The Standard Model as an Effective Field Theory

Ilaria Brivio and Michael Trott[a]

[a] *Niels Bohr International Academy & Discovery Center, Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK-2100, Copenhagen, Denmark*

Abstract: Projecting measurements of the interactions of the known Standard Model (SM) states into an effective field theory (EFT) framework is an important goal of the LHC physics program. The interpretation of measurements of the properties of the Higgs-like boson in an EFT allows one to consistently study the properties of this state, while the SM is allowed to eventually break down at higher energies. In this review, basic concepts relevant to the construction of such EFTs are reviewed pedagogically. Electroweak precision data is discussed as a historical example of some importance to illustrate critical consistency issues in interpreting experimental data in EFTs. A future precision Higgs phenomenology program can benefit from the projection of raw experimental results into consistent field theories such as the SM, the SM supplemented with higher dimensional operators (the SMEFT) or an Electroweak chiral Lagrangian with a dominantly $J^P = 0^+$ scalar (the HEFT). We discuss the developing SMEFT and HEFT approaches, that are consistent versions of such EFTs, systematically improvable with higher order corrections, and comment on the pseudo-observable approach. We review the challenges that have been overcome in developing EFT methods for LHC studies, and the challenges that remain.

Contents

1 Introduction

Our understanding of the nature of fundamental interactions can advance through a direct discovery of a new particle, or indirectly. Knowledge gathered through indirect methods has historically been the leading indication of a new particle or theoretical framework. It has also been the case that such indirect knowledge is usually ambiguous in that it can be an indication of several possible models. This is essentially due to the decoupling theorem [1] which formalizes how the non-analytic structure of correlation functions due to heavy states are projected out when matching onto a low energy effective field theory (EFT). The discovery of a new particle is a clarifying event, as it usually removes such ambiguities.

The Large Hadron Collider (LHC) was constructed based on the expectation that the functional mechanism by which $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1) \to \mathrm{U_{em}}(1)$ below the unitarity violation scale(s) dictated by the massive $W^\pm, Z$ vector bosons [2–7] would be revealed by probing the TeV energy range, with strong theoretical prejudice in favour of the Brout-Englert-Higgs mechanism influencing design choices [8]. In addition, it was expected that other beyond the Standard Model (SM) states involved in this mechanism could also be discovered in the characteristic energy range that LHC is exploring. The first expectation for LHC has been met to date with the discovery of a dominantly $J^P = 0^+$ boson [9–11] consistent with the SM Higgs boson [12, 13].

The lack of additional new state discoveries at LHC (to date) is perhaps unsurprising considering the large global data set consistent with the SM. In recent years, the direct discovery of new[1] states has become less frequent; the last three such discoveries being the top quark in 1995 [14, 15], the reporting of direct evidence of the tau neutrino in 2000 [16], and

[1] At least arguably fundamental.

the Higgs-like boson discovered in 2012 [12, 13]. The possibility that the next direct discovery of a new particle is a prospect for the far experimental future is manifest. This expectation is supported by the lack of statistically significant deviations from SM predictions in the global data set, which can be largely a result of at least a moderate degree of decoupling of physics beyond the SM to higher energy scales ($\gg m_{Z,W,h}$). Avoiding unproductive melancholy, this is motivation for increasing our understanding of all manner of SM physics to improve our theoretical predictions of experimental results. Thereby we sharpen the theoretical tools that allow us to indirectly search for physics beyond the SM.

This motivation is supported by the fact that all of these latest discoveries of new states $\{t, \nu_\tau, h\}$ were preceded by decades of indirect evidence gathered using EFT techniques, requiring precise SM predictions. Further, this argument also supports developing EFT methods to capture the low energy effect of physics beyond the SM, as only focusing efforts on improving SM predictions is insufficient the moment a real deviation is discovered. In addition, the unavoidable theoretical ambiguity associated with indirect knowledge of physics beyond the SM means that a singular theoretical explanation of such a deviation from the SM is unlikely to be epistemologically assured. It is important to be able to systematically understand such an anomalous measurement in a well defined field theory framework, that also dictates correlated deviations in other processes to distinguish between competing explanations. After all, any successful explanation of such a deviation must be consistent with the global data set, not just the observable deviating from the SM.

It can be remarkably efficient to approach such tests of consistency by projecting a particular model into an EFT framework in the presence of some degree of decoupling. This is the case so long as the EFT is well developed, so that properly interfacing with the global data set can be done in a one time matching calculation. To this end, it is essential to systematically improve our understanding of the EFTs that can accommodate SM deviations in advance of any such discovery of the SM breaking down in describing the data. It is also critical to encode the current data set into a form that maximizes its future utility when the SM can no longer successfully describe higher energy measurements.

This review is focused on these tasks. We discuss the recent developments in using indirect methods to study the Higgs-like scalar, related signals, and the development of two EFT frameworks. Both of these frameworks describe the known SM particles that lead to non-analytic structure in the correlation functions measured in particle physics experiments to date, in some region of phase space. These theories are distinguished by the nature of the low energy (infrared -IR) limit of physics beyond the SM being assumed. When the SM Higgs doublet is present in the EFT construction, the EFT is known as the Standard Model Effective Field Theory (the SMEFT). Conversely, when the SM Higgs doublet is not present, the Higgs Effective Field Theory (the HEFT) is constructed. Due to the lack of any clear experimental indication to choose between these approaches at this time, it is important to minimize theoretical bias when reporting LHC data. For this reason, it can also be advantageous to project raw experimental data in terms of cross section measurements into gauge invariant pseudo-observables in some cases, that are constructed by expanding around the poles of the SM states. These pseudo-observable decompositions can be related to multiple theoretical frameworks, such as these two EFT's. We also discuss this developing paradigm and the relationship between these various approaches.

The outline of this review is as follows. In Section 2 we discuss the motivation for a precision Higgs phenomenology program. In Section 3 we review pedagogically the key points leading to the structure of EFTs. We then turn to discussing the candidate field theories to use to interpret the global data set. First the SM is reviewed in Section 4.1 to fix notation. We then discuss the SMEFT in Section 4.2 and the HEFT in Section 4.3. Some issues that are being currently debated in the literature are reviewed in Section 5. In Section 6 we discuss and review pedagogically the distinction between S matrix elements, Lagrangian parameters and pseudo-observables with an emphasis on the differences between these concepts that are accentuated in the presence of an EFT such as the SMEFT or HEFT. We apply this understanding to LEPI-II pseudo-observable measurements and interpretations in Section 7. In Section 8 we discuss how many of these concepts and subtleties appear again in the interpretation of the measurements of the Higgs-like scalar at LHC, and review the κ formalism and proposals to go beyond this formalism in the long term LHC program. In Section 9 we discuss the application of EFT's to top quark measurements at LHC. Finally, in Section 10 we summarize the state of affairs early in LHC RunII and sketch out some expected future developments. The Appendix presents a series of LO results in a unified notation for SMEFT shifts to $\bar{\psi}\psi \to V \to \bar{\psi}\psi$ scattering through $V = \{Z, W\}$ gauge boson scattering, Higgs production and decay, vector boson scattering and hV production using a $\{\hat{\alpha}_{ew}, \hat{G}_F, \hat{m}_Z\}$ input parameter set.

The intended audience for this review is a mixture of experts and novices and both theorists and experimentalists. The presentation is geared to aid a new Ph.D. student with a solid quantum field theory background to jump into this area of research. We hope that experts in the field will also benefit from some of the discussion on the conceptual and technical aspects of these interesting examples of EFT's incorporating the presence of the Higgs-like boson.[2]

2 The need for a precision Higgs phenomenology program

Further developing the theoretical methods discussed in this review is not an idle pursuit. It is reasonable to expect that the properties of the dominantly $J^P = 0^+$ scalar boson could be perturbed by physics beyond the SM, and a precision Higgs phenomenology program could uncover such perturbations.

The reason for this expectation is the idea that the SM Higgs mechanism describing $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1) \to \mathrm{U_{em}}(1)$ is likely to be only an effective description. This belief is deeply rooted in the historical origin of the Higgs mechanism itself. The Higgsed phase of the SM (see discussion in Refs. [17, 18]) can be understood to be analogous to the ideas that first emerged in the Landau-Ginzburg effective model of superconductivity [19]. The Landau-Ginzburg

[2]We apologize in advance for overlooking any references in this literature.

action functional is given by [18]

$$LG(s) = \int_{\mathrm{Re}^3} dx^3 \left[\frac{1}{2}|(d - 2\,i\,e\,A)s|^2 + \frac{\gamma}{2}\left(|s|^2 - a^2\right)\right], \tag{2.1}$$

where A is the vector potential of Electromagnetism, d is a derivative defined on Re^3, e is the electric charge and s is a section of the (squared) unitary complex line bundle of Electromagnetism. This action has an energetically favoured minimum for $|s| = a$ and $(d - 2\,i\,e\,A)s = 0$ when $\gamma > 0$, leading to a topologically flat line bundle in the superconducting phase, and the exclusion of the magnetic field from the superconducting material.[3]

The (partial) action of the SM Higgs [20–22] is directly analogous

$$S_H = \int d^4x \left(|D_\mu H|^2 - \lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2\right), \tag{2.2}$$

with H is the Higgs doublet and D is the covariant derivative of the $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1)$ theory. This theory has an energetically favoured minimum at $\langle H^\dagger H\rangle = v^2/2$. Expanding around the minimum of the potential (that defines the EW vacuum background field) leads to the massive $W^\pm_\mu, Z_\mu$ vector bosons, due to the Higgsing of $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1) \to \mathrm{U_{em}}(1)$. Massless $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1)$ vector boson field configurations are then energetically excluded.

Landau-Ginzburg theory is not a fundamental theory. It is a functional effective description of superconductivity that can be related to a theory of Cooper pairs, such as BCS [23] theory. The connection drawn between Landau-Ginzburg theory and the Higgsed phase in Yang-Mills theory by Anderson [24], leads to an expectation of a shorter distance (or higher energy i.e ultraviolet – UV) completion/origin of the Higgs mechanism.[4] In this manner, the curious appearance of a classical Higgs potential with a chosen "mexican hat" form and an explicit scale v in the SM Lagrangian is understandable as a general low energy parameterization of underlying physics leading to an effective $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1) \to \mathrm{U}(1)_{\mathrm{em}}$. It is possible that this parameterization is not appropriate as an IR limit of a UV sector leading to the observed massive gauge bosons. This is a way to understand the difference between HEFT and SMEFT that will be discussed in more detail below.

2.1 Quantum corrections to Higgs properties and potential

The previous section advanced an argument in support of a more fundamental description of Electroweak symmetry breaking (EWSB) in that the Higgs potential has no quantum or dynamical origin in the SM; it is a parameterization. Functionally, it is a directly assumed classical potential – that is extremely sensitive to UV physics including quantum corrections. The reason for this is that the dimension of the $(H^\dagger H)$ operator is two. Dimensional analysis indicates that this operator can receive dimensionful corrections due to heavy states that

[3] Interestingly, the topology of the scalar manifold defined by the Higgs doublet H will form a crucial discriminant between the SMEFT and HEFT theories in the discussion that follows.

[4] A direct analogy would have the Higgs as a composite field, similar to the Cooper pair of BCS theory.

extend the SM. Such heavy states generally couple to the field H, or composite operators involving H, to perturb the properties of the Higgs when integrated out,[5] see Fig. 1. The

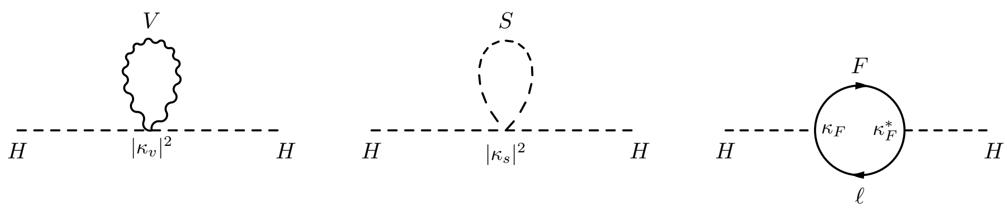


Figure 1. One loop corrections giving threshold matching contributions to $H^\dagger H$. We have used a common notation to simplify presentation in Eqn. 2.5, but note that $|\kappa_v|^2, |\kappa_s|^2$ can be negative.

Higgs field transforms as an $\mathrm{SU_L}(2)$ doublet and has hypercharge $\mathsf{y}_H = 1/2$, so composite operators involving H that allow dimension ≤ 4 couplings to singlet fermion (F), scalar bilinear ($S^\dagger S$), and (non-gauged) vector fields V_μ are of the form[6]

$$\Delta\mathcal{L} = |\kappa_v|^2 \, (H^\dagger H) \, V_\mu^\dagger \, V^\mu - |\kappa_s|^2 \, (H^\dagger H) \, (S^\dagger S) + \kappa_F \, \overline{F} \, \tilde{H}^\dagger \ell_L + h.c. \tag{2.3}$$

where ℓ is a lepton $\mathrm{SU_L}(2)$ doublet with hypercharge $\mathsf{y}_\ell = -1/2$. These interaction terms lead to threshold matching contributions in the Higgs potential[7]

$$\Delta V(H^\dagger H) \simeq H^\dagger H \left(\frac{3\,|\kappa_v|^2 \, m_v^2 \; N_v}{16\,\pi^2} + \frac{|\kappa_s|^2 \, m_s^2 \, N_s}{16\,\pi^2} - \frac{|\kappa_F|^2 \, m_F^2 \, N_F}{16\,\pi^2} \right) + \cdots , \tag{2.4}$$

where m_i is the mass of the corresponding field, and N_i can result from the sum over the degrees of freedom in an internal (flavour) symmetry group of the field i. When avoiding tuning the bare Higgs mass in the classical Lagrangian against quantum corrections,[8] then it follows that $m_h \sim |\kappa_i| \, m_i \sqrt{N_i}/4\,\pi$. This is the reason the mass of the SM Higgs is expected to be proximate (up to a loop factor) to beyond the SM mass scales.

One can turn the relation between m_h and m_i around. Then corrections to cross sections that are probed through a measurement exploiting a propagating SM state (that goes on-shell) scale as

$$\frac{\sigma_{SM+i}}{\sigma_{SM}} \simeq \frac{1}{16\,\pi^2} \left(\frac{N_i^2 \, |\kappa_i|^2 \kappa'}{g_{SM} \, \lambda} \right) + \cdots \tag{2.5}$$

[5]In some cases low energy effects can be present modifying Higgs properties that do not satisfy this requirement, due to reducing the field theory with the Equations of Motion (EOM) to a minimal basis reshuffling the appearance of IR physics effects.

[6]$H^\dagger D^\mu H V_\mu + h.c$ can be directly shown to not lead to a threshold correction, by integrating by parts.

[7]The threshold matching contributions are obtained by calculating in dimensional regularization (DR) in $d = 4 - 2\,\epsilon$ dimensions using $\overline{\mathrm{MS}}$ subtraction, and Taylor expanding the resulting amplitudes in the limit $v^2/m_i^2 < 1$.

[8]Tuning parameters at the Lagrangian level to avoid these conclusions can be understood to be best avoided in the following way. As Lagrangian parameters can always be related to measured quantities through S matrix elements, and eliminated in relationships between S matrix elements, parameter tuning can be understood to be equivalent to assuming precise relationships between a series of independent measured quantities, in order to have a further S matrix element take on a value not expected by naive dimensional analysis.

Here we have introduced another coupling between the new physics state and the SM (κ') and Taylor expanded out the non-analytic structure of the tree level propagating state i. The non-analytic structure of the propagating SM state is essentially unchanged in this limit and cancels out in the ratio. g_{SM} corresponds to a generic SM coupling. This result argues that $\sim$ % level deviations in Higgs properties can occur in scenarios that avoid parameter tuning. This estimate is subject to the following qualifications:

- Differences between coupling constants can lead to further enhancements/suppressions.
- This estimate implicitly assumed one local contact operator was introduced (at tree level) interfering with the SM. It has been proven [25] that when considering tree level effects[9] multiple operators are always present if the UV scales have a dynamical origin.
- If the state i does not lead to any corrections to σ_{SM+i} at tree level, then a further $\sim 1/16\pi^2$ suppression occurs. On the other hand, when considering one loop effects, the multiplicity of operators present is generically very large due to one loop mixing, see Section 4.2.4 for more detail.[10]

This rough and schematic understanding is nevertheless validated in exact calculations in some popular new physics models, see Refs. [29–31] for more discussion.

2.2 Higgs substructure

EFTs capture the Taylor expanded effects of particle exchange at tree and loop level, but also encode multi-pole expansions of underlying structure [32–35] that are generic in field configurations set up by charge distributions that are spatially separated. The classic example is the multi-pole expansion of an electrostatic charge distribution [36, 37]. This physics is not directly or trivially identified in general with tree or loop level particle exchange diagrams in the presence of non-perturbative bound states (such as a composite Higgs), and offers further hopes for perturbations of Higgs properties that could be experimentally measured.

When an EFT is capturing the consistent low energy limit of a strongly interacting light composite Higgs, the multi-pole expansion should be considered [34, 35].[11] The possibility that the Higgs field is composite underlies a significant fraction of the interest in the EFT methods discussed in this review. Composite Higgs models are still of experimental interest. These ideas emerged in the 80's in Refs. [39–44] with early studies also exploring the possibility of dynamical mass generation in extra dimensional scenarios [45, 46]. These ideas re-emerged and were extended in the late 90's in the context of Little Higgs constructions [47, 48], and extra dimensional models aimed at dynamical mass generation [49–52] with the Holographic composite Higgs models consolidating many of these developments in Refs. [53, 54].

[9] Subject to the conditions that a flavour symmetry is not explicitly broken and the mass scales of the heavy UV states have a dynamical origin.

[10] In some exceptional cases, $\mathcal{L}_6$ operators do not mix. This can be understood at an operator level using operator weights and related helicity and unitarity arguments [26–28].

[11] This point has recently been re-emphasized in Ref. [38].

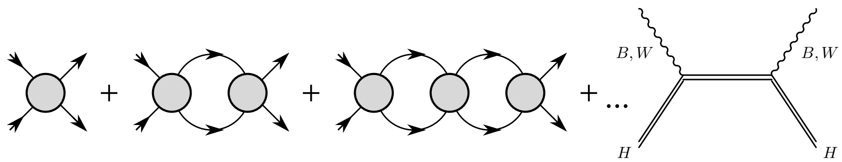


Figure 2. Diagrams relevant for the multi-pole expansion. The left figure illustrates the infinite sum of bubble diagrams in a "ladder approximation" in the field theory equivalent of the non-relativistic Schrödinger equation, while the right diagram illustrates the scattering of the $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1)$ gauge fields off of the constituent particles of a composite Higgs.

In analyzing scattering off of non-perturbative bound states, leading to a multi-pole expansion, perturbative methods fail by definition. One can gain some intuition on how this scattering is represented in an EFT by considering the solutions to the time independent Schrödinger equation of a non-local potential of a fixed target, represented as $V(\mathbf{r}, \mathbf{r}')$. Such a potential can mimic the extended nature of the composite particle. With appropriate boundary conditions this scattering is described by the Lippmann–Schwinger equation [55]; outgoing wavefunctions are related to those incoming by a partial wave transition matrix that satisfies the integral equation

$$T_\ell(\mathbf{k}, \mathbf{k}'; E) = V_\ell(\mathbf{k}, \mathbf{k}') + \frac{2}{\pi} \int_0^\infty d|\mathbf{q}|\, q^2 \frac{V_\ell(\mathbf{k}', \mathbf{q}) T_\ell(\mathbf{q}, \mathbf{k}; E)}{E - q^2/\mu + i\epsilon}. \tag{2.6}$$

Here $\mathbf{k}, \mathbf{k}'$ are the Fourier momentum of the radial coordinate $\mathbf{r}, \mathbf{r}'$ and μ is the reduced mass. The asymptotic scattering is described by a partial wave scattering matrix: $S_\ell(k) = 1 + 2ikf_\ell(k)$, which can be parameterized by a partial wave phase shift $S_\ell(k) = e^{2i\delta_\ell(k)}$, for spherically symmetric potentials. The multi-pole expansion in this case is the fact that the phase shift parameter characterizing the scattering matrix has a power series expansion in k^2. For the partial wave $\ell = 0$, this expansion is given as

$$k \cot \delta_0(k) = -\frac{1}{a_0} + \frac{1}{2} r_0\, k^2 - C_2 r_0^3\, k^4 + \cdots \tag{2.7}$$

There is an expansion in derivatives acting on a field F of the state associated with asymptotic wavefunction scattering off of the fixed target field $\mathcal{T}$ as a series of interactions of the form $\sim \{\mathcal{T}\, F\, F, \mathcal{T}\, F \nabla^2 F, \mathcal{T}\, F \nabla^4 F\}$. The effective range expansion in the parameters $\{a_0, r_0, C_2 r_0^3\}$ are analogous to Wilson coefficients in the relativistic EFT. The bound state substructure generates a series of scales that characterize the multi-pole expansion. This is generic in EFTs describing the scattering off of bound states. See Refs. [56, 57] for related discussion on non-relativistic bound states in EFTs.

In nucleon-nucleon (NN) EFT [58–60] an analogy to the non-relativistic scattering case is extensive. The time independent Schrödinger equation corresponds to a summation of an infinite set of Feynman diagrams in the EFT, defining a scattering amplitude $\mathcal{A}$ as shown in Fig. 2. The Born series expansion of the related Lippmann–Schwinger equation can be mapped to an infinite sum of ladder diagrams that depends on the particles exchanged in the EFT and the kinematics of the poles dominating the convolution integrals between the nucleon potential and the non-relativistic propagators. The amplitude defined by this Born series is related to the phase shift [59]

$$|\mathbf{p}|\,\cot\delta(\mathbf{p}) = i\,|\mathbf{p}| + \frac{4\,\pi}{M}\frac{1}{\mathcal{A}}. \tag{2.8}$$

Here $\mathbf{p}$ is the three momentum of the NN system and M is the mass scale of the nucleon. Here $\mathcal{A}$ is a scattering amplitude the corresponds to the infinite sum of ladder diagrams. Again a power series expansion of the phase shift leads to the multi-pole expansion. The parameters characterizing the effective range expansion of nucleon scattering take on values that differ by an order of magnitude [59] and depends in a non-trivial manner on the spectrum of states retained in the EFT as the bound state is near threshold.

For a composite Higgs, one can consider the Higgs to be analogous to the nucleon, and the convolution integral of the Higgs self interaction potential to be made with Higgs propagators and beyond the SM states involved in EW symmetry breaking, such as vector resonances analogous to the ρ meson in chiral perturbation theory. Considering current LHC data, there is little motivation to assume that such a ρ state is accidentally lighter than the remaining states in the strong sector. Scattering results developed in analogy to NN scattering EFT then lead to a multi-pole expansion in derivative operators involving the Higgs field.

