producing this sequence of transistors from a plain string through a series of intermediary formats, until a visual representation is produced.

Initially, this document will discuss the requirements of and use cases of a program to reason about CMOS, and we will then define key concepts, such as the fundamental propositional logic syntax and the semantic meanings usually associated with logic gates, and the definition of the two different transistors that form CMOS. After this, the algorithms to be used will be defined. On closer inspection, their composition has a behaviour not entirely dissimilar to that of a compiler, and the subsequent design of the program upon realising this was the paradigm being used will then be analysed. The program that has then been produced will finally be compared against the stated objectives, with possible further work then listed.

Projects of a much greater scope already exist in the real world, for example, the third iteration of the Simulation Program with Integrated Circuit Emphasis

(SPICE3) simulates circuits with up to 43 different types of transistor, however, requires a specifically formatted input that is rather difficult to read for new users. There are also many Very-large-scale integration programs, which deal with CMOS, however, as the name suggests, these are more appropriate for large projects, such as CPU design. As previously discussed, there are also many applications where you may use logic gates to implement larger functions, however, these rarely include details about the transistors used.

This program will attempt to combine the ease of use of simple logical expressions while producing an easily understandable representation of the output needed. As such, it will not concern itself with the precise layout of wires between the different inputs.

1.1 Typical Use

On starting the application, the user will typically enter a well-formed logical expression, as described using the grammar above, with a non-trivial canonical representation (i.e. $expr \neq \top, \perp$). This will then be parsed and internally converted to a canonical form (DNF) which will then be processed to produce a series of transistors to implement the desired logical expression. This will then be rendered, with gates highlighted red to indicate drivenness, and a list of variables provided. The user can then change the assignment of these variables by toggling a checkbox, and this change will then be reflected in the rendered set of gates.

1.2 Document Conventions

By convention, when class diagrams are used, ovals will represent traits/abstract classes, with rectangles representing concrete implementations. Objects are indicated using bold lines in the diagram, and are similar in behaviour to statically declared objects in many languages.

Whenever **this font face** is used, the text is referring to either some implementation level object or class, or the grammar used by the parser as opposed to the standard blackboard mathematical symbols, or text entered into the application or used for testing.

2 Requirements and Design

2.1 Requirements

The program would be expected to satisfy the following minimal requirements in order for it to be considered successful:

Accuracy any representation of CMOS produced must correctly implement the logical expression that has been entered, and the visual output of the expression should match the model that has been stored.

Representation the produced visualisation should be intuitive to read: each element should be presented separately from the others, and clearly labelled. Any gate carrying potential should be clearly highlighted.

Portability the visual output produced by the program should be available to the user in a variety of formats.

Interactivity the program should produce sensible warnings when an expression that is not possible to parse has been entered. Once a CMOS representation has been constructed, it should be possible for the user to interact with the publicly visible inputs (the variables named in the initial expression), with updates delivered with a minimal amount of latency. Should the user wish to view the change in state due to the latency of the transistors, this should be visible, and at a variable speed.

2.2 Design

In order to fulfil the above specifications, it was obvious that a graphical interface would be required, so that the user could quickly interact with the program and determine the variable assignments intuitively, and to have their actions quickly reflected in the circuit diagram. Initially, due to the quality of the output produced by L^AT_EX diagrams, using the Tikz engine to produce the output and importing temporary files was considered, given the success of projects such as TikzEdt[11], however, as it would not be possible to assume that any system would have the necessary libraries and programs installed, it was determined that this would not be a feasible method to consider. Additionally, by using a Java-based drawing library, animations would be easier, should the project be extended to include them, as opposed to a L^AT_EX based solution, which is, by definition, designed for static documents.

Given the nature of the project, the model-view-controller paradigm allows a sensible division of labour, also allowing for extensibility and clear separation of the purpose of the model and the interface.

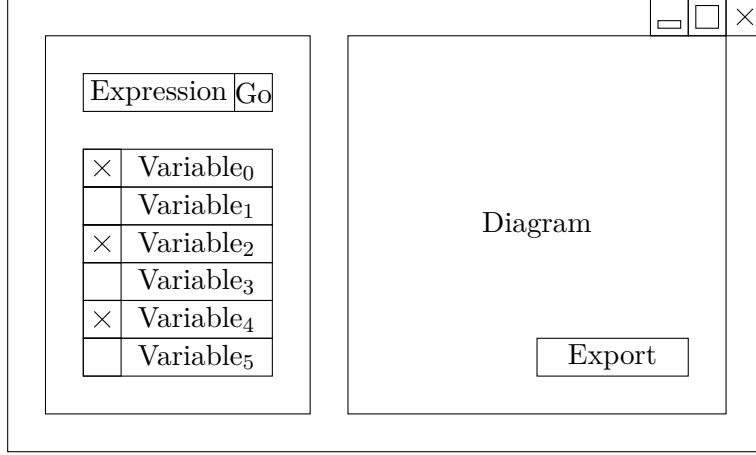


Figure 1: Initial Design

3 Definitions

3.1 Logical Expressions

A logical expression is built from a combination of constants, literals, conjunctions, disjunctions and negations[10], using a grammar described as follows:

$$e = \neg e = !e \quad (1)$$

$$| e \vee e = e \text{ or } e \quad (2)$$

$$| e \wedge e = e \text{ and } e \quad (3)$$

$$| c \quad (4)$$

$$| v \quad (5)$$

$$v \in \Sigma \quad (6)$$

$$c = \top | \perp = \mathbf{0} | \mathbf{1} \quad (7)$$

where Σ is some accepted alphabet of variables (in this program, as described by the Perl-Compatible Regex `^(?!out|and|or$)([A-Za-z0-9]+)`, that is, every alphanumeric phrase, with the exception of the single word **out**, which has been reserved for specific use within the program, as well as **and** and **or** which are keywords in the logical expression language). \top and \perp are used within this text to denote the notions true and false, respectively, and are represented in the program by the strings **1** and **0** respectively. Additionally, the order that the various production rules are listed also gives an order of precedence, with negations taking a higher precedence in the parse tree

than disjunctions and conjunctions. Each variable within the expression that is used is given an assignment by an evaluation function, commonly called $\mathcal{A} : \mathcal{V} \rightarrow \{\top, \perp\}$, with the set of variables typically denoted as \mathcal{V} . A logical expression E is then evaluated recursively based on the following rules:

- $E = \neg F$: E is evaluated as true iff F is false (corresponding to equation 1)
- $E = F \vee G$: E is evaluated as true iff F is true or G is true (corresponding to equation 2)
- $E = F \wedge G$: E is evaluated as true iff F is true and G is true (corresponding to equation 3)
- $E = v$ for some $v \in \Sigma$ is evaluated as true iff $\mathcal{A}(v) = \top$ (corresponding to equation 5)
- $E = c$ for some constant is evaluated as the constant (corresponding to equation 4)

3.2 CMOS Transistors

As the name would indicate, CMOS is made up of two distinct, complementary parts[4]: a P-transistor can only carry high potential from their source to their drain when the gate (connected to some wire representing a variable) is not driven, whereas an N-transistor can only carry low potential from their drain to their source when the gate is driven. Two power rails then provide a low potential (commonly referred to as V_{dd} , and in this document, referred to at an implementation level as **Drain**), and a high potential (commonly referred to as V_{ss} , and in this document, referred to at an implementation level as **Source**), with a third wire used as the output wire, referred to in this document as **Result**.

$$P(g, s, d) = \neg g \rightarrow (s \leftrightarrow d) \quad (8)$$

$$N(g, s, d) = g \rightarrow (s \leftrightarrow d) \quad (9)$$

Equation 8 describes a simplified version of the P-transistor: the potential is only carried (s matches d) when the input g is not powered, losing the notion that only high potential flows from the source to the drain. Equation 9 similarly describes a simplified version of the N-transistor: the potential is only carried (s matches d) when the input g is powered, losing the notion that only low potential flows from the drain to the source.

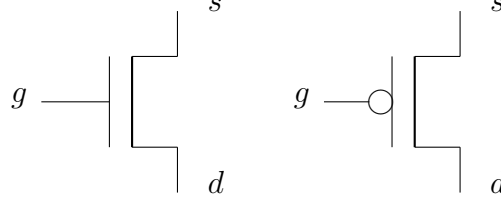


Figure 2: N-transistor and P-transistor, respectively

Therefore, when using these transistors, it is not sufficient to specify the implementation of a logical expression in a positive sense (carrying potential when the valuations of the inputs causes the expression to be evaluated as true), but rather, both the high potential must be driven to the output, and the low potential, using a complementary set of networks, made up solely of P-transistors and N-transistors. Throughout this report, and in the implementation, these are drawn with the high potential being carried from the top of a diagram to the middle by a network of P-transistors, and the low potential being carried from the bottom of a diagram to the middle by a network of N-transistors. To model conjunctions, a pair of P-transistors should be placed in series, indicating that the second transistor will only carry the potential if its gate is not driven, and the first transistor is carrying a high potential. Similarly, a disjunction is modelled by a pair of P-transistors placed in parallel: potential will be carried if at least one of the transistors' gate is not driven.

In a more general sense, for some expression e , we want a circuit $c = (p, n)$ such that $p \rightarrow e$ and $n \leftarrow e \equiv \neg n \rightarrow \neg e$, where p and n are entirely made of P- and N- transistors respectively.

Suppose that $a \wedge b$ is part of a network of P-transistors, such that p is the drain immediately above the implementation of $a \wedge b$, and q is the source immediately below the implementation. Therefore, p is acting as the source for $a \wedge b$, and q is the drain, and we therefore want a series of transistors such that $a \wedge b \rightarrow (p \leftrightarrow q)$. As described above, we will need the two transistors in series to give the necessary result, and they must be connected somehow, so we therefore introduce a new variable to act as the point between them:

$$\begin{aligned}
& a \wedge b \rightarrow (p \leftrightarrow q) \\
= & \exists x \cdot ((a \rightarrow (p \leftrightarrow x)) \wedge (b \rightarrow (x \leftrightarrow q))) \\
= & \exists x \cdot P(\neg a, p, x) \wedge P(\neg b, x, q)
\end{aligned}$$

To define a similar construction in N-transistors, we have

$$\begin{aligned}
& a \wedge b \leftarrow p \leftrightarrow q \\
\equiv & \neg(a \wedge b) \rightarrow (p \leftrightarrow q) \\
\equiv & \neg a \vee \neg b \rightarrow (p \leftrightarrow q)
\end{aligned}$$

and we therefore derive

$$\begin{aligned}
& \neg a \vee \neg b \rightarrow (p \leftrightarrow q) \\
= & (\neg a \rightarrow (p \leftrightarrow q)) \wedge (\neg b \rightarrow (p \leftrightarrow q)) \\
= & N(\neg a, p, q) \wedge N(\neg b, p, q)
\end{aligned}$$

Alternatively, suppose that $a \vee b$ is part of a network of P-transistors, with p and q behaving as before, and we therefore want a series of transistors such that $a \vee b \rightarrow (p \leftrightarrow q)$. As described above, we will need the two transistors in parallel to give the necessary result:

$$\begin{aligned}
& a \vee b \rightarrow (p \leftrightarrow q) \\
= & (a \rightarrow (p \leftrightarrow q)) \wedge (b \rightarrow (p \leftrightarrow q)) \\
= & P(\neg a, p, q) \wedge P(\neg b, p, q)
\end{aligned}$$

To define a similar construction in N-transistors, we have

$$\begin{aligned}
& a \vee b \leftarrow p \leftrightarrow q \\
\equiv & \neg(a \vee b) \rightarrow (p \leftrightarrow q) \\
\equiv & \neg a \wedge \neg b \rightarrow (p \leftrightarrow q)
\end{aligned}$$

and we therefore derive

$$\begin{aligned}
& \neg a \wedge \neg b \rightarrow (p \leftrightarrow q) \\
= & \exists x \cdot (\neg a \rightarrow (p \leftrightarrow x)) \wedge (\neg b \rightarrow (x \leftrightarrow q)) \\
= & N(\neg a, p, x) \wedge N(\neg b, p, x)
\end{aligned}$$

The networks seen in fig. 3 on the following page are standard implementations of nand- and nor-gates.

4 Implementation

4.1 Model

4.1.1 Logical Expressions

Every object that is used to represent logical expressions inherits from some common **Node** object that requires accessors to determine if the given fragment

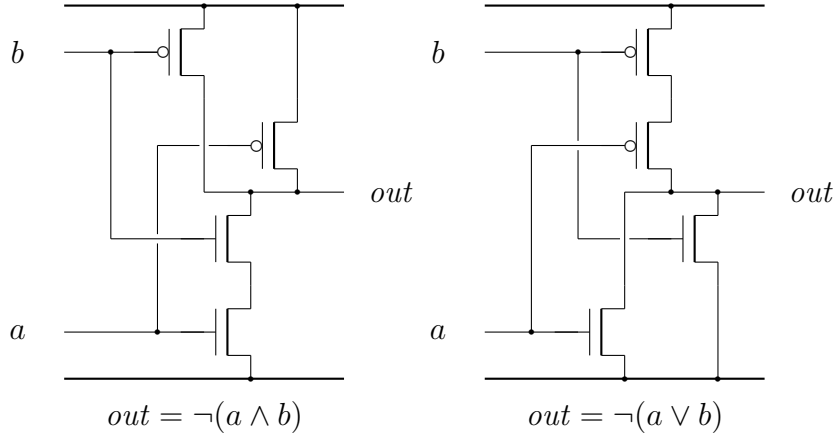


Figure 3: Standard nor- and nand- gate implementations

evaluates to true, as defined in eq. (1) on page 7 onwards. As would be expected given the recursive nature of the design of the grammar of the language of propositional logic, the parameters taken by each class that is not an atom are nodes, and these are then evaluated recursively.

