
Study the effect of training set noise over different regression algorithms and settings

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Abstract

It is very common in machine learning techniques to deal with noisy data, which may affect the accuracy of the resulting data models. For that reason, effective dealing with noise is a key aspect in machine learning to obtain reliable models from data. In this assignment, we address this issue by comparing how the noise affect linear, Lasso and Ridge regression on the final outcomes of 3 different datasets. We will compare the final outcomes from the original datasets with the outcomes from the noisy dataset using the root mean squared error as metric. We saw that adding noise either on target feature or on the remaining features of the dataset, the RMSE is getting bigger and bigger and the “fit” of the regression line is getting worst. We also noticed that noise on target feature affect more the predicted values than the noise at the other features of the dataset. For this

Keywords

Attribute noise, noisy data, performance of regression algorithms, statistical test, noise effects

1. Introduction

Machine learning techniques, and specifically supervised learning techniques, are applied to extract novel and interesting information from data collected out of real-world problems. An important characteristic of these datasets is that the data frequently contains noise. Noisy data may be the consequence of human error due to mistakes in the translation phase or due errors while collecting them. Noisy data, may produce bias to the learning process, and as a result, is more difficult for learning algorithms to form accurate models from them. Developing learning techniques that effectively deal with noisy data is a key aspect in machine learning.

In this assignment we provide a comparison of the effect of attribute noise and class noise on three regression models (a) Linear Regression, (b) Lasso Regression, (c) Ridge Regression. We are going to use 3 different datasets from different domains, each of them describes a different regression problem. The first dataset is the *air Quality* dataset that contains instances of hourly averaged responses from metal oxide chemical sensors embedded in an Air Quality Chemical Multisensor Device. [10] The second dataset is the *Computer Hardware Data Set* That contains some characteristics like cache memory, machine cycle time, etc for different computer models. [11] The third dataset is *Facebook metrics Data Set*, data is related to post's published during the year of 2014 on the Facebook's page of a renowned cosmetics brand. [12] We used these datasets to generate the "noisy" version of them. Finally, having the clean and the noisy version of each dataset, we compared the effect of noise on the models created (9 different models, for each dataset 2 different kind of noise).

Training a linear regression model is the process of finding out the most suitable coefficients for the linear function that best describes the input variables. On each step, the algorithm seeks to eliminate the error produced by the predictions and the real values. It does so by constantly seeking for a way to limit the value of a function (the so-called loss function) that helps us measure the error. In regression problems, we need evaluation metrics designed for comparing continuous values. Root Mean Square Error (RMSE) is the most commonly used regression loss function. RMSE is the sum of squared distances between our target variable and predicted values. We would like to minimize the RMSE function as much as possible, so the prediction will be as close as possible to the ground truth by updating the coefficients.

The results enable us to highlight the characteristics of the different approaches. The whole analysis is developed from the following null (initial) hypothesis. We are going to use the statistical test, two paired t-test to consider the null hypothesis.

Initial hypothesis: Given the characteristics of each learning technique, we initially propose that noisy training data don't affect the outcome of regression algorithms: Linear Regression, Ridge Regression and Lasso Regression

The structure of the present paper is as follows: in Sect. 2 summarization related work in noise analysis. In Sect. 3 presentation of the three Regression algorithms and the learning techniques. Sect. 4 description of the data sets, and explanation of the methodology used for the noise generation. Sect. 5 presents and analyzes the results of the different experiments. Sect. 6 describe and run hypothesis tests based on null hypothesis. Sect. 7 summarization the overall conclusions.

2. Related work

Understanding the impact of noisy data in the performance of machine learning algorithms is a key issue for improving algorithms reliability. Below will be described some techniques of noise generation. Noise can be added only on the target feature, at all features or a combination of them. [1] In general, label noise (known as variance at which any factor is shrunked by a noise factor) affects significantly the model. [7] In current researches, some machine learning techniques are considered more “robust” to noise, errors and missing values than others. [1] Except from that, there also many alternative multivariate linear regression methods conceived to consider data uncertainties [8], whereas other approaches like Lasso Regression known as “ ℓ_1 -penalized regression” are not equipped to deal with noisy or missing data. The idea of a statistical test and more specifically the concept of P-value as a measure of the likelihood of the observed value of the statistic under the “null hypothesis” has been introduced [16] in many researches. This kind of statistical test will also be used at this case.

2.1. Noise Generation

Noise generation can be described and aim at multiple aspects of the dataset. Not only that, but also noise can be characterized by the statistical aspects of itself. We summarize them and present the following bullets of the various aspects of noise:

- Noise can be introduced in the input attributes or/and the target feature of the data
- Noise follows a specific distribution, for example, normal or Gaussian
- Noise have values that can be relative to min, max, std (standard deviation) of each variable or to the variable value itself.

We have implemented 1 python script which generates 2 kinds of noisy datasets, noisy features and noisy target features, based on an input dataset. The process is described below.

2.1.1. Attribute noise

This noise category adds noisy data at a specific feature of the train dataset. As we want to study the effect of noise when trained with a clean dataset versus being trained with noisy train dataset, we will not alter test dataset.

Firstly, we define the percentage of noise which could be between 0%-100%, let that be the F_c variable. After using this percentage, we find out how many random variables will be generated by multiplying the percentage of noise F_c with the number of instances, so we have N random values. Now, for each feature, we generate N random numbers R_v with a Gaussian distribution and within a range between the max and min of the corresponding feature. Then we generate another random set of ids with a uniform distribution within the range 1 to N , which produces the instance ids whose data value should be overwritten by the generated noisy value. [1]

Specifically, we assign create a dictionary of lists. Each element of the dictionary is of the type {Feature name: list}, where each list contains N (number of

instances multiplied by the fraction of noise) elements. For every such element of a feature list we generate two values:

- i) Noisy value: Value that will be part of a gaussian distribution, with min and max the feature's min max value respectively, and mean value the mean value of the feature. Furthermore, we define also standard deviation to be $(\text{max}-\text{min})/6$.
- ii) Id: Each feature noisy value will have a corresponding id of a record to be altered. The id's are randomly selected using a uniform distribution that will generate unique (and not duplicate id's)

Using (i) and (ii), we assign to the record with id the (ii) id and we replace for the specific feature the noisy (i) feature value.

