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EFFECTIVE LAGRANGIAN ANALYSIS OF NEW INTERACTIONS AND FLAVOUR CONSERVATION

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New interactions with a scale Λ larger than the Fermi scale $G_F^{-1/2}$ will manifest themselves at energies below Λ through small deviations from the standard model, which can be described by an effective lagrangian containing non-renormalizable $SU(3) \times SU(2) \times U(1)$ invariant operators. We construct the first two terms of this lagrangian in an expansion in powers of $1/\Lambda$ and study systematically possible effects of new interactions such as anomalous magnetic moments, deviations from universality in weak interactions and rare processes. Among the flavour conserving processes the universality of the charged current weak interactions yields the strongest bound on the new interaction scale, $\Lambda > 5$ TeV, whereas flavour non-conserving processes imply the bound $\Lambda > 3000$ TeV. We derive conditions for natural flavour conservation. Although it cannot be excluded that all flavour changing operators are dynamically suppressed, this appears difficult to understand without a symmetry for which the standard electroweak theory seems to provide no hint. We emphasize the importance of searching for rare decays of D-mesons.

1. Introduction

The standard model [1] of strong and electroweak interactions has been very successful phenomenologically; even the interesting "new events" from high-energy $p\bar{p}$ collisions at CERN are now likely to be explained by conventional background [2].

Nevertheless, the model does not have the trademark of a really fundamental theory: many arbitrary parameters and no prediction of the number of particles are only two of the unsatisfactory aspects. Consequently there could be further particles and interactions as one probes higher energies. These might come only at the Planck scale ($\sim 10^{19}$ GeV), and then remain unobservable, or at an intermediate mass scale Λ.

If we assume that the standard model indeed describes physics well in the energy range up to the W-mass, but, in view of the above, take it to be an *effective* low-energy theory in which heavy fields have been integrated out, then it is compelling to describe the phenomena up to energies of order Λ by an effective lagrangian technique, where fields are considered as classical. [This is done for the low-energy interactions of (light) pions [3] where f_π corresponds to Λ.] Such a procedure is quite general and independent of the new interactions at scale Λ; all one must do is to impose $SU(3)\times SU(2)\times U(1)$ invariance (and possibly further conservation laws, such as baryon number conservation, etc.). It contains, however, the assumption that no additional fields are present, such as coloured scalars with masses $O(m_W)$ (which could, of course, be included in a further analysis).

In this spirit, the total lagrangian valid up to energies of order m_W can be written as an expansion in $(1/\Lambda)$:

$$\mathcal{L}_{\text{eff}}=\mathcal{L}_0+\frac{1}{\Lambda}\mathcal{L}_1+\frac{1}{\Lambda^2}\mathcal{L}_2+\cdots, \tag{1.1}$$

where $\mathcal{L}_0$ is the standard lagrangian of dimension four [$SU(3)\times SU(2)\times U(1)$ gauge fields, the usual fermion fields and one Higgs doublet], $\mathcal{L}_1$ is of dimension five, etc. All $\mathcal{L}_i$ are $SU(3)\times SU(2)\times U(1)$ invariant. This follows from requiring that $SU(2)\times U(1)$ breaking is indeed a phenomenon connected to the Fermi scale and not to Λ, i.e., that for energies $E>G_F^{-1/2}$ the complete lagrangian is $SU(2)\times U(1)$ symmetric.

Such an approach has been used [4] to classify baryon and lepton number violating operators and to study the effects of four-fermion contact interactions [5]. Extensive lists of operators in $\mathcal{L}_2$ have been given by Burges and Schnitzer, and Leung, Love and Rao [6]. In this paper we pursue this approach further, with particular emphasis on bounds on Λ and on the problem of flavour-changing neutral interactions in $\mathcal{L}_2$. Similar questions have previously been investigated in the context of horizontal interactions [7, 8], extended technicolour theories [9] and composite models [10].

In the following section we define our conventions and $\mathcal{L}_0$; in sect. 3 we develop $\mathcal{L}_1$ and $\mathcal{L}_2$ (imposing lepton and baryon number conservation). Then we consider deviations from the standard model in sect. 4; sect. 5 gives the bounds on Λ and in sect. 6 we discuss the conditions needed to ensure flavour conservation. Finally we present our conclusions in sect. 7. The proof of two results of sect. 6 is given in the appendix.

2. The fields and the standard lagrangian $\mathcal{L}_0$

In order to introduce the notation and to establish the necessary equations of motion we begin with $\mathcal{L}_0$ in (1.1). The fields are (α, i, p are colour, weak isospin and flavour* indices respectively):

Matter fields:

left-handed lepton doublets:	ℓ_p^i,	
right-handed charged leptons:	e_p,	
left-handed quark doublets:	$q_p^{\alpha i}$,	
right-handed quarks:	u_p^α,	d_p^α,
Higgs boson doublet:	φ^i,	$\tilde{\varphi}^i=\varepsilon^{ij}\varphi_j^*$.

* We will also use the terms family or generation.

Gauge fields:

$$\begin{aligned}
&\text{gluons:} && G_\mu^A\,, \qquad A=1\cdots 8\,,\\
& && G_{\mu\nu}^A=\partial_\mu G_\nu^A-\partial_\nu G_\mu^A+g_s f_{ABC}G_\mu^B G_\nu^C\,, && (2.1)\\
&\text{W bosons:} && W_\mu^I\,, \qquad I=1\cdots 3\,,\\
& && W_{\mu\nu}^I=\partial_\mu W_\nu^I-\partial_\nu W_\mu^I+g\varepsilon_{IJK}W_\mu^J W_\nu^K\,, && (2.2)\\
&\text{B boson:} && B_\mu\,,\\
& && B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu\,. && (2.3)
\end{aligned}$$

As mentioned above, we assume that all physics up to energies of order m_W is described by these fields. The effective lagrangian we are going to construct will only contain them. Clearly, theories with new interactions, in particular models with composite quarks and leptons will have new particles. Here, the view is that all experimental evidence is against such particles and that they are likely to be too heavy to appear as physical particles in low energy processes. The standard SU(3) × SU(2) × U(1) lagrangian is:

$$\begin{aligned}
\mathcal{L}_0=&-\tfrac14 G_{\mu\nu}^A G^{A\mu\nu}-\tfrac14 W_{\mu\nu}^I W^{I\mu\nu}-\tfrac14 B_{\mu\nu}B^{\mu\nu}\\
&+(D_\mu\varphi)^\dagger(D^\mu\varphi)+m^2\varphi^\dagger\varphi-\tfrac12\lambda(\varphi^\dagger\varphi)^2\\
&+i\bar{\ell}\not{D}\ell+i\bar{e}\not{D}e+i\bar{q}\not{D}q+i\bar{u}\not{D}u+i\bar{d}\not{D}d\\
&+(\Gamma_e\bar{\ell}e\varphi+\Gamma_u\bar{q}u\tilde{\varphi}+\Gamma_d\bar{q}d\varphi+\text{h.c.})\,, \qquad (2.4)
\end{aligned}$$

where

$$D_\mu=1\partial_\mu-ig_s\tfrac12\lambda^A G_\mu^A-ig\tfrac12\tau^I W_\mu^I-ig'YB_\mu \qquad (2.5)$$

is the covariant derivative; λ^A acts on colour indices, τ^I on SU(2) indices, and Y is assigned as follows: $Y(\ell)=-\frac12$, $Y(\mathrm{e})=-1$, $Y(\mathrm{q})=\frac16$, $Y(\mathrm{u})=\frac23$, $Y(\mathrm{d})=-\frac13$, $Y(\varphi)=\frac12$. The terms with λ^A or τ^I are of course only there if the respective fields are colour triplets or SU(2) doublets. In (2.4) we have suppressed the flavour indices of the fields. The Yukawa couplings Γ_e, etc., are matrices in flavour space; the kinetic terms are proportional to the unit matrix in flavour space.

Since we are going to treat all of these fields as classical, we will make use of the (classical) equations of motion. Variation with respect to the fields gives:

$$\bar{\ell}:\ i\not{D}\ell+\Gamma_e e\varphi=0\,, \qquad (2.6)$$

$$\bar{e}:\ i\not{D}e+\Gamma_e^\dagger\varphi^\dagger\ell=0\,, \qquad (2.7)$$

$$\bar{q}:\ i\not{D}q+\Gamma_u u\tilde{\varphi}+\Gamma_d d\varphi=0\,, \qquad (2.8)$$

$$\bar{u}:\ i\not{D}u+\Gamma_u^\dagger\tilde{\varphi}^\dagger q=0\,, \qquad (2.9)$$

$$\bar{d}:\ i\not{D}d+\Gamma_d^\dagger\varphi^\dagger q=0\,, \qquad (2.10)$$

$$\varphi^{\dagger}:\quad D^2\varphi-\Gamma_{\mathrm{e}}^{\dagger}\bar{e}\ell-\Gamma_{\mathrm{u}}\bar{q}\varepsilon u-\Gamma_{\mathrm{d}}^{\dagger}\bar{d}q=0\,, \tag{2.11}$$

$$G_{\mu}^{A}:\quad (D_{\nu}G^{\nu\mu})^{A}+g_{\mathrm{s}}(\bar{q}\gamma^{\mu}\lambda^{A}q+\bar{u}\gamma^{\mu}\lambda^{A}u+\bar{d}\gamma^{\mu}\lambda^{A}d)=0\,, \tag{2.12}$$

$$W_{\mu}^{I}:\quad (D_{\nu}W^{\nu\mu})^{I}+g(i\varphi^{\dagger}\overleftrightarrow{D}^{\mu}\tau^{I}\varphi+\bar{\ell}\gamma^{\mu}\tau^{I}\ell+\bar{q}\gamma^{\mu}\tau^{I}q)=0\,, \tag{2.13}$$

$$B_{\mu}:\quad \partial_{\nu}B^{\nu\mu}+g'(-\tfrac{1}{2}\bar{\ell}\gamma^{\mu}\ell-\bar{e}\gamma^{\mu}e+\tfrac{1}{6}\bar{q}\gamma^{\mu}q+\tfrac{2}{3}\bar{u}\gamma^{\mu}u-\tfrac{1}{3}\bar{d}\gamma^{\mu}d)=0\,. \tag{2.14}$$

In (2.6)-(2.14), we have suppressed all indices; the covariant derivatives of the field strengths in (2.12) and (2.13) read explicitly:

$$(D_{\mu}G_{\nu\lambda})^{A}=\partial_{\mu}G_{\nu\lambda}^{A}+g_{\mathrm{s}}f_{ABC}G_{\mu}^{B}G_{\nu\lambda}^{C}\,, \tag{2.15}$$

$$(D_{\mu}W_{\nu\lambda})^{I}=\partial_{\mu}W_{\nu\lambda}^{I}+g\varepsilon_{IJK}W_{\mu}^{J}W_{\nu\lambda}^{K}\,. \tag{2.16}$$

From these equations various useful relations can be obtained. Consider for instance (2.6):

$$i\not{D}\ell+\Gamma_{\mathrm{e}}e\varphi=0\,.$$

Hermitian conjugation and multiplication with γ_0 from the right yields

$$-i\bar{\ell}\overleftarrow{\not{D}}+\Gamma_{\mathrm{e}}^{\dagger}\varphi^{\dagger}\bar{e}=0\,, \tag{2.17}$$

where $\overleftarrow{D}_{\mu}$ indicates that the derivative acts on $\bar{\ell}$ and that in the covariant derivative $i\lambda^{A}$, etc., are to be replaced by $-i(\lambda^{A})^{\dagger}=-i\lambda^{A}$ since we work in an hermitian basis. Multiplying (2.6) by $\bar{\ell}$ and (2.17) by ℓ, and subtracting, gives

$$\partial_{\mu}\bar{\ell}\gamma^{\mu}\ell=i(\Gamma_{\mathrm{e}}\bar{\ell}_{e}\varphi-\Gamma_{\mathrm{e}}^{\dagger}\varphi^{\dagger}\bar{e}\ell)\,. \tag{2.18}$$

Relations of this kind will prove useful later in the following section. We close by noting that the vacuum expectation value of φ is:

$$v=\langle\varphi\rangle=174\ \mathrm{GeV}\,. \tag{2.19}$$

3. Construction of $\mathscr{L}_{\mathrm{eff}}$

Considering now $\mathscr{L}_{\mathrm{eff}}$ in (1.1) we come first to $\mathscr{L}_1$. It cannot be constructed with fermions only or scalars only, because of dimensional reasons and because scalars are doublets. A possible operator with scalars and fermions has two fermions and two scalars. If the scalars are φ and φ^*, then the two fermions must have total hypercharge zero, which is only possible if we take a multiplet and its charge conjugate. But then the fermion-antifermion operator cannot be a Lorentz scalar. If the scalars are two φ's, then they must combine to an SU(2) triplet, since the singlet product of two equal doublets vanishes, and also the two fermions must both be SU(2) doublets. The resulting operator may be written as a Majorana mass term for a gauge singlet "composite fermion" formed from a φ and a lepton doublet ℓ [4]:

$$\mathscr{L}_1=\varepsilon_{ij}\bar{\ell}_{\mathrm{R}}^{ci}\varphi^{j}\varepsilon_{kl}\ell_{\mathrm{L}}^{k}\varphi^{l}+\mathrm{c.c.}\,. \tag{3.1}$$

$\mathscr{L}_1$ violates lepton number, giving rise to a Majorana mass for the neutrino of order $G_F^{-1}\Lambda^{-1}$. With $m_\nu^M \leq 5$ eV, Λ would be of the order of 10^{13} GeV, and hence uninteresting here since we want to test Λ's in a much lower range. Dimension-five operators consisting of only fermions and gauge bosons, gauge bosons only, etc., cannot be constructed. Addition of new fields such as right-handed neutrinos, changes of course this result.

Imposing baryon and lepton number conservation, we next turn to $\mathscr{L}_2$. Most of the operators of $\mathscr{L}_2$ have already been given in ref. [6]; however, constant use of the classical equations of motion restricts the number of independent forms considerably and thus allows a simpler classification.