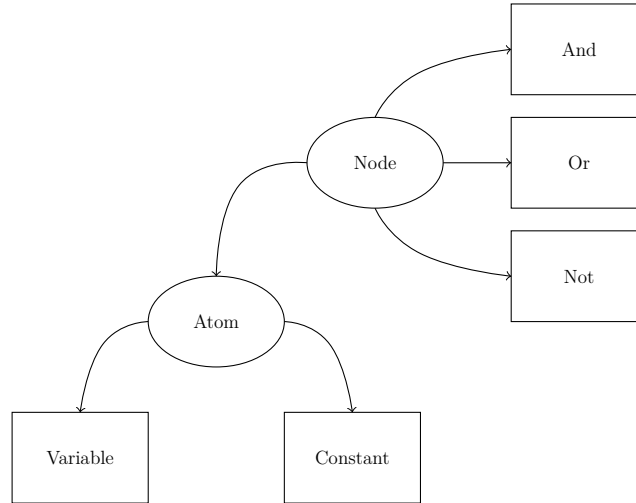


Figure 4: Logical Expression Class Relations

4.1.2 Transistors and Wires

The transistors are similarly structured: inheriting common attributes, where possible, through a central **Transistor** class, sub-typing as appropriate. There are five different types of wire, all inheriting from a central **Wire** class.

Source and **Drain** objects are responsible solely for delivering high and low potentials respectively, and a **Result** object acting as the central link. Finally, there are two wire classes responsible for carrying high and low potential downwards and upwards called **WireHigh** and **WireLow**, respectively.

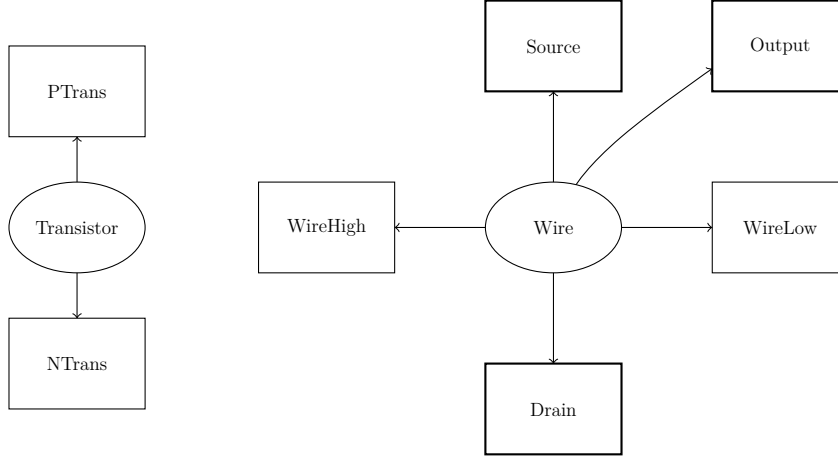


Figure 5: Gate and Wire Class Relations

Algorithm 1: Converting DNF expression to transistors

Input : Expression $E = \bigvee_{i=1}^n \left(\bigwedge_{j=1}^m P_i \right)$ in Disjunctive Normal Form

Output : None

Side Effects : Transistor network produced on **Result**

clear **Result**;

previous-gate $\leftarrow ()$;

foreach $c_i \in E = c_1 \vee c_2 \vee \dots \vee c_n$ **do**

foreach $l_i \in c_i = l_1 \wedge l_2 \wedge \dots \wedge l_m$ **do**

if previous-gate *is defined* **then**

Create a new gate and connect it to previous-gate, updating previous-gate to the new gate;

else

Create a new gate and connect it to **Result**, setting previous-gate to this gate;

Connect previous-gate to **Source/Drain** as appropriate;

The pseudo-code in algorithm 1 describes the final implementation of the gate creation algorithm, however, initially, a different approach was attempted,

which expanded the number of classes inheriting from **Node** to include implications and a representation of iff, as well as the classes for the transistors. After parsing the formula, a function then proceeded to try and convert the formula and its negation into a series of transistors with literals as inputs, joined by conjunctions and disjunctions. Dealing with the large number of special cases quickly became impractical, especially when small repeated changes had to be made, and therefore was abandoned in favour of a separation of the logical and “physical” models, where the transistors were joined by wires and received potentials as inputs.

The operation is, in the worst case, exponential in the number of variables used, $\mathcal{O}(2^n)$. The expression $\bigoplus_{i=1}^n v_i$ will have 2^{n-1} clauses, each containing n variables which must each be individually processed by the transistor-generation algorithm.

Once an initial array of transistors has been generated, the program then minimises the number of transistors produced, by traversing from **Source** to **Result**, followed by from **Drain** to **Result**, looking for transistors operating on the same literal that may be combined into one group, by removing the redundant transistor, and extending the wire below, as described in algorithm 2 on the next page. Alternatively, rather than working inwards from the outer nodes, it could work outwards from the **Result** wire, however, it is likely that only one of these passes may be performed, as applying it in both directions may result in transistors being connected that should not be, e.g. $a \oplus b \oplus c$ will result in a and $\neg a$ being connected to the **Source** twice, and c and $\neg c$ being connected to the drain twice, however, by merging both sets of gates, connections are then formed such that **Result** is always driven, either with a low or high potential.

Additionally, it may be more expedient to negate the expression that needs finding and then negate the output to determine a final drivenness result, as if several input variables need negating, this will require an extra two transistors per variable. For example, $a \wedge b$ requires 8 gates ($\neg a$ from **Source** to intermediate wire, $\neg b$ from intermediate wire to **Result**, $\neg a$ from **Drain** to **Result**, $\neg b$ from **Drain** to **Result**, and a pair of transistors to produce each of $\neg a$, and $\neg b$), whereas $\neg(\neg a \vee \neg b)$ requires 6 transistors (a from **Drain** to intermediate wire, b from intermediate wire to **Result**, a from **Source** to **Result**, b from **Source** to **Result**, and a pair of transistors to produce the final negation). It is therefore necessary to generate the transistor networks at least twice, however, it is far more useful to the program’s structure to implement **Result** as a single static object, and in half of the cases, the worst-case of three passes will not be needed. This is discussed in more detail in algorithm 3 on page 15.

In the worst case, for the drain having 2^n connections of n variables, as seen

Algorithm 2: Minimising the number of transistors

Input : Starting wire, top/bottom network indicator
Output : None
Side Effects: Minimised number of transistors in network
Updated totals to reflect this

```
if wire  $\neq$  Result then
    gates :  $\mathcal{V} \rightarrow \text{Transistor} \leftarrow \emptyset$ ;
    if checkingTopNetwork then
        | gate-list  $\leftarrow$  wire.getDrains
    else
        | gate-list  $\leftarrow$  wire.getSources
    foreach gate  $\in$  gate-list do
        if gate.input  $\in$  dom(gates) then
            if checkingTopNetwork then
                | foreach drain  $\in$  gate.drain.getDrains do
                | | gates(gate.input).drain.addDrain(drain)
            else
                | foreach source  $\in$  gate.source.getSources do
                | | gates(gate.input).source.addSource(source)
            remove gate;
            total-gates  $\leftarrow$  total-gates - 1;
        else
            | gates  $\leftarrow$  gates  $\cup \{(gate.input, gate)\}$ ;
```

in the case of repeated application of the xor function, at the i^{th} stage, 2^{i-1} transistors may be combined, effectively forming a binary tree. Let $T : \mathbb{N} \rightarrow \mathbb{R}$ be the number of steps required to process n nodes in the worst case, this give $T(2^n) = T(k) = \mathcal{O}(k) + 2T(k/2)$, which by the Master Theorem, first discussed in [2], therefore runs in $\Theta(k \log k) = \mathcal{O}(n 2^n)$. However, by application of the Birthday Problem, assuming that $\mathcal{O}(2^{\sqrt{n}})$ entries are needed for a collision in a space of 2^n elements, this makes finding duplicate gates much less likely, and substantially reduces the number of wires that must be explored.

Initially, potential was going to be modelled using the **Option[Boolean]** type, with **Some(true)** and **Some(false)** to indicate high and low potential separately, and **Nothing** to simulate a transistor or wire that is not driven. This approach has also been reworked, to further separate the differences between the abstract logical expressions with precise true/false values, and the

Algorithm 3: Determining the optimal sequence of negations

Input : Expression to convert to transistors *expr*
Output : Integer value of number of gates needed
Side Effects: Transistor network produced on *Result*

```
normal ← try-layout(expr);  
double-negate ← try-layout(¬expr) + 2;  
if normal ≤ doubleNegate then  
    try-layout(expr);  
    return normal;  
else  
    return double-negate;
```

CMOS-specific high and low potentials where the transistors/wires are driven, or a specific **Undriven** value. Functionally, this does not differ significantly in practise from the originally proposed **Option[Boolean]** implementation. The pseudo-code in algorithm 4 on the next page, and algorithm 5 on page 17 describe in further detail the steps required to determine drivenness.

This runs in constant time, within some factor of $3/2$, as each side of the network must be processed.

In the worst case, the entirety of the network of gates must be explored, as it may be that the side carrying high potential is undriven, and the “last” sequence of transistors on the side carrying low potential is the only one to be driven, and by similar reasoning as seen above, for the worst case of $\oplus_{i=1}^n v_i$, will require iterating over $\mathcal{O}(n2^{n/2})$ gates

4.1.3 Helpers

Parsing Expressions are then parsed using Scala’s Packrat parsing library [5]. During parsing, each new variable is registered with a static map stored in the **Variable** object, that takes the place of the assignment function \mathcal{A} , as discussed in section 3.1 on page 8. After parsing, but before the gate implementation (see section 4.1.2 on page 11), a modified version of the Quine-McCluskey algorithm (see section 4.1.3 on the next page) is applied to the parsed expression to minimise it and produce a canonical form.

The running time of the Packrat parser is consistently linear in the size of the input received, albeit at the cost of increased storage capacity, regardless of the specific grammar used. This library has the advantage of allowing efficient parsing by using memoization to reduce the number of times that

Algorithm 4: Traversing transistors to determine drivenness

Input : None (necessary information contained in static object)
Output : True/false value to indicate if the output is driven
Side Effects: None

```
r ← Undriven;  
foreach transistor ∈ Result sources do  
  | if transistor-Status(gate) ≠ Undriven then  
  |   | r ← transistor-Status(transistor);  
if r = Undriven then  
  | foreach transistor ∈ Result drains do  
  |   | if transistor-Status(gate) ≠ Undriven then  
  |   |   | r ← transistor-Status(transistor);  
if r = Undriven then  
  | report error  
else  
  | return r
```

a production rule needs testing. This parsing library uses a domain-specific language to assist in the definition of the grammar to be parsed to ensure that the abstract representation of the grammar matches the implementation as closely as possible.

Normal Forms After an expression is parsed, it is then converted to a normal form to allow the transistor conversion tool to process it, in this case, as a disjunction of conjunctions, by using the Quine-McCluskey Algorithm, and algorithm developed by McCluskey[7], building on work by Quine[8, 9]. The algorithm has the following sections:

1. Convert logical expression to boolean function:

$$f : \{0, 1\}^{|A|} \rightarrow \{0, 1\}$$
$$f(v_1, v_2, \dots, v_n) = \sum m(n_1, n_2, \dots, n_m) + d(n_{m+1}, \dots, n_k)$$

where m is the set of minterms, and d is the set of “don’t care” values (not needed in this implementation, as we are only considering logical expressions that are defined in the syntax given above). The minterms give a non-minimal canonical representation of a formula (e.g. for $f(A, B) = \sum m(0)$, f represents the boolean formula $\neg A \wedge \neg B$). This is described in more detail in algorithm 6 on page 18.

Algorithm 5: Helper methods to determine drivenness

```
def transistor-Status (transistor: Transistor) as
  Input   : transistor to check for drivenness
  Output  : Potential on transistor
   $r \leftarrow$  Undriven;
  if transistor is P-transistor then
    if  $\neg$ transistor.input then
      |  $r \leftarrow$  Wire-Status(transistor.source)
    else
      |  $r \leftarrow$  Undriven
  else
    | By symmetry for N-transistor
  return  $r$ ;

def Wire-Status (wire: Wire) as
  Input   : wire to check for driven-ness
  Output  : Potential on wire
   $r \leftarrow$  Undriven;
  if wire is Source then
    |  $r \leftarrow$  High
  else if wire is Drain then
    |  $r \leftarrow$  Low
  else if wire is WireHigh then
    if  $\neg$ transistor.input then
      |  $r \leftarrow$  Wire-Status(transistor.source)
    else
      |  $r \leftarrow$  Undriven
  else if wire is WireLow then
    | By symmetry for WireLow
  return  $r$ ;
```

2. Find the prime implicants of the function (P is an *implicant* for F if P is a conjunction of literals, and if $\mathcal{A} \models F$ whenever $\mathcal{A} \models P$. A *prime implicant* is an implicant that is minimal in the number of terms that it contains (i.e. no more literals can be removed without it becoming a non-implicant)). This is described in more detail in algorithm 7 on page 19.
3. Find the essential prime implicants, as well as any others that are necessary to cover the function (a prime implicant is *essential* when it

is the only prime implicant to cover some minterm). This is described in more detail in algorithm 8 on page 20.

4. Print/export the results using the list of variables to assign values, and iterating through each of the essential implicants. This will also result in the returned expression's clauses already being in alphabetical order, making later optimisations based on connections of gates easier.

Algorithm 6: Quine-McCluskey – Logical Expression to Boolean Function

Input : Logical Expression
Output : List of minterms

Get from \mathcal{V} from Expression;
Variable list \leftarrow “”;

foreach $v \leftarrow \mathcal{V}$ **do**
| Variable list $\leftarrow v +$ Variable list;

foreach $i \in \{0, 1, \dots, 2^{|\mathcal{V}|}\}$ **do**
| **foreach** $j \in \mathbb{Z}_{|\mathcal{V}|}$ **do**
| | $\mathcal{A}(v_j) = i/2^j \bmod 2$;
| **if** $\mathcal{A} \models \text{Expression}$ **then**
| | Minterms $\leftarrow \text{Minterms} \cup \{i\}$

return Minterms

As discussed in Quine and McCluskey's papers, this algorithm runs in $\mathcal{O}(3^n)$, in the worst case, ($\mathcal{O}(3^n/n)$) prime implicants will be generated alone. The problem that it solves is effectively in NP, though sub-exponential approximation algorithms are available.

When processing a logical expression by hand, the usual method is to produce a Karnaugh map, first discussed in [6], and use manual techniques to quickly find minterms that cover as much as possible. For example, the Karnaugh maps for $(a \wedge b) \vee (\neg a \wedge \neg b \wedge c)$, and $a \oplus b \oplus c$ are produced in table 1 on page 21. While the first expression, and its negation, decomposes nicely into a series of clauses where each clause covers at least one other box, and usually two, $a \oplus b \oplus c$ is an example of a degenerate case without any such nice decomposition, but rather, each clause produced covers one box at a time.

