One-loop non-renormalization results in EFTs

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Abstract

In Effective Field Theories (EFTs) with higher-dimensional operators many anomalous dimensions vanish at the one-loop level. With the use of supersymmetry, and a classification of the operators according to their embedding in super-operators, we are able to understand why many of these anomalous dimensions are zero. The key observation is that one-loop contributions from superpartners trivially vanish in many cases under consideration, making the superfield formalism a powerful tool even for non-supersymmetric models. We show this in detail in a simple $U(1)$ model with a scalar and fermions, and explain how to extend this to SM EFTs and the QCD Chiral Langrangian. This provides an understanding of why most "current-current" operators do not renormalize "loop" operators at the one-loop level, and allows to find the few exceptions to this ubiquitous rule.

1 Introduction

Quantum Effective Field Theories (EFTs) provide an excellent framework to describe physical systems, most prominently in particle physics, cosmology and condensed matter. With the recent discovery of the Higgs boson and the completion of the SM, EFTs have provided a systematic approach to smartly parametrize our ignorance on possible new degrees of freedom at the TeV scale. Any theory beyond the SM, with new heavy degrees of freedom, can be matched into an EFT that consists of operators built out solely with the SM degrees of freedom.

Recently, there has been much effort put into the determination of the one-loop anomalous dimensions of the dimension-six operators of the SM EFT [1, 2, 3, 4, 5]. This has revealed a rather intriguing structure in the anomalous-dimension matrix, with plenty of vanishing entries that are a priori allowed by all symmetries. Some vanishing entries are trivial since no possible diagram exist. Nevertheless, some of them show intricate cancelations without any apparent reason. Similar cancelations had been observed before in other EFTs (see for example [6, 7]).

To make manifest the pattern of zeros in the matrix of anomalous dimensions, it is crucial to work in the proper basis. Refs. [2, 3] pointed out the importance of working in bases with operators classified as "current-current" operators and "loop" operators. The first ones, which we call from now on JJ-operators, were defined to be those operators that can be generated as a product of spin-zero, spin-1/2 or spin-one currents of renormalizable theories [8, 9, 3], while the rest were called "loop" operators. [1] In this basis it was possible to show [2] that some class of loop-operators were not renormalized by JJ-operators, suggesting a kind of generic non-renormalization rule. The complete pattern of zeros in the SM EFT was recently provided in Ref. [10] in the basis of [11], a basis that also maintains the separation between JJ- and loop-operators. A classification of operators based on holomorphy was suggested to be a key ingredient to understand the structure of zeros of the anomalous-dimension matrix [10].

In the present paper we provide an approach to understand in a simple way the vanishing of anomalous-dimensions. The reason behind many cancelations is the different Lorentz structure of the operators that makes it impossible to mix them at the one-loop level. Although it is possible to show this in certain cases by simple inspection of the one-loop diagrams, we present a more compact and systematic approach based on the superfield formalism. For this reason we embed the EFT into an effective superfield theory (ESFT), and classify the operators depending on their embedding into super-operators. Using the ESFT, we are able to show by a simple spurion analysis (the one used to prove non-renormalization theorems in supersymmetric theories) the absence, in certain cases, of mixing between operators of different classes. We then make the important observation that the superpartner contributions to the one-loop renormalization under consideration trivially vanish in many cases. This allows us to conclude that some of the non-renormalization results of the ESFTs apply to the

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[1]This classification is well-defined regardless of the specific UV-completion. Field redefinitions (or use of the equations of motion) do not mix JJ-operators and loop-operators.

non-supersymmetric EFTs as well. In other words, we will show that in many cases super-symmetry allows to relate a *non-trivial* calculation to a *trivial* one (that of the superpartner loops). This also provides a way to understand the few exceptions to the ubiquitous rule that JJ-operators do not renormalize loop-operators at the one-loop level.

The paper is organized as follows. In Sec. 2 we start with a simple theory, the EFT of scalar quantum electrodynamics, to illustrate our approach for obtaining one-loop non-renormalization results. In later subsections, we enlarge the theory including fermions, and present an exceptional type of JJ-operator that renormalizes loop-operators. In Sec. 3 we show how to generalize our approach to derive analogous results in the SM EFT and we also discuss the holomorphic properties of the anomalous dimensions. In Sec. 4 we show the implications of our approach for the QCD Chiral Lagrangian. We conclude in Sec. 5.

2 Non-renormalization results in a $U(1)$ EFT

Let us start with the simple case of a massless scalar coupled to a $U(1)$-gauge boson with charge Q_ϕ, assuming for simplicity CP-conservation. The corresponding EFT is defined as an expansion in derivatives and fields over a heavy new-physics scale Λ: $\mathcal{L}_{\text{EFT}} = \sum_d \mathcal{L}_d$, where $\mathcal{L}_d$ denotes the terms in the expansion made of local operators of dimension d. The leading terms ($d \leq 6$) in the EFT are given by

$$\mathcal{L}_4 = -|D_\mu \phi|^2 - \lambda_\phi |\phi|^4 - \frac{1}{4g^2} F_{\mu\nu}^2 \,, \qquad \mathcal{L}_6 = \frac{1}{\Lambda^2} \left[c_r \mathcal{O}_r + c_6 \mathcal{O}_6 + c_{FF} \mathcal{O}_{FF} \right] \,, \qquad (1)$$

where the dimension-six operators are

$$\mathcal{O}_r = |\phi|^2 |D_\mu \phi|^2 \,, \qquad \mathcal{O}_6 = |\phi|^6 \,, \qquad \mathcal{O}_{FF} = |\phi|^2 F_{\mu\nu} F^{\mu\nu} \,. \qquad (2)$$

We can use different bases for the dimension-six operators although, when looking at operator mixing, it is convenient to work in a basis that separates JJ-operators from loop-operators, as we defined them in the introduction. Using field redefinitions (or, equivalently, the equation of motion (EOM) of ϕ) we can reduce the number of JJ-operators to only two: for instance, $\mathcal{O}_T = \frac{1}{2} J^\mu J_\mu$ and $\mathcal{O}_6 = J^* J$, where $J_\mu = \phi^* \overset{\leftrightarrow}{D}_\mu \phi$ and $J = |\phi|^2 \phi$. It is convenient, however, to set a one-to-one correspondence between operators and supersymmetric D-terms, as we will show below. For this reason, we choose for our basis $\mathcal{O}_6$ and $\mathcal{O}_r$. [2] The only loop-operator, after requiring CP-invariance, is $\mathcal{O}_{FF}$.

Many of the one-loop non-renormalization results that we discuss can be understood from arguments based on the Lorentz structure of the vertices involved. Take for instance the non-renormalization of $\mathcal{O}_{FF}$ by $\mathcal{O}_r$. Integrating by parts and using the EOM, we can eliminate $\mathcal{O}_r$ in favor of $\mathcal{O}'_r = (\phi D_\mu \phi^*)^2 + h.c..$ Now, it is apparent that $\mathcal{O}'_r$ cannot renormalize $\mathcal{O}_{FF}$ because either $\phi D_\mu \phi^*$ or $\phi^* D_\mu \phi$ is external in all one-loop diagrams, and these Lorentz

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[2]In the $U(1)$ case we are considering, $\mathcal{O}_r = \frac{1}{2} (\mathcal{O}_H - \mathcal{O}_T)$ where $\mathcal{O}_H = \frac{1}{2} (\partial_\mu |\phi|^2)^2$.

structures cannot be completed to form $\mathcal{O}_{FF}$. Since, in addition, there are no possible one-loop diagrams involving $\mathcal{O}_6$ that contribute to $\mathcal{O}_{FF}$, we can conclude that in this EFT the loop-operator cannot be renormalized at the one-loop level by the JJ-operators. As we will see, similar Lorentz-based arguments can be used for other non-renormalization results. This approach, however, requires a case by case analysis and it is not always guaranteed that one can find an easy argument to see that the loop is zero without a calculation. In this paper we present a more systematic and unified understanding of such vanishing anomalous dimensions based on a superfield approach that we explain next.

We first promote the model of Eq. (1) to an ESFT and study the renormalization of the dimension-six operators in this supersymmetric theory. The superfield formalism makes it transparent to determine which operators do not mix at the one-loop level. Although in this theory the renormalization of operators involves also loops of superpartners, we will show in a second step that either the ordinary loop (involving ϕ and A_μ) is already trivially zero or it is the superpartner loops which trivially vanish. Therefore, having ensured that there are no cancellations between loops of ordinary matter and supermatter, we are able to extend the supersymmetric non-renormalization results to the non-supersymmetric case. In other words, the advantage of this approach is that we can turn a loop calculation with the ordinary ϕ and A_μ into a calculation with superpartners, where the Lorentz structure of the vertex can make it easier to see that the one-loop contributions are zero.