To see this, consider (1.1):

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_0 + \frac{1}{\Lambda^2}\mathscr{L}_2 + \cdots .$$

The equations of motion, derived from varying $\mathscr{L}_{\text{eff}}$ with respect to the fields, are given by (2.6)-(2.14) where the zero on the right-hand side is replaced by terms of the order of $1/\Lambda^2$. This implies that in $\mathscr{L}_2$, all the terms of the form $\not{D}\ell$, etc., can be replaced by $\Gamma_e e\varphi$, etc., up to order $1/\Lambda^4$. Also in other cases terms with derivatives can often be eliminated.

We thus set

$$\mathscr{L}_2 = \sum_i \alpha_i O_i \tag{3.2}$$

and derive the operators O_i of dimension six which are $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ invariant. As in ref. [6], we divide them according to the fields they contain. The hermitian conjugate operators are understood to be included.

3.1. VECTORS ONLY

In order to obtain gauge invariant operators, we must construct them out of the field strengths and their duals; one needs two or three of them to form a gauge singlet. In the case of two, two (covariant) derivatives are needed to realize a dimension-six operator. Using partial integration* and the equations of motion all the derivative terms can be eliminated. Then we get:

$$O_G = f_{ABC} G_\mu^{A\nu} G_\nu^{B\lambda} G_\lambda^{C\mu} , \tag{3.3}$$

$$O_{\tilde{G}} = f_{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\lambda} G_\lambda^{C\mu} , \tag{3.4}$$

$$O_W = \varepsilon_{IJK} W_\mu^{I\nu} W_\nu^{J\lambda} W_\lambda^{K\mu} , \tag{3.5}$$

$$O_{\tilde{W}} = \varepsilon_{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\lambda} W_\lambda^{K\mu} . \tag{3.6}$$

Terms such as $d_{ABD} G_\mu^{A\nu} G_\nu^{B\lambda} G_\lambda^{C\mu}$ vanish.

* For gauge invariant quantities, one can perform partial integration on the factors also for the covariant derivative.

3.2. FERMIONS ONLY

These are obviously four-fermion operators. Denoting by L the left-handed, and by R the right-handed fields, the operators which satisfy fermion number conservation are of the form $\bar{L}L\bar{L}L$, $\bar{R}R\bar{R}R$, $\bar{L}R\bar{R}L$, $\bar{L}R\bar{L}R$. The order of the fields is irrelevant because of the Fierz transformations. We will choose the order we find most convenient.

(i) $\bar{L}L\bar{L}L$ operators are of the form $(\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L)$, where the parentheses stand for contracting spinor indices; additional group indices depend on the fields. We easily find

$$O_{\ell\ell}^{(1)} = \tfrac{1}{2}(\bar{\ell}\gamma_\mu \ell)(\bar{\ell}\gamma^\mu \ell)\,, \qquad O_{\ell\ell}^{(3)} = \tfrac{1}{2}(\bar{\ell}\gamma_\mu \tau^I \ell)(\bar{\ell}\gamma^\mu \tau^I \ell)\,, \tag{3.7}$$

$$O_{qq}^{(1,1)} = \tfrac{1}{2}(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)\,, \qquad O_{qq}^{(8,1)} = \tfrac{1}{2}(\bar{q}\gamma_\mu \lambda^A q)(\bar{q}\gamma^\mu \lambda^A q)\,, \tag{3.8}$$

$$O_{qq}^{(1,3)} = \tfrac{1}{2}(\bar{q}\gamma_\mu \tau^I q)(\bar{q}\gamma^\mu \tau^I q)\,, \qquad O_{qq}^{(8,3)} = \tfrac{1}{2}(\bar{q}\gamma_\mu \lambda^A \tau^I q)(\bar{q}\gamma^\mu \lambda^A \tau^I q)\,, \tag{3.9}$$

$$O_{\ell q}^{(1)} = (\bar{\ell}\gamma_\mu \ell)(\bar{q}\gamma^\mu q)\,, \qquad O_{\ell q}^{(3)} = (\bar{\ell}\gamma_\mu \tau^I \ell)(\bar{q}\gamma^\mu \tau^I q)\,. \tag{3.10}$$

λ^A and τ^I are the SU(3) and SU(2) matrices, appropriate contractions are understood. In all these operators, we have suppressed generation (flavour) indices; in principle there is an operator for any combination of four flavours. All operators not listed violate lepton or baryon number; they have been considered previously [4]. These remarks also apply to the following operators.

(ii) $\bar{R}R\bar{R}R$ operators are also of the form $(\bar{R}\gamma_\mu R)(\bar{R}\gamma^\mu R)$. We have:

$$O_{ee} = \tfrac{1}{2}(\bar{e}\gamma_\mu e)(\bar{e}\gamma^\mu e)\,, \tag{3.11}$$

$$O_{uu}^{(1)} = \tfrac{1}{2}(\bar{u}\gamma_\mu u)(\bar{u}\gamma^\mu u)\,, \qquad O_{uu}^{(8)} = \tfrac{1}{2}(\bar{u}\gamma_\mu \lambda^A u)(\bar{u}\gamma^\mu \lambda^A u)\,, \tag{3.12}$$

$$O_{dd}^{(1)} = \tfrac{1}{2}(\bar{d}\gamma_\mu d)(\bar{d}\gamma^\mu d)\,, \qquad O_{dd}^{(8)} = \tfrac{1}{2}(\bar{d}\gamma_\mu \lambda^A d)(\bar{d}\gamma^\mu \lambda^A d)\,, \tag{3.13}$$

$$O_{eu} = (\bar{e}\gamma_\mu e)(\bar{u}\gamma^\mu u)\,, \tag{3.14}$$

$$O_{ed} = (\bar{e}\gamma_\mu e)(\bar{d}\gamma^\mu d)\,, \tag{3.15}$$

$$O_{ud}^{(1)} = (\bar{u}\gamma_\mu u)(\bar{d}\gamma^\mu d)\,, \qquad O_{ud}^{(8)} = (\bar{u}\gamma_\mu \lambda^A u)(\bar{d}\gamma^\mu \lambda^A d)\,. \tag{3.16}$$

(iii) $\bar{L}R\bar{R}L$ operators. In principle, we could imagine two Lorentz structures, $(\bar{L}\sigma_{\mu\nu}R)(\bar{R}\sigma^{\mu\nu}L)$ and $(\bar{L}R)(\bar{R}L)$. A Fiertz transformation, however, transforms the first term into the second and hence the tensor terms will be dropped.

$$O_{\ell e} = (\bar{\ell}e)(\bar{e}\ell)\,, \tag{3.17}$$

$$O_{\ell u} = (\bar{\ell}u)(\bar{u}\ell)\,, \tag{3.18}$$

$$O_{\ell d} = (\bar{\ell}d)(\bar{d}\ell)\,, \tag{3.19}$$

$$O_{qe} = (\bar{q}e)(\bar{e}q)\,, \tag{3.20}$$

$$O_{qu}^{(1)} = (\bar{q}u)(\bar{u}q)\,, \qquad O_{qu}^{(8)} = (\bar{q}\lambda^A u)(\bar{u}\lambda^A q)\,, \tag{3.21}$$

$$O_{qd}^{(1)} = (\bar{q}d)(\bar{d}q)\,, \qquad O_{qd}^{(8)} = (\bar{q}\lambda^A d)(\bar{d}\lambda^A q)\,, \tag{3.22}$$

$$O_{qde} = (\bar{\ell}e)(\bar{d}q)\,. \tag{3.23}$$

(iv) Finally, we come to the $\bar{L}R\bar{L}R$ terms. With conserved baryon and lepton number, we can write only

$$O_{qq}^{(1)} = (\bar{q}u)(\bar{q}d)\,, \tag{3.24}$$

$$O_{qq}^{(8)} = (\bar{q}\lambda^A u)(\bar{q}\lambda^A d)\,, \tag{3.25}$$

$$O_{\ell q} = (\bar{\ell}e)(\bar{q}u)\,. \tag{3.26}$$

3.3. SCALARS ONLY

Here, there are either six bosons (three φ's and three $\varphi^\dagger$'s) or four bosons and two derivatives, which must act on group invariants. There are only two operators

$$O_{\varphi} = \tfrac{1}{3}(\varphi^\dagger\varphi)^3\,, \tag{3.27}$$

$$O_{\partial\varphi} = \tfrac{1}{2}\partial_\mu(\varphi^\dagger\varphi)\,\partial^\mu(\varphi^\dagger\varphi)\,. \tag{3.28}$$

3.4. FERMIONS AND VECTORS

A dimension-six operator requires two fermions and three other powers of momentum. Since all vectors have hypercharge zero, and all fermion fields have different hypercharge, the fermions are always a field and its conjugate, which requires a γ_μ matrix. The gauge fields can come via covariant derivatives or as field strengths; we can either have three covariant derivatives (on fermions) or one field strength and one covariant derivative.

Operators with three covariant derivatives acting on fermions or one covariant derivative acting on the field strength can be transformed by means of the equations of motion into operators discussed in subsects. 3.2 and 3.7. Operators with dual field strengths can also be omitted because for fermions of definite chirality, one has the identity

$$\begin{gathered}\tilde{F}^{\mu\nu}\gamma_\mu D_\nu\psi_\pm = \mp(iF^{\mu\nu}\gamma_\mu D_\nu - \tfrac{1}{2}F^{\mu\nu}\sigma_{\mu\nu}\not{D})\psi_\pm\,,\\ \gamma_5\psi_\pm = \psi_\pm\,.\end{gathered} \tag{3.29}$$

The remaining operators of dimension six containing fermions and vectors read

$$O_{\ell W} = i\bar{\ell}\tau^I\gamma_\mu D_\nu \ell W^{I\mu\nu}\,, \qquad O_{\ell B} = i\bar{\ell}\gamma_\mu D_\nu \ell B^{\mu\nu}\,, \tag{3.30}$$

$$O_{eB} = i\bar{e}\gamma_\mu D_\nu e B^{\mu\nu}\,, \tag{3.31}$$

$$O_{qG} = i\bar{q}\lambda^A\gamma_\mu D_\nu q G^{A\mu\nu}\,, \tag{3.32}$$

$$O_{qW} = i\bar{q}\tau^I\gamma_\mu D_\nu q W^{I\mu\nu}\,, \qquad O_{qB} = i\bar{q}\gamma_\mu D_\nu q B^{\mu\nu}\,, \tag{3.33}$$

$$O_{uG} = i\bar{u}\lambda^A\gamma_\mu D_\nu u G^{A\mu\nu}\,, \tag{3.34}$$

$$O_{uB} = i\bar{u}\gamma_\mu D_\nu u B^{\mu\nu}\,, \tag{3.35}$$

$$O_{dG} = i\bar{d}\lambda^A\gamma_\mu D_\nu d G^{A\mu\nu}\,, \tag{3.36}$$

$$O_{dB} = i\bar{d}\gamma_\mu D_\nu d B^{\mu\nu}\,. \tag{3.37}$$

3.5. SCALARS AND VECTORS

In these operators, the vectors arise either through their field strengths or through covariant derivatives. As to the scalars, φ and $\varphi^\dagger$ must come in equal numbers to ensure $SU(2)\times U(1)$ invariance. With one φ and one $\varphi^\dagger$, we can combine two field strengths, one field strength and two (covariant) derivatives, or four covariant derivatives.

In the case of four covariant derivatives, partial integrations always allow us to write the corresponding operator as $(D^2\varphi)^\dagger(D^2\varphi)$ which, by (2.11), can be replaced by terms without derivatives. If there are two covariant derivatives and two scalar fields, there is one on φ and the other on $\varphi^\dagger$; otherwise one can use again the equations of motion.

In the case of two φ's and two $\varphi^\dagger$'s, one can have two covariant derivatives which act on two different fields. They can either act on φ and $\varphi^\dagger$, or on two φ's. In the latter case, however, the only operator is $(\varphi^\dagger D^\mu\varphi)(\varphi^\dagger D_\mu\varphi)$. Using the relation

$$\partial_\mu(\varphi^\dagger\varphi)=(D_\mu\varphi^\dagger)\varphi+\varphi^\dagger D_\mu\varphi \tag{3.38}$$

one obtains after partial integration

$$\begin{aligned}(\varphi^\dagger D^\mu\varphi)(\varphi^\dagger D_\mu\varphi)&=-(\varphi^\dagger D^\mu\varphi)(D_\mu\varphi^\dagger\varphi)+(\varphi^\dagger\varphi)\,\partial_\mu(\varphi^\dagger D^\mu\varphi)\\&=-(\varphi^\dagger D^\mu\varphi)(D_\mu\varphi^\dagger\varphi)+(\varphi^\dagger\varphi)(D_\mu\varphi^\dagger D^\mu\varphi)\\&\quad+(\varphi^\dagger\varphi)(\varphi^\dagger D^2\varphi)\,.\end{aligned} \tag{3.39}$$

Applying the equations of motion to the last term in (3.39), we see that operators with both derivatives acting on φ's need not be considered.

