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Reduced Effective Lagrangians

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ABSTRACT

Effective Lagrangians, including those that are spontaneously broken, contain redundant terms. It is shown that the classical equations of motion may be used to simplify the effective Lagrangian, even when quantum loops are to be considered.

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1. INTRODUCTION

Within the framework of accelerator physics, there are two ways we may obtain information on physics above the weak scale. By building higher and higher energy machines, we hope to directly observe new particles as the energy of the machine passes the threshold of particle production. At the SSC, for example, we hope to find some direct evidence of a mass-generating mechanism or other new phenomena. We may also hope that high-precision measurements at lower energies will provide indirect evidence of high-energy physics. In these measurements we search for deviations from the Standard Model; any such deviation is either evidence that the Standard Model is incorrect or a signal of some new physics on a higher energy scale.

If we accept the validity of the Standard Model, it becomes important to ask how to characterize any possible deviations from it. Although we do not know the structure of physics beyond the Standard Model, we know that the low-energy effects of the full higher-energy theory (valid at energies above the mass Λ of some new particle) can be incorporated into an effective Lagrangian [1]. For any given extension of the Standard Model we can write an effective Lagrangian composed of only low-energy (Standard-Model) fields in a series of terms of higher and higher dimension:

$$\mathcal{L}_{\text{eff}} = \sum_{n=-2}^{\infty} \frac{1}{\Lambda^n} \alpha_{\mathcal{O}} \, \mathcal{O}^{(n+4)} \tag{1.1}$$

The operators $\mathcal{O}^{(n+4)}$ have dimension $[\text{mass}]^{(n+4)}$ and contain only derivatives and fields with masses below Λ. If the high-energy physics decouples [2], then the sum starts at $n = 0$, and $\mathcal{O}^{(4)}$ is the Standard Model. All operators (not

including those related by the equations of motion) up to $\mathcal{O}^{(6)}$ have been listed by Buchmüller and Wyler [3]. If the physics above scale Λ does not decouple from the low energy physics, then we can write our effective theory as a gauged chiral model [4,5,6]. In this case $\Lambda \sim 4\pi v$ [5], and there exist terms $\mathcal{O}^{(2)}$ of chiral dimension 2. The (renormalization-scale-dependant) constants $\alpha_{\mathcal{O}}$ determine the strength of the contribution of $\mathcal{O}$; they are calculated by matching the Green's functions (or S-matrix elements) of the effective Lagrangian to those of the full high-energy theory (for an explanation see, for example, [7]).

The situation at hand is somewhat different. We do not know the high-energy theory, and so we cannot perform the matching to find the values for the α's. Whatever the high-energy theory is, though, it will generate in the effective Lagrangian a tower of terms $\mathcal{O}^{(n)}$ each obeying the symmetries of the theory. So we may parametrize all possible forms of new high-energy physics by writing down an effective Lagrangian containing all operators $\mathcal{O}$ which respect the symmetries of the theory. Since higher order terms are all suppressed by higher powers of $\frac{1}{\Lambda}$, we can terminate the series at some point with negligible effects.

This still leaves a large number of terms to be considered. The number can of course be reduced by integration by parts; the action S is usually unchanged by such a manipulation. Further, the classical equations of motion can clearly be used to remove terms when the effective Lagrangian is only to be used at tree level [3,6,8,9]. Recently Georgi has shown that in certain cases the classical equations of motion can be used even in the quantum theory (in loop diagrams) [7]. Explicitly exempted from this in [7] are terms quadratic in the fields. Also, gauge theories and spontaneously broken theories are not fully considered. These

limitations preclude the use of this simplification in the Standard Model. In this paper we will show that, quite generally, terms in an effective Lagrangian that are connected by the equations of motion are redundant. They may be dropped from the effective Lagrangian without changing observables. A generalization of the equivalence theorem [10,11,12] shows that this is also true for quadratic terms. Spontaneously broken gauge theories present no new problems. The result is that any effective Lagrangian can be brought into a canonical form consisting of a reduced set of operators that are gauge invariant and unrelated by the classical equations of motion. We may choose such a set to minimize the number of higher derivative terms; this is often (but not always - for example see [9,13]) the most useful form.

