Notebook

June 26, 2019

Statement I $\dfrac{\sum_{i=1}^{n} a_i x_i}{\sum_{i=1}^{n} a_i} = \sum_{i=1}^{n} x_i$

$$\frac{\sum_{i=1}^{n} a_i x_i}{\sum_{i=1}^{n} a_i} = \frac{a_1 x_1 + a_2 x_2 + \ldots + a_n x_n}{a_1 + a_2 + \ldots + a_n} \neq x_1 + x_2 + \ldots x_n$$

Thus,

$$\frac{\sum_{i=1}^{n} a_i x_i}{\sum_{i=1}^{n} a_i} \neq \sum_{i=1}^{n} x_i$$

Statement II $\sum_{i=1}^{n} x_1 = nx_1$

It is not true because there is no 'i', so

$$\sum_{i=1}^{n} x_1 = x_1$$

Statement III $\sum_{i=1}^{n} a_3 x_i = na_3\bar{x}$

It is true because

$$\sum_{i=1}^{n} a_3 x_i = a_3(x_1 + x_2 + \ldots + x_n) = a_3(\bar{x}n) = na_3\bar{x}$$

Statement IV $\sum_{i=1}^{n} a_i x_i = n \bar{a} \bar{x}$

It is not true because

$$\sum_{i=1}^{n} a_i x_i = a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = n(\bar{ax}) \neq n \bar{a} \bar{x}$$

Question 4a   Suppose we have the following scalar-valued function on x and y:

$$f(x,y) = x^2 + 4xy + 2y^3 + e^{-3y} + \ln(2y)$$

Compute the partial derivative of $f(x,y)$ with respect to x.

$$2x + 4y$$

Now compute the partial derivative of $f(x, y)$ with respect to y:

$$4x + 6y^2 - 3e^{-3y} + \frac{1}{y}$$

Finally, using your answers to the above two parts, compute $\nabla f(x,y)$ (the gradient of $f(x,y)$) and evaluate the gradient at the point $(x = 2, y = -1)$.

$$(0, 13 - 3e^3)$$

Question 4b Find the value(s) of x which minimizes the expression below. Justify why it is the minimum.

$\sum_{i=1}^{10}(i-x)^2$

Answer:

x = 5.5, I calculate the derivative of it and make it equal to 0, then I calculate the x value of that equation and that is the minimum point. The reason is that when the derivatice equals to 0, the line goes to the minimum point.

Question 4c Let $\sigma(x) = \dfrac{1}{1+e^{-x}}$. Show that $\sigma(-x) = 1 - \sigma(x)$.

$$\sigma(-x) = \frac{1+e^x - e^x}{1+e^{-x}} = 1 - \frac{e^x}{1+e^x} = 1 - \frac{1}{1+e^{-x}} = 1 - \sigma(x)$$

Question 4d Show that the derivative can be written as:

$$\frac{d}{dx}\sigma(x) = \sigma(x)(1 - \sigma(x))$$

$$\frac{d\sigma(x)}{dx}$$

$$= \frac{e^x + 2e^{2x}}{(1 + e^x)^2}$$

$$= \frac{e^x}{1 + e^x} \frac{1 + 2e^x}{1 + e^x}$$

$$= \frac{1}{1 + e^{-x}}(1 - \frac{1}{1 + e^{-x}})$$

$$= \frac{1}{1 + e^{-x}} \frac{1}{1 + e^x})$$

$$= \sigma(x)(1 - \sigma(x))$$

Question 4e   Write code to plot the function $f(x) = x^2$, the equation of the tangent line passing through $x = 8$, and the equation of the tangent line passing through $x = 0$.