We then obtain:

$$O_{\varphi G}=\tfrac{1}{2}(\varphi^\dagger\varphi)G^A_{\mu\nu}G^{A\mu\nu}\,,\qquad O_{\varphi\tilde G}=(\varphi^\dagger\varphi)\tilde G^A_{\mu\nu}G^{A\mu\nu}\,, \tag{3.40}$$

$$O_{\varphi W}=\tfrac{1}{2}(\varphi^\dagger\varphi)W^I_{\mu\nu}W^{I\mu\nu}\,,\qquad O_{\varphi\tilde W}=(\varphi^\dagger\varphi)\tilde W^I_{\mu\nu}W^{I\mu\nu}\,, \tag{3.41}$$

$$O_{\varphi B}=\tfrac{1}{2}(\varphi^\dagger\varphi)B_{\mu\nu}B^{\mu\nu}\,,\qquad O_{\varphi\tilde B}=(\varphi^\dagger\varphi)\tilde B_{\mu\nu}B^{\mu\nu}\,, \tag{3.42}$$

$$O_{WB}=(\varphi^\dagger\tau^I\varphi)W^I_{\mu\nu}B^{\mu\nu}\,,\qquad O_{\tilde WB}=(\varphi^\dagger\tau^I\varphi)\tilde W^I_{\mu\nu}B^{\mu\nu}\,, \tag{3.43}$$

$$O^{(1)}_\varphi=(\varphi^\dagger\varphi)(D_\mu\varphi^\dagger D^\mu\varphi)\,,\qquad O^{(3)}_\varphi=(\varphi^\dagger D^\mu\varphi)(D_\mu\varphi^\dagger\varphi)\,. \tag{3.44}$$

3.6. FERMIONS AND SCALARS

In this case we must have two fermions and three bosons, or two fermions, two bosons and a derivative which must act on a gauge invariant quantity. Two derivatives are not possible. Since a derivative must act on a gauge invariant quantity, and no field is a singlet, it must be of the form $\partial_\mu(\varphi_1\varphi_2)\,\partial^\mu(\varphi_3\varphi_4)$ or $\partial^2(\varphi_1\varphi_2)\varphi_3\varphi_4$ where the products $\varphi_1\varphi_2$, $\varphi_3\varphi_4$ are gauge singlets and φ_i any of the fields of interest. But then, any such operator has dimension seven.

The operators with one derivative which we can form must contain a γ_μ to be a Lorentz scalar. We can then form terms such as $\partial_\mu(\varphi^\dagger\varphi)(\bar{\ell}\gamma_\mu\ell)$; however, by partial integration and eq. (2.18), we see that it does not give rise to a new term. Terms with two φ's are impossible, because they would imply both fermions to be a lepton doublet, from which no Lorentz vector with vanishing lepton number can be formed.

We are thus left with three operators containing no derivatives:

$$O_{e\varphi}=(\varphi^\dagger\varphi)(\bar{\ell}e\varphi)\,, \tag{3.45}$$

$$O_{u\varphi}=(\varphi^\dagger\varphi)(\bar{q}u\tilde{\varphi})\,, \tag{3.46}$$

$$O_{d\varphi}=(\varphi^\dagger\varphi)(\bar{q}d\varphi)\,, \tag{3.47}$$

3.7. VECTORS, FERMIONS AND SCALARS

Finally we consider combinations of all three kinds of fields. Clearly, there are two fermions and one or two scalars. If there are two scalars, we must have one covariant derivative and one γ_μ. From the hypercharge assignments the only possibility to have two φ's (and not one φ and one $\varphi^\dagger$) is $\varphi\varphi\bar{u}\gamma_\mu d$, where the covariant derivative must act on one φ. For other fermion pairs we have one φ and one $\varphi^\dagger$, where the covariant derivative acts on φ or $\varphi^\dagger$; if it acted on the fermions, it could be elminated by the equation of motion. This then gives the operators:

$$O_{\varphi\ell}^{(1)}=i(\varphi^\dagger D_\mu\varphi)(\bar{\ell}\gamma^\mu\ell)\,, \tag{3.48}$$

$$O_{\varphi\ell}^{(3)}=i(\varphi^\dagger D_\mu\tau^I\varphi)(\bar{\ell}\gamma^\mu\tau^I\ell)\,, \tag{3.49}$$

$$O_{\varphi e}=i(\varphi^\dagger D_\mu\varphi)(\bar{e}\gamma^\mu e)\,, \tag{3.50}$$

$$O_{\varphi q}^{(1)}=i(\varphi^\dagger D_\mu\varphi)(\bar{q}\gamma^\mu q)\,, \tag{3.51}$$

$$O_{\varphi q}^{(3)}=i(\varphi^\dagger D_\mu\tau^I\varphi)(\bar{q}\gamma^\mu\tau^I q)\,, \tag{3.52}$$

$$O_{\varphi_u}=i(\varphi^\dagger D_\mu\varphi)(\bar{u}\gamma^\mu u)\,, \tag{3.53}$$

$$O_{\varphi_d}=i(\varphi^\dagger D_\mu\varphi)(\bar{d}\gamma^\mu d)\,, \tag{3.54}$$

$$O_{\varphi\varphi}=i(\varphi^\dagger\varepsilon D_\mu\varphi)(\bar{u}\gamma^\mu d)\,. \tag{3.55}$$

We note that, if φ is replaced by its vacuum expectation value, the neutral gauge boson contributing in D_μ is just the Z-boson of the standard model, i.e., $gW_3-g'B$.

We now come to the operators with one scalar. There are either two covariant derivatives or one field strength. The two covariant derivatives can act (i) both on the scalar, (ii) one on the scalar and one on a fermion, (iii) both on the fermions. We discuss these cases in turn.

(i) The two possibilities are $D_\mu D_\nu\varphi g^{\mu\nu}\bar{\psi}_1\psi_2$ and $D_\mu D_\nu\varphi\bar{\psi}_1\sigma_{\mu\nu}\psi_2$, where ψ_1 and ψ_2 are the fermion fields allowed by $SU(2)\times U(1)$ invariance. The first one need not be considered because of the equations of motion, the second is equivalent to the operator $\bar{\psi}\sigma_{\mu\nu}\psi_2\varphi F^{\mu\nu}$, where $F_{\mu\nu}$ stands for any of the field strengths.

(ii) The two possibilities are now $D_\mu\varphi\bar{\psi}_1 D^\mu\psi_2$ and $D_\mu\varphi\bar{\psi}_1\sigma^{\mu\nu}D_\nu\psi_2$ (plus terms where D_μ acts on $\bar{\psi}$). Using the equations of motion, the second is equivalent to the first and a term without vectors (subsect. 3.6.).

(iii) The two derivatives can act both on ψ_2, on $\bar{\psi}_1$, or one on $\bar{\psi}_1$ and the other on ψ_2. If they both act on ψ_2, the relation

$$\not{D}\not{D} = D_\mu D_\nu g^{\mu\nu} - iD_\mu D_\nu\sigma^{\mu\nu} \tag{3.56}$$

shows that the corresponding operator is, by the equation of motion, equivalent to an operator with a field strength. If the D_μ's act on $\bar{\psi}_1$ and ψ_2, the general form $\varphi D_\mu\bar{\psi}_1(ag^{\mu\nu}+b\sigma^{\mu\nu})D_\nu\psi_2$ reduces by means of partial integration to operators already discussed.

We then get:

$$O_{D_e} = (\bar{\ell}D_\mu e)D^\mu\varphi\,, \qquad O_{\bar{D}_e} = (D_\mu\bar{\ell}e)D^\mu\varphi\,, \tag{3.57}$$

$$O_{Du} = (\bar{q}D_\mu u)D^\mu\tilde{\varphi}\,, \qquad O_{\bar{D}u} = (D_\mu\bar{q}u)D^\mu\tilde{\varphi}\,, \tag{3.58}$$

$$O_{Dd} = (\bar{q}D_\mu d)D^\mu\varphi\,, \qquad O_{\bar{D}d} = (D_\mu\bar{q}d)D^\mu\varphi\,, \tag{3.59}$$

$$O_{eW} = (\bar{\ell}\sigma^{\mu\nu}\tau^I e)\varphi W^I_{\mu\nu}\,, \qquad O_{eB} = (\bar{\ell}\sigma^{\mu\nu}e)\varphi B_{\mu\nu}\,, \tag{3.60}$$

$$O_{uG} = (\bar{q}\sigma^{\mu\nu}\lambda^A u)\tilde{\varphi}G^A_{\mu\nu}\,, \tag{3.61}$$

$$O_{uW} = (\bar{q}\sigma^{\mu\nu}\tau^I u)\tilde{\varphi}W^I_{\mu\nu}\,, \qquad O_{uB} = (\bar{q}\sigma^{\mu\nu}u)\tilde{\varphi}B_{\mu\nu}\,, \tag{3.62}$$

$$O_{dG} = (\bar{q}\sigma^{\mu\nu}\lambda^A d)\varphi G^A_{\mu\nu}\,, \tag{3.63}$$

$$O_{dW} = (\bar{q}\sigma^{\mu\nu}\tau^I d)\varphi W^I_{\mu\nu}\,, \qquad O_{dW} = (\bar{q}\sigma^{\mu\nu}d)\varphi B_{\mu\nu}\,. \tag{3.64}$$

This concludes the list of all operators in $\mathcal{L}_2$. There are 80 independent operators. These terms are also generated radiatively in the standard model, but there they are of course expected to be small.

4. Deviations from the standard model

The standard model predicts a number of relations between masses and coupling constants such as the connection between the Weinberg angle and the W and Z masses, or the Higgs couplings and the fermion masses. Such relations exist at tree level and are modified in a well-defined and testable way by radiative corrections. In this section we will discuss modifications of these relations due to non-perturbative effects which are described by the operators listed in the previous section. The new relations are obtained by supplementing $\mathcal{L}_0$ with $\mathcal{L}_2$, where φ is replaced by its vacuum expectation value. Obviously they depend on the - *a priori* arbitrary - coefficients α_i of the operators. We label the coefficients by the same indices as the operators.

4.1. THE MASSES OF W AND Z

In order to obtain the masses of the W and Z bosons, one has to compute the complete quadratic part of the gauge boson lagrangian. From eqs. (2.4) and (3.41)-(3.44) one obtains the result

$$\begin{aligned}\mathcal{L}^{(2)}_{W,B} = & -\tfrac{1}{4}\left(1-2\alpha_{\varphi W}\frac{v^2}{\Lambda^2}\right)W^I_{\mu\nu}W^{I\mu\nu}-\tfrac{1}{4}\left(1-2\alpha_{\varphi B}\frac{v^2}{\Lambda^2}\right)B_{\mu\nu}B^{\mu\nu}-\alpha_{WB}\frac{v^2}{\Lambda^2}W^3_{\mu\nu}B^{\mu\nu}\\ & +\tfrac{1}{4}v^2\left(1+\alpha^{(1)}_{\varphi}\frac{v^2}{\Lambda^2}\right)(g^2W^I_\mu W^{I\mu}+g'^2B_\mu B^\mu-2gg'W^3_\mu B^\mu)\\ & +\tfrac{1}{4}\alpha^{(3)}_{\varphi}\frac{v^4}{\Lambda}(gW^3_\mu-g'B_\mu)(gW^{3\mu}-g'B^\mu)\,. \end{aligned} \tag{4.1}$$

Contrary to the standard model, the lagrangian (4.1) contains a "mass mixing" between the vector fields W^3_μ and B_μ as well as a "current mixing" between the field strengths $W^3_{\mu\nu}$ and $B_{\mu\nu}$. For arbitrary coefficients α_i, one cannot diagonalize both non-diagonal terms simultaneously by means of an orthogonal transformation. One possibility is to diagonalize the "mass mixing" term and to use for the resulting mass eigenstates the propagator formalism [11]. Here we diagonalize directly [12] up to terms of order $(v/\Lambda)^4$:

$$W^3_\mu=\sin\theta^0_{\mathrm{w}}\left(1+\alpha_{AA}\frac{v^2}{\Lambda^2}\right)A_\mu+\left[\cos\theta^0_{\mathrm{w}}\left(1+\alpha_{ZZ}\frac{v^2}{\Lambda^2}\right)-\sin\theta^0_{\mathrm{w}}\alpha_{AZ}\right]Z_\mu\,, \tag{4.2a}$$

$$B_\mu=\cos\theta^0_{\mathrm{w}}\left(1+\alpha_{AA}\frac{v^2}{\Lambda^2}\right)A_\mu-\left[\sin\theta^0_{\mathrm{w}}\left(1+\alpha_{ZZ}\frac{v^2}{\Lambda^2}\right)+\cos\theta^0_{\mathrm{w}}\alpha_{AZ}\right]Z_\mu\,, \tag{4.2b}$$

where

$$\alpha_{AA}=\sin^2\theta^0_{\mathrm{w}}\alpha_{\varphi W}+\cos^2\theta^0_{\mathrm{w}}\alpha_{\varphi B}-2\sin\theta^0_{\mathrm{w}}\cos\theta^0_{\mathrm{w}}\,\alpha_{WB}\,, \tag{4.2c}$$

$$\alpha_{ZZ}=\cos^2\theta^0_{\mathrm{w}}\alpha_{\varphi W}+\sin^2\theta^0_{\mathrm{w}}\alpha_{\varphi B}+2\sin\theta^0_{\mathrm{w}}\cos\theta^0_{\mathrm{w}}\alpha_{WB}\,, \tag{4.2d}$$

$$\alpha_{AZ}=2[\sin\theta^0_{\mathrm{w}}\cos\theta^0_{\mathrm{w}}(\alpha_{\varphi B}-\alpha_{\varphi W})+(\cos^2\theta^0_{\mathrm{w}}-\sin^2\theta^0_{\mathrm{w}})\alpha_{WB}]\,. \tag{4.2e}$$

θ^0_{w} denotes the tree-level standard model weak angle:

$$\cos\theta^0_{\mathrm{w}}=\frac{g}{(g^2+g'^2)^{1/2}}\,. \tag{4.3}$$

The physical vector fields $W^\pm_\mu$, A_μ and Z_μ are described by the lagrangian

$$\mathcal{L}^{(2)}_{W^\pm,A,Z}=-\tfrac{1}{2}W^+_{\mu\nu}W^{-\mu\nu}+m^2_{\mathrm{W}}W^+_\mu W^{-\mu}-\tfrac{1}{4}A_{\mu\nu}A^{\mu\nu}-\tfrac{1}{4}Z_{\mu\nu}Z^{\mu\nu}+\tfrac{1}{2}m^2_{\mathrm{Z}}Z_\mu Z^\mu\,, \tag{4.4}$$

where the physical masses are given by

$$m_W = m_W^0\left[1+(\alpha_{\varphi W}+\tfrac{1}{2}\alpha_{\varphi}^{(1)})\frac{v^2}{\Lambda^2}\right],$$

$$(m_W^0)^2=\tfrac{1}{2}g^2v^2\,, \tag{4.5a}$$

$$m_Z = m_Z^0\left[1+(\alpha_{ZZ}+\tfrac{1}{2}\alpha_{\varphi}^{(1)}+\tfrac{1}{2}\alpha_{\varphi}^{(3)})\frac{v^2}{\Lambda^2}\right],$$

$$(m_Z^0)^2=\tfrac{1}{2}(g^2+g'^2)v^2\,. \tag{4.5b}$$

As expected, both the W and the Z masses receive corrections $O(v^2/\Lambda^2)$.