2. PROOF

The purpose of this section is to show that we may use the equations of motion on any terms of dimension d $\geq$ 5 in the effective Lagrangian. In particular, we show that any such term which contains $D^2\phi$, $D^\mu F^I_{\mu\nu}$, or $\slashed{D}\psi$ gives contributions to the S matrix identical to those from a term with fewer derivatives, with which we may therefore replace it. (Here ϕ is a scalar, $F^I_{\mu\nu}$ is a gauge field strength, ψ is a fermion, and D_μ is a covariant derivative.) The generalization will be clear.

Let $\mathcal{L}$ be an effective Lagrangian valid for energies below Λ. It can be written as

$$\mathcal{L}_{\text{eff}} = \sum_{n=-2}^{\infty} \frac{1}{\Lambda^n} \alpha_{\mathcal{O}} \, \mathcal{O}^{(n+4)} \equiv \sum_{n=0}^{\infty} \eta^n \mathcal{L}_n \tag{2.1}$$

where η is a small parameter (such as $1/\Lambda$) and $\mathcal{L}_0$ includes any terms with negative

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powers of η. The operators $\mathcal{O}$ can all be chosen to be local; in the following we assume this choice has been made.

As an example, consider $\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \lambda\phi^4 + \eta g_1\phi^6 + \eta g_2\phi^3\partial^2\phi$. By making the shift of variables $\phi \to \phi + \eta g_2\phi^3$ we induce $\mathcal{L}_{\text{eff}} \to \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \lambda'\phi^4 + \eta g_1'\phi^6 + \mathcal{O}(\eta^2)$. By the equivalence theorem [10,11,12] the S matrix is unaffected by the change in variables, and so (to first order in η) we may choose to use the new effective Lagrangian in place of the original one. In the following, we will generalize the equivalence theorem, showing that a similar procedure is possible for any effective Lagrangian.

In the following φ_i stands for any of the various fields in the theory. First consider an effective Lagrangian containing a term like $\eta T[\varphi]D^2\phi$ where $T[\varphi]$ is any local function of any of the fields and their derivatives (η is some appropriate power of $1/\Lambda$). Let $Z'[j_i]$ be the generating functional for the Green's functions with the j_i sources for each of the fields. Then (working for now only to first order in η)

$$Z'[j_i] = \int \prod_l \mathcal{D}\varphi_l' \, \exp i \int d^4x \left[\mathcal{L}_0' + (\mathcal{L}_1' - \eta T D^2\phi') + \eta T D^2\phi' + \sum_i j_i\varphi_i'\right] + \mathcal{O}(\eta^2).$$

$$(2.2)$$

The term in $\mathcal{L}_1'$ to be removed (that is, $\eta T D^2\phi'$) has been written explicitly. We can now change variables so that $(\phi')^\dagger = \phi^\dagger + \eta T$. (If the scalar is real, then we let $\phi' = \phi + \eta T$.) This change is only a redefinition of a variable of integration, so we expect Z' (and therefore all Green's functions) to be unchanged.

Written in the shifted variable

$$Z'[j_i] = \int \prod_l \mathcal{D}\varphi_l \left| \frac{\delta(\phi')^\dagger}{\delta\phi^\dagger} \right| \exp i \int d^4x \left[\mathcal{L}'_0 + \eta T(\frac{\delta\mathcal{L}'_0}{\delta\phi^\dagger} - \partial_\mu \frac{\delta\mathcal{L}'_0}{\delta\partial_\mu\phi^\dagger}) + (\mathcal{L}'_1 - \eta T D^2 \phi) \right.$$
$$\left. + \eta T D^2 \phi + \sum_i j_i \varphi_i + j_{\phi^\dagger} \eta T \right] + \mathcal{O}(\eta^2).$$

$$(2.3)$$

In this equation we have expanded $\mathcal{L}\left(\phi^\dagger + \eta T\right)$ in a Taylor series about $\phi^\dagger$. The shifted Z' differs from (2.2) in three ways: there is a new Lagrangian, a Jacobian of the transformation, and a new coupling to the source $j_{\phi^\dagger}$.

