UCRHEP–T270

A short course in effective Lagrangians. *

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Abstract

These lectures provide an introduction to effective theories concentrating on the basic ideas and providing some simple applications

I. INTRODUCTION.

When studying a physical system it is often the case that there is not enough information to provide a fundamental description of some of its properties. In such cases one must parameterize the corresponding effects by introducing new interactions with coefficients to be determined phenomenologically. Experimental limits or measurement of these parameters then (hopefully) provides the information needed to provide a more satisfactory description.

A standard procedure for doing this is to first determine the dynamical degrees of freedom involved and the symmetries obeyed, and then construct the most general Lagrangian, the *effective Lagrangian* for these degrees of freedom which respects the required symmetries. The method is straightforward, quite general and, most importantly, *it works!*

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*Lectures delivered at the *VII Mexican Workshop on Particles and Fields*, Merida, Yucatan, Mexico, 10-17 November, 1999.

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In following this approach one must be wary of several facts. Fist it is clear that the relevant degrees of freedom can change with scale (e.g. mesons are a good description of low-energy QCD, but at higher energies one should use quarks and gluons); in addition, physics at different scales may respect different symmetries (e.g. mass conservation is violated at sufficiently high energies). It follows that the effective Lagrangian formalism is in general applicable only for a limited range of scales. It is often the case (but no always!) that there is a scale Λ so that the results obtained using an effective Lagrangian are invalid for energies above Λ.

The formalism has two potentially serious drawbacks. First, effective Lagrangian has an infinite number of terms suggesting a lack of predictability. Second, even though the model has an UV cutoff Λ and will not suffer from actual divergences, simple calculations show that is is a possible for this type of theories to generating radiative corrections that grow with Λ, becoming increasingly important for higher and higher order graphs. Either of these problems can render this approach useless. It is also necessary verify that the model is unitary.

I will discuss below how these problems are solved, an provide several applications of the formalism. The aim is to give a flair of the versatility of the approach, not to provide an exhaustive review of all known applications.

II. FAMILIAR EXAMPLES

A. Euler-Heisenberg effective Lagrangian

This Lagrangian summarizes QED at low energies (below the electron mass) [1]. At these energies only photons appear in real processes and the effective Lagrangian will be then constructed using the photon field A_μ, and will satisfy a $U(1)$ gauge and Lorenz invariances. Thus it can be constructed in terms of the field strength $F_{\mu\nu}$ or the loop variables $\mathcal{A}(\Gamma) =$

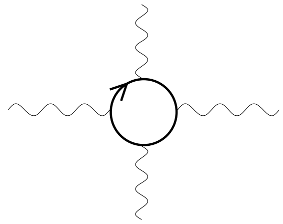

FIG. 1. Graph generating the leading terms in the Euler-Heisenberg effective Lagrangian

$\oint_\Gamma A\cdot dx$. The latter are non-local, so that a local description would involve only F, namely [1]

$$\begin{aligned}\mathcal{L}_{\rm eff} &= \mathcal{L}_{\rm eff}(F)\\ &= aF^2 + bF^4 + c(F\tilde{F})^2 + dF^2(F\tilde{F})\cdots \end{aligned} \tag{1}$$

One can arbitrarily normalize the fields and so choose $a = -1/4$. The constants $b,\ c$ and d have units of mass^{-2}.

Note that the term $\propto d$ violates CP. Though we know QED respects C and P, it is possible for other interactions to violate these symmetries, there is nothing in the discussion above that disallows such terms and, in fact, weak effects will generate them. For this system we are in a privileged position for we know the underlying physics, and so we can calculate $b,\ c,\ d, \ldots$. The leading effects come form QED which yields $b, c \sim 1/(4\pi m_e)^2$ at 1 loop [1]. The parameters b and c summarize all the leading *virtual* electron effects. (see Fig. II A). Forgetting about this underlying structure we could have simply *defined* a scale M and taken $b,\ c \sim 1/M^2$ (so that $M = 4\pi m_e$), and while this is perfectly viable, M is not relevant phenomenologically speaking as it does not corresponds of a physical scale. In order to extract information about the physics underlying the effective Lagrangian from a measurement of b and c we must be able to at least estimate the relation between these constants and the underlying scales.

In addition we also know that $d \sim \xi/(4\pi v)$ with $v \sim 246$GeV and ξ is a very small constant proportional to the Jarlskog determinant [2]. The effective Lagrangian can hold

[1] There is no $F\tilde{F}$ terms since it is a total derivative.

terms with radically different scales and limits on some constants cannot, in general, translate to others. In this case the terms are characterized by different CP transformation properties, and it is often the case that such global symmetries are useful in differentiating terms in the effective Lagrangian. The point being that a term violating a given global symmetry at scale Λ will generate all terms in the effective Lagrangian with the same symmetry properties through radiative corrections. The caveat in the argument being that the underlying theory might have some additional symmetries not apparent at low energies which might further segregate interactions and so provide different scales for operators with the same properties under all low energy symmetries.

When calculating with the effective Lagrangian the effects produced by the new terms proportional to b, c are suppressed by a factor $\sim (E/4\pi m_e)^2$, where E is the typical energy on the process and $E \ll m_e$. Thus the effects of these terms are tiny, yet they are noticeable because they generate a *new* effect: $\gamma - \gamma$ scattering.

B. (Standard) Superconductivity

This is a brief summary of the very nice treatment provided by Polchinski [3]. The system under consideration has the electron field ψ as its only dynamical variable (the phonons are assumed to have been integrated out, generating a series of electron self-interactions), it respects $U(1)$ electromagnetic gauge invariance, as well as Galilean invariance and Fermion number conservation.

Assuming a local description, the first few terms in the effective Lagrangian expansion are (neglecting those containing photons for simplicity)

$$\mathcal{L}_{\rm eff} = \int_k \psi^*_{\mathbf{k}} \left[i\partial_t - e_{\mathbf{k}} + \mu \right] \psi_{\mathbf{k}} + \int \psi^*_{\mathbf{k}} \psi_{\mathbf{l}} \psi_{\mathbf{q}} \psi^*_{\mathbf{p}} \delta(\mathbf{k} - \mathbf{l} - \mathbf{q} + \mathbf{p}) V_{\mathbf{klq}} + \cdots \tag{2}$$

In this equation the relation $e_{\mathbf{k}} = \mu$ determines the Fermi surface, while $V \sim \frac{(\text{electron-photon coupling})^2}{(\text{phonon mass})^2}$ summarizes the virtual phonon effects. In order to determine the importance of the various terms we need the dimensions of the field ψ. A vector $\mathbf{k}$ lies on the Fermi Surface (FS) if $e_{\mathbf{k}} = \mu$, if $\mathbf{p}$ is near the FS one can write $\mathbf{p} = \mathbf{k} + \ell\hat{\mathbf{n}}$ (with $e_{\mathbf{k}} = \mu$). Scaling towards the FS implies $\ell \to s\ell$ with $s \to 0$. Then assuming $\psi \to s^d\psi$ the quadratic terms in the action will be scale invariant provided $d = -1/2$. The quartic terms in the action then scales as s and becomes negligible near the FS *except* when the pairing condition $\mathbf{q} + \mathbf{l} = 0$ is obeyed. In this case the quartic term scales as s^0 and cannot be ignored. In fact this term determines the most interesting behavior of the system at low temperatures (see [3] for full details).