In order to relate the ratio of the W and Z masses to the measured weak angle we first have to discuss the corrections to the Fermi constant and to neutral current couplings which is the subject of the following two subsections. Let us note that also the relation between the gauge couplings g, g' and the electromagnetic charge e is modified. From eq. (4.2) one obtains

$$e = g\sin\theta_w^0\left(1+\alpha_{AA}\frac{v^2}{\Lambda^2}\right). \tag{4.6}$$

4.2. THE COUPLINGS OF W AND Z TO FERMIONS

The couplings of the massive vector bosons to quarks and leptons are described by the lagrangian

$$\mathscr{L}_J=\sqrt{\tfrac{1}{2}}\,g(J_\mu^C W^{+\mu}+J_\mu^{C\dagger}W^{-\mu})+\frac{g}{\cos\theta_w^0}J_\mu^N Z^\mu\,, \tag{4.7}$$

where J_μ^C and J_μ^N are the charged and neutral currents. Both of them are affected by the operators of subsect. 3.7. One easily obtains from (3.48)–(3.55) and (4.2):

$$J_\mu^C=\eta(\nu_L)\bar\nu_L\gamma_\mu e_L+\eta(u_L)\bar u_L\gamma_\mu d_L+\eta(u_R)\bar u_R\gamma_\mu d_R\,, \tag{4.8a}$$

with

$$\eta(\nu_L)=1+(\alpha_{\varphi W}+2\alpha_{\varphi\ell}^{(3)})\frac{v^2}{\Lambda^2}\,, \tag{4.8b}$$

$$\eta(u_L)=1+(\alpha_{\varphi W}+2\alpha_{\varphi q}^{(3)})\frac{v^2}{\Lambda^2}\,, \tag{4.8c}$$

$$\eta(u_R)=-\alpha_{\varphi\varphi}\frac{v^2}{\Lambda^2}\,; \tag{4.8d}$$

and

$$J_\mu^N=\varepsilon(\nu_L)\bar\nu_L\gamma_\mu\nu_L+\varepsilon(e_L)\bar e_L\gamma_\mu e_L+\varepsilon(e_R)\bar e_R\gamma_\mu e_R$$
$$+\varepsilon(u_L)\bar u_L\gamma_\mu u_L+\varepsilon(d_L)\bar d_L\gamma_\mu d_L+\varepsilon(u_R)\bar u_R\gamma_\mu u_R+\varepsilon(d_R)\bar d_R\gamma_\mu d_R\,, \tag{4.9a}$$

with

$$\varepsilon(\nu_{\mathrm{L}})=\tfrac{1}{2}+\tfrac{1}{2}(\alpha_{ZZ}-\alpha_{\varphi\ell}^{(1)}+\alpha_{\varphi\ell}^{(3)})\frac{v^2}{\Lambda^2}, \tag{4.9b}$$

$$\varepsilon(e_{\mathrm{L}})=-\tfrac{1}{2}+x-\tfrac{1}{2}(\alpha_{ZZ}+\alpha_{\varphi\ell}^{(1)}+\alpha_{\varphi\ell}^{(3)}-2x\alpha_{ZZ}-2y\alpha_{AZ})\frac{v^2}{\Lambda^2}, \tag{4.9c}$$

$$\varepsilon(e_{\mathrm{R}})=x-\tfrac{1}{2}(\alpha_{\varphi_e}-2x\alpha_{ZZ}-2y\alpha_{AZ})\frac{v^2}{\Lambda^2}, \tag{4.9d}$$

$$\varepsilon(u_{\mathrm{L}})=\tfrac{1}{2}-\tfrac{2}{3}x+\tfrac{1}{2}(\alpha_{ZZ}-\alpha_{\varphi q}^{(1)}+\alpha_{\varphi q}^{(3)}-\tfrac{4}{3}x\alpha_{ZZ}-\tfrac{4}{3}y\alpha_{AZ})\frac{v^2}{\Lambda^2}, \tag{4.9e}$$

$$\varepsilon(d_{\mathrm{L}})=-\tfrac{1}{2}+\tfrac{1}{3}x-\tfrac{1}{2}(\alpha_{ZZ}+\alpha_{\varphi q}^{(1)}+\alpha_{\varphi q}^{(3)}-\tfrac{2}{3}x\alpha_{ZZ}-\tfrac{2}{3}y\alpha_{AZ})\frac{v^2}{\Lambda^2}, \tag{4.9f}$$

$$\varepsilon(u_{\mathrm{R}})=-\tfrac{2}{3}x-\tfrac{1}{2}(\alpha_{\varphi u}+\tfrac{4}{3}x\alpha_{ZZ}+\tfrac{4}{3}y\alpha_{AZ})\frac{v^2}{\Lambda^2}, \tag{4.9g}$$

$$\varepsilon(d_{\mathrm{R}})=\tfrac{1}{3}x-\tfrac{1}{2}(\alpha_{\varphi d}-\tfrac{2}{3}x\alpha_{ZZ}-\tfrac{2}{3}y\alpha_{AZ})\frac{v^2}{\Lambda^2}, \tag{4.9h}$$

$$x=\sin^2\theta_{\mathrm{w}}^0, \qquad y=\sin\theta_{\mathrm{w}}^0\cos\theta_{\mathrm{w}}^0. \tag{4.9i}$$

The most important deviations from the standard model are the appearance of right-handed charged currents and the absence of quark-lepton universality in charged and neutral currents. Furthermore, we note that the coefficients α_i are matrices in generation space which in general will contain arbitrary flavour mixing.

4.3. THE FERMI CONSTANT

In the standard model the Fermi constant, measured in μ-decay, reads at tree level

$$G_{\mathrm{F}}^0=\frac{1}{4\sqrt{2}}\frac{g^2}{(m_{\mathrm{W}}^0)^2}. \tag{4.10}$$

The non-perturbative corrections described by $\mathscr{L}_2$ modify the W-lepton couplings, the W-mass and also directly the four-fermion operator for μ-decay. From eqs. (3.7), (4.5a) and (4.8b), one easily finds

$$G_{\mathrm{F}}=G_{\mathrm{F}}^0\left[1+(2\alpha_{\ell\ell}^{(3)}+4\alpha_{\varphi\ell}^{(3)}-\alpha_{\varphi}^{(1)}-4\alpha_{\varphi W})\frac{v^2}{\Lambda^2}\right], \tag{4.11}$$

where, for simplicity, we have assumed different elements of the same matrix appearing in (4.11) to be equal.

4.4. THE WEAK ANGLE

A particularly convenient quantity to determine the weak angle is the ratio [13]

$$R=\frac{\sigma^{\nu\mathrm{NC}}-\sigma^{\bar{\nu}\mathrm{NC}}}{\sigma^{\nu\mathrm{CC}}-\sigma^{\bar{\nu}\mathrm{CC}}}, \tag{4.12}$$

where $\sigma^{(\bar{\nu})\mathrm{NC}}$ and $\sigma^{(\bar{\nu})\mathrm{CC}}$ denote the neutrino (antineutrino) nucleon cross sections for an isoscalar target. In terms of charged and neutral current couplings defined through

$$\mathscr{L}_{\nu q}^{\mathrm{CC}}=\frac{g^2}{2m_{\mathrm{W}}^2}\bar{e}_{\mathrm{L}}\gamma^{\mu}\nu_{\mathrm{L}}[\eta_{\nu}(u_{\mathrm{L}})\bar{u}_{\mathrm{L}}\gamma_{\mu}d_{\mathrm{L}}+\eta_{\nu}(u_{\mathrm{R}})\bar{u}_{\mathrm{R}}\gamma_{\mu}d_{\mathrm{R}}]+\text{c.c.}, \tag{4.13a}$$

$$\begin{aligned}\mathscr{L}_{\nu q}^{\mathrm{NC}}=\frac{g^2}{2\cos^2\theta_{\mathrm{w}}^0 m_{\mathrm{Z}}^2}\bar{\nu}_{\mathrm{L}}\gamma^{\mu}\nu_{\mathrm{L}}[&\varepsilon_{\nu}(u_{\mathrm{L}})\bar{u}_{\mathrm{L}}\gamma_{\mu}u_{\mathrm{L}}+\varepsilon_{\nu}(d_{\mathrm{L}})\bar{d}_{\mathrm{L}}\gamma_{\mu}d_{\mathrm{L}}\\ &+\varepsilon_{\nu}(u_{\mathrm{R}})\bar{u}_{\mathrm{R}}\gamma_{\mu}u_{\mathrm{R}}+\varepsilon_{\nu}(d_{\mathrm{R}})\bar{d}_{\mathrm{R}}\gamma_{\mu}d_{\mathrm{R}}],\end{aligned} \tag{4.13b}$$

one has

$$R=\left(\frac{m_{\mathrm{W}}^2}{\cos^2\theta_{\mathrm{w}}^0 m_{\mathrm{Z}}^2}\right)^2\frac{\varepsilon_{\nu}(u_{\mathrm{L}})^2+\varepsilon_{\nu}(d_{\mathrm{L}})^2-\varepsilon_{\nu}(u_{\mathrm{R}})^2-\varepsilon_{\nu}(d_{\mathrm{R}})^2}{\eta_{\nu}(u_{\mathrm{L}})^2-\eta_{\nu}(u_{\mathrm{R}})^2}. \tag{4.14}$$

The weak angle is determined from

$$R=\tfrac{1}{2}(1-2\sin^2\theta_{\mathrm{w}}^{\mathrm{LE}}) \tag{4.15}$$

through a measurement of R. We denote this angle by $\theta_{\mathrm{w}}^{\mathrm{LE}}$ as it is defined in terms of low-energy neutral current processes.

The couplings η and ε in the lagrangians (4.13) follow from eqs. (4.8), (4.9) and the neutrino-quark four-fermion operators (3.10), (3.18) and (3.19). Neglecting the scalar contribution (3.23), one easily finds up to $\mathrm{O}(v^4/\Lambda^4)$:

$$\eta_{\nu}(u_{\mathrm{L}})=\eta(\nu_{\mathrm{L}})\eta(u_{\mathrm{L}})+2\alpha_{\ell q}^{(3)}\frac{v^2}{\Lambda^2}, \tag{4.16a}$$

$$\eta_{\nu}(u_{\mathrm{R}})=\eta(\nu_{\mathrm{L}})\eta(u_{\mathrm{R}}), \tag{4.16b}$$

$$\varepsilon_{\nu}(u_{\mathrm{L}})=2\varepsilon(\nu_{\mathrm{L}})\varepsilon(u_{\mathrm{L}})+(\alpha_{\ell q}^{(1)}+\alpha_{\ell q}^{(3)})\frac{v^2}{\Lambda^2}, \tag{4.16c}$$

$$\varepsilon_{\nu}(d_{\mathrm{L}})=2\varepsilon(\nu_{\mathrm{L}})\varepsilon(d_{\mathrm{L}})+(\alpha_{\ell q}^{(1)}-\alpha_{\ell q}^{(3)})\frac{v^2}{\Lambda^2}, \tag{4.16d}$$

$$\varepsilon_{\nu}(u_{\mathrm{R}})=2\varepsilon(\nu_{\mathrm{L}})\varepsilon(u_{\mathrm{R}})+\tfrac{1}{2}\alpha_{\ell u}\frac{v^2}{\Lambda^2}, \tag{4.16e}$$

$$\varepsilon_{\nu}(d_{\mathrm{R}})=2\varepsilon(\nu_{\mathrm{L}})\varepsilon(d_{\mathrm{R}})+\tfrac{1}{2}\alpha_{\ell d}\frac{v^2}{\Lambda^2}. \tag{4.16f}$$

Inserting the expressions (4.16) into (4.14) we obtain from (4.5) and (4.15) up to $O(v^4/\Lambda^4)$:

$$\sin^2\theta_w^{LE} = \sin^2\theta_w^0 + \delta\,,$$

$$\delta = [(\alpha_{\varphi\ell}^{(1)} + \alpha_{\varphi\ell}^{(3)} + \alpha_{\varphi q}^{(3)} + \alpha_{\varphi}^{(3)})(1-2x) - \tfrac{2}{3}x(\alpha_{\varphi q}^{(1)} + \alpha_{\varphi q}^{(3)} - \alpha_{\varphi u} - 2\alpha_{\ell q}^{(1)} + 2\alpha_{\ell q}^{(3)} + \alpha_{\ell u})$$
$$+ \tfrac{1}{3}x(\alpha_{\varphi q}^{(1)} - \alpha_{\varphi q}^{(3)} - \alpha_{\varphi d} - 2\alpha_{\ell q}^{(1)} - 2\alpha_{\ell q}^{(3)} + \alpha_{\ell d}) + y\alpha_{AZ}]\frac{v^2}{\Lambda^2}\,, \tag{4.17}$$

which deviates again from the standard model angle by terms linear in $1/\Lambda^2$.

