V.a.c

X v.a.c → densité

[la loi Uniforme]

$$f(x) = \begin{cases} \frac{1}{b-a} & x \in [a;b] \\ 0 & \text{sinon} \end{cases}$$

$P(X \leq t) = F(t)$

$\boxed{\mathcal{E}(\lambda) \; ; \lambda > 0}$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geqslant 0 \\ 0 & \text{sinon} \end{cases}$$

Normale

$N(0,1)$

centré réduite

Générale

moyenne

$N(\mu, \sigma)$

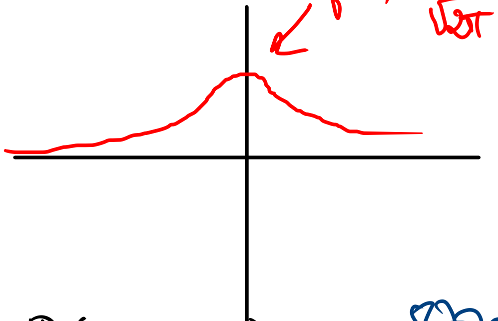
écart type

standard deviation

	Densité	Fonction de répartition	Espérance	Variance
$\mathcal{U}([a;b])$	$f(t) = \begin{cases} \frac{1}{b-a} & \text{si } t \in [a;b] \\ 0 & \text{sinon} \end{cases}$	$F(x) = \begin{cases} 0 & \text{si } x < a \\ \frac{x-a}{b-a} & \text{si } x \in [a;b] \\ 1 & \text{si } x > b \end{cases}$	$E(X) = \frac{a+b}{2}$	$V(X) = \frac{(b-a)^2}{12}$
$\mathcal{E}(\lambda)$	$f(t) = \begin{cases} 0 & \text{si } t < 0 \\ \lambda e^{-\lambda t} & \text{si } t \geqslant 0 \end{cases}$	$F(x) = \begin{cases} 0 & \text{si } x < 0 \\ 1 - e^{-\lambda x} & \text{si } x \geqslant 0 \end{cases}$	$E(X) = \frac{1}{\lambda}$	$V(X) = \frac{1}{\lambda^2}$

$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

Normale centrée réduite $X \sim N(0,1)$



$\boxed{a \geq 0}$

$P(X \leq a)$

Tableau

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8557	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

$P(X \leq 1,17) = 0,8790$

$1.17 = 1.10 + 0,07$

$P(X > 1,32)$

$\|$

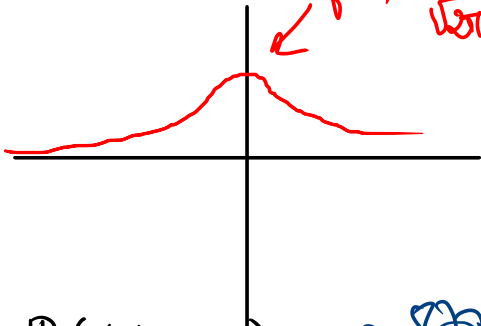
$1 - P(X \leq 1,32)$

$= 1 - 0,9066$

$= 0,0934$

Normale centré réduite $X \sim N(0,1)$

$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$



$\boxed{a \geq 0}$

$P(X \leq a)$ → Tableau

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
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1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