C. Electroweak interactions

Again I will follow the general recipe. I will concentrate only on the (low energy) interactions involving lepton fields, which are then the degrees of freedom. Since I assume the energy to be well below the Fermi scale, the only relevant symmetries are $U(1)$ gauge and Lorenz invariances. In addition there is the question whether the heavy physics will respect the discrete symmetries C, P or CP; using perfect hindsight I will retain terms that violate these symmetries

Assuming a local description I have [1]

$$\mathcal{L}_{\text{eff}} = \sum \bar{\psi}_i(i\not{D} - m_i)\psi_i + \sum f_{ijkl}\left(\bar{\psi}_i\Gamma^a\psi_j\right)\left(\bar{\psi}_k\Gamma_a\psi_l\right) + \cdots \tag{3}$$

where the ellipsis indicate terms containing operators of higher dimension, or those involving the electromagnetic field. The matrices Γ are to be chosen among the 16 independent basis $\Gamma^a = \{1, \gamma_\mu, \sigma_{\mu\nu}, \gamma_\mu\gamma_5, \gamma_5\}$

The coefficients for the first two terms are be fixed by normalization requirements. While a SM calculation gives $f \sim g^2/m_W^2 = 1/v^2$ ($v \simeq 246$GeV) and is generated by tree-level graphs (see Fig. II C) because of this the scale $1/\sqrt{f}$ is, in fact, the scale of the heavy physics and so the model is applicable at energies swell below v. The four fermion interactions summarize the leading virtual gauge boson effects. The contributions of the four-fermion operators to processes with typical energy E are suppressed by a factor E^2/v^2. These can be observed (or bounded) despite the $E \ll v$ condition because they generate *new* effects: C and P (and some of them chirality) violation.

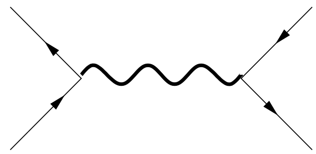

FIG. 2. Standard model processes generating four fermion interactions at low energies (e.g.. Bhaba scattering)

D. Strong interactions at low energies

In this case we are interested in the description of the interactions among the lightest hadrons, the meson multiplet. The most convenient parameterization of these degrees of freedom is in terms of a unitary field [9] U such that $U = \exp(\lambda_a \pi^a / F)$ where π^a denote the eight meson fields, λ^a the Gell-Mann matrices and F is a constant (related to the pion decay constant). The symmetries obeyed by the system are chiral $SU(3)_L \times SU(3)_R$, Lorenz invariance, C and P.

With these constraints the effective Lagrangian takes the form

$$\mathcal{L}_{\rm eff} = a \, \mathrm{tr} \partial U^\dagger \cdot \partial U + \left[b \, \mathrm{tr} \partial_\mu U^\dagger \partial_\nu U \partial^\mu U^\dagger \partial^\nu U + \cdots \right] + \cdots \tag{4}$$

I can set $a \sim F^2$ by properly normalizing the fields. In this case the leading term in the effective Lagrangian will determine all (leading) low-energy pion interactions in terms of the single constant F. The effects form the higher-order terms have been measured and the data requires $b \sim 1/(4\pi)^2$. This result is also predicted by the consistency of this approach which requires that radiative corrections to a, b, etc. should be at most of the same size as their tree-level values.

III. BASIC IDEAS ON THE APPLICABILITY OF THE FORMALISM

Being a model with intrinsic an cutoff there are no actual ultraviolet divergences in most effective Lagrangian computations. Still there are interesting renormalizability issues that arise when doing effective Lagrangian loop computations.

Imagine doing a loop calculation including some vertices terms of (mass) dimension higher than the dimension of space-time. These must have coefficients with dimensions of mass to some negative power. The loop integrations will produce in general terms growing with Λ the UV cutoff which are polynomials in the external momenta [2] and will preserve the symmetries of the model [4]. Hence these terms which may *grow* with Λ correspond to vertices appearing in the most general effective Lagrangian and can be absorbed in a renormalization of the corresponding coefficients. They have no observable effects (though they can be used in naturality arguments [5].

Effective theories will also be unitary *provided* one stays within the limits of their applicability. Should one exceed them new channels will open (corresponding to the production of the heavy excitations) and unitarity violating effects will occur. This is *not* produced by real unitarity violating interactions, but due to our using the model beyond its range of applicability (e.g. it the typical energy of the process under consideration reaches of exceeds Λ). One can, of course, *extend* the model, but this necessarily introduces *ad-hoc* elements and will dilute the generality gained using effective theories.

For example consider WWZ interactions with an effective Lagrangian of the form

$$\mathcal{L}_{\rm eff} = \lambda(p,k) W_{\mu\nu}(k) W^{\nu\rho}(p) Z_\rho{}^\mu(-p-k) + \cdots ; \tag{5}$$

(where $V_{\alpha\beta} = \partial_\alpha V_\beta - \partial_\beta V_\alpha$) One can then choose λ to insure unitarity is preserved (at least in some processes), for example [6]

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[2]Since a graph can be rendered convergent by taking sufficient number of derivatives with respect to the external momenta.

$$\lambda(p,k) = \frac{\lambda_0}{(p\cdot k + \Lambda)^n} \tag{6}$$

which, for n sufficiently large insures that the cross section for the reaction $e^+e^- \to Z \to WW$ is unitary, since it behaves as s^{2-2n} for a CM energy= $s \gg \Lambda^2$. But the very same effective vertex also modifies other reactions such as, for example $u\bar{d} \to W \to ZW$ where the cross section now has a factor $(s-\Lambda^2)^{-2n}$ and will exhibit resonant behavior if $s \sim \Lambda^2$. If one requires $s \ll \Lambda^2$ (as required by the consistency of the formalism) there are neither unitarity violations nor resonance effects. If, however, one uses the above Ansatz to extend the range of applicability to $s \sim \Lambda^2$ and beyond then very clear resonances should be observed in hadron colliders. Given that these have not been observed one *must* use for Λ a value significantly larger than the average CM energy for the hard W pair production cross section.