The dimension-six operators of Eq. (2) can be embedded in different types of super-operators. As it will become clear in what follows, it is important for our purposes to embed the dimension-six operators into super-operators with the lowest possible dimension. This corresponds to an embedding into the highest θ-component of the super-operator (notice that we can always lower the θ-component by adding derivatives in superspace). This provides a classification of the dimension-six operators that is extremely useful in analyzing the one-loop mixings. Let us start with the loop-operator $\mathcal{O}_{FF}$. Promoting ϕ to a chiral supermultiplet Φ and the gauge boson A_μ to a vector supermultiplet V, one finds that $\mathcal{O}_{FF}$ can be embedded into the θ^2-component (F-term) of the super-operator

$$\Phi^\dagger e^{V_\Phi} \Phi \, \mathcal{W}^\alpha \mathcal{W}_\alpha = -\frac{1}{2} \theta^2 \mathcal{O}_{FF} + \cdots , \tag{3}$$

where we have defined $V_\Phi \equiv 2Q_\phi V$, $\mathcal{W}^\alpha$ is the field-strength supermultiplet, and we follow the notation of [12] (using a mostly-plus metric). Since the super-operator in Eq. (3) is non-chiral, the $\mathcal{O}_{FF}$ cannot be generated in a supersymmetry-preserving theory at any loop order. For the embedding of the JJ-operators, the situation is different. Some of them can be embedded in a D-term (a $\bar{\theta}^2\theta^2$-component), while for others this is not possible. In the example discussed here, we have

$$\left(\Phi^\dagger e^{V_\Phi} \Phi\right)^2 = -4\theta^2\bar{\theta}^2 \mathcal{O}_r + \cdots , \tag{4}$$

and therefore $\mathcal{O}_r$ is allowed by supersymmetry to appear in the Kähler potential and is not-protected from one-loop corrections. Nevertheless $\mathcal{O}_6$ must arise from the θ^0-component of

the super-operator

$$\left(\Phi^\dagger e^{V_\Phi} \Phi\right)^3 = \mathcal{O}_6 + \cdots , \tag{5}$$

and then must be zero in a supersymmetry-preserving theory at any loop order.

We can now embed Eq. (1) in a ESFT. We use a supersymmetry-breaking (SSB) spurion superfield $\eta \equiv \theta^2$ (of dimension $[\eta] = -1$) to incorporate the couplings of Eq. (1) that break supersymmetry. We have [3]

$$
\begin{aligned}
\mathcal{L}_4 &\subset \int d^4\theta \left[\Phi^\dagger e^{V_\Phi} \Phi + \lambda_\phi \eta\eta^\dagger (\Phi^\dagger e^{V_\Phi} \Phi)^2\right] + \left[\int d^2\theta \, \mathcal{W}^\alpha \mathcal{W}_\alpha + h.c.\right] , \\
\mathcal{L}_6 &\subset \frac{1}{\Lambda^2} \int d^4\theta \left\{ \tilde{c}_r \left(\Phi^\dagger e^{V_\Phi} \Phi\right)^2 + \tilde{c}_6 \, \eta\eta^\dagger (\Phi^\dagger e^{V_\Phi} \Phi)^3 + \left[\tilde{c}_{FF} \, \eta^\dagger (\Phi^\dagger e^{V_\Phi} \Phi)\mathcal{W}^\alpha \mathcal{W}_\alpha + h.c.\right] \right\} .
\end{aligned} \tag{6}
$$

It is very easy to study the one-loop mixing of the dimension-six operators in the above ESFT using a simple η-spurion analysis. For example, it is clear that there cannot be renormalization from terms with no SSB spurions, such as $\tilde{c}_r$, to terms with SSB spurions, such as $\tilde{c}_{FF}$. Also, corrections from $\tilde{c}_r$ to $\tilde{c}_6$ are only possible through the insertion of λ_ϕ, that carries a $\eta\eta^\dagger$. Similarly, terms with a SSB spurion $\eta^\dagger$ cannot renormalize terms with two SSB spurions $\eta^\dagger\eta$, unless they are proportional to λ_ϕ. This means that $\tilde{c}_{FF}$ can only renormalize $\tilde{c}_6$ with the insertion of a λ_ϕ. The inverse is however not guaranteed: terms with more SSB spurions can in principle renormalize terms with less spurions. For example, $\tilde{c}_{FF}$, that carries a spurion $\eta^\dagger$, could generate at the loop level the operator

$$\int d^4\theta \eta^\dagger \bar{\mathcal{D}}^2 \tilde{\mathcal{O}}_r = \int d^4\theta (\bar{\mathcal{D}}^2 \eta^\dagger)\tilde{\mathcal{O}}_r = \int d^4\theta \tilde{\mathcal{O}}_r , \tag{7}$$

where $\tilde{\mathcal{O}}_r = \left(\Phi^\dagger e^{V_\Phi} \Phi\right)^2$ and we have defined $\mathcal{D}^2 \equiv \mathcal{D}_\alpha \mathcal{D}^\alpha$, with $\mathcal{D}_\alpha \Phi = e^{-V_\Phi} D_\alpha(e^{V_\Phi} \Phi)$ being the gauge-covariant derivative in superspace. Therefore one has to check it case by case. For example, $\tilde{c}_6$ could in principle renormalize $\tilde{c}_{FF}$, but it is not possible to write the relevant diagram since it involves a vertex with too many Φ's. This implies that $\tilde{c}_{FF}$ is only renormalized by itself at the one-loop level.

This simple renormalization structure is the starting point from which, by examining more closely the loops involved at the field-component level, we will derive the following non-renormalization results in the non-supersymmetric EFT of Eq. (1):

Non-renormalization of $\mathcal{O}_{FF}$ by $\mathcal{O}_r$: The differences between our original EFT in Eq. (1) and its supersymmetric version, Eq. (6), are the presence of the fermion superpartners for the gauge and scalar: the gaugino, λ, and "Higgsino", ψ. We will show, however, that the contributions from superpartners trivially vanish in the mixing of JJ- and loop-operators. In

$$\int d^4\theta \left(\Phi^\dagger e^{V_\Phi} \Phi\right)^2 = -4\mathcal{O}_r + 2(i\phi^* \overset{\leftrightarrow}{D}_\mu \phi)\psi^\dagger \bar{\sigma}^\mu \psi + 2|\phi|^2(i\psi^\dagger \bar{\sigma}^\mu \overset{\leftrightarrow}{D}_\mu \psi) + \cdots , \tag{8}$$

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[3] Anomaly cancelation requires the inclusion of additional fields that do not play any role in our discussion. We ignore them in what follows.

we have only the 3 terms shown that can potentially contribute to $\mathcal{O}_{FF}$ at the one-loop level. These terms can be considered as part of a supersymmetric JJ-operator generated from integrating-out a heavy vector superfield that contains a scalar, a vector and a fermion. Other terms not shown in Eq. (8) involve too many fields (see Appendix) and therefore are only relevant for an analysis beyond one-loop. The first term of Eq. (8) can potentially give a contribution to $\mathcal{O}_{FF}$ from a loop of ϕ's, while the second and third term could from a loop of Higgsinos. It is very easy to see that the loop of Higgsinos does not contribute to $\mathcal{O}_{FF}$. Indeed, if in the second term of Eq. (8) we close the Higgsinos in a loop, the current $J_\mu = i\phi^* \overset{\leftrightarrow}{D}_\mu \phi$ is left as an external factor, and it is then clear that we can only generate the JJ-operator $J_\mu J^\mu$. Moreover, the third term of Eq. (8) vanishes by using the EOM: $\bar{\sigma}^\mu D_\mu \psi = 0$ (up to gaugino terms that are not relevant here). Therefore, Higgsinos do not contribute at the one-loop level to the renormalization of the loop-operator $\mathcal{O}_{FF}$. We can then extend the non-renormalization result from the ESFT of Eq. (6) to the non-supersymmetric EFT of Eq. (1) and conclude that *the loop-operator cannot be renormalized at the one-loop level by the JJ-operators.*

Non-renormalization of $\mathcal{O}_r$ by $\mathcal{O}_{FF}$: It remains to study the renormalization from $\mathcal{O}_{FF}$ to $\mathcal{O}_r$. This can arise in principle from a loop of gauge bosons. In the supersymmetric theory, Eq. (6), $\tilde{c}_r$ does not carry any SSB spurion and therefore its renormalization by $\tilde{c}_{FF}$ cannot be prevented on general grounds, as we explained before. Nevertheless, we find that operators induced by $\tilde{c}_{FF}$, through a loop of V's, must leave an external factor $\eta^\dagger \Phi^\dagger e^{V_\Phi} \Phi$ from the vertex and then, the only operator that could potentially contribute to $\tilde{c}_r$ must have the form [4]

$$\frac{1}{\Lambda^2} \int d^4\theta\, \eta^\dagger \left(\Phi^\dagger e^{V_\Phi} \Phi\right) \bar{\mathcal{D}}^2 \left(\Phi^\dagger e^{V_\Phi} \Phi\right) + h.c.. \tag{9}$$

From the EOM for Φ, we have that $\bar{\mathcal{D}}^2 \Phi^\dagger = 0$ up to λ_ϕ terms that bring too many powers of Φ, so that the projection of Eq. (9) into $\mathcal{O}_r$ vanishes. Finally, one also has to ensure that redundant JJ-super-operators, that can give $\left(\Phi^\dagger e^{V_\Phi} \Phi\right)^2$ through superfield redefinitions, are not generated at the one-loop level. In particular, the redundant super-operator

$$\frac{1}{\Lambda^2} \int d^4\theta\, \left(\Phi^\dagger e^{V_\Phi} \Phi\right) \mathcal{D}_\alpha \mathcal{W}^\alpha\,, \tag{10}$$

if generated at the loop level, can give a contribution to $\tilde{c}_r$ after superfield redefinitions, or equivalently, after using the EOM of V: $\mathcal{D}_\alpha \mathcal{W}^\alpha + h.c. = -g Q_\phi \Phi^\dagger e^{V_\Phi} \Phi$. We do not find, however, any non-zero contribution from $\eta^\dagger (\Phi^\dagger e^{V_\Phi} \Phi) \mathcal{W}^\alpha \mathcal{W}_\alpha$ to the operator in Eq. (10), as such contributions, coming from a V/Φ loop, must be proportional to $\eta^\dagger \mathcal{W}^\alpha \Phi$. [5]

Having shown that supersymmetry guarantees zero contributions to $\tilde{c}_r$ from $\tilde{c}_{FF}$, we must

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[4] Notice that the presence of $\eta^\dagger$, arising from the vertex, requires that the super-operator must have two derivatives $\bar{\mathcal{D}}$ in order to potentially contain $\mathcal{O}_r$.