Algorithm 7: Quine-McCluskey – Prime Implicants

Input : List of minterms `minterms`

Output : List of prime implicants

```
foreach  $i \in \text{minterms}$  do
  trueCount = 0;
  foreach  $j \in \mathbb{Z}_{|\mathcal{V}|}$  do
    implicant-table $_{i,v_j} \leftarrow i/2^j \bmod 2$ ;
    trueCount  $\leftarrow \text{trueCount} + i/2^j \bmod 2$ ;
  implicant-table $_{i,\text{trueCount}} \leftarrow \text{trueCount}$ ;
modified  $\leftarrow \top$ ;
while modified =  $\top$  do
  modified  $\leftarrow \perp$ ;
  foreach  $i \in \text{implicant-table}$  do
    foreach  $j \in \text{implicant-table}, j \neq i$  do
      if  $i$  and  $j$  differ in one position in implicant-table then
        implicant-table $_{\langle i,j \rangle} \leftarrow \text{implicant-table}_i \oplus \text{implicant-table}_j$ 
        where  $i \oplus j$  produces  $i \wedge j$  with any differences replaced
        with  $-$ ;
        modified  $\leftarrow \top$ ;
      if modified =  $\perp$  then
        mark  $i$  as prime;
  return marked entries in implicant-table
```

4.2 View and Controller

The view and controller are handled by two separate classes, one with an overall responsibility for drawing and updating the UI view presented to the user, written in Java and named **Gui**, which then invokes the **DrawCircuit** Scala class to initially draw and then update the displayed circuit as the inputs are changed. The following algorithm is a naïve version, designed for drawing transistors where each wire, other than the static **Source**, **Drain**, and **Result** has precisely one transistor attached. This is described in more detail in algorithm 9 on page 22.

In order to modify the above algorithm to draw transistors where one wire may have multiple transistors attached, the drawing algorithm checks for the presence of a pre-drawn transistor, and if it is found, connects the two branches and finishes drawing (as it would otherwise duplicate work). This

Algorithm 8: Quine-McCluskey – Essential and covering implicants

Input : List of prime implicants, and minterms

Output : Minimised expression in DNF

```
foreach  $m \in \text{Minterms}$  do
    foreach  $p \in \text{prime-implicants}$  do
        if  $m \models p$  then
            minterm-imp-table $_{m,p} \leftarrow \top$ ;

covered-minterms  $\leftarrow \emptyset$ ;
chosen-implicants  $\leftarrow \emptyset$ ;
foreach  $m \in \text{Minterms}$  do
    if  $|x = \{i \in \text{minterm-imp-table} : \text{minterm-imp-table}_{m,i} = \top\}| = 1$ 
    then
        minimised-minterms  $\leftarrow \text{minimised-minterms} \cup x$ ;
        foreach  $\text{minterm} \in x$  do Find all essential implicants, and add
            them to the list of required implicants
            covered-minterms  $\leftarrow \text{covered-minterms} \cup \text{minterm}$ ;

foreach  $m \in \text{Minterms} \setminus \text{covered-minterms}$  do
    Find  $c \in \text{prime-implicants}$  such that  $c$  covers  $m$  and as many other
    values as possible;
    minimised-minterms  $\leftarrow \text{minimised-minterms} \cup \{c\}$ ;
return minimised-minterms
```

does increase the work necessary before drawing, however, as the presence of drawn transistors needs to be removed, using a visitation pattern not dissimilar to the method used to determine drivenness. This is described in more detail in algorithm 10 on page 23.

Unfortunately, this algorithm does not properly take into account overlapping transistors/wires, as they are drawn from the **Result** wire outwards. Techniques to allow both the optimised drawing algorithm, and the gate-reduction algorithms to work together more effectively are discussed in section 7 on page 30.

		<i>ab</i>			
		00	01	11	10
<i>c</i>	0	0	0	1	0
	1	1	0	1	0

		<i>ab</i>			
		00	01	11	10
<i>c</i>	0	0	1	0	1
	1	1	0	1	0

		<i>ab</i>			
		00	01	11	10
<i>c</i>	0	0	0	1	0
	1	1	0	1	0

		<i>ab</i>			
		00	01	11	10
<i>c</i>	0	0	1	0	1
	1	1	0	1	0

Table 1: Karnaugh Maps

Algorithm 9: Transistor Drawing Algorithm

Input : Graph object to modify

Output : None

Side Effects: Graph updated with visual representation of transistors

Clear Graph;

Add Result to graph as long thin horizontal line going through $(0, 0)$;

$x \leftarrow 0$;

foreach *transistor* : N-transistor attached to Result **do**

$y \leftarrow 0$;

 current-transistor \leftarrow transistor;

while *current-transistor* \neq Source **do**

 Add current-transistor to Graph at (x, y) with size (δ_x, δ_y) ;

if transistor-Status(*current-transistor*) \neq Undriven **then**

 | Highlight current-transistor

 current-transistor \leftarrow transistor.next;

$y \leftarrow y + \delta_y$;

 last-transistors \leftarrow last-transistors \cup current-transistor;

$x \leftarrow x + \delta_x$;

The above, by symmetry, for the bottom transistor network;

foreach *transistor* \in last-transistors **do**

if *transistor* : N-transistor **then**

 | Connect transistor to Source;

else

 | Connect transistor to Drain;

Algorithm 10: Optimised Transistor Drawing Algorithm

Input : Graph object to modify

Output : None

Side Effects: Graph updated with visual representation of transistors

Clear Graph;

Traverse transistors to reset drawn-transistor property;

drawn-transistors $\leftarrow \emptyset$;

Add Result to graph as long thin horizontal line going through $(0, 0)$;

$x \leftarrow 0$;

foreach *transistor* : N-transistor attached to Result **do**

$y \leftarrow 0$;

 current-transistor \leftarrow transistor;

while *current-transistor* \neq Source **do**

if *current-transistor*.drawn-node = \emptyset **then**

 Add current-transistor to Graph at (x, y) with size (δ_x, δ_y) ;

 drawn-transistors \leftarrow drawn-transistors \cup {current-transistor};

if transistor-Status(*current-transistor*) \neq Undriven **then**

 | Highlight current-transistor

 current-transistor \leftarrow transistor.next;

$y \leftarrow y + \delta_y$;

else

 Connect previous-transistor to current-transistor.drawn-node;

 current-transistor \leftarrow Source;

 last-transistors \leftarrow last-transistors \cup current-transistor;

$x \leftarrow x + \delta_x$;

The above, by symmetry, for the bottom transistor network;

foreach *transistor* \in last-transistors **do**

if *transistor* : N-transistor **then**

 | Connect transistor to Source;

else

 | Connect transistor to Drain;

5 Comparison to a Compiler

The usual definition of a compiler is “a program that can read a program in one language and translate it into an equivalent program in another language” [1] usually with some sort of intermediate feedback if errors are encountered; with most compilers usually translating from a human-readable, high-level language into something low-level that can then be directly executed by a machine. The structure of a compiler can be thought of as the composition of a series of functions, each with a distinct focus, as shown in Figure 6 on page 24.

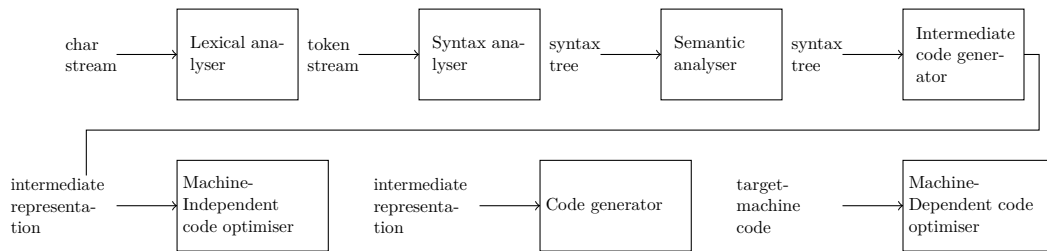


Figure 6: Compiler Structure

In a conventional compiler, the lexical analysis stage splits up the entered text into variables to be recognised within the program, or as keywords/reserved tokens within the language, usually achieved using some deterministic finite automata to recognise a regular expression, with some order of precedence established to allow keywords to be treated as such, rather than as variables. This is followed by syntax analysis, where the discrete tokens are then used to form a *syntax tree*, where the nodes are production rules or terminals in the context-free grammar, and the edges correspond to variables leading to further production rules, similar to the structure of logical expressions defined in (1) to (5) in eq. (4) on page 7. Semantic analysis varies in its depth, depending on the language, however, most compilers will usually perform some sort of check for proper declaration, as well as ensuring that variables are not used outside of the scope that they were defined in.

The lexical, syntax, and semantic analysis stages can all be thought to be covered by the parsing stages in the application, as the various tokens in the language (**and**, **or**, **!**, **(**, and **)**) are isolated and then used to form the syntax tree, while variables are added to the assignment map in the **Variable** object. Additionally, should any stage of the parsing module fail, the first of these errors is then reported back to the user to allow them to correct their input, and the assignment map emptied so that their previous state does not then affect their next input. Owing to the simplicity of the propositional

logic language, there is obviously no need for formal semantic analysis of the entered expression, however, it is useful to register each variable, with the minimal semantic check that the reserved keywords **out**, **and**, and **or** are not used in the entered expression.

Once no more work can occur on the syntax tree, the compiler then starts to focus on generating code, directed by the structure of the syntax tree, following by platform-agnostic optimisations that focus on repeated patterns within the code, or redundant operations often focusing on a small number of operations at any one time. This process is often called *peep-hole optimisation* owing to the metaphor of only being able to see a small part of a scene at any one time through a peep-hole camera.

This is replaced in the CMOS calculator by the Quine-McCluskey algorithm before transistor generation, effectively putting the optimisation before the code generation, though in a more general software engineering metaphor, this could be considered as a refactoring to remove redundancy before actually compiling the code, resulting in different byte-code, but with equivalent effects. Additionally, this has the advantage of ensuring that expressions such as $\bigwedge_{i=0}^n a$ get reduced properly to a , rather than attempting to produce n N-transistors in series for a , followed by n P-transistors in parallel, so the number of gates is strictly bound by the number of variables, rather than the size of the entered expression.

Once no more work can take place on the intermediate code, a compiler then starts producing platform-specific code, followed by any further platform-specific operations. In the CMOS calculator's case, this would then be the transistor generation algorithm, taking the canonical form generated by the Quine-McCluskey algorithm, and turning it into the CMOS implementation.

The next stage in the pipeline produces optimisations to the machine-readable code that has been produced for the relevant platform, with a specific focus on architecture specific optimisations. For Intel-based systems, with their larger instruction sets, and instructions capable of processing multiple operations on data at a time, this may focus on maximising the number of pieces of data operated upon by combining them into vectors, or on ARM machines with large numbers of registers, this frequently translates into taking advantage of the large caches available to minimise the number of secondary memory lookups and therefore the number of cycles spent idling while the RAM is queried. Within the context of the CMOS transistors, this then focuses on maximising the number of transistors that may be eliminated while retaining the same original expression. Just as the optimisation stages of a compiler will focus on the structure of the program to find subtle optimisations, so too does this program produce a minimised final output by examining the wires nearest to it.

6 Testing

6.1 General Testing Strategy

The output from the logical expression classes was deliberately designed so that the output that they would produce would be compatible with the parser, thus making testing the output of any given module producing logical expressions as output much easier, as they could then be compared to the expected values, once the behaviour of the parser was verified.

Where possible the testing is designed to test error cases, simple cases (e.g. atoms in the parser, or simple expressions for the Quine-McCluskey algorithm), followed by more complicated cases, usually based on the inductive definitions seen above.

6.2 Parsing

Care must be taken, as already discussed to ensure that only valid inputs are parsed, without affecting the ability of the program to operate using predefined constants that have been set aside for specific purposes. Furthermore, the parser must ensure that statements are parsed with due care paid to the order of operations, with negations binding correctly. The contents of table 2 on the following page list the sample test data and expected outcomes.

6.3 Quine-McCluskey Algorithm

The contents of table 3 on page 28 list the sample test data and expected outcomes.

6.4 Transistor Generation and Drawing

Due to the subjective nature of a “good” layout, the diagrams in table 4 on page 29 demonstrate the result of entering the following expressions. When the number of transistors needed is formatted as $n + 2$, this is used to indicate that less transistors are needed to produce the negation of the expression and negate this, rather than to produce the original expression. For clarity, in the case of a tie, the original expression is preferred. The visual output ¹ shown is used to demonstrate the difference between the networks of transistors when the output is driven high or driven low, with arbitrary assignments used to indicate this.

