HW 4

Di Zhen

ID: 3035272293

1

(a)

$$E(X) = \sum_{i=1}^{n=5} E(X_i) = \sum_{i=1}^{n=5} p \times 1 = 5p$$

(b)

$$Var(X) = \sum_{i=1}^{n=5} Var(X_i) = 5p(1-p)$$

2

$$\begin{aligned}
Var[X-Y] &= E[((X-Y)-E[X-Y])^2] \\
&= E[((X-Y)-E[X]+E[Y])^2] \\
&= E[((X-E[X])-(Y-E[Y]))^2] \\
&= E[(X-E[X])^2+(Y-E[Y])^2-2(X-E[X])(Y-E[Y])] \\
&= E[(X-E[X])^2]+E[(Y-E(Y))^2]-2E[(X-E[X])(Y-E[Y])] \\
&= Var[X]+Var[Y]-0 \\
&= Var[X]+Var[Y]
\end{aligned}$$

3

(a)

$$\begin{aligned}
E[X_1] &= 1\times\frac{1}{6}+2\times\frac{1}{6}+3\times\frac{1}{6}+4\times\frac{1}{6}+5\times\frac{1}{6}+6\times\frac{1}{6}=\frac{7}{2} \\
E[X_2] &= 1\times\frac{1}{16}+2\times\frac{1}{16}+3\times\frac{3}{16}+4\times\frac{3}{16}+5\times\frac{4}{16}+6\times\frac{4}{16}=\frac{17}{4} \\
E[Y] &= E[X_1X_2]=E[X_1]E[X_2]=\frac{7}{2}\times\frac{17}{4}=\frac{119}{8}\approx 14.9
\end{aligned}$$

(b)

$$
\begin{aligned}
Var[Y] &= E[Y^2] - E[Y]^2 \\
&= E[X_1^2]E[X_2^2] - E[Y]^2 \\
&= \frac{1}{6}(1^2+2^2+3^2+4^2+5^2+6^2) \times [\frac{1}{16}(1^2+2^2) + \frac{3}{16}(3^2+4^2) + \frac{4}{16}(5^2+6^2)] - \frac{119}{8}^2 \\
&\approx 85.9
\end{aligned}
$$

(c)

After simulating 10,000 rolls of the pair of dice in python. The mean of Y is 14.8664, the variance of Y is 85.56415104.

While in 3(a) and 3(b), the mean is around 14.9 and the variance is around 85.9, which are close to the values from simulation.