[5] Of these, the only one that cannot be put to zero by the EOM of Φ is $\int d^4\theta\, \eta^\dagger \mathcal{W}^\alpha \Phi [\bar{\mathcal{D}}_{\dot{\alpha}}, \{\mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}\}] e^{V_\Phi} \Phi^\dagger$ but, from the identity $[\bar{\mathcal{D}}_{\dot{\alpha}}, \{\mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}\}] \sim i\mathcal{W}_\alpha$ [13] , one can see that this only contributes to $\tilde{c}_{FF}$.

check what are the effects of superpartner loops. From (see Appendix)

$$\int d^4\theta \eta^\dagger (\Phi^\dagger e^{V_\Phi}\Phi)\mathcal{W}^\alpha \mathcal{W}_\alpha + h.c. = -\mathcal{O}_{FF} + \left(2i|\phi|^2 \, \lambda\sigma^\mu\partial_\mu\lambda^\dagger - \frac{1}{\sqrt{2}}\phi^*\lambda\sigma^{\mu\nu}\psi F_{\mu\nu} + h.c. \right) + \dots ,$$
(11)

where $\sigma^{\mu\nu} = \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)$, it is clear that a gaugino/Higgsino loop cannot give a contribution to $\mathcal{O}_r$: the second term of Eq. (11), after using the EOM for the gaugino, $\sigma^\mu\partial_\mu\lambda^\dagger = g\phi\psi^\dagger$, can only give a contribution proportional to $|\phi|^2\phi$; while the contribution from the third term must be proportional to $\phi^*F_{\mu\nu}$. None of them have the right Lorentz structure to contribute to $\mathcal{O}_r$. Therefore, we conclude that the *loop-operator $\mathcal{O}_{FF}$ can only renormalize at the one-loop level the JJ-operators that break supersymmetry, like $\mathcal{O}_6$, and not those that can be embedded in a D-term, like $\mathcal{O}_r$.*

2.1 Including fermions

Let us extend the previous EFT to include two charged Weyl fermions, q and u, with $U(1)$-charges Q_q and Q_u, such that $Q_\phi + Q_q + Q_u = 0$. We have now extra terms in the Lagrangian (respecting CP-invariance): [6]

$$\Delta\mathcal{L}_4 = iq^\dagger\bar{\sigma}^\mu D_\mu q + iu^\dagger\bar{\sigma}^\mu D_\mu u + y_u \left(\phi qu + h.c.\right) ,$$

$$\Delta\mathcal{L}_6 = \frac{1}{\Lambda^2}\left[c_{\phi f}\mathcal{O}_{\phi f} + c_{4f}\mathcal{O}_{4f} + c_{y_u}\left(\mathcal{O}_{y_u} + h.c.\right) + c_D\left(\mathcal{O}_D + h.c.\right)\right] ,$$
(12)

where $f = q, u$. The JJ-operators are

$$\mathcal{O}_{y_u} = |\phi|^2\phi qu , \quad \mathcal{O}_{\phi f} = i(\phi^* f^\dagger)\bar{\sigma}^\mu D_\mu(f\phi) , \quad \mathcal{O}_{4f} = (f^\dagger\bar{\sigma}_\mu f)(f^\dagger\bar{\sigma}^\mu f) .$$
(13)

Instead of $\mathcal{O}_{\phi f}$, we could have chosen the more common JJ-operator $i(\phi^*\overleftrightarrow{D}_\mu\phi)(f^\dagger\bar{\sigma}^\mu f)$ for our basis. Both are related by

$$\mathcal{O}_{\phi f} = \frac{i}{2}(\phi^*\overleftrightarrow{D}_\mu\phi)(f^\dagger\bar{\sigma}^\mu f) + \frac{i}{2}|\phi|^2 f^\dagger\bar{\sigma}^\mu\overleftrightarrow{D}_\mu f ,$$
(14)

where the last term could be eliminated by the use of the EOM. Our motivation for keeping $\mathcal{O}_{\phi f}$ in our basis is that, as we will see later, it is in one-to-one correspondence with a supersymmetric D-term. The only additional loop-operator for a $U(1)$ model with fermions is the dipole operator

$$\mathcal{O}_D = \phi(q\sigma^{\mu\nu}u)F_{\mu\nu} .$$
(15)

Let us consider the operator mixing in this extended EFT. We will discuss all cases except those for which no diagram exists at the one-loop level. As we said before, in principle, many vanishing entries of the anomalous-dimensions can be simply understood from inspection of the Lorentz structure of the different vertices. For example, it is relatively simple to check

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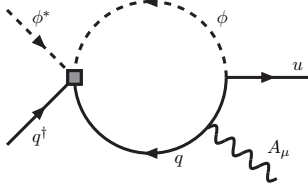[6]Similar remarks to those made in footnote 3 about anomalies apply to this extended model.

Figure 1: *A potential contribution from $\mathcal{O}_{\phi q}$ to $\mathcal{O}_D$.*

that the JJ-operators $\mathcal{O}_{4f}$ and $\mathcal{O}_{\phi f}$ do not renormalize the loop-operators. For this purpose, it is important to recall that we can write four-fermion operators, such as $(q^\dagger \bar\sigma_\mu q)(u^\dagger \bar\sigma^\mu u)$, in the equivalent form $q^\dagger u^\dagger q u$. From this, it is obvious that closing a loop of fermions can only give operators containing the Lorentz structure $f^\dagger f$ or qu that cannot be completed to give a dipole operator (nor its equivalent forms, $q\sigma_{\mu\nu}\sigma_\rho D^\rho q^\dagger F^{\mu\nu}$ or $D_\mu \phi q D^\mu u H$). For the case of $\mathcal{O}_{\phi f}$, the absence of renormalization of the dipole operator, as for example from diagrams like the one in Fig. 1, can be proved just by realizing that we can always keep the Lorentz structure $\bar\sigma^\mu D_\mu (\phi f)$ external to the loop; this Lorentz structure cannot be completed to form a dipole operator. The contribution of $\mathcal{O}_{\phi f}$ to $\mathcal{O}_{FF}$ is also absent, as can be deduced from Eq. (14): the first term, after closing the fermion loop, gives the wrong Lorentz structure to generate $\mathcal{O}_{FF}$, while the second term gives an interaction with too many fields if we use the fermion EOM. Finally, $\mathcal{O}_{y_u}$ can only contribute to the Lorentz structure $\phi q u$, not to the dipole one in Eq. (15).

We can be more systematic and complete using our ESFT approach. Let us see first how the operators of Eq. (12) can be embedded in super-operators. By embedding q and u in the chiral supermultiplets Q and U, we find that the dipole loop-operator must arise from the θ^2-term of a non-chiral superfield:

$$\Phi \left(Q \overset{\leftrightarrow}{\mathcal{D}}_\alpha U \right) \mathcal{W}^\alpha = -\theta^2 \mathcal{O}_D + \cdots. \tag{16}$$

Among the JJ-operators of Eq. (13), two of them can arise from supersymmetric D-terms and are then supersymmetry-preserving:

$$\left(\Phi^\dagger e^{V_\Phi} \Phi \right) \left(Q^\dagger e^{V_Q} Q \right) = \bar\theta^2 \theta^2 \mathcal{O}_{\phi q} + \cdots, \quad \left(Q^\dagger e^{V_Q} Q \right) \left(Q^\dagger e^{V_Q} Q \right) = -\frac{1}{2} \bar\theta^2 \theta^2 \mathcal{O}_{4q} + \cdots, \tag{17}$$

and similar operators for $Q \to U$, where we again use the short-hand notation $V_Q = 2Q_q V$. Nevertheless, one of the JJ-operators must come from the θ^2-component of a non-chiral superfield that is not invariant under supersymmetry:

$$\left(\Phi^\dagger e^{V_\Phi} \Phi \right) \Phi Q U = \theta^2 \mathcal{O}_{y_u} + \cdots. \tag{18}$$

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We can now promote Eq. (12) to a ESFT:

$$
\begin{aligned}
\Delta\mathcal{L}_4 \;\subset\;& \int d^4\theta \left(Q^\dagger e^{V_Q} Q + U^\dagger e^{V_U} U\right) + \left[\int d^2\theta\, y_u \Phi Q U + h.c.\right], \\
\Delta\mathcal{L}_6 \;\subset\;& \frac{1}{\Lambda^2}\int d^4\theta \Big\{ \tilde{c}_{\phi f}(\Phi^\dagger e^{V_\Phi}\Phi)(F^\dagger e^{V_F} F) + \tilde{c}_{4f}(F^\dagger e^{V_F} F)(F^\dagger e^{V_F} F) \\
& + \left[\eta^\dagger\left(\tilde{c}_{y_u}(\Phi^\dagger e^{V_\Phi}\Phi)\Phi Q U + \tilde{c}_D \Phi\,(Q\overset{\leftrightarrow}{\mathcal{D}}_\alpha U)\mathcal{W}^\alpha\right) + h.c.\right]\Big\},
\end{aligned} \tag{19}
$$

where $F = Q, U$.