IV. USING EFFECTIVE LAGRANGIANS

Effective Lagrangians provide an efficient way of summarizing some (perhaps very complex) interactions. The idea is simply to include all the effective vertices produces by those excitations which are not directly observed.

For example given a real scalar field ϕ and assume that all Fourier components above a scale Λ are not directly observable (i.e. the available energies lie all below Λ), then the effective Lagrangian is obtained by integrating over the variables observable at energies $\ge \Lambda$; writing $\phi = \phi_0 + \phi_1$, with

$$\phi_0(\mathbf{k}):\ |\mathbf{k}| < \Lambda \qquad \phi_1(\mathbf{k}):\ \Lambda \le |\mathbf{k}| < \Lambda_1 \tag{7}$$

then by definition

$$e^{iS_{\rm eff}} = \int [d\phi_1] e^{iS(\phi_0,\phi_1)}, \qquad S_{\rm eff} = \int d^n x \mathcal{L}_{\rm eff} \tag{8}$$

where $\mathcal{L}_{\rm eff}$ is obtained by expanding $S_{\rm eff}$ in powers of Λ which gives an infinite tower of local operators.

Another common situation where effective Lagrangians appear occurs when some heavy excitations are integrated out. This can be illustrated by the following toy model [3]

$$S = \int d^n x \left[\bar\psi (i \not\partial - m)\psi + \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\Lambda^2 \phi_1^2 + f\phi\bar\psi\psi \right] \tag{9}$$

where ϕ is heavy. A simple calculation gives

$$S_{\rm eff} = \int d^n x \left[\bar\psi (i \not\partial - m)\psi + \frac{1}{2} f^2 \bar\psi\psi \frac{1}{\Box + \Lambda^2} \bar\psi\psi \right] \tag{10}$$

and

$$\mathcal{L}_{\rm eff} = \bar\psi (i \not\partial - m)\psi + \frac{f^2}{2\Lambda^2} \sum_{l=1}^{\infty} \bar\psi\psi \left(\frac{\Box}{\Lambda^2} \right)^n \bar\psi\psi \tag{11}$$

Note that terms with large number of derivatives will be suppressed by a large power of the small factor (E/Λ), if we are interested in energies $E \sim \Lambda$ the *whole* infinite set of vertices must be included in order to reproduce the ϕ pole.

A. How to parameterize ignorance

If one knows the theory we can, in principle, calculate $\mathcal{L}_{\rm eff}$ (or do a full calculation). Yet there are many cases where the underlying theory is not known. In these cases an effective theory if obtained by writing *all* possible interactions among the light excitations. The model then has an infinite number of terms each with an unknown parameter, and these constants then parameterize *all* possible underlying theories. The terms which dominate are those usually called renormalizable (or, equivalently, marginal or relevant). The other terms are called non-renormalizable, or irrelevant, since their effects become smaller as the energy decreases

This recipe for writing effective theories must be supplemented with some symmetry restrictions. The most important being that the all the terms in the effective Lagrangian must

[3] I'm cheating in order to get a closed form for the effective action, a more realistic model should include a term $\propto \phi_1^4$

respect the local gauge invariance of the low-energy physics (more technically, the one respected by the renormalizable terms in the effective action) [7]. The reason is that the presence of a gauge variant term will generate *all* gauge variant interactions thorough renormalization group evolution.

a. Gauge invariantizing Using a simple argument it is possible to turn any theory into a gauge theory [8] and so it appears that the requirement of gauge invariance is empty. That this is not the case is explained here. I first describe the trick which grafts gauge invariance onto a theory and then discuss the implications.

Consider an arbitrary theory with matter fields (spin 0 and 1/2) and vector fields V_μ^n, $n = 1, \ldots N$. Then

- Choose a (gauge) group G with N generators $\{T^n\}$. Define a covariant derivative $D_\mu = \partial_\mu + V_\mu^n T^n$ and *assume* that the V_μ^n are gauge fields.

- Invent a unitary field U transforming according to the fundamental representation of G and construct the gauge invariant composite fields

$$\mathcal{V}_\mu^n = -\mathrm{tr} T^n U^\dagger D_\mu U \tag{12}$$

  Taking $\mathrm{tr} T^n T^m = -\delta_{nm}$, it is easy to see that in the unitary gauge $U = 1$, $\mathcal{V}_\mu^n = V_\mu^n$.

Thus if simply replace $V \to \mathcal{V}$ in the original theory we get a gauge theory. Does this mean that gauge invariance irrelevant since it can be added at will? In my opinion this is not the case.

In the above process *all matter fields are assumed gauge singlets* (none are minimally coupled to the gauge fields).In the case of the standard model , for example, the universal coupling of fermions to the gauge bosons would be accidental in this approach. In order to recover the full predictive power commonly associated with gauge theories, the matter fields must transform non-trivially under G which can be done only if there are strong correlations among some of the couplings. It is *not* trivial to say that the standard model group is

$SU(3)\times SU(2)\times U(1)$ with left-handed quarks transforming as $(3,2,1/6)$, left-handed leptons as $(1,2,-1/2)$, etc., as opposed to a $U(1)^{12}$ with all fermions transforming as singlets [10].

B. How to estimate ignorance

A problem which I have not addressed so far is the fact that effective theories have an infinite number of coefficients, with the (possible) problem or requiring an infinite number of data points in order to make any predictions. On the other hand, for example, if this is the case why is it that the Fermi theory of the weak interactions is so successful?

The answer to this question lies in the fact that not all coefficients are created equal, there is a *hierarchy* [9,10]. As a result, given any desired level of accuracy, only a finite number of terms need to be included. Moreover, even though the effective Lagrangian coefficients cannot be calculated without knowing the underlying theory, they can still be *bounded* using but a minimal set of assumptions about the heavy interactions. It is then also possible to estimate the errors in neglecting all but the finite number of terms used.

As an example consider the standard model at low energies and calculate two processes: Bhaba cross section and the anomalous magnetic moment of the electron. For Bhaba scattering there is a contribution due the Z-boson exchange (see Fig. II C)

$$e^+e^- \to Z \to e^+e^- \quad \text{generates} \quad \mathcal{O} = \frac{1}{2m_Z^2}\left(\bar e\Gamma\gamma^\mu e\right)\left(\bar e\Gamma\gamma_\mu e\right) \tag{13}$$

where $\Gamma = g_V + g_A\gamma_5$. The coefficient of the effective operator $\mathcal{O}$ is then $\sim$ (coupling/physical mass)$^2 \sim 1/v^2$

The electron anomalous magnetic moment receives contributions from virtual W, Z and H exchanges (see Fig. IV B). The corresponding low-energy operator is

$$\mathcal{O} = \bar e\sigma_{\mu\nu}eF^{\mu\nu} \tag{14}$$

In this case the coefficient $\sim \{\text{coupling}/[4\pi(\text{physical mass})]\}^2 \sim 1/(4\pi v)^2$ [4].