In addition, when considering the multi-pole expansion in terms of the $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1)$ gauge fields, scattering off the bound constituents that make up the composite Higgs can occur. This is the case if the constituents are charged under the SM gauge groups, or a larger group that contains the SM as a subgroup as illustrated in Fig.2 (right).[12]

A composite Higgs has an associated multi-pole expansion. Unfortunately, using crossing symmetry (i.e. rotating Fig.2 (right) 90° counterclockwise) to consider the interaction potential as only describing the composite Higgs state is inconsistent with a non-relativistic EFT approach related to the Schrödinger equation.[13] Furthermore, the summation of subsets of diagrams in "ladder approximations" to the convolution integrals is not valid in general. Noting all of these concerns, the multi-pole expansion can be associated with the suppression scale and Wilson coefficients of the $\mathrm{U}(3)^5$ symmetric operators[14]

$$\lambda_{Mul}^2 \simeq \{\frac{C_{H\Box}}{\Lambda^2}, \frac{C_{HD}}{\Lambda^2}, \frac{C_{HWB}}{\Lambda^2}, \frac{C_{HW}}{\Lambda^2}, \frac{C_{HB}}{\Lambda^2}\}. \tag{2.9}$$

When these operators are all constrained so that $\lambda_{Mul} \ll \lambda_h$, where the Compton wavelength of the Higgs $\lambda_h = \hbar/m_h\,c$, the Higgs boson is effectively interacting as a point-like particle, when considering these dimension six operators. As we have scaled the operators by Λ associated with particles integrated out of the spectrum, the Wilson coefficients of the operators involved in the multi-pole expansion can be expected to differ from order one values – if the

[12] We restrict our attention to the EW interactions due to the expectation that new physics underlying Higgs compositeness would be associated with EW symmetry breaking.

[13] As crossing symmetry relations are between Mandelstam variables constructed out of full four vectors.

[14] See Table 1 for the operator definitions corresponding to these Wilson coefficients.

scales characterizing the effective range expansion are distinct from the mass scale of the states integrated out of the theory constructing the EFT.

To summarize: considering the possibility of compositeness and the related multi-pole expansion, the UV sensitivity of Higgs properties, and the classical nature of the assumed SM EW symmetry breaking potential, a precision Higgs phenomenology program to probe for indirect hints of physics beyond the SM is well motivated.

3 Basics of EFT

A Taylor expansion in dimensionless ratios was used in the previous sections to simplify the results. That such a simplification can occur is consistent with the intuitive understanding that IR physics can be calculated without reference to the details of all UV physics. This is generic in observables calculated in a Quantum Field Theory (QFT) so long as limited theoretical precision is all that is demanded. Manifestly this is true for the SM; which despite being a QFT that is not well defined to arbitrarily high energies,[15] has still been validated to be an adequate description of LHC data considering current experimental precision.

3.1 Separation of scales, renormalization and local/analytic expansions

EFT is a set of ideas that justifies why this systematic separation of the physics of different scales can be true in field theory.[16] Renormalization also separates IR and UV physics, but EFT is more than a statement that QFTs are systematically renormalizable. Furthermore, the success of renormalization programs in QFTs can be understood as an EFT consequence in an intuitive way.[17] When renormalizing a QFT the short distance physics in the theory one calculates in is modified, and the effects of regularizing such physics is absorbed into the low energy parameters of the effective theory. How this modification takes place is illustrative of scale separation in EFTs.

Consider calculating an amplitude at one loop using dimensional regularization in $d = 4 - 2\,\epsilon$ dimensions [74].[18] The amplitude can be expressed by using the master formula for Minkowski space momentum integrals

$$M_I = \int \frac{d^d q}{(2\,\pi)^d (\hat{\mu}^2)^{n\,(d-4)/4}} \, \frac{(q^2)^\alpha}{(q^2 - \Delta^2)^\beta} = \frac{i\,(-1)^{\alpha-\beta}}{(\Delta^2)^{\beta-\alpha-d/2}} \, \frac{\Gamma(\alpha + d/2)\Gamma(\beta-\alpha-d/2)}{\Gamma(d/2)\Gamma(\beta)}, \quad (3.1)$$

[15]Due to the presence of Landau poles in the $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1)$ theory.

[16]For excellent reviews on EFT see Refs. [33, 61–67]. The pioneering works developing the modern understanding of EFT include Refs. [1, 68–73].

[17]The old field theory approach of stressing of the distinction between bare and renormalized parameters is drawn when correlation functions involving the parameters are considered to be measured or predicted to arbitrary precision. In this sense, the EFT understanding of renormalization is consistent with such lore.

[18]We use $\overline{\mathrm{MS}}$ subtraction, by introducing n powers of $\hat{\mu}^{(4-d)/2} = (\mu\,\sqrt{e^\gamma/4\pi})^{(4-d)/2}$ for each power of the coupling present defining the amplitude, so that the renormalized coupling remains dimensionless. Here γ is the Euler-Mascheroni constant. The arguments in this section are formulated for a one loop example but they generalize to higher loop orders directly.

with a four momentum q^μ and a factor Δ introduced for the characteristic scales (masses, kinematic invariants) in the amplitude. Following the discussion of Georgi [33], consider integrating over the ϵ momentum space of such an amplitude after Wick rotating to Euclidean momentum space and factorizing the loop momentum as $q^2 = q_\epsilon^2 + q_E^2$. Restricting one's attention to divergent terms one finds [33]

$$M_I \propto \int \frac{d^4 q_E}{(2\,\pi)^4} \frac{(q_E^2)^\alpha}{(q_E^2+\Delta^2)^\beta} \left[\frac{\Gamma(\beta+\epsilon)}{\Gamma(\beta)}\right] \left[\frac{4\,\pi\mu^2}{q_E^2+\Delta^2}\right]^\epsilon . \tag{3.2}$$

The last two factors in square brackets are both finite as $\epsilon \to 0$, but there is an important difference between them. The final term does not significantly change the amplitude so long as all the scales are similar $\mu^2 \sim \Delta^2 \sim q_E^2$. On the other hand, this term leads to a modification (an introduced regularization) of the amplitude that can become significant for small ϵ if $\mu^2 \gg \Delta^2 + q_E^2$ or $\mu^2 \ll \Delta^2 + q_E^2$, i.e. when highly separated scales are present in the amplitude. In this manner, the universal subtractions present in systematically renormalizing a QFT are understood to correspond to UV physics effects that have been systematically removed out of the lower energy theory by such a regularization for $\mu^2 \ll \Delta^2 + q_E^2$. That such a separation of IR and UV physics can occur is the key idea of EFT and this can be understood to be an underlying reason for renormalization to work. The case $\mu^2 \gg \Delta^2 + q_E^2$ has a different meaning, it corresponds to an IR divergence, and we discuss this divergence below.

Counterterms are of a simple universal analytic form when using DR. This is also the case in other regularization schemes, such as schemes with dimensionful regulators that directly satisfy the decoupling theorem [1]. One might doubt if the regularization of divergences due to arbitrary UV physics sectors can be subtracted out of a prediction of a lower energy observable in this simple manner. Formally, this can be understood to follow from a proof supplied by Bogoliubov and Parasiuk[19] on the analytic nature of counterterms in 1957 [75]. Renormalization Group (RG) based arguments also support this understanding, as demonstrated by Polchinski in Ref. [76], as do the diagrammatic arguments of Weinberg's power counting theorem [77]. Recently the formal proof of the renormalizability in effective field theories has also been studied with increased mathematical rigor in Ref. [78].

A less formal and more intuitive understanding of the universal nature of the subtractions follows from considering the constraints of the global symmetries in the EFT, Lorentz invariance, and the fact that the non-analytic structure of correlation functions[20] is only generated when intermediate states propagate on-shell. When subtracting the effects of UV physics acting to regularize divergences in the full theory systematically out of a lower energy amplitude, far below the characteristic mass scales of such UV states, these states are off-shell. The correlation functions can be simplified by Taylor expanding in the ratio of the separated scales, and are well approximated by the first few analytic terms in the expansion. Any divergence thereby has an analytic form. The locality of the subtractions is because off-shell exchange of

[19] We thank F. Herzog for this reference.

[20] Here we refer to the poles and cuts in the momentum space of the spectral function defined in analogy to the Källén-Lehmann [79, 80] two point spectral function.

the virtual particles (of mass $\sim M$) occurs, but it is local as it is limited to short times and distances by the uncertainty principle [67]

$$\Delta t\, \Delta E \sim \Delta t\, M > 1 \to \Delta t \sim \frac{1}{M}, \qquad \Delta |x|\, \Delta |p| \sim \Delta |x|\, M > 1 \to \Delta |x| \sim \frac{1}{M}. \tag{3.3}$$

Here we are using units where $\hbar = 1 = c$.[21] Renormalization understood in this manner does not draw any fundamental distinction between theories with only interaction terms limited to mass dimension $d \leq 4$ and EFT's with a tower of higher dimensional operators. The renormalizability is understood to be possible due to the fact that the only way that high energy physics integrated out of the low energy theory can modify the lower energy construction is through a tower of local analytic operators. In both cases the renormalizability of the theories follows from the separation of scales that allows the Taylor expansion.

This reasoning also holds for the non-divergent contributions of UV physics approximated in an EFT by expanding in a ratio of scales. As a result, an EFT is a field theory with a tower of local analytic operators of dimension d divided by $d-4$ powers of a suppression scale characteristic of the UV physics removed from the EFT construction.[22] A well constructed EFT is designed to capture the relevant low energy physics to predict a set of experimental measurements, while exploiting the simplifications that result from such expansions as soon as possible. The essential and key idea is to separate the description of the processes under study into IR (i.e. infrared or long distance) propagating states and their interactions, captured by the local and analytic operator expansion, and the UV dependent short distance Wilson coefficients, i.e. construct

$$\mathcal{L}_{EFT} \simeq \sum_i C_i^{UV}(\mu)\, O_i^{IR}(\mu). \tag{3.4}$$

Taking this reasoning to its logical conclusion gives a commonly held set of "prime directives" of effective field theorists:[23]

- Isolate and separate a series of characteristic scales in observables.
- Construct the Lagrangian of the EFT only out of the degrees of freedom that lead to non-trivial structure in the correlation functions. These are propagating on-shell states (i.e. with $p^2 \sim m^2$), ideally with only one scale defining the EFT closely related to the scales identified in the previous step. In short, expand ASAP, at the Lagrangian level.
- Calculate in the EFT without unnecessary reference to the UV physics that is is decoupled. Some UV dependence is present, but it is sequestered in the EFT into the short

[21] Unless otherwise noted we use such "God-given" units in this review.

[22] In some exceptional cases EFT's can be constructed with non-local operators. This is usually due to distinguishing field excitations as retained or removed from the EFT by assigning a four momentum of a particle excitation of a field (not the p^2 Lorentz invariant used to distinguish on or off-shell) some scaling rules. See Refs. [81–85] for famous examples.

[23] We acknowledge M. Luke for this nomenclature.

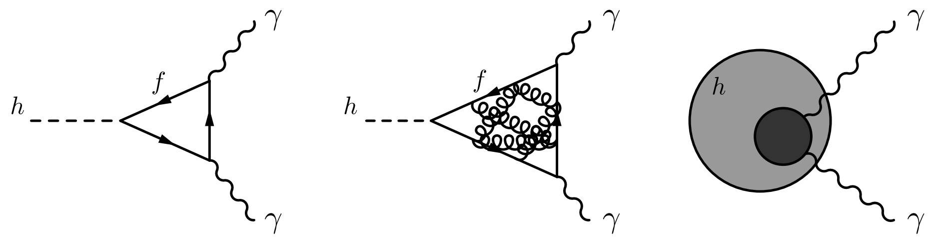


Figure 3. Illustration of various UV physics scenarios captured in the SMEFT for $h \to \gamma\gamma$. The leftmost figure illustrates perturbative mediators (a fermion f) leading to the decay $h \to \gamma\gamma$. The middle figure illustrates this decay being mediated by UV states in a strongly interacting field theory where the diagram sum does not converge. The rightmost figure illustrates the possibility of Higgs substructure leading to the SMEFT multi-pole expansion with the outer circle indicating the Compton wavelength of the Higgs λ_h and the shaded interior region a substructure scale characterized by λ_{Mul}.

distance Wilson coefficients in the matching procedure. In contrast, in the EFT the operators encode the IR physics describing long distance propagating states.

- Use a mass independent renormalization scheme, such as dimensional regularization.

The last two points are related to the requirement of matching and some technical consequences that result from a renormalization and subtraction scheme choice that we discuss in more detail in the following sections.

The physics that can be captured in the SMEFT in this manner as a consistent IR limit is only limited by the assumptions that $\Lambda > v$, and the existence of a Higgs doublet in the EFT construction. Various cases of beyond the SM physics are illustrated in Fig. 3. When this strict separation of scales is maintained, i.e. the SMEFT is treated as a general EFT retaining all of the operators that are allowed by the assumed symmetries, powerful model independent conclusions can result. All of the cases in Fig. 3, and combinations of such cases in possible UV physics sectors, have to project onto a series of local and analytic operators with various Wilson coefficients in the EFT expansion. This point holds even when the UV physics cannot be calculated with known field theory techniques.

If this separation of scales is violated, then the resulting statements and analysis, even if constructed and framed in EFT language, are not EFT conclusions. Such conclusions can be model dependent or simply ill-defined. This issue is very well known in research areas where EFT techniques have been dominant for decades, but is less appreciated when applying EFT techniques to characterize and constrain new physics at LHC, which is a continual source of debates in the literature. The key problem that can be introduced when going beyond EFT is the introduction of an assumption heavy hypothetical UV physics sector, which can result in the lack of a consistent IR limit, rendering the EFT framework used inconsistent and without meaning. To avoid this problem the key requirements that UV assumptions must address are

- The IR limit of a UV theory must be well defined, which requires that the UV theory is written down. In particular, the origin of the scales in a UV theory should be specified to have a possibility of a meaningful IR limit.

- If a strong interaction is present in a UV completion, and the mass scale characterizing bound states is $\Lambda \gg v$, then non-perturbative contributions can exist (see Fig.3 right) and should not be assumed to vanish without a precise justification. Assuming that such non-perturbative effects are absent, or negligible, in the EFT projects into a strong, and at times undefined, condition on UV completions that can generate the EFT. Again we emphasize that one of the core strengths of the standard approach to EFT is the ability to characterize and constrain such physics rendering UV assumptions on strongly coupled physics completely avoidable.

3.2 The decoupling theorem

The previous section outlines the basic intuition underlying EFT methods that is formalized in the Appelquist-Carazzone decoupling theorem [1] (see also Symanzik [86]). This theorem played an important role in the emergence of EFT methods in the 1970's. Examining this result in detail shows how renormalization scheme choice is also a technical issue of some importance when calculating in EFTs, as in the SM. The decoupling theorem is developed studying a set of massless gauge fields, denoted A_μ (and referred to as "vector mesons" at times in Ref. [1]), that are coupled to a set of massive fermions, denoted Ψ. The Lagrangian is

$$\mathcal{L}_{dc} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a - \bar{\Psi} i \not{D} \Psi - \bar{\Psi} m \Psi - \delta m \bar{\Psi}\Psi, \tag{3.5}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A_{b,\mu} A_{c,\nu}, \tag{3.6}$$

$$(D_\mu \Psi)_n = \partial_\mu \Psi_n + i [T_a A^a_\mu]_n, \tag{3.7}$$

and finally $[T_a, T_b] = i f_{abc} T^a$ defines the Lie algebra of the gauge group, with coupling g.[24] Here δm explicitly denotes the mass counterterm. The decoupling theorem is stated as [1]:

For any 1PI Feynman graph with external vector mesons only but containing internal fermions, when all external momenta (i.e. p^2) are small relative to m^2, then apart from coupling constant and field strength renormalization the graph will be suppressed by some power of m relative to a graph with the same number of external vector mesons but no internal fermions.[25]

Removing the field whose quanta is a heavy particle from the Lagrangian used to calculate experimental observables, based on the decoupling theorem, is known as "integrating out"

[24] We have modified some notational conventions compared to Ref. [1] to maintain a common notation throughout the review.

[25] Here the exact wording of Ref. [1] is edited for clarity.

a particle from the theory. The decoupling theorem is stated and proven for the specific field theory in Eqn. 3.5, but the arguments used to prove it generalize directly to other theories.[26] The generalization of this result to arbitrary field theories can be given in terms of all n-point Green's functions G^n as

$$\prod_i^N Z_i G^n_{full}(p_1, p_2 \cdots p_n; \mu) = \prod_j^M Z_j \, G^n_{EFT}(p_1, p_2 \cdots p_n; \mu) + \frac{1}{m^2} \prod_i^k Z_k \, G'^n_{EFT}(p_1, p_2 \cdots p_n; \mu) + \cdots \quad (3.8)$$

where $Z_{i..N}$ is the set of renormalizations required to render the full theory finite, $Z_{i..M}$ is the set of renormalizations required to render the leading $d \leq 4$ terms in the effective theory finite in an on-shell scheme. $Z_{i..M..k}$ includes these renormalizations and the additional renormalizations required to also render the local contact operators suppressed by $1/m^2$ finite. This theorem is formally establishing that if the intermediate heavy fields do not go on-shell (i.e never satisfy $p^2 \simeq m^2$) they modify the leading local analytic operator structures through renormalization in the lower energy Lagrangian, or add additional interactions suppressed by powers of $1/m$. This is expected considering the schematic arguments of the previous section.

The proof of the decoupling theorem is non-trivial. The statement is for *any 1PI graph*, i.e. can be an arbitrarily high order in perturbation theory. Due to this, the renormalization scheme chosen has an important impact on the arguments required to establish the proof. In Ref. [1] the scheme used defines δm to fix the fermion self energy to vanish at $\not{k} = m$. The remaining counterterms are subtractions defined at off-shell Euclidean momentum subtraction points ($p^2 = -\mu^2$). Wavefunction renormalization conditions fix the propagator to have its tree level form. Using this scheme Ref. [1] considered divergent and finite terms in arbitrary 1PI Feynman graphs and established the decoupling theorem exhaustively. Careful attention is paid in the proof to ensure that a well defined IR limit ($p^2/m^2 < 1$) is under consideration, by examining IR safe observables consistent with the KLN theorem [88, 89]. Equally important is a careful consideration of sub-divergences (that are sensitive to the regularization scheme used) in establishing the theorem. An implicit dependence on an off-shell subtraction scheme is present in the decoupling theorem.

One of the "prime directives" of EFT is a direct consequence of the decoupling theorem: calculate in the EFT without unnecessary reference to the UV physics. This is required as the UV physics is decoupled and simply removed from an EFT in a controlled fashion. Its IR effects are reproduced to a limited precision and encoded in the matching procedure in the Wilson coefficients of the EFT.

3.3 Non-decoupling physics

The decoupling theorem has some exceptions. This should be surprising considering the generality of the arguments that have been advanced in the previous sections. Calculating in field theories to an approximate precision, in the presence of separated scales, is usefully

[26] For example, for a detailed discussion and proof on decoupling for scalar field theory with two fields when $d = 6$ see Collins [87].

thought of using the techniques of EFT. Such EFT's are constructed based on the separation of scales that underlies decoupling. However, no theorem can escape the constraints of its exact wording and assumptions, and this is also true for the decoupling theorem. Several examples of "non-decoupling" effects are discussed in the literature. Heavy physics of this form does not imply that an EFT is impossible to construct to capture an IR limit of some UV physics. It just enforces the construction of the EFT to take on a particular form, usually by requiring that a non-linear representation of a symmetry be used.[27]

3.3.1 The ρ parameter

Non-decoupling effects can occur when heavy states and the light states are embedded in the same representations of a symmetry group in the full theory. Divergences can be forbidden by the linearly realized symmetry, due to cancellation between the particles of different masses embedded in such a (softly broken) representation of a symmetry group. When some of the states are no longer in the spectrum in the EFT, the counterterms are no longer forbidden by the linearly realized symmetry. Then perturbative corrections can grow with the mass of the state removed from the theory.

The practical signal of this physics can be the appearance of numerically larger perturbative corrections when the heavy state is still retained in the theory, and at times an additional mass dependence outside of logarithms in such corrections. This can be the case as in the loop corrections the masses sometimes act to regulate the divergences when the symmetry is linearly realized. Several historical examples of this form of non-decoupling are present in the literature, in νe scattering [90], in large $\mathcal{O}(\alpha_s)$ corrections (due to quark doublet mass splittings) to the axial neutral current [91] and in the behavior of one loop corrections [92–95] to the ratio of charged and neutral currents in the SM, due to the diagrams shown in Fig. 4.

We discuss this latter case of the ρ parameter [96], defined as the ratio of charged and neutral currents at low energies, as an example. The ρ parameter has the perturbative expansion, with one loop contributions shown in Fig.4 (in $\overline{\mathrm{MS}}$) which give

$$\rho \simeq \frac{\bar{g}_Z^2\, \bar{m}_W^2}{\bar{g}_2^2\, \bar{m}_Z^2} + \frac{N_c\, \hat{G}_F}{8\,\pi^2\sqrt{2}}\left(m_t^2 + m_b^2 - 2\frac{m_t^2\, m_b^2}{m_t^2 - m_b^2}\log\left(\frac{m_t^2}{m_b^2}\right)\right) - \frac{11\hat{G}_F\, \hat{M}_Z^2 s_{\hat{\theta}}^2}{24\sqrt{2}\pi^2}\log\left(\frac{m_h^2}{m_Z^2}\right). \tag{3.9}$$

The Higgs mass dependent correction is not exceedingly large and it is not related in mass to another particle in the spectrum by a linear realization of a symmetry. The limit $m_h \to \infty$ can be taken, which still leads to a non-linear realization of $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ as the Higgs field and its vacuum expectation value are related when this symmetry is linearly realized. The effective theory construction of Refs [97–101] results when the limit $m_h \to \infty$ is taken. The corrections due to the heavy Higgs matched onto this EFT are not suppressed by explicit powers of m_h^2 in their leading contributions. The full results of this form are given in Refs. [102, 103]. This is an example of non-decoupling effects that deviate from a naive expectation formed from the decoupling theorem.

[27] Again the existence of the SMEFT and the HEFT can be understood to be related to this fact, as non-decoupling effects in the scalar sector are a possibility.

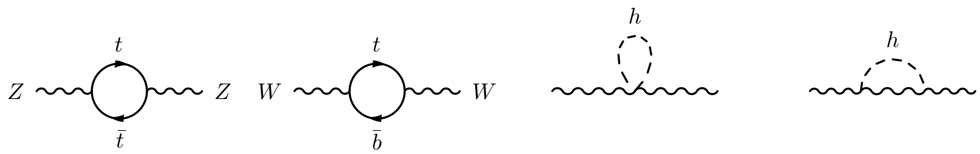


Figure 4. SM one loop corrections to the ρ parameter.