Non-renormalization of loop-operators from JJ-operators: The embedding of the EFT into the ESFT shows the following rule. Loop-operators ($\mathcal{O}_{FF}$ and $\mathcal{O}_D$) cannot be supersymmetrized, while some JJ-operators can be supersymmetrized ($\mathcal{O}_r$, $\mathcal{O}_{4f}$ and $\mathcal{O}_{\phi f}$) and others cannot ($\mathcal{O}_{y_u}$ and $\mathcal{O}_6$). Supersymmetry then guarantees that loop-operators can at most be generated from the latter ones, $\mathcal{O}_{y_u}$ and $\mathcal{O}_6$, embedded respectively in $\eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)\Phi Q U$ and $\eta\eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)^3$. By simple inspection of these latter vertices, however, we find that neither of them is possible at the one-loop level. Therefore, in the ESFT the loop-operators are not renormalized at one-loop level by the JJ-operators.

To extend the above results to the non-supersymmetric EFT, we must ensure that these non-renormalization results do not arise from cancellations between loops involving "ordinary" fields (A_μ, ϕ, q and u) and loops involving superpartners (λ, ψ, $\tilde{q}$ and $\tilde{u}$). This can be proved by showing that either the former or the latter are zero. In certain cases it is easier to look at the loop of ordinary fields, while in others it is easier to look at the superpartner loops. For example, we have (see appendix)

$$
\int d^4\theta \left(Q^\dagger e^{V_Q} Q\right)\left(Q^\dagger e^{V_Q} Q\right) = -\frac{1}{2}\mathcal{O}_{4q} + 2q^\dagger\bar{\sigma}^\mu q(i\tilde{q}^\dagger\overset{\leftrightarrow}{D}_\mu\tilde{q}) + 2(iq^\dagger\bar{\sigma}^\mu\overset{\leftrightarrow}{D}_\mu q)|\tilde{q}|^2 + \cdots, \tag{20}
$$

where we see that a renormalization to $\mathcal{O}_D$ can arise either from the first term (by a loop of "quarks" q) or the second and third term by a loop of "squarks" $\tilde{q}$. It is easier to see that the loops of squarks are zero: they can only generate operators containing $q^\dagger\bar{\sigma}^\mu q$ or $q^\dagger\bar{\sigma}^\mu\overset{\leftrightarrow}{D}_\mu q$, that do not have the structure necessary to contribute to the dipole operator $\mathcal{O}_D$ nor to operators related to this one by EOMs, such as $q\sigma_{\mu\nu}\sigma_\rho D^\rho q^\dagger F^{\mu\nu}$. We could proceed similarly for the other operators. For the case of $\mathcal{O}_{\phi f}$, however, the one-loop contribution to $\mathcal{O}_D$ contains scalars and fermions (see Fig. 1) and the corresponding graph with superpartners has a similar structure, and therefore is not simpler. Nevertheless, both can be showed to be zero by realizing that $\bar{\sigma}^\mu D_\mu(\phi f)$ can always be kept as external to the loop, and that this Lorentz structure cannot be completed to form a dipole operator. We can conclude that the absence of renormalization of loop-operators by JJ-operators valid in the ESFT also applies to the EFT.

Class of JJ-operators not renormalized by loop-operators: Following the same approach, we can also check whether loop-operators can generate JJ-operators. Let us first work within the ESFT. We have shown already that the loop-super-operator $\eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)\mathcal{W}^\alpha\mathcal{W}_\alpha$

cannot generate the JJ-super-operator $(\Phi^\dagger e^{V_\Phi}\Phi)^2$. The same arguments apply straightforwardly to $(F^\dagger e^{V_F}F)(\Phi^\dagger e^{V_\Phi}\Phi)$. For the case of the dipole super-operator, $\eta^\dagger\Phi(Q\overset{\leftrightarrow}{\mathcal{D}}_\alpha U)\mathcal{W}^\alpha$, we have a potential contribution to $\left(Q^\dagger e^{V_Q}Q\right)\left(U^\dagger e^{V_U}U\right)$ coming from a Φ/V loop. Nevertheless, as the factor $\eta^\dagger Q\overset{\leftrightarrow}{\mathcal{D}}_\alpha U$ remains in the external legs, it is clear that such contribution can only lead to operators containing $\eta^\dagger\mathcal{D}^2$, which are not JJ-super-operators. Similarly, contributions to $\left(\Phi^\dagger e^{V_\Phi}\Phi\right)\left(Q^\dagger e^{V_Q}Q\right)$ could arise from a U/V loop, but one can always arrange it to leave either $\eta^\dagger\mathcal{D}_\alpha\Phi$ or $\eta^\dagger\mathcal{D}_\alpha Q$ in the external legs [7], which again does not have the structure of a JJ-super-operator (the same applies for $Q\leftrightarrow U$). Finally we must check whether redundant JJ-super-operators, as the one in Eq. (10), can be generated by the dipole. Similar arguments as those below Eq. (10) can be used to prove that this is not the case. Notice, however, that we cannot guarantee the absence of renormalization by loop-super-operators neither of $\eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)\Phi QU$ nor of $\eta\eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)^3$. We then conclude that only the JJ-super-operators that preserve supersymmetry (with no SSB-spurions) are safe at the one-loop level from the renormalization by loop-super-operators.

It remains to show that this result extends also to non-supersymmetric EFT. From Eq. (41) of the Appendix, we have, after using the gaugino EOM and eliminating the auxiliary fields F_i, that loops from superpartners can only give contributions proportional to ϕff, $|\phi|^2 f$, ff or $F_{\mu\nu}f$ (for $f = q, u$). None of these terms can lead to the Lorentz structure of $\mathcal{O}_r$, $\mathcal{O}_{4f}$ nor $\mathcal{O}_{\phi f}$. These are exactly the same JJ-operators that could not be generated (at one loop) from loop-operators in the ESFT.

2.1.1 An exceptional JJ-operator

Let us finally extend the EFT to include an extra fermion, a "down-quark" d of charge Q_d, such that $Q_\phi = Q_q + Q_d$. The following extra terms are allowed in the Lagrangian:

$$
\begin{aligned}
\Delta\mathcal{L}_4 &= id^\dagger\bar{\sigma}^\mu D_\mu d + y_d\left(\phi^* qd + h.c.\right), \\
\Delta\mathcal{L}_6 &= \frac{1}{\Lambda^2}\left[c_{y_d}\mathcal{O}_{y_d} + c_{y_u y_d}\mathcal{O}_{y_u y_d} + h.c.\right],
\end{aligned}
\tag{21}
$$

where we have the additional JJ-operators

$$
\mathcal{O}_{y_d} = |\phi|^2\phi^* qd, \quad \mathcal{O}_{y_u y_d} = quqd,
\tag{22}
$$

apart from operators similar to the ones in Eq. (12) with f including also the d.

Following the ESFT approach, we embed the d-quark in a chiral supermultiplet D and the operators of Eq. (21) into the super-operators:

$$
\begin{aligned}
\Phi^\dagger e^{V_\Phi}QD &= \theta^2\phi^* qd + \cdots, \\
\left(\Phi^\dagger e^{V_\Phi}\Phi\right)\Phi^\dagger e^{V_\Phi}QD &= \theta^2\mathcal{O}_{y_d} + \cdots, \\
(QU)\mathcal{D}^2(QD) &= -4\theta^2\mathcal{O}_{y_u y_d} + \cdots.
\end{aligned}
\tag{23}
$$

---

[7] Using integration by parts and the EOM of V, we can write the dipole super-operator as $\int d^4\theta\eta^\dagger\Phi(Q\overset{\leftrightarrow}{\mathcal{D}}_\alpha U)\mathcal{W}^\alpha = -\int d^4\theta\eta^\dagger[(\mathcal{D}_\alpha\Phi)QU\mathcal{W}^\alpha + 2\Phi(\mathcal{D}_\alpha Q)U\mathcal{W}^\alpha + O(\Phi_i^5)]$ where $\Phi_i = \Phi, Q, U$.

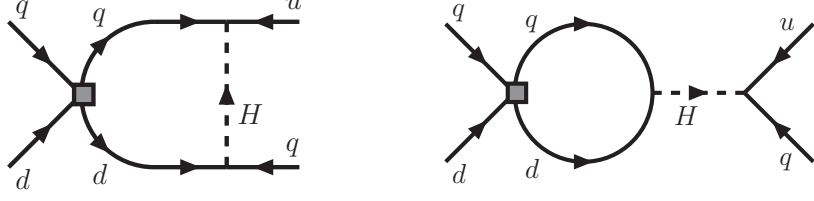Figure 2: *Contributions to $c_{y_u y_d}$ proportional to $y_d y_u$.*

As all of these operators come from a θ^2-term of non-chiral super-operators, we learn that they can only be generated from supersymmetry-breaking. We can promote Eq. (21) into a ESFT in the following way:

$$
\begin{aligned}
\Delta\mathcal{L}_4 &\subset \int d^4\theta \left[D^\dagger e^{V_D} D + \left(\eta^\dagger y_d \Phi^\dagger e^{V_\Phi} Q D + h.c. \right) \right] \, , \\
\Delta\mathcal{L}_6 &\subset \frac{1}{\Lambda^2} \int d^4\theta \, \eta^\dagger \left[\tilde{c}_{y_d} \left(\Phi^\dagger e^{V_\Phi} \Phi \right) \Phi^\dagger e^{V_\Phi} Q D + \tilde{c}_{y_u y_d} \left(QU \right) \mathcal{D}^2 \left(QD \right) \right] + h.c. \, .
\end{aligned}
\tag{24}
$$

Now, and this is very important, when considering only d, q, ϕ in isolation (without the u fermion), we can always change the supersymmetric embedding of ϕ by considering $\phi^* \in \bar{\Phi}$, where $\bar{\Phi}$ is a chiral supermultiplet of charge $-1/2$. By doing this, we can write the Yukawa-term for the d in a supersymmetric way, $\int d^2\theta \, y_d \bar{\Phi} QD$, and guarantee that the renormalization of operators involving only ϕ, q, d is identical to the one of ϕ, q, u explained in the previous section.