4.5. THE ρ-PARAMETER

A quantity, which is very sensitive to deviations from the standard model, is the ρ-parameter

$$\rho = \frac{m_W^2}{m_Z^2\cos^2\theta_w^{LE}}\,. \tag{4.18}$$

In the standard model one has $\rho = 1$ up to radiative corrections which decrease ρ by about 2% [14]. The effect of the non-perturbative corrections on ρ can be obtained from eqs. (4.5) and (4.17) which yield

$$\rho = 1 + 2[x(\alpha_{\varphi W} - \alpha_{\varphi B}) - 2y\alpha_{WB} - \tfrac{1}{2}\alpha_{\varphi}^{(3)}]\frac{v^2}{\Lambda^2} + \frac{\delta}{\cos^2\theta_w^0}\,. \tag{4.19}$$

It is well known that in the standard model $\rho = 1$ is guaranteed by the "custodial" SU(2)' symmetry [15] under which $\Phi = (\varphi, \tilde\varphi)$, with $\tilde\varphi^i = \varepsilon^{ij}\varphi_j^*$, transforms as a doublet. The part of $\mathscr{L}_0$ containing scalars and W-vector bosons is invariant under SU(2)', whereas the inequality of up and down Yukawa coupling matrices violates SU(2)'. Therefore, one-loop radiative corrections involving fermion loops lead to a deviation from $\rho = 1$. The v^2/Λ^2 correction in eq. (4.19) depends on coefficients which violate the SU(2)' symmetry, such as $\alpha_{\varphi}^{(3)}$, $\alpha_{\varphi q}^{(1)}$, $\alpha_{\varphi q}^{(3)}$, etc. For $g' = 0$, (4.19) does not depend on the SU(2)' invariant coefficients $\alpha_{\varphi}^{(1)}$, $\alpha_{\varphi W}$ and $\alpha_{\varphi B}$. As expected the custodial SU(2)' symmetry controls deviations from $\rho = 1$ also for the non-perturbative effects described by $\mathscr{L}_2$. We emphasize that contrary to the standard model already the part of $\mathscr{L}_2$ with scalars and vectors only contains a term which violates SU(2)', $\alpha_{\varphi}^{(3)}$, and thus leads to a deviation from $\rho = 1$.

4.6. THE FERMION HIGGS COUPLINGS

The couplings of the neutral Higgs boson H^0 to quarks and leptons is also modified by the operators of subsect. 3.6. From

$$\mathscr{L}_{e\varphi} = \Gamma_e \bar{\ell}_e\varphi + \frac{1}{\Lambda^2}\alpha_{e\varphi}(\varphi^\dagger\varphi)(\bar{\ell}e\varphi) + \text{c.c.} \tag{4.20}$$

and

$$\varphi = \begin{pmatrix} 0 \\ v + H^0 \end{pmatrix}, \tag{4.21}$$

one obtains

$$\mathcal{L}_{e\varphi} = M_e \bar{e}_L e_R + g_{eH} \bar{e}_L e_R H^0 + \cdots, \tag{4.22}$$

where

$$M_e = v\left(\Gamma_e + \frac{v^2}{\Lambda^2}\alpha_{e\varphi}\right), \tag{4.23a}$$

$$g_{eH} = \Gamma_e + 3\frac{v^2}{\Lambda^2}\alpha_{e\varphi}. \tag{4.23b}$$

As a consequence of the dimension-six operators, masses and Higgs couplings are no longer proportional. The modified relations for quarks and leptons read

$$g_{eH} = \frac{1}{v} M_e + 2\frac{v^2}{\Lambda^2}\alpha_{e\varphi}, \tag{4.24a}$$

$$g_{uH} = \frac{1}{v} M_u + 2\frac{v^2}{\Lambda^2}\alpha_{u\varphi}, \tag{4.24b}$$

$$g_{dH} = \frac{1}{v} M_d + 2\frac{v^2}{\Lambda^2}\alpha_{d\varphi}. \tag{4.24c}$$

We emphasize again that Γ_i and $\alpha_{i\varphi}$, $i =$ e, u, d, are matrices in generation space. Therefore the Higgs couplings are only flavour diagonal if in the change from weak eigenstates to mass eigenstates Γ_i and $\alpha_{i\varphi}$ are diagonalized simultaneously. We will come back to this point in more detail in sect. 6. Furthermore we notice that the Higgs couplings will in general be *CP* nonconserving if Γ_e and $\alpha_{e\varphi}$ have different phases.

4.7. THE HIGGS MASS AND SELF-INTERACTION

The deviations from the standard model also modify the self-interactions of the scalar particles. Although Higgs–Higgs scattering is unlikely to be studied experimentally in the foreseeable future, the φ^4 coupling manifests itself indirectly through radiative corrections.

Including (3.27) the scalar potential reads

$$V = m^2\varphi^\dagger\varphi + \tfrac{1}{2}\lambda(\varphi^\dagger\varphi)^2 + \frac{1}{3\Lambda^2}\alpha_\varphi(\varphi^\dagger\varphi)^3. \tag{4.25}$$

The coefficient α_φ has to be positive in order for V to be bounded from below and to have a local minimum satisfying $v/\Lambda \ll 1$. The minimization yields for the vacuum expectation value

$$v^2 = -\frac{m^2}{\lambda}\left(1+\alpha_\varphi \frac{m^2}{\lambda^2\Lambda^2}+\mathrm{O}\left(\frac{m^4}{\lambda^4\Lambda^4}\right)\right). \tag{4.26}$$

For the mass and the quartic coupling of the Higgs scalar one finds up to terms linear in $1/\Lambda^2$:

$$m_{\mathrm{H}}^2 = 4\lambda v^2 + 8\alpha_\varphi \frac{v^4}{\Lambda^2}, \tag{4.27}$$

$$\lambda_{\mathrm{H}} = \lambda + 10\alpha_\varphi \frac{v^2}{\Lambda^2}. \tag{4.28}$$

Eqs. (4.5) and (4.28) yield for the mass ratio of the Higgs boson and the W vector boson:

$$\frac{m_{\mathrm{H}}^2}{m_{\mathrm{W}}^2} = 8\frac{\lambda}{g^2}\left[1+(2\alpha_\varphi - \alpha_\varphi^{(1)} - 4\alpha_{\varphi W})\frac{v^2}{\Lambda^2}\right]. \tag{4.29}$$

4.8. MAGNETIC COUPLINGS

In the subsects. 4.1 to 4.7, we have considered the effect of dimension-six operators on mass relations and coupling constants which are already present in the standard model. The operators listed in sect. 3 contain, of course, also many new interactions. Of particular interest are the magnetic moment type interactions of subsect. 3.7. Replacing φ by its vacuum expectation value one obtains for the magnetic couplings of gauge bosons to fermions:

$$\begin{aligned}
\mathcal{L}_{\mathrm{M}} = {} & g_s \frac{v}{\Lambda^2}[\alpha_{uG}\bar{u}_{\mathrm{L}}\sigma^{\mu\nu}\lambda^A u_{\mathrm{R}} + \alpha_{dG}\bar{d}_{\mathrm{L}}\sigma^{\mu\nu}\lambda^A d_{\mathrm{R}}]G^A_{\mu\nu} \\
& + e\frac{v}{\Lambda^2}[(\alpha_{uB}+\alpha_{uW})\bar{u}_{\mathrm{L}}\sigma^{\mu\nu}u_{\mathrm{R}} + (\alpha_{dB}-\alpha_{dW})\bar{d}_{\mathrm{L}}\sigma^{\mu\nu}d_{\mathrm{R}} \\
& + (\alpha_{eB}-\alpha_{eW})\bar{e}_{\mathrm{L}}\sigma^{\mu\nu}e_{\mathrm{R}}](A_{\mu\nu} - 2\tan\theta^0_{\mathrm{w}}\partial_\mu Z_\nu) \\
& + 2\frac{g}{\cos\theta^0_{\mathrm{w}}}\frac{v}{\Lambda^2}[\alpha_{uW}\bar{u}_{\mathrm{L}}\sigma^{\mu\nu}u_{\mathrm{R}} - \alpha_{dW}\bar{d}_{\mathrm{L}}\sigma^{\mu\nu}d_{\mathrm{R}} \\
& - \alpha_{eW}\bar{e}_{\mathrm{L}}\sigma^{\mu\nu}e_{\mathrm{R}}](\partial_\mu Z_\nu - ig\cos\theta^0_{\mathrm{w}} W^+_\mu W^-_\nu) \\
& + 2\sqrt{2}\, g\frac{v}{\Lambda^2}[\alpha_{dW}\bar{u}_{\mathrm{L}}\sigma^{\mu\nu}d_{\mathrm{R}} + \alpha^\dagger_{uW}\bar{u}_{\mathrm{R}}\sigma^{\mu\nu}d_{\mathrm{L}} \\
& + \alpha_{eW}\bar{\nu}_{\mathrm{L}}\sigma^{\mu\nu}e_{\mathrm{R}}](D_\mu - ig\cos\theta^0_{\mathrm{w}}Z_\mu)W^+_\nu + \text{c.c.},
\end{aligned}$$

where

$$D_\mu = \partial_\mu - ieA_\mu\,, \qquad A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \tag{4.30}$$

The lagrangian (4.30) contains the familiar magnetic moment couplings to the photon which give rise to anomalous magnetic moments for leptons and quarks. All couplings will in general be flavour non-diagonal which leads to transitions such as $\mu \to e\gamma$, $c \to uG$, etc. The operators in subsect. 3.4, which are not of magnetic type, can also contribute to these processes. They contain, however, one more derivative, and the corresponding amplitudes are therefore suppressed with respect to the magnetic amplitudes by $m_f/v \sim g_f$, where f denotes the heaviest fermion in the process. The lagrangian (4.30) contains also couplings between two fermions and two gauge bosons which can contribute for instance to the decay $Z \to W^- e^+ \nu_e$ [16]. We note that no operator exists for the decay $Z \to e^+ e^- \gamma$.

In this section we have not considered all deviations from the standard model. We have not discussed, for instance, deviations from the V-A structure in μ-decay. Furthermore we notice that the presence of right-handed neutrinos would lead to operators simulating right-handed currents.

5. Bounds on the new interaction scale Λ

The operators discussed in the previous sections lead to deviations from standard model predictions for masses and couplings, and also to new effects, especially with respect to flavour changing processes. In this section we study the bounds on Λ which can be obtained from the present agreement between experiment and the standard electroweak theory.

In general quantities such as the effective neutral current lagrangian or the ρ parameter depend on a number of different operator coefficients whose relative sizes and signs are unknown. Here we do not attempt a complete analysis, which would have to combine the experimental constraints on all coefficients. However, in order to compare bounds obtained from different quantities in a consistent manner, we proceed as follows. For a measured quantity the deviation from the standard model prediction has the generic form

$$A = A_0\left(1 + \sum_i c_i \alpha_i \frac{v^2}{\Lambda^2}\right), \tag{5.1}$$

where α_i are the coefficients of the various operators contributing to A. We will set $|\sum_i c_i \alpha_i| = 1$ and thereby convert bounds on $(A - A_0)/A_0$ into bounds on Λ. If the bound is obtained from a combination of measured quantities whose non-perturbative corrections are not known to be correlated we will assume them to be additive. The coefficients of operators causing new effects are chosen to be 1.

5.1. WEAK INTERACTION PARAMETERS

(*a*) *V–A structure of the leptonic charged current.* The operator $O_{\ell e}$ (3.17) contributes to μ-decay through

$$\mathscr{L}_{\mu e} = \frac{1}{\Lambda^2}\, \bar{e}_R \nu_{eL} \bar{\nu}_{\mu L} \mu_R + \text{h.c.}\,. \tag{5.2}$$

The recent analysis of Mursula and Scheck yields for a possible scalar contribution the bound [17]

$$\frac{2m_W^2}{g^2\Lambda^2} < 0.076 \tag{5.3}$$

and therefore

$$\Lambda > 630\ \text{GeV}\,. \tag{5.4}$$

A similar bound follows for a tensor contribution to the charged weak current.

(*b*) *Quark-lepton universality.* A stringent test of the universality of the charged current weak interactions of quarks and leptons is the equality of the Fermi constants $G_F^{(\mu)}$ and $G_F^{(\beta)}$ determined in μ-decay and β-decay or, equivalently, the unitarity of the Kobayashi–Maskawa matrix. The experimental determinations [18] $V_{ud} = 0.973 \pm 0.001$, $V_{us} = 0.231 \pm 0.003$, $V_{ub} < 0.0067$ yield

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 - 1 = \pm 0.003\,. \tag{5.5}$$

A deviation from universality (cf. subsect. 4.2) which changes each KM matrix element by v^2/Λ^2 leads, together with (5.5), to the bound

$$\Lambda > 4.9\ \text{TeV}\,. \tag{5.6}$$

We emphasize that in models of "nearby compositeness", where quarks and leptons are supposed to have a different internal structure at a scale below 1 TeV, a special dynamical mechanism will be necessary to guarantee that all contact interactions of quarks and leptons as well as their couplings to (composite) W-bosons will be compatible with the universality constraint (5.5).

An alternative deviation from Cabibbo universality is possible if there are new very heavy fermions which mix with the usual ones but are too heavy to be produced in an experiment [19]. In such models one obtains bounds on the mixing angles of light and heavy fermions of about 0.1 (in ref. [19] this was only considered for neutrinos but of course it also holds for other particles). If the deviation from Cabibbo universality is due to a mixing with heavy fermions rather than additional contact interactions, the observed states are not orthogonal.

(*c*) *ρ-parameter.* The ρ-parameter (4.18) involves the three observables m_W, m_Z and $\cos\theta_W^{LE}$. Assuming a v^2/Λ^2 correction for each of them, we obtain from $\delta\rho/\rho < 2\%$ [14] (cf. ref. [67]).

$$\Lambda > 2.8\ \text{TeV}\,, \tag{5.7}$$

(*d*) *Neutron dipole moment.* If the new interactions are not CP conserving the operator $O_{\varphi\tilde{G}}$ (3.40) contributes to the strong CP violating θ parameter [20] ($g_s^2\alpha_{\varphi\tilde{G}} = 1$):

$$\delta\theta = 32\pi^2 \frac{v^2}{\Lambda^2}. \tag{5.8}$$

From $\theta < 10^{-9}$ [20] one obtains the stringent bound

$$\Lambda > 10^5\ \mathrm{TeV}. \tag{5.9}$$

All our bounds from weak interaction parameters are summarized in table 1.