Even larger corrections come about due to splitting the quark masses in the limit $m_t \gg m_b$. Note that

$$m_t^2 + m_b^2 - 2\frac{m_t^2\, m_b^2}{m_t^2 - m_b^2}\log\left(\frac{m_t^2}{m_b^2}\right) \to 0, \tag{3.10}$$

in the limit $m_t \to m_b$. Integrating out the top, while leaving the b quark in the spectrum, leads to a theory without a linearly realized $\mathrm{SU}(2)_\mathrm{L}$ symmetry. Furthermore, m_t is acting to regulate an integral, which is a reason that it appears as a polynomial outside of the logarithm. As $m_t = y_t\, v/\sqrt{2}$ the limit $m_t \to \infty$ must correspond to $v \to \infty$, $y_t \to \infty$, or both. The former limit is interesting, as the corrections given Eqn. 3.10 vanish if $m_t/m_b \to 1$ as $v \to \infty$. Then $\mathrm{SU}(2)_\mathrm{L}$ can again be linearly realized. The limit $y_t \gg 1$ is a strong coupling limit, leading to a breakdown of perturbation theory. Then the decoupling theorem's implicit assumption of a valid perturbation theory no longer holds. In the case of the ρ parameter, the non-decoupling effects come about due to this strong coupling limit in addition to the differences in the realization of the symmetries of the full theory and the low energy EFT.[28]

3.3.2 Weak interactions

When exact symmetries are present in a subset of interactions in the EFT, such symmetries can first be broken explicitly by the heavy fields integrated out. Then the leading operator mediating a process can be due to the local contact operator correction to the EFT suppressed by m^2 (in the case of weak interactions a suppression by m_W^2), but with no *relative* suppression compared to any leading order effect, which is absent. This is another way in which non-decoupling effects can come about.

The weak interactions are an important example of this form of non-decoupling. The SM is defined in Section 4.1. Flavour violating effects in the SM due to the weak interactions having an intricate pattern that encodes non-decoupling physics of this form. In the limit that the Yukawa interactions of the SM vanish, $Y_{u,d,e} \to 0$ a $\mathrm{U}(3)^5$ global flavour symmetry group of the SM is present. We define this group through the relation between the weak (unprimed)

[28]In the case of the ρ parameter the corrections shown are also the leading violations of custodial symmetry, as an additional subtlety.

basis and the mass (primed) basis as

$$u_L = \mathcal{U}(u,L)\,u'_L, \qquad u_R = \mathcal{U}(u,R)\,u'_R, \qquad \nu_L = \mathcal{U}(\nu,L)\,\nu'_L, \tag{3.11}$$

$$d_L = \mathcal{U}(d,L)\,d'_L, \qquad d_R = \mathcal{U}(d,R)\,d'_R, \qquad e_L = \mathcal{U}(e,L)\,e'_L, \qquad e_R = \mathcal{U}(e,R)\,e'_R. \tag{3.12}$$

Each $\mathcal{U}$ rotation defines a U(3) flavour group. The $\mathrm{U}(3)^5$ group of the SM is defined as

$$\mathrm{U}(3)^5 = \mathcal{U}(u,R)\times\mathcal{U}(d,R)\times\mathcal{U}(Q,L)\times\mathcal{U}(\ell,L)\times\mathcal{U}(e,R). \tag{3.13}$$

The relative $\mathcal{U}$ rotations between components of the lepton and quark $\mathrm{SU}_L(2)$ doublet fields define the PMNS and CKM matrices as

$$V_{\rm CKM} = \mathcal{U}(u,L)^\dagger\,\mathcal{U}(d,L), \qquad U_{\rm PMNS} = \mathcal{U}(e,L)^\dagger\,\mathcal{U}(\nu,L). \tag{3.14}$$

Consistent with the discussion in the previous section, the unbroken $\mathrm{U}(3)^5$ flavour symmetry of the SM forbids divergences corresponding to flavour violating interactions.

The $\mathcal{U}(u,R)\times\mathcal{U}(d,R)\times\mathcal{U}(e,R)$ rotations commute with the weak interaction generators. At tree level the neutral current interactions to the left handed doublet fields (ψ_L) couple to the diagonal generators

$$\mathsf{y_i}\psi_L = y_i\begin{pmatrix}1&0\\0&1\end{pmatrix}\psi_L, \quad \tau^3\psi_L = \begin{pmatrix}1&0\\0&-1\end{pmatrix}\psi_L. \tag{3.15}$$

which also commutes with the $\mathcal{U}(Q,L)\times\mathcal{U}(\ell,L)$ rotations between the weak and mass eigenstates. No tree level flavour changing neutral currents follows.

This symmetry is broken when the charged currents that interact in the weak eigenbasis of the SM propagate, and quark mass differences are retained in the resulting amplitudes. This distinguishes the mass and weak eigenstates of the SM. Flavour violating effects come about due to the relative rotation of the SM states in ψ_L proportional to $V_{\rm CKM}, U_{\rm PMNS}$, that distinguishes the components of the ψ_L doublets, and appear in interactions proportional to $\tau^{1,2}$ through which the charged currents couple. This leads to flavour changing charged currents at tree level in the SM. Nevertheless, if all weak or mass eigenstates are summed over, and flavour violating spurions are neglected, the flavour symmetry is again restored due to the rotation between the eigenbases being unitary.

Consider the diagrams in Fig. 5 as an illustrative example. The leftmost diagram gives a contribution to the decay $K_L \to \mu^+\mu^-$.[29] Constructing the EFT useful for the measurement scale $\mu^2 \simeq m_K^2$, both the top quark and the W boson are not on-shell fields propagating for longer distances (compared to the measurement scale), and hence integrated out. The leading operator mediating the decay is given by [104]

$$\mathcal{L}_{K_L\to\mu^+\mu^-} = \frac{\hat{G}_F}{\sqrt{2}}\frac{\hat{\alpha}_{ew}}{2\,\pi s_{\hat{\theta}}^2}\,V_{ts}^\star\,V_{td}Y(x_t)\;(\bar{s}\,\gamma^\mu P_L\,d)\,(\bar{\mu}\,\gamma^\mu P_L\,\mu) + h.c + \cdots \tag{3.16}$$

[29] The Kaon mesons are defined by their quark content as $K^0 = d\,\bar{s}, \bar{K^0} = \bar{d}\,s$ and $K_L = (d\,\bar{s} - s\bar{d})/\sqrt{2}$, $K_S = (d\,\bar{s} + s\bar{d})/\sqrt{2}$.

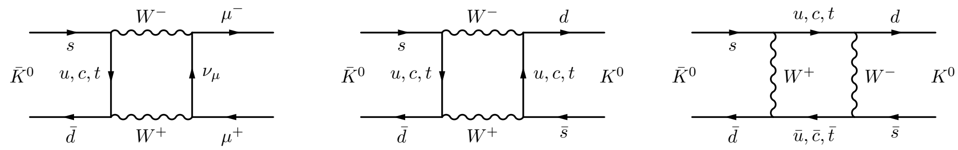


Figure 5. SM one loop corrections to kaon decay and mixing.

Here $x_t = m_t^2/m_W^2$ and retained is the contribution from the top quark in the loop that breaks flavour symmetry. We neglect higher order (and penguin diagram) contributions to the decay. See Refs. [105–108] for more discussion on such corrections.

The "non-decoupling" effects are indicated by the presence of polynomial powers of m_t^2 in the numerator, similar to the case of the ρ parameter. Again, integrating out the top quark will lead to a non-linearly realized $\mathrm{SU(2)_L}$ symmetry, but the situation is different than in the case of the ρ parameter, where the top mass scale also regulates the integral. Fig. 5 (left) is a naively convergent integral (in an appropriately chosen gauge). The $m_{t,c,u}$ dependence as a polynomial mass contribution outside of a logarithm. This dependence follows from the flavour breaking pattern of the SM matched onto the lower scale EFT.

The result reflects the Glashow-Iliopoulos-Maiani [109] (GIM) mechanism describing the relevant phenomenological suppression by powers of the quark masses in addition to the weak couplings as a result of the $\mathrm{U}(3)^5$ symmetry breaking pattern of the SM. The GIM mechanism is also a statement that in the limit of vanishing quark masses, Eqn. 3.16 (and similar flavour changing amplitudes for other processes) exactly vanish as due to unitarity

$$\sum_i V_{is}^\star V_{id} = 0. \tag{3.17}$$

One loop contributions to Kaon mixing also respect the GIM mechanism, and include the contributions shown in Fig. 5 (right two diagrams). Reproducing the SM amplitudes following from the weak interactions at low energies in an EFT is non-trivial. A proper treatment summing all logarithms integrates out a series of SM fields $\{h, W, Z, t, c\}$ in sequence, in the renormalization group evolution down to the experimental scale of $K^0 - \bar{K^0}$ mixing. For a modern discussion on the reproduction of the SM result in the low energy EFT see Ref. [62].

The sum of these graphs (combined with Goldstone boson diagrams for gauge independence) gives the leading result [72, 110–112]

$$\mathcal{L}_{K^0-\bar{K}^0} = \frac{\hat{G}_F}{\sqrt{2}} \frac{\hat{\alpha}_{ew}}{4\,\pi s_{\hat{\theta}}^2} \sum_i V_{is}^\star V_{id} \sum_j V_{js} V_{jd}^\star \bar{E}(x_i, x_j) \, (\bar{s}\,\gamma^\mu P_L\, d) \left(\bar{d}\,\gamma^\mu P_L\, s\right) + h.c + \cdots \tag{3.18}$$

where

$$\begin{aligned} \bar{E}(x_i, x_j) = & -x_i\, x_j \left(\frac{1}{x_i - x_j} \left[\frac{1}{4} - \frac{3}{2}\frac{1}{x_i - 1} - \frac{3}{4}\frac{1}{(x_i-1)^2} \right] \log x_i, \right. \\ & \left. + \frac{1}{x_j - x_i} \left[\frac{1}{4} - \frac{3}{2}\frac{1}{x_j - 1} - \frac{3}{4}\frac{1}{(x_j-1)^2} \right] \log x_j - \frac{3}{4}\frac{1}{(x_i-1)(x_j-1)} \right) \end{aligned} \tag{3.19}$$

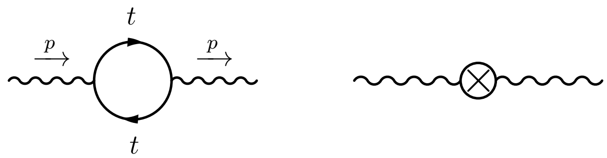


Figure 6. One loop contribution to the running of the QED coupling constant due to a fermion; shown is the case when the fermion is the top.

Again $x_{i,j} = m_{i,j}^2/m_W^2$ and these indicies sum over up quark flavours. We neglect here higher order corrections, see Refs. [72, 108, 110–112] for further discussion. The "non-decoupling" flavour breaking structure of the SM interactions is present in that the result is proportional to four powers of quark masses in Eqn. 3.18.

The many instances of non-decoupling effects in the SM should generate caution when choosing between the SMEFT and HEFT formalisms to capture the low energy limit of physics beyond the SM. The possibility of non-decoupling effects related to the discovered 0^+ boson with mass $\sim 125\,\mathrm{GeV}$ is pressing and well motivated. One of the key distinctions between the SMEFT and HEFT constructions is the latter is arguably more appropriate to capture the IR limit of such non-decoupling UV physics coupled to the 0^+ boson.

3.3.3 Renormalization scheme dependence and decoupling

A renormalization scheme is composed of a method to regulate divergent integrals, and a subtraction scheme choice. When relationships between physically measured S matrix elements are determined in perturbation theory, the regularization and subtraction scheme choice has no physical effect. When discussing renormalized Lagrangian parameters per se and intermediate results for observables in terms of these parameters, scheme dependence is present. Ref. [1] used a renormalization scheme which performs subtractions at an off-shell Euclidean momentum point. This approach to renormalization has the benefit of making decoupling manifest, which is not the case in dimensional regularization (DR) when $\overline{\mathrm{MS}}$ is used as a subtraction scheme.

Consider the Lagrangian for quantum electrodynamics (QED), and the running of the QED coupling e due to fermions ψ_f. The Lagrangian is given by

$$\mathcal{L}^0_{QED} = -\frac{1}{4}\, F_0^{\mu\,\nu}\, F^0_{\mu\,\nu} + \bar{\psi}^0_f \gamma_\mu (i\,\partial^\mu - e_0\, Q_f\, A_0^\mu)\,\psi^0_f - m_f^0\,\bar{\psi}^0_f\,\psi^0_f, \tag{3.20}$$

with $F_0^{\mu\,\nu} = \partial^\mu\, A_0^\nu - \partial^\nu\, A_0^\mu$ the QED field strength tensor composed of bare fields, indicated with 0 labels. The one loop diagram shown in Fig. 6 (left) gives

$$i\,\mathcal{A} = -e_0^2\, Q^2\, \mu^{4-d} \int \frac{d^d q}{(2\pi)^d} \frac{\mathrm{Tr}\left(\gamma^\mu(\not{q} + \not{p} + m_f)\gamma^\nu(\not{q} + m_f)\right)}{((q+p)^2 - m_f^2)(q^2 - m_f^2)}, \tag{3.21}$$

which is divergent. The bare fields are related to the renormalized fields (denoted with r labels) by introducing the renormalization constants Z_i

$$A_0^\nu = \sqrt{Z_A} A_r^\nu, \qquad \psi_f^0 = \sqrt{Z_{\psi_f}} \psi_f^r, \tag{3.22}$$

$$e^0 = Z_e \, \mu^\epsilon \, e^r, \qquad m_f^0 = Z_{m_f} \, m_f^r. \tag{3.23}$$

μ^ϵ is introduced as dimensional regularization with $d = 4 - 2\epsilon$ is used to regulate the divergent integrals. The counterterm that performs the subtraction for $\Pi_{AA}(p^2)$ is indicated in Fig. 6 (right) and gives

$$-\frac{i}{4} \, Z_A \, (p^\mu \, p^\nu - p^2 g^{\mu\,\nu}). \tag{3.24}$$

Choosing the manner in which the divergence is subtracted fixes Z_A and defines the subtraction scheme. Defining a renormalization condition for $\Pi_{AA}(p^2)$ where the diagram is subtracted at the Euclidean momentum point $p^2 = -M_f^2$ removes the divergence, and gives the sum for Fig. 6

$$i \, \mathcal{A}_{M_f} = -i \, \frac{e_0^2 \, Q^2 \, \mu^{2\epsilon}}{2 \, \pi^2} \, (p^\mu \, p^\nu - p^2 g^{\mu\,\nu}) \int_0^1 dx \log \frac{m_f^2 - p^2 \, x \, (1-x)}{m_f^2 + M_f^2 \, x \, (1-x)}. \tag{3.25}$$

Alternatively, the $\overline{\mathrm{MS}}$ scheme subtracts the ϵ poles and a set of constant terms due to the rescaling $\mu \to \mu \, (e^\gamma / 4\pi)^{1/2}$. Using $\overline{\mathrm{MS}}$ the sum for Fig. 6 is given by

$$i \, \mathcal{A}_{\overline{\mathrm{MS}}} = -i \, \frac{e_0^2 \, Q^2}{2 \, \pi^2} \, (p^\mu \, p^\nu - p^2 g^{\mu\,\nu}) \int_0^1 dx \log \frac{m_f^2 - p^2 \, x \, (1-x)}{\mu^2}. \tag{3.26}$$

In either case, the Ward identities of the theory due to unbroken $\mathrm{U}(1)_{\mathrm{em}}$ fix

$$Z_e = 1/\sqrt{Z_A}, \tag{3.27}$$

to order e_0^2. The bare coupling is independent of the renormalization scheme choice so expanding the derivative of the bare coupling with respect to M_f gives

$$\beta(e) = \frac{e_0^3 \, Q^2}{2 \, \pi^2} \int_0^1 dx \, \frac{M_f^2 \, x^2 \, (1-x)^2}{m_f^2 + M_f^2 \, x \, (1-x)}. \tag{3.28}$$

The running of $\beta(e)$ calculated in this manner exhibits manifest decoupling. For $m_f < M_f$ one finds

$$\beta(e) \simeq \frac{e_0^3 \, Q^2}{12 \, \pi^2}, \tag{3.29}$$

while for $M_f < m_f$ one has

$$\beta(e) \simeq \frac{e_0^3 \, Q^2}{60 \, \pi^2} \, \frac{M_f^2}{m_f^2}. \tag{3.30}$$

When renormalizing the theory for measurements made at scales $\sim -M_f^2 < m_f^2$, decoupling of the effects of the heavy fermion is manifest. For $\overline{\rm MS}$ one finds the result in Eqn. 3.28 for all μ. In this case, to implement decoupling appropriately one must set $\beta(e) \simeq 0$ for $\mu \lesssim m_f$ by hand.[30]

The effects of heavy particles contributing to an experimental measurement that do not propagate on shell are still encoded in local contact operators in the EFT, no matter what subtraction scheme is chosen. Connecting back to the initial regularization result in Eqn. 3.2, large logarithms can be present expanding this equation when using DR and $\overline{\rm MS}$, if $\mu^2 \ll m_f^2$ (or $\mu^2 \ll \Delta^2$ in the notation of Section 3.1), indicating the regularization of an amplitude. A poorly behaved perturbative expansion results if decoupling is not imposed by hand in this scheme.

Considering the requirement of modifying the beta functions by hand, it could be surprising that using $\overline{\rm MS}$ and DR is strongly preferred in modern EFT calculations. This renormalization scheme makes the power counting of the EFT manifest and directly preserved in loop calculations. This is an important technical simplification that overwhelms the drawback of having to impose decoupling by hand. See Section 3.5.1 for further discussion on this point.

3.4 Matching

An EFT's dynamics is defined without the need to extensively reference the details of any UV completion. This is fortunate, as the EFT and the UV completion are quite different. They do not have the same high energy behavior and each theory is renormalized separately, with a different set of counterterms.

An EFT is a self-consistent field theory capable of predicting S matrix elements for a range of energies where the expansion leading to the EFT is convergent. At times an EFT can be constructed to faithfully reproduce the predictions of a UV completion in a low energy limit. As the correspondence between the EFT and the UV theory is limited, when this is done, the theories must be matched to ensure the predictions agree. This procedure fixes the free parameters (the Wilson coefficients) of the tower of higher dimensional operators that make up $\mathcal{L}_{EFT}$. When the UV completion is weakly coupled, the matching procedure can be directly carried out in perturbation theory. To fix a set of n free parameters in the EFT, a set of n linearly independent S matrix elements[31] are calculated in both the EFT and in the UV completion. The results are equated in the IR limit that defines the EFT. Denote this limit as $p^2 \ll M^2$ with M^2 some heavy mass scale of a state in the UV completion and not in the EFT. Fixing

$$\langle p_1 \cdots p_a | S_{1..n} | k_1 \cdots k_b \rangle^{\rm UV}_{p^2 \ll M^2} \equiv \langle p_1 \cdots p_a | S_{1..n} | k_1 \cdots k_b \rangle^{\rm EFT}, \tag{3.31}$$

so defines the matching conditions that fixes the Wilson coefficients in terms of the parameters of the UV completion, and the couplings of the perturbative expansion. This procedure works

[30]This point holds for β functions, and also for other theoretical quantities such as the effective potential [113–115].

[31]We define the S matrix precisely below in Section 6.1.

at tree level, and order by order in perturbation theory where both UV and IR divergences can occur.

The divergences can be neglected in practice and the matching condition is defined by the finite parts of Eqn. 3.31. This follows from the UV divergences being subtracted by the corresponding counterterms on each side of Eqn. 3.31. The IR divergences correspond to the case $\mu^2 \gg \Delta^2 + q_E^2$ in Section 3.1. These divergences are also regulated in dimensional regularization but a key defining condition of the EFT construction is that the IR physics of the EFT and the UV completion is the same. Matching fixes the Wilson coefficients to reflect the short distance UV physics integrated out of the EFT. The reason is simple and intuitive, the IR physics that is not modified in transitioning from the UV theory to an EFT description cancels in the matching. This includes the IR divergences themselves. The correction that remains to match onto the Wilson coefficients is then only due to the UV physics integrated out of the EFT. When the matching procedure is carried out in DR and $\overline{\rm MS}$, the following technical simplifications also occur:

- Scaleless integrals vanish and the IR and UV divergences in such integrals cancel. A simple example of this is given by the wavefunction renormalization factor of a massless fermion in QCD with the ψ two point function having divergences
$$\frac{\alpha_s\, C_F}{4\,\pi}\, i\not{p} \left[\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right]. \tag{3.32}$$
This can simplify matching calculations dramatically, as diagrams that are scaleless in the EFT can be neglected when using DR.

- In calculating one loop matchings, expanding intermediate results in small ϵ, or dimensionless ratios, before Feynman parameter integrals are carried out, can be justified so long as the modifications in the results cancel in the matching condition.

- Quadratic divergences are represented as ϵ poles. Dimensionful threshold corrections in the matching conditions occur at tree level (in the EFT), and also as one loop running corrections to EFT parameters [116].

- Usually the matching conditions are evaluated at the scale of particles integrated out of the theory to define the EFT. This is not required, but is advantageous as it acts to minimize potentially large logs in the perturbatively expanded matching equations, when such matching is combined with Renormalization Group Evolution (RGE) running.

3.4.1 Matching examples

“Integrating out a field" as nomenclature follows from the path integral approach to defining an effective action as developed by Wilson [117, 118]. The effective action $S_{\rm eff}[\phi]$ retaining a light field ϕ, and removing the heavy field Φ is defined such that
$$e^{iS_{\rm eff}[\phi]} = \int d\Phi\, e^{iS[\phi,\Phi]} / \int d\Phi\, e^{iS[\phi,0]}\,, \tag{3.33}$$

where the integral is over all field values Φ. Hence the heavy field is “integrated out". At times this procedure can be carried out formally in the path integral while using the Equations of Motion (EOM) for the theory. Such manipulations can require the interactions to be of a limited form to be formally justified.

Integrating out a heavy field and determining the matching conditions for the Wilson coefficients at tree level can always be done using Feynman diagram techniques directly. Again the EOM are used and this approach can be easier in some cases than directly determining S matrix elements in the full and effective theories as in Eqn. 3.31, and solving the resulting system of equations.

