The Higgs mechanism on the lattice[*]

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ABSTRACT

The lattice regularization of the Higgs sector of the standard model is summarized. The triviality bound and vacuum instability bound are described. The question of chiral gauge theories is discussed. Some aspects of the numerical simulations of the electroweak phase transition are considered.

1. Lattice regularization of the Higgs sector

The masses of the elementary particles in the standard model are due to the Higgs mechanism which is the consequence of the spontaneous symmetry breaking in the scalar field sector of the theory. The spontaneous symmetry breaking is a non-perturbative phenomenon which can be well described in the framework of lattice regularization.

1.1. Lattice action

The formulation of the Higgs mechanism on the lattice is well known. For this and a general introduction to quantum field theory on the lattice see [1]. The basic ingredients of the Higgs sector in the standard model are the Higgs scalar field and the non-abelian SU(2) gauge field of weak interactions. The Euclidean lattice action of this *SU(2) Higgs model* is conventionally written as

$$S[U, \varphi] = \beta \sum_{pl} \left(1 - \frac{1}{2} \mathrm{Tr}\, U_{pl} \right) +$$

$$+ \sum_x \left\{ \frac{1}{2} \mathrm{Tr}\, (\varphi_x^+ \varphi_x) + \lambda \left[\frac{1}{2} \mathrm{Tr}\, (\varphi_x^+ \varphi_x) - 1 \right]^2 - \kappa \sum_{\mu=1}^{4} \mathrm{Tr}\, (\varphi_{x+\hat{\mu}}^+ U_{x\mu} \varphi_x) \right\} . \qquad (1)$$

In the first line here we see the Wilson action for the SU(2) gauge field, with $U_{x\mu}$ denoting the SU(2) gauge link variable and U_{pl} as the product of four U's around a plaquette. In the second line there is the action piece describing the Higgs scalar field and its interaction with the gauge field. The Higgs field is represented by the complex $2 \otimes 2$ matrix in isospin space φ_x. This satisfies the condition $\varphi_x^+ = \tau_2 \varphi_x^T \tau_2$ which ensures that φ_x has only four independent real components.

The bare parameters in the lattice action in Eq. (1) are: $\beta \equiv 4/g^2$ for the gauge coupling g, λ for the scalar quartic coupling and κ for the scalar hopping parameter

related to the bare mass square μ_0^2 by

$$\mu_0^2 = (1 - 2\lambda)\kappa^{-1} - 8 \ . \tag{2}$$

For the understanding of the Higgs mechanism one has to investigate the phase structure of the model, first as a function of the bare parameters. It turns out that there are two "phases", namely the *Higgs-phase* where the Higgs mechanism is operational and the *confining phase* which is qualitatively similar to QCD with scalar quarks. There is a phase transition surface $\kappa_{cr} = \kappa_{cr}(\beta, \lambda)$ separating the two phases: for $\kappa > \kappa_{cr}$ there is the Higgs-phase and for $\kappa < \kappa_{cr}$ the confining phase. In the standard model the relavant phase is the Higgs-phase, where the gauge bosons acquire a non-zero mass due to the non-zero vacuum expectation value of the scalar field.

The phase transition surface is everywhere of first order with a discontinuous change of physical quantities, however this surface has a boundary at small β and large λ, where the phase transition becomes second order with a continuous change and infinite correlation lengths. Beyond this boundary the two "phases" are analytically connected. Correspondingly, there exist no (local, gauge invariant) order parameters which would distinguish them. This is in agreement with Elitzur's theorem on the impossibility of spontaneously broken local symmetries [2]. In fact, strictly speaking, we are dealing with one single phase on the two sides of the transition surface [3]. The two regions are, however, quantitatively different.

Since the lattice serves only as a regularization scheme, in order to obtain a continuum quantum field theory, one has to remove the cut-off, i. e. one has to perform the *continuum limit*. If $a(\kappa, \beta, \lambda)$ denotes the lattice spacing, one requires for any physical mass $a \cdot Mass \rightarrow 0$.