It is then clear that supersymmetry breaking from Yukawas can only arise through the combination $y_u y_d$. This allows to explain why contributions to $\mathcal{O}_{y_u y_d}$ from $(q^\dagger \bar{\sigma}_\mu q)(d^\dagger \bar{\sigma}^\mu d)$ must be proportional to $y_u y_d$, as explicit calculations have shown in the SM context [10].

In the ESFT, the operator $(q^\dagger \bar{\sigma}_\mu q)(d^\dagger \bar{\sigma}^\mu d)$ is embedded in a supersymmetry-preserving super-operator and therefore can only generate supersymmetry-breaking interactions, such as $\mathcal{O}_{y_u y_d}$, via the SSB couplings $y_u y_d$. The one-loop contributions from superpartners do not affect this result, as Eq. (20) shows that they are trivially zero. The explicit SM diagrams are shown in Fig. 2, and we have checked that indeed they give a non-zero result.

The operators $\mathcal{O}_{y_u y_d}$ and $\mathcal{O}_{y_{u,d}}$ are the only JJ-operators that are embedded in the ESFT with the same SSB-spurion dependence as the loop-operators – see Eq. (24). Therefore, they can potentially renormalize $\mathcal{O}_D$. Although this was not the case for $\mathcal{O}_{y_{u,d}}$ due to its Lorentz structure, as we explained above, we have confirmed by explicit calculation that $\mathcal{O}_{y_u y_d}$ indeed renormalizes $\mathcal{O}_D$. This is then an exception to the ubiquitous rule that JJ-operators do not renormalize loop-operators.

10

	Operators	SSB spurion	Super-operators
JJ-operators	$\mathcal{O}_+ = D_\mu(H_i^\dagger H_j^\dagger)D^\mu(H^i H^j)$	η^0	$(H^\dagger e^{V_H} H)^2$
	$\mathcal{O}_{4f} = \left(\bar{f}\gamma^\mu t^a f\right)\left(\bar{f}\gamma_\mu t^a f\right)$		$(F^\dagger t^a e^{V_F} F)(F^\dagger t^a e^{V_F} F)$
	$\mathcal{O}_{Hf} = i(H^\dagger t^a)_i(\bar{f}t^a)_j\gamma^\mu D_\mu\left(H^i f^j\right)$		$(H^\dagger t^a e^{V_H} H)(F^\dagger t^a e^{V_F} F)$
	$\mathcal{O}_R^{ud} = (iH^\dagger\overset{\leftrightarrow}{D}_\mu \tilde{H})(\bar{d}_R\gamma^\mu u_R)$	$\bar{\mathcal{D}}_{\dot\alpha}\eta^\dagger$	$H^\dagger\bar{\mathcal{D}}^{\dot\alpha}\tilde{H}U^\dagger e^{V_D} D$
	$\mathcal{O}_- = |H^\dagger D_\mu H|^2$	$|\bar{\mathcal{D}}_{\dot\alpha}\eta^\dagger|^2$	$|H^\dagger e^{V_H}\mathcal{D}_\alpha H|^2$
	$\mathcal{O}_6 = |H|^6$	$|\eta|^2$	$(H^\dagger e^{V_H} H)^3$
	$\mathcal{O}_y = |H|^2 H\bar{f}_R f_L$	$\eta^\dagger$	$(H^\dagger e^{V_H} H)HFF$
	$\mathcal{O}_{yy} = \left(\bar{f}_R t^a f_L\right)\left(\bar{f}_R t^a f_L\right)$		$(Ft^a F)\mathcal{D}^2(Ft^a F)$
Loop-operators	$\mathcal{O}_D = H^\dagger\bar{f}_R\sigma^{\mu\nu}t^a f_L F_{\mu\nu}^a$		$H(Ft^a\overset{\leftrightarrow}{\mathcal{D}}_\alpha F)\mathcal{W}^{a\,\alpha}$
	$\mathcal{O}_{FF_+} = H^\dagger t^a t^b H F_{\mu\nu}^a(F^{b\,\mu\nu} - i\tilde{F}^{b\,\mu\nu})$		$(H^\dagger t^a t^b e^{V_H} H)\mathcal{W}^{a\,\alpha}\mathcal{W}_\alpha^b$
	$\mathcal{O}_{3F_+} = f^{abc}F_\mu^{a\,\nu}F_\nu^{b\,\rho}(F_\rho^{c\,\mu} - i\tilde{F}_\rho^{c\,\mu})$		$f^{abc}\mathcal{D}^\beta\mathcal{W}^{a\,\alpha}\mathcal{W}_\beta^b\mathcal{W}_\alpha^c$

Table 1: *Left: Basis of dimension-six SM operators classified as JJ-operators and loop-operators. We also distinguish those that can arise from a supersymmetric D-term (η^0) from those that break supersymmetry either by an spurion $\bar{\mathcal{D}}_{\dot\alpha}\eta^\dagger$, $\eta^\dagger$, $|\bar{\mathcal{D}}_{\dot\alpha}\eta^\dagger|^2$ or $|\eta|^2$. We denote by $F_{\mu\nu}^a$ ($\tilde{F}_{\mu\nu}^a$) any SM gauge (dual) field-strength. The t^a matrices include the $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ generators, depending on the quantum numbers of the fields involved. Fermion operators are written schematically with $f = \{Q_L, u_R, d_R, L_L, e_R\}$. Right: For each operator in the left column, we provide the super-operator at which it is embedded.*

3 Generalization to the Standard Model EFT

We can generalize the previous analysis to dimension-six operators in the SM EFT. We begin by constructing an operator basis that separates JJ-operators from loop-operators. We then classify them according to their embedding into a supersymmetric model, depending on whether they can arise from a super-operator with no SSB spurion (η^0), which therefore preserves supersymmetry, or whether they need SSB spurions, either $\bar{\mathcal{D}}_{\dot\alpha}\eta^\dagger$, $\eta^\dagger$, $|\bar{\mathcal{D}}_{\dot\alpha}\eta^\dagger|^2$ or $\eta\eta^\dagger$ (that selects the $\bar{\theta}\theta^2$, θ^2, $\bar{\theta}\theta$ and $\bar{\theta}^0\theta^0$ component of the super-operator, respectively), or their Hermitian-conjugates. The supersymmetric embedding naturally selects a SM basis that we present in Table 1. In this basis, the non-renormalization results between the different classes of operators discussed in the previous section will also hold.

The operator basis of Table 1 is close to the basis defined in Ref. [11]. One significant

difference is our choice of the only-Higgs JJ-operators, that we take to be $\mathcal{O}_\pm$ and $\mathcal{O}_6$, and of the Higgs-fermion JJ-operator $\mathcal{O}_{Hf}$. As in the $U(1)$ case, this choice is motivated by the embedding of operators into super-field operators, as we have just mentioned (see more details below). Concerning the classification of 4-fermion operators, our $\mathcal{O}_{4f}$ operators correspond not only to types $(\bar{L}L)(\bar{L}L)$, $(\bar{R}R)(\bar{R}R)$ and $(\bar{L}L)(\bar{R}R)$ of Ref. [11], but also to the operator $Q_{ledq} = (\bar{L}_L e_R)(\bar{d}_R Q_L)$ classified as $(\bar{L}R)(\bar{R}L)$ in [11], since this latter can be written as a $\mathcal{O}_{4f}$ by Fierz rearrangement. Finally, our $\mathcal{O}_{yy}$ operators correspond to the four operators of type $(\bar{L}R)(\bar{L}R)$ in [11].

To embed the SM fields in supermultiplets we follow the common practice of working with left-handed fermion fields so that Q_L, u_R^c and d_R^c are embedded into the chiral supermultiplets Q, U and D (generically denoted by F). With an abuse of notation, we use H for the SM Higgs doublet as well as for the chiral supermultiplet into which it is embedded. Finally, gauge bosons are embedded in vector superfields, V^a, and we use the notation $V_\Phi \equiv 2t^a V^a$ where t^a include the generators of the SM gauge-group in the representation of the chiral-superfield Φ.

Concerning the embedding of operators into super-operators, there are a few differences with respect to the $U(1)$ model discussed in the previous section, as we discuss below. Starting with the JJ-operators, we have a new type of operator not present in the $U(1)$ case, $\mathcal{O}_R^{ud} = (iH^\dagger \overset{\leftrightarrow}{D}_\mu \tilde{H})(\bar{d}_R \gamma^\mu u_R)$, where $\tilde{H} \equiv i\sigma_2 H^*$. This operator cannot be embedded as the others in a D-term due to $\tilde{H}^\dagger H = 0$ and must be embedded as a $\theta^2\bar{\theta}$ term of a spinor super-operator:

$$\int d^4\theta \; \bar{\mathcal{D}}_{\dot{\alpha}}\eta^\dagger (H^\dagger \bar{\mathcal{D}}^{\dot{\alpha}} \tilde{H}) U^\dagger e^{V_D} D = \mathcal{O}_R^{ud} + \cdots . \tag{25}$$