As a set of examples, consider the case of a heavy SM singlet scalar field (S) and a heavy SM singlet (Weyl) fermion (N). The Lagrangian for the former case for $d \leq 4$ interactions is

$$\mathcal{L}_{SM+S} = \mathcal{L}_{SM} + \frac{1}{2}\left((\partial_\mu S)(\partial^\mu S) - m_S^2 S^2\right) - \frac{\kappa_1}{2}\, S^2 H^\dagger H - \Lambda_1\, S\, H^\dagger\, H - \Lambda_2\, S^3 - \kappa_2\, S^4 \tag{3.34}$$

The SM is defined in Section 4.1, $\kappa_{1,2}$ are dimensionless couplings while $\Lambda_{1,2}$ have mass dimension one. For field values $\langle H^\dagger\, H\rangle < \langle S^2\rangle < m_S^2$ and $p^2 < m_S^2$ one can solve the equation of motion for S and Taylor expand around the classical solution finding

$$S \simeq -\frac{\Lambda_1\, H^\dagger H}{m_S^2} + \cdots\,, \tag{3.35}$$

which substituted back into the initial Lagrangian gives

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \frac{\Lambda_1^2}{2\, m_S^2}\,(H^\dagger\, H)^2 + \mathcal{L}^{(6)} + \cdots\,. \tag{3.36}$$

The leading correction term shown can be absorbed into a finite shift of the SM Higgs self coupling, consistent with the decoupling theorem. The terms in $\mathcal{L}_6$ are given by

$$\mathcal{L}^{(6)} = -\frac{\Lambda_1^2}{m_S^4}\,\mathcal{Q}_{H\Box} + \left(\frac{\Lambda_2\,\Lambda_1}{m_S^2} - \frac{\kappa_1}{2}\right)\frac{\Lambda_1^2}{m_S^4}\,\mathcal{Q}_H \tag{3.37}$$

using the Warsaw basis for $\mathcal{L}_6$.[32] The matching contributions are organized due to the IR operator forms that result, not naive scalings in m_S. The coefficients of terms in $\mathcal{L}_6$ are expected to be overall $\propto 1/m_S^2$. The naive expectation of the ordering of the operator forms in powers of m_S is generically upset due to the presence of dimensionful couplings, as purposefully illustrated here. Naturalness considerations imply that $\Lambda_{1,2}^2 \lesssim m_S^2\, 16\,\pi^2$. When this bound is saturated, large Wilson coefficients result. Even when the bound is not saturated, the presence of such dimensionful couplings can lead to $\mathcal{O}(1)$ Wilson coefficients ($\times 1/m_S^2$). This is a particular concern when considering matching to strongly interacting UV physics sectors, below the scale present in the confining phase of such a sector. Such dimensionful couplings can then be present in the interactions of the resulting bound states, unless forbidden by

[32]See Section 4.2.1 for details on basis choice and $\mathcal{L}_6$.

a symmetry, and can directly upset any intuition based on perturbative matching in a UV coupling $g^\star$ and taking a limit $g^\star \to 4\,\pi$.[33]

As another example, consider the case of a SM singlet Weyl fermion integrated out in the UV. This scenario corresponds to the Seesaw model [119–122] for generating massive Neutrino's, and has been studied in an EFT context in Refs.[123–133]. The Lagrangian can be defined as $\mathcal{L}_{SM} + \mathcal{L}_{N_p}$ where

$$2\,\mathcal{L}_{N_p} = \overline{N_p}(i\partial\!\!\!/ - m_p)N_p - \overline{\ell_L^{\beta}}\tilde{H}\omega_\beta^{p,\dagger}N_p - \overline{\ell_L^{c\beta}}\tilde{H}^*\,\omega_\beta^{p,T}N_p - \overline{N_p}\,\omega_\beta^{p,*}\tilde{H}^T\ell_L^{c\beta} - \overline{N_p}\,\omega_\beta^{p}\tilde{H}^\dagger\ell_L^{\beta} \tag{3.38}$$

The couplings $\omega_\beta^p = \{x_\beta, y_\beta, z_\beta\}$ are complex vectors in flavour space that absorbed the Majorana phases. $p = \{1,2,3\}$ is summed over. Integrating out the N_p at tree level by taking the $p^2 < m_p^2$ limit of the tree level exchange diagram gives the result $\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \cdots$ where

$$\mathcal{L}^{(5)} = \frac{c_{\beta\,\kappa}}{2}\left(\overline{\ell_L^{c,\beta}}\,\tilde{H}^\star\right)\left(\tilde{H}^\dagger\,\ell_L^\kappa\right) + h.c. \tag{3.39}$$

and $c_{\beta\,\kappa} = (\omega_\beta^p)^T\,\omega_\kappa^p/m_p$. The matching is onto the leading correction to the SM dimension four Lagrangian [134, 135]. See Ref. [136] for notational details. Expanding the result around the vacuum expectation value for the Higgs field gives experimentally required Neutrino masses.

Calculating higher order corrections to the matching results can also be directly determined using standard Feynman diagram techniques. Conversely, a naive path integral approach to integrating out a field can be practically limited to leading order calculations. Determining higher order perturbative matching corrections is straightforward in the case of SM singlet fields in a UV sector. The required loop corrections are only present on the left or the right hand side of Eqn. 3.31 due to the different symmetry groups of the SM and the UV sector in this case. Alternatively, when UV field content integrated out is charged under the SM gauge groups, loop corrections in the EFT and in the full theory (on both sides of Eqn. 3.31) are required at each order in perturbation theory. The S matrix elements are calculated to higher orders in perturbation theory in the full theory and the EFT. UV divergences are canceled by counterterms dictated by the subtraction and regularization scheme chosen. IR divergences and constant terms cancel in the matching calculations, and the Wilson coefficients are then determined to the desired order by the UV physics removed from the EFT. Higher order terms in the operator expansion (which is usually referred to as the non-perturbative expansion of the EFT in the literature) can also be determined using these techniques, and mixed perturbative and non-perturbative contributions.

For a sample of excellent examples of matching see Ref. [137–146]. Of particular note, recently a comprehensive tree level matching dictionary has been constructed in Ref. [147].

3.4.2 Covariant Derivative Expansion matching

Matching typically requires the computation of a large number of diagrams in the full theory and the EFT, which is done choosing a convenient gauge, and subsequently recombining the

[33] See Section 5.1.2 for more discussion on such an approach.

results into gauge invariant effective operators. This can be cumbersome when determining matching calculations to higher orders in the expansions present in the EFT. A recently developed technique, that goes under the name of the Covariant Derivative Expansion (CDE), is aimed at simplifying this computation by resorting to more advanced functional methods. This technique has two main advantages: it does not require the evaluation of Feynman diagrams because the matching is done at the action level and, at the same time, it seeks to preserve manifest gauge invariance at all the stages of the calculation. The CDE method has been introduced in the modern EFT context in Ref. [139] reviving an approach previously explored in the 80's [148, 149] for other applications. In the following we summarize the main argument of [139]. At the action level, the matching of a theory containing both heavy fields Φ and light fields ϕ onto an EFT that describes only the ϕ degrees of freedom again amounts to constructing an effective action $S_{\text{eff}}[\phi]$ such that

$$e^{iS_{\text{eff}}[\phi]} = \int d\Phi\, e^{iS[\phi,\Phi]} \Big/ \int d\Phi\, e^{iS[\phi,0]} \,. \tag{3.40}$$

Using a saddle-point approximation, which is valid for a perturbative expansion up to one loop, and expanding the heavy fields around their background values $\Phi = \Phi_c + \eta$ one obtains

$$S_{\text{eff.}}[\phi] \simeq S(\Phi_c) + \frac{i}{2}\,\text{Tr}\log\left(- \left.\frac{\delta^2 S}{\delta \Phi^2}\right|_{\Phi_c}\right), \tag{3.41}$$

where the first term contains the structures obtained integrating out the heavy field in tree level diagrams, while the second encodes the contributions generated at one-loop. Eqn. 3.41 can be evaluated explicitly assuming a generic (universal) structure for the Lagrangian of the UV model. For example, if the heavy field is a complex scalar, one has

$$\mathcal{L}_{UV} \supseteq -\Phi^\dagger(D^2 + M^2 + U(x))\Phi + \Big(\Phi^\dagger B(x) + \text{h.c.}\Big) + \mathcal{O}(\Phi^3) \tag{3.42}$$

where $D_\mu = \partial_\mu - iA_\mu$ is a covariant derivative and $U(x)$, $B(x)$ are arbitrary model-dependent expressions containing light fields. The tree-level matching contribution $S(\Phi_c)$ is derived in a standard fashion replacing $\Phi \to \Phi_c$ in $\mathcal{L}_{UV}$, where Φ_c is the solution of the EOM for Φ, and expanding the resulting Lagrangian in inverse powers of M. The final result is

$$\begin{aligned}\Delta\mathcal{L}_{\text{eff,tree}} =& - B^\dagger \left[-D^2 - M^2 - U\right]^{-1} B + \mathcal{O}(\Phi_c^3)\\ \simeq& \, B^\dagger M^{-2} B + B^\dagger M^{-2}[-D^2 - U]M^{-2}B + \ldots\end{aligned} \tag{3.43}$$

where we have dropped the x dependence. Note that the field Φ is generally a multiplet, so that the mass term M is a matrix, which does not necessarily commute with $(D^2 + U)$.

The evaluation of the one loop piece is slightly more involved. The most general result can be found in Ref. [139] together with a detailed derivation. The final expression obtained expanding up to dimension 6 is

$$\begin{aligned}
\Delta\mathcal{L}_{\text{eff,1-loop}} = \frac{c_s}{(4\pi)^2}\,\text{Tr}\,\Bigg\{ &+ M^4\left[-\frac{1}{2}\left(\log\frac{M^2}{\mu^2}-\frac{3}{2}\right)\right] + M^2\left[-\left(\log\frac{M^2}{\mu^2}-1\right)U\right] \\
&+ M^0\left[-\frac{1}{12}\left(\log\frac{M^2}{\mu^2}-1\right)G'^2_{\mu\nu} - \frac{1}{2}\log\frac{M^2}{\mu^2}\,U^2\right] \\
&+ \frac{1}{M^2}\left[\frac{1}{60}\left(D_\mu G'_{\mu\nu}\right)^2 - \frac{1}{90}G'_{\mu\nu}G'_{\nu\sigma}G'_{\sigma\mu} + \frac{1}{12}\left(D_\mu U\right)^2 - \frac{1}{6}U^3 - \frac{1}{12}UG'_{\mu\nu}G'_{\mu\nu}\right] \\
&+ \frac{1}{M^4}\Bigg[\frac{1}{24}U^4 - \frac{1}{12}U\left(D_\mu U\right)^2 + \frac{1}{120}\left(D^2U\right)^2 + \frac{1}{24}\left(U^2G'_{\mu\nu}G'_{\mu\nu}\right) \\
&\qquad + \frac{1}{120}\left[(D_\mu U),(D_\nu U)\right]G'_{\mu\nu} - \frac{1}{120}\left[U[U,G'_{\mu\nu}]\right]G'_{\mu\nu}\Bigg] \\
&+ \frac{1}{M^6}\left[-\frac{1}{60}U^5 + \frac{1}{20}U^2\left(D_\mu U\right)^2 + \frac{1}{30}\left(UD_\mu U\right)^2\right] + \frac{1}{M^8}\left[\frac{1}{120}U^6\right]\Bigg\}.
\end{aligned} \tag{3.44}$$

Here $c_s = \{1/2, 1\}$ for a real and complex scalar Φ respectively and $G'_{\mu\nu} = [D_\mu, D_\nu]$. Finally, the trace is over internal indices, i.e. Lorentz, flavour, gauge indices etc. Inserting into Eqn. 3.44 the expressions of U and $G_{\mu\nu}$ defined in a specific model, one immediately obtains a sum of dimension six operators whose coefficients are automatically matched with the UV model. Eqn. 3.44 is universal in the sense that it can be applied not only to the case of scalar Φ but also when the heavy field is a fermion or vector boson, as detailed in Ref. [139]. Although extremely practical, this expression has two main defects:

1. It holds only in the case of degenerate heavy states, in which the mass matrix M is diagonal and commutes with the other structures;

2. The $\Delta\mathcal{L}_{\text{eff,1-loop}}$ computed in this way accounts only for loop diagrams in which all the internal lines are heavy. Mixed heavy-light loops are missing because the light field have been treated as background fields and therefore only enter as external lines [150, 151].

Point 1 was addressed in Ref. [152], that generalized Eqn. 3.44 to the case of a non-degenerate multiplet Φ obtaining an expression for the effective action at one loop that was named the UOLEA (Universal One Loop Effective Action). Point 2 represents a deeper problem in the basic CDE technique described above, which requires a modification of the functional treatment. Different solutions have been proposed in Refs. [153–155]. Both Ref. [153] and Ref. [154] expand the functional analysis of Ref. [139] with the inclusion of the fluctuations around the background fields for the light degrees of freedom ϕ and both suggest a method that requires the subtraction of non-local terms from the functional determinant. This step is avoided with the alternative method proposed in Refs. [155], that builds upon Refs. [103, 156] and employs the "expansion-by-regions" technique for the evaluation of the loop integrals

[157–159]. One defines the multiplet $\varphi = (\Phi, \phi)$ and

$$\Delta\mathcal{L}_{\text{eff,1-loop}} = \frac{1}{2}\varphi^\dagger \left.\frac{\delta^2\mathcal{L}_{UV}}{\delta\varphi^*\delta\varphi}\right|_{\varphi_c} = \varphi^\dagger \begin{pmatrix} \Delta_H & X^\dagger_{HL} \\ X_{HL} & \Delta_L \end{pmatrix} \varphi, \tag{3.45}$$

where the last term contains a block matrix so that the heavy fields Φ are contracted by Δ_H, the light ones by Δ_L and X_{HL} is a mixed term. The key idea of Ref. [155] is to perform a field transformation that brings the matrix to a block diagonal form diag$(\tilde{\Delta}_H, \Delta_L)$, shifting the effect of X_{HL} into the heavy-particle contribution while leaving the Δ_L unchanged. This procedure gives $\tilde{\Delta}_H = \Delta_H - X^\dagger_{HL}\Delta_L^{-1}X_{HL}$ which, in the scalar case, can be expressed in the notation of Ref. [139] as

$$\tilde{\Delta}_H = -(D^2 + M^2 + \tilde{U}), \qquad\qquad \tilde{U} = U(x) + U_{HL}(x,p)\,, \tag{3.46}$$

where U_{HL} comes from the field redefinition and carries a dependence on the loop momentum p. The effective action takes the form $S_{\text{eff}}[\phi] = ic_s \,\mathrm{Tr}\log\tilde{\Delta}_H$ and it generates all the loop diagrams with at least one heavy internal propagator. The desired result for the mixed heavy-light one loop contributions to the EFT matching are obtained performing the loop momentum integrals in $\tilde{\Delta}_H$ only in the "hard" region, i.e. first expanding out all the low-energy scales, that are small in the limit $p \sim M$, and then integrating over the full d-dimensional p space. This method makes use of dimensional regularization and is known as "integration-by-regions" [157–159].[34] As a result, the contribution to the dimension six effective Lagrangian in Eqn. 3.44 is extended by the inclusion of [155]:

$$\begin{aligned}
\Delta\mathcal{L}^{HL}_{\text{eff,1-loop}} = -ic_s \int \frac{d^dp}{(2\pi)^d}\Bigg\{ & \frac{1}{p^2-M^2}\,\mathrm{tr_s}\left(\tilde{U}\right) + \frac{1}{2}\frac{1}{(p^2-M^2)^2}\,\mathrm{tr_s}\left(\tilde{U}^2\right) \\
& + \frac{1}{3}\frac{1}{(p^2-M^2)^3}\left[\mathrm{tr_s}\left(\tilde{U}^3\right) + \mathrm{tr_s}\left(\tilde{U}D^2\tilde{U}\right) + 2ip^\mu\,\mathrm{tr_s}\left(\tilde{U}D_\mu\tilde{U}\right)\right] \\
& + \frac{1}{4}\frac{1}{(p^2-M^2)^4}\Big[\mathrm{tr_s}\left(\tilde{U}^4\right) + 2ip^\mu\,\mathrm{tr_s}\left(\tilde{U}^2D_\mu\tilde{U}\right) + 2ip^\mu\,\mathrm{tr_s}\left(\tilde{U}D_\mu\tilde{U}^2\right) \\
& \qquad + \mathrm{tr_s}\left(\tilde{U}^2D^2\tilde{U}\right) + \mathrm{tr_s}\left(\tilde{U}D^2\tilde{U}^2\right) - 4\,p^\mu p^\nu\,\mathrm{tr_s}\left(\tilde{U}D_\mu D_\nu\tilde{U}\right) \\
& \qquad + 2ip^\mu\,\mathrm{tr_s}\left(\tilde{U}D^2D_\mu\tilde{U}\right) + 2ip^\mu\,\mathrm{tr_s}\left(\tilde{U}D_\mu D^2\tilde{U}\right) + \mathrm{tr_s}\left(\tilde{U}(D^2)^2\,\tilde{U}\right)\Big] \\
& + \frac{1}{5}\frac{1}{(p^2-M^2)^5}\Big[\mathrm{tr_s}\left(\tilde{U}^5\right) + 2ip^\mu\,\mathrm{tr_s}\left(\tilde{U}^3D_\mu\tilde{U}\right) + 2ip^\mu\,\mathrm{tr_s}\left(\tilde{U}^2D_\mu\tilde{U}^2\right) \\
& \qquad + 2ip^\mu\,\mathrm{tr_s}\left(\tilde{U}D_\mu\tilde{U}^3\right) - 4p^\mu p^\nu\,\mathrm{tr_s}\left(\tilde{U}^2D_\mu D_\nu\tilde{U}\right) \\
& \qquad - 4p^\mu p^\nu\,\mathrm{tr_s}\left(\tilde{U}D_\mu\tilde{U}D_\nu\tilde{U}\right) - 4p^\mu p^\nu\,\mathrm{tr_s}\left(\tilde{U}D_\mu D_\nu\tilde{U}^2\right) \\
& \qquad - 8i\,p^\mu p^\nu p^\rho\,\mathrm{tr_s}\left(\tilde{U}D_\mu D_\nu D_\rho\tilde{U}\right\}\Big]\Bigg\} + \mathcal{L}^F_{\text{EFT}} + \mathcal{O}\left(M^{-3}\right)\,.
\end{aligned} \tag{3.47}$$

[34] This approach has been debated in the literature and reservations have been raised on this technique in an EFT context. See for example the discussion in Ref. [160].

Here $\mathrm{tr_s}$ is a subtracted trace defined as

$$\mathrm{tr_s} f(\tilde{U}, D_\mu) \equiv \mathrm{tr}\left(f(\tilde{U}, D_\mu) - f(U, D_\mu) - \Theta_f \right) , \tag{3.48}$$

where f is an arbitrary function of $\tilde{U}$ and covariant derivatives, while Θ_f generically denotes all the terms with covariant derivatives at the rightmost of the original trace. The subtraction of the terms containing only U avoids a double counting, as these contributions are already contained in Eqn. 3.44. Finally, $\mathcal{L}^F_{\mathrm{EFT}}$ contains all the terms containing open derivatives, that eventually combine into operators with field-strength tensors. The evaluation of this piece truncated at dimension-six terms is quite complex: a universal expression, although achievable in principle, is not yet available to date.

Although not definitive, the results presented in Ref. [155] represented a key step in the development of CDE techniques. In particular, they highlighted that the heavy-light structures could be directly inferred from the heavy-only ones and they paved the way for the development of a covariant diagrammatic formalism [161]. The latter allows an immediate and efficient evaluation of functional quantities in terms of gauge-invariant operators, while providing a graphical representation that keeps track of the CDE expansion[35]. This powerful formalism has been adopted recently in Ref. [162] to compute explicitly all the universal terms (operators and coefficients) in the UOLEA functional in [152], with the addition of the heavy-light ones and for both the degenerate and non-degenerate cases. These results extend the construction in Ref. [152] and supersede those of Ref. [155], providing universal expressions for CDE matching that can be applied to a given model without the need of reiterating the functional procedure.

As mentioned above, the evaluation of open derivative terms is still missing at this stage, but it is expected to become available in the near future, again thanks to covariant diagrams techniques. The same holds for mixed bosonic-fermionic loops. The calculation of these two categories of effects will complete the "UOLEA program" to supply a completely universal, gauge-invariant result for EFT matching.

The power of the CDE approach has recently been illustrated in many examples in the literature. A stand out example is the demonstration of how the known one loop matching MSSM results of Ref. [163] can be elegantly determined using the modern CDE approach [164].

3.5 Choose any scheme, so long as it is dimensional regularization and $\overline{\mathrm{MS}}$

In matching calculations, divergences can be dropped as the UV divergences in each theory are canceled by UV counterterms, and the IR divergences in the full theory and the EFT coincide by definition. If such divergences are retained in the calculation, they must be regulated. Any physical conclusion is independent of a regulator choice, in the limit the regulator is taken to infinity.[36] Nevertheless, the ease of obtaining physical conclusions, and performing loop

[35] Analogously, Feynman diagrams allow to compute correlation functions and make manifest the organization of different contributions in a perturbative expansion.

[36] For a recent discussion on this point in more detail see Ref.[165].

calculations depends on the regulator choice. The discussions in the previous sections were made using DR. Using a dimensionless regulator is now standard in most EFT studies, and can be essential in EFT studies of the Higgs boson. The reason for this is that the SM is classically scaleless in the limit $v \to 0$ ($m_h \to 0$ as a result) and a dimensionful regulator explicitly breaks this (anomalous) symmetry by introducing a dimensionful cut off scale.

It took decades for the benefits of DR to be fully appreciated in the EFT community. This was due to some history and some misunderstanding. The history is due to the key initial ideas of the RG emerging into broad use[37] following the pioneering condensed matter studies of Kadanoff [167], Wilson [117, 118] and Wilson and Fisher [168]. The partition function in the early cases studied using the RG was constructed out of a reduced number of degrees of freedom by integrating out a series of high energy modes as

$$Z = \int^{\Lambda_1} \mathcal{D}\,\phi_i^1 \, e^{-S_1(\phi_i^2)} = \int^{\Lambda_2} \mathcal{D}\,\phi_i^2 \, e^{-S_2(\phi_i^2)} \cdots = \int^{\Lambda_n} \mathcal{D}\,\phi_i^n \, e^{-S_n(\phi_i^n)}, \tag{3.49}$$

where $\Lambda^n > \Lambda^{n-1}$. At each step in integrating out a momentum shell, the action is matched by fixing its parameters and redefining the fields, to reproduce the low energy physics of the action with the higher energy modes removed. The differential version of this procedure gives the RG Equations which are local and analytic, as can be understood from the general arguments of the previous sections. That the initial applications [117, 118, 167] of the RG was formulated with a dimensionful regulator makes perfect sense as the modes integrated out were discretizing physically separated spin configurations, with a corresponding dimensionful Fourier transformed momentum. The misunderstanding that slowed the adoption of DR was the perception that as it integrates over all momenta, it does not faithfully limit the EFT to momenta where it is defined, introducing an inconsistency. This is incorrect, as discussed in Section 3.