2.1.2. Target feature noise

This kind of noise adds noisy data at the target feature of the train dataset. We don't modify test data at this case also. The process is the same as before, as we seek to generate noise values for one feature instead of the multiple features we did on the previous feature case. Thus, we generate noisy values and ids for the target feature only.

2.1.4. Level of noise

We have added 3 different level of noise: 15%, 35%, 50%, so we will generate 6 noisy datasets for each different dataset.

2.2 Stratified split for test train dataset

We wanted to create two different datasets for the implementation of noise effect. The first dataset would be the train one and the second one would be the test one. The test dataset will not contain any noise as stated previously. For the train dataset, we create two different versions of it, one with noise for the attribute and one with noise for the class dataset.

What is important here is the fact that we needed to perform the split in a stratified way. Thus, we wanted for the test and the train distribution to be as similar as possible. Due to the fact that for regression purposes, sciikit learning was not able to perform the split, we decided to make it manually, implementing it as explained below:

1. We sort the dataset based on the target feature
2. We split it the sorted dataset to lists of 10 items.
3. The proportion of split to test and train is the number of items we select out of the 10 items. We decided to split the dataset based on 80 % train – 20% test, so the number of elements taken from the sorted 10 items would be 2 items. Those 2 items would be added to the test dataset, whereas the remaining 8 are part of the training dataset.

As we see by that, we force with the way we create the datasets that the distribution of the train dataset would be preserved and be present also to the test dataset.

3. Regression algorithms

3.1 Regression techniques

Regression analysis is a form of predictive modelling technique which investigates the relationship between a dependent and independent variable. This technique is used for forecasting, time series modelling and finding the curve / line to the data points, in such a manner that the differences between the distances of data points from the curve or line is minimized.

One of the first steps in a regression analysis is to determine if multicollinearity is a problem. Multicollinearity, or collinearity, is the existence of near-linear relationships among the independent variables. Multicollinearity causes two basic types of problems. The coefficient estimates can swing rapidly based on which other independent variables are in the model, so coefficients become very sensitive to small changes in the model. Multicollinearity reduces the precision of the estimate coefficients, which makes the statistical power of the regression model weak.

In regression problems, we need evaluation metrics designed for comparing continuous values. We would like to minimize the MSE (loss function) for a specific data-point as much as possible so the prediction will be as close as possible to the ground truth. This is done by learning the parameters, executing an iterative process that updates coefficients at every step and reduce the loss function as much as possible until the minimum point of the loss function has been reached.

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$$

Where n: number of instances, \hat{Y} : predicted values, Y: observed values

3.2. Linear regression

Linear Regression establishes a relationship between dependent variable (Y) and one or more independent variables (X) using a best fit straight line. The dependent variable is continuous, independent variable(s) can be continuous or discrete, and the regression line is linear.

Its equation is $Y = a + b_1X_1 + b_2X_2 + b_3X_3 + \dots$

where Y: the response, a: the intercept, b: model coefficients, X_n (the nth feature)

This equation can be used to predict the value of target variable based on given predictor variable(s). We would like to minimize the MSE (loss function) for a specific data-point as much as possible. Linear Regression is very sensitive to Outliers. Having noisy data can influence on the computation of MSE and as a result the regression line. Prediction errors can occur due to two sub components or combination of them. First component is the bias and the second is the variance (multicollinearity problem). [2]

3.3. Ridge Regression

Ridge regression adds “squared magnitude” of coefficient as penalty term to the loss function, it is known as L2 regularization. It is another regression method and used when the data suffers from multicollinearity, independent variables are highly correlated. In multicollinearity, even though the least squares estimates are unbiased, their variances are large. Large variances mean that the observed value is far from the true value. Adding a degree of bias, standard errors are reduced. Ridge regression solves the multicollinearity problem through shrinkage parameter λ (lambda). Ridge regression is a well-known approach to dealing with noisy data. [4]

Its equation is $Y=a+b*X+\epsilon$

where Y: the response, a: the intercept, b: model coefficients, X_n (the nth feature), ϵ : the error. Error term is the value needed to correct the prediction error between the observed and predicted value. The highlighted part below represents L2 regularization element. [6]

$$\sum_{i=1}^n (y_i - \sum_{j=1}^p x_{ij}\beta_j)^2 + \lambda \sum_{j=1}^p \beta_j^2$$

If *lambda* is very large then it will add too much weight and it will lead to under-fitting, if *lambda* equals to zero then we have linear regression. We can understand that it's important how *lambda* is chosen. Minimizing the sum of the squares of the vertical deviations from each data point to the line enforce β coefficients to be lower but it does not enforce them to be zero.

3.4. Lasso Regression

Lasso Regression (Least Absolute Shrinkage and Selection Operator) adds “*absolute value of magnitude*” of coefficient as penalty term to the loss function, its known as L1 regularization. It penalizes the absolute size of the regression coefficients, reduces the variability and improving the accuracy of linear regression models.

Lasso regression differs from ridge regression in a way that it uses absolute values in the penalty function, instead of squares. This leads to penalizing values which causes some of the parameter estimates to turn out exactly zero. Larger the penalty applied, further the estimates get shrunk towards absolute zero. Therefore, you might end up with fewer features included in the model than you started with, which is a huge advantage. Lasso regressor is not equipped to deal with noisy or missing data. [2]

Its equation is $Y=a+b*X+\epsilon$

where Y: the response, a: the intercept, b: model coefficients, X_n (the nth feature), ϵ : the error. Error term is the value needed to correct the prediction error between the

observed and predicted value. The highlighted part below represents L1 regularization element. [6]

$$\sum_{i=1}^n (Y_i - \sum_{j=1}^p X_{ij} \beta_j)^2 + \lambda \sum_{j=1}^p |\beta_j|$$

If λ is a large value will make coefficients zero hence it will under-fit, if λ equals to zero then we have linear regression. The key difference between Lasso and Ridge techniques is that Lasso shrinks the less important feature's coefficient to zero thus, removing some feature altogether. So, this is effective for feature selection in case we have a very big number of features.