For the JJ-operators involving only the Higgs field, there is also an important difference with respect to the $U(1)$ case. We have now two independent operators, [8] but only one can arise from a supersymmetric D-term: [9]

$$(H^\dagger e^{V_H} H)^2 = -\bar{\theta}^2\theta^2 \mathcal{O}_+ + \cdots , \tag{26}$$

where

$$\mathcal{O}_+ = [2\mathcal{O}_r + \mathcal{O}_H - \mathcal{O}_T] = D_\mu(H_i^\dagger H_j^\dagger) D^\mu(H^i H^j) , \tag{27}$$

with $\mathcal{O}_r$, $\mathcal{O}_H$ and $\mathcal{O}_T$ being the SM analogues of the $U(1)$ operators, obtained simply by replacing ϕ by H. The other independent only-Higgs operator must arise from a SSB term. We find that this can be the $\theta\bar{\theta}$-component of the superfield

$$\bar{\mathcal{D}}^{\dot{\alpha}}(H^\dagger e^{V_H} H)\mathcal{D}_\alpha(H^\dagger e^{V_H} H) = -4(\bar{\sigma}^\mu\theta)^{\dot{\alpha}}(\sigma^\nu\bar{\theta})_\alpha \left(D_\mu H^\dagger H\right)\left(H^\dagger D_\nu H\right) + \cdots . \tag{28}$$

We can write this operator in a superfield Lagrangian by using the spurion $|\bar{\mathcal{D}}_{\dot{\alpha}}\eta^\dagger|^2$:

$$\int d^4\theta \; \bar{\mathcal{D}}_{\dot{\alpha}}\eta^\dagger \mathcal{D}^\alpha\eta \; \bar{\mathcal{D}}^{\dot{\alpha}}(H^\dagger e^{V_H} H)\mathcal{D}_\alpha(H^\dagger e^{V_H} H) = -16 \; \mathcal{O}_- + \cdots , \tag{29}$$

---

[8] The $U(1)$-case identity $\mathcal{O}_r = (\mathcal{O}_H - \mathcal{O}_T)/2$ does not hold in the SM due to the fact that H is a doublet.
[9] The operator $(H^\dagger \sigma^a e^{V_H} H)^2$ can be reduced to $(H^\dagger e^{V_H} H)^2$ by using $\sigma_{ij}^a \sigma_{kl}^a = 2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl}$.

where

$$\mathcal{O}_- = \frac{1}{2}\left[\mathcal{O}_H - \mathcal{O}_T\right] = |H^\dagger D_\mu H|^2 \ . \tag{30}$$

Concerning loop-operators, we have the new operators $\mathcal{O}_{3F} = f^{abc} F_\mu^{a\,\nu} F_\nu^{b\,\rho} F_\rho^{c\,\mu}$ and $\mathcal{O}_{3\tilde{F}} = f^{abc} F_\mu^{a\,\nu} F_\nu^{b\,\rho} \tilde{F}_\rho^{c\,\mu}$, possible now for the non-Abelian groups $SU(2)_L$ and $SU(3)_c$, which again can only arise from a θ^2-term:

$$f^{abc}\mathcal{D}^\beta \mathcal{W}^{a\,\alpha} \mathcal{W}_\beta^b \mathcal{W}_\alpha^c = i\theta^2 \mathcal{O}_{3F_+} + \cdots , \tag{31}$$

where we have defined $\mathcal{O}_{3F_\pm} = \mathcal{O}_{3F} \mp i\mathcal{O}_{3\tilde{F}}$. To contain $\mathcal{O}_{3F_+}$, Eq. (31) must then appear in the ESFT multiplying the SSB-spurion $\eta^\dagger$, as the rest of loop-operators.

For the loop-operators $\mathcal{O}_{FF} = H^\dagger t^a t^b H F_{\mu\nu}^a F^{b\,\mu\nu}$ and their CP-violating counterparts, $\mathcal{O}_{F\tilde{F}} = H^\dagger t^a t^b H F_{\mu\nu}^a \tilde{F}^{b\,\mu\nu}$, we can proceed as above and embed them together in the super-operators

$$(H^\dagger t^a t^b e^{V_H} H)\mathcal{W}^{a\,\alpha} \mathcal{W}_\alpha^b = -\frac{1}{2}\theta^2 \mathcal{O}_{FF_+} + \cdots . \tag{32}$$

where $\mathcal{O}_{FF_\pm} = \mathcal{O}_{FF} \mp i\mathcal{O}_{F\tilde{F}}$.

3.1 One-loop operator Mixing

It is straightforward to extend the $U(1)$ analysis of section 2 to the operators of Table 1 to show that, with the exception of $\mathcal{O}_{yy}$, the JJ-operators do not renormalize the loop-operators. The only important differences arise from the new type of JJ-operators, $\mathcal{O}_R^{ud}$ and $\mathcal{O}_-$. Concerning $\mathcal{O}_R^{ud}$, it is very simple to see that this operator cannot renormalize loop-operators (from a loop of quarks one obtains operators with the Lorentz structure $(i\tilde{H}^\dagger D_\mu H)$; while the Higgs-loop gives operators containing $\bar{d}_R \gamma_\mu u_R$, and none of them can be loop-operators). Concerning $\mathcal{O}_-$, we only need to worry about the renormalization of $\mathcal{O}_{FF}$. This can be studied directly in the ESFT, as superpartner contributions from JJ-operator to loop-operators can be shown to trivially vanish. In the ESFT, the operator $\mathcal{O}_-$ is embedded in a super-operator containing the SSB-spurion $|\mathcal{D}_\alpha\eta|^2$. This guarantees the absence of renormalization of loop-super-operators as these latter contain the SSB-spurion $\eta^\dagger$. Besides this direct contribution, there is an indirect route by which $\mathcal{O}_-$ could renormalize $\mathcal{O}_{FF}$: by generating $\mathcal{O}_{HF} = i(D^\mu H)^\dagger t^a (D^\nu H) F_{\mu\nu}^a$ which, via integration by parts, can give $\mathcal{O}_{FF}$. The operator $\mathcal{O}_{HF}$ can come from the super-operator $\tilde{\mathcal{O}}_{HF} = \bar{\mathcal{D}}_{\dot{\alpha}}\eta^\dagger \bar{\mathcal{D}}^{\dot{\alpha}} H^\dagger e^{V_H} \mathcal{D}_\alpha H \mathcal{W}^\alpha$ that in principle is not protected by a simple SSB-spurion analysis from being generated by super-operators $\propto |\mathcal{D}_\alpha\eta|^2$. Nevertheless, contributions to $\tilde{\mathcal{O}}_{HF}$ must come from Eq. (29) with derivatives acting on the two Higgs superfields external to the loop, and due to the derivative contractions, this can only give $\bar{\mathcal{D}}_{\dot{\alpha}}\eta^\dagger \mathcal{D}^\alpha \eta \bar{\mathcal{D}}^{\dot{\alpha}} H^\dagger \mathcal{D}_\alpha H \mathcal{D}_\beta \mathcal{W}^\beta$; by the use of the EOM of V, however, this gives a JJ-super-operator and not $\tilde{\mathcal{O}}_{HF}$.

In the SM case, the exceptional $\mathcal{O}_{yy}$ operators (than can in principle renormalize the dipole

operators) are (following the notation in [3])

$$
\begin{aligned}
\mathcal{O}_{y_u y_d} &= (\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{Q}_L^s d_R)\,, \\
\mathcal{O}_{y_u y_d}^{(8)} &= (\bar{Q}_L^r T^A u_R)\epsilon_{rs}(\bar{Q}_L^s T^A d_R)\,, \\
\mathcal{O}_{y_u y_e} &= (\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{L}_L^s e_R)\,, \\
\mathcal{O}'_{y_u y_e} &= (\bar{Q}_L^{r\,\alpha} e_R)\epsilon_{rs}(\bar{L}_L^s u_R^\alpha)\,,
\end{aligned}
\tag{33}
$$

where r, s are $SU(2)_L$ indices and T^A are $SU(3)_c$ generators. Although in principle all of these four operators could renormalize the SM dipoles, it is easy to realize that $\mathcal{O}_{y_u y_e}$ will not: the only possible way of closing a loop ($\bar{Q}_L u_R$ or $\bar{L}_L e_R$) does not reproduce the dipole Lorentz structure for the external fermion legs. One concludes that only the three remaining operators in Eq. (33) renormalize the SM dipole operators and we have verified this by an explicit calculation. These are the only dimension-six JJ-operator of the SM that renormalize loop-operators. Some of these exceptions were also pointed out in [4]. Our analysis completes the list of these exceptions and helps to understand the reason behind them. From the analysis of the $U(1)$ case, we can also explain the presence of $y_u y_d$ in the renormalization of $\mathcal{O}_{yy}$ from $\mathcal{O}_{4f}$ [10].

It is obvious that no operator other than itself renormalizes $\mathcal{O}_{3F_+}$: no adequate one-loop 1PI diagram can be constructed from other dimension-six operators, since they have too many fermion and/or scalar fields. Nevertheless $\mathcal{O}_{3F_+}$ can in principle renormalize JJ-operators. Let us consider, for concreteness, the case of $\mathcal{O}_{3F_+}$ made of $SU(2)_L$ field-strengths. SM-loop contributions from $\mathcal{O}_{3F_+}$ can generate the JJ-operators $(D_\nu F^{a\,\mu\nu})^2$ and $J_\mu^a D_\nu F^{a\,\mu\nu}$ (where J_μ^a is the weak current), and indeed these contributions have been found to be nonzero by an explicit calculation [5]. By using the EOM, $D_\nu F^{a\,\mu\nu} = g J^{a\,\mu}$, we can reduce these two operators to $(J_\mu^a)^2$. Surprisingly, one finds that the total contribution from $\mathcal{O}_{3F_+}$ to $(J_\mu^a)^2$ adds up to zero [5, 10]. We can derive this result as follows. From inspection of Eq. (42), one can see that the superpartners cannot give any one-loop contribution to these JJ-operators. Therefore the result must be the same in the SM EFT as in the corresponding ESFT. Looking at the Higgs component of $(J_\mu^a)^2 = (H^\dagger \sigma^a \overset{\leftrightarrow}{D}_\mu H)^2 + \cdots$, we see that this operator must arise from the ESFT term $\int (\mathcal{D}^\alpha \eta \mathcal{J}_\alpha^a + h.c.)^2$ where $\mathcal{J}_\alpha^a = H^\dagger \sigma^a \mathcal{D}_\alpha H$. This super-operator, however, cannot be generated from the super-operator in Eq. (31), as this latter appears in the ESFT with a different number of SSB-spurions, $\eta^\dagger$. This proves that $\mathcal{O}_{3F_+}$ cannot generate JJ-operators with Higgs. Now, if current-current super-operators with H are not generated, those with Q cannot be generated either, since in the ESFT the $SU(2)_L$ vector does not distinguish between different $SU(2)_L$-doublet chiral superfields. This completes the proof that $\mathcal{O}_{3F_+}$ does not renormalize any JJ-operator in the basis of Table 1.