A key step forward in the development of EFTs was the realization that such a dimensionful regularization is not necessary, and best avoided. The main reasons for this are that such regulators make power counting suspect, and calculations technically challenging.

3.5.1 Z decay and dimensionful regulators

The problems of dimensionful regulators are well illustrated by the example of four fermion operators generating the decay $Z \to \bar{\ell}\,\ell$ at one loop. The results dependent on y_t are known [169] in the $\mathrm{U}(3)^5$ limit. The Effective Lagrangian generated has the terms

$$\mathcal{L}_{Z,eff} = -\,2\,2^{1/4}\,\sqrt{\hat{G}_F}\,\hat{m}_Z\,\bar{\ell}_s\,\gamma_\mu \left[(\bar{g}_L^\ell)_{ss}\,P_L + (\bar{g}_R^\ell)_{ss}\,P_R\right]\ell_s, \tag{3.50}$$

where at one loop the four fermion operators give a correction [169]

$$\Delta(g_L^\ell)_{ss} = \frac{N_c\,\hat{m}_t^2}{16\,\pi^2\,\Lambda^2}\,\log\left[\frac{\mu^2}{\hat{m}_t^2}\right]\,\left[C^{(1)}_{\substack{\ell q\\ ss33}} - C^{(3)}_{\substack{\ell q\\ ss33}} - C_{\substack{\ell u\\ ss33}}\right], \tag{3.51}$$

$$\Delta(g_R^\ell)_{ss} = \frac{N_c\,\hat{m}_t^2}{16\,\pi^2\,\Lambda^2}\,\log\left[\frac{\mu^2}{\hat{m}_t^2}\right],\,\left[-C_{\substack{eu\\ ss33}} + C_{\substack{qe\\ 33ss}}\right]. \tag{3.52}$$

[37] A precursor of these ideas were presented by Stuckelberg and Petermann in a prescient work [166].

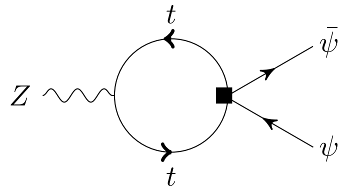


Figure 7. Four fermion operator contribution to $Z \to e^+ e^-$ at one loop.

Here we are using the notation of Ref. [169] where hat superscripts correspond to measured quantities at tree level. These results come about in DR and $\overline{\rm MS}$ in the following manner. The anomalous dimensions required were determined in Ref. [170] using DR and exploiting the Background Field method [171–173]. This method preserves the symmetries of the SMEFT when gauge fixing as fields in the action are split into classical (ϕ) and quantum (Q) components

$$S(Q) \to S(Q + \phi). \tag{3.53}$$

A gauge fixing term then breaks the gauge invariance of the quantum fields while maintaining the gauge invariance of the classical background fields, this makes gauge independent counterterms easier to determine. The Background Field method can be directly implemented in DR which also preserves the symmetries of the Lagrangian [172]. The direct calculation of Fig. 7 leads to contributions of the form

$$\begin{aligned} i\,\mathcal{A} \propto\; & 4\,N_c\,(\bar{g}^{\ell}_{L,SM}\,\bar{g}_{L,Q} + \bar{g}^{\ell}_{R,SM}\,\bar{g}_{R,Q}) \int \frac{d^D l}{(2\pi)^D} \frac{(D-2)}{D} \frac{l^2}{(l^2 - \hat{m}_t^2)^2}, \\ & - 4\,N_c\,(\bar{g}^{\ell}_{L,SM}\,\bar{g}_{R,Q} + \bar{g}^{\ell}_{R,SM}\,\bar{g}_{L,Q}) \int \frac{d^D l}{(2\pi)^D} \frac{\hat{m}_t^2}{(l^2 - \hat{m}_t^2)^2}. \end{aligned} \tag{3.54}$$

The notation $\bar{g}_{L/R,Q}$ corresponds to $P_{L/R}$ in the four fermion operators Q_i inserted in the loop diagrams. In DR and $\overline{\rm MS}$ the only scale in the loops to make up the dimensions of the $1/\Lambda^2$ suppression is the top quark mass, and the results combine in a non-trivial manner to cancel the pole of Ref. [170], leaving the finite terms in Eqn. 3.51.

In the case of a dimensionful regulator where $D = 4$ a cut off is introduced to the loop momentum $\sim \Lambda$. Generally such regulators violate gauge invariance, as they regulate p^μ and not D^μ. Furthermore, translation invariance of the momenta in the propagators can be broken, which stands in the way of using the Feynman/Schwinger trick to combine propagators in loop calculations. This makes identifying gauge independent counter-terms a challenge and the use of the Background Field method unfeasible. Assuming that the gauge invariant anomalous dimensions were determined using a dimensionful regulator (somehow), the first term in Eqn. 3.54 gives

$$\int^{\Lambda} \frac{d^4 l}{(2\,\pi)^4} \frac{l^2}{(l^2 - \hat{m}_t^2)^2} \simeq \frac{\Lambda^2}{16\,\pi^2}. \tag{3.55}$$

The Λ^2 dependence acts to cancel the $1/\Lambda^2$ suppression of the operators in $\bar{g}_{L/R,Q}$. This results in an $\mathcal{O}(1)$ shift of the amplitude, due to a violation of the power counting.[38] Dimensionful regulators are a nuisance to be avoided.

3.5.2 Avoiding regulator dependence

In DR, power counting violations due to the regulator choice do not occur in the insertion of higher dimensional operators in loop diagrams. The μ scale introduced in the regulation procedure only appears in the Logs as in Eqn. 3.51. Large mass scales that are present in the UV theory can appear outside of Logs, in threshold matching corrections to $(H^\dagger\, H)$, as shown in Section 2. This is the appearance of the Hierarchy problem using DR.

The Hierarchy problem is the need to stabilize the scale invariance violating coefficient of $(H^\dagger\, H)$ against perturbations proportional to scales $\Lambda \gg \hat{m}_h$. Using a hard cut off regulator makes it challenging to disentangle the perturbation to $(H^\dagger\, H)$ due to UV physics from unphysical regulator dependence. Explicit breaking of scale invariance by the regulator is unfortunate, as the mass parameter of the Higgs is the only classical source of the violation of scale invariance in the SM.[39] This can lead to a different point of view as to what the Hierarchy problem is, and what can solve the Hierarchy problem.

A regulator dependent argument can be formulated focused on the effect of the top quark on the operator $(H^\dagger H)$ at one loop. The relevant diagram is the leftmost entry in Fig. 1 which gives a coefficient to this operator of the form

$$i\,\mathcal{A} = -\frac{|y_t|^2\,N_c}{2}\int^\Lambda \frac{d^4l}{(2\pi)^4}\frac{l^2+4\,\hat{m}_t^2}{(l^2-\hat{m}_t^2)^2}, \tag{3.56}$$

$$= -\frac{|y_t|^2\,N_c\,\Lambda^2}{32\,\pi^2}+\cdots \tag{3.57}$$

Such a regulated quadratic divergence in the case of hard cut off can be compared to DR where the mass scale in the loop is $\hat{m}_t^2$ and the ϵ pole is subtracted. The appearance of the quadratic divergence can be interpreted as a signal of the Hierarchy problem that is regulator independent, when it is indicating a threshold correction in a manner consistent with a dimensionless regulator such as DR.

The standard conclusion that TeV scale states are motivated to appear in multiplets that stabilize $(H^\dagger H)$ is valid and regulator independent. In DR, this is the statement that one loop threshold corrections to $(H^\dagger H)$ scale $\propto$ $(y_a^2\,m_a^2 \pm y_b^2\,m_b^2)/(16\,\pi^2)$ where $m_{a,b}$ are states in such a multiplet with couplings $y_{a,b}$ to H. Symmetries can be built into models to suppress such threshold corrections, and stabilize the dimension two operator. For example, unbroken supersymmetry by construction fixes $y_a = y_b$ and $m_a = m_b$ with the difference in sign between terms being present due to the different spin states in supermultiplets. In a regulator independent manner, a precision Higgs phenomenology program is well motivated

[38] DR is not without its own challenges, in particular the definition of γ_5 in d dimensions requires a scheme choice. See the recent discussion in Ref. [169] on the appearance of this issue in Fig. 7.

[39] See Refs. [174–181] for related discussion on scale invariance and the Hierarchy problem.

to search for the low energy signatures of physics beyond the SM that acts to stabilize the Higgs mass.

4 Candidate field theories: the SM, SMEFT and HEFT

The main field theories discussed in this review used to interpret LHC, LEP and other low energy data, are the SM, the SMEFT or the HEFT. The choice of which field theory to use is distinguished by the assumption on the size of possible new physics effects compared to the achievable experimental resolution (ΔE_r) at current and future facilities, and an assessment of the propagating states in the particle spectrum. By using the SM, one assumes that it will always hold when interpreting the data that

$$\Delta E_r \gg \frac{C_i\, v^2}{g_{SM}\, \Lambda^2}, \frac{C_i\, p^2}{g_{SM}\, \Lambda^2}. \tag{4.1}$$

Here each C_i is a Wilson coefficient in the SMEFT or HEFT that corresponds to the “pole expansion” ratio v^2/Λ^2 or the derivative expansion ratio p^2/Λ^2. We define the SM to fix our notation in Section 4.1. Both the HEFT and the SMEFT follow from the expectation that it is possible that

$$\Delta E_r \lesssim \frac{C_i\, v^2}{g_{SM}\, \Lambda^2}, \frac{C_i\, p^2}{g_{SM}\, \Lambda^2}, \tag{4.2}$$

will occur in the near future, or in the longer term. This assumption is reasonable to adopt. These EFTs are further distinguished by the presence of a Higgs doublet (or not) in the construction. In the SMEFT (see Section 4.2) the EFT is constructed with an explicit Higgs doublet, while in the HEFT (see Section 4.3) no such doublet is included.

4.1 The Standard Model

We define the SM Lagrangian [182–184], with conventions consistent with Refs. [116, 170, 185], as

$$\begin{aligned}\mathcal{L}_{\rm SM} = & -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \sum_{\psi=q,u,d,\ell,e} \overline{\psi}\, i\not{D}\, \psi \\ & + (D_\mu H)^\dagger(D^\mu H) - \lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2 - \left[H^{\dagger j}\overline{d}\, Y_d\, q_j + \widetilde{H}^{\dagger j}\overline{u}\, Y_u\, q_j + H^{\dagger j}\overline{e}\, Y_e\, \ell_j + \text{h.c.}\right],\end{aligned} \tag{4.3}$$

where H is an $\mathrm{SU_L}(2)$ scalar doublet.[40] With this normalization convention, the Higgs boson mass is $m_H^2 = 2\,\lambda\, v^2$. The vacuum expectation value (vev) acts to break $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1) \to$

[40] The alert reader will notice the lack of dual field strength terms of the form $\mathrm{Tr}\left[F^{\mu\,\nu}\tilde{F}_{\mu\,\nu}\right]$ for the Yang-Mills fields $F = \{W, G\}$, which can be present [186, 187]. Here and below the dual fields are defined with the convention $\widetilde{F}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ with $\epsilon_{0123} = +1$. The measurements of the electric dipole moment of the neutron indicate that such topological terms [188–190] are strongly suppressed for QCD, and the accidental conservation of $B + L$ in the SM allows the neglect of such terms for electroweak theory in the SM [189, 191].

$\mathrm{U_{em}}(1)$ and is defined as $\langle H^\dagger H\rangle = v^2/2$ in the SM, with $v \sim 246$ GeV. The gauge covariant derivative is defined by the states its $\mathrm{SU_c}(3)\times\mathrm{SU_L}(2)\times\mathrm{U_Y}(1)$ generators act on, and we use the conventional form

$$D_\mu = \partial_\mu + i g_3 T^A A^A_\mu + i g_2 t^I W^I_\mu + i g_1 \mathsf{y}_\mathbf{i} B_\mu. \tag{4.4}$$

The $\mathsf{y}_\mathbf{i}$ is the $\mathrm{U_Y}(1)$ hypercharge generator. The T^A are the $\mathrm{SU_c}(3)$ generators, the Gell-Mann matrices, with normalization $\mathrm{Tr}(T^A T^B) = 2\delta^{AB}$. The $t^I = \tau^I/2$ are the $\mathrm{SU_L}(2)$ generators, the Pauli matrices, taken to be

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4.5}$$

For example, H has hypercharge $\mathsf{y}_H = 1/2$, is a $\mathrm{SU_c}(3)$ singlet and a $\mathrm{SU_L}(2)$ doublet so that D acting on H is given by the matrix equation $D_\mu H = (\partial_\mu + i g_2 t^I W^I_\mu + i g_1 B_\mu/2)H$. $\mathrm{SU_L}(2)$ indices are usually denoted as $\{i,j,k\}$ and $\{I,J,K\}$ in the fundamental and adjoint representations respectively. The $\mathrm{SU_c}(3)$ indices in the adjoint representation are instead denoted as $\{A,B,C\}$, each of which runs from $\{1..8\}$. $\widetilde{H}$ is defined by $\widetilde{H}_j = \epsilon_{jk} H^{\dagger\, k}$ where the $\mathrm{SU_L}(2)$ invariant tensor ϵ_{jk} is defined by $\epsilon_{12} = 1$ and $\epsilon_{jk} = -\epsilon_{kj}$, $j,k = \{1,2\}$.

All fermion fields have a suppressed flavour index in Eqn. (4.3). We conventionally denote these indicies by $\{p,r,s,t\}$ that each run over $\{1,2,3\}$ for the three generations. The fermion mass matrices are $M_{u,d,e} = Y_{u,d,e}\, v/\sqrt{2}$. $Y_{u,d,e}$ and $M_{u,d,e}$ are complex Yukawa matrices in flavour space. Explicitly reintroducing flavour indicies one has

$$H^{\dagger j}\, \overline{d}\, Y_d\, q_j = H^{\dagger j}\, \overline{d}_p\, [Y_d]_{pr}\, q_{rj}. \tag{4.6}$$

The fermion fields q and ℓ are left-handed fields, i.e. they transform as $(1/2,0)$ under the restricted Lorentz group $\mathrm{SO}^+(3,1)$.[41] The u, d and e are right-handed fields and transform as $(0,1/2)$. The chiral projectors have the convention $\psi_{L/R} = P_{L/R}\,\psi$ where $P_{R/L} = (1 \pm \gamma_5)/2$. The matter fields of the SM have the charges and representations

Field	$\mathrm{SU_c}(3)$	$\mathrm{SU_L}(2)$	$\mathrm{U_Y}(1)$	$\mathrm{SO}^+(3,1)$
$q_i = (u^i_L, d^i_L)^T$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$(1/2,0)$
$u_i = \{u_R, c_R, t_R\}$	$\mathbf{3}$	$\mathbf{1}$	$2/3$	$(0,1/2)$
$d_i = \{d_R, s_R, b_R\}$	$\mathbf{3}$	$\mathbf{1}$	$-1/3$	$(0,1/2)$
$\ell_i = (\nu^i_L, e^i_L)^T$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$(1/2,0)$
$e_i = \{e_R, \mu_R, \tau_R\}$	$\mathbf{1}$	$\mathbf{1}$	-1	$(0,1/2)$
H	$\mathbf{1}$	$\mathbf{2}$	$1/2$	$(0,0)$

(4.7)

[41] Formally the spinors are defined as finite dimensional representations of the orthochronous Lorentz transformations. The label (a,b) corresponds to the spin of the representation, where the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ group that is locally isomorphic to $\mathrm{SO}^+(3,1)$ has been used to label the 2 component Weyl spinor subgroups of a four component Dirac spinor. As the Lorentz group is connected it is related to the universal cover group $\mathrm{SL}(2,\mathbf{C})$ which is sometimes itself referred to as the Lorentz group in other applications.

The fields in Eqn. 4.3 are in the weak eigenbasis, where

$$q_1 = \begin{bmatrix} u_L \\ d'_L \end{bmatrix}, \quad q_2 = \begin{bmatrix} c_L \\ s'_L \end{bmatrix}, \quad q_3 = \begin{bmatrix} t_L \\ b'_L \end{bmatrix}, \quad \ell_1 = \begin{bmatrix} \nu_L^{e\,\prime} \\ e_L \end{bmatrix}, \quad \ell_2 = \begin{bmatrix} \nu_L^{\mu\,\prime} \\ \mu_L \end{bmatrix}, \quad \ell_3 = \begin{bmatrix} \nu_L^{\tau\,\prime} \\ \tau_L \end{bmatrix}, \tag{4.8}$$

$$\begin{bmatrix} d'_L \\ s'_L \\ b'_L \end{bmatrix} = V_{\rm CKM} \begin{bmatrix} d_L \\ s_L \\ b_L \end{bmatrix}, \qquad\qquad \begin{bmatrix} \nu_L^{e\,\prime} \\ \nu_L^{\mu\,\prime} \\ \nu_L^{\tau\,\prime} \end{bmatrix} = U_{\rm PMNS} \begin{bmatrix} \nu_L^{e} \\ \nu_L^{\mu} \\ \nu_L^{\tau} \end{bmatrix}. \tag{4.9}$$

To date, the SM is a successful EFT for physics at and below the energies probed by LHC $\sqrt{s} \lesssim 3\,\mathrm{TeV}$. The SM does not explain the experimental evidence for dark matter [192–194], Baryogenesis [195, 196] and neutrino masses [197–201]. These experimental facts, and the theoretical arguments of Section 2, argue for embedding the SM in a more complete model of fundamental interactions and particles – of some unknown form.

4.1.1 The Standard Model EOM

The SM equations of motion (EOM) play a role in the choice of a SMEFT operator basis, and the removal of redundant operators. We summarize the well known SM EOM here, which follow from demanding a stationary action $S_{SM} = \int \mathcal{L}_{SM}\, d^4x$ with respect to a variation due to each SM field. We follow the notation and presentation of Ref. [116]. For the Higgs field one has

$$D^2 H_k - \lambda v^2 H_k + 2\lambda (H^\dagger H) H_k + \overline{q}^j\, Y_u^\dagger\, u \epsilon_{jk} + \overline{d}\, Y_d\, q_k + \overline{e}\, Y_e\, l_k = 0\,, \tag{4.10}$$

while the fermion field EOM are given by

$$\begin{aligned} &i\not{D}\, q_j = Y_u^\dagger\, u\, \widetilde{H}_j + Y_d^\dagger\, d\, H_j\,, \qquad && i\not{D}\, d = Y_d\, q_j\, H^{\dagger\, j}\,, \qquad && i\not{D}\, u = Y_u\, q_j\, \widetilde{H}^{\dagger\, j}\,, \\ &i\not{D}\, l_j = Y_e^\dagger\, e H_j\,, && i\not{D}\, e = Y_e\, l_j H^{\dagger\, j}\,, \end{aligned} \tag{4.11}$$

and the gauge field EOM are given as

$$[D^\alpha, G_{\alpha\beta}]^A = g_3 j_\beta^A, \qquad [D^\alpha, W_{\alpha\beta}]^I = g_2 j_\beta^I, \qquad D^\alpha B_{\alpha\beta} = g_1 j_\beta. \tag{4.12}$$

Note that $[D^\alpha, F_{\alpha\beta}]$ is the covariant derivative in the adjoint representation in the notation above. Hermitian derivative notation is introduced as

$$H^\dagger\, i\overleftrightarrow{D}_\beta H = i H^\dagger (D_\beta H) - i (D_\beta H)^\dagger H\,, \quad H^\dagger\, i\overleftrightarrow{D}^I_\beta H = i H^\dagger \tau^I (D_\beta H) - i (D_\beta H)^\dagger \tau^I H. \tag{4.13}$$

Using this notation, the gauge currents are

$$\begin{aligned} j_\beta &= \sum_{\psi=u,d,q,e,l} \overline{\psi}\, \mathsf{y}_i \gamma_\beta \psi + \frac{1}{2} H^\dagger\, i\overleftrightarrow{D}_\beta H, \qquad j_\beta^I = \frac{1}{2} \overline{q}\, \tau^I \gamma_\beta q + \frac{1}{2} \overline{l}\, \tau^I \gamma_\beta l + \frac{1}{2} H^\dagger\, i\overleftrightarrow{D}^I_\beta H\,, \\ j_\beta^A &= \sum_{\psi'=u,d,q} \overline{\psi}\, T^A \gamma_\beta \psi. \end{aligned} \tag{4.14}$$

We use the notation $\psi = \{u, d, q, e, l\}$ to sum over all SM fermions, and $V = \{B, W, G\}$ to sum over the SM gauge fields. Note that these EOM have corrections due to $\mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \cdots$ in the SMEFT, that must be included for a consistent matching to higher orders in the non-perturbative expansion. Such corrections are also $\mathcal{L}^{(n)}$ basis dependent.

4.2 The Standard Model Effective Field Theory

The SMEFT is a consistent EFT generalization of the SM constructed out of a series of $\mathrm{SU_C}(3) \times \mathrm{SU_L}(2) \times \mathrm{U_Y}(1)$ invariant higher dimensional operators, built out of SM fields and including an H field as defined in Table 4.7. The idea of the SMEFT is that extensions to the SM are assumed to involve massive particles heavier than the measured vev, which sets the scale (up to coupling suppression) of the SM states. In addition, it is assumed that any non-perturbative matching effects are characterized by a scale parametrically separated from the EW scale and the observed Higgs-like boson is embedded in the $\mathrm{SU_L}(2)$ Higgs doublet.

The SMEFT follows from these assumptions and is defined as

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + ..., \qquad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4. \tag{4.15}$$

The operators $Q_i^{(d)}$ are suppressed by $d-4$ powers of the cutoff scale Λ, and the $C_i^{(d)}$ are the Wilson coefficients. The number of non-redundant operators in $\mathcal{L}^{(5)}$, $\mathcal{L}^{(6)}$, $\mathcal{L}^{(7)}$ and $\mathcal{L}^{(8)}$ is known [134, 135, 202–207]. Furthermore, the general algorithm to determine operator bases at higher orders developed in Refs. [206–209] makes the SMEFT defined to all orders in the expansion in local operators. Note that when transitioning to the SMEFT, symmetry arguments leading to a neglect of dual field strength terms in $\mathcal{L}_{SM}$ should be reformulated, as such terms multiplied by $(H^\dagger H)$ appear in $\mathcal{L}_6$. The dual field strength terms should not be casually neglected.