The change of the variable of integration we have performed in (2.2) presumably has no effect. It is commonly known as the equivalence theorem [10,12] that in many cases we may make the change of variables in *only* the Lagrangian without changing the S matrix. In other words, we may remove the Jacobian of the transformation and the additional coupling to $j_{\phi^\dagger}$ in (2.3) without changing the S matrix. The only effect on mass shell is to renormalize the Green's functions (though in general the off-shell Green's functions are changed). Statements of the equivalence theorem for $\phi \to F(\phi)$ usually require F to be a point transformation, or $\phi \to \phi + F(\phi)$, where the expansion of $F(\phi)$ begins with the term second order in ϕ. For applications to spontaneously broken theories, this would require the use of the shifted field, destroying the (broken) symmetry. These requirements are in fact too demanding; we will see that the transformation $(\phi')^\dagger = \phi^\dagger + \eta T$ leaves the S matrix unchanged for any function T to any order in η. In the next three paragraphs we consider the three differences between equations (2.2) and (2.3), respectively: the change in the Lagrangian, the Jacobian of the transformation, and the new coupling to the source $j_{\phi^\dagger}$.

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The new Lagrangian is just the original Lagrangian plus ηT times the classical equation of motion for $\phi^\dagger$. The variable shift we have performed respects the symmetries of the theory; since $\phi^\dagger \phi$ and $T\phi$ are both invariant under symmetry operations, $\phi^\dagger + \eta T$ transforms as $\phi^\dagger$ does. Because of this the new Lagrangian explicitly retains all the symmetries of the original. If $\mathcal{L}_0$ has the usual quadratic terms, then the new Lagrangian is $\mathcal{L}_{\text{eff}} + \eta T(-D^2\phi - m^2\phi + \text{terms with two or more fields})$. The first term cancels $\eta T D^2 \phi$ in $\mathcal{L}_1$. Georgi [7] has pointed out that since the effective Lagrangian contains *all* terms allowed by the symmetries of the theory, each of remaining terms is of a form already present. We can absorb them by changing the coefficients $\alpha_{\mathcal{O}}$ of some terms already present in $\mathcal{L}_{\text{eff}}$ (this is true to all orders in η).

Regardless of the the structure of $T[\varphi]$ (but assuming it is local) the presence of the Jacobian has no effect on the theory. It can be written as a ghost coupling $\bar{c}c + \eta\bar{c}\frac{\delta T}{\delta \phi}c$. We can see that in any diagram containing ghosts, there will always be at least one loop containing only ghost propagators. Assume without loss of generality that T has only one term.[#1] The ghost Lagrangian from the shift will have exactly two terms; one is $\bar{c}c$, and the other will be a kinetic term only if $T = \partial^m \phi^\dagger$ for any m. So there can be *either* a kinetic term for c (in which case the ghosts will not couple to physical fields) *or* the ghosts will couple to physical fields (in which case there will be no kinetic term for the ghosts, which can therefore be consistently disregarded when dimensional regularization is employed), but not both. Another way to see the point is to note that the effective theory is valid

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#1 If, on the contrary, $T = T_1 + T_2$, we can break the shift of variables into two parts: $(\phi')^\dagger = (\phi'')^\dagger + \eta T_1$, and then $(\phi'')^\dagger = \phi^\dagger + \eta T_2$. The net effect of the two transformations is $(\phi')^\dagger = \phi^\dagger + \eta T + \mathcal{O}(\eta^2)$.

only up to energies of order $\Lambda = 1/\sqrt{\eta}$. Let $T = (\partial^2\phi + \lambda\phi^3)$, so the ghost Lagrangian is $\bar{c}(1 + \eta\partial^2 + 3\eta\lambda\phi^2)c$. Now rescale $c \to c/\sqrt{\eta}$. Even though the ghosts propagate, their mass is on the order of the cutoff! In any loop we can therefore expand the ghost propagators into the numerator, and so loops consisting purely of ghosts will contribute only quadratic (or more highly divergent) terms. Any diagrams containing ghosts will therefore not contribute to the S matrix. Note that for a more general field transformation, the equivalence theorem does not hold because the ghosts do not decouple [11]. For instance, for $\phi \to \partial^2\phi + \lambda\phi^3$ the ghost Lagrangian will be $\bar{c}\partial^2 c + 3\lambda\bar{c}c\phi^2$, which will have physical effects. The transformation necessary for the case at hand avoids this pitfall.