1.2. Triviality bound

The couplings in the SU(2) Higgs model are not asymptotically free, therefore the Gaussian fixed point at zero couplings is not appropriate for defining the continuum limit. For a non-trivial continuum limit one would need a non-trivial ultraviolet fixed point at non-zero couplings. This would resolve the problems related to the *Landau pole* in perturbation theory, which seem to imply that the running quartic coupling becomes infinite at high energy scales.

The non-perturbative lattice investigations show, however, that there exists no non-trivial fixed point. The consequence is that the renormalized quartic coupling λ_R is zero in the continuum limit:

$$\lim_{a \rightarrow 0} \lambda_R(\kappa, \beta, \lambda) = 0 \ . \tag{3}$$

The path along which the limit is performed here is quite arbitrary, but a usual way to perform continuum limits is along *lines of constant physics,* with some dimensionless

physical quantities fixed. The consequence of Eq. (3) is that in the SU(2) Higgs sector the continuum limit is trivial, that is non-interacting.

Another, more detailed, formulation of the triviality of the continuum limit is that, considering the cut-off theory at non-zero lattice spacings ($a \cdot Mass > 0$), there exists a cut-off dependent upper bound on the quartic coupling and hence on the ratio M_H/M_W (M_H denotes the physical Higgs boson mass, M_W the W-boson mass). One can show that, as a consequence of renormalizability, the upper bound on λ_R goes only logarithmically to zero for decreasing lattice spacing. As a consequence, there exist effective theories with very high cut-off's and non-zero renormalized quartic and gauge couplings. In these theories with high cut-off's the cut-off effects are negligibly small.

If one tolerates larger cut-off effects the upper bound on the Higgs boson mass becomes higher. At some point, of course, the cut-off effects become so large that the effective theory looses sense. In this way one obtains an absolute upper bound [4]. This absolute upper bound is, of course, somewhat uncertain because it depends on ones requirements on the quality of the effective theory. The results of numerical simulations in the SU(2) Higgs model give an absolute upper bound [5,6]:

$$M_H \leq 9 M_W \ . \tag{4}$$

Similar results can also be obtained in the pure scalar ϕ^4 model, taking into account the SU(2) gauge coupling only perturbatively [4,7]. A recent systematic investigation of the effects of the choice of the scalar lattice action gave as a best estimate for the absolute upper bound $M_H \leq (710 \pm 60)$ GeV [8].

1.3. Vacuum instability bound

The triviality of couplings remains if Yukawa couplings of the type present in the standard model are included. The cut-off dependent upper bounds on the renormalized couplings are such that in phenomenologically relevant models there is no strongly interacting Higgs-Yukawa sector, provided that the cut-off scale is chosen to be high, say near the scale of grand unification. The absolute upper bounds are in general quantitatively similar to the upper bounds obtained from the assumption of the unitarity of perturbative scattering amplitudes. Similarly, the position of the Landau pole in the one-loop β-function can also be taken as a rough estimate for the upper bounds.

Another cut-off dependent restriction on the renormalized couplings originates from the fact that the radiative corrections generated by the Yukawa couplings make the renormalized scalar quartic coupling larger. This obviously implies a lower bound on the Higgs boson mass which depends on the strength of the Yukawa couplings. In a renormalization group treatment à la Coleman-Weinberg [9] this manifests itself by the fact that the vacuum appears to be unstable at large scales. The largest scales in
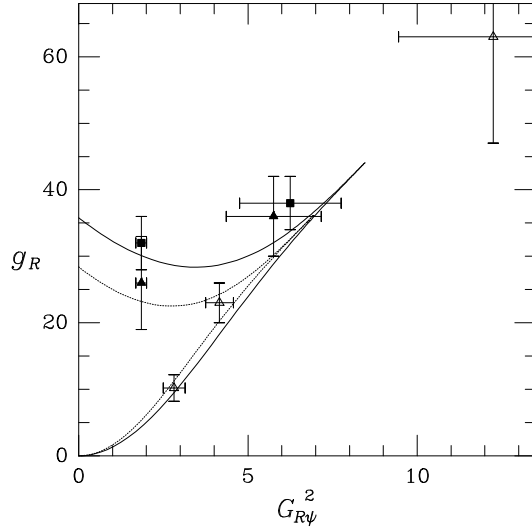
Fig. 1. Upper and lower bounds on g_R as a function of $G^2_{R\psi}$ together with the one-loop perturbative estimates for cut-off ratios $\Lambda/M_H = 3$ (solid curve) and $\Lambda/M_H = 4$ (dotted curve). Open points denote the data for the lower bound, whereas full symbols are data for the upper bound. The $6^3 \cdot 12$-lattice is represented by triangles, whereas points on $8^3 \cdot 16$ are denoted by squares.

a cut-off quantum field theory are at the cut-off, hence the instability occurs at the cut-off.