In the SMEFT $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1) \to \mathrm{U}(1)_{\mathrm{em}}$ is Higgsed as in the SM. The minimum of the Higgs potential is now determined including the effect of the operator $Q_H \equiv \left(H^\dagger H\right)^3$, which modifies the scalar doublet potential to the form [185]

$$V(H^\dagger H) = \lambda \left(H^\dagger H - \frac{1}{2} v^2\right)^2 - C_H \left(H^\dagger H\right)^3, \tag{4.16}$$

yielding the new minimum

$$\langle H^\dagger H \rangle = \frac{v^2}{2} \left(1 + \frac{3 C_H v^2}{4\lambda}\right) \equiv \frac{1}{2} v_T^2, \tag{4.17}$$

on expanding the exact solution $(\lambda - \sqrt{\lambda^2 - 3 C_H \lambda v^2})/(3 C_H)$ to first order in C_H. This expansion assumes a mass gap to the scale(s) of new physics (referred to schematically as Λ) which leads to the expansion parameter $\sim C_H \, \bar{v}_T^2/\Lambda^2 < 1$. The dependence on Λ was suppressed in the previous equations. We absorb the cut off scale into the Wilson coefficients as a notational choice unless otherwise noted.

The SMEFT is an enormously powerful consistent field theory to use to characterize the low energy limit of physics beyond the SM. Even if a full model extension of the SM becomes experimentally supported in the future, the SMEFT can still be a useful and appropriate tool to use to interface with large swaths of experimental data below the characteristic scale(s) Λ of a new physics sector. It cannot be emphasized too strongly that the systematic development of this framework is expected to have an important return on investment of the time expended on it. The payoff in terms of improved scientific conclusions being enabled from the ever growing data set of measurements of SM states below the scale Λ is clear. This payoff can be starkly contrasted to the return on time invested when developing the predictions of a particular model, or even a set of models, if the many assumptions of the model are not experimentally validated. Considering the current global data set of particle physics, adopting the IR assumptions that define the SMEFT seems to be a very reasonable compromise between utility and generality of the theoretical framework assumed to accommodate the certain fact that the SM is an incomplete description of reality, while the LHC data set is indicating some degree of decoupling is present to the scales involved with extending the SM.

4.2.1 Operator bases in the SMEFT

In this work we use the first[42] non-redundant operator basis for $\mathcal{L}^{(6)}$ determined in the literature, as given in Ref. [203]. This construction has come to be known as the "Warsaw basis". To fix notation we include the complete summary of the baryon number conserving operators in this basis in Table 1, defined using notational conventions consistent with the previous sections.

The development of the Warsaw basis [203] underlies the systematic development of the SMEFT in recent years. It is a surprising fact that the full reduction of an operator basis for $\mathcal{L}^{(6)}$ to a non-redundant form took to 2010, a full twenty four years after Ref. [202] was published. Preceding the development of the Warsaw basis, innumerable works employed subsets of operators to perform phenomenological studies of the low energy effects of various models. Examples of this form of analysis include Refs. [212–219]. In the literature, the Buchmuller and Wyler result [202] is frequently referred to as an operator basis, we note that it is fully specified and well defined, but overcomplete, unlike the Warsaw basis. In addition the SILH subset of operators [218] is sometimes referred to as an operator basis in some literature, as is the HISZ subset of operators [212]. This is perhaps due to the influential nature of these works. To avoid confusion we stress that neither of these works contains a complete set of $\mathcal{L}^{(6)}$ operators, and certainly not a well defined minimal non-redundant basis. Subtleties involving flavour indicies are present in extending the subset of operators in Ref. [218] to a full basis, see Ref. [185] for a discussion. As a result, the emergence of a full basis including these "SILH operators" required some further efforts to resolve these issues [220, 221]. A full basis was defined in Ref. [221] for the first time incorporating these flavour index subtleties.

[42]This basis was built using the foundation laid down in Refs. [202, 210, 211].

$1: X^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^*\left(H^\dagger D_\mu H\right)$

$5: \psi^2H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu}B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$
Q_{HWB}	$H^\dagger\tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger\tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2XH + \text{h.c.}$	
Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)H\, B_{\mu\nu}$

$7: \psi^2H^2D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p\gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{l}_p\tau^I\gamma^\mu l_r)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p\gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{q}_p\tau^I\gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p\gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p\gamma^\mu d_r)$

$8: (\bar{L}L)(\bar{L}L)$	
$Q_{\ell\ell}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$
$Q_{\ell q}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{\ell q}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$

$8: (\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$
Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$
Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8: (\bar{L}L)(\bar{R}R)$	
Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8: (\bar{L}R)(\bar{R}L) + \text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

$8: (\bar{L}R)(\bar{L}R) + \text{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$

$8: (\bar{L}R)(\bar{L}R) + \text{h.c.}$	
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\epsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)$

Table 1. $\mathcal{L}_6$ of Refs. [203] as given in Ref. [185]. The flavour labels p, r, s, t on the Q operators are suppressed on the left hand side of the tables.

4.2.2 Removing redundant operators

Ref. [210] reported a complete set of operators that satisfies $\mathrm{SU_c}(3) \times \mathrm{SU_L}(2) \times \mathrm{U_Y}(1)$ symmetry. This was the first step in constructing an $\mathcal{L}_6$ operator basis, but such a construction is overcomplete. Combinations of operators are present, whose Wilson coefficients vanish when observables are calculated. This is due to the EOM relating the field variables when external states go on-shell. Making small field redefinitions on the SM fields $\mathcal{O}(1/\Lambda^2)$ one can remove the combination of $\mathcal{L}_6$ operators that will vanish in this manner directly in the Lagrangian, instead of having the cancellation occur when the S matrix element is constructed.[43] When this is done, the SM fields have a meaning that is contextual in the SMEFT and are fixed in a particular operator basis with a chosen set of $\mathcal{L}_6$ operators. The set of $\mathcal{O}(1/\Lambda^2)$ bosonic field redefinitions that preserve $\mathrm{SU_c}(3) \times \mathrm{SU_L}(2) \times \mathrm{U_Y}(1)$ are[44]

$$H'_j \to H_j + h_1 \frac{D^2 H_j}{\Lambda^2} + h_2 \frac{\bar{e}\,\ell_j\, Y_e}{\Lambda^2} + h_3 \frac{\bar{d}\, q_j\, Y_d}{\Lambda^2} + h_4 \frac{(\bar{u}\epsilon\, q_j)^\star\, Y_u^\star}{\Lambda^2} + h_5 \frac{H^\dagger H\, H_j}{\Lambda^2}, \tag{4.18}$$

$$B'_\mu \to B_\mu + b_1 \frac{\bar{\psi}\gamma_\mu\psi}{\Lambda^2} + b_2 \frac{H^\dagger\, i \overleftrightarrow{D}_\mu H}{\Lambda^2} + b_3 \frac{D^\alpha B_{\alpha\mu}}{\Lambda^2} + b_4\, \frac{H^\dagger H\, B_\mu}{\Lambda^2}, \tag{4.19}$$

$$W^{I\prime}_\mu \to W^I_\mu + w_1 \frac{\bar{q}\sigma^I\gamma_\mu q}{\Lambda^2} + w_2 \frac{\bar{\ell}\sigma^I\gamma_\mu \ell}{\Lambda^2} + w_3 \frac{H^\dagger \overleftrightarrow{D}^I_\mu H}{\Lambda^2} + w_4 \frac{[D^\alpha, W_{\alpha\mu}]^I}{\Lambda^2} + w_5 \frac{H^\dagger H W^I_\mu}{\Lambda^2}, \tag{4.20}$$

$$G^{A\prime}_\mu \to G^A_\mu + g_1 \frac{\bar{q}T^A\gamma_\mu q}{\Lambda^2} + g_2 \frac{\bar{d}T^A\gamma_\mu d}{\Lambda^2} + g_3 \frac{\bar{u}T^A\gamma_\mu u}{\Lambda^2} + g_4 \frac{[D^\alpha, G_{\alpha\mu}]^A}{\Lambda^2} + g_5\, \frac{H^\dagger H\, G^A_\mu}{\Lambda^2}, \tag{4.21}$$

while the corresponding transformations on the right handed fermion fields are

$$e' \to e + e_1 \frac{\bar{\ell} i \not{D} H Y_e^\dagger}{\Lambda^2} + e_2 \frac{\bar{\ell} i \overleftarrow{\not{D}} H Y_e^\dagger}{\Lambda^2} + e_3 \frac{H^\dagger H e}{\Lambda^2} + e_4 \frac{D^2 e}{\Lambda^2}, \tag{4.22}$$

$$d' \to d + d_1 \frac{\bar{q} i \not{D} H Y_d^\dagger}{\Lambda^2} + d_2 \frac{\bar{q} i \overleftarrow{\not{D}} H Y_d^\dagger}{\Lambda^2} + d_3 \frac{H^\dagger H d}{\Lambda^2} + d_4 \frac{D^2 d}{\Lambda^2}, \tag{4.23}$$

$$u' \to u + u_1 \frac{\bar{q} i \not{D} \tilde{H} Y_u^\dagger}{\Lambda^2} + u_2 \frac{\bar{q} i \overleftarrow{\not{D}} \tilde{H} Y_u^\dagger}{\Lambda^2} + u_3 \frac{H^\dagger H u}{\Lambda^2} + u_4 \frac{D^2 u}{\Lambda^2}, \tag{4.24}$$

and finally the redefinitions of the left handed fermion fields are

$$q'_j \to q_j + q_1 \frac{u i \not{D} \tilde{H}_j Y_u^\dagger}{\Lambda^2} + q_2 \frac{u i \overleftarrow{\not{D}} \tilde{H}_j Y_u^\dagger}{\Lambda^2} + q_3 \frac{d i \not{D} H_j Y_d^\dagger}{\Lambda^2} + q_4 \frac{d i \overleftarrow{\not{D}} H_j Y_d^\dagger}{\Lambda^2} + q_5 \frac{H^\dagger H q_j}{\Lambda^2} + q_4 \frac{D^2 q_j}{\Lambda^2}, \tag{4.25}$$

$$\ell'_j \to \ell + l_1 \frac{e i \not{D} H_j Y_e^\dagger}{\Lambda^2} + l_2 \frac{e i \overleftarrow{\not{D}} H_j Y_e^\dagger}{\Lambda^2} + l_3 \frac{H^\dagger H \ell_j}{\Lambda^2} + l_4 \frac{D^2 \ell_j}{\Lambda^2}. \tag{4.26}$$

[43] This procedure can be understood heuristically as aligning the field variables in the SMEFT more directly with classical (asymptotic) poles and the residues of the propagators at these poles, when the external particles go on-shell.

[44] Here we restrict ourselves to field redefinitions that have only dynamical field content, neglecting explicit v^2/Λ^2 corrections.

Here $\{h_a, b_a, w_a, e_a, d_a, u_a, q_a, l_a\}$ are free variables. Performing field redefinitions with only a single $\mathcal{O}(1/\Lambda^2)$ term on the right hand side of each equation one can choose to cancel an operator out of a full set of operators reported in Ref. [210]. This is known as removing redundant operators. An example of this procedure is as follows. The B^μ dependent, flavour symmetric terms in an overcomplete $\mathcal{L}_{SMEFT}$ are

$$
\begin{aligned}
\mathcal{L}_{B'} = &-\frac{1}{4}B'_{\mu\nu}B'^{\mu\nu} - g_1\, \mathsf{y}_\psi\, \overline{\psi}\, \not{B}'\, \psi + (D^\mu H)^\dagger (D_\mu H) + \mathcal{C}_B (H^\dagger \overleftrightarrow{D}^\mu H)(D^\nu B_{\mu\nu}), \\
&+ \mathcal{C}_{BH}(D^\mu H)^\dagger\, (D^\nu H)\, B'_{\mu\nu} + C^{(1)}_{\underset{tt}{Hl}} Q^{(1)}_{\underset{tt}{Hl}} + C_{\underset{tt}{He}}\, Q_{\underset{tt}{He}} + C^{(1)}_{\underset{tt}{Hq}} Q^{(1)}_{\underset{tt}{Hq}} + C_{\underset{tt}{Hu}}\, Q_{\underset{tt}{Hu}}, \\
&+ C_{\underset{tt}{Hd}}\, Q_{\underset{tt}{Hd}} + C_{HB}\, Q_{HB} + C_T\, (H^\dagger \overleftrightarrow{D}^\mu H)\, (H^\dagger \overleftrightarrow{D}^\mu H).
\end{aligned} \tag{4.27}
$$

Performing the small field redefinition

$$
B'_\mu \to B_\mu + b_2 \frac{H^\dagger\, i \overleftrightarrow{D}_\mu H}{\Lambda^2}, \tag{4.28}
$$

yields the result $\mathcal{L}_B - g_1\, b_2 \Delta B$ where

$$
\begin{aligned}
\Delta B = &\, \mathsf{y}_l Q^{(1)}_{\underset{tt}{Hl}} + \mathsf{y}_e Q_{\underset{tt}{He}} + \mathsf{y}_q Q^{(1)}_{\underset{tt}{Hq}} + \mathsf{y}_u Q_{\underset{tt}{Hu}} + \mathsf{y}_d Q_{\underset{tt}{Hd}}, \\
&+ \mathsf{y}_H\, (Q_{H\Box} + 4\, Q_{HD}) + \frac{1}{g_1} B^{\mu\nu} \partial_\mu (H^\dagger i \overleftrightarrow{D}_\nu H).
\end{aligned} \tag{4.29}
$$

Choosing b_2 to cancel one of the $\mathcal{L}_{B'}$ operators introduces a shift in the Wilson coefficients of the remaining operators. When the full set of such field redefinitions has been performed, this corresponds to choosing a non-redundant basis. The removal of redundant operators is always done in a gauge independent manner, as this procedure is only justified by the invariance of observables (i.e. S matrix elements) under gauge independent field redefinitions.[45]

Many field redefinitions are possible and Eqn. 4.18-4.25 can introduce or remove the same operators. It is essential to have a gauge independent algorithm to employ to systematically remove operator forms to obtain a minimal non-redundant basis. In the Warsaw basis, the algorithm is the systematic removal of derivative operators and an equally careful application of Fierz identities to reduce out redundant four fermion operators (see also Ref. [211]). This reflects the approach of an on-shell EFT construction [222, 223], so named because the on-shell EOM are used to reduce out explicit factors of $D^2 H$ and $\not{D}\psi$ in the higher order terms. The derivative removing algorithm of the Warsaw basis was used to help develop the results defining higher order corrections in the SMEFT operator expansion reported in Refs. [206–209]. Building on past works enumerating flavour invariants in the SM [224, 225] and group invariants in SUSY theories [226] it has been found to be beneficial to employ Hilbert series and a conformal algebra to systematize the counting of the number of operators at even higher orders than $\mathcal{L}_6$ in the SMEFT, while systematically removing derivative operators as in the Warsaw basis construction. Finally, the Warsaw basis algorithm was also essential to enabling

[45] See Section 6.1 for more details.

the one loop renormalization of the operators in $\mathcal{L}_6$, that was developed in Refs. [116, 170, 185, 227, 228], as discussed in Section 4.2.4.

Although the Warsaw basis removes derivative terms systematically, not all of the derivative invariants acting on H can be removed with such $\mathcal{O}(1/\Lambda^2)$ field redefinitions. For example, $\Box H$ does not appear in Eqn. 4.18 as it does not satisfy $\mathrm{SU_L}(2)_\mathrm{L} \times \mathrm{U_Y}(1)$ invariance as H is not a singlet. The distinction between $D^2 H$ and $\Box H$ is a way to understand the relevance of the scalar manifold topology in determining what derivative terms can be removed.[46] Defining the doublet field H to be decomposed into the real scalar fields $\vec{\phi}^T = \{\phi_1, \phi_2, \phi_3, \phi_4\}$ as

$$H = \begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix}, \tag{4.30}$$

the Lagrangian derivative terms can be expressed as

$$\mathcal{L}_{derv} = \frac{1}{2}(\partial_\mu \vec{\phi}) \cdot (\partial^\mu \vec{\phi}) + \frac{C_{H\Box}}{\Lambda^2} \vec{\phi}^2 \Box \vec{\phi}^2 + \frac{C_{HD}}{\Lambda^2} (\vec{\phi} \cdot (\partial^\mu \vec{\phi}))^2 + \cdots \tag{4.31}$$

This defines a target space metric for the scalar manifold that acts on $\partial^\mu \phi_i \partial_\mu \phi_j / 2$ as

$$R_{ij} = \delta_{ij} + 2\frac{\phi_i\, \phi_j}{\Lambda^2}(C_{HD} - 4 C_{H\Box}) + \cdots \tag{4.32}$$

The Riemann tensor R^i_{jkl} associated with this manifold can be directly determined from R_{ij}, and it does not vanish [229–231]. Since the target space metric is not flat, there does not exist a field redefinition which everywhere sets $R_{ij} = \delta_{ij}$. Not all of the H self interaction derivative terms can be removed with a gauge independent field redefinition as a result.

When expanding around the vev in unitary gauge, a distinction between $D^2 h$ and $\Box h$ is absent. As expected, the on-shell effective field theory approach of removing all p^2 invariants for h can be achieved with a *gauge dependent* field redefinition, and h can be canonically normalized for calculations with [185, 232]

$$h \to h\Big(1 + (C_{H\Box} - \frac{1}{4} C_{HD}) \bar{v}_T^2 \Big(1 + \frac{h}{\bar{v}_T} + \frac{h^2}{3\bar{v}_T^2}\Big)\Big). \tag{4.33}$$

This distinction between the ability to remove derivative terms on h and not H is problematic as $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1)$ is Higgsed, and one uses $\mathrm{SU_L}(2)$ global symmetry rotations to rotate the theory to a form where only the h field takes on a vev. This is also done when including corrections in the EFT. If this symmetry is broken by assumption, or an ill defined Lagrangian convention choice, a mismatch between the vev and the fluctuations around the vev can be introduced in a gauge dependent manner. This in turn can render a parameterization of new physics effects intrinsically gauge dependent and unsuitable to use in the event that real (gauge independent) deviations from the SM are found using EFT techniques. Such a parameterization can break structures in the EFT intrinsic to its construction and well defined

[46] It is possible to misunderstand Ref. [223] on this point as only a singlet scalar field is discussed in detail.

nature, leading to stronger constraints that are misleading on the Wilson coefficient space. See Section 7.1.4 for a discussion on such a structure - that is known as a SMEFT reparameterization invariance, which when not broken by assumption, or an inconsistent parameterization, requires that combination of data sets be used to lift degeneracies between parameters. This issue can change bounds on Wilson coefficients by orders of magnitude [233], so great care should be taken to avoid introducing inconsistencies of this form in SMEFT studies.

In addition to preforming the field redefinitions in Eqns. 4.18-4.25 one can directly translate between operator bases once a minimal non-redundant basis is found. This is done by constructing $\mathcal{L}_6$ operator relations directly out of the SM EOM's in Section 4.1.1 and then employing them to transition between bases. As illustrated by Eqn. 4.29 there is a gauge independent field redefinition that underlies the EOM relations in Eqn. 4.35.[47] An example of these relationships is between the flavour singlet operator forms appearing in Ref. [218]

$$\begin{aligned}
\mathcal{P}_{HW} &= -i\, g_2\, (D^\mu H)^\dagger\, \tau^I\, (D^\nu H)\, W^I_{\mu\nu}, & \mathcal{P}_{HB} &= -i\, g_1\, (D^\mu H)^\dagger\, (D^\nu H)\, B_{\mu\nu}, \\
\mathcal{P}_{W} &= -\frac{i\, g_2}{2}\, (H^\dagger\, \tau^I \overleftrightarrow{D}^\mu H)\, (D^\nu W^I_{\mu\nu}), & \mathcal{P}_{B} &= -\frac{i\, g_1}{2}\, (H^\dagger \overleftrightarrow{D}^\mu H)\, (D^\nu B_{\mu\nu}), \\
\mathcal{P}_{T} &= (H^\dagger \overleftrightarrow{D}^\mu H)\, (H^\dagger \overleftrightarrow{D}^\mu H), & &
\end{aligned} \tag{4.34}$$

and the Warsaw basis operators in Table 1, given by [185]

$$\begin{aligned}
\mathcal{P}_B &= \frac{1}{2}\mathsf{y}_H g_1^2 Q_{H\Box} + 2 g_1^2 \mathsf{y}_H Q_{HD} + \frac{1}{2} g_1^2 \left[\mathsf{y}_l Q^{(1)}_{\underset{tt}{Hl}} + \mathsf{y}_e Q_{\underset{tt}{He}} + \mathsf{y}_q Q^{(1)}_{\underset{tt}{Hq}} + \mathsf{y}_u Q_{\underset{tt}{Hu}} + \mathsf{y}_d Q_{\underset{tt}{Hd}}\right], \\
\mathcal{P}_W &= \frac{3}{4} g_2^2 Q_{H\Box} - \frac{1}{2} g_2^2 m_H^2 (H^\dagger H)^2 + 2 g_2^2 \lambda Q_H + \frac{1}{4} g_2^2 \left[Q^{(3)}_{\underset{tt}{Hl}} + Q^{(3)}_{\underset{tt}{Hq}}\right] \\
&\quad + \frac{1}{2} g_2^2 \left([Y_u^\dagger]_{rs} Q_{\underset{rs}{uH}} + [Y_d^\dagger]_{rs} Q_{\underset{rs}{dH}} + [Y_e^\dagger]_{rs} Q_{\underset{rs}{eH}} + h.c.\right), \\
\mathcal{P}_{HB} &= \frac{1}{2} g_1^2 \mathsf{y}_H Q_{H\Box} + 2 g_1^2 \mathsf{y}_H Q_{HD} - \frac{1}{2}\mathsf{y}_H g_1^2 Q_{HB} - \frac{1}{4} g_1 g_2 Q_{HWB}, \\
&\quad + \frac{1}{2} g_1^2 \left[\mathsf{y}_l Q^{(1)}_{\underset{tt}{Hl}} + \mathsf{y}_e Q_{\underset{tt}{He}} + \mathsf{y}_q Q^{(1)}_{\underset{tt}{Hq}} + \mathsf{y}_u Q_{\underset{tt}{Hu}} + \mathsf{y}_d Q_{\underset{tt}{Hd}}\right], \\
\mathcal{P}_{HW} &= \frac{3}{4} g_2^2 Q_{H\Box} - \frac{1}{2} g_2^2 m_H^2 (H^\dagger H)^2 + 2 g_2^2 \lambda Q_H - \frac{1}{4} g_2^2 Q_{HW} - \frac{1}{2}\mathsf{y}_H g_1 g_2 Q_{HWB} + \frac{1}{4} g_2^2 \left[Q^{(3)}_{Hl} + Q^{(3)}_{Hq}\right] \\
&\quad + \frac{1}{2} g_2^2 \left([Y_u^\dagger]_{rs} Q_{\underset{rs}{uH}} + [Y_d^\dagger]_{rs} Q_{\underset{rs}{dH}} + [Y_e^\dagger]_{rs} Q_{\underset{rs}{eH}} + h.c.\right).
\end{aligned} \tag{4.35}$$

Note that the operators in Eqn. 4.34 do not carry flavour indicies while the operators in Eqn. 4.35 do carry flavour indicies. One needs to define the flavour indices of the operators removed when changing basis in order to avoid ambiguities [185]. Respecting flavour symmetry is the reason Yukawa matrices appear in Eqns. 4.18-4.25 instead of arbitrary flavour matrices.