Concerning the non-renormalization of JJ-operators by loop-operators, the last new case left to discuss is that of $\mathcal{O}_-$ by $\mathcal{O}_{FF}$. The SSB-spurion analysis forbids such renormalization in the ESFT and the result can be extended to the SM EFT as no superpartner-loop contributes either (see Eq. (40) in the Appendix).

At energies below the electroweak scale, we can integrate out W, Z, Higgs and top, and
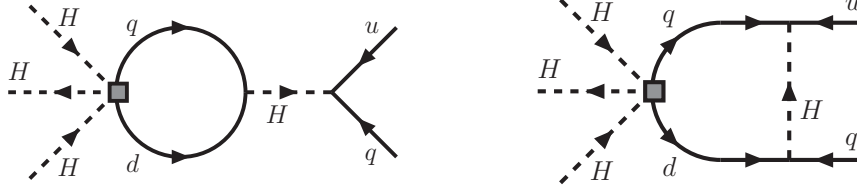
Figure 3: *Non-holomorphic mixing between $\mathcal{O}_{y_u}$ and $\mathcal{O}_{y_d}$.*

write an EFT with only light quarks and leptons, photon and gluons. This EFT contains four-fermion operators of type $\mathcal{O}_{4f}$, generated at tree-level, that are JJ-operators, and other operators of dipole-type that are loop-operators. Following the above approach we can prove that these four-fermion operators cannot renormalize the dipole-type operators, and this is exactly what is found in explicit calculations [7].

3.2 Holomorphy of the anomalous dimensions

It has been recently shown in Ref. [10], based on explicit calculations, that the anomalous dimension matrix respects, to a large extent, holomorphy. Here we would like to show how to derive some of these properties using our ESFT approach. In particular, we will derive that, with the exception of one case, the one-loop anomalous dimensions of the complex Wilson-coefficients $c_i = \{c_{3F_+}, c_{FF_+}, c_D, c_y, c_{yy}, c_R^{ud}\}$ do not depend on their complex-conjugates c_j^*:

$$\frac{\partial \gamma_{c_i}}{\partial c_j^*} = 0 \,. \tag{34}$$

We start by showing when Eq. (34) is satisfied just by simple inspection of the SM diagrams. For example, it is easy to realize that holomorphy must be respected in contributions from dimension-six operators in which fermions with a given chirality, e.g., f_α or $f_\alpha f_\beta'$, are kept as external legs; indeed, the corresponding Hermitian-conjugate operator can only contribute to operators with fermions in the opposite chirality. Interestingly, we can extend the same argument to operators with field-strengths if we write the loop-operators as

$$\mathcal{O}_{3F_+} = -\frac{1}{4}\text{tr}\,\mathcal{F}_\alpha{}^\beta \mathcal{F}_\beta{}^\lambda \mathcal{F}_\lambda{}^\alpha\,, \quad \mathcal{O}_{FF_+} = \frac{1}{4}H^\dagger t^a t^b H (\mathcal{F}^a)_{\alpha\beta}(\mathcal{F}^b)^{\beta\alpha}\,, \quad \mathcal{O}_D = H^\dagger f_\alpha (\mathcal{F}^a)^{\alpha\beta} t^a f_\beta'\,, \tag{35}$$

where we have defined $\mathcal{F}^{\alpha\beta} \equiv (F_{\mu\nu}^a t^a \sigma^{\mu\nu})^{\alpha\beta}$ that transforms as a $(\mathbf{1}, \mathbf{0})$ under the Lorentz group, and write the Hermitian-conjugate of Eq. (35) with $\mathcal{F}^{\dot\alpha\dot\beta}$, a $(\mathbf{0}, \mathbf{1})$ under the Lorentz group, as for example, $\mathcal{O}_{3F+}^\dagger = \mathcal{O}_{3F-} = -\frac{1}{4}\text{tr}\,\mathcal{F}_{\dot\alpha}{}^{\dot\beta}\mathcal{F}_{\dot\beta}{}^{\dot\lambda}\mathcal{F}_{\dot\lambda}{}^{\dot\alpha}$. From Eq. (35) it is clear that any diagram with an external $\mathcal{F}_{\alpha\beta}$ respects holomorphy, as it can only generate the operators of Eq. (35) and not their Hermitian conjugates. One-loop contributions from $\mathcal{O}_{FF_+}$ in which $H^\dagger t^a t^b H$ is kept among the external fields, however, do not necessarily respect holomorphy. An explicit calculation is needed, and while contributions to $\mathcal{O}_{FF_+}$ vanish by the reasoning
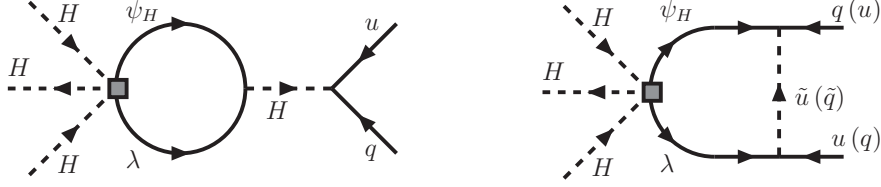
15

Figure 4: *Contributions from $\mathcal{O}_{FF+}^{\dagger}$ to $\mathcal{O}_y$.*

given in [1], contributions to $\mathcal{O}_y$ are found to be holomorphic. Furthermore, potentially non-holomorphic contributions can also be identified by tracking the fermion chirality. For instance, in the $\mathcal{O}_{y_u} \leftrightarrow \mathcal{O}_{y_d}$ operator mixing, the fermion chirality flips due to contributions proportional to $y_u y_d$, as shown in Fig. 3. These contributions have the same loop structure as the contributions of Fig. 2.

Following our previous supersymmetric approach, it is quite simple to check whether or not loop contributions are holomorphic. In the ESFT, holomorphy is trivially respected as super-operators with an $\eta^{\dagger}$-spurion renormalize among themselves and cannot induce the Hermitian-conjugate super-operators since those contain an η, and vice versa. The only exception to this rule can arise from supersymmetry-breaking, which requires the combination $y_u y_d$. We find that the only possible contributions $\propto y_u y_d$ are the supersymmetric versions of the diagrams in Fig. 3, that generate, as we said, a mixing between $\mathcal{O}_{y_u}$ and $\mathcal{O}_{y_d}$.

Given this holomorphy property of the ESFT, we still must check whether at the field-component level this property is preserved, and assure that it is not due to cancellations between SM contributions and those of superpartners. This can be easily done by looking at either one or the other loop (whenever holomorphy in the ESFT implies that the sum is zero). In this way, as one of the loops (either the SM or the superpartner one) involves fermions, we can always relate holomorphy to fermion chirality. We find that the only potentially non-holomorphic contributions come from the diagrams in Fig. 4, which correspond to the superpartner one-loop contributions to $\mathcal{O}_y$ arising from $\mathcal{O}_{FF_+}^{\dagger}$ (illustrated for the $U(1)_Y$ case). These contributions are induced by the vertex $|H|^2 \lambda^{\dagger} \bar{\sigma}^{\mu} \partial_{\mu} \lambda \sim |H|^2 H \lambda^{\dagger} \psi_H^{\dagger}$ of Eq. (11), where we have used the EOM of λ (and replaced the $U(1)$ ϕ and ψ by the SM Higgs and Higgsino). An explicit calculation of these diagrams shows however that they cancel each other, the sum being proportional to $Y_H + Y_q + Y_u = 0$. A similar cancellation occurs in the $SU(2)$ case. This could have been anticipated as there are no such diagrams for the $SU(3)_c$ case, and this cancellation should be independent of the gauge group.

We conclude that the only non-holomorphic anomalous dimension is in the $\mathcal{O}_{y_u} \leftrightarrow \mathcal{O}_{y_d}$ mixing, and its origin can be tracked to the supersymmetry-breaking combination $y_u y_d$.