Set the range of the x-axis to (-15, 15) and the range of the y axis to (-100, 300) and the figure size to (4,4). Your resulting plot should look like this:

You should use the `plt.plot` function to plot lines. You may find the following functions useful:

- `plt.plot(..)`
- `plt.figure(figsize=..)`
- `plt.ylim(..)`
- `plt.axhline(..)`

```
In [17]: import matplotlib.pyplot as plt

         def f(x):
             return x**2

         def df(x):
             return x*2

         def plot(f, df):
             plt.figure(figsize = (4,4))
             x = np.linspace(-15,15,500)
             plt.xlim(-15,15)
             plt.ylim(-100,300)
             plt.axhline(0,-15,15,color = 'r')
             plt.plot(x,f(x))
             h = lambda x: df(8)*x-f(8)
             plt.plot(x,h(x),'orange')

         plot(f,df)
```
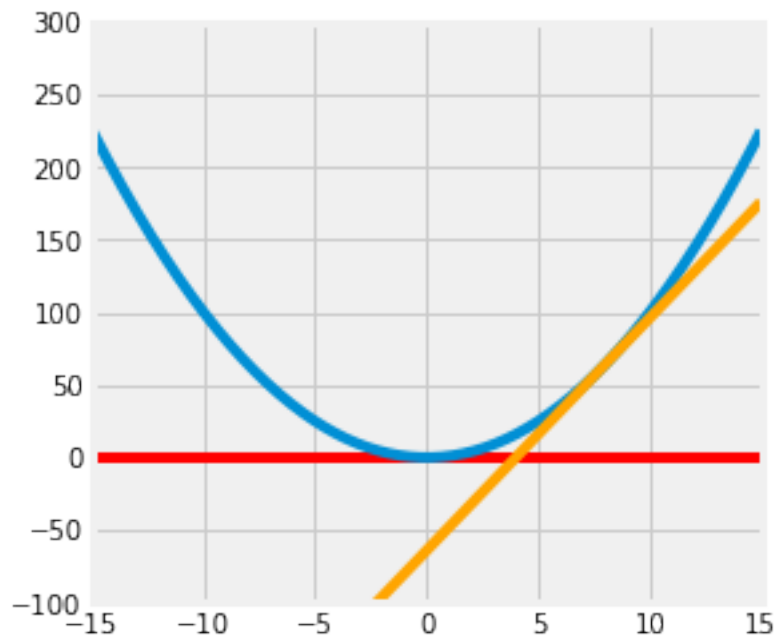
0.0.1 Question 5

Consider the following scenario:

Only 1% of 40-year-old women who participate in a routine mammography test have breast cancer. 80% of women who have breast cancer will test positive, but 9.6% of women who don't have breast cancer will also get positive tests.

Suppose we know that a woman of this age tested positive in a routine screening. What is the probability that she actually has breast cancer?

Hint: Use Bayes' rule.

$$P = \frac{80\%1\%}{80\%1\% + 9.6\%99\%} = 7.8\%$$

0.0.2 Question 6

We should also familiarize ourselves with looking up documentation and learning how to read it. Below is a section of code that plots a basic wireframe. Replace each `# Your answer here` with a description of what the line above does, what the arguments being passed in are, and how the arguments are used in the function. For example,

```
np.arange(2, 5, 0.2)
# This returns an array of numbers from 2 to 5 with an interval size of 0.2
```

Hint: The `Shift + Tab` tip from earlier in the notebook may help here. Remember that objects must be defined in order for the documentation shortcut to work; for example, all of the documentation will show for method calls from np since we've already executed `import numpy as np`. However, since z is not yet defined in the kernel, `z.reshape()` will not show documentation until you run the line `z = np.cos(squared)`.

```
In [18]: from mpl_toolkits.mplot3d import axes3d

         u = np.linspace(1.5*np.pi, -1.5*np.pi, 100)
         # This returns an array of 100 numbers from 1.5*np.pi to -1.5*np.pi with same intervals
         [x,y] = np.meshgrid(u, u)
         # This returns a matrix containing two vectors of u
         squared = np.sqrt(x.flatten()**2 + y.flatten()**2)
         z = np.cos(squared)
         # This returns an array calculated x and y by square, adding together, sqrt and cos
         z = z.reshape(x.shape)
         # This returns a (100,100) matrix reshaped from 1D array
         fig = plt.figure()
         ax = fig.add_subplot(111, projection='3d')
         # This returns a 3d space with the range of x,y,z axises are all 0 to 1.0
         ax.plot_wireframe(x, y, z, rstride=10, cstride=10)
         # This returns a 3d plot
         ax.view_init(elev=50., azim=30)
         # This set the elevation to 50 and azimuth of the axes to 30.
         plt.savefig("figure1.png")
         # This save the figure
```