4.2.3 Phenomenological Lagrangians

There is a distinction between the concept of an operator basis in the SMEFT and an incomplete ad-hoc phenomenological Lagrangian used to characterize a subset of some SM

[47] We explicitly separate out the integration by parts identity $P_T = -Q_{H\Box} - 4Q_{HD}$.

deviations. Such an ad-hoc formalism can be constructed in a manner akin to a coordinate system basis choice for $\mathrm{Re}^{\mathbf{3}}$, in a beyond the SM "deviation space". A core characteristic of ad-hoc constructions is that the full Lagrangian is not specified, only a few terms, or a sector are defined. An ad-hoc approach can have some uses as discussed in Refs. [234–236], but referring to such a construction as a SMEFT operator basis has lead to enormous confusion in recent literature. The following problems can also occur when using ad-hoc phenomenological Lagrangians:

- Utilizing unitary gauge to perform gauge dependent field redefinitions on only parts of a full SMEFT Lagrangian does not satisfy the equivalence theorem [237].[48] There is no formal expectation of gauge independent results being obtained making such a transformation, even in LO results. An ad-hoc construction developed in unitary gauge is very susceptible to gauge dependence for this reason. See Section 6.1 and Refs. [238, 239] for more discussion.

- Particular UV scenarios can lead to the expectation that only certain operators (times Wilson coefficients) are present due to a vanishing of all other Wilson coefficients at tree level, and can also lead to the expectation that the Wilson coefficients of different operators differ by a loop factor. This cannot formally break the field redefinition relations in Eqns. 4.18-4.25 or the EOM relations Eqn. 4.35 when the separation of scales present in the EFT construction is adhered to, as these are IR operator relations. At times such UV assumptions are imposed removing operators, before a non-redundant basis is defined as the suppression is associated with the operator, not the UV dependent Wilson coefficient. This is a mistake to avoid.

- A clear sign of a phenomenological Lagrangian is the inability of such an approach to accommodate and aid in determining loop corrections.

To decide if a construction is truly an operator basis one can examine if the field redefinitions in Eqns. 4.18-4.25 can be used to transform a complete set of $\mathrm{SU_c}(3) \times \mathrm{SU_L}(2) \times \mathrm{U_Y}(1)$ operators to the particular chosen set or form. If this is not possible, then such a construction is not an operator basis according to the definition above. The "Higgs Basis" construction in Section II.2.1 of Ref. [31] has not been shown to satisfy this definition of a basis.

4.2.4 One loop running of $\mathcal{L}_{SM} + \mathcal{L}_6$

Determining the closure of the full one loop anomalous dimension matrix of $\mathcal{L}_6$ is an important check of the consistency of a basis. The counterterm structure of $\mathcal{L}_6$ can be determined without expanding around the vev of the Higgs boson, as the scales introduced when the Higgs takes on a vev regulate the IR of the theory. Using DR and $\overline{\mathrm{MS}}$ the full renormalization of the $\mathcal{L}_6$ Warsaw basis was reported in Refs. [116, 170, 185, 227, 228] using this approach. These results built upon the past results reported in Refs. [72, 212, 240–260]. It was found that the

[48] See Section 6.1 for more discussion on the equivalence theorem.

Warsaw basis closes at one loop and the full 2499×2499 anomalous dimension matrix is now determined. This is the only basis for which this has been demonstrated to date.[49]

Several aspects of the results in Refs. [116, 170, 185, 227, 228] can be understood to follow directly from the procedure to construct an operator basis described in Section 4.2.1. The renormalization of operators in $\mathcal{L}_6$ mix down, modifying the running of the $\mathcal{L}_{SM}$ parameters [116]. This is a scale dependent result that indicates that the $\mathcal{L}^{(6)}$ operator bases are only defined when performing small field redefinitions on the SM fields of $\mathcal{O}(1/\Lambda^2)$. Similarly, as is well known in past calculations in NRQCD [263, 264] the anomalous dimensions of redundant operators exhibit scheme and gauge dependence that only cancel once an operator basis is reduced to its non-redundant form [116]. This is another scale dependent sign that the field variables and operators are not independent and related by the EOM (as discussed in Section 4.2.2) when a redundant basis is used. This is a reason to report results in a non-redundant basis.

For the same reason, one cannot use only the results of Eqn. 4.35 to translate the anomalous dimensions determined in the Warsaw basis to an alternate basis constructed out of the operators in Eqn. 4.34. The removal of operator forms when defining the Warsaw basis used the full set of EOM results, not just the EOM field redefinitions related to Eqn. 4.35. To map the anomalous dimension results of the Warsaw basis to another basis, all of these EOM reductions must be undone, mapping the complete set of divergences to an overcomplete $\mathcal{L}_{SMEFT}$. Subsequently, a gauge independent algorithm must be defined that maps the full set of overcomplete divergences to a chosen non-redundant basis. If no such algorithm exists, then this procedure cannot be carried out. This is probably the reason that only the Warsaw basis has been completely renormalized to date.

Gauge independent field redefinitions have a central role in defining the SMEFT, and this is manifest in the counterterm structure of $\mathcal{L}_6$. An ad-hoc phenomenological Lagrangian as discussed Section 4.2.3, that is not obtained with such field redefinitions, is a challenge to ever renormalize for this reason.[50]

4.2.5 Functional redundancy/factorizability of the SMEFT

A consequence of how operator bases are defined is that specifying an incomplete parameterization in only a few possible interaction terms, or in a sector of interactions (like the Higgs sector) while simultaneously assuming “SM-like" interactions in other sectors, generally introduces inconsistencies into the SMEFT.

[49] Interest in the renormalization of $\mathcal{L}_6$ continues. These results were distilled into the tool reported in Ref. [261]. Some results reported in an alternate scheme appeared in Ref. [262] as did some partial results in an alternate basis in Ref. [221]. This continuing interest is also due to the curious structure of the anomalous dimension matrix [27, 185, 243, 245]. This structure was vigorously misunderstood in the literature until it was explained in Ref. [28] as being due to helicity and unitarity in the SMEFT. The explanation in Ref. [28] is UV independent, it is a statement on the IR physics captured in the operator forms in the SMEFT, so it is a valid EFT understanding of this structure.

[50] See also the discussion in Ref. [238].

For example, the parameter b_2 can be chosen in Eqn. 4.29 so that the operator $Q^{(1)}_{Hl}$ is absent in a basis. This can only be done at the cost of shifting the Wilson coefficients of the remaining operators in Eqn. 4.29. Attempting to study the effects of the operator $Q_{H\Box}$ in the process $h \to \bar{\psi}\,\psi$, that receives such a shift, must be done with care. This operator's Wilson coefficient leads to a shift in the effective Yukawa coupling $\mathcal{L} = -\mathcal{Y}_\psi\, h\,\bar{\psi}\,\psi$ of the form

$$\mathcal{Y}_\psi = \frac{m_\psi}{v_T}\left[1 + C_{H\Box}\, v_T^2\right] + \cdots . \tag{4.36}$$

Studying deviations in Higgs properties due to this operator, while simultaneously assuming a "SM-like" Z boson interaction with $\bar{\psi}\,\psi$ to accommodate Electroweak precision data (EWPD) constraints in a global analysis is inconsistent in general. The same shift in the Wilson coefficients of the operators $Q_{He}, Q^{(1)}_{Hq}, Q_{Hu}, Q_{Hd}$ related to the shift of $Q_{H\Box}$, that is introduced to remove the operator $Q^{(1)}_{Hl}$ from a basis, could be uncovered in a study of $h \to \bar{\psi}\,\psi$ or $Z \to \bar{\psi}\,\psi$. This shift is required for consistency to be present in anomalous Z couplings, unless a series of other Wilson coefficients are tuned to cancel out the expected correction.

If this cancellation is assumed one has stepped outside of a general SMEFT analysis. If a parameterization is constructed that is designed to directly hide such correlations, enormous confusion can be introduced into SMEFT studies. Considering such shifts as independent one must impose the constraint of the chosen Wilson coefficients canceling in all other processes in a global analysis. Even more arduous is to impose this assumption on all other equivalent combinations of Wilson coefficients (due to EOM relations) when using a non-redundant minimal basis. Not imposing this assumption introduces an inconsistency which is referred to as a functional redundancy in Ref. [265]. Due to the large number of parameters present in the SMEFT, it is required to combine a series of measurements to constrain the SMEFT Wilson coefficient space. Functional redundancies can block this combination of measurements being performed in a consistent manner and lead to spurious results.

Directly following from this subtlety is in what sense the SMEFT is factorizable into a subset of contributions when predicting S matrix elements. Approximations must be made in interfacing with experimental results. The approximations that are safe to impose while maintaining a model independent SMEFT analysis are *IR assumptions*. For more discussion on this point see Section 6.5

Conversely, naively imposing UV assumptions can make the SMEFT inconsistent as a field theory construction, and incapable of capturing the IR limit of a UV physics sector. This can render such a SMEFT study ambiguous, with unclear implications for a UV physics sector, as it only uncovers a distorted approximation of the true low energy constraints that a UV sector will face. SMEFT studies walk a fine line between being powerful model independent conclusions, and statements based on inconsistent field theory without any true UV implication or meaning. This fine line is defined by consistency in the SMEFT analysis being enforced (or violated) to the precision and accuracy demanded by the data.

4.3 The Higgs Effective Field Theory

4.3.1 Minimal assumptions in the scalar sector

Both the SM and the SMEFT constructions are field theories that assume the existence of the scalar complex field H defined in Table 4.7 in the constructed Lagrangian. The introduction of such a field is a consequence of requiring:

- (i) Three Goldstone bosons π^I, the longitudinal components of the EW gauge bosons.
- (ii) One singlet scalar h, corresponding to the physical Higgs boson, that ensures the *exact* unitarity at all energies of scattering amplitudes with external π^I fields. Pedagogical illustrations of this argument can be found in Refs. [266–268].

Condition (i) is an IR assumption that is imperative for a correct description of the EW symmetry breaking. Requirement (ii) can be relaxed and the EFT can remain self-consistent for lower energies scattering events where the EFT is well defined. An EFT does not have to exactly preserve unitarity, it only has to be unitary up to the cut off scale where the Taylor expansion used in its construction breaks down. An EFT is dictated by describing the long distance propagating states that lead to non-analytic structure in the correlation functions of scattering amplitudes. From this perspective, the ideal theoretical tool to describe scenarios without assumption (ii) being made is the Higgs Effective Field Theory (HEFT).

In the HEFT the long distance propagating states are again the massive SM fermions and gauge bosons. Instead of a H field, a dominantly $J^P = 0^+$ singlet scalar state h is included in the EFT with free couplings to the remaining SM states. The HEFT is based on the Callan-Coleman-Wess-Zumino (CCWZ) formalism [68, 69] and provides a parameterization of the scalar sector with minimal IR assumptions.

This approach has been continually rediscovered over the years as it adheres to the EFT "prime directive" in the scalar sector. Several ad-hoc parameterizations have independently emerged over the years along these lines [101, 219, 269, 270] but none of these works developed a complete self-consistent EFT.[51] It is an important development that in recent years in Refs. [101, 271–287] this theory has been advanced to a degree that it is now a consistent EFT description.

The size and pattern of any deviations from the SM discovered using EFT methods can indicate whether a HEFT or a SMEFT description is appropriate. Any deviation from the SMEFT expectation that follows from the exact H doublet structure can carry significant information about the possible UV physics matched onto the lower energy EFT description. Theoretical tools that can consistently account for the possibility that the Higgs boson may not be part of an exact $\mathrm{SU_L}(2)$ multiplet with the π^I, and to probe experimentally this hypothesis as a part of a wider precision measurements program are essential. The experimental collaborations have already started to address this issue, providing independent measurements

[51] Ref. [267] appears after Refs. [101, 219, 269, 270] but states that it introduces this parameterization to the literature, which has caused some confusion, as Ref. [267] cites Ref. [270].

of several Higgs couplings and testing for the presence of anomalous Lorentz structures (see e.g. Ref. [288]). At the present moment, all the observations are compatible with the SM, and the SMEFT extension, but the evidence in favor of this option is still not compelling. The presence of large uncertainties (roughly of order 10–20% as discussed in Section 8), together with the fact that some decay and production channel are still not accessible, allows for significant deviations.

4.3.2 Topology of the scalar manifold

In parallel to the HEFT formalism being continually rediscovered over the years, there has been a continual rediscovery of the resistance to this idea in the theoretical community. This resistance is usually due to the field reparameterization equivalence theorem [237] of S matrix elements. A consequence of this theorem is that a coordinate choice for a scalar manifold does not matter for S matrix elements. In the SM, the HEFT or the SMEFT, the scalar manifold of interest is $\mathcal{M}(\pi^I, h)$. A coordinate choice for this manifold has no physical effect.

The HEFT literature is not due to a misunderstanding of this point. This EFT is designed to describe a set of possible low energy IR limits that are not consistent with a predictive version of the SMEFT (i.e. when the operator expansion in the EFT converges). Specific examples are given by the Dilaton constructions of Refs. [289, 290], which have long been known to be represented by the HEFT. Nevertheless, a formulation of the HEFT/SMEFT discriminant in terms of scalar manifold topology was not present in the literature until recently.

It is not surprising that this discriminant is topological in nature. Manifold topology plays an important role in theories of symmetry breaking such as in the SM, or in the Landau-Ginzburg effective theory of superconductivity (see Section 2). Topological distinctions of vacuum states are common in field theory, and have long been understood to underlie the properties of the scattering of pions [291, 292] based on the curvature of the scalar manifold describing the Goldstone bosons of chiral perturbation theory. The heuristic embedding of possible deviations from SM expectations given by SM $\subset$ SMEFT $\subset$ HEFT is also well known for many years, but the precise theoretical statement is given as

- The SM has a flat scalar manifold $\mathcal{M}$ and an O(4) fixed point which is the unbroken global custodial group $\mathrm{O}(4) \sim \mathrm{SU_L}(2) \times \mathrm{SU_R}(2)$. This group is respected by a subset of the SM Lagrangian [293–295]. When $\langle H^\dagger H\rangle = v^2/2$ a $\mathrm{SU_{L+R}}(2) = \mathrm{SU_C}(2)$ subgroup (also generally referred to the custodial group) is unbroken that leads to the prediction $m_W = m_Z \cos\theta_W$. The linearization lemma [68] then allows for a local linear transformation of the scalar manifold coordinates, which results in the embedding of the scalar fields into a linear multiplet H. This is the group theory equivalent of assuming a H field in the SM construction.

- The SMEFT has a curved scalar manifold [229–231] due to the presence of two derivative Higgs operators. In the Warsaw basis these operators are $Q_{HD}, Q_{H\Box}$. The SMEFT also has a O(4) fixed point which allows the EFT to be constructed with the linear multiplet H. The presence of this scalar curvature is a key point of the SMEFT construction and is the reason that one cannot remove all the two derivative Higgs operators from the SMEFT basis with gauge independent field redefinitions.

- HEFT has a curved scalar manifold $\mathcal{M}$ and does not contain a O(4) fixed point [230, 231].

This distinction between the SM, SMEFT and HEFT is field redefinition invariant.[52]

4.3.3 UV embeddings of HEFT

The HEFT is a general field theory that can describe a wide variety of scenarios, including composite Higgs models [40–44, 300–304] and Dilaton constructions [289, 290] while reproducing the SM in a specific limit of parameter space. The HEFT formalism is of most interest, if it captures the IR limit of a UV completion of the SM. Determining the necessary and sufficient conditions on UV dynamics that leads to the HEFT construction at low energies is an unsolved problem. The naive assumption that the distinction between SMEFT and HEFT is a statement of the presence of a linear multiplet in the UV sector integrated out of the theory is not correct. The assumption is an IR assumption on the nature of the EFT that corresponds to an (unknown) set of criteria on UV dynamics.

This can be directly seen in the non-minimal coupling to gravity of the Higgs doublet in a UV sector. In this case the classical background field scalar manifold also plays a central role. This Lagrangian is given as a tower of higher dimensional operators as

$$\mathcal{L}_{H,inf} = \sqrt{-\hat{g}}\left(\mathcal{L}_{SM} - \frac{M_p^2\,\hat{R}}{2} - \xi H^\dagger H\,\hat{R}\right). \tag{4.37}$$

This Lagrangian is known as the Higgs-Inflation Lagrangian, and was popularized in recent literature in Ref. [305]. Here $\hat{g}$ is the determinant of the Jordan frame metric $\hat{g}_{\mu\nu}$, $M_p = 2.44\times 10^{18}\,\mathrm{GeV}$ is the reduced Planck mass, $\hat{R}$ is the Ricci scalar and ξ is a dimensionless coupling. When $\langle H^\dagger H\rangle \equiv \hat{v}^2/2 \gg v^2/2$ this Lagrangian leads to a flat Higgs potential due to mixing between the scalar state h and a scalar component of the graviton.[53] This can be seen expanding the metric about Minkowski space as $\hat{g}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_p$ giving

$$\delta\mathcal{L} = \frac{\xi}{M_p}(h+\hat{v})^2\,\eta^{\mu\,\nu}\partial^2 h_{\mu\,\nu}. \tag{4.38}$$

This mixing was analyzed in a complete fashion in Ref. [306] for the first time and becomes significant when $\sqrt{\langle H^\dagger H\rangle} \sim M_p/\xi$. At this scale the canonical normalization of the h field and the graviton leads to significant non-linearities introduced into an EFT description of the propagating degrees of freedom – resulting in the HEFT. This explains the scattering results

[52] Note that in Refs. [296–299] and some other works this field redefinition invariant distinction between the HEFT and SMEFT is obscured and Ref. [218] is incorrectly credited for the introduction of a nonlinear realization.

[53] The scaler degree of freedom of gravity can only be removed in the usual manner with diffeomorphism invariance in Minkoswki space when this mixing is not present, which requires the theory is first canonically normalized.

found related to the cut-off scale behavior of this theory [229, 307–310]. Although the UV dynamics includes a H doublet, the IR EFT construction useful for scattering calculations around background fields $\sqrt{\langle H^\dagger H\rangle} \gtrsim M_p/\xi$ is a version of the HEFT. As this operator is induced renormalizing the SM in curved space, this physics is formally always present and the (small) non-linearities due to gravity are present in EFT descriptions of Higgs physics. Of course these effects are below the current (and most likely future!) experimental resolution.

The relevant point for LHC phenomenology is the possibility that dynamics present in a UV sector that explains EWSB leads to similar IR effects, that can be accommodated in the HEFT. In composite Higgs theories, some new physics states may mix with the h field, thus weakening the unitarity constraint as captured by the HEFT formalism, and permitting deviations from an exact doublet structure. Due to our inability to calculate the low energy limit of all possible strongly interacting sectors to understand the precise conditions on dynamics that can lead to such IR effects, this formalism should not be casually dismissed. For recent discussions on IR limits associated with TeV scale physics that require the HEFT, see Refs. [280, 311, 312].

4.3.4 The HEFT Lagrangian, preliminaries

In analogy with the formalism employed in chiral perturbation theory (χPT) for the pions of QCD, in HEFT the three Goldstone bosons π^I are embedded into a dimensionless unitary matrix

$$\mathbf{U} = \exp\left(i\tau^I\pi^I/v\right)\,, \qquad \mathbf{U} \mapsto L\mathbf{U}R^\dagger\,, \tag{4.39}$$

which transforms as a bi-doublet under global $\mathrm{SU_L}(2)\times\mathrm{SU_R}(2)$ transformations. Inside the exponential, the π^I fields appear suppressed by the scale v rather than by a heavier cutoff Λ. This is a reasonable choice, as v can be regarded as an order parameter of EWSB and the characteristic scale of the π^I. This implies that, unlike in the SMEFT case, π^I insertions are not suppressed in general. Additional suppressions may be induced if the HEFT is matched to specific UV models that associate these degrees of freedom with a heavier scale. This is the case of Composite Higgs models, in which both the π^I and h fields arise weighted by a scale f that satisfies the relation $4\pi f \geq \Lambda$, with Λ being the cutoff of the EFT [313]. In this case, the scalar fields are always accompanied in the EFT by factors of $\xi = (v/f)^2 < 1$.

In the EW vacuum $\langle\mathbf{U}\rangle = \mathbf{1}$ the global chiral symmetry is spontaneously broken down to the custodial group $\mathrm{SU_C}(2) = \mathrm{SU_{L+R}}(2)$. In addition, the $\mathrm{SU_R}(2)$ component is explicitly broken in the EW sector by the Yukawa couplings and the gauged $U(1)$ subgroup with coupling g_1 generated by τ^3. It is convenient then to define objects that transform in the adjoint of $\mathrm{SU_L}(2)$, to be employed as building blocks of a $\mathrm{SU_C}(3)\times\mathrm{SU_L}(2)\times\mathrm{U_Y}(1)$ invariant Lagrangian. It is possible to construct a scalar and a vector as follows:

$$\begin{aligned} \mathbf{T} &= \mathbf{U}\tau^3\mathbf{U}^\dagger & \qquad \mathbf{T} &\mapsto L\mathbf{T}L^\dagger \\ \mathbf{V}_\mu &= (D_\mu\mathbf{U})\mathbf{U}^\dagger & \qquad \mathbf{V}_\mu &\mapsto L\mathbf{V}_\mu L^\dagger\,, \end{aligned} \tag{4.40}$$

where the covariant derivative of the $\mathbf{U}$ field is

$$D_\mu \mathbf{U} = \partial_\mu \mathbf{U} + \frac{ig_2}{2} W_\mu^I \tau^I \mathbf{U} - \frac{ig_1}{2} B_\mu \mathbf{U} \tau^3 \,. \tag{4.41}$$

Note that the field $\mathbf{T}$ is invariant under hypercharge transformations but not under the global $\mathrm{SU_R}(2)$ group so it can be treated as a custodial symmetry breaking spurion.