[4] In addition the coefficient is suppressed by a factor of m_e since it violates chirality.

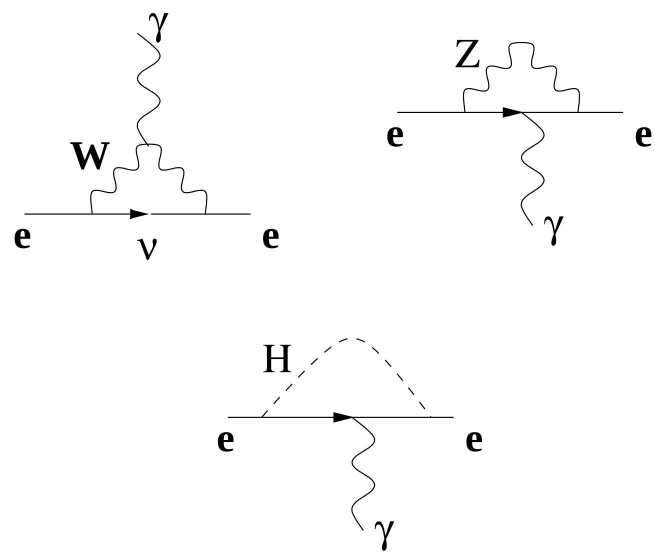

FIG. 3. Weak contributions to the electron anomalous magnetic moment

The point of this exercise is to illustrate the fact that, for weakly coupled theories, loop-generated operators have smaller coefficients than operators generated at tree level. Leading effects are produced by operators which are generated at tree level.

C. Coefficient estimates

In this section I will provide arguments which can be used to estimate (or, at least bound) the coefficients in the effective Lagrangian. These are order of magnitude calculations and might be off by a factor of a few; it is worth noting that no single calculation has provided a significant deviation from these results.

The estimate calculations should be done separately for weakly and strongly interacting theories. I will characterize the first as those where radiative corrections are smaller than the tree-level contributions. Strongly interacting theories will have radiative corrections of the same size at any order [5]

[5] Should the radiative corrections increase with the order of the calculation, it is likely that the dynamic variables being used are not appropriate for the regime where the calculation is being

In this case leading terms in the effective Lagrangian are those which can be generated at tree level by the heavy physics. Thus the dominating effects are produced by operators which have the lowest dimension (leading to the smallest suppression from inverse powers of Λ) and which are tree-level generated (TLG) operators can be determined [11].

When the heavy physics is described by a gauge theory it is possible to obtained all TLG operators [11]. The corresponding vertices fall into 3 categories, symbolically

- vertices with 4 fermions.
- vertices with 2 fermions and k bosons; $k = 2, 3$
- vertices with n bosons; $n = 4, 6$.

A particular theory may not generate one or more of these vertices, the only claim is that there is *a* gauge theory which does.

In the case of the standard model with lepton number conservation the leading operators have dimension 6 [12,11]. Subleading operators are either dimension 8 and their contributions are suppressed by an additional factor $(E/\Lambda)^2$ in processes with typical energy E. Other subleading contributions are suppressed by a loop factor $\sim 1/(4\pi)^2$. Note that it is possible to have situations where the only two effects are produced by either dimension 8 TLG operators or loop generated dimension 6 operators. In this case the former dominates only when $\Lambda > 4\pi E$.

a. Triple gauge bosons The terms in the electroweak effective Lagrangian which describe the interaction of the W and Z bosons generated by some heavy physics underlying the standard model has received considerable attention recently [13]. In terms of the $SU(2)$ and $U(1)$ gauge fields W and B and the scalar doublet ϕ these interactions are

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done.

$$
\begin{aligned}
\mathcal{L}_{\rm eff} &= \frac{1}{\Lambda^2}\left(\alpha_W \mathcal{O}_W + \alpha_{BW}\mathcal{O}_{BW}\right)\\
\mathcal{O}_W &= \epsilon_{IJK} W^I_{\mu\nu} W^{J\nu}{}_\lambda W^{K\lambda\mu}\\
\mathcal{O}_{WB} &= \phi^\dagger \tau^I \phi W^I_{\mu\nu} B^{\mu\nu}
\end{aligned}
\tag{15}
$$

The above arguments inly that there is no TLG operator containing three gauge bosons. This means that all effective contributions to the WWZ and $WW\gamma$ interactions are loop generated, so their coefficients *necessarily* take the form $\prod$(coupling constants)$/(16\pi^2)$. Thus the parameters κ and λ commonly used to parameterize these interactions are of order 5×10^{-3}. *Experiments providing limits significantly above this value provide no information about the heavy physics.*

2. Strongly interacting theories

I will imagine a theory containing scalars and fermions which interact strongly. Gauge couplings are assumed to be small and will be ignored. This calculation is useful for low energy chiral theories but not for low energy QCD [14,15,9].

A generic effective operator in this type of theories takes the form

$$
\mathcal{O}_{abc} \sim \lambda \Lambda^4 \left(\frac{\phi}{\Lambda_\phi}\right)^a \left(\frac{\psi}{\Lambda_\psi}^{3/2}\right)^b \left(\frac{\partial}{\Lambda}\right)^c \tag{16}
$$

Then the condition that these dynamic variables appropriately describe the physics below Λ implies that radiative corrections to the couplings are at most as large as the tree-level values, namely $\delta_{\rm rad}\lambda \le \lambda$. A straightforward estimate (including a factor of $1/(16\pi^2)$ for each loop) shows that this condition is satisfied only if

$$
\Lambda_\psi = \frac{1}{(4\pi)^{2/3}}\Lambda, \quad \Lambda_\phi = \frac{1}{4\pi}\Lambda, \quad \lambda = \frac{1}{16\pi^2} \tag{17}
$$

In terms of $U \sim \exp(\phi/\Lambda_\phi)$, the operators take the form

$$
\mathcal{O}_{abc} = \frac{1}{(4\pi)^{2-b}} \Lambda^{4-c-3b/2} \partial^c U^{a'} \psi^b \tag{18}
$$

In particular the coefficient of the two derivative operators ${\rm tr}\partial U^\dagger \partial U$ is $\propto \Lambda_\phi^2$.

For the case where ϕ represents the interpolating field for the lightest mesons PCAC implies $\Lambda_\phi = f_\pi$ [14,9]. Then

$$\psi^4 \propto \frac{1}{f_\pi^2} \qquad \partial^4 U^4 \propto \frac{1}{16\pi^2} \qquad \psi^2 \partial^2 U^2 \propto \frac{1}{4\pi f_\pi} \tag{19}$$

(note that these are upper bounds). The extensive data on low energy meson reactions can be used to gauge the validity of these predictions, they are indeed satisfied. In particular the $(\partial U)^4$ terms have coefficients $\sim 1/(16\pi^2)$.