4. Design of the dataset

In our experiments we will use the 3 different datasets. All of them are based on real data (data descriptions of real objects or events), may contain a certain amount of background noise, erroneous values, and so on. All the data sets have as target numerical attributes. Forced noise will be added only at the train data. Before adding noise, we have splitted the dataset into train and test.

4.1. Air-Quality

The air quality dataset contains 9358 instances of hourly averaged responses from an array of 5 metal oxide chemical sensors embedded in an Air Quality Chemical Multisensor Device and 15 features. The device was located on the field in a significantly polluted area, at road level, within an Italian city. Data were recorded from March 2004 to February 2005 (one year) representing the longest freely available recordings of on field deployed air quality chemical sensor devices responses. At this case, we try to find the best regression model at which the predict the amount of C6H6(GT) will be predicted. There is a readme file available at which there are detailed information about the dataset. [10]

As said before, this is a real dataset so for sure it contains noisy data. Because we don't know exactly the percentage of noise we generated other datasets based on this at which we will include several noise levels on the features and on the target feature. The generated datasets contain noise data at the target class – C6H6(GT) - whereas the other noisy datasets contains noise data at all features of the dataset except the target one. The generation of noise has been analyzed on Sect. [2.1](#).

4.2. Computer Hardware

The computer hardware dataset contains 209 instances and 10 features which describe the characteristic of a computer: model name, cache memory, machine cycle, published performance, etc. At this case, we try to find the best regression model at which the performance of the computer will be predicted. There is a readme file available at which there are detailed information about the dataset. [11] The process of noise generation is the same with the *Air-quality dataset*.

4.3. Facebook metrics

The Facebook metrics dataset contains 500 instances and 19 attributes. Some of them are known prior to post publication and the 12 remaining features used to evaluate the post impact. The data is related to posts' published during the year of 2014 on the Facebook's page of a renowned cosmetics brand. At this case, we try to find the best regression model at which the -Total Interactions -with a Facebook page will be predicted. There is a readme file available at which there are detailed information about the dataset. [12] The process of noise generation is the same with the *Air-quality dataset*.

5. Results of the experiments

At all cases we haven't added noise on test data. Test dataset is the same for all different cases. We have run all regression problems for 6 different noisy data.

x_f_train_noisy.xlsx → x is the percentage of noise at all the features of the dataset

x_c_train_noisy.xlsx → x is the percentage noise at the target feature of the dataset

5.1. Air-quality

Regarding the air-quality houses we run 3 regression algorithms for different noise levels and types of noise which are listed below. We have used the root mean squared error as metric. We can see that using noisy data for training the model we result in bigger RMSE, so the “fit” of the regression line is getting worst.

Linear

Table 1: RMSE Linear Regression

Datasets	RMSE
clean data:	1.1389
0.15_c_train_noisy.xlsx	19.9225
0.15_f_train_noisy.xlsx	12.4318
0.35_c_train_noisy.xlsx	47.5515
0.35_f_train_noisy.xlsx	22.5007
0.50_c_train_noisy.xlsx	68.4762
0.50_f_train_noisy.xlsx	28.1405

Lasso

Table 2: RMSE Lasso Regression

Datasets	RMSE
clean data:	1.1908
0.15_c_train_noisy.xlsx	19.9185
0.15_f_train_noisy.xlsx	12.4300
0.35_c_train_noisy.xlsx	47.5579
0.35_f_train_noisy.xlsx	22.4879
0.50_c_train_noisy.xlsx	68.4858
0.50_f_train_noisy.xlsx	28.1286

Ridge

Table 3: RMSE Ridge Regression

Datasets	RMSE
clean data:	1.1389
0.15_c_train_noisy.xlsx	19.9225
0.15_f_train_noisy.xlsx	12.4318
0.35_c_train_noisy.xlsx	47.5515
0.35_f_train_noisy.xlsx	22.5007
0.50_c_train_noisy.xlsx	68.4762
0.50_f_train_noisy.xlsx	28.1405

From the next plots, Figure 1, Figure 2 we can see that adding several levels of noise, RMSE is getting bigger.