It is worth to emphasize that this type of instability is by no means a phase transition, which is usually associated to the instability of the vacuum in the infrared. "Tunneling" considerations are in this case out of context because they would refer to the possibility of mathematically defining the quantum field theory with a cut-off.

The numerical Monte Carlo simulations in Higgs-Yukawa theories involve fermions and are, therefore, much more demanding than the bosonic Higgs sector discussed in the previous subsection. In addition to the practical difficulties implied by the necessity to evaluate the Grassmannian path integrals, there is also the question of principle about the possibility of chiral gauge couplings of the fermion fields. This will be separately discussed below.

Neglecting the gauge couplings, there have been several numerical studies aiming to determine both the triviality upper bound and the vacuum instability lower bound as a function of a strong Yukawa coupling. For a typical result see Fig. 1 from Ref. [10]. The renormalized quartic coupling is denoted in this figure by g_R, the renormalized Yukawa coupling by $G_{R\psi}$. The numerical data are compared to the perturbative estimates obtained from the one-loop β-function at different cut-off's.

1.4. No chiral gauge theories?

It is an obvious question whether the lattice formulation of the Higgs-sector in

Eq. (1) can also be extended to the full standard model? From the practical point of view concerning the upper and lower bounds discussed above this is not important, because the bounds depend essentially only on the strong couplings: the lattice formulation of QCD is well known [1] and the chiral electroweak $SU(2) \otimes U(1)$ couplings are weak. Nevertheless, at least as a question of principle, a complete non-perturbative formulation of the standard model is obviously interesting. In fact, this is more than just a question of principle: although in most applications the perturbative treatment of the chiral electroweak gauge couplings is sufficient, there are some questions where the non-abelian non-perturbative nature of the $SU(2)$ coupling becomes relevant. For instance, this is the case for the electroweak phase transition discussed in the next section and also for some instanton induced processes in high energy scattering [11].

It turns out that, in contrast to vector-like gauge theories as QCD where the lattice formulation is easy and elegant, the non-perturbative lattice formulation of chiral gauge theories seems impossible. After many years of struggle one can conjecture that beyond the perturbative framework no chiral gauge theories exist! The assumptions, which are used as a basis for this conjecture, are:

- quantum field theories are defined as limits of regularized cut-off theories;

- the infinite cut-off limit exists and is independent of the choice of the regularization;

- there exists an explicitly gauge invariant local action;

- the formulation can be given by a Euclidean path integral satisfying reflection positivity, which implies unitarity after Wick rotation to Minkowski space.

Under these, rather natural, assumptions all attempts for constructing a chiral gauge theory seem to fail (see, for instance, Ref. [12]). Relaxing some of the assumptions allows for some formulations. An example is the "overlap formalism" in Ref. [13].

The root of all difficulties is the Nielsen-Ninomiya theorem, which tells that under rather general assumptions in a lattice formulation there are always *fermion doublers* with opposite chirality which render the lattice theory non-chiral (vector-like) [14]. Another characteristic feature of the failure of formulating chiral gauge theories on the lattice is the emergence of mirror fermion pairs due to the dynamics, even if naively the fermion doublers seem to be removed from the physical spectrum. There is an intimate relation also to the triviality of the continuum limit: the lattice fermion doublers required by the Nielsen-Ninomiya theorem cannot be given an arbitrarily large mass because of the triviality of the Yukawa couplings.