Again, whether or not T is linear in ϕ, the term $\eta j_{\phi^\dagger} T$ has no effect on the S matrix. Instead of

$$G^{(n)'} = \langle 0| \, T \, [\phi(x_1)...\phi(x_n)] \, |0\rangle$$

we have

$$G^{(n)} = \langle 0| \, T \, [(\phi(x_1) + \eta T(x_1))...(\phi(x_n) + \eta T(x_n))] \, |0\rangle \, .$$

It can be seen diagrammatically that $G^{(n)} = f^n(p)G^{(n)'} +$ (terms with fewer than n poles). The term $\eta j_{\phi^\dagger} T$ has only the effect (on-shell) of multiplying each n-point Green's function $G^{(n)}$ by $f^n(p)$, the nth power of some function of momentum (see [12] for a full explanation in a slightly more restricted context). Indeed, if T is linear in ϕ, even the off-shell Green's functions are related to the original ones in this way. In any case, this multiplicative factor cancels out in the definition of the S matrix and leaves all S-matrix elements unaffected.

The result is that Z' gives the same S matrix as the generating functional

$$Z[j_i] = \int \prod_l \mathcal{D}\varphi_l \, \exp i \int d^4x \left[\mathcal{L}_0 + (\mathcal{L}_1 - \eta T D^2 \phi) + \sum_i j_i \varphi_i \right] + \mathcal{O}(\eta^2). \quad (2.4)$$

The term $\eta T D^2 \phi$ has been removed, the on-shell Green's functions are the same up to a renormalization, and the (unknown) values of some $\alpha_{\mathcal{O}}$'s have changed to linear combinations of the original $\alpha_{\mathcal{O}}$'s (changing $\mathcal{L}'$ to $\mathcal{L}$). This equivalence is true regardless of the structure of the local function T.

The preceding comments are true also for a term $\eta T \, \slashed{D}\psi$. In this case one makes the change of variables $\bar{\psi} \to \bar{\psi} + \eta T$. The equations of motion contain $\slashed{D}\psi$ rather than $D^2\phi$, and the rest follows similarly.

Finally, a term like $\eta T_a^\nu D^\mu F_{\mu\nu}^a$ is also redundant. Here we must make the change of variables $A_a^\nu \to A_a^\nu + \eta T_a^\nu$ (where A is any abelian or non-abelian Yang-Mills gauge field, μ and ν are Lorentz indices, and a,b, and c are symmetry-group indices). Note that this change respects the local gauge symmetries; since the term $\eta T_a^\nu D^\mu F_{\mu\nu}^a$ is gauge invariant, T_a^ν transforms like $F_a^{\mu\nu}$. Under a gauge transformation

$$A_a^\nu \to A_a^\nu + \partial^\nu \Lambda_a + g f_{abc} \Lambda_b A_c^\nu$$
$$T_a^\nu \to T_a^\nu + g f_{abc} \Lambda_b T_c^\nu \quad (2.5)$$

(f_{abc} are the structure constants of the symmetry group), so $A_a^\nu + \eta T_a^\nu$ transforms just like A_ν^a:

$$(A + \eta T)_a^\mu \to (A + \eta T)_a^\mu + \partial^\mu \Lambda_a + g f_{abc} \Lambda_b (A + \eta T)_c^\mu. \quad (2.6)1$$

The proof follows through unchanged from above, but now the action also contains pieces whose job it is to fix the gauge. The variable change $A_a^\nu \to (A + \eta T)_a^\nu$ which

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takes us from (2.2) to (2.3) produces the following change in the gauge-fixing term (using a simple choice as an example):

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}[f]^2$$

$$f = \partial_\mu A_a^\mu \quad \rightarrow \quad f = \partial_\mu (A + \eta T)_a^\mu, \tag{2.7}$$

and the following change in the Faddeev-Popov ghost term:

$$\mathcal{L}_{FP} = \partial_\mu \omega_a^* (\partial^\mu \omega_a + g f_{abc} \omega_b A_c^\mu) \quad \rightarrow \quad \partial_\mu \omega_a^* (\partial^\mu \omega_a + g f_{abc} \omega_b (A + \eta T)_c^\mu). \tag{2.8}$$

The new FP ghost term is exactly that needed $(\omega_a^* \frac{\delta f}{\delta \Lambda} \omega_a)$ for the new gauge-fixing term, and so the symmetry is consistently fixed. After making the change of variables we can choose instead to gauge-fix with the original gauge-fixing term, with the original FP ghost term - the net effect is that our change of variables hasn't changed $\mathcal{L}_{GF} + \mathcal{L}_{FP}$.