For the case of the standard model the field U can be used to provide masses for the W and Z bosons without a physical Higgs being present (the price is that the model breaks down at energies $\sim 4\pi v = 3$TeV). In this case the gauge fields are introduced minimally and it is the term $(DU)^2$ gives a mass to the W and Z which fixes $\Lambda_\phi = v = 246$GeV whence $\Lambda = 3$TeV; as before, the model makes no sense beyond this scale [6] In addition, when the gauge fields are reintroduced, the terms with 4 derivatives will generate triple-vector boson couplings, again leading to the estimates $\lambda, \kappa \sim 5 \times 10^{-3}$ [10].

D. Radiative corrections

Despite the presence of higher-dimensional operators radiative corrections can be calculated in the usual way. As an example imagine calculating the corrections to the cross section for the reaction $e^+e^- \to e^+e^-$ using the standard model with the addition of a 4-fermion interaction

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff}^{\rm SM} + \frac{f}{\Lambda^2}\left(\bar\psi\gamma^\mu\psi\right)\left(\bar\psi\gamma_\mu\psi\right) + \cdots \tag{20}$$

where ψ denotes the electron field.

The calculation is illustrated in Fig. IV D where the loops involving the 4-fermion operator are cut-off at a scale Λ. The SM and new physics (NP) contributions are, symbolically,

[6]Tough it is conceivable that a full non-perturbative calculation would show that the theory cures itself and can be extended beyond this scale, there is no indication that this miracle occurs.

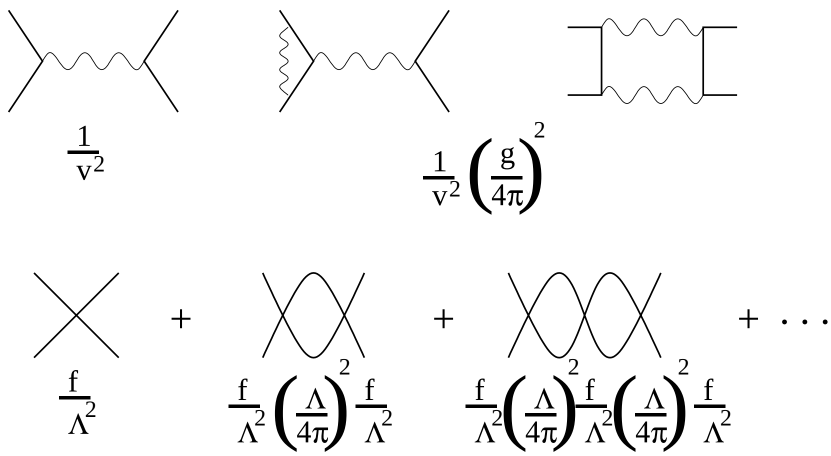

FIG. 4. Radiative corrections to Bhaba scattering in the presence of a 4-fermion interaction

$$\begin{aligned}&\text{SM:}\frac{1}{v^2}\left[1+\frac{g^2}{16\pi^2}+\cdots\right]\\&\text{NP:}\frac{f}{\Lambda^2}\left[1+\frac{f}{16\pi^2}+\cdots\right]\end{aligned} \tag{21}$$

Note that this consistent behavior (that the new physics effects disappear as $\Lambda \to \infty$) results form having the physical scale of new physics Λ in the coefficient of the operator. Had we used f'/v^2 instead of f/Λ^2 the new physics effects would appear to be enormous, and growing with each new loop. It is not that the use of f'/v^2 is wrong, it is only that it is misleading to believe f' can be of order one; it *must* be suppressed by the small factor $(v/\lambda)^2$.

Using these results we see that this reaction is sensitive to Λ provided $f(v/\Lambda)^2 >$sensitivity. If the sensitivity is, say 1% this corresponds to $\Lambda/\sqrt{f} > 2.5$TeV [16].

This perturbative calculation is manageable provided $f < 16\pi^2$, otherwise the underlying physics is strongly coupled. It is still possible in that case to provide estimates of the new physics contributions, though these are less reliable, these estimates imply that $1+f/(4\pi)^2+\cdots \sim 1$ when $f \sim 16\pi^2$.

V. APPLICATIONS TO ELECTROWEAK PHYSICS

With the above results one can determine, for any given process, the leading contributions (as parameterized by the various effective operator coefficients). Using then the coefficient estimates one can provide the expected magnitude of the new physics effects with only Λ as an unknown parameter, and so estimate the sensitivity to the scale of new physics.

It is important to note that this is sometimes a rather involved calculation as all contributing operators must be included. For example, in order to determine the heavy physics effects on the oblique parameters one must calculate not only these affecting the vector boson polarization tensors, but also this which modify the Fermi constant, the fine structure constant, etc. as these quantities are used when extracting S, T and U from the data [18].

A. Effective lagrangian

In the following I will assume that the underlying physics is weakly coupled and derive the leadingoperators that can be expected form the existence of heavy excitations at scale Λ.

The complete list of dimension 6 operators was cataloged a long time ago for the case where the low energy spectrum includes a single scalar doublet [12] [7]. It is then straightforward to determine the subset of operators which can be TLG, they are [11]

- Fermions: $\left(\bar\psi_i \Gamma^a \psi_j\right)\left(\bar\psi_k \Gamma^a \psi_l\right)$
- Scalars: $|\phi|^6, \ (\partial|\phi|^2)^2$
- Scalars and fermions: $|\phi|^2 \times$ Yukawa term
- Scalars and vectors: $|\phi|^2|D\phi|^2, \ |\phi^\dagger D\phi|^2$

---

[7] More complicated scalar sectors have also been studied [17], though not exhaustively.

- Fermions, scalars and vectors: $\left(\phi^\dagger T^n D^\mu \phi\right)\left(\bar\psi_i T^n \gamma_\mu \psi_j\right)$

where T denotes a group generator and Γ a product of a group generator and a gamma matrix.

Observables affected by the operators in this list provide the highest sensitivity to new physics effects provided that the standard model effects are themselves small (or that the experimental sensitivity is large enough to observe small deviations). I will illustrate this with two (incomplete) examples

B. b-parity

This is a proposed method for probing new flavor physics [19]. Its virtue lies in the fact that it is very simple and sensitive (though it does not provide the highest sensitivity for all observables). The basic idea is based on the observation that the standard model acquires an additional global $U(1)_b$ symmetry in the limit $V_{ub} = V_{cb} = V_{td} = V_{ts} = 0$ (given the experimental values $0.002 < |V_{ub}| < 0.005$, $0.036 < |V_{cb}| < 0.046$, $0.004 < |V_{td}| < 0.014$, $0.034 < |V_{ts}| < 0.046$, deviations form exact $U(1)_b$ invariance will be small). Then for any standard model interaction a reaction to the type

$$n_i\ b - \text{jet} + X \to n_f\ b - \text{jet} + Y \tag{22}$$

will obey

$$(-1)^{n_i} = (-1)^{n_f} \tag{23}$$

to very high accuracy. The number $(-1)^{\#\text{ of } b \text{ jets}}$ defines the **b-parity** of a state (it being understood that the top quarks have decayed).