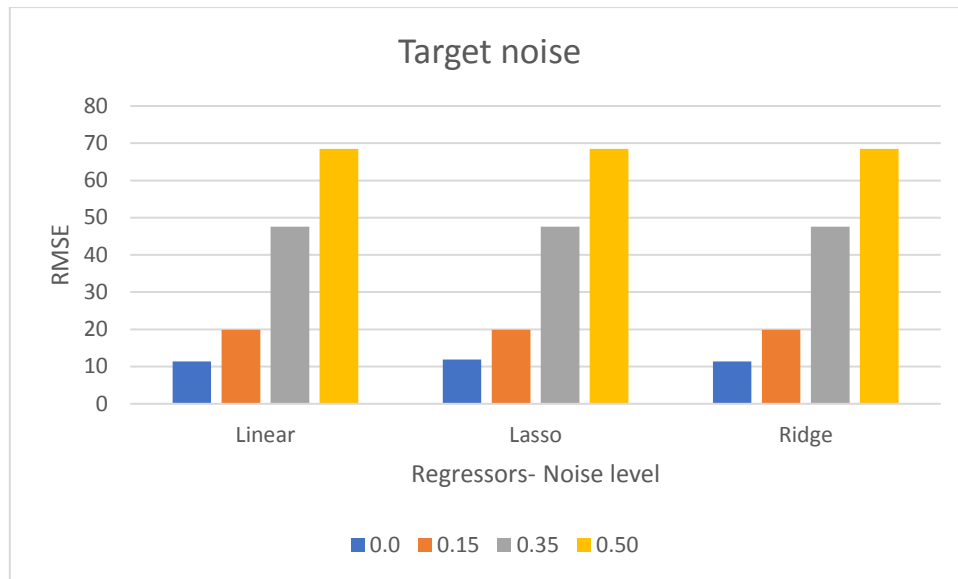


Figure 1:RMSE when target noise added

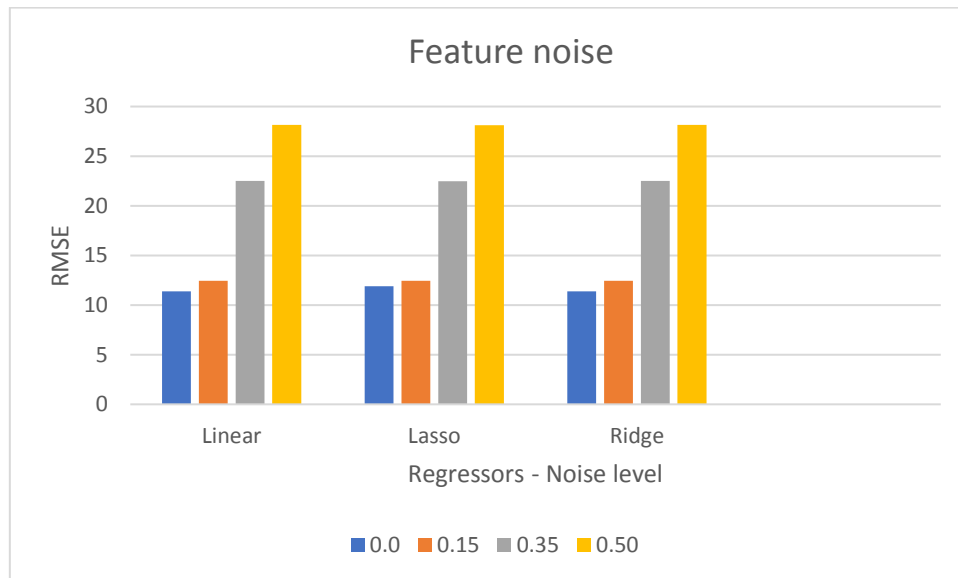


Figure 2:RMSE when feature noise added

Also, we can see that target noise affects more the outcome of the model than the feature noise. Where the noise level is big ($> 35\%$) RMSE of the dataset with noisy target feature is two times the RMSE of the datasets with noisy features and clean the target feature, see Figure 3.

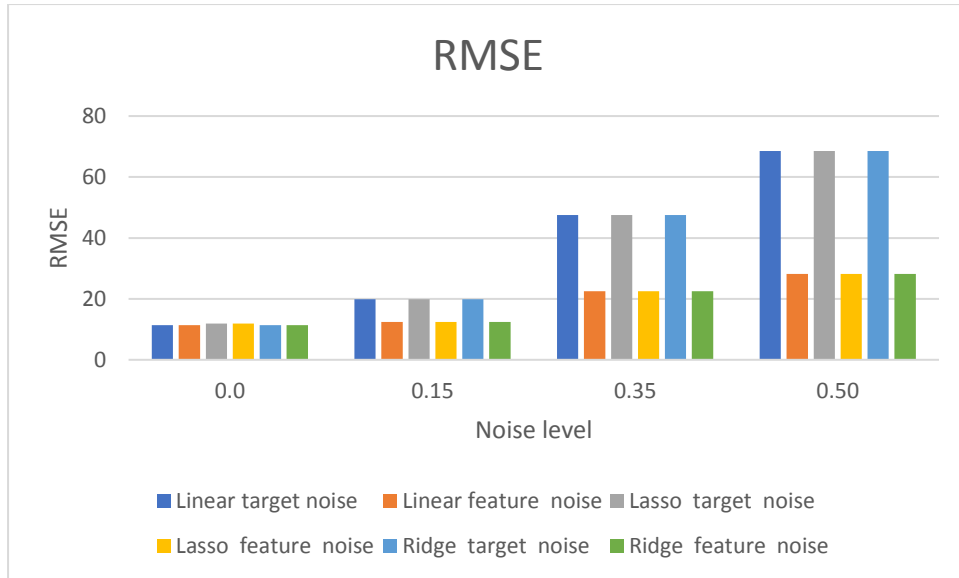


Figure 3: RMSE with target - features noise

5.2. Computer Hardware

We can notice the same behavior on computer hardware dataset. Running 3 regression algorithms for different noise levels and types of noise and using the root mean squared error as metric, we can see that using noisy data for training the model we result in bigger RMSE, so the “fit” of the regression line is getting worst.

Linear

Table 4: RMSE Linear Regression

Datasets	RMSE
clean data:	38.5487
0.15_c_train_noisy.xlsx	93.7774
0.15_f_train_noisy.xlsx	65.2908
0.35_c_train_noisy.xlsx	212.6189
035_f_train_noisy.xlsx	110.6693
0.50_c_train_noisy.xlsx	280.8337
0.50_f_train_noisy.xlsx	147.3978

Lasso

Table 5: RMSE Lasso Regression

Datasets	RMSE
clean data:	38.5239
0.15_c_train_noisy.xlsx	93.7374
0.15_f_train_noisy.xlsx	65.2561
0.35_c_train_noisy.xlsx	212.6021
035_f_train_noisy.xlsx	110.6317
0.50_c_train_noisy.xlsx	280.7999
0.50_f_train_noisy.xlsx	147.3213

Ridge

Table 6: RMSE Ridge Regression

Datasets	RMSE
clean data:	38.5486
0.15_c_train_noisy.xlsx	93.7772
0.15_f_train_noisy.xlsx	65.2904
0.35_c_train_noisy.xlsx	212.6189
035_f_train_noisy.xlsx	110.6692
0.50_c_train_noisy.xlsx	280.8332
0.50_f_train_noisy.xlsx	147.3973

The above results Table 4, Table 5, Table 6, are displayed also at the below diagrams. It is obvious that adding noise, RMSE is getting bigger.