By repeatedly shifting variables, we can continue the process outlined above for all redundant terms to all orders. First we remove all order η terms containing the appropriate derivative forms. Once we have removed all possible derivative terms to order η, we can continue the process with derivative terms of order η^2; since the change of variables is $\phi \rightarrow \phi + \eta^2 T$, all order η terms will be unaffected. In this way we can successively remove derivative terms of the given form order-by-order in η.

For a spontaneously broken theory the proof above carries through unchanged. We may write the Lagrangian in terms of the shifted field ϕ (for example $\phi = \binom{\phi_1 + v}{\phi_2}$)) so that the (broken) symmetries of the theory are still apparent. Then a term $\eta T[\varphi] D^2 \phi$ is redundant, with the required shift in fields again being $\phi^\dagger \rightarrow$

$\phi^\dagger + \eta T$ (in the example this would mean $\phi_1 + v \to \phi_1^* + v^* + \eta T$ and $\phi_2^* \to \phi_2^* + \eta T$). In this case it is crucial that $T[\varphi]$ linear in the fields not be disallowed, since upon expanding any shifted scalars ϕ, $T[\varphi]$ may in general be a sum of terms, some of which are linear in the fields ϕ_1 or ϕ_2. That $T[\varphi]$ may be linear means all of these terms in $\eta T[\varphi] D^2 \phi$ are redundant, not just those with two or more fields.

3. TECHNICAL CONSIDERATIONS

The preceeding manipulations are only formal, for two reasons. Firstly, we have treated the path integral as if it were a simple integral in the variable φ. A rigorous treatment would discretize the variables $\varphi(x)$ and write the path integral as an infinite product of integrals over the discrete variables φ_x. We would then perform the shift in variables, and rewrite the result as a path integral over continuous variables. Alternately, we could change variables in the canonical operator formalism. In this way it is found [14,15] that, in general, the formal manipulations above are actually incorrect; a discrepancy arises at the two-loop level. It will turn out, however, that this complication can be dealt with.

In the operator formalism, the operator Hamiltonian $\hat{H}$ contains non-commuting factors $\hat{Q}$ and $\hat{P}$. It is related to the classical Hamiltonian H by

$$\langle q | \hat{H}(\hat{Q}, \hat{P}) | p \rangle_{QP} = \frac{e^{ipq/\hbar}}{\sqrt{2\pi\hbar}} H(p, q) \tag{3.1}$$

The subscript QP indicates that H is "QP ordered", so that all factors of $\hat{Q}$ are placed to the left of all factors of $\hat{P}$. When we make a change of variables, the new Hamiltonian $\hat{H}' = \hat{H}(f(\hat{Q}, \hat{P}), g(\hat{Q}, \hat{P}))$ is no longer QP ordered. Because of the

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non-commuting factors,

$$\langle q | \hat{H} | p \rangle_{QP} = \langle q | \hat{H}' | p \rangle = \langle q | (\hat{H}' + \hat{H}'_{new}) | p \rangle_{QP} = \frac{e^{ipq/\hbar}}{\sqrt{2\pi\hbar}} (H' + H'_{new}) \qquad (3.2)$$

where the term after the first equals sign is not QP ordered, and $\hat{H}'_{new}$ arises because $\hat{P}$ and $\hat{Q}$ do not commute. We see that the Hamiltonian in the new variables is not just $H'(f(Q, P), g(Q, P))$; there is an additional term.

If, rather than the operator form, we consider a path integral over classical fields, we see the same effect differently expressed. Because of the stochastic nature of the path integral, terms of order ϵ which we have naively neglected must be considered [14]. This is important, because the chain rule is not valid to order ϵ; this can be inferred from $\frac{dq}{dt} = \lim_{\epsilon \to 0} \frac{q(t+\epsilon) - q(t)}{\epsilon} = \lim_{\epsilon \to 0} [\frac{dq}{dt} + \frac{1}{2}\epsilon\frac{d^2q}{dt^2} + \mathcal{O}(\epsilon^2)]$. When we change variables in the continuous Lagrangian, we ignore the $\mathcal{O}(\epsilon)$ term. But when we work carefully, with discrete variables, these order ϵ terms in the integral are seen to contribute an additional potential term in the limit $\epsilon \to 0$. This is identical to H'_{new}.