5.2. MAGNETIC TRANSITIONS

The magnetic couplings between fermion pairs and gauge bosons arise from operators (cf. subsect. 3.7) whose fermion-scalar part is similar to the Yukawa couplings of $\mathscr{L}_0$. The same is true for the operators of subsect. 3.6. The operator $O_{e\varphi}$ (3.45), for instance, contributes to the electron mass:

$$\delta m_e = \frac{v^3}{\Lambda^2}. \tag{5.10}$$

Demanding $\delta m_e < m_e$ yields the bound

$$\Lambda > 100\ \mathrm{TeV}. \tag{5.11}$$

One may argue that this bound is artificial because the coefficient $\alpha_{e\varphi}$, as well as the coefficients of the magnetic transition operators, should be of the order of the Yukawa coupling g_e. In the following we will therefore always quote a bound for Λ, corresponding to $\alpha = 1$, and also for Λ', which is obtained with $\alpha = (m_1 + m_2)/v$, where $m_{1,2}$ are the masses of the two fermions.

From eq. (4.30) one obtains for the anomalous magnetic moments of electron and muon

$$\delta a_{e\mu} = 4\frac{m_{e,\mu} v}{\Lambda^2}. \tag{5.12}$$

TABLE 1

Bounds from weak interaction parameters (subsect. 5.1)

	Λ [TeV]	exp. bound	
S/(V-A), μ-decay	0.65	<0.076	[17]
$\lvert G_F^{(\beta)} - G_F^{(\mu)}/G_F^{(\mu)}\rvert$	5.0	<0.003	[18]
$\delta\rho$	2.0	<0.02	[14]
d_n, strong CPV	10^5	$\theta < 10^{-9}$	[20]

References are in square brackets.

For the flavour changing transitions $\mu \to e\gamma$, $\tau \to \mu\gamma$ $(e\gamma)$ one has (cf. ref. [21])

$$\Lambda = \left(\frac{3\times 2^{10}\pi^3\alpha_{EM}}{BR(e_i \to e_j\gamma)}\right)^{1/4}\left(\frac{v}{m_\mu}\right)^{1/2} v. \tag{5.13}$$

An interesting bound on the colour magnetic transition $c \to uG$ can be obtained by assuming that the corresponding branching ratio is smaller than the Cabibbo forbidden transition $c \to d\bar{d}u$:

$$\Lambda = \left(\frac{2^{14}\pi^3\alpha_s}{5BR(c \to d\bar{d}u)}\right)^{1/4}\left(\frac{v}{m_c}\right)^{1/2} v \tag{5.14}$$

where

$$BR(c \to d\bar{d}u) \approx \tfrac{3}{5}|V_{cd}|^2 . \tag{5.15}$$

The bounds following from eqs. (5.12)–(5.15) are collected in table 2. We note that

$$\Lambda \sim \delta a^{-1/2}, \qquad \Lambda \sim BR(f_i \to f_j\gamma, G)^{-1/4}, \tag{5.16}$$

i.e., the anomalous magnetic moments are proportional to an amplitude whereas the branching ratios scale like the square of an amplitude. As can be seen in table 2, the current experimental bounds for heavy flavours impose already stringent constraints on their flavour changing couplings.

5.3. FOUR-FERMION INTERACTIONS

Obviously the four-fermion operators in subsect. 3.2 can contribute to rare processes. They have already been studied in detail especially by Cahn and Harari [8] and Dimopoulos and Ellis [9].

Of particular interest for composite models is the process $K_L^0 \to e^-\mu^+$ because it conserves baryon and lepton number as well as generation number. The operators

TABLE 2

Bounds on Λ from magnetic transitions (subsect. 5.2)

Process	Λ [TeV]	Λ' [TeV]	exp. bound	
δa_e	40	0.1	$<2\times10^{-10}$	[22]
δa_μ	50	1.5	$<3\times10^{-8}$	[22]
$\mu\to e\gamma$	10^4	250	$BR<1.9\times10^{-10}$	[23]
$\tau\to\mu\gamma$	60	6	$BR<5.5\times10^{-4}$	[23]
$\tau\to e\gamma$	60	6	$BR<6.4\times10^{-4}$	[23]
$c\to uG$	60	5	$BR<0.03$	

$\Lambda' = \Lambda((m_1+m_2)/v)^{1/2}$; $\alpha_s = 0.2$; $m_c = 1.2$ GeV; references are in square brackets.

$O_{\ell q}$ (3.10), $O_{\ell d}$ (3.19), O_{eq} (3.20) and O_{ed} (3.15) can contribute. For a pure V–A transition one has

$$\frac{\Gamma(\mathrm{K_L^0}\to \mathrm{e}^-\mu^+)}{\Gamma(\mathrm{K}^+\to\mu^+\nu_\mu)}=\left(\frac{v^2}{\Lambda^2\sin\theta_c}\right)^2=\left(\frac{363\ \mathrm{GeV}}{\Lambda}\right)^4. \tag{5.17}$$

Similar bounds are obtained for other operators. Note, however, that the linear combination $O_{\ell q}+O_{\ell d}\sim(\bar{e}_L\gamma^\mu\mu_L)(\bar{s}\gamma_\mu d)$ is not bounded since $\langle \mathrm{K_L^0}|\bar{s}\gamma_\mu d|0\rangle=0$. The same operators which contribute to $\mathrm{K_L^0}\to\mathrm{e}^-\mu^+$ also affect the process $\mathrm{K}^+\to\pi^+\mathrm{e}^-\mu^+$. For a pure V–A transition, one obtains

$$\frac{\Gamma(\mathrm{K}^+\to\pi^+e^-\mu^+)}{\Gamma(\mathrm{K}^+\to\pi^0\mu^+\nu_\mu)}=\left(\frac{v^2}{\Lambda^2\sin\theta_c}\right)^2=\left(\frac{363\ \mathrm{GeV}}{\Lambda}\right)^4. \tag{5.18}$$

The conversion $\mu\mathrm{N}\to\mathrm{eN}$ can occur via $O_{\ell q}$ (3.10), $O_{\ell u}$ (3.18), $O_{\ell d}$ (3.19), O_{eq} (3.20), O_{eu} (3.14) and O_{ed} (3.15). To obtain a coherence effect only the hadronic vector part will contribute sizeably; in the non-relativistic limit also only the leptonic vector part is important. For a pure V–A transition,

$$\mathscr{L}_{\mu\mathrm{N}\to\mathrm{eN}}=\frac{1}{\Lambda^2}\bar{e}_L\gamma^\mu\mu_L(\bar{u}_L\gamma_\mu u_L+\bar{d}_L\gamma_\mu d_L). \tag{5.19}$$

We obtain from the calculation of Cahn and Harari [8]

$$\frac{\Gamma(\mu\mathrm{N}\to\mathrm{eN})}{\Gamma(\mu\mathrm{N}\to\nu\mathrm{N})}=\frac{27\times 2^4}{1+3g_A^2}\left(\frac{v}{\Lambda}\right)^4, \tag{5.20}$$

where $g_A=-1.25$ is the axial nucleon form factor.

The purely lepton process $\mu\to 3\mathrm{e}$ receives contributions from the operators $O_{\ell\ell}$ (3.7), $O_{\ell e}$ (3.17) and O_{ee} (3.13). For a pure V–A transition one has

$$\frac{\Gamma(\mu\to 3\mathrm{e})}{\Gamma(\mu\to \mathrm{e}\nu\bar{\nu})}=\left(\frac{v}{\Lambda}\right)^4=\left(\frac{174\ \mathrm{GeV}}{\Lambda}\right)^4. \tag{5.21}$$

Finally we consider the mass differences in the $\mathrm{K}^0-\bar{\mathrm{K}}^0$, $\mathrm{D}^0-\bar{\mathrm{D}}^0$ and $\mathrm{B_s}-\bar{\mathrm{B}}_\mathrm{s}$ system. The operators which can contribute are O_{qq} (3.8, 9), O_{qu} (3.21), O_{qd} (3.22), O_{uu} (3.12) and O_{dd} (3.13). The simplest case is $O_{qq}^{(1,1)}$ since it contains the same operators as those arising in the standard model (from the box diagram [24]).

In order to obtain bounds on Λ from the kaon and D-mesons systems we use the experimental value of $m_{\mathrm{K_L}}-m_{\mathrm{K_S}}$ and the experimental bound on $|m_{\mathrm{D_1^0}}-m_{\mathrm{D_2^0}}|$ [25], and require the effects from the $O_{qq}^{(1,1)}$ operators to be less than these numbers. The matrix elements are evaluated in the vacuum insertion approximation; we take $f_\mathrm{K}\simeq 0.12$ GeV, $f_\mathrm{D}=0.10$ GeV (higher values for f_D raise the bound). One has:

$$\frac{\Delta m_{\mathrm{K,D}}}{m_{\mathrm{K,D}}}=\frac{1}{3}\frac{f_{\mathrm{K,D}}^2}{\Lambda^2}. \tag{5.22}$$

In the case of the B_s meson system we compare against the theoretical value in the standard model. Using approximate values for the KM angles [26], we obtain

$$\frac{1}{\Lambda^2} \leqslant \sin^4\theta_c \left(\frac{m_t}{37\ \mathrm{GeV}}\right)^2 \frac{\alpha_{EM}}{4\pi v^2}. \tag{5.23}$$

(In the case of the B_s meson this should give the right order of magnitude, since $m_t \gg m_b$.) All the bounds on Λ, for pure V-A four-fermion operators [eqs. (5.17)-(5.22)] are summarized in table 3.

A slightly better constraint follows for the operators O_{qd} (3.22) because they have a different Lorentz structure,

$$O_{qd}^{(1)} = \bar{s}_L d_R \bar{s}_R d_L + \cdots. \tag{5.24}$$

This operator has been studied in the context of left-right symmetric models and has been shown to yield an extra factor of ~7.5 in the vacuum approximation [27]. This then gives a bound on Λ of

$$\Lambda > 3200\ \mathrm{TeV}, \tag{5.25}$$

Similar factors can be calculated for other operators in the same way as in ref. [27]. We have listed them in table 4.

It is instructive to compare the bounds obtained in this section with bounds obtained from scattering processes. From an analysis of Bhabha scattering, Eichten, Lane and Peskin have obtained bounds on Λ for $(\bar{e}e)(\bar{e}e)$ operators with different helicity structure [5]

$$\Lambda > \begin{cases} 210\ \mathrm{GeV} & (LL,\ RR \text{ models}) \\ 420\ \mathrm{GeV} & (VV,\ AA \text{ models}), \end{cases} \tag{5.26}$$

where, for consistency, we have rescaled the bounds of Eichten et al. by $1/\sqrt{4\pi}$. Deep inelastic μ-scattering can probe the operator

$$\mathscr{L}_{\mu e q} = \frac{1}{\Lambda^2} \bar{\mu}\gamma^\mu e \bar{q}\gamma_\mu q + \mathrm{c.c.} \tag{5.27}$$

TABLE 3

Bounds on Λ for $(\mathrm{V-A})^2$ four-fermion operators (see text)

Process	Λ [TeV]	exp. bound	
$K_L^0 \to e^-\mu^+$	9 (65)	$BR < 3\times10^{-6}$	[23]
$K^+ \to \pi^+ e^- \mu^+$	20	$BR < 5\times10^{-9}$	[23]
$\mu N \to eN$	250	$\dfrac{\Gamma(\mu N \to eN)}{\Gamma(\mu N \to \nu N)} < 1.6\times10^{-11}$	[28]
$\mu \to 3e$	150	$BR < 2.4\times10^{-12}$	[29]
$\Delta M(K^0-\bar{K}^0)$	1000	$< 3.5\times10^{-12}$ MeV	[23]
$\Delta M(D^0-\bar{D}^0)$	150	$< 3.3\times10^{-10}$ MeV	[25]
$\Delta M(B_s-\bar{B}_s)$	2000	< standard model	

The number in brackets corresponds to $BR(K_L^0 \to e^-\mu^+) < 10^{-9}$; references are in square brackets.

TABLE 4

Values of $R_{12}=\langle \mathrm{K}^0|(\bar{s}O_1 d)(\bar{s}O_2 d)|\bar{\mathrm{K}}^0\rangle/\langle \mathrm{K}^0|(\bar{s}\gamma^{\mu}(1-\gamma_5)d) \times(\bar{s}\gamma_{\mu}(1-\gamma_5)d)|\bar{\mathrm{K}}^0\rangle$ in the vacuum insertion approximation

Operators		R_{12}
$O_1=1\pm\gamma_5$,	$O_2=1\pm\gamma_5$	-6.3
$O_1=1-\gamma_5$,	$O_2=1+\gamma_5$	7.6
$O_1=\gamma_{\mu}(1-\gamma_5)$,	$O_2=\gamma^{\mu}(1+\gamma_5)$	-5.8
$O_1=\lambda^A\gamma_{\mu}(1-\gamma_5)$,	$O_2=\lambda^A\gamma^{\mu}(1-\gamma_5)$	$\frac{4}{3}$
$O_1=\lambda^A(1\pm\gamma_5)$,	$O_2=\lambda^A(1\pm\gamma_5)$	-6.7
$O_1=\lambda^A(1-\gamma_5)$,	$O_2=\lambda^A(1+\gamma_5)$	$\frac{2}{3}$
$O_1=\lambda^A\gamma_{\mu}(1-\gamma_5)$,	$O_2=\lambda^A\gamma^{\mu}(q+\gamma_5)$	-26.7

We use $m_{\mathrm{K}}^2/(m_{\mathrm{d}}+m_{\mathrm{s}})^2=10$.

through the reaction $\mu^-\mathrm{N}\to\mathrm{e}^-\mathrm{X}$. A planned experiment of the New Muon Collaboration is expected to reach a lower bound on Λ of 640 GeV [30] (or to find an effect).

The accelerators of the next decade can improve the present bounds almost by a factor of 4 [31]. These bounds are interesting and complementary to those discussed in this section. Our results show, however, that the most stringent bounds on Λ come from low-energy precision experiments, in particular the test of quark-lepton universality of the charged weak current.