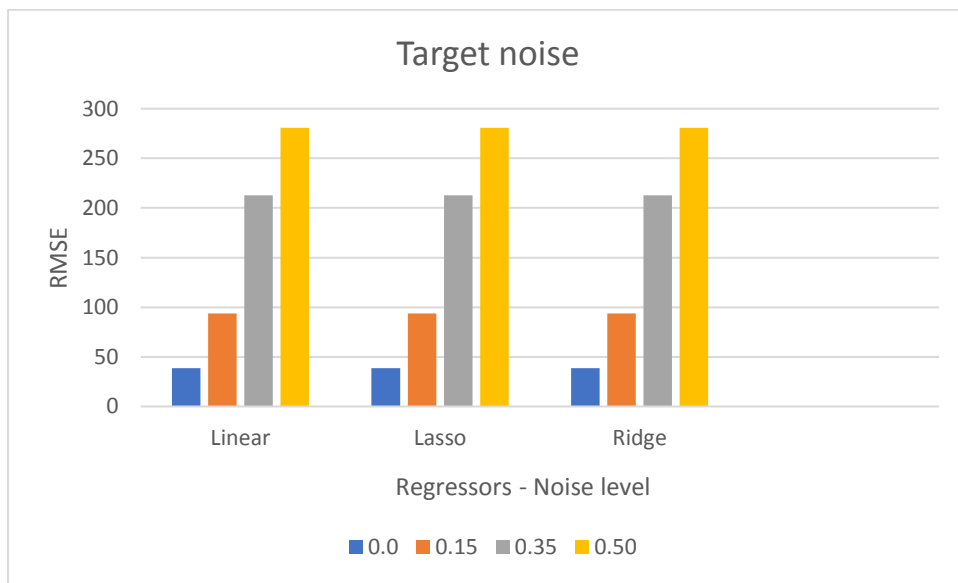


Figure 4: RMSE when target noise added

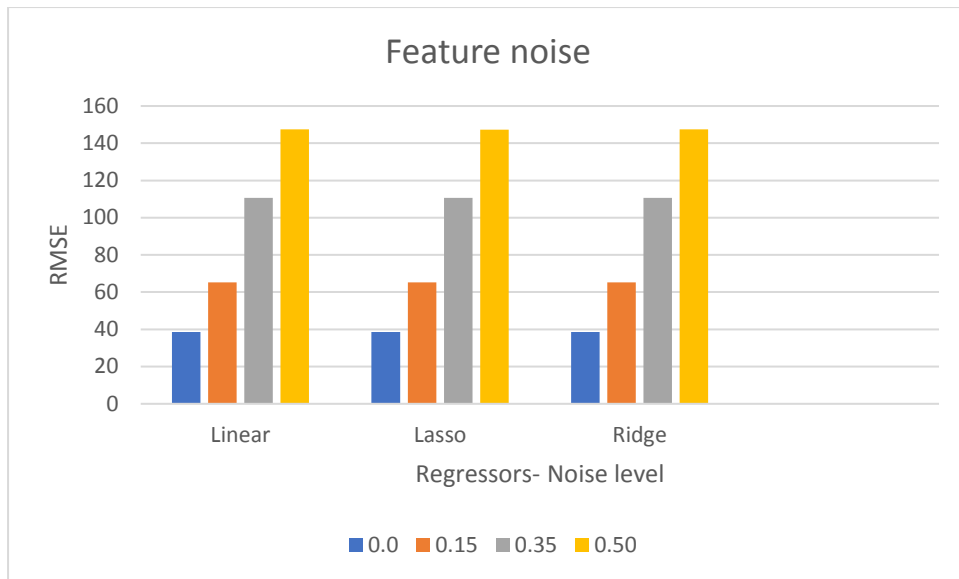


Figure 5: RMSE when feature noise added

Also, we can see that target noise affects more the outcome of the model than the feature noise. Where the noise level is big ($> 35\%$) RMSE of the dataset with noisy target feature is two times the RMSE of the datasets with noisy features and clean the target feature, see Figure 6.

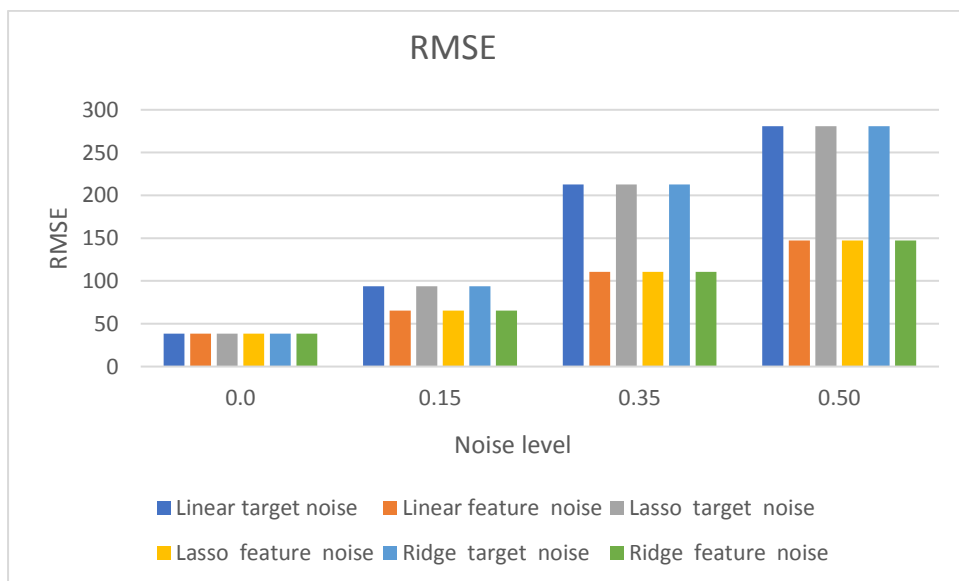


Figure 6: RMSE with target - features noise

5.3. Facebook Metrics

The same behavior is noticed on Facebook metrics dataset, also. Running 3 regression algorithms for different noise levels and types of noise and using the root mean squared error as metric, we can see that using noisy data for training the model we result in bigger RMSE, so the “fit” of the regression line is getting worst.