In fact, this extra potential term is regulator dependant. Salomonson [16] noted this by explicitly comparing Feynman graphs calculated before and after a change of variables. When using a cutoff, the extra potential was needed, but when using dimensional regularization, no new term appeared. It can be shown for a local field theory that the additional potential term is proportional to $\delta^n(0)$, where $n+1$ is the number of space-time dimensions. (In operator language, this factor comes from the commutator of $\hat{P}$ and $\hat{Q}$. In the path integral version, it enters through additional volume elements connected with the space integrals.) The quantum mechanical path integral, equivalent to a 0+1 dimensional field theory, has no

such delta functions factors. If the theory is regulated dimensionally, it would appear that these terms leave the S matrix unchanged, and so the extra potential can be disregarded.

Another possible source of error in the last section is the manipulation of divergent integrals. The conclusions reached above are strictly true only if we assume the theory to have been regularized beforehand. Let us assume the theory in (2.2) has been rendered finite by dimensional regularization; as ϵ approaches zero, the regulation is removed. In this case the generating functionals in (2.2) and (2.4) give identical S-matrix elements with identical ϵ dependence. We can write the effective Lagrangian in (2.2) as $\mathcal{L}'_{\text{eff}} + \mathcal{L}'_{ct}$ (a counter-term Lagrangian $\mathcal{L}'_{ct}$ has been extracted), so that the S-matrix elements from Z' are UV finite (*i.e.*, they have no ϵ dependence). We can then use the results of the last section to show that all terms in $\mathcal{L}'_{ct}$ of dimension d ≥ 5 which contain $D^2\phi$, $D_\mu F^I_{\mu\nu}$, or $\slashed{D}\psi$ can be dropped in favor of terms with fewer derivatives. The reduced form of the full Lagrangian will contain no terms with these derivatives, either in $\mathcal{L}_{\text{eff}}$ or $\mathcal{L}_{ct}$, but the S matrix will be the same as the that from $\mathcal{L}'_{\text{eff}}$ and $\mathcal{L}'_{ct}$, and so it will be finite (ϵ-independent). The reduced set of effective operators is therefore renormalizable, in the sense that counter terms of the form already present in the reduced Lagrangian are sufficient to renormalize all S-matrix elements.

In practice this type of renormalization may be cumbersome, as Green's functions are divergent, and it is not until S-matrix elements are calculated that the coefficients of the counter terms are apparent. In some cases it may be easier to imagine employing all possible counter terms, including those in the form of terms not in the reduced $\mathcal{L}_{\text{eff}}$. In this way we can clearly make all Green's functions

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finite. This approach has the disadvantage that terms removed at one scale may reappear at another. When running the couplings, some that we removed will be reintroduced. Of course, we may remove them again, at the new scale, in the same way they were removed originally. The relationships among the $\alpha's$ will therefore be different at different scales, in just such a way as to give the same $\alpha(\mu)'s$ in this method of renormalization as in the other.

4. CONCLUSIONS

Not all terms in an effective Lagrangian contribute independently to the S matrix. By using the classical equations of motion in the Lagrangian, the number of terms which must be considered can be reduced while maintaining all symmetries of the effective Lagrangian (including any broken symmetries). This is true for all terms in an effective Lagrangian (including those quadratic in fields) and for gauged and spontaneously broken theories.

A result of the discussion above is that, in the full effective Lagrangian, some of the parameters α are redundant; it is only the the coefficients of the terms in the maximally reduced effective Lagrangian that are completely determined by the high-energy theory. For each equation of motion, there is one arbitrary parameter, and we may exploit this ambiguity to choose values as we see fit. Additionally, it is clear that that the value of some α may not be the same in two effective Lagrangians with differing sets of terms, and so care must be taken when comparing estimates of an α to also compare the forms of the Lagrangians used in calculations.