$P(X \leq 1,17) = 0,8790$

$1.17 = 1.10 + 0,07$

$P(X > 1,32)$

$= 1 - P(X \leq 1,32)$

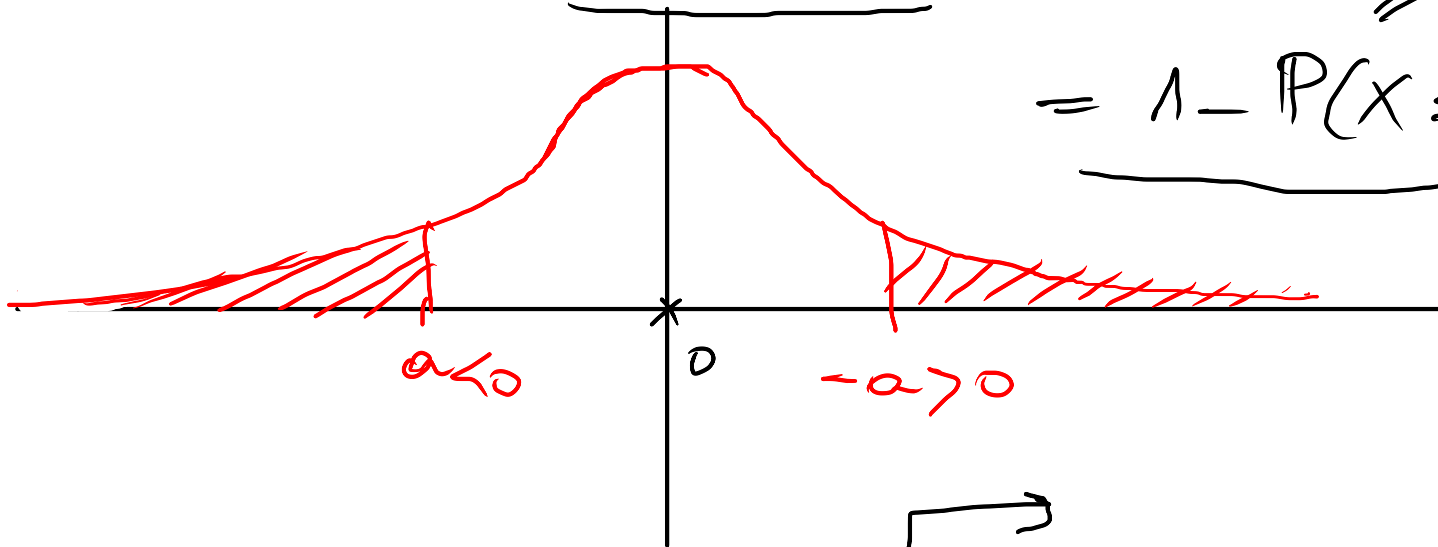
$= 1 - 0,9066$

$= 0,0934$

$X \sim N(0,1)$

$\underline{\underline{a<0}}$ $\quad \mathbb{P}(X \le a) = \mathbb{P}(X \ge -a)$

$= 1 - \mathbb{P}(X \le -a)$



$\underline{\underline{\text{ex}}}$ $\quad \mathbb{P}(X \le -0{,}85) = 1 - \mathbb{P}(X \le 0{,}85)$

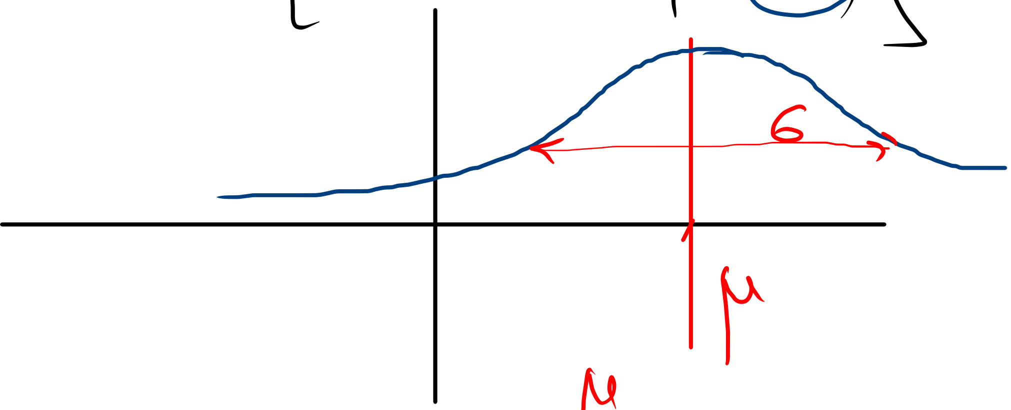
$= 1 - 0.8023 \dots$

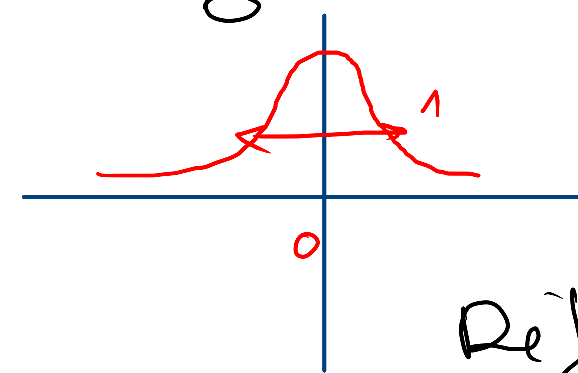
$$\mathbb{P}(\alpha \le X \le \beta) = \mathbb{P}(X \le \beta) - \mathbb{P}(X \le \alpha)$$

$\boxed{a>0}$ $\quad \left[\mathbb{P}(-a \le X \le a) = 1 - 2\mathbb{P}(X \le -a) \dots \right.$

$\left. = 2\mathbb{P}(X \le a) - 1 \right]$

$$[X \sim N(\mu, \sigma)] \xrightarrow{\text{Transformation}} \frac{X-\mu}{\sigma} \sim N(0,1)$$





Reference

$$X \sim N(\overset{\mu}{10}, \underset{\sigma}{3})$$

$N(0,1)$

$$P(X \leqslant 12) = P\left(\frac{X-10}{3} \leqslant \frac{12-10}{3}\right)$$

$$\simeq P(Z \leqslant 0,67) \longrightarrow \text{Table}$$

$$= 0,7486 \ \boxtimes$$

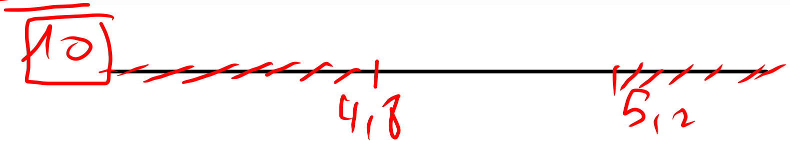
$X \sim N(5, 0{,}2)$ unité centimètre

95. A machine produces bolts the length of which (in centimeters) obeys a normal probability law with mean 5 and standard deviation $\sigma = 0.2$. A bolt is called defective if its length falls outside the interval (4.8, 5.2).