The way out of this dilemma is either to give up some of the above fundamental assumptions, or to assume the existence of heavy *mirror leptons and quarks* in nature which render the standard model non-chiral at high energy. The standard model extended by three mirror pairs of fermion families can be formulated without
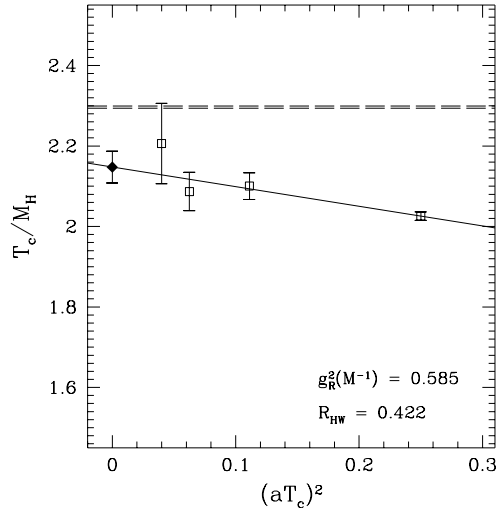
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Fig. 2. The numerical results for the ratio of the transition temperature and the Higgs boson mass T_c/M_H versus $(aT_c)^2 = L_t^{-2}$. The straight line is the extrapolation to very small lattice spacings, which gives the continuum value shown by the filled symbol. The dashed horizontal lines are the perturbative predictions at order g^3 (upper) and g^4 (lower), respectively.

difficulties on the lattice [15]. The mirror fermions can mix with their normal fermion partners, which imply small breakings of universality of the fermion couplings to gauge bosons [16]. The mixing also allows for associate production of fermion-mirror-fermion pairs at high energy colliders.

2. Electroweak phase transition

At high temperatures, above the scale of the vacuum expectation value, the Higgs mechanism is not operational, the symmetry of the vacuum gets restored. In the early Universe matter first existed in the symmetry restored confinement phase and went through a phase transition into the Higgs-phase.

The properties of the *electroweak phase transition* between the confinement and Higgs-phase might have a substantial influence on the later history of the Universe. The number of baryons is not conserved in the standard model, therefore the small baryon asymmetry of the Universe could perhaps be created in non-equilibrium processes during a strong enough first order electroweak phase transition [17].

2.1. Four dimensional lattice simulations

Near the electroweak phase transition and in the confinement phase with restored symmetry infrared singularities render the perturbation theory uncertain. Non-perturbative numerical simulations are useful in providing numerical control of the
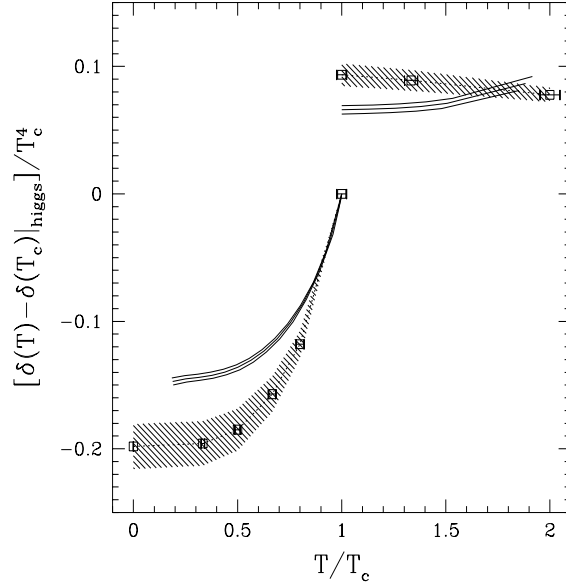
Fig. 3. Comparison of the numerical results for $\delta \equiv \epsilon/3 - P$ with those from two-loop perturbation theory. The shaded areas show the uncertainty of the numerical simulation results due to the uncertainty in the derivatives of bare parameters along the lines of constant physics. The solid lines show the perturbative results together with the uncertainties induced by the errors on R_{HW} and g_R^2.

perturbative resummation techniques.

One way of performing the numerical simulations is to first reduce the theory by the use of perturbation theory to three dimensions and do the numerical simulations there. For this approach see the contribution of Shaposhnikov at this workshop [18]. It is also possible to perform numerical simulations directly in four dimensions, without reduction. For some recent results see Refs. [19,20]. In the Higgs boson mass range below 50 GeV the agreement with two-loop resummed perturbation theory is reasonably good. An example is shown by Fig. 2 from Ref. [20], where the ration of the phase transition temperature to the Higgs boson mass T_c/M_H is shown at $M_H \simeq 35$ GeV. At higher M_H perturbation theory becomes quite uncertain but numerical simulations in four dimensions are still possible and informative [21,22].