In practice this result is quite useful. It is of great utility to be able to work with as few terms in $\mathcal{L}_{\text{eff}}$ as possible when calculating loop diagrams. That the results

apply in spontaneously broken theories, without destroying the broken symmetry, makes them useful for calculating in the Standard Model [17], and these results have been assumed in, for example, [9]. Generally, it is easiest to calculate with an effective Lagrangian in which the number of derivatives is minimized, though other choices are possible. Indeed other choices of operators are sometimes more useful, for example when the derivative terms are more tightly constrained by experiment than are those with which we would replace them [9], or when the derivative terms in the effective Lagrangian are expected, based on some knowledge of the high-energy theory, to be small [13].

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REFERENCES

1. H. Georgi and H. Politizer, *Phys. Rev.* **D14** (1976) 1829. E. Witten, *Nucl. Phys.* **B104** (1976) 445. E. Witten, *Nucl. Phys.* **B122** (1977) 109. J. Collins, F. Wilczek, and A. Zee *Phys. Rev.* **D18** (1978) 242. S. Weinberg, *Phys. Rev. Lett.* **43** (1979) 1566. F. Wilczek and A. Zee, *Phys. Rev. Lett.* **43** (1979) 1571. Y. Kazama and Y.P. Yao, *Phys. Rev.* **D21** (1980) 1116. B. Ovrut and H. Schnitzer, *Phys. Rev.* **D21** (1980) 3369. S. Weinberg, *Phys. Lett.* **91B** (1980) 51.

2. T. Appelquist and J. Carazzone, *Phys. Rev.* **D11** (1975) 2856. K. Symanzik, *Comm. Math. Phys.* **34** (1973) 7.

3. W. Buchmüller and D. Wyler, *Nucl. Phys.* **B268** (1986) 621; *Phys. Lett.* **B197** (1987) 379. See also: C.N. Leung *et al. Z. Phys.* **C31** (1986) 433. C.J.C. Burgess and H.J. Schnitzer, *Nucl. Phys.* **B228** (1983) 464.

4. T. Appelquist and C. Bernard, *Phys. Rev.* **D23** (1981) 425. H. Georgi, *Weak Interactions and Modern Particle Theory*, Benjamin / Cummings Publishing Co., Menlo Park, CA, USA (1984).

5. A. Longhitano *Nucl. Phys.* **B188** (1981) 118. S. Weinberg, *Physica* **A96** (1979) 327.

6. J. Gasser and H. Leutwyler, *Ann. Phys.* **158** (1984) 142.

7. H. Georgi, *Nucl. Phys* **B339** (1991) 339.

8. J. Gasser and H. Leutwyler, *Nucl. Phys.* **B250** (1985) 93.

9. A. De Rújula, M.B. Gavela, P. Hernandez, and E. Maasó, CERN-TH.6272/91.

10. J.S.R. Chisholm, *Nucl. Phys* **26** (1961) 469. S. Kamefuchi, L. O'Raifeartaigh, and A. Salam, *Nucl. Phys.* **28** (1961) 529. P.P. Divakaran, *Nucl. Phys.* **42** (1963) 235. M.C. Bergère and Y.M.P. Lam, *Phys. Rev. D* **D13** (1976) 3247. H.S. Sharatchandra *Ann. Phys.* **116** (1978) 408.

11. A. Salam and J. Strathdee, *Phys. Rev.* **D2** (1970) 2869.

12. R.E. Kallosh and I.V. Tyutin, *Yad. Fiz.* **17** (1973) 190 (reprinted as *Sov. J. Nucl. Phys.* **17** (1973) 98).

13. B. Grinstein and M. Wise, *Phys. Lett.* **B265** (1991) 326.

14. J.L. Gervais and A. Jevicki, *Nucl. Phys.* **B110** (1976) 93. S.F. Edwards and Y.V. Gulyaev, *Proc. Roy. Soc.* **A279** (1964) 229.

15. H. Fukutaka and T. Kashiwa, *Ann. Phys.* **185** (1988) 301. J. Hietarinta, *Phys. Rev.* **D25** (1982) 2103. W. Kerler, *Nucl. Phys.* **B129** (1978) 312. W. Kerler, *Lett. Nuov. Cim.* **23** (1978) 523. R. Marnelius, *Nucl. Phys.* **B142** (1978) 108.

16. P. Salomonson, Nucl. Phys. **B121** (1977) 433

17. C. Arzt, M.B. Einhorn, and J. Wudka, University of Michigan preprint UM-TH-92-17.