Linear

Table 7: RMSE Linear Regression

Datasets	RMSE
clean data:	0.0000
0.15_c_train_noisy.xlsx	498.5506
0.15_f_train_noisy.xlsx	247.5395
0.35_c_train_noisy.xlsx	1183.3034
035_f_train_noisy.xlsx	314.5667
0.50_c_train_noisy.xlsx	1712.6426
0.50_f_train_noisy.xlsx	379.1050

Lasso

Table 8: RMSE Lasso Regression

Datasets	RMSE
clean data:	5.7487
0.15_c_train_noisy.xlsx	499.0478
0.15_f_train_noisy.xlsx	247.5375
0.35_c_train_noisy.xlsx	1156.2907
035_f_train_noisy.xlsx	314.5598
0.50_c_train_noisy.xlsx	1694.8640
0.50_f_train_noisy.xlsx	379.0970

Ridge

Table 9: RMSE Ridge Regression

Datasets	RMSE
clean data:	0.0002
0.15_c_train_noisy.xlsx	498.5505
0.15_f_train_noisy.xlsx	247.5395
0.35_c_train_noisy.xlsx	1183.3022
035_f_train_noisy.xlsx	314.5667
0.50_c_train_noisy.xlsx	1712.6423
0.50_f_train_noisy.xlsx	379.1050

All these results are also displayed at the next plots, where adding noise the RMSE is getting bigger and far away from the best fit.

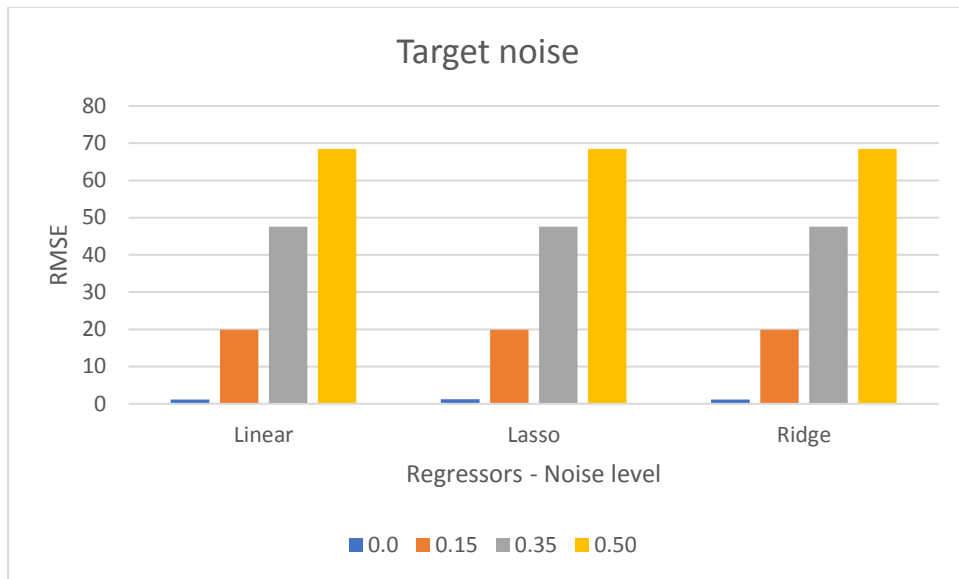


Figure 7: RMSE when target noise added

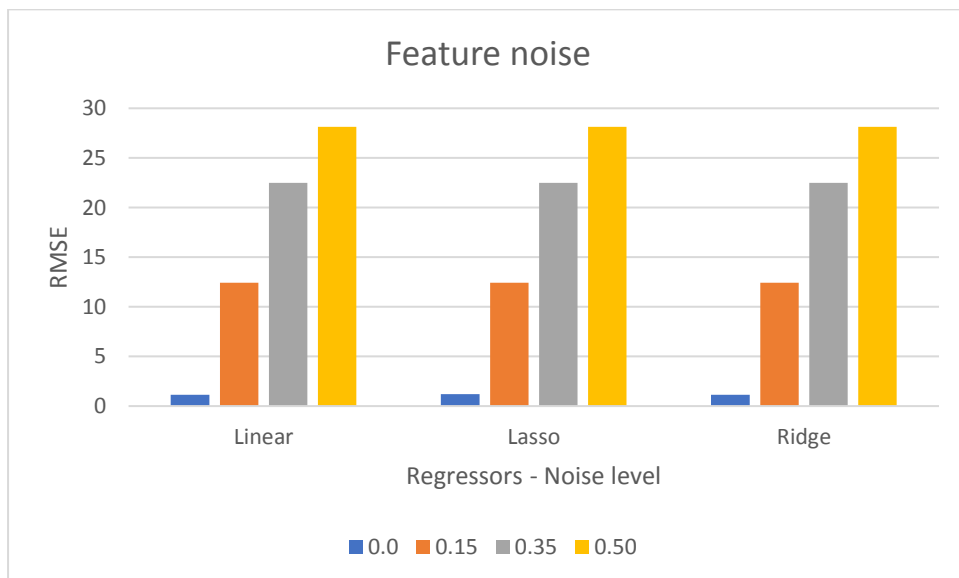


Figure 8: RMSE when feature noise added

RMSE, where noise level is bigger than 35% is two times bigger on dataset that have noisy target feature in comparison with dataset which have just noisy features and clean target value, where target is the predicted value from the regression model, see Figure 9.