6. Flavour conservation

In the previous section we have derived a number of strong lower bounds on the new interaction scale Λ from rare processes such as $\mathrm{K}_{\mathrm{L}}^0\to\mathrm{e}^-\mu^+$, $\mu\to\mathrm{e}\gamma$, $\mathrm{K}^0-\bar{\mathrm{K}}^0$ mixing, etc. In the standard model such processes are allowed in the quark sector, although suppressed by the GIM mechanism [32]; in the lepton sector they do not appear due to the masslessness of the neutrinos.

The observed strong suppression of neutral flavour changing processes can be understood in two ways. Either the scale of new interactions is very large, $\Lambda > 10^4$ TeV, or the flavour changes in the renormalizable part $\mathscr{L}_0$ and the non-renormalizable part $\mathscr{L}_2$ of the lagrangian are "aligned", i.e., simultaneously diagonalizable in the transformation from weak to mass eigenstates.

In this section we discuss the conditions for flavour conservation and the resulting constraints on new interaction scales. Flavour violation can occur in the two-fermion operators of subsects. 3.4, 3.6 and 3.7, and in the four-fermion operators of subsect. 3.2. We consider them in turn.

Before we do so let us repeat the connection between the (physical) mass eigenstates and the eigenstates of the weak interactions. The complete mass term of the lagrangian reads

$$\mathscr{L}_{\mathrm{M}} = M_e \bar{e}_{\mathrm{L}} e_{\mathrm{R}} + M_u \bar{u}_{\mathrm{L}} u_{\mathrm{R}} + M_d \bar{d}_{\mathrm{L}} d_{\mathrm{R}} + \text{c.c.}\,. \tag{6.1}$$

The mass matrices are diagonalized by the bi-unitary transformations

$$V_{\mathrm{L}}^{e\dagger} M_e V_{\mathrm{R}}^{e} = \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix}, \tag{6.2a}$$

$$V_{\mathrm{L}}^{u\dagger} M_u V_{\mathrm{R}}^{u} = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}, \tag{6.2b}$$

$$V_{\mathrm{L}}^{d\dagger} M_d V_{\mathrm{R}}^{d} = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}, \tag{6.2c}$$

and thus the relations between weak eigenstates and mass eigenstates (denoted by a tilde) read

$$e_{\mathrm{L}} = V_{\mathrm{L}}^{e} \tilde{e}_{\mathrm{L}}\,, \qquad e_{\mathrm{R}} = V_{\mathrm{R}}^{e} \tilde{e}_{\mathrm{R}}\,, \qquad \text{etc.} \tag{6.3}$$

This means that in all operators considered previously we must replace the fields e, u, d by $\tilde{e}$, $\tilde{u}$, $\tilde{d}$ in order to obtain the interactions between the physical particles. We note that under the transformations (6.2) the product of a mass matrix and its hermitian conjugate transforms with a single unitary matrix, e.g.,

$$\begin{aligned} M_e M_e^\dagger &\to V_{\mathrm{L}}^{e\dagger} M_e M_e^\dagger V_{\mathrm{L}}^{e}\,, \\ M_e^\dagger M_e &\to V_{\mathrm{R}}^{e\dagger} M_e^\dagger M_e V_{\mathrm{R}}\,. \end{aligned} \tag{6.4}$$

In the standard model it is always possible to choose a basis for the weak eigenstates such that $V_{\mathrm{L}}^{e} = V_{\mathrm{R}}^{e} = V_{\mathrm{L}}^{u} = V_{\mathrm{R}}^{u} = V_{\mathrm{R}}^{d} = 1$. Then the Kobayashi–Maskawa matrix is given by $V_{\mathrm{KM}} = V_{\mathrm{L}}^{d}$.

6.1. TWO-FERMION OPERATORS

Many of the relations discussed in the following between the coefficients α_i in $\mathscr{L}_2$ and the Yukawa couplings $\Gamma_{e,d,u}$ in $\mathscr{L}_0$ are identical for leptons, u-quarks and d-quarks. We will always give the explicit formulae for d-quarks and comment on occasional differences for u-quarks and leptons.

There are two couplings which change chirality, the interaction with the Higgs scalar H^0 and the magnetic coupling to gauge fields (cf. subsects. 4.6 and 4.8; $F^A_{\mu\nu} = G^A_{\mu\nu}$, $W^I_{\mu\nu}$, $B_{\mu\nu}$):

$$\mathscr{L}_{d\varphi} = M_d \bar{d}_{\mathrm{L}} d_{\mathrm{R}} + g_{dH} \bar{d}_{\mathrm{L}} d_{\mathrm{R}} H^0 + \text{c.c.}\,, \tag{6.5}$$

$$\mathscr{L}_{d\mathrm{F}} = \alpha_{d\mathrm{F}} \frac{v}{\Lambda^2} \bar{d}_{\mathrm{L}} \sigma^{\mu\nu} \mathrm{d}_R F_{\mu\nu} + \text{c.c.}\,, \tag{6.6}$$

where

$$M_{\mathrm{d}}=v\left(\Gamma_{\mathrm{d}}+\frac{v^2}{\Lambda^2}\alpha_{\mathrm{d}\varphi}\right), \tag{6.7}$$

$$g_{\mathrm{dH}}=\Gamma_{\mathrm{d}}+3\frac{v^2}{\Lambda^2}\alpha_{\mathrm{d}\varphi}. \tag{6.8}$$

Flavour conservation requires that the matrices Γ_{d}, $\alpha_{\mathrm{d}\varphi}$ and α_{dF} are simultaneously diagonalized by the bi-unitary transformation (6.2c). This means that these matrices are normal, i.e., satisfy all relations of the kind [33]:

$$[\Gamma_{\mathrm{d}}\alpha^{\dagger}_{\mathrm{d}\varphi}, \Gamma_{\mathrm{d}}\Gamma^{\dagger}_{\mathrm{d}}]=0\,, \qquad [\alpha_{\mathrm{d}\varphi}\alpha^{\dagger}_{\mathrm{d}\varphi}, \Gamma_{\mathrm{d}}\Gamma^{\dagger}_{\mathrm{d}}]=0\,, \tag{6.9a}$$

$$[\Gamma_{\mathrm{d}}\alpha^{\dagger}_{\mathrm{d}F}, \Gamma_{\mathrm{d}}\Gamma^{\dagger}_{\mathrm{d}}]=0\,, \qquad [\alpha_{\mathrm{d}F}\alpha^{\dagger}_{\mathrm{d}F}, \Gamma_{\mathrm{d}}\Gamma^{\dagger}_{\mathrm{d}}]=0\,, \qquad \text{etc.} \tag{6.9b}$$

The simplest solution to eqs. (6.9) is that the matrices $\alpha_{\mathrm{d}\varphi}$ and α_{dF} are proportional to the Yukawa coupling matrix,

$$\alpha_{\mathrm{d}\varphi}\propto\Gamma_{\mathrm{d}}\,, \qquad \alpha_{\mathrm{dF}}\propto\Gamma_{\mathrm{d}}\,, \tag{6.10}$$

which appears sensible from the point of view of composite models. We notice that, for a KM matrix different from the unit matrix, there exists no weak eigenstate basis with $\alpha_i\propto 1$ for all matrices α_i of u- *and* d-quarks.

The two fermion operators of subsect. 3.4 conserve chirality. For d-quarks one has

$$\mathscr{L}'_{\mathrm{qF}}=\frac{\alpha'_{\mathrm{qF}}}{\Lambda^2}\bar{d}_{\mathrm{L}}\gamma_{\mu}D_{\nu}d_{\mathrm{L}}F^{\mu\nu}+\text{c.c.}\,, \tag{6.11}$$

$$\mathscr{L}'_{\mathrm{dF}}=\frac{\alpha'_{\mathrm{dF}}}{\Lambda^2}\bar{d}_{\mathrm{R}}\gamma_{\mu}D_{\nu}d_{\mathrm{R}}F^{\mu\nu}+\text{c.c.}\,. \tag{6.12}$$

Flavour conservation requires that $V^{\mathrm{d}\dagger}_{\mathrm{R}}\alpha'_{\mathrm{dF}}V^{\mathrm{d}}_{\mathrm{R}}$ is diagonal which is equivalent to the commutator condition:

$$[\alpha'_{\mathrm{dF}}, \Gamma_{\mathrm{d}}\Gamma^{\dagger}_{\mathrm{d}}]=0\,. \tag{6.13}$$

The simplest solutions of (6.13) are

$$\alpha'_{\mathrm{dF}}\propto\Gamma_{\mathrm{d}}\Gamma^{\dagger}_{\mathrm{d}} \tag{6.14a}$$

or

$$\alpha'_{\mathrm{dF}}\propto 1\,. \tag{6.14b}$$

Because of the SU(2) invariance the α'_{qF} have to satisfy (6.13) *and* the same equation where Γ_{d} is replaced by Γ_{u}. As we show in the appendix, this implies for a non-trivial KM matrix:

$$\alpha'_{\mathrm{qF}}\propto 1\,. \tag{6.15}$$

We thus find a connection between the flavour and the Lorentz structure of the bilinear fermion couplings. Whereas the helicity changing vertices must have a non-trivial flavour dependence, the preferred structure of the helicity conserving vertices is the unit matrix.

6.2. FOUR-FERMION OPERATORS

Our investigation of the four-fermion operators will be based on the hypothesis that they originate from the exchange of intermediate states with masses of order Λ and with all possible internal and Lorentz quantum numbers. Favour conservation is a constraint only on the sum of all these contributions. In the spirit of "natural flavour conservation" [34] we will assume, however, that the contribution of each intermediate state conserves flavour separately. This implies that the exchanged particles carry *definite flavour*, which may be zero or non-zero. By this we mean that the couplings of the heavy states to the light fermion mass eigenstates respect a $U(1)^3$ family symmetry generated by the charges Q_p, $p=1,\dots,3$, which are 1 for the fermions of the pth family and zero for all others. The $U(1)^3$ transformations are an approximate symmetry of $\mathscr{L}_0$ which is broken by the off-diagonal elements of the KM matrix.

The effective lagrangian for the coupling of the heavy scalar (S) tensor (T) and vector (V_L, V_R) particles to the light quarks and leptons reads

$$\begin{aligned}\mathscr{L}_X &= \beta_{S}{}^{p}_{q}\bar{L}^q R_p S + \beta_{T}{}^{p}_{q}\bar{L}^q \sigma^{\mu\nu} R_p T_{\mu\nu} \\ &\quad + \beta_{V_L}{}^{p}_{q}\bar{L}^q \gamma^\mu L_p V_{L\mu} + \beta_{V_R}{}^{p}_{q}\bar{R}^q \gamma^\mu R_p V_{R\mu} + \text{c.c.}\,,\end{aligned} \tag{6.16}$$

where L and R stand generically for left- and right-handed fields and $p, q=1,\dots,3$ are generation indices; the $SU(3)\times SU(2)\times U(1)$ quantum numbers of S, T, V_L, V_R are dictated by L and R. Obviously, the exchange of the heavy particles yields the four-fermion operators of sect. 3. The operator $O_{\ell\ell}^{(1)}$ of (3.7), for instance, arises from the exchange of an isoscalar vector boson $V_{L\mu}$ coupled to lepton doublets:

$$\frac{1}{\Lambda^2}\alpha_{\ell\ell}^{(1)} O_{\ell\ell}^{(1)} = \frac{1}{2M_{V_L}^2}(\beta_{V_L}\bar{\ell}\gamma_\mu \ell)^2\,, \tag{6.17}$$

which yields

$$\begin{aligned}\Lambda &= M_{V_L}\,, \\ \alpha_{\ell\ell}^{(1)}{}^{pr}_{qs} &= \beta_{V_L}{}^{p}_{q}\beta_{V_L}{}^{r}_{s}\,.\end{aligned} \tag{6.18}$$

The conditions for the matrices β_X, $X=S, T, V_L, V_R$, which guarantee the absence of rare processes, depend on the quantum numbers of the exchanged particles X. For X fields with zero U(1) charges, i.e., $Q_p(X)=B(X)=L(X)=0$, the interactions in the effective lagrangian (6.16) are analogous to those discussed in subsect. 6.1, i.e., the Higgs couplings and the chirality changing and conserving couplings to field strengths. Thus the conditions for flavour conservation for β_X are identical with eqs. (6.9) and (6.13), if X carries no global U(1) charges. As in subsect. 6.1, chirality changing couplings have to mix different flavours whereas for chirality conserving couplings $\beta_X \propto 1$ is the simplest solution*. In composite models of quarks

* Similar conclusions have been reached independently by Mizrachi and Peccei [35].

and leptons*, however, one expects heavy states with non-zero baryon and lepton number, and possibly also non-zero generation number. If such particles are exchanged the requirement of flavour conservation in the sense of generation number conservation for the couplings β_X is not sufficient in order to prevent unwanted rare decays.

The classic example, which illustrates this, is the decay $K^0_L \to e^- \mu^+$ shown in the fig. 1. The exchanged particles have the $(Q_1, Q_2, 3B, L)$ charges

$$\begin{aligned} X_1 &\sim (1, -1, 0, 0)\,, \\ X_2 &\sim (0, 0, 1, -1)\,, \\ Y &\sim (1, 1, 1, 1)\,. \end{aligned} \tag{6.19}$$

The exchange of $X_{1,2}$ and Y causes a "flip" of the U(1) charges for the fermions at each vertex. Clearly, to prevent the process $K^0_L \to e^- \mu^+$, it is not enough to require the conservation of $Q_{1,2}$, B and L, but one has to demand that $X_{1,2}$ and Y couple only to a *single fermion pair* in the mass eigenstate basis. Then the X_1 exchange, for instance, can only lead to $(\bar{s}\tilde{d})(\bar{\tilde{d}}\tilde{s})$ or $(\bar{\tilde{e}}\tilde{\mu})(\bar{\tilde{\mu}}\tilde{e})$, but not to $(\bar{\tilde{s}}\tilde{d})(\bar{\tilde{e}}\tilde{\mu})$.