The standard model is then b-parity even, and the idea is to consider a lepton collider [8] and simply count the number of b jets in the final state; new physics effects will show up as events with odd number of b jets.

---

[8]In hadron colliders there are sea-b quarks which foul-up the argument

The standard model produces no measurable irreducible background, yet there are significant *reducible* backgrounds which reduced the sensitivity to Λ. To estimate these effects I define

- $\epsilon_b = b-$jet tagging efficiency
- $t_c = c-$jet **mis**tagging efficiency (probability of mistaking a $c-$jet jet for a $b-$jet
- t_j =light-jet **mis**tagging efficiency (probability of mistaking a light-jet for a $b-$jet

so that the *measured* cross section with k-b-jets is

$$\bar{\sigma}_k = \sum_{u+v+w=k} \left[\binom{n}{u} \epsilon_b^u (1-\epsilon_b)^{n-u} \right] \left[\binom{m}{v} t_c^v (1-t_c)^{m-v} \right] \left[\binom{\ell}{w} t_j^w (1-tj)^{\ell-w} \right] \sigma_{nm\ell} \tag{24}$$

where $\sigma_{nm\ell}$ denotes the cross section for the final state with n b-jets, m c-jets, and ℓ light jets. Note that $\left[\binom{n}{u} \epsilon_b^u (1-\epsilon_b)^{n-u} \right]$ is the probability of tagging u and missing $n-u$ b-jets out of the n available.

As an example consider

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm sm} + \frac{f_{ij}}{\Lambda^2} \left(\bar{\ell} \gamma^\mu \ell \right) \left(\bar{q}_i \gamma_\mu q_j \right) \tag{25}$$

where $i \neq j$ denote family indices. Taking $m_H = 100$GeV $|f| = 1$ $t_c = t_j = 0$ the sensitivity to Λ is summarized by the following table

Limits from $e^+e^- \to t\bar{c} + \bar{t}c + b\bar{s} + \bar{b}s \to 1b-$jet $+ X$				
$\sqrt{s}$	L	$\epsilon_b = 50\%$	$\epsilon_b = 60\%$	$\epsilon_b = 70\%$
200 GeV	2.5 fb^{-1}	1.4 TeV	1.5 TeV	1.6 TeV
500 GeV	75 fb^{-1}	5.0 TeV	5.2 TeV	5.5 TeV
1000 GeV	200 fb^{-1}	9.5 TeV	10.0 TeV	10.7 TeV

These results are promising yet they will be degraded in a realistic calculation. First one must include the effects of having $t_{c,j} \neq 0$. In addition there are complications in using inclusive reactions such as $e^+e^- \to b + X$ since the contributions form events with large number of jets can be very hard to evaluate (aside from the calculational difficulties there are additional complications when *defining* what a jet is). A more realistic approach is to restrict the calculation to a sample with a fixed number of jets (2 and 4 are the simplest) and determine the sensitivity to Λ for various choices of ϵ_b and t_j using this population only.

C. CP violation

Just as for b-parity the CP violating effects are small within the standard model and so precise measurements of CP violating observable might be very sensitive to new physics effects.

In order to study CP violations it is useful to first define what the CP transformation *is*. In order to do this in general denote the Cartan group generators by H_i and the root generators by E_α, then it is possible to find a basis where *all* the group generators are real and, in addition, the H_i are diagonal [20]. Define then CP transformation by Transformations

$$\begin{aligned} \psi &\rightarrow C\psi^* \text{ (fermions)} \\ \phi &\rightarrow \phi^* \text{ (scalars)} \\ A_\mu^{(i)} &\rightarrow -A_\mu^{(i)}, \ (i: \text{ Cartan generator}) \\ A_\mu^{(\alpha)} &\rightarrow -A_\mu^{(-\alpha)}, \ (\alpha: \text{ root}) \end{aligned}$$

it is easy to see that the field strengths and currents transform as A_μ, while $D\phi \rightarrow (D\phi)^*$. It then follows that in this basis the whole gauge sector of *any* gauge theory is CP conserving; CP violation can arise *only* in the scalar potential and fermion-scalar interactions using this basis.

In order to apply this to electroweak physics I will need the list of TLG operators of dimension 6 which violate CP, they are given by [9]

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[9] The notation is the following: ℓ and q denote the left-handed lepton and quark doublets; u, d and e denote the right handed quark and charged lepton fields. *lambda* denote the Gell Mann matrices, τ the Pauli matrices, and $\epsilon = i\tau^2$. D represents the covariant derivatives and ϕ the scalar doublet.

$$
\begin{array}{lll}
\left(\bar{\ell}e\right)\left(\bar{d}q\right)-\text{h.c.} & \left(\bar{q}u\right)\varepsilon\left(\bar{q}d\right)-\text{h.c.} & \left(\bar{q}\lambda^A u\right)\varepsilon\left(\bar{q}\lambda^A d\right)-\text{h.c.} \\
\left(\bar{\ell}e\right)\varepsilon\left(\bar{q}u\right)-\text{h.c.} & \left(\bar{\ell}u\right)\varepsilon\left(\bar{q}e\right)-\text{h.c.} & |\phi|^2\left(\bar{\ell}e\phi-\text{h.c.}\right) \\
|\phi|^2\left(\bar{q}u\tilde{\phi}-\text{h.c.}\right) & |\phi|^2\left(\bar{q}d\phi-\text{h.c.}\right) & |\phi|^2\partial_\mu\left(\bar{\ell}\gamma^\mu\ell\right) \\
|\phi|^2\partial_\mu\left(\bar{e}\gamma^\mu e\right) & |\phi|^2\partial_\mu\left(\bar{q}\gamma^\mu q\right) & |\phi|^2\partial_\mu\left(\bar{u}\gamma^\mu u\right) \\
|\phi|^2\partial_\mu\left(\bar{d}\gamma^\mu d\right) & & \\
\mathcal{O}_1=\left(\phi^\dagger\tau^I\phi\right)D_\mu^{IJ}\left(\bar{\ell}\gamma^\mu\tau^J\ell\right) & & \\
\mathcal{O}_2=\left(\phi^\dagger\tau^I\phi\right)D_\mu^{IJ}\left(\bar{q}\gamma^\mu\tau^J q\right) & & \\
\mathcal{O}_3=\left(\phi^\dagger\varepsilon D_\mu\phi\right)\left(\bar{u}\gamma^\mu d\right)-\text{h.c} & &
\end{array}
$$

All operators except $\mathcal{O}_{1,2,3}$ violate chirality and their coefficients are strongly bounded by their contributions to the strong CP parameter θ; in addition some chialiry violating operators contribute to meson decays (which again provide strong bounds for fermions in the first generation) and, finally, in natural theories some contribute radiatively to fermion masses and will be then suppressed by the smaller of the corresponding Yukawa couplings. For these reasons I will not consider them further. Moreover, since I will be interested in limits that can be obtained using current data, I will ignore operators whose only observable effects involve Higgs particles.