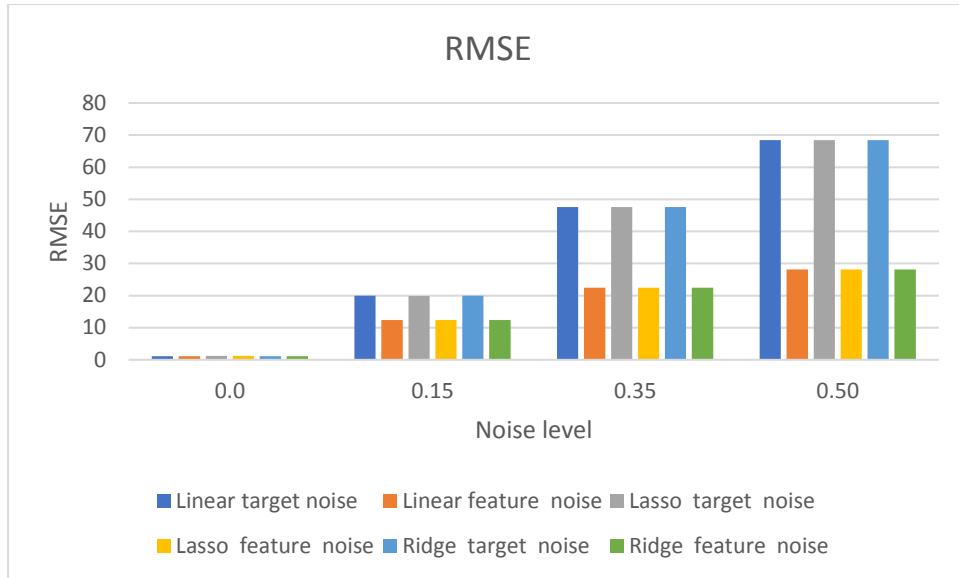


Figure 9: RMSE with target - features noise

6. Hypothesis testing

6.1. Null hypothesis

Hypothesis testing is a way to test the results of a survey or experiment. As first step the null hypothesis should be defined. The null hypothesis is a general statement or default position that there is not, or there is a relationship between some measured phenomena. Hypothesis testing using some statistical tests which will reject or no the null hypothesis. If the null hypothesis will be rejected then the initial hypothesis we have done is fault, otherwise is true. We have defined the null hypothesis that include some main questions on the dataset.

Null hypothesis

Null hypothesis – H_0 : Given the characteristics of each learning technique, we initially propose that noisy training data, don't affect negatively the predictions of regression algorithms: Linear Regression, Ridge Regression and Lasso Regression, using 3 clean datasets and the noisy versions of them.

Alternative hypothesis – H_1 : Given the characteristics of each learning technique, noisy training data, affect the final outcome of regression algorithms: Linear Regression, Ridge Regression and Lasso Regression, using 3 datasets an the noisy versions of them. It is equivalent to different means for the samples of the dataset.

Some questions are listed below:

1. Does the noise level affect the final outcomes?
2. Does the noise level in combination with the dataset size affect the final outcomes?
3. Which type of noise affect most the final outcomes, feature noise or target noise?
4. Which algorithm is more robust to noisy data?

6.2. Two sample t-test

As mentioned before, we are interested in comparing the performance of the different learning techniques on data with different proportions of attribute noise and class feature noise in the training data set, using the same test dataset. For that scope we will use the Independent t -test that is used to determine if two population means are equal or not. More specifically a paired t -test. It is about doing two different tests on same dataset (the test dataset is the same, we just change the train dataset).

With a two-sample t test, we are comparing the means for two different samples. Paired t -test is used when there are two measurements on the same item. T score is a ratio between the difference between two groups. The larger the t score, the more difference there is between groups. Each t -value has a p -value to go with it. A p -value is the probability that the results from your sample data occurred by chance. P -values are from 0% to 100%. In most cases, a p -value of 0.05 (5%) is accepted to mean the data is valid.

As we would like to compare the noisy datasets with the clean version of them we will use this type of statistical test, two sample t -test. At all cases we will use the clean and the noisy version of a dataset and we will do a two samples t -test with the predicted target values of these datasets (test dataset) to reject the null hypothesis and accept the alternative hypothesis.

6.3. Running the hypothesis testing

Using the predicted values of the target feature, the means and the variances from clean and noisy data, we calculated the t -test using the *ttest* from *spicy*.

The above null hypothesis will be rejected if the calculated p -value is less than the significance level, at point 0.05 then the test concludes that there is a statistically significant difference between the two populations. In any other case, there is no statistically significant difference between the two populations and the test fails to reject the null hypothesis.

We run hypothesis testing between the predicted values from a model which has been trained with clean data and the predicted values from a model which has been trained with noisy data. For these two different lists we have calculated the t -score and the p -value.

6.3.1. Air-quality

At the next tables are displayed the results from t-test. At all different cases the p-value is significantly less than 0.05 which is the significance level and as a result the null hypothesis is rejected, means that the two predicted populations have totally different mean. Different means result in different predicted values between the clean and the noisy datasets.

Linear

Table 10: T-test Linear Regression

Datasets	t-test	Null hypothesis
0.15_c_train_noisy.xlsx	t = 15.03362190118589 p = 2.4227230583356263e-49	Rejected
0.15_f_train_noisy.xlsx	t = -7.228143293466815 p = 1.1813135455456348e-12	Rejected
0.35_c_train_noisy.xlsx	t = 39.5475794804161 p = 8.266978887459529e-286	Rejected
035_f_train_noisy.xlsx	t = -14.685989904088236 p = 3.252659908298373e-47	Rejected
0.50_c_train_noisy.xlsx	t = 61.20928403696941 p = 0.0	Rejected
0.50_f_train_noisy.xlsx	t = -18.988919219794894 p = 1.551783631564541e-76	Rejected

Lasso

Table 11: T-test Lasso Regression

Datasets	t-test	Null hypothesis
0.15_c_train_noisy.xlsx	t = -15.043846576693332 p = 2.0943736404231374e-49	Rejected
0.15_f_train_noisy.xlsx	t = 7.224901410377084 p = 1.2094416317402355e-12	Rejected
0.35_c_train_noisy.xlsx	t = -39.561663832262965 p = 5.579900047636842e-286	Rejected
035_f_train_noisy.xlsx	t = 14.678922135650181 p = 3.5896096720653385e-47	Rejected
0.50_c_train_noisy.xlsx	t = -61.23352152280937 p = 0.0	Rejected
0.50_f_train_noisy.xlsx	t = 18.98518611916189 p = 1.6557187465791873e-7	Rejected

Ridge

Table 12: T-test Ridge Regression

Datasets	t-test	Null hypothesis
0.15_c_train_noisy.xlsx	t = -15.033622988255889 p = 2.4226855565123957e-49	Rejected
0.15_f_train_noisy.xlsx	t = 7.228143787255435 p = 1.181309310534366e-12	Rejected
0.35_c_train_noisy.xlsx	t = -39.54757995436419 p = 8.266869540334386e-286	Rejected
035_f_train_noisy.xlsx	t = 14.68599046285735 p = 3.252634555515201e-47	Rejected
0.50_c_train_noisy.xlsx	t = -61.209285335015494 p = 0.0	Rejected
0.50_f_train_noisy.xlsx	t = 18.988919778258154 p = 1.5517685805312463e-76	Rejected

Null hypothesis is rejected for the datasets *Air-Quality*.