¹Reproduced in appendix

Input	Expected Output
“”	None
0	False
1	True
out	None
test	Variable(test)
(None
)	None
()	None
a	Variable(a)
(a)	Variable(a)
!	None
!a	Not(Variable(a))
(!a)	Not(Variable(a))
!(a)	Not(Variable(a))
and	None
a and	None
and a	None
a and b	And(Variable(a), Variable(b))
!a and b	Not(And(Variable(a), Variable(b)))
a and !b	And(Variable(a), Not(Variable(b)))
!a and !b	Not(And(Variable(a), Not(Variable(b))))
(!a) and b	And(Not(Variable(a)), Variable(b))
a and (!b)	And(Variable(a), Not(Variable(b)))
(!a) and !b	And(Not(Variable(a)), Not(Variable(b)))
or	None
a or	None
or a	None
a or b	Or(Variable(a), Variable(b))
!a or b	Not(Or(Variable(a), Variable(b)))
a or !b	Or(Variable(a), Not(Variable(b)))
!a or !b	Not(Or(Variable(a), Not(Variable(b))))
(!a) or b	Or(Not(Variable(a)), Variable(b))
a or (!b)	Or(Variable(a), Not(Variable(b)))
(!a) or !b	Or(Not(Variable(a)), Not(Variable(b)))

Table 2: Parsing Test Data

Input	Expected Output
(!a) and a	None
(!a) or a	None
a and and a and a	Variable(a)
a or a or a or a	Variable(a)
((!a) and b) or (a and (!b))	Or(And(Variable(a), Not(Variable(b))), And(Not(Variable(a)), Variable(b)))
!((a and b) or ((!a) and (!b) and c))	Or(Or(And(Not(Variable(b)), Variable(a)), And(Not(Variable(c)), Not(Variable(a)))), And(Variable(b), Not(Variable(a))))
a and b and c	And(Variable(c), And(Variable(b), Variable(a)))
c and b and a	And(Variable(c), And(Variable(b), Variable(a)))
(a and b and c) or (a and b and c)	And(Variable(c), And(Variable(b), Variable(a)))

Table 3: Quine-McCluskey Algorithm Test Data

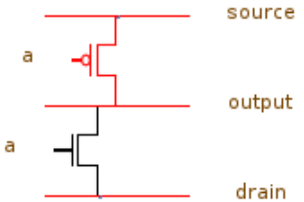
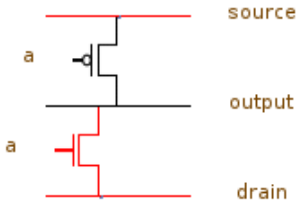
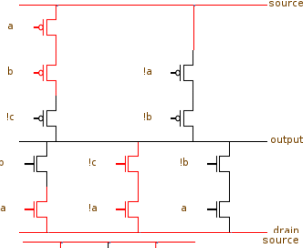
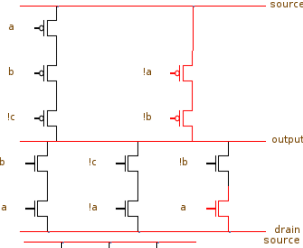
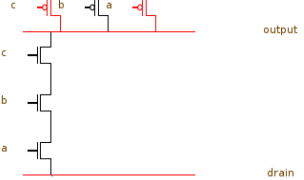
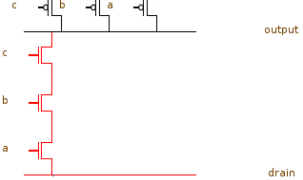
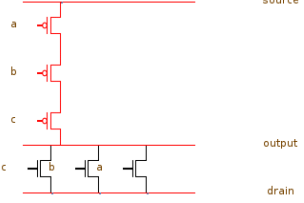
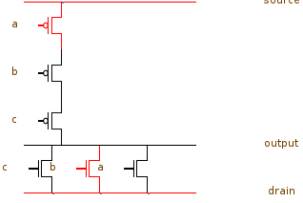
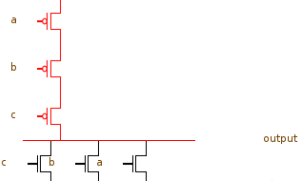

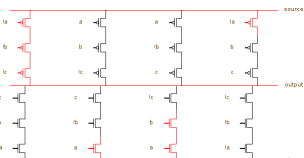
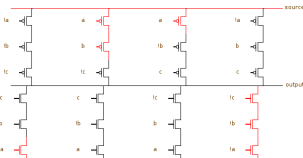
Input	Driven Output	Undriven Output	Transistors
$\neg a$			2
$(a \wedge b) \vee (\neg a \wedge \neg b \wedge c)$			17
$a \wedge b \wedge c$			$6 + 2$
$a \vee b \vee c$			$6 + 2$
$(a \vee b \vee c) \wedge (a \vee b \vee c)$			$6 + 2$
$a \oplus b \oplus c$			$6 + 2$

Table 4: Program Output

7 Conclusions and Further Work

7.1 Conclusions

The project has achieved much of the scope that it had originally set out to, however, the order of completion of different subcomponents of the program needed changing quite substantially. Initially, the order of completion of the modules was intended to be parser, then transistor-generator, followed by gate-drawer, and then optimisations (including Quine-McCluskey-related functions), however, it quickly became apparent that laying out the transistors would be far quicker and simpler if the input was in some sort of normal form first, therefore, finding an implementation of the Quine-McCluskey algorithm became a far higher priority, with drawing and a finalised transistor-generation algorithm being slightly delayed. On the other hand, as predicted, once a gate-drawing algorithm was in place, it became far easier to troubleshoot issues relating to local optimisation of gates, as the output of the optimisation sequence was far easier to visualise, rather than attempting to traverse the representation of wires and transistors stored in the currently executing program's memory.

Accuracy The program's visual output, as the drawing algorithm is structurally based on the representation within the model, the rendering is accurate. Owing to constraints on the drawing algorithm, it was necessary to sacrifice an optimal number of gates in lieu of ensuring that the drawn output remains consistent and easily interpreted: due to the way that mxGraph positions vertices and edges, any attempts to reduce overlap would have caused the edges to shift and become disconnected.

Representation The program produces a clearly labelled network of transistors, with driven transistors indicated in red, and undriven transistors in black. Unlike the diagrams seen in fig. 3 on page 11, wires linking common variables are not shown, to further enhance clarity, as in more complicated networks, a large number of overlapping wires may be needed. Additionally, when a variable needs negating, the small not-gate is not shown, nor are the transistors necessary, as this would only complicate the number of diagrams needed, as well as their layout.

Portability Beyond the bitmap based images that are produced by the program, which are then easily converted between similar formats, there is no other output produced by the program, to either allow reloading old networks, nor raster-based images to allow simple scaling without loss of resolution. While a number of workarounds exist to solve the

latter problem, through software that is already in existence, these are likely to produce images that are less than optimally encoded, as they will not necessarily take into account. Additionally, unlike 9-patch image files, stretching in one dimension will produce a skewed image, rather than sub-components resizing themselves intelligently.

Interactivity The program produces several different types of warning, corresponding to each stage of the compilation, and then the current runtime state of the circuit. The earliest warning may be invoked during parsing if a incorrectly parsed expression is seen, followed by after the completion of the first stage of the Quine-McCluskey algorithm, if the entered logical expression has a trivial normal form (\top or \perp). Finally, in the event that the circuit produced does not always drive the output with a high or low potential, this will cause a warning to be thrown as it occurs, however, this may be a transient error, as the circuit should only be driven high or low at any time. If in the future, it becomes possible to import pre-produced circuits, some of these may not be properly formed, so it is a helpful check to have in place, as well as when prototyping the transistor production algorithm to ensure that the output was driven consistently.

7.2 Further Work

7.2.1 Model

This project has deliberately focused on defining logical expressions using strictly binary values, and precisely adhering to the standard rules for laying out CMOS diagrams, without any deviations. It has been assumed that the output wire will always be driven, with an undriven state being invalid, whereas the standard tri-state buffer [4], seen in fig. 7 on the following page, where $enb \rightarrow (in \leftrightarrow out)$, and $\neg enb$ results in the output wire being undriven, is a commonly used gate.

Additionally, the structure of the transistors produced by the transistor placing algorithm in algorithm 1 on page 12 is more constrained than the transistors that may be drawn.

Decreasing the verbosity of the required input should also be a priority for future development. While operators such as \rightarrow , \leftrightarrow , \oplus do not add any additional expressiveness to the language, they are not currently parsed by the parsing module.

Furthermore, exploring limits on the size of the circuits produced to more accurately simulate the effects of resistance could be a further expansion

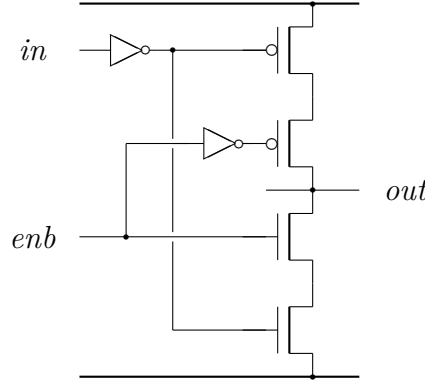


Figure 7: Tri-state buffer

for the system, which would also necessitate changes to the procedure by which the visual output is created in order to facilitate animation to properly demonstrate the effect of resistance in the time taken for the current to flow across a transistor or through wires, assuming non-negligible resistance. A naïve approach would involve the height of series of transistors being artificially limited, however, this would not take into account the length of connecting wires.

The model could also be expanded to include a more formalised notion of Tony Hoare’s drivenness analysis[3], as discussed in a draft paper, where formulae (8) and (9) in section 3.2 on page 8 are updated as follows:

$$\delta P(g, s, d) = \neg g \wedge s \wedge d \wedge \delta g \rightarrow (\delta s \leftrightarrow \delta d) \quad (10)$$

$$\delta N(g, s, d) = g \wedge \neg s \wedge \neg d \wedge \delta g \rightarrow (\delta s \leftrightarrow \delta d) \quad (11)$$

where the presence of δv is used to indicate that v is driven, either high or low, if δv also holds.

Finally, a future project may focus on converting the model and various helpers to use multi-threaded implementations to speed up the various algorithms, especially the transistor-generation algorithm, as each sequential group of transistors could be independently generated and can therefore be added in some thread-safe manner to the shared **Result**, **Source**, and/or **Drain** objects. Operations traversing are also prime candidates for parallelisation, using a controller-slave pattern.

7.2.2 View

The data produced by the view is currently only exported in the PNG format, which is easily convertible to other bitmap formats, whereas a future version of this software should be able to produce output in a wider variety of formats, and ideally, especially for reports like this, L^AT_EX-compatible output, or Dia compatible output.

A future implementation of the transistor-drawing algorithm should be reworked in order to draw the transistors from the **Source** and **Drain** objects respectively, so as to ensure that grouped transistors are handled properly, without overlapping, however, this would also require a more detailed knowledge as to the structure and bounds of the generated network of transistors.

7.2.3 Controller

Due to the implementation of the pointer-based mxGraph that I was using to produce the visual output, it was not possible for the user to interact with the graph itself directly, rather, and changes to the assignments or expression triggered a redraw where any changes were then made visible. In larger graphs, if they were particularly complicated, this may not have always been particularly obvious, therefore, a future iteration of this project, a brief highlighting effect should be considered, in order to further improve visibility. Additionally, due to the indirect way of accessing nodes in the graph, a useful addition would be to include the ability to directly interact with the graph to see the result of changing specific variables.

Adding the ability to drag-and-drop transistors onto the graph, connect them and derive an expression for a valid arrangement of transistors would be a logical extension of the current functionality, though it is likely that any output produced would, unless the number of variables was limited, not be in any normal form, due to the cost of running the Quine-McCluskey algorithm (in the worst case, $\mathcal{O}(3^n/n)$ prime implicants will be generated alone). Additionally, unless the graph was encoded with the rules required for valid CMOS structures, it would be trivial for users to produce graphs that may have undriven output, which then cannot be validly encoded as a logical expression.

8 Acknowledgements

There are a number of people, without whose assistance over the past year, this project would not have been possible.

Professor Peter Jeavons, my college tutor, both for his support over the course of my degree and in the past year.

Professor Geraint Jones, my supervisor, for his advice and support, as well as some of the standard diagrams of the N-transistors and P-transistors seen in this document.

James Wallis, and Andrew Wright, the other two Computer Science students in St Anne's, for their unwavering friendship over the last three year, and continued support.

Matthew Sjödin, Eleanor Kirk, Lucy Wright, and Stephen Heap for general support throughout the project.

The rest of my family and friends, and all others without whom this project would not have been possible.

References

- [1] Alfred V Aho. *Compilers: Principles, Techniques and Tools (for Anna University)*, 2/e. Pearson Education India, 2003. 24
- [2] Jon Louis Bentley, Dorothea Haken, and James B Saxe. A general method for solving divide-and-conquer recurrences. *ACM SIGACT News*, 12(3):36–44, 1980. 14
- [3] C. A. R. Hoare. A calculus for the derivation of C-MOS switching circuits. Two drafts; appeared as ‘A Theory for the Derivation of Combinational C-MOS Circuit Designs’, *Theoretical Computer Science* 90:235–251, 1991, doi 10.1016/0304-3975(91)90309-P, April – October 1988. 32
- [4] Geraint Jones. Digital Systems, CMOS. Lecture notes, April 2013. Compiled by report author. 8, 31
- [5] Manohar Jonnalagedda. Packrat parsing in scala. Online, January 2009. 15
- [6] M. Karnaugh. The map method for synthesis of combinational logic circuits. *American Institute of Electrical Engineers, Part I: Communication and Electronics, Transactions of the*, 72(5):593–599, Nov 1953. 18
- [7] Edward J. McCluskey. Minimization of boolean functions. *Bell System Technical Journal*, 35:28, November 1956. 16

- [8] W. V. Quine. The problem of simplifying truth functions. *The American Mathematical Monthly*, 59(8):pp. 521–531, 1952. 16
- [9] W. V. Quine. A way to simplify truth functions. *The American Mathematical Monthly*, 62(9):pp. 627–631, 1955. 16
- [10] Bernard Sufrin. Introduction to Formal Proof. Lecture notes, April 2013. Compiled by report author. 7
- [11] T Willwacher. Tikzedt – a semigraphical Tikz editor. Online. 6

A Listings

Listings

1	And.scala	36
2	Atom.scala	37
3	Constant.scala	37
4	Drain.scala	37
5	Driven.scala	37
6	High.scala	37
7	Implicant.scala	37
8	LogicalFunction.scala	39
9	Low.scala	40
10	Node.scala	40
11	Not.scala	40
12	NTrans.scala	40
13	Or.scala	40
14	Potential.scala	41
15	PTrans.scala	41
16	Result.scala	41
17	Source.scala	42
18	Transistor.scala	42
19	Undriven.scala	42
20	Variable.scala	42
21	WireHigh.scala	43
22	WireLow.scala	43
23	Wire.scala	44

24	ParserTest.scala	45
25	PITable.scala	46
26	PrimeImplicant.scala	48
27	qmm.scala	49
28	QuineMcCluskeyTest.scala	51
29	DrawCircuit.scala	52
30	Gui.java	57
31	CMOSLayout.scala	64
32	LogicalExpression.scala	70
33	Parser.scala	72

A.1 Model

Listing 1: And.scala

```

0 package model

  /**
   * Created by joshua on 17/12/14.
   */
5 case class And(lhs : Node, rhs : Node) extends Node {
  def get = lhs.get && rhs.get

  // Allows for commutativity of operators
  override def equals(that : Any) : Boolean = that match {
10   case And(lhs_, rhs_) => (lhs_ == lhs && rhs_ == rhs) || (
    lhs_ == rhs && rhs_ == lhs)
    case _ => false
  }

  override def toString() = (lhs match {

```

```

15   case Variable(x) => x;
      case _ => "(" + lhs.toString + ")";
    }) + " and " + rhs match {
      case Variable(x) => x;
      case _ => "(" + rhs.toString + ")";
20  })
  }

```

Listing 2: Atom.scala

```

0 package model

  /**
   * Created by joshua on 17/12/14.
   */
5 trait Atom {
  def get() : Boolean
  }