Contrary to $X_{1,2}$, Y has non-zero fermion number $F = 3B + L$. Couplings of $F = -2$ fields are described by the effective lagrangian

$$\begin{aligned} \mathscr{L}_Y = {} & \gamma^{pq}_{S_L} \bar{L}^c_p L_q S_L + \gamma^{pq}_{S_R} \bar{R}^c_p R_q S_R \\ & + \gamma^{pq}_{T_L} \bar{L}^c_p \sigma^{\mu\nu} L_q T_{L\mu\nu} + \gamma^{pq}_{T_R} \bar{R}^c_p \sigma^{\mu\nu} R_q T_{R\mu\nu} + \gamma^{pq}_{\nu} \bar{R}^c_p \gamma^{\mu} L_q V_{\mu} + \text{c.c.}\,. \end{aligned} \tag{6.20}$$

We observe that because each of the states in (6.19) couples only to one pair of fermions it is convenient to arrange them in a row of length $3 \times 3 + 3 = 12$, and to

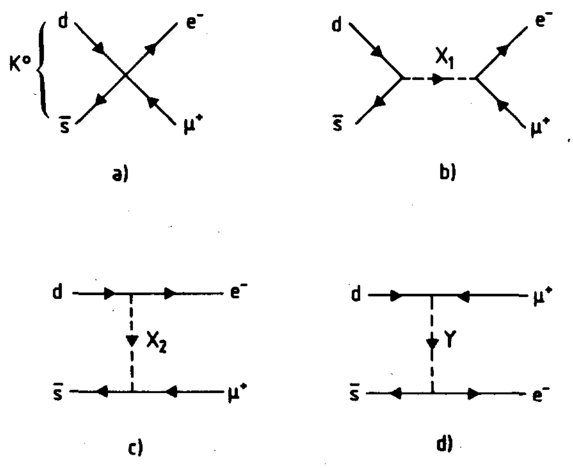

Fig. 1. Contact interaction (a) for the decay $K^0_L \to e^- \mu^+$ and the corresponding contributions of heavy intermediate states [(b) – (d)]. The generation-, baryon- and lepton-numbers of X_1, X_2 and Y are given in eq. (6.19).

* For a recent review see, e.g., ref. [36].

view the couplings of $X_{1,2}$ and Y as entries in a 12×12 matrix. This can be done for each exchanged spin separately. Thus the three families of d-quarks and charged leptons are viewed as a U(12) multiplet:

$$D^a_{\mathrm{L,R}} = (d_{\mathrm{L,R}}{}^{\alpha}_{p},\, e_{\mathrm{L,R}p})\,, \qquad a = 1\cdots 12\,. \tag{6.21}$$

The unitary transformations (6.12) induce the unitary U(12) transformations

$$V^D_{\mathrm{L,R}} = \begin{pmatrix} \mathbb{1}_c \times V^{\mathrm{d}}_{\mathrm{L,R}} & 0 \\ 0 & V^{\mathrm{e}}_{\mathrm{L,R}} \end{pmatrix}, \tag{6.22}$$

which act on $D_{\mathrm{L,R}}$ as

$$D^a_{\mathrm{L,R}} \to (V^{\mathrm{d}}_{\mathrm{L,R}}{}^{p'}_{p} d_{\mathrm{L,R}}{}^{\alpha}_{p'},\, V^{\mathrm{e}}_{\mathrm{L,R}}{}^{p'}_{p} e_{\mathrm{L,R}p'})\,. \tag{6.23}$$

X and Y fields with flip-couplings are characterized by two indices a and b, which denote the fermion pair to which they couple in the mass eigenstate basis. Thus we have fields X^a_b, Y^{ab} and coupling matrices $\beta_{X}{}^a_b$, γ_{Yab}. The matrices β_X and γ_Y satisfy the conditions:

$$V^{D\dagger}_{\mathrm{L}}(\beta_{\mathrm{S,T}}{}^a_b)\, V^D_{\mathrm{R}} \propto E^a_b\,, \tag{6.24a}$$

$$V^{D\dagger}_{\mathrm{L,R}}(\beta_{\mathrm{L,R}}{}^a_b)\, V^D_{\mathrm{L,R}} \propto E^a_b \tag{6.24b}$$

and

$$(\gamma_{Y_{\mathrm{L,R}}ab})\, V^D_{\mathrm{L,R}} V^D_{\mathrm{L,R}} \propto F_{ab}\,, \qquad Y = S, T, \tag{6.24c}$$

$$(\gamma_{Vab})\, V^D_{\mathrm{R}} V^D_{\mathrm{L}} \propto F_{ab}\,, \tag{6.24d}$$

where

$$(E^a_b)^c_d = \delta^a_d \delta^c_b\,, \qquad (F_{ab})^{cd} = \delta^c_a \delta^d_b\,. \tag{6.24e}$$

Given the connection between weak eigenstates and mass eigenstates, eqs. (6.24) determine all allowed flip-couplings. We notice that E^a_b and F_{ab} label the U(12) generators. If there exist mass degenerate X and Y fields for each generator we have a globally U(12) invariant interaction.

At present there exists no experimental information, such as a bound on the branching ratio for $\mathrm{D}^0 \to \mathrm{e}^-\mu^+$, which would constrain flip-transitions involving u-quarks. If they follow the pattern of the d-quarks it is natural to extend (6.21) to a left-handed SU(2) doublet Q_{L} and two right-handed SU(2) singlets,

$$Q_{\mathrm{L}} = \begin{pmatrix} U_{\mathrm{L}} \\ D_{\mathrm{L}} \end{pmatrix},\ U_{\mathrm{R}},\ D_{\mathrm{R}}\,, \tag{6.25}$$

where

$$U^a_{\mathrm{L,R}} = (u_{\mathrm{L,R}}{}^{\alpha}_{p},\, \nu_{\mathrm{L,R}p})\,, \tag{6.26}$$

which would require right-handed neutrinos. Then the couplings of X and Y fields would have to satisfy also eqs. (6.24) with $V^D_{\mathrm{L,R}}$ replaced by $V^U_{\mathrm{L,R}}$. For couplings involving Q_{L} this would lead to *two* conditions which are clearly incompatible for a non-trivial KM matrix (cf. the appendix). On the contrary, there is only one condition for the couplings of U_{R} and D_{R}, which can always be satisfied.

Let us summarize the main results of this section. The requirement of natural flavour conservation imposes constraints on the couplings of fermion pairs to the Higgs scalar, to gauge fields and to heavy fields with masses of order Λ.

The couplings to fields with zero baryon-, lepton- and generation-number depends on the Lorentz structure. Chirality changing interactions should be proportional to Yukawa couplings, chirality conserving interactions proportional to the unit matrix.

New dynamics at the scale Λ is likely to be associated with heavy fields carrying non-zero baryon-, lepton- and generation-number. Such fields can couple only to a single fermion pair in the mass eigenstate basis. This is in sharp contrast to the scalar and vector fields of the standard model which couple to fermion pairs whenever this is allowed by $SU(3) \times SU(2) \times U(1)$ invariance. The requirements for the couplings of heavy fields with non-zero U(1) charges can be naturally fulfilled if the underlying theory has a U(12) invariance.

Composite models, where a Pati-Salam SU(4) symmetry is extended to SU(12), have been considered in the literature [37, 38], especially in connection with the suppression of rare processes [39]. These models, however, shift the problem of understanding the replication of generations to the preon level, which is contrary to the motivation for quark-lepton substructure.

Of particular theoretical interest are bounds on rare decay modes of D mesons. They can shed light on the question whether the U(12) invariance, which is suggested by the suppression of rare K decays, is compatible with the weak isospin symmetry of the standard model.

7. Summary and conclusions

Using an effective lagrangian technique we have studied systematically deviations from the standard model which could be the low-energy manifestations of $SU(3) \times SU(2) \times U(1)$ invariant new interactions with scales Λ in the TeV range. The effective lagrangian contains 1 dimension-five operator, which violates lepton number, and 80 baryon- and lepton-number conserving dimension-six operators.

The present agreement between experiment and the standard electroweak theory can be used to obtain lower bounds on Λ from various operators, whose presence would affect the V-A structure of the charged weak current, quark-lepton universality, the ρ-parameter, anomalous magnetic moments, rare processes, etc. There is a hierarchy of bounds depending on the quantities under consideration. The best bound from flavour conserving processes results from quark-lepton universality, $\Lambda > 5$ TeV. The experimental limits for branching ratios of rare decays yield lower bounds on Λ of almost 10^4 TeV, and from the upper limit on the θ parameter of strong CP violation one even obtains the bound $\Lambda > 10^5$ TeV.

Obviously, one can only hope for new interactions at TeV energies, if all the possible flavour changing processes are strongly suppressed. Such processes arise from flavour changing interactions of fermions with the Higgs scalar, gauge bosons or heavy fields with masses of order Λ, whose exchange will lead to effective four-fermion interactions at energies below Λ. In sect. 6 we have given sufficient conditions for the couplings of fermion pairs, which guarantee the absence of unobserved rare decay modes. Of particular interest is the unobserved decay mode $K_L^0 \to e^- \mu^+$ which conserves baryon-, lepton- and generation-number. The absence of this decay appears to require either the absence of heavy intermediate states with baryon- and lepton-number (which is very unlikely in composite models), or a U(12) extension of the Pati–Salam SU(4) symmetry. There is, however, no trace of such a symmetry in the standard model lagrangian (i.e., the dimension-four part of the complete effective lagrangian).

If flavour conservation is extended to include u-quarks (implying also the absence of $D^0 \to e^- \mu^+$), then, because of the incompatibility of conditions (6.24) for left-handed d- and u-quarks, heavy states with non-vanishing lepton-, baryon- or generation-number cannot couple to left-handed quarks. As in composite models such couplings appear unavoidable, strong suppression of $K_L^0 \to e^- \mu^+$ and $D^0 \to e^- \mu^+$ seems to be incompatible with a substructure of left-handed quarks and leptons. Furthermore, because heavy states with zero baryon and lepton number, but non-zero generation number cannot simultaneously couple to quarks and leptons, preons carrying generation number must be different for quarks and leptons.

Maybe the main point of our analysis is that it demonstrates explicitly how remarkable the standard electroweak theory is. Given the standard model field content, the *most general renormalizable* lagrangian gives a satisfactory description of all experimental results. Giving up renormalizability and allowing dimension-six operators with scales Λ in the TeV range destroys this success completely. This may indicate that there are no new interactions at TeV energies, and that the puzzles of the standard model, such as the replication of generations, the smallness of Yukawa couplings, etc., will find their explanation only at much higher energies.

Obviously, our analysis cannot exclude new interactions in the TeV range. It is conceivable that flavour-changing processes are suppressed for symmetry reasons. The most promising window to new physics are rare processes, and especially limits on rare decay modes of D-mesons would provide valuable constraints on all attempts to find a solution of the flavour puzzle.

During the course of this work we have benefitted from discussions with A.J. Buras, R.N. Cahn, C.A. Heusch, J. Kambor, J.H. Kühn, J. Missimer, H.P. Nilles, R.D. Peccei, F. Scheck and C. Schmid.

Note added

After this paper was completed we became aware of a preprint of Bigi, Köpp and Zerwas [40] where also bounds for the mass scales of various flavour changing operators are given.

Appendix

Here we prove two results of sect. 6. Let us first consider the chirality conserving coupling α_q of ($\bar{q}q$) pairs to vector bosons [cf. eqs. (6.13)-(6.15)]. Flavour conservation requires that α_q is diagonalized by V_L^u and V_L^d:

$$V_L^{u\dagger}\alpha_q V_L^u = \alpha_1^D, \tag{A.1a}$$

$$V_L^{d\dagger}\alpha_q C_L^d = \alpha_2^D. \tag{A.1b}$$

From (A.1) one easily obtains

$$V_{KM}\alpha_1^D = \alpha_2^D V_{KM}, \tag{A.2}$$

where $V_{KM} = V_L^{u\dagger} V_L^d$ is the Kobayashi-Maskawa matrix. This implies for the matrix elements of $\alpha_{1,2}^D$ for all p and $q(p, q \cdots = 1 \cdots 3)$:

$$(V_{KM})_p^q[(\alpha_1^D)_q^q - (\alpha_2^D)_p^p] = 0. \tag{A.3}$$

If all matrix elements of V_{KM} are non-zero, one obtains

$$\alpha_1^D = \alpha_2^D \propto 1, \tag{A.4}$$

or, equivalently, [cf. eq. (6.15)]:

$$\alpha_q \propto 1. \tag{A.5}$$

Similarly one can show that "flip" transitions are not allowed for the left-handed quark doublets. Let β_V be a chirality conserving coupling of ($\bar{q}q$) pairs with $\Delta B = \Delta L = 0$, $\Delta Q_i \neq 0$. Flavour conservation requires [cf. (6.24b)]

$$V_L^{u\dagger}(\beta_{V_q^p}) V_L^u \propto E_q^p, \tag{A.6a}$$

$$V_L^{d\dagger}(\beta_{V_q^p}) V_L^d \propto E_{q'}^{p'}, \tag{A.6b}$$

where E_q^p is the 3×3 matrix defined in (6.24e). Eqs. (A.6) imply

$$V_L^{u\dagger}(\beta_{V_q^p})(\beta_{V_q^p})^\dagger V_L^u \propto E_q^p(E_q^p)^\dagger, \tag{A.7a}$$

$$V_L^{d\dagger}(\beta_{V_q^p})(\beta_{V_q^p})^\dagger V_L^d \propto E_{q'}^{p'}(E_{q'}^{p'})^\dagger. \tag{A.7b}$$

$$(E_q^p(E_q^p)^\dagger)_s^r = \delta_q^r \delta_s^q. \tag{A.7c}$$

Eqs. (A.7) are in contradiction with eqs. (A.1) and (A.5). We thus conclude $\beta_{V_q^p} = 0$, i.e., "generation flip" couplings of left-handed quark doublets are incompatible with flavour conservation.

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