With these restrictions only $\mathcal{O}_{1,2,3}$ remain; their terms not involving scalars are

$$
\begin{aligned}
\mathcal{O}_1 &\rightarrow -\frac{igv^2}{\sqrt{2}}\left(\bar{\nu}_L\, \not{W}^+ e_L-\text{h.c.}\right)\\
\mathcal{O}_2 &\rightarrow -\frac{igv^2}{\sqrt{2}}\left(\bar{u}_L\, \not{W}^+ d_L-\text{h.c.}\right)\\
\mathcal{O}_3 &\rightarrow -\frac{igv^2}{\sqrt{8}}\left(\bar{u}_R\, \not{W}^+ d_R-\text{h.c.}\right)
\end{aligned}
$$

The contributions from $\mathcal{O}_{1,2}$ can be absorbed in a renormalization of standard model coefficients whence only $\mathcal{O}_3$ produces observable effects, corresponding to a right-handed quark current. Existing data (from τ decays and m_W measurements) implies $\Lambda \gtrsim 500\text{GeV}$

One can also determine the type of new interactions which might be probed using these operators [11]. The heavy physics which can generate $\mathcal{O}_3$ at tree level is described in Fig. V C. If the underlying theory is natural we conclude that there will be no super-

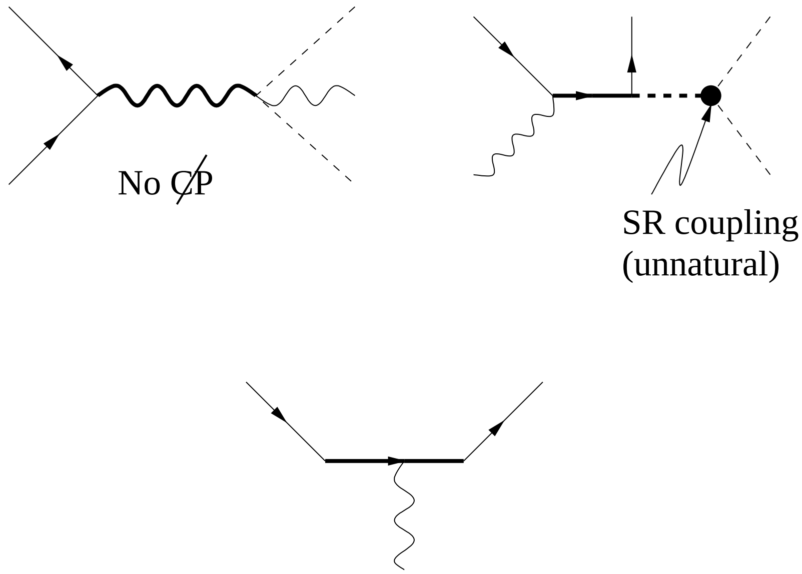

FIG. 5. Heavy physics contributing to CP violating operators. Wavy lines denote vectors, solid lines fermions, and dashed ones scalars. Heavy lines denote heavy excitations.

renormalizable couplings; in this case $\mathcal{O}_3$ will be generated by heavy fermion exchanges only [10]

Note finally that these arguments are only valid for weakly coupled heavy physics. For strongly coupled theories other CP violating operators can be important, e.g.

$$\frac{f}{\Lambda^2} B^{\mu\nu} \left(\bar{e} \gamma_\mu D_\nu e - \mathrm{h.c} \right) \tag{26}$$

since $|f| \sim 1$.

VI. OTHER APPLICATIONS

The effective Lagrangian approach can be applied in many other situations such as gravity and high temperature field theory. I will briefly consider the latter.

---

[10] It is true that vertices involving light fermions, light scalars and heavy fermions produce mixings between the light and heavy scales, but this occurs at the one loop level. In contrast cubic terms of order Λ in the scalar potential would shift v at tree level.

A. Large temperatures

It is a well-known fact that the thermodynamics of a system with Hamiltonian H can be derived form the partition function $\mathrm{tr} e^{-\beta H}$. This resembles closely the (trace of the) quantum evolution operator e^{-iHt} hence we can obtain the thermodynamics of a system by the replacement $-it \to \beta$: non-zero temperature field theory corresponds to Euclidean field theory on a cylinder of perimeter$= \beta$, I will denote the corresponding Euclidean time by τ [21]

Since the time direction is finite the fields are expanded in a Fourier series. For bosons one obtains

$$\phi = \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \phi_n(\mathbf{k}) e^{i(2n\pi T\tau + \mathbf{k}\cdot\mathbf{r})} \tag{27}$$

and the corresponding free propagator is given by

$$\frac{1}{(2n\pi T)^2 + \mathbf{p}^2 + m^2} \tag{28}$$

The field is periodic in τ due to the commutativity of the variables in the functional integral (there is a much more physical reason, called the Kubo-Martin-Schwinger condition) [21].

Note that the $n \neq 0$ modes become heavy as $T \to \infty$ so that in this limit only the $n = 0$ modes remain and the theory reduces to a 3-D Euclidean field theory (there might be some subtleties involved, see below).

Fro fermions the expansion is in odd Fourier modes since the corresponding integration variables anticommute. explicitly

$$\psi = \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \psi_n(\mathbf{k}) e^{i(2n+1)\pi T\tau + \mathbf{k}\cdot\mathbf{r})} \tag{29}$$

with fee propagator

$$\frac{1}{[i(2n+1)\pi T + \mu]\,\gamma^0 - \mathbf{k}\cdot\gamma - m} \tag{30}$$

which shows that all modes become heavy as $T \to \infty$. There will be then no fermions in the spectrum at very large temperatures. Note that this occurs independently of the fermion mass [21].

Despite the absence of heavy fermions and scalars (effective mass $\sim T$) at large temperatures, we can still ask what is their effect on the scalar modes that survive in this regime. To this end we can construct the corresponding effective theory. I will illustrate the procedure using a simple example.