```

Listing 3: Constant.scala

```

0 package model

  /**
   * Created by joshua on 17/12/14.
   */
5 case class Constant(val truth : Boolean) extends Node with Atom
  {
    def get = truth
  }

```

Listing 4: Drain.scala

```

0 package model

  /**
   * Created by joshua on 17/12/14.
   */

```

```

5 object Drain extends Wire {
  override def removeGate (transistor : Transistor) : Unit = {
    sources = sources diff Array(transistor)
  }

10 def get() : Potential = Low()

  override def clear : Unit = clearSources

  override def toString() = "Drain"
15  override def resetDrawnGates () : Unit = ()
  }

```

Listing 5: Driven.scala

```

0 package model

  /**
   * Created by joshua on 17/12/14.
   */
5 trait Driven {
  }

```

Listing 6: High.scala

```

0 package model

  /**
   * Created by joshua on 17/12/14.
   */
5 case class High() extends Driven with Potential {
  override def isHigh : Boolean = true
  }

```

Listing 7: Implicant.scala

```

0 package model

```

```

/**
 * Created by joshua on 23/12/14.
 */
5 class Implicant(val minterm : Int, val tag : Int = 1, val group :
    List[Int] = Nil) {
    var prime : Boolean = true

    def cost(order : Int) : Int = {
10     order - group.size
    }

    def order() : Int = {
        return group.length
    }
15
    def canCombine(other : Implicant) : Boolean = {
        ///--- if the other one is less than this, don't bother
        comparing
        ///if (other.minterm < minterm)
        ///return false
20
        ///--- only include ones that exist in at least one function
        if ((other.tag & tag) == 0)
            return false

        ///--- if differences are not equivalent, don't bother
        comparing
        if (group != other.group)
            return false

        def bitdist(x : Int, y : Int) = qmm.bitcount(x ^ y)
30
        ///--- difference needs to be just one bit
        if (bitdist(other.minterm, minterm) != 1)
            return false

        return true
35    }

    override def equals(that : Any) = that match {
        case other : Implicant => {

```

```

40         hashCode == other.hashCode
        }
        case _ => false
    }

45    override def hashCode = terms().hashCode

    def combine(other : Implicant) : Implicant = {
        val newtag = other.tag & tag;
        val diff = math.abs(other.minterm - minterm)
        val newgroup = (group :: List(diff)).sorted
        val newmt = if (minterm > other.minterm) other.minterm else
            minterm

        return new Implicant(newmt, newtag, newgroup)
    }
55
    override def toString() = terms().mkString("<", ",", ">")

    def printTerms() = println(terms().mkString("(", ",", ")"))

60    def terms() = {
        var terms : List[Int] = List(minterm)
        for (difference <- group) {
            terms = terms :: terms.map(_ + difference)
        }
        terms
65    }

    def print() {
        printf("%d %s tag=%d %s\n", minterm, group.mkString("(", ",",
            ")"), tag, if (prime) {
70             "prime"
        } else {
            ""
        })
    }

75    def withVars(vars : List[String]) : String = {
        val weights = (0 until vars.length).map(1 << _).reverse
        val varByWeight = (weights zip vars).toMap
        //println(varByWeight)
    }

```

```

80     val expression = for (w <- weights) yield {
        if (!group.contains(w)) {
            if ((minterm & w) != 0) {
                varByWeight(w)
85            } else {
                "!(%s)".format(varByWeight(w))
            }
        } else {
            ""
        }
90    }
    expression.filter(_ != "").reduceLeft(_ + " and " + _)
95 }

```

Listing 8: LogicalFunction.scala

```

0 package model

import helper.Parser

/**
5  * Represents the current logical expression as a sum of
    products
    * (sum of minterms)
    */
object LogicalFunction {
    private[this] var minterms = List[List[Variable]]()
10    private[this] var satisfying = Set[List[Variable]]()

    def get() : Boolean = {
        var result = false;
        for (term <- minterms) {
15            var minres = true;
            for (atom <- term) {
                minres = minres && atom.get
            }
            result = result || minres
20        }
    }

```

```

    result
}

def quineMcCluskey(expr : Node) : Option[Node] = {
25    try {
        Parser.variableParser(convertToMinTerms(expr))
    } catch {
        case _ => return None
    }
30 }

def convertToMinTerms(expr : Node) : String = {
    var numericalValues = List[Int]()
    val identMap = Variable.getMap - "out"
35    val size = 1 << identMap.size
    var variables = ""
    for (term <- identMap) {
        variables = term._1 + variables
    }
40    // iterate over every combination of values (2^n time)
    for (i <- 0 until size) {
        var j = i
        var terms = List[Variable]()
        for (term <- identMap) {
45            if (j % 2 == 1) {
                terms = Variable(term._1) +: terms
                Variable.setValue(term._1, true)
            } else {
                Variable.setValue(term._1, false)
50            }
            j = j / 2
        }
        if (expr.get) {
            satisfying = satisfying + terms
55            numericalValues = i +: numericalValues
        }
    }
    return qmm.method(numericalValues, Nil, qmm.letters(
        variables))
60 }

```

Listing 9: Low.scala

```

0 package model

  /**
   * Created by joshua on 17/12/14.
   */
5 case class Low() extends Driven with Potential {
  override def isHigh : Boolean = false
}

```

Listing 10: Node.scala

```

0 package model

  /**
   * Created by joshua on 17/12/14.
   */
5 abstract class Node {
  var isInNormalForm = false;

  def get() : Boolean
}

```

Listing 11: Not.scala

```

0 package model

  /**
   * Created by joshua on 17/12/14.
   */
5 case class Not(negated : Node) extends Node with Atom {
  def get = !negated.get

  override def toString = "!" + (negated match {
    case Variable(_) => negated.toString
10 case _ : Node => "(" + negated.toString + ")"
  })
}

```

Listing 12: NTrans.scala

```

0 package model

  /**
   *  $N(g, s, d) = g \rightarrow (s \leftrightarrow d)$ 
   *
   *  $N$  gates can only carry low potential from their drain to
   * their source
   */
5 case class NTrans(input : Node, drain : Wire, source : Wire)
  extends Transistor {
  override def get : Potential = if (input.get) drain.get() else
    Undriven()
  var drawnGate : Option[AnyRef] = None

10 def sourceDriven = input.get && (drain.get == Low())

  override def resetDrawnGates() : Unit = {
    drawnGate = None
15 drain.resetDrawnGates()
  }
}

```

Listing 13: Or.scala

```

0 package model

  /**
   * Created by joshua on 17/12/14.
   */
5 case class Or(lhs : Node, rhs : Node) extends Node {
  def get = lhs.get || rhs.get

  // Allows for commutativity of operators
  override def equals(that : Any) : Boolean = that match {
10 case Or(lhs_, rhs_) => (lhs_ == lhs && rhs_ == rhs) || (lhs_
    == rhs && rhs_ == lhs)
    case _ => false
  }
}

```

```

    override def toString() = (lhs match {
15   case Variable(x) => x;
      case _ => "(" + lhs.toString + ")";
    }) + " or " + (rhs match {
      case Variable(x) => x;
      case _ => "(" + rhs.toString + ")";
20  })
  }
}

```

Listing 14: Potential.scala

```

0 package model

trait Potential {
  def isHigh : Boolean
}

```

Listing 15: PTrans.scala

```

0 package model

/**
 * P(g, s, d) = !g -> (s <=> d)
 *
5  * P gates can only carry high potential from the source to
   their drain
 */
case class PTrans(input : Node, drain : Wire, source : Wire)
  extends Transistor {
  override def get : Potential = if (!input.get) source.get()
    else Undriven()
  var drawnGate : Option[AnyRef] = None
10  def drainDriven = input.get && (source.get == High())

  override def resetDrawnGates() : Unit = {
    drawnGate = None
15    source.resetDrawnGates()
  }
}

```

```

}

```

Listing 16: Result.scala

```

0 package model

object Result extends Wire {
  override def removeGate (transistor : Transistor) : Unit = {
    sources = sources diff Array(transistor)
5    drains = drains diff Array(transistor)
  }

  override def get() : Potential = {
    var result : Potential = Undriven()
10    for (gate <- getSources) {
      if (gate.get != Undriven()) {
        result = gate.get
      }
    }
15    if (result == Undriven()) {
      for (gate <- getDrains) {
        if (gate.get != Undriven()) {
          result = gate.get
        }
20      }
    }
    if (result == Undriven()) {
      result//throw new RuntimeException("Result not driven")
    } else {
25      result
    }
  }

  override def clear : Unit = {
30    clearDrains
    clearSources
    Source.clear
    Drain.clear
  }
35  override def toString() = "Result"
}

```

```

    override def resetDrawnGates() : Unit = {
      for (gate <- getSources) {
40      gate.resetDrawnGates()
      }
      for (gate <- getDrains) {
        gate.resetDrawnGates()
      }
45      ()
    }
  }
}

```

Listing 17: Source.scala

```

0 package model

/**
 * Created by joshua on 17/12/14.
 */
5 object Source extends Wire {
  override def removeGate (transistor : Transistor) : Unit = {
    drains = drains diff Array(transistor)
  }

10 def get() : Potential = High()

  override def clear : Unit = clearDrains

  override def toString() = "Source"

15 override def resetDrawnGates () : Unit = ()
}

```

Listing 18: Transistor.scala

```

0 package model

/**
 * Created by joshua on 17/12/14.

```

```

*/
5 abstract class Transistor {
  def remove () = {
    drain.removeGate(this)
    source.removeGate(this)
  }

10 val input : Node
  val drain : Wire
  val source : Wire
  var drawnGate : Option[AnyRef]
15 def resetDrawnGates() : Unit

  def get() : Potential
}

```

Listing 19: Undriven.scala

```

0 package model

/**
 * Created by joshua on 17/12/14.
 */
5 case class Undriven() extends Potential {
  override def isHigh : Boolean = false
}

```

Listing 20: Variable.scala

```

0 package model

import scala.collection.immutable.TreeMap

case class Variable(val ident : String) extends Node with Atom {
5   def get = Variable.lookup(ident)

  override def toString = ident
}

```

```

10 object Variable {
    def negateAll(expr : Node) : Unit = {
        for (variable <- identMap) {
            setValue(variable._1, false)
        }
15     setValue("out", expr.get())
    }

    private var identMap = new TreeMap[String, Boolean]
    identMap += Tuple2("out", false)
20     private var intermediate = 0;

    def create(ident : String, value : Boolean) : Variable = {
        if (!(identMap contains ident))
            identMap += Tuple2(ident, value)
25     Variable(ident)
    }

    def lookup(ident : String) = identMap apply ident

30     def setValue(ident : String, value : Boolean) = identMap +=
        Tuple2(ident, value)

    def clear() = {
        identMap = new TreeMap[String, Boolean]
        identMap += Tuple2("out", false)
35     intermediate = 0
    }

    def getMap = identMap
}

```

Listing 21: WireHigh.scala

```

0 package model

/**
 * Created by joshua on 17/04/15.
 */
5 class WireHigh extends Wire {
    override def removeGate (transistor : Transistor) : Unit = {

```

```

        sources = sources filter (x => !(x eq transistor))
        drains = drains filter (x => !(x eq transistor))
    }
10     override def get() : Potential = {
        var res : Potential = Undriven()
        for (source <- getSources) {
            if (source.get != Undriven()) {
15             res = source.get
            }
        }
        res
    }

20     override def clear : Unit = {
        clearSources
        clearDrains
    }

25     override def toString() = ""

    override def resetDrawnGates() : Unit = {
        for (source <- getSources) {
30         source.resetDrawnGates()
        }
    }
}

```

Listing 22: WireLow.scala

```

0 package model

/**
 * Created by joshua on 17/04/15.
 */
5 class WireLow extends Wire {
    override def removeGate (transistor : Transistor) : Unit = {
        sources = sources filter (x => !(x eq transistor))
        drains = drains filter (x => !(x eq transistor))
    }
10

```

```

    override def get() : Potential = {
      var res : Potential = Undriven()
      for (drain <- getDrains) {
        if (drain.get != Undriven()) {
15         res = drain.get
        }
      }
      res
    }
20
    override def clear : Unit = {
      clearSources
      clearDrains
    }
25
    override def toString() = ""

    override def resetDrawnGates() : Unit = {
      for (drain <- getDrains) {
30        drain.resetDrawnGates()
      }
    }
  }

```

Listing 23: Wire.scala

```

0 package model

abstract class Wire {
  def removeGate (transistor : Transistor) : Unit

```

```

5   protected var sources = Array[Transistor]()
   protected var drains = Array[Transistor]()

   def get() : Potential

10  def addSource(node : Transistor) = {
      sources = node +: sources
    }

   def getSources = sources
15
   def clearSources : Unit = {
      sources = Array[Transistor]()
    }

20  def clear() : Unit

   def getDrains = drains

   def addDrain(node : Transistor) = {
25     drains = node +: drains
   }

   def clearDrains : Unit = {
      drains = Array[Transistor]()
30   }

   def resetDrawnGates() : Unit
}

```

Listing 24: ParserTest.scala

```

0  /**
   * Created by joshua on 31/12/14.
   */

   package model.test

5
   import helper._
   import model._
   import org.scalatest.prop.{GeneratorDrivenPropertyChecks, TableDrivenPropertyChecks}
   import org.scalatest.{FlatSpec, Matchers}

10  class ParserTest extends FlatSpec with Matchers with GeneratorDrivenPropertyChecks with TableDrivenPropertyChecks {

       val varNameA ="a"
       val b ="b"
15  val test = "test"
       val tests = Table(("input", "output"),
                           (!a) and !b", Some(And(Not(Variable(varNameA)), Not(Variable(b))))),
                           (!a) and b", Some(And(Not(Variable(varNameA)), Variable(b))))),
                           ("a and !b", Some(And(Variable(varNameA), Not(Variable(b))))),
                           ("a and (!b)", Some(And(Variable(varNameA), Not(Variable(b))))),
20  ("a and b", Some(And(Variable(varNameA), Variable(b))),
                           ("0", Some(Constant(false))),
                           ("1", Some(Constant(true))),
                           ("", None),
                           ("out", None),
25  ("(", None),
                           (")", None),
                           ("()", None),
                           ("!", None),
                           ("and", None),
30  ("a and", None),
                           ("and a", None),
                           ("or", None),
                           ("a or", None),
                           ("or a", None),
35  ("!a and !b", Some(Not(And(Variable(varNameA), Not(Variable(b))))),
                           ("!a and b", Some(Not(And(Variable(varNameA), Variable(b))))),
                           ("!a or !b", Some(Not(Or(Variable(varNameA), Not(Variable(b))))),
                           ("!a or b", Some(Not(Or(Variable(varNameA), Variable(b))))),
40  ("!a", Some(Not(Variable(varNameA)))),