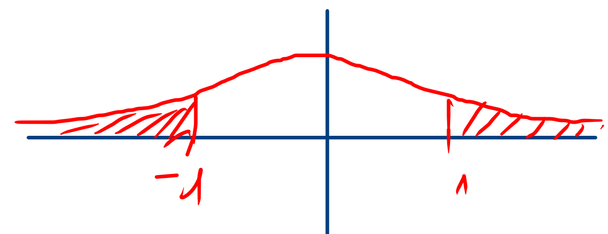
(a) What is the proportion of defective bolts that this machine produces?

(b) What is the probability that among ten bolts none will be defective?

$D = \{X \notin [4{,}8 , 5{,}2]\}$



(a) $\mathbb{P}(D) = \mathbb{P}(X \notin [4{,}8 , 5{,}2])$



$= \mathbb{P}(X < 4{,}8) + \mathbb{P}(X > 5{,}2)$

$Z = \frac{X-5}{0{,}2} \sim N(0,1)$

$= \mathbb{P}\left(\frac{X-5}{0.2} < \frac{4.8-5}{0.2}\right) + \mathbb{P}\left(\frac{X-5}{0.2} > \frac{5.2-5}{0.2}\right)$

1.0 | 0.8413

$= \mathbb{P}(Z < -1) + \mathbb{P}(Z > 1) = 2\mathbb{P}(Z > 1) = 2[1 - \mathbb{P}(Z \le 1)]$

$= 2[1 - 0{,}8413] \ldots$

$= 0{,}3174$ $\boxed{\times}$

$$\underbrace{B_1 \quad B_2 \quad \text{—————} \quad B_{10}}$$

$$B_1 \downarrow \bar{D} \qquad B_2 \downarrow \bar{D}$$

$$\mathbb{P}((b)) = \mathbb{P}(\bar{D})^{10} = [1 - \mathbb{P}(D)]^{10}$$

$$= [0{,}6826]^{10}$$

$$= 0{,}02196$$

94. The height of men is normally distributed with mean $\mu = 167$ cm and standard deviation $\sigma = 3$ cm.

(I) What is the percentage of the population of men that have height, (a) greater than 167 cm, $\boxed{\text{(b) greater than 170 cm, (c)}}$ between 161 cm and 173 cm?

(II) In a random sample of four men what is the probability that:

(i) all will have height greater than 170 cm;

(ii) two will have height smaller than the mean (and two bigger than the mean)?

on note $X \sim N(167, 3)$ $\quad Z \sim N(0,1)$ $\qquad$ <u>Rq</u> $\mathbb{P}(Z<0) = 0{,}5$

(a) $\mathbb{P}(X > 167) = \mathbb{P}\left(Z > \frac{167-167}{3}\right) = \mathbb{P}(Z>0)$

$= 0{,}5$

(b) $\mathbb{P}(X > 170) = \mathbb{P}\left(Z > \frac{170-167}{3}\right) = \mathbb{P}(Z > 1)$

$= 1 - \mathbb{P}(Z < 1) = 1 - 0{,}8413 = 0{,}1587$ ⊠

c) $\mathbb{P}(161 \leqslant X \leqslant 173) = \mathbb{P}\left(\frac{161-167}{3} \leqslant Z \leqslant \frac{173-167}{3}\right)$

$$= \mathbb{P}(-2 \leqslant Z \leqslant 2)$$

$$= 2\underbrace{\mathbb{P}(Z \leqslant 2)} - 1$$

$$= 2 \times [0,9772] - 1$$

$$\mathbb{P}(161 \leqslant X \leqslant 173) = 0,9544$$

II) $\{H_1, H_2, H_3, H_4\}$

(b) (b) (b) (b)

$\binom{4}{2} = C_4^2 = \boxed{6}$

II a) $\mathbb{P}$(les 4 H ont une taille > 170) $= \mathbb{P}(b)^4 = 6.34 \times 10^{-4}$

II) b) $\mathbb{P}(2H > 1{,}67$ et $2H < 1{,}67) = \binom{4}{2} \cdot (0{,}5)(0{,}5) \cdot (0{,}5)(0{,}5)$

$= 6 \times (0{,}5)^4$

$= 0{,}375$