Consider the following scalar theory

$$\mathcal{L}^{(4)} = \frac{1}{2}\left(\partial\phi^2\right) - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \tag{31}$$

Then the excitations which survive at large T are

$$\varphi(\mathbf{x}) = \sqrt{T}\int_0^\beta d\tau\ \phi(\mathbf{x},\tau) \tag{32}$$

where φ is the dynamical variable of a 3 dimensional Euclidean field theory (in 3 dimensions the scalar fields have units of $\sqrt{\text{mass}}$ which explains the $\sqrt{T}$ factor). The only symmetry (aside form Euclidean invariance) is the reflection symmetry $\varphi \to -\varphi$. The scale of the new theory is set by $\Lambda = T$, but in this case the model is supposed to describe physics *above* Λ

With these considerations we can write the effective theory for φ,

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\left(\nabla\varphi\right)^2 + \frac{1}{2}a\varphi^2 + \frac{1}{4!}b\varphi^4 + \frac{c}{6!}\varphi^6 + O(1/T) \tag{33}$$

note that b is a super-renormalizable coupling and may lead to infrared problems.

The coefficients a, b, c, etc. can be calculated from the original theory. At one loop one obtains

$$a = \frac{\lambda T^2}{24} \qquad b = -\frac{m}{2\pi}\left(\frac{\lambda T}{4m}\right)^2 \qquad c = \frac{1}{4\pi}\left(\frac{\lambda T}{4m}\right)^3$$

But this calculation has some potential problems. Consider the $2k$ point function at zero external momentum; the corresponding graphs are given in Fig. VI A A simple estimate (verified by explicit calculation) shows that

$$\text{Graph} \propto \underbrace{\frac{\lambda^k}{m^{2k-4}}}_{\text{prefactors+dim. analysis}} \times \underbrace{\left(\frac{T}{m}\right)^{k+1}}_{\text{integral+sums}} \tag{34}$$

which corresponds to the operator

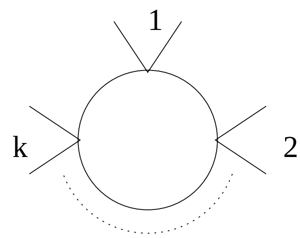

FIG. 6. Graphs exhibiting interesting infrared behavior at high temperatures

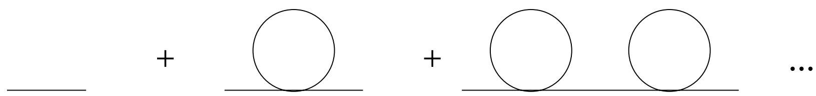

FIG. 7. Radiative corrections to the $n = 0$ propagator which cure the infrared divergences in the effective coefficients.

$$\mathcal{O}^{(k)} \sim \frac{\lambda^k}{m^{2k-4}} \left(\frac{T}{m}\right)^{k+1} \left(\sqrt{T}\,\varphi\right)^{2k} \frac{1}{T} = m^3 \left(\frac{\sqrt{\lambda}\,T\,\varphi}{m^{3/2}}\right)^{2k} \tag{35}$$

whose coefficient has *positive* powers of Λ $(= T)$ and are not suppressed at large temperatures. In fact, should this be correct the, effective theory expansion would be useless.

The solution to this infrared problem (diverging effective coefficients as $m \to 0$ is well known for this type of theories [21]: the propagator for the $n = 0$ mode gets dressed and in so doing the m^2 gets shifted by an amount $\propto T^2$. Explicitly, the graphs in Fig. VI A shift

$$m^2 \to m^2 + \frac{\lambda}{24} T^2 \tag{36}$$

so that the previous expression for the effective operator coefficient becomes

$$\mathcal{O}^{(k)} \sim \frac{\lambda^{(3-k)/2}}{T^k} \varphi^{2k} \tag{37}$$

which vanishes as $T \to \infty$. Note that there is still a remnant of the infrared properties of the theory in that the coefficients still diverge as $\lambda \to 0$.

a. QCD at high temperatures The previous arguments can be applied to the case of gauge theories. Just as for the scalar field, the gauge field is periodic in β and can be expanded in Fourier modes. At high temperatures, all but the $n = 0$ modes are heavy with masses $\sim T$. The remaining light modes are

$$\mathbf{A}^A_{n=0} \equiv \mathbf{a}^A \qquad A^{0A}_{\ \ n=0} \equiv \varphi^A \tag{38}$$

leaving a 3-D Euclidean $SU(3)$ model with gauge fields $\mathbf{a}^A$ and with a scalar octet (the φ^A). The 3-D gauge coupling constant is $g\sqrt{T}$ (where g denotes the QCD gauge coupling)

The simplest infrared divergences are cured by the dressing the gluon propagator at one loop [21]; the ϕ^A propagator at large T then becomes

$$\frac{1}{p^2} \rightarrow \frac{1}{p^2 + cg^2T^2} \tag{39}$$

for some numerical constant c. But this effect is not extended to the $\mathbf{a}^A$ for the corresponding vacuum polarization obeys $\Pi_{ii}(p \rightarrow 0) \rightarrow 0$ [21].

The fact that the $\mathbf{a}$ remain massless leads to various interesting problems. For example the higher order corrections to the free energy, provided by graphs in Fig. a. Suppose that the gauge bosons have a (dynamically generated) mass m. In this case a graph with ℓ loops behaves as [21]

$$g^6T^4(g^2T/m)^{\ell-3} \qquad (\ell > 3) \tag{40}$$

For the case where internal lines correspond to A^0 (or φ^A) $m \sim gT$ and the graph is well behaved, $\sim g^{\ell+3}T^4$. On the other hand when the internal lines represent A^i (or, equivalently, $\mathbf{a}^A$) propagator a problem will arise unless $m \sim g^2T$ is generated (we already know there is no $O(g)$ correction to m). This so-called magnetic mass has not been obtained perturbatively though it is widely believed to be generated.

Additional problems arise since the gauge coupling constant in the 3-d theory has dimensions of $\sqrt{\text{mass}}$ leading to super-renormalizable interactions with the related infrared divergences [22].

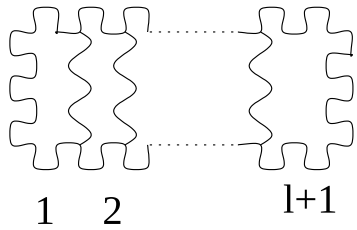

FIG. 8. Some radiative corrections to the QCD free energy

VII. CONCLUSIONS

In these lectures I have provided a review of some of the very many aspects and properties of effective theories, as well as some of their application. Despite this drawback I hope it does give a flair for the strength of the approach.

Effective theories will be used in deriving the implications of new data on the properties of the physics which underlies the standard model , but in addition it can be applied to a wide variety of phenomena ranging form QCD to superconductivity. It is this flexibility which makes the formalism so attractive.

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