```

```

        (!a)", Some(Not(Variable(varNameA)))),
        (!a)", Some(Not(Variable(varNameA)))),
        (!a) or !b", Some(Or(Not(Variable(varNameA)), Not(Variable(b)))))),
        (!a) or b", Some(Or(Not(Variable(varNameA)), Variable(b)))))),
45      ("a or !b", Some(Or(Variable(varNameA), Not(Variable(b)))))),
        ("a or (!b)", Some(Or(Variable(varNameA), Not(Variable(b)))))),
        ("a or b", Some(Or(Variable(varNameA), Variable(b)))))),
        ("a", Some(Variable(varNameA))),
        ("a)", Some(Variable(varNameA))),
50      ("test", Some(Variable(test)))
    )

    "The table of results" should "be parsed as expected" in forAll (tests) {
      (input : String, res : Option[Node]) => {
55        Parser.variableParser(input) should be (res)
        Variable.clear()
      }
    }
  }
}

```

46

Listing 25: PITable.scala

```

0 package model

import scala.annotation.tailrec

/**
5  * Created by joshua on 05/01/15.
  */
object PITable {
  def solve(primeImplicants : List[Implicant], minterms : List[Int], vars : List[String]) = {
10    val start = new PITable(
      primeImplicants.map(x => new PrimeImplicant(x, x.terms().toSet)),
      minterms.toSet,
      Set[Implicant](),
      vars
15    )

    bestSolution(start)
  }
}

```

```

20 // @tailrec
    def bestSolution(t : PITable) : PITable = t.finished match {
        case true => t
        case false => {
            val branches = for (row <- t.rows) yield bestSolution(reduceTable(t.selectRow(row)))
25         branches.minBy(_.cost(t.vars.length))
        }
    }

    @tailrec def reduceTable(t : PITable) : PITable = t.selectEssential match {
30     case (true, newTable) => reduceTable(newTable.reduceRows)
        case (false, _) => t.reduceRows
    }

35 case class PITable(val rows : List[PrimeImplicant], val cols : Set[Int], val results : Set[Implicant], val vars :
    List[String]) {

    def cost(order : Int) = {
        results.foldLeft(0) {
40         _ + _.cost(order)
        }
    }

    def finished = cols.size == 0

45 def selectRow(row : PrimeImplicant) = {
        val nRows = rows.filter(_ != row).map(_.reduce(row.terms))
        val nCols = cols -- row.terms
        val nRes = results + row.implicant
50     this.copy(rows = nRows, cols = nCols, results = nRes)
    }

    def reduceRows = {
        var nRows = rows.filter(!_.empty())
55     for (List(a, b) <- rows.combinations(2)) {
        if (a dominates b) {
            nRows = nRows.filter(_ != b)
        } else if (b dominates a) {
            nRows = nRows.filter(_ != a)
60     }
    }

```

```

    }
    this.copy(rows = nRows)
  }

65 def rowsForMinterm(m : Int) = (for (row <- rows if row.covers(m)) yield row).filter(_ !=())

def selectEssential = {
  var newTable = this
  var done = false
70  var effective = false

  while (!done) {
    done = true
    for (m <- cols) {
75      newTable.rowsForMinterm(m) match {
        case List(x : PrimeImplicant) => {
          done = false
          effective = true
          newTable = newTable.selectRow(x)
80        }
        case _ => ()
      }
    }
  }
85  (effective, newTable)
}

def toSumOfProducts(vars : List[String]) = {
90  assert(finished)

  results.map(_._withVars(vars)).toList.sorted.foldLeft("")(l, r) => if (l != "") "(%s) or (%s)".format(l, r) else r
}
}

```

Listing 26: PrimeImplicant.scala

```

0 package model

/**
 * Created by joshua on 05/01/15.

```



```

    */
5  ///--- class that slowly covers less and less
    class PrimeImplicant(val implicant : Implicant, val terms : Set[Int]) {
        val tag = implicant.tag
        val order = implicant.order()

10  ///--- remove given terms from this data's effect
        def reduce(coveredTerms : Set[Int]) = {
            new PrimeImplicant(implicant, terms -- coveredTerms)
        }

15  ///--- if this is higher order and contains all the minterms of the other
        def dominates(other : PrimeImplicant) = ((other.order <= order) && (other.terms.subsetOf(terms)))

        ///--- whether this contains a given minterm or not
        def covers(minterm : Int) = terms.contains(minterm)

20  ///--- whether this is now empty
        def empty() = terms.size == 0

        override def equals(that : Any) = that match {
25  case other : PrimeImplicant => hashCode == other.hashCode
        case _ => false
        }

        //override def hashCode = implicant.hashCode
30  override def toString() = terms.mkString("", ", ", "")
    }

```

Listing 27: qmm.scala

```

0  // Based on github.com/jjfiv/qmm-scala

    package model

    import helper.Parser

5  import scala.annotation.tailrec
    import scala.collection.immutable.List

    object qmm {

```

```

10 def method(minterms : List[Int], dontcares : List[Int], vars : List[String]) : String = {
    val implicants = (minterms ::: dontcares).sorted.sortBy(bitcount(_)).map(new Implicant(_))
    val order = vars.length

    val prime_implicants = genImplicants(implicants, order).filter(_.prime)
15    val results = PITable.solve(prime_implicants, minterms, vars)

    val res = results.toSumOfProducts(vars)

20    return res
}

def bitcount(x : Int) : Int = {
    @tailrec def bcrec(accum : Int, n : Int) : Int = n match {
25        case 0 => accum
        case x => bcrec(accum + (x % 2), (x >> 1))
    }
    bcrec(0, x)
}

30 def genImplicants(zero_cubes : List[Implicant], order : Int) : List[Implicant] = {

    import scala.collection.mutable.{Set => MutableSet}

35    ///-- generate list and populate with zero-cubes
    var implicants = MutableSet[Implicant]()
    for (i <- zero_cubes) implicants += i

    ///-- operate on current order until highest reached
40    for (currentOrder <- 0 until order) {
        ///-- grab all implicants of the current order
        val data = implicants.toList.filter(_.order == currentOrder)

        for (List(a, b) <- data.combinations(2)) {
45            if (a.canCombine(b)) {
                a.prime = false
                b.prime = false

                val n = a.combine(b)

50                implicants += n
            }
        }
    }
}

```

```

    }
  }
}
55   implicants.toList
}

def letters(x : String) : List[String] = x.split("").filter(_ != "").toList
60 }

```

Listing 28: QuineMcCluskeyTest.scala

```

0 package model.test

import helper.Parser
import model._
import org.scalatest.prop.{TableDrivenPropertyChecks, GeneratorDrivenPropertyChecks}
5 import org.scalatest.prop.Tables.Table
import org.scalatest.{FlatSpec, Matchers}

/**
 * Created by joshua on 04/02/15.
10 */
class QuineMcCluskeyTest extends FlatSpec with Matchers with GeneratorDrivenPropertyChecks with TableDrivenPropertyChecks {
  val varNameA = "a"
  val b = "b"
  val c = "c"
15  val test = "test"
  val tests = Table(("input", "output"),
    ("(!a) and a", None),
    ("(!a) or a", None),
    ("a and a and a and a", Some(Variable(varNameA))),
    ("a or a or a or a", Some(Variable(varNameA))),
20    ("a and b", Some(And(Variable(varNameA), Variable(b)))),
    ("(a and (!b)) or ((!a) and b)",
      Some(Or(And(Variable(varNameA), Not(Variable(b))), And(Not(Variable(varNameA)), Variable(b))))),
    ("!((a and b) or ((!a) and (!b) and c))",
25    Some(Or(Or(And(Not(Variable(b)), Variable(varNameA)), And(Not(Variable(c)), Not(Variable(varNameA)))), And(
      Variable(b), Not(Variable(varNameA))))),
    ("a and b and c", Some(And(Variable(c), And(Variable(b), Variable(varNameA))))),
    ("c and b and a", Some(And(Variable(c), And(Variable(b), Variable(varNameA))))),

```

```

        ("(a and b and c) or (a and b and c)", Some(And(Variable(c),And(Variable(b),Variable(varNameA)))))
    )
30  "The table of results" should "be minimised as expected" in forAll (tests) { (input : String, res : Option[Node]) => {
        LogicalFunction.quineMcCluskey(Parser.variableParser(input).get) should be (res)
        Variable.clear()
    }
35  }
}

```

A.2 View

Listing 29: DrawCircuit.scala

52

```

0  package view

    import com.mxgraph.util.mxConstants
    import com.mxgraph.view.mxGraph
    import model._
5
    /**
     * Created by joshua on 08/04/15.
     */
    case class DrawCircuit (val graph : mxGraph) {
10  val nodes = scala.collection.mutable.Stack[AnyRef]()
        val deltaY = 50
        val deltaX = 25
        val lastSources = scala.collection.mutable.MutableList[AnyRef]()
        val lastDrains = scala.collection.mutable.MutableList[AnyRef]()
15  var parent = graph.getDefaultParent
        var currentX = 0;
        var sourcesYLimit = 0;
        var drainsYLimit = 0;
        var maxX = 0;
20
        def draw () : Unit = {
            parent = graph.getDefaultParent
            // No gates are currently drawn, so reset

```

```

Result.resetDrawnGates()
25 styleSheet("nmos_de")
   styleSheet("pmos_de")
   styleSheet("nmos_en")
   styleSheet("pmos_en")
   wireStyle(true)
30 wireStyle(false)
   otherWireStyle()
   graph.getModel.beginUpdate()
   try { {
       val node = graph.insertVertex(parent, null, "output", 0, 0, 0, 1)
35   val sources = Result.getSources
       // every gate in sources should connect to the output wire
       for (sources <- sources) {
           nodes push node
       }
40   currentX = 0
       drawNetwork(sources, 0, true)
       maxX = currentX
       currentX = -10
       val drains = Result.getDrains
45   for (drain <- drains) {
       nodes push node
   }
       drawNetwork(drains, 0, false)
       maxX = Math.max(maxX, currentX)
50   val newOut = graph.insertVertex(parent, null, "output", -15, 0, maxX + 30, 1, "wireUndriven")

       val drain = graph.insertVertex(parent, null, "drain", -15, drainsYLimit, maxX + 30, 1,
                                       "wireDriven")

55   val source = graph.insertVertex(parent, null, "source", -15, -1.0 * sourcesYLimit, maxX + 30, 1,
                                       "wireDriven")

       if (Result.get() == High()) {
60   graph.setCellStyle("wireDriven", Array(newOut))
       }

       for (edge <- graph.getEdges(node)) {
65   graph.splitEdge(edge, Array(newOut))
       }

```

```

    for (sourceGate <- lastSources) {
        val x = graph.getCellGeometry(sourceGate).getCenterX
        val dot = graph.insertVertex(parent, null, "", x, -1.0 * sourcesYLimit, 1, 1)
70     graph.insertEdge(parent, null, "", sourceGate, dot, "wireOther")
    }

    for (drainGate <- lastDrains) {
        val x = graph.getCellGeometry(drainGate).getCenterX
75     val dot = graph.insertVertex(parent, null, "", x, drainsYLimit, 1, 1)
        graph.insertEdge(parent, null, "", drainGate, dot, "wireOther")
    }

    lastSources.clear()
80    lastDrains.clear()

    graph.removeCells(Array(node))
    }
    } finally {
85     graph.getModel.endUpdate()
    }
}

private def styleSheet (name : String) : Unit = {
90    // add gate as node at (xPos, y+1)
    // get graph stylesheet
    val stylesheet = graph.getStylesheet

    // define image style name
95    val styleName = name

    // define image style
    val style = new java.util.HashMap[String, AnyRef]()
    style.put(mxConstants.STYLE_SHAPE, mxConstants.SHAPE_IMAGE)
100    style.put(mxConstants.STYLE_IMAGE, "file:resources/%s.png".format(name))
    style.put(mxConstants.STYLE_LABEL_POSITION, mxConstants.ALIGN_LEFT)

    stylesheet.putCellStyle(styleName, style)
}

105 private def otherWireStyle() : Unit = {
    // get graph stylesheet

```

```

    val stylesheet = graph.getStylesheet

110 // define image style name
    val styleName = "wireOther"

    // define image style
    val style = new java.util.HashMap[String, AnyRef]()
115 style.put(mxConstants.STYLE_ENDARROW, mxConstants.NONE)
    style.put(mxConstants.STYLE_STARTARROW, mxConstants.NONE)
    style.put(mxConstants.STYLE_STROKECOLOR, "red")
    style.put(mxConstants.STYLE_FILLCOLOR, "red")
    style.put(mxConstants.STYLE_LABEL_POSITION, mxConstants.ALIGN_RIGHT)
120
    stylesheet.putCellStyle(styleName, style)
}

125 private def wireStyle (driven : Boolean) : Unit = {
    // get graph stylesheet
    val stylesheet = graph.getStylesheet

    // define image style name
130 val styleName = "wire" + (if (driven) {
        "Driven"
    } else {
        "Undriven"
    })

135
    // define image style
    val style = new java.util.HashMap[String, AnyRef]()
    style.put(mxConstants.STYLE_SHAPE, mxConstants.SHAPE_LINE)
    style.put(mxConstants.STYLE_STROKECOLOR, if (driven) {
140 "red"
    } else {
        "black"
    })
    style.put(mxConstants.STYLE_FILLCOLOR, if (driven) {
145 "red"
    } else {
        "black"
    })
    style.put(mxConstants.STYLE_LABEL_POSITION, mxConstants.ALIGN_RIGHT)