4 Implications for the QCD Chiral Lagrangian

We can extend the above analysis also to the QCD Chiral Lagrangian [6]. At $O(p^2)$, we have

$$\mathcal{L}_2 = \frac{f_\pi^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle. \tag{36}$$

This is an operator that can be embedded in a D-term as $\int d^4\theta \langle \mathcal{U}^\dagger \mathcal{U}\rangle$, where U and its superpartners are contained in $\mathcal{U} \equiv e^{i\Phi}$, with Φ being a chiral superfield. At $O(p^4)$, the QCD Chiral Lagrangian is usually parametrized by the L_i coefficients [6] in a basis with operators that are linear combinations of JJ-operators and loop-operators. These are

$$\mathcal{L}_4 = -iL_9\langle F_R^{\mu\nu}D_\mu U D_\nu U^\dagger + F_L^{\mu\nu}D_\mu U^\dagger D_\nu U\rangle + L_{10}\langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\rangle + \cdots. \tag{37}$$

A more convenient basis is however

$$\mathcal{L}_4 = iL_{JJ}\langle D_\mu F_L^{\mu\nu}(U^\dagger \overset{\leftrightarrow}{D}_\nu U) + (U\overset{\leftrightarrow}{D}_\nu U^\dagger)D_\mu F_R^{\mu\nu}\rangle + L_{loop}\langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\rangle + \cdots, \tag{38}$$

where $L_{JJ} = L_9/2$ and $L_{loop} = L_9 + L_{10}$. It is easy to see that the first operator of Eq. (38) is a JJ-operator, while the second is a loop-operator. This latter can only be embedded in a θ^2-term of a super-operator (i.e., $\langle \mathcal{U}^\dagger \mathcal{W}_R^\alpha \mathcal{U} \mathcal{W}_{\alpha L}\rangle$), and therefore it cannot be renormalized by the operator in Eq. (36) in the supersymmetric limit. As contributions from superpartner loops can be easily shown to vanish, we can deduce that Eq. (36) cannot renormalize L_{loop} at the one-loop level. This is indeed what one finds from the explicit calculation [6]: $\gamma_{L_{loop}} = \gamma_{L_9} + \gamma_{L_{10}} = 1/4 - 1/4 = 0$.

5 Conclusions

In EFTs with higher-dimensional operators the one-loop anomalous dimension matrix has plenty of vanishing entries apparently not forbidden by the symmetries of the theory. In this paper we have shown that the reason behind these zeros is the different Lorentz structure of the operators that does not allow them to mix at the one-loop level. We have proposed a way to understand the pattern underlying these zeros based on classifying the dimension-six operators in JJ- and loop-operators and also according to their embedding in super-operators (see Table 1 for the SM EFT). We have seen that all loop-operators break supersymmetry, [10] while we have two classes of JJ-operators, those that can be supersymetrized and those that cannot. This classification is very useful to obtain non-renormalization results based in a pure SSB-spurion analysis in superfields, that can be extended to non-supersymmetric EFTs. In terms of component fields, the crucial point is that the vanishing of the anomalous-dimensions does not arise from cancellations between bosons and fermions but from the underlying Lorentz structure of the operators.

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[10]This is not true in general. For instance, in models with two Higgses of opposite hypercharge, H and $\bar{H}$, one can have the supersymmetric loop-operator $\int d^2\theta H\bar{H}\mathcal{W}^\alpha\mathcal{W}_\alpha$. Notice that in such a case supersymmetry also protects that operator from being renormalized in the ESFT.
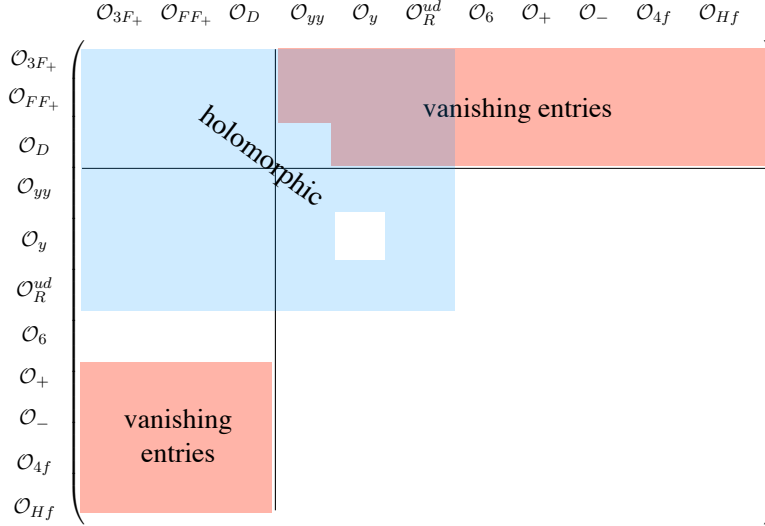
Figure 5: *Anomalous-dimension matrix of the dimension-six SM operators showing which entries (red-shaded) vanish following the present analysis. We also show the entries (light blue-shaded) that respect the holomorphic condition Eq. (34). Solid lines separate loop-operators from JJ-operators.*

We have presented how this approach works in a simple $U(1)$ model with a scalar and fermions, and have explained how to extend this to SM EFTs and the QCD Chiral Langrangian. The main results are summarized in Fig. 5 that shows which entries of the anomalous-dimension matrix for the SM EFTs operators we have proved to vanish. We have also explained how to check if holomorphy is respected by the complex Wilson-coefficients, a property that is fulfilled in most cases, as Fig. 5 shows. Our approach can be generalized to other theories as well as to the analysis of other anomalous dimensions, a work that we leave for a further publication.

Note added

We have corrected the holomorphic structure of the anomalous dimension matrix with respect to the previous version of this paper (see Fig. 5) profitting from the analysis of Ref. [14].

Acknowledgments

Appendix

In this Appendix we show the expansion in component fields of some of the super-operators discussed in the text. We work in the Wess-Zumino gauge. In particular, for the $U(1)$ case, we show the supersymmetry-preserving super-operator

$$
\int d^4\theta \left(\Phi^\dagger e^{V_\Phi}\Phi\right)\left(Q^\dagger e^{V_Q}Q\right) = -|\tilde{q}|^2|D_\mu\phi|^2 - |\phi|^2|D_\mu\tilde{q}|^2 - \frac{1}{2}\partial_\mu|\tilde{q}|^2\partial^\mu|\phi|^2 + \frac{i}{2}|\tilde{q}|^2(\psi^\dagger\bar{\sigma}^\mu\overset{\leftrightarrow}{D}_\mu\psi)
$$

$$
+\frac{i}{2}|\phi|^2(q^\dagger\bar{\sigma}^\mu\overset{\leftrightarrow}{D}_\mu q) + \frac{1}{2}\left[(\psi^\dagger\bar{\sigma}^\mu q)(i\tilde{q}^*\overset{\leftrightarrow}{D}_\mu\phi) + h.c.\right] + \frac{1}{2}\left[\phi\tilde{q}^*(i\psi^\dagger\bar{\sigma}^\mu\overset{\leftrightarrow}{D}_\mu q) + h.c.\right]
$$

$$
-\frac{1}{2}\left(i\phi^*\overset{\leftrightarrow}{D}_\mu\phi - \psi^\dagger\bar{\sigma}^\mu\psi\right)\left(i\tilde{q}^*\overset{\leftrightarrow}{D}_\mu\tilde{q} - q^\dagger\bar{\sigma}^\mu q\right)
$$

$$
-\left[(\psi^\dagger q^\dagger)\phi F_q + (\psi^\dagger q^\dagger)\tilde{q}F_\phi - \phi F_\phi^*\tilde{q}^*F_q + h.c.\right] + |\phi|^2|F_q|^2 + |\tilde{q}|^2|F_\phi|^2
$$

$$
-\sqrt{2}g(Q_\phi + Q_q)\left[|\phi|^2\tilde{q}\lambda^\dagger q^\dagger - |\tilde{q}|^2\phi\lambda^\dagger\psi^\dagger + h.c.\right] + g(Q_\phi + Q_q)|\phi|^2|\tilde{q}|^2D \,, \tag{39}
$$

where boundary terms have been dropped out in integration by parts rearrangements. The fields are embedded in the super-multiplets as $\Phi \sim \{\phi, \psi, F_\phi\}$, $Q \sim \{\tilde{q}, q, F_q\}$ and $V \sim \{\lambda, A_\mu, D\}$. The D and $F_{q,\phi}$ auxiliary fields are irrelevant in the discussion of the renormalization of loop-operators by JJ-operators because they are necessarily involved in vertices with too many scalar and/or fermion fields.

The loop-super-operators for the $U(1)$ case are given by

$$
\int d^4\theta\,\eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)\mathcal{W}^\alpha\mathcal{W}_\alpha = -\frac{1}{2}\mathcal{O}_{FF_+} + |\phi^2|\left(D^2 + 2i\lambda\sigma^\mu\partial_\mu\lambda^\dagger\right)
$$

$$
-\frac{1}{\sqrt{2}}\phi^*\lambda\sigma^{\mu\nu}\psi F_{\mu\nu} - \sqrt{2}\phi^*\psi\lambda D + \lambda\lambda\phi^*F_\phi \,, \tag{40}
$$

$$
\int d^4\theta\,\eta^\dagger\Phi(Q\overset{\leftrightarrow}{\mathcal{D}}_\alpha U)\mathcal{W}^\alpha = -\mathcal{O}_D + \Big\{ -\sqrt{2}i\phi\tilde{q}(u\sigma^\mu\partial_\mu\lambda^\dagger) + 2\tilde{u}\phi F_q D + 2\sqrt{2}\phi F_u\lambda q
$$

$$
+\sqrt{2}\tilde{u}F_\phi\lambda q + \sqrt{2}F_u\tilde{q}\psi\lambda + D\tilde{q}\psi u
$$

$$
-\frac{1}{2}\tilde{q}\psi\sigma_{\mu\nu}uF^{\mu\nu} + \sqrt{2}(\psi q)(\lambda u) - (u \leftrightarrow q)\Big\} \,. \tag{41}
$$

For the non-Abelian case, there is also the loop-super-operator

$$
\int d^4\theta\,\eta^\dagger\text{tr}[\mathcal{D}^\beta\mathcal{W}^\alpha\mathcal{W}_\beta\mathcal{W}_\alpha] = \frac{1}{4}\mathcal{O}_{3F^+} + i\,\text{tr}\left[\frac{1}{2}F_{\mu\nu}\lambda\sigma^{\mu\nu}(\sigma^\gamma\partial_\gamma\lambda^\dagger) + \lambda\sigma^\mu\partial_\mu\lambda^\dagger D\right] \,. \tag{42}
$$

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