6.3.2. Computer Hardware

At the next tables are displayed the results from t-test for the *computer-hardware* dataset. At all different cases the p-value is significantly less than 0.05 which is the significance level except the case of adding 15% noise on target features in all regressors. Nevertheless, at all other cases the null hypothesis is rejected, means that the two predicted populations have totally different mean.

Linear

Table 13: T-test Linear Regression

Datasets	t-test	Null hypothesis
0.15_c_train_noisy.xlsx	t = 2.59937155659267 p = 0.022146104149973568	Rejected
0.15_f_train_noisy.xlsx	t = -1.7196334142059024 p = 0.17854324839801744	Accepted
0.35_c_train_noisy.xlsx	t = 7.781581311296905 p = 3.777893986536076e-11	Rejected
035_f_train_noisy.xlsx	t = -3.6423354316616776 p = 0.0009441221620696245	Rejected
0.50_c_train_noisy.xlsx	t = 9.949216557025935 p = 1.881149068254537e-15	Rejected
0.50_f_train_noisy.xlsx	t = -5.263466491543089 p = 2.221484469902549e-06	Rejected

Lasso

Table 14: T-test Lasso Regression

Datasets	t-test	Null hypothesis
0.15_c_train_noisy.xlsx	t = -2.6000261121447457 p = 0.022107184605789983	Rejected
0.15_f_train_noisy.xlsx	t = 1.718335842968734 p = 0.17901828870808645	Accepted
0.35_c_train_noisy.xlsx	t = -7.782902373199898 p = 3.755272298040809e-11	Rejected
035_f_train_noisy.xlsx	t = 3.6408050431594625 p = 0.0009489609706677256	Rejected
0.50_c_train_noisy.xlsx	t = -9.953183777599909 p = 1.8474882062332565e-15	Rejected
0.50_f_train_noisy.xlsx	t = 5.260850785621607 p = 2.2452270655321374e-06	Rejected

Ridge

Table 15: T-test Ridge Regression

Datasets	t-test	Null hypothesis
0.15_c_train_noisy.xlsx	t = -2.5993734616277293 p = 0.02214599078766579	Rejected
0.15_f_train_noisy.xlsx	t = 1.7196205488090526 p = 0.17854795333477622	Accepted
0.35_c_train_noisy.xlsx	t = -7.781580832954946 p = 3.7779022021471245e-11	Rejected
035_f_train_noisy.xlsx	t = 3.6423282812016886 p = 0.000944144715924029	Rejected
0.50_c_train_noisy.xlsx	t = -9.949261316941232 p = 1.8807658833138953e-15	Rejected
0.50_f_train_noisy.xlsx	t = 5.263445287288487 p = 2.2216759470570315e-06	Rejected

Except one case, null hypothesis is rejected for the datasets *computer-hardware*.

6.3.3. Facebook Metrics

At the next tables are displayed the results from t-test for the *Facebook-metrics* dataset. At all different cases the p-value is significantly less than 0.05 which is the significance level means that the null hypothesis is rejected, and the two predicted populations have totally different mean.

Linear

Table 16: T-test Linear Regression

Datasets	t-test	Null hypothesis
0.15_c_train_noisy.xlsx	t = 11.95211307103593 p = 7.685570854667642e-25	Rejected
0.15_f_train_noisy.xlsx	t = 4.9005157995393684 p = 3.9688644287990675e-06	Rejected
0.35_c_train_noisy.xlsx	t = 18.421168972540805 p = 1.6899228875090092e-44	Rejected
035_f_train_noisy.xlsx	t = -6.994060996076745 p = 8.027350844185886e-11	Rejected
0.50_c_train_noisy.xlsx	t = 23.585304041706397 p = 4.114654940321093e-59	Rejected
0.50_f_train_noisy.xlsx	t = -9.636375039246415 p = 5.790043884750027e-18	Rejected

Lasso

Table 17: T-test Lasso Regression

Datasets	t-test	Null hypothesis
0.15_c_train_noisy.xlsx	t = -12.01185537818384 p = 5.061022059851135e-25	Rejected
0.15_f_train_noisy.xlsx	t = 4.9005157995393684 p = 3.9688644287990675e-06	Rejected
0.35_c_train_noisy.xlsx	t = -19.967974118806612 p = 5.1184010814082e-49	Rejected
035_f_train_noisy.xlsx	t = 7.033961534942495 p = 6.387493849907074e-11	Rejected
0.50_c_train_noisy.xlsx	t = -24.55296331684178 p = 1.0800501108763166e-61	Rejected
0.50_f_train_noisy.xlsx	t = 9.704632326051211 p = 3.6809095413735244e-18	Rejected

Ridge

Table 18: T-test Ridge Regression

Datasets	t-test	Null hypothesis
0.15_c_train_noisy.xlsx	t = -11.952122879452345 p = 7.685043912704029e-25	Rejected
0.15_f_train_noisy.xlsx	t = 4.88804279690139 p = 4.199789518869917e-06	Rejected
0.35_c_train_noisy.xlsx	t = -18.421228483074326 p = 1.6892384020422573e-44	Rejected
035_f_train_noisy.xlsx	t = 6.994060404455819 p = 8.027377999079204e-11	Rejected
0.50_c_train_noisy.xlsx	t = -23.58532495271241 p = 4.1141213211074254e-59	Rejected
0.50_f_train_noisy.xlsx	t = 9.636374069662848 p = 5.7900810963047114e-18	Rejected

Null hypothesis is also rejected for the datasets *Facebook-metrics*.

7. Conclusions

Comparing the outcomes from the original datasets with the outcomes from the noisy dataset using the two-sample paired t-test and 3 datasets from different domains, we conclude that adding noise either on target feature or on the remaining features of the dataset, the noise in the end will affect the outcomes of the regression problem. More specifically, the RMSE is getting bigger and bigger and the “fit” of the regression line is getting worst. Another important notice is that noise on target feature affect more the predicted values than the noise at the other features of the dataset, results in bigger RMSE.

7.1. Repository

Code Is available on GitHub on that link: <https://github.com/JoHNNyB92/applied/>

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