```

```

150     stylesheet.putCellStyle(styleName, style)
    }

    private[this] def drawNetwork (gates : Array[Transistor], y : Int, drawingTopNetwork : Boolean) : Unit = {
155     if (gates.isEmpty) {
        // if last node on the chain, nothing to connect it to, so forget it
        if (drawingTopNetwork) {
            lastSources += nodes.pop()
            sourcesYLimit = scala.math.max(sourcesYLimit, y)
160        } else {
            lastDrains += nodes.pop()
            drainsYLimit = scala.math.max(drainsYLimit, y)
        }
    } else {
165     for (gate <- gates) {
        gate.drawnGate match {
            case Some(node : AnyRef) => {
                val box = graph.getCellGeometry(node)
                val x = box.getCenterX - deltaX * 2
170                val tempy = box.getCenterY + (box.getHeight * (if (drawingTopNetwork) {
                    -0.5
                } else {
                    0.5
                })) - deltaY
175                val point = graph.insertVertex(parent, null, "", x, tempy, 0, 0)
                graph.insertEdge(parent, null, "", nodes.pop(), node,
                    mxConstants.STYLE_SHAPE + "=" + mxConstants.SHAPE_LINE)
            }
            case None => {
180                val previousNode = nodes.pop()
                val node =
                    graph.insertVertex(parent, null, gate.input.toString, currentX, if (drawingTopNetwork) {
                        -(y + deltaY)
                    } else {
185                        y
                    }, deltaX * 2.0, deltaY, (if (!drawingTopNetwork) {
                        "nmos"
                    } else {
                        "pmos"
                    }) + (if (gate.get() == Undriven()) {
190                        "_de"
                    })
            }
        }
    }

```



```

        } else {
            "_en"
        })
    )))
195     gate.drawnGate = Some(node)
        graph.insertEdge
            (parent, null, "", previousNode, node, mxConstants.STYLE_SHAPE + "=" + mxConstants.SHAPE_IMAGE)
            nodes.push node
        drawNetwork(if (drawingTopNetwork) {
200             gate.source.getSources
        } else {
            gate.drain.getDrains
        }, y + deltaY, drawingTopNetwork)
        currentX += deltaX * 2
205     }
    }
}
210 }

```

57

A.3 Controller

Listing 30: Gui.java

```

0  package controller;

    import com.mxgraph.io.mxCodec;
    import com.mxgraph.layout.mxGraphLayout;
    import com.mxgraph.layout.mxParallelEdgeLayout;
5  import com.mxgraph.model.mxGraphModel;
    import com.mxgraph.swing.mxGraphComponent;
    import com.mxgraph.util.mxCellRenderer;
    import com.mxgraph.util.mxXmlUtils;
    import com.mxgraph.util.png.mxPngEncodeParam;
10  import com.mxgraph.util.png.mxPngImageEncoder;
    import com.mxgraph.view.mxGraph;
    import com.mxgraph.view.mxGraphView;
    import helper.CMOSLayout;

```

```

import helper.Parser;
15 import model.Node;
import model.Result;
import model.Variable;
import scala.Option;
import scala.Tuple2;
20 import scala.collection.Iterator;
import scala.collection.immutable.TreeMap;
import view.DrawCircuit;

import javax.swing.*;
25 import java.awt.*;
import java.awt.event.ActionEvent;
import java.awt.event.ActionListener;
import java.awt.event.ItemEvent;
import java.awt.event.ItemListener;
30 import java.awt.image.BufferedImage;
import java.io.File;
import java.io.FileOutputStream;
import java.net.URLEncoder;
import java.util.HashMap;
35
/**
 * Created by joshua on 08/04/15.
 */
public class Gui {
40 /**
 * Handler for the whole graph.
 */
public mxGraph graph;
/**
45 * Handler for the graphic component.
 */
public mxGraphComponent graphComponent = null;
/**
 * A handler for the graph's layout.
50 */
public mxGraphLayout layout;
private JPanel mainPanel;
private JPanel visPanel;
private JPanel inputPanel;
55 private JTextField textInput;

```

```

private JButton goButton;
private JCheckBox outputCheckBox;
private JPanel variableHolder;
private JButton exportButton;
60 private HashMap<String, JCheckBox> nameBoxMap = new HashMap<String, JCheckBox>();
private DrawCircuit drawCircuit;

public Gui () {
    goButton.addActionListener(new ActionListener() {
65         @Override public void actionPerformed (ActionEvent actionEvent) {
            // Clear checkboxes
            variableHolder.removeAll();
            variableHolder.add(outputCheckBox);
            Variable.clear();

70             Option<Node> parseResult = Parser.variableParser(textInput.getText());

            if (parseResult.isEmpty()) {
                JOptionPane.showMessageDialog(mainPanel, "Could not parse expression", "Error",
75                 JOptionPane.ERROR_MESSAGE);
            } else {
                final Node result = parseResult.get();
                String message = CMOSLayout.layout(result);
                JOptionPane.showMessageDialog(mainPanel, message, "Finished", JOptionPane.INFORMATION_MESSAGE);
80                 final TreeMap<String, Object> map = Variable.getMap();
                final Iterator<Tuple2<String, Object>> it = map.iterator();
                visPanel.setSize(400, 300);
                if (graph != null) {
                    ((mxGraphModel) graph.getModel()).clear();
85                 }
                graphComponent = initGraph();
                visPanel.add(graphComponent, BorderLayout.CENTER);
                mainPanel.updateUI();
                mainPanel.validate();
                mainPanel.repaint();
90                 while (it.hasNext()) {
                    Tuple2<String, Object> value = it.next();
                    String variableName = value._1();
                    final JCheckBox variableBox = new JCheckBox(variableName, false);
                    if (!variableName.equals("out")) {
95                         variableBox.addItemListener(new ItemListener() {
                            @Override public void itemStateChanged (ItemEvent e) {

```

```

        Variable.setValue(variableBox.getText(), variableBox.isSelected());
        outputCheckBox.setSelected(Result.get().isHigh());
        // Clear Graph
        if (graph != null) {
            ((mxGraphModel) graph.getModel()).clear();
        }
        drawCircuit.draw();

        resizeGraphView(graphComponent);
    }
});
variableBox.setSelected(Variable.lookup(variableName));
variableHolder.add(variableBox);
nameBoxMap.put(variableName, variableBox);
variableHolder.validate();
variableHolder.repaint();
    }
}
}
outputCheckBox.setSelected(parseResult.get().get());
});
exportButton.addActionListener(new ActionListener() {
    @Override public void actionPerformed (ActionEvent actionEvent) {
        try {
            JFileChooser fileChooser = new JFileChooser();
            if (fileChooser.showSaveDialog(mainPanel) == JFileChooser.APPROVE_OPTION) {
                File file = fileChooser.getSelectedFile();
                String filename = file.getAbsolutePath();

                BufferedImage image = mxCellRenderer.createBufferedImage(graph, null, 1, Color.WHITE,
                                                                    graphComponent.isAntiAlias(), null,
                                                                    graphComponent.getCanvas());

                mxPngEncodeParam param = null;
                if (image != null) {
                    param = mxPngEncodeParam.getDefaultEncodeParam(image);
                } else {
                    JOptionPane.showMessageDialog(mainPanel, "Could not create image", "Error", JOptionPane
                                                                    .ERROR_MESSAGE);
                }

                // Creates the URL-encoded XML data

```

```

140         mxCodec codec = new mxCodec();
        String xml = URLEncoder.encode(mxXmlUtils.getXml(codec.encode(graph.getModel())), "UTF-8");

        param.setCompressedText(new String[] { "graph", xml });

145         // Saves as a PNG file
        FileOutputStream outputStream = new FileOutputStream(new File(filename));

        try {
            mxPngImageEncoder encoder = new mxPngImageEncoder(outputStream, param);

150             if (image != null) {
                encoder.encode(image);
            } else {
                System.out.println("No Image");
155             }
        } finally {
            outputStream.close();
        }
160     } catch (Exception e) {
        }
    }
    });
165 }

public static void main (String[] args) {
    try {
        UIManager.setLookAndFeel(UIManager.getSystemLookAndFeelClassName());
170    } catch (Exception e1) {
        e1.printStackTrace();
    }

    JFrame frame = new JFrame("CMOS Calculator");
175    frame.setContentPane(new Gui().mainPanel);
    frame.setDefaultCloseOperation(JFrame.EXIT_ON_CLOSE);
    frame.pack();
    frame.setVisible(true);
    }
180
    /**

```

```

        * This will initialize the graph component to draw the shapes on.
        *
        * @return The graph component which was drawn on.
185    */
    public final mxGraphComponent initGraph () {
        mxGraphComponent localGraphComponent = null;
        try {
190            graph = new mxGraph();
            localGraphComponent = new mxGraphComponent(graph);

            // Allow negative co-ordinates (makes drawing easier as bottom half can be negatively positioned)
            graph.setAllowNegativeCoordinates(true);
            // dangling edges are bad and result in all kinds of nasty things
195            graph.setAllowDanglingEdges(false);
            // edge source and target are the same
            graph.setAllowLoops(true);
            // don't need this
            graph.setCellsResizable(false);
            // don't allow movement
            graph.setCellsMovable(false);
            // don't allow new connections
            graph.setConnectableEdges(false);
            // make editing labels more comfortable
200            localGraphComponent.setEnterStopsCellEditing(true);
            // antialiasing \o/
            localGraphComponent.setAntiAlias(true);
            // set size
            localGraphComponent.setSize(visPanel.getWidth(), visPanel.getHeight());
210            graph.setAutoOrigin(true);

            // define a parallel layout for the edges
            layout = new mxParallelEdgeLayout(graph);
215            drawCircuit = new DrawCircuit(graph);
            drawCircuit.draw();

            resizeGraphView(localGraphComponent);
        } catch (Exception e) {
220            JOptionPane.showMessageDialog(mainPanel, e.getMessage(), "Error", JOptionPane.ERROR_MESSAGE);
        } finally {
            return localGraphComponent;
        }
    }

```

```

    }
225
    private void resizeGraphView (mxGraphComponent localGraphComponent) {
        mxGraphView view = localGraphComponent.getGraph().getView();
        int compLenH = localGraphComponent.getHeight();
        int viewLenH = (int) view.getGraphBounds().getHeight();
230        double scaleH = (double) (compLenH - 1) / viewLenH * view.getScale();

        int compLenW = localGraphComponent.getWidth();
        int viewLenW = (int) view.getGraphBounds().getWidth();

235        double scaleW = (double) (compLenW - 1) / viewLenW * view.getScale();

        view.setScale(Math.min(scaleH, scaleW));
    }

240    {
        // GUI initializer generated by IntelliJ IDEA GUI Designer
        // >>> IMPORTANT!! <<<
        // DO NOT EDIT OR ADD ANY CODE HERE!
        setupUI();
245    }

    /**
     * Method generated by IntelliJ IDEA GUI Designer >>> IMPORTANT!! <<< DO NOT edit this method OR call it in your
     * code!
250     *
     * @noinspection ALL
     */
    private void setupUI () {
        mainPanel = new JPanel();
255        mainPanel.setLayout(new BorderLayout(0, 0));
        visPanel = new JPanel();
        visPanel.setLayout(new BorderLayout(0, 0));
        visPanel.setMinimumSize(new Dimension(400, 300));
        visPanel.setPreferredSize(new Dimension(400, 300));
260        mainPanel.add(visPanel, BorderLayout.CENTER);
        visPanel.setBorder(BorderFactory.createTitledBorder("CMOS Output"));
        inputPanel = new JPanel();
        inputPanel.setLayout(new BorderLayout(0, 0));
        mainPanel.add(inputPanel, BorderLayout.WEST);
265        final JPanel panel1 = new JPanel();

```

```

        panel1.setLayout(new FlowLayout(FlowLayout.CENTER, 5, 5));
        panel1.setEnabled(true);
        inputPanel.add(panel1, BorderLayout.NORTH);
        textInput = new JTextField();
270    textInput.setMinimumSize(new Dimension(70, 26));
        textInput.setPreferredSize(new Dimension(100, 26));
        textInput.setText("");
        textInput.setToolTipText("Enter the text to parse here");
        panel1.add(textInput);
275    goButton = new JButton();
        goButton.setText("Parse!");
        panel1.add(goButton);
        final JScrollPane scrollPanel = new JScrollPane();
        inputPanel.add(scrollPanel, BorderLayout.CENTER);
280    variableHolder = new JPanel();
        variableHolder.setLayout(new FlowLayout(FlowLayout.CENTER, 5, 5));
        scrollPanel.setViewportView(variableHolder);
        outputCheckBox = new JCheckBox();
        outputCheckBox.setActionCommand("");
285    outputCheckBox.setEnabled(false);
        outputCheckBox.setText("out");
        variableHolder.add(outputCheckBox);
        final JPanel panel2 = new JPanel();
        panel2.setLayout(new BorderLayout(0, 0));
290    inputPanel.add(panel2, BorderLayout.SOUTH);
        exportButton = new JButton();
        exportButton.setText("Export");
        panel2.add(exportButton, BorderLayout.NORTH);
    }
295    /** @noinspection ALL */
    public JComponent getRootComponent () {
        return mainPanel;
    }
300 }

```

A.4 Helper

Listing 31: CMOSLayout.scala

```

0 package helper

import model._

import scala.collection.mutable
5 import scala.collection.mutable.{HashSet, Stack}

/**
 * Created by joshua on 08/01/15.
 */
10 object CMOSLayout {

    private var totalGates = 0

    def main(args: Array[String]): Unit = {
15     layout(helper.Parser.variableParser("(x and y) or ((!x) and (!y) and z)") match {
        case Some(v) => {
            println(v); v
        }
    })
    println(Result.getSources.length)
    20     for (source <- Result.getSources) println(source.toString)
    println(Result.getDrains.length)
    for (drain <- Result.getDrains) println(drain.toString)
    }

    25     private val negations = new HashSet[Variable]()

    def checkForDuplicateGates (wire : Wire)(checkingTopNetwork : Boolean) : Unit = {
        // if we're at the result, we've finished
    30     if (wire != Result) {
        val wiresToCheck = new mutable.HashSet[Wire]()
        val gates = new mutable.HashMap[Node, Transistor]()
        for (gate <- (if(checkingTopNetwork) wire.getDrains else wire.getSources)) {
            if (gates.contains(gate.input)) {
    35                 if (checkingTopNetwork) {
                    for (drain <- gate.drain.getDrains) gates(gate.input).drain.addDrain(drain)
                    wiresToCheck += gates(gate.input).drain
                } else {
                    for (source <- gate.source.getSources) gates(gate.input).source.addSource(source)
    40                     wiresToCheck += gates(gate.input).source
                }
            }
        }
    }
}