2.2. A thermodynamical equation of state

An important, recently developing, area for numerical simulations is the investigation of thermodynamical equations of state in the electroweak plasma. This is particularly interesting in the high temperature confining phase, where perturbation theory is quite uncertain (strictly speaking useless), due to the so called "magnetic mass" problem. A general attitude is to assume a simple ideal gas behaviour. Numerical simulations could be very valuable for controlling the magnitude of the deviations

from this. A first example is shown by Fig. 3 from Ref. [20], where the thermodynamical quantity $\delta \equiv \epsilon/3 - P$ is depicted (P denotes the pressure, ϵ the energy density). As one can see, the deviation from the ideal gas behaviour $\delta = 0$ is small in the confining phase. The relevant ratio $\delta/T^4 = \delta/T_c^4 \cdot T_c^4/T^4$ is in fact non-zero mainly near the phase transion temperature $T = T_c$.

3. References

1. I. Montvay, G. Münster, *Quantum Fields on a Lattice* (Cambridge University Press, Cambridge, 1994)
2. S. Elitzur, *Phys. Rev.* **D12** (1975) 3978.
3. E. Fradkin, S. Shenker, *Phys. Rev.* **D19** (1979) 3682.
4. R. Dashen, H. Neuberger, *Phys. Rev. Lett.* **50** (1983) 1897.
5. W. Langguth, I. Montvay, *Z. Phys.* **C36** (1987) 725.
6. A. Hasenfratz, T. Neuhaus, *Nucl. Phys.* **B297** (1988) 205.
7. M. Lüscher, P. Weisz, *Nucl. Phys.* **B318** (1989) 705.
8. U.M. Heller, M. Klomfass, H. Neuberger, P. Vranas, *Nucl. Phys.* **B405** (1993) 555.
9. S. Coleman, E. Weinberg, *Phys. Rev.* **D7** (1973) 1888.
10. L. Lin, I. Montvay, G. Münster, M. Plagge, H. Wittig, *Phys. Lett.* **B317** (1993) 143.
11. A. Ringwald, *Nucl. Phys.* **B330** (1990) 1;
    O. Espinosa, *Nucl. Phys.* **B343** (1990) 310.
12. *Nonperturbative Aspects of Chiral Gauge Theories,* Rome, 1992; published in *Nucl. Phys. Proc. Suppl.* **29BC** (1992).
13. R. Narayanan, H. Neuberger, *Nucl. Phys.* **B443** (1995) 305.
14. H.B. Nielsen and H. Ninomiya, *Nucl. Phys.* **B185** (1981) 20; erratum ibid. **B195** (1981) 541; *Nucl. Phys.* **B193** (1981) 173.
15. I. Montvay, *Phys. Lett.* **199B** (1987) 89; *Nucl. Phys. Proc. Suppl.* **4** (1988) 443; *Phys. Lett.* **B205** (1988) 315; *Nucl. Phys. Proc. Suppl.* **29BC** (1992) 159.
16. I. Montvay, *Phys. Lett.* **B382** (1996) 104.
17. V.A. Kuzmin, V.A. Rubakov, M.E. Shaposhnikov, *Phys. Lett.* **B155** (1985) 36.
18. M.E. Shaposhnikov, these Proceedings.
19. Z. Fodor, J. Hein, K. Jansen, A. Jaster, I. Montvay, *Nucl. Phys.* **B439** (1995) 147.
20. F. Csikor, Z. Fodor, J. Hein, A. Jaster, I. Montvay, *Nucl. Phys.* **B474** (1996) 421.
21. Y. Aoki, *Nucl. Phys. Proc. Suppl.* **53** (1997) 609; preprint hep-lat/9612023.
22. F. Csikor, Z. Fodor, J. Hein, J. Heitger, A. Jaster, I. Montvay, *Nucl. Phys. Proc. Suppl.* **53** (1997) 612.