```

```

        }
        gate.remove()
        totalGates -= 1
    } else {
45     gates.put(gate.input, gate)
    }
}
for (wire <- wiresToCheck) checkForDuplicateGates(wire)(checkingTopNetwork)
50 }

def layout(expr: model.Node): String = {
    val normal = tryLayout(expr)
    val doubleNegate = tryLayout(Not(expr)) + 2
55     if (normal <= doubleNegate) {
        tryLayout(expr)
        return normal + "transistors used"
    } else {
60     return doubleNegate + "transistors used, with two to negate the output"
    }
}

def tryLayout(expr : model.Node) : Integer = {
65     Result.clear;
    totalGates = 0;
    negations.clear()

    LogicalFunction.quineMcCluskey(expr) match {
70     case Some(x) => {
        execute(x)(true)
    }
    case None => {
75     throw new RuntimeException("Expression not minimised correctly")
    }
    }
    LogicalFunction.quineMcCluskey(Not(expr)) match {
    case Some(x) => {
80     execute(x)(false)
    }
    case None => {
        throw new RuntimeException("Expression not minimised correctly")
    }
    }
}

```

```

    }
  }
85  Variable.negateAll(expr)
    //checkForDuplicateGates(Source)(true)
    //checkForDuplicateGates(Drain)(false)
    (totalGates + 2 * negations.size)
  }
90
  /**
   * pre: expr has been converted to sum of products form (DNF)
   * @param expr - the expression to be implemented in Gui
   * @param buildingTopNetwork - indicates if the network we are building is to be used to carry a high potential or
95  *                          a low potential
   * @return - side effects on the Result object
   */
  private def execute(expr: model.Node)(buildingTopNetwork: Boolean): Unit = {
    // I : \ / stack or subExpr = expr
100  // Initially stack = [], subExpr = expr => I
    val stack = Stack[model.Node]()
    var subExpr = expr

    while (subExpr != Constant(false)) {
105      subExpr match {
        case Or(x, y) => {
          stack push y
          subExpr = x
        }
110      case Variable(_) => {
          println("Processing: " + subExpr.toString)
          convertToGates(subExpr, buildingTopNetwork)
          if (!stack.isEmpty) {
            subExpr = stack.pop()
115          } else {
            subExpr = Constant(false)
          }
        }
      }
120      case Not(_) => {
          println("Processing: " + subExpr.toString)
          convertToGates(subExpr, buildingTopNetwork)
          if (!stack.isEmpty) {
            subExpr = stack.pop()
          } else {

```

```

125     subExpr = Constant(false)
    }
  }
  case And(_, _) => {
    println("Processing: " + subExpr.toString)
130    convertToGates(subExpr, buildingTopNetwork)
    if (!stack.isEmpty) {
      subExpr = stack.pop()
    } else {
      subExpr = Constant(false)
135    }
  }
}
}
}
}

140 /**
 * As the parser operates using a fold, the right hand node must always be an Or(x,y), with the last node being made
 * up of a Variable(x) or Not(Variable(x))
 *
145 * For some expression, assuming buildingTopNetwork,  $x_0 \wedge x_1 \wedge \dots \wedge x_n$ , with literals  $x_0, x_1, \dots, x_n$ ,  $x_0$ 's
 * drain is attached to the output wire,  $x_0$ 's source is attached to a new wire, which is then attached to  $x_1$ 's
 * drain, working along the conjunction until the last node.  $x_n$  is attached to the source wire.
 *
 * If not buildingTopNetwork, replace all instances of source with drain, and attach  $x_n$  to the drain wire.
150 *
 * @param node the logical expression to be converted to a series of gates
 * @param buildingTopNetwork indicates if the top half, or bottom is being evaluated
 *       (fromBottom => PGate, !fromBottom => NGate)
 *
155 *
 *       If a connection is made above a wire, then a source is being added, and if it is being made below a
 *       wire, then a
 *       drain is added
 */
private def convertToGates(node: Node, buildingTopNetwork: Boolean) = {
  var subNode: model.Node = node
160  var currentWire: Wire = Result

  while (subNode != Constant(false)) subNode match {
    case And(x, y) => {
      val wire = if (buildingTopNetwork) { new WireHigh() } else { new WireLow() }
165      x match {

```

```

    case Not(Variable(v)) => {
      if (buildingTopNetwork) {
        val gate = new PTrans(Variable(v), currentWire, wire)
        currentWire addSource gate
170      wire addDrain gate
        println("Adding source to " + currentWire.toString())
      } else {
        val gate = new NTrans(Not(Variable(v)), wire, currentWire)
        negations add Variable(v)
175      currentWire addDrain gate
        wire addSource gate
        println("Adding drain to " + currentWire.toString())
      }
    }
180  case Variable(v) => {
    if (buildingTopNetwork) {
      val gate = new PTrans(Not(Variable(v)), currentWire, wire)
      negations add Variable(v)
      currentWire addSource gate
185      wire addDrain gate
      println("Adding source to " + currentWire.toString())
    } else {
      val gate = new NTrans(Variable(v), wire, currentWire)
      currentWire addDrain gate
190      wire addSource gate
      println("Adding drain to " + currentWire.toString())
    }
  }
}
195 currentWire = wire
    subNode = y
}
// Create a new gate and finish
case Not(Variable(v)) => {
200   if (buildingTopNetwork) {
    val gate = new PTrans(Variable(v), currentWire, Source)
    currentWire addSource gate
    Source addDrain gate
    println("Adding source to " + currentWire.toString())
205   } else {
    val gate = new NTrans(Not(Variable(v)), Drain, currentWire)
    negations add Variable(v)
  }
}

```

```

        currentWire addDrain gate
        Drain addSource gate
        println("Adding drain to " + currentWire.toString())
    }
    subNode = Constant(false)
}
case Variable(v) => {
215   if (buildingTopNetwork) {
        val gate = new PTrans(Not(Variable(v)), currentWire, Source)
        negations add Variable(v)
        currentWire addSource gate
        Source addDrain gate
220       println("Adding source to " + currentWire.toString())
    } else {
        val gate = new NTrans(Variable(v), Drain, currentWire)
        currentWire addDrain gate
        Drain addSource gate
225       println("Adding drain to " + currentWire.toString())
    }
    subNode = Constant(false)
}
}
230 }

private def exprNotAtom(expr: Node) : Boolean = expr match {
    case And(_, _) => true
    case Or(_, _) => true
235    case _ : Atom => false
    case _ => true
}

def exprNotConjunction(node: Node): Boolean = node match {
240    case And(_, _) => false
    case _ => true
}
}

```

Listing 32: LogicalExpression.scala

```

0 package helper

```

11

```

import model._

import scala.util.parsing.combinator.{PackratParsers, RegexParsers}
5
/**
 * \Sigma = [A-Za-z0-9]*
 * expr -> conj | disj | literal
 * conj -> literal | literal "and" conj
10 * disj -> conj | conj "or" disj
 * literal -> "!" simp | simp
 * simp -> variable | constant | "(" expr ")" | expr
 * variable -> \Sigma\Sigma*\{"out", "0", "1"}
 * constant -> 0 | 1
15 */
class LogicalExpression extends RegexParsers with PackratParsers {
  lazy val variable : PackratParser[Node] = """"^(?!out|and|or$)([A-Za-z0-9]+)""".r ^^ {
    v => {
      Variable.create(v, false)
20    }
  }

  lazy val const : PackratParser[Node] = ("1" | "0") ^^ {
    case "1" => {
      Constant(true)
25    }
    case "0" => {
      Constant(false)
    }
30  }

  lazy val literal : PackratParser[Node] = ("!" ~ simp | simp) ^^ {
    case "!" ~ (n : Node) => {
      Not(n)
35    }
    case n : Node => {
      n
    }
  }
40

  lazy val conj : PackratParser[Node] = replsep(literal, "and") ^^ {
    _..reduceRight(And)
  }

```

```

45  lazy val disj : PackratParser[Node] = replsep(conj, "or") ^^ {
    _._reduceRight(0r)
  }

    lazy val simp : PackratParser[Node] = (variable
50      ||| const
      ||| ("(" ~ expr ~ ")") ^^ {

        case "(" ~ e ~ ")" => {
          e
        }
55    })

      ||| expr)

    lazy val expr : PackratParser[Node] = (conj
60      ||| disj
      ||| literal)
  }

```

72

Listing 33: Parser.scala

```

0  package helper

    import model.Node

    object Parser extends LogicalExpression {
5    def variableParser(x : String) : Option[Node] = {
      val result = parseAll(expr, x)
      if (result.successful)
        Some(result.get)
      else
10      None
    }
  }

```

B Test Results

The following are the images seen in table 4 on page 29, with details of the assignment required.

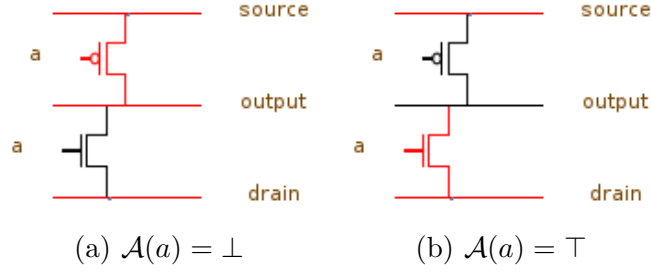


Figure 8: $\neg a$

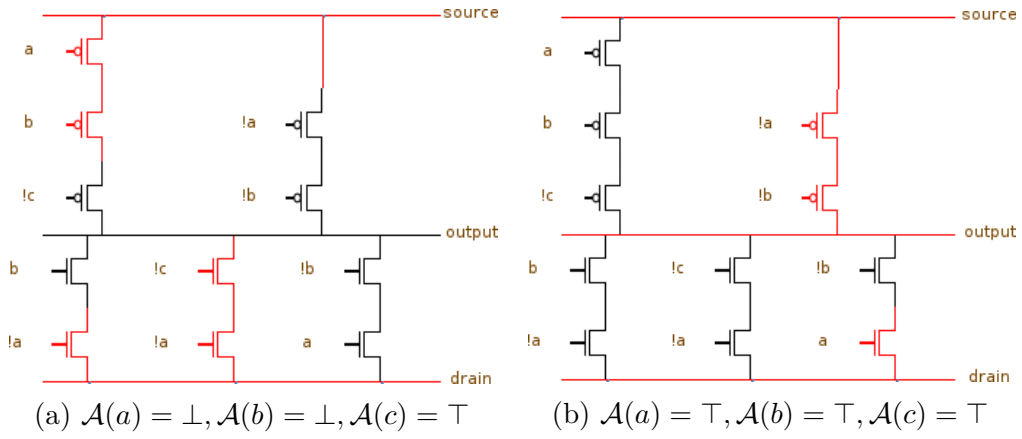


Figure 9: $(a \wedge b) \vee (\neg a \wedge \neg b \wedge c)$

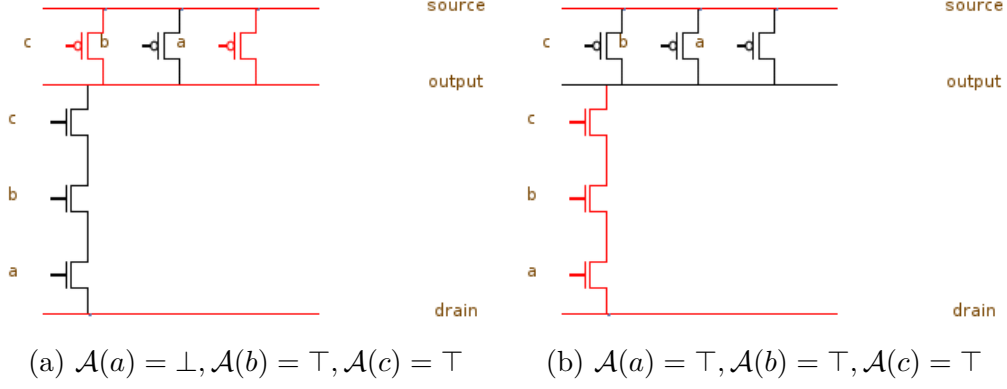


Figure 10: $a \wedge b \wedge c$ with negation applied to the result

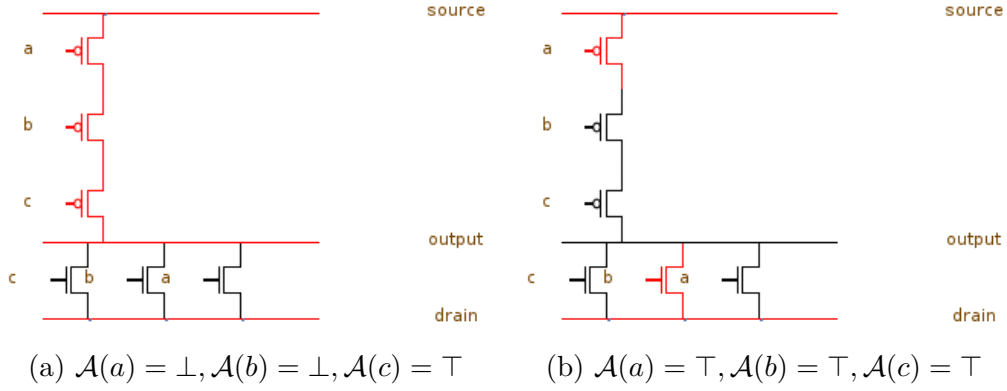


Figure 11: $a \vee b \vee c$ and $(a \vee b \vee c) \wedge (a \vee b \vee c)$ with negation applied to the result

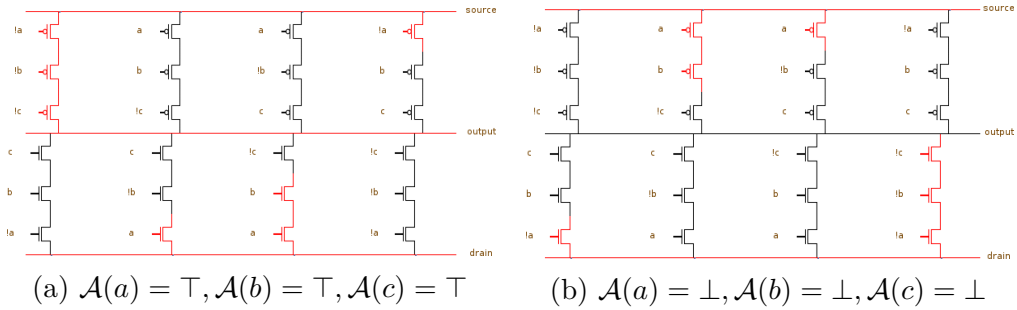


Figure 12: $a \oplus b \oplus c$