The physical Higgs scalar is introduced as a gauge singlet h. This choice ensures the most general approach to the physics of this state and makes the HEFT a versatile tool which includes the $\mathrm{SU_L}(2)$ doublet case as a particular limit. Specifically, the SM Higgs doublet H can be written as a fixed combination of the fields h and $\mathbf{U}$ according to:

$$\begin{pmatrix} \tilde{H} & H \end{pmatrix} = \frac{v+h}{\sqrt{2}} \mathbf{U} \,. \tag{4.42}$$

Being a singlet, the h field has completely arbitrary couplings, that are customarily encoded in generic functions [101, 219, 267]

$$\mathcal{F}_i(h) = 1 + 2a_i \frac{h}{v} + b_i \frac{h^2}{v^2} + \dots \tag{4.43}$$

that constitute another building block for the HEFT Lagrangian. Note that here the polynomial structure should be interpreted as the result of a Taylor expansion in (h/v), which may include an infinite number of terms.

4.3.5 The HEFT Lagrangian

The HEFT Lagrangian is composed of the gauge and fermion fields in Section 4.1, and the scalar fields $\mathbf{U}$, h defined in the previous section. Its construction historically builds upon that of the EW chiral Lagrangian [97–101], in which only the three π^I fields were retained in the spectrum, while the physical Higgs was assumed to be sufficiently heavy to be integrated out. Extensions with addition of extra (pseudo-)scalar fields were also explored more recently [312, 314, 315]. The HEFT Lagrangian can be defined as

$$\mathcal{L}_{HEFT} = \mathcal{L}_0 + \Delta\mathcal{L} + \dots \,, \tag{4.44}$$

where $\mathcal{L}_0$ contains the leading order terms and $\Delta\mathcal{L}$ includes first order deviations. Unlike the SMEFT, it is not possible to classify the HEFT invariants based on just the canonical dimension. This is due to the fact that the HEFT is a fusion of χPT (in the scalar sector) with the SMEFT (in the fermions and gauge sector). Because these two theories follow different counting rules, the structure of the mixed expansion is complex. A rigorous and self-contained method to determine the expected suppression of a given HEFT invariant represents a non-trivial and subtle task that has been intensely debated in the literature. The discussion of this technical point is postponed to Section 5.2.

The leading Lagrangian $\mathcal{L}_0$ can be written as[54]

$$\begin{aligned}\mathcal{L}_0 = &-\frac{1}{4}G^A_{\mu\nu}G^{A\,\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \sum_\psi \bar{\psi}i\not{D}\psi + \\ &+ \frac{1}{2}\partial_\mu h\partial^\mu h - \frac{v^2}{4}\,\mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}^\mu)\mathcal{F}_C(h) - V(h) + \\ &- \frac{v}{\sqrt{2}}\left(\bar{q}\,\mathbf{U}\mathcal{Y}_Q(h)q_R + \text{h.c.}\right) - \frac{v}{\sqrt{2}}\left(\bar{\ell}\,\mathbf{U}\mathcal{Y}_Q(h)\ell_R + \text{h.c.}\right)\,,\end{aligned} \tag{4.45}$$

where the right-handed fermions have been collected into doublets of the global $\mathrm{SU_R}(2)$ symmetry

$$q_R = \begin{bmatrix} u_R \\ d_R \end{bmatrix}, \qquad \ell_R = \begin{bmatrix} 0 \\ e_R \end{bmatrix} \tag{4.46}$$

and the Yukawa couplings include a dependence on h:

$$\mathcal{Y}_Q(h) = \mathrm{diag}\left(\sum_n Y_u^{(n)}\frac{h^n}{v^n},\, \sum_n Y_d^n\frac{h^n}{v^n}\right), \quad \mathcal{Y}_L(h) = \mathrm{diag}\left(0,\, \sum_n Y_e^{(n)}\frac{h^n}{v^n}\right). \tag{4.47}$$

The first term in the sum ($n = 0$) generates fermion masses, while higher orders describe the couplings with an arbitrary number of h insertions. The term $\mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}^\mu)$ in the second line of Eqn. 4.45 contains the kinetic terms of the π^I and the mass terms of the gauge bosons. Because the matrix $\mathbf{U}$ is adimensional, this invariant has canonical dimension 2 and therefore appears in the Lagrangian multiplied by a factor v^2. The reason why it is the scale v, and not the cutoff Λ, that multiplies this term is that this choice ensures a canonically normalized kinetic term for the π^I, given the definition of $\mathbf{U}$ in Eqn. 4.39. The same principle applies to the Yukawa couplings.

The Lagrangian $\mathcal{L}_0$ is equivalent to the SM Lagrangian up to the presence of an arbitrarily large number of Higgs and π^I insertions and to the fact that the Higgs couplings (which includes the scalar potential) are parameterized by independent coefficients rather than being fixed by the doublet structure. Despite being allowed by symmetry, Higgs couplings to kinetic term structures are absent in $\mathcal{L}_0$. In the case of the fermion and h kinetic terms, this is because any extra $\mathcal{F}(h)$ factor can be removed via field redefinitions [274, 284] and reabsorbed into the coefficients that parameterize the Higgs interactions. This can be understood to follow

[54] Due to the lack of a unique criterion for the classification of the invariants, there is some freedom in the definition of the LO itself. To define one universal rule that uniquely determines the order a certain HEFT operator should be assigned, in a UV independent manner, is the essential challenge. This difficulty has been overcome in the literature by identifying a set of heuristic criteria, which essentially follow from internal consistency requirements and some common sense considerations. Unfortunately, some UV dependence is generally also present. Nevertheless, there is substantial agreement in the community with respect to the organization of the HEFT expansion which we stress here, despite disagreements on the definition of counting rules. For definiteness, here we use the conventions of Ref. [284]. We stress this is not a value judgment, simply a convention. The same LO Lagrangian has been previously adopted in Refs. [272, 274], although in a slightly different notation. Different conventional choices were made in Refs. [271, 276, 278].

directly from the general arguments on constructing on-shell EFTs given in Refs. [222, 223]. In the case of gauge kinetic terms such couplings are chosen to be retained in the basis, but they are customarily classified as higher order effects due to a suppressed Wilson coefficient, under the assumption that the transverse components of the gauge bosons do not couple strongly to the Higgs sector. The same choice has been made for the suppression of the Wilson coefficient of the operator $\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)^2$, which would belong to $\mathcal{L}_0$ according to the derivative expansion of χPT, but that must carry an implicit suppression in its matching if the custodial symmetry is assumed to be only weakly broken, as suggested by experimental observations.

The Lagrangian $\Delta\mathcal{L}$ contains the leading deviations from $\mathcal{L}_0$. Again, it is not possible to infer in a rigorous and universal way which classes of operators belong to this order. The relative impact of two given invariants depends in general on the kinematic regime considered (see Section 5.2 for further details). It is possible to identify a set of invariants that *can* be responsible for the largest deviations at least at energies up to a few TeV. They amount to 148 independent invariants in total [284]. A complete basis of operators encompassing both bosonic and fermionic sector has been first proposed in Ref. [274] and an alternative set has been derived independently in Ref. [284]. The two works agree on the classes of operators, although there are differences in the choice of individual terms retained. We describe the operators present sector by sector as follows.

Bosonic operators: This must include chiral invariants with four derivatives, which are next-to-leading terms in the chiral expansion and can be reduced to an independent set with the structures[55]

$$\begin{aligned}&(\mathbf{V}_\mu)^4, \quad (\mathbf{V}_\mu)^3\partial_\nu\mathcal{F}_i(h), \quad (\mathbf{V}_\mu)^2(\partial_\nu\mathcal{F}_j(h))^2, \quad (\partial_\mu\mathcal{F}_k(h))^4\\ &X_{\mu\nu}(\mathbf{V}_\rho)^2, \quad X_{\mu\nu}\mathbf{V}_\rho\partial_\sigma\mathcal{F}_l(h), \quad (X_{\mu\nu})^2\mathcal{F}_m(h)\,.\end{aligned}$$

Note that operators in this category are not suppressed by any powers of Λ in the Lagrangian, as they have formally canonical dimension 4. The two-derivative operator $v^2\,\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)^2/4$ also belongs to $\Delta\mathcal{L}$ as explained above. Finally, operators with the structure $(X_{\mu\nu})^3$ can be included at this order despite containing 6 derivatives. This can be justified assuming that, being composed of only transverse gauge boson, these operators follow the ordering rules of the SMEFT (see Section 5.2).

A complete set of interactions for the bosonic sector, constructed avoiding reduction via the EOM, was presented and studied phenomenologically in Ref. [276] (CP conserving terms) and Ref. [278] (CP odd terms).

Operators with fermions: $\Delta\mathcal{L}$ contains operators with one fermionic current and up to two derivatives, as at least a subset of this category of invariants is required as counter-terms in the one-loop renormalization of $\mathcal{L}_0$. Schematically, they can be reduced to the set of independent structures

$$(\bar\psi\gamma^\mu\psi)\mathbf{V}_\mu, \quad (\bar\psi\psi)(\mathbf{V}_\mu)^2, \quad (\bar\psi\psi)\mathbf{V}_\mu\partial^\mu\mathcal{F}_n(h), \quad (\bar\psi\sigma^{\mu\nu}\psi)(\mathbf{V}_\mu)\partial_\nu\mathcal{F}_o(h), \quad (\bar\psi\sigma^{\mu\nu}\psi)(X_{\mu\nu})\,.$$

[55] Not all of the Lorentz indicies in these expressions match. This indicates the possibility of multiple Lorentz contractions.

Here operators with one derivative are unsuppressed, while operators with two derivatives are multiplied by Λ^{-1}, having canonical dimension 5. Four-fermion operators appear in $\Delta\mathcal{L}$ as well, because a subset of terms in this class is required for the renormalization of $\mathcal{L}_0$.

These considerations lead to the construction of a consistent basis of independent operators in the HEFT, reported by multiple groups, which is closed under the EOM relations.

4.4 SMEFT vs. HEFT

S matrix elements and relations between S matrix elements constructed in the SMEFT and the HEFT expansions are potentially distinguishable, as the theories are distinct in a field redefinition invariant manner [230, 231]. The ordering of the theories in their IR assumptions is given by $\mathrm{SM(H,\Lambda \to \infty) \subset SMEFT(H,\Lambda \neq \infty) \subset HEFT(h,\Lambda \neq \infty)}$.

Early statements on the need to experimentally check the assumption of a H field in the EFT used, appeared in the literature before the turn on of LHC [219, 267, 269]. Developing a precise statement on what exact experimental result would decide between these two frameworks conclusively is an ongoing challenge. The differences between SMEFT and HEFT identified to date, and existing proposals to take advantage of these differences are as follows.

4.4.1 Higgs/Triple Gauge Couplings

While in SMEFT the Higgs couplings follow a polynomial dependence in $(v+h)^n$ due to the H doublet, in the HEFT they do not. Current LHC results are compatible with H in the spectrum, but with large uncertainties that allow for significant deviations. A fundamental probe of this point would be the observation of double-Higgs production, which is challenging at the LHC. A discussion of this process in relation to the HEFT can be found in Refs. [219, 267, 316–324].

Another distinctive prediction of the HEFT is the decorrelation of triple gauge couplings and Higgs-gauge bosons interactions, due to the fact that in this EFT the covariant derivative $D_\mu H \sim \partial_\mu h \mathbf{U} + (v+h) D_\mu \mathbf{U}$ is formally split into two independent pieces. The possibility of isolating this effect experimentally has been studied in Refs. [276, 284].

The appearance of Hermitian derivative terms acting on Higgs fields in the SMEFT of the form $H^\dagger\, i\overleftrightarrow{D}_\beta H$ and $H^\dagger\, i\overleftrightarrow{D}^I_\beta H$ in $\mathcal{L}_6$ relates anomalous Z couplings in $\bar\psi\psi \to Z \to \bar\psi\psi$ to contact operator contributions to $pp \to h \to Z\, Z^\star \to \bar\psi\psi\,\bar\psi\psi$. This correlation can be probed as a consistency test of the SMEFT accommodating any discovered deviations in the kinematic spectra of $h \to Z\,\bar\psi\psi$ and $h \to Z\, Z^\star \to \bar\psi\psi\,\bar\psi\psi$ determined in the narrow width approximation [276, 325]. For recent analysis of this spectra in the SMEFT/HEFT, see Refs. [275, 325–330].

4.4.2 Organization of the expansion

As detailed in Section 5.2, the HEFT Lagrangian is not organized as an expansion in canonical dimensions as in the SMEFT, but has a more complex structure. This is because the physical Higgs h and the π^I containing matrix $\mathbf{U}$ are independent in HEFT. A consequence is the re-shuffling the orders at which interactions appear in the expansion.

Insertions of longitudinal gauge bosons are less suppressed in HEFT. Couplings of these fields or, equivalently, of the π^I can be probed in high energy scattering. These processes are also potentially sensitive to the presence of strong interactions in the EWSB sector. Several channels have been analyzed in HEFT, including $V_L V_L \to V'_L V'_L$ $(V, V' = Z, W^\pm)$ [331], $V_L V_L \to hh$ [267, 332], $V_L V_L \to t\bar{t}$ [333] and $\gamma\gamma \to V_L V_L$ [334]. The latter is more promising, having been already observed at the LHC [335].

Some interesting differences in the allowed interactions in HEFT compared to the SMEFT at fixed order in the power counting of each theory have been identified. For example, the operator $\epsilon_{\mu\nu\rho\sigma}\,\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\,\mathrm{Tr}(\mathbf{V}^\nu W^{\rho\sigma})\mathcal{F}(h)$, introduces triple and quartic gauge couplings with an anomalous antisymmetric Lorentz structure in HEFT.[56]. A phenomenological study of this term has been carried out in Ref. [276].

4.4.3 Number of operators/operator structure

HEFT contains a larger number of invariants compared to the SMEFT order by order in the expansions of each theory. In the flavour-blind limit, the complete HEFT basis $\Delta\mathcal{L}$ contains 148+h.c operators [284], compared to the 76 parameters when $n_f = 1$ in the SMEFT at $\mathcal{L}^{(6)}$ [185, 203].

Most of the operators appearing in HEFT contribute to the same interaction vertices in the SMEFT due to $\mathcal{L}^{(6)}$. When this is the case, the larger number of invariants is then present in the corrections to predicted S matrix elements with the same interaction vertices. This does not lead to a measurable difference in a single observable as multiple observables are required to fix the free parameters. Potentially, patterns of allowed deviations in multiple processes could be more easily accommodated in HEFT than the SMEFT.

4.4.4 Tails vs poles

Differences between SMEFT and HEFT identified to date exist in interactions involving at least three boson fields: hhh, $hhhh$, hVV, VVV. In each EFT, one can work in the canonically normalized basis of fields. For pole observables, using the different EFTs label different unknown parameters in $V\bar{\psi}\psi$ and the mass terms VV. Studying $\bar{\psi}\psi \to V \to \bar{\psi}\psi$ observables the power counting of the corrections in each EFT scales as $C_i\, v^2/\Lambda^2$. At present, no SM-resonant exchange (i.e. pole process) measured at LEP, the Tevatron, or LHC has deviated in a statistically significant manner from the SM expectation, see Section 7.

Future RunII data, the High Luminosity phase of the LHC, and future facilities will offer more precise experimental results on pole observables, and S matrix elements that receive contributions from hhh, $hhhh$, hVV, VVV. These off-shell vertices have a non-trivial relation to LHC observables, with other possible deviations simultaneously present at each order in the EFT expansions in vertex corrections. When studying these processes, two expansions

[56]This operator violates explicitly the custodial symmetry group ($\mathrm{SU_C}(2)$) Therefore it may be suppressed in scenarios where this symmetry is broken only through standard effects (non-homogeneous Yukawas and $g' \neq 0$).

are present $\{v^2/\Lambda^2,\, p^2/\Lambda^2\}$ where p^2 stands for a general kinematic invariant, and in general $p^2/\Lambda^2 \to 1$ in the tail of a distribution.

It is an unsolved problem to develop a method for consistent SMEFT/HEFT analysis of the tails of kinematic distributions and to project the global constraints of each EFT into these distributions.[57] Defining the interference of the SM for an observable O with a tail expansion parameter factored out with HEFT or SMEFT as $\langle\rangle_O$, to distinguish these theories and conclude that the SMEFT cannot accommodate a discovered deviation from the SM, while the HEFT can accommodate such a deviation, requires

$$\Delta\langle \mathcal{L}_6 - (\mathcal{L}_0 + \Delta\mathcal{L})\rangle_O \frac{p^2}{\Lambda^2} > \langle \mathcal{L}^8\rangle_O \frac{(p^2)^2}{\Lambda^4}. \tag{4.48}$$

Due to the large multiplicity of operators appearing in the SMEFT expansion at $\mathcal{L}_8$ [206–209], examining the distinguishability of these theories in the limit $p^2/\Lambda^2 \lesssim 1$ (as opposed to $p^2/\Lambda^2 << 1$) is difficult. An additional challenge is the suppression of the interference terms due to $\mathcal{L}_6$ in several tail observables of this form following from the Helicity non-interference arguments of Ref. [340–344]. It remains to be conclusively shown if HEFT and SMEFT can be functionally distinguished in the tails of distributions. Determining the global constraints on these theories from the LEP and LHC data sets is an essential step to examine this question quantitatively.

5 Power counting

The SMEFT and the HEFT are constructed out of an infinite series of operators based on a separation of scales. It is required that these theoretical descriptions are consistent, and they are of interest if they are predictive. For this reason, it is important to establish:

- How a Lagrangian term scales in a consistent manner with the dimensionful quantities in the theory, i.e. the normalization of terms.
- A prescription for ordering such invariants within the EFT expansion to estimate the relative physical impact of any given Lagrangian term on a measurement, so that the *approximate theoretical precision* of the EFT can be well defined.

The procedure of establishing the consistent scaling/ordering of the terms in the Lagrangian is usually called "power counting". There is a canonical interpretation of this concept that runs through Refs. [26, 33, 57, 68, 69, 71, 223, 345–348] and many other works. We adopt this interpretation in this review. A different usage of "power counting" also exists in the literature, as we stress below.

[57] For recent results aimed at this problem, see Refs. [38, 336–341].

5.1 Counting rules for the SMEFT

The SMEFT is built out of the SM fields and couplings that have the well defined canonical mass dimensions [..]. Defining the total mass dimensions of the $[\mathcal{L}_{SM}] = d$, then

$$[\psi] = \frac{d-1}{2}, \quad [Y_i] = \frac{4-d}{2}, \quad [V] = \frac{d-2}{2}, \quad [g_i] = \frac{4-d}{2}, \quad [H] = \frac{d-2}{2}, \quad [\lambda] = 4-d. \tag{5.1}$$

A power counting in canonical mass dimension leads to an operator built out of field insertions with total mass dimension d multiplied by a factor Λ^{4-d}. According to its canonical dimension, operators with $d \leq 4$ (the SM Lagrangian) are leading terms (LT), operators with $d = 6$ are next-to-leading terms (NLT) in the SMEFT and so on. This power counting rule is an ordering and normalization statement on the EFT.

For the EFT to be predictive it is also required that the expansion parameters are perturbative

$$C_i\, v^2/\Lambda^2 < 1, \qquad C_i\, p^2/\Lambda^2 < 1. \tag{5.2}$$

The accompanying Wilson coefficients are UV dependent. This condition translates into a condition on the UV completion by insisting that the IR EFT construction is predictive. A power counting in Naive Mass Dimension allows an estimate of the neglected higher order terms in a calculation by varying the unknown $C_i^d v^{d-4}/\Lambda^{d-4}, C_i^d |p|^{d-4}/\Lambda^{d-4}$ over a declared range of values, allowing an interpretation of the data in the SMEFT.

5.1.1 NDA counting

One can determine a consistent set of power counting rules that are a property of the SMEFT (and other field theories) that are distinct from just using the canonical mass dimension estimate. This alternative approach is known as Naive Dimensional Analysis (NDA) [313]. NDA normalization has a physical intuition underlying it. One can determine the generation of operators Q_i from operators Q_j using topological relations due to the full set of connected diagrams in the EFT. Applying this idea to the SMEFT, one can assume roughly homogeneous matching $C_i \sim C_j$ for operators in $\mathcal{L}_6$, and demand that parameter tuning is avoided. Then one expects $|C_i| \gtrsim |\Delta C_i|$ where $|\Delta C_i|$ is the absolute value of the induced Wilson coefficient for C_i due to $\int \mathcal{L}_{SM} \times C_j\, Q_j$ from the allowed connected diagrams.

The generalized version of NDA [348] of this form, which builds upon the work in Refs. [26, 313, 345–347, 349] gives a normalization of Lagrangian terms as

$$\frac{\Lambda^4}{16\pi^2}\left(\frac{\partial_\mu}{\Lambda}\right)^D \left(\frac{4\pi V_\mu}{\Lambda}\right)^A \left(\frac{4\pi\psi}{\Lambda^{3/2}}\right)^F \left(\frac{4\pi H}{\Lambda}\right)^S \left(\frac{g}{4\pi}\right)^{N_g} \left(\frac{y}{4\pi}\right)^{N_y} \left(\frac{\lambda}{16\pi^2}\right)^{N_\lambda}, \tag{5.3}$$

for an interaction term with D derivatives, V gauge fields , F fermion insertions and H scalar fields, accompanied by N_g gauge coupling constants, N_y Yukawas and N_λ Yukawas.

NDA was first determined in the context of the chiral quark [313] model, but its approach has been successfully applied to many other EFT descriptions. This is because it is a consistency condition on a EFT construction. NDA does not change the Lagrangian. It makes manifest an ordering of the Wilson coefficients that corresponds to a lack of tuning of parameters, consistent with its assumptions in the EFT. This normalization is a property of the EFT and the EOM generated by varying the action of the EFT (see Section 4.1.1) obeys this power counting as a result.

NDA supplies a reasonable *estimate* of the normalization of a Lagrangian term, which still multiplies an unknown Wilson coefficient. Eqn. 5.3 characterizes the scales in a UV sector as $\sim \Lambda$. If a distinct set of scales exists in matching onto a UV sector, this can lead to the Wilson coefficients having to differ significantly if a NDA normalization is used. Applying a NDA normalization to global fits in the SMEFT the Wilson coefficients should be treated as free parameters for this reason.

5.1.2 UV matching scenarios

A distinct approach to ordering parameters in the Lagrangian also goes under the name power counting in the literature. To clarify this difference we will refer to this approach as "Meta-matching". The idea is to make more assumptions on the UV sector, that leads to broad conditions on the Wilson coefficients. The difficulty is in making UV assumptions in a well defined, precise, and consistent manner. Meta-matching is closely related to power counting in that it defines a normalization of Lagrangian terms in the EFT, this is why it is also referred to as power counting in some literature. It is also a distinct UV assumption heavy approach that blurs the strict separation of scales defining an EFT construction. Despite this, it can still be a useful tool.

Topological tree/loop Meta-matching: The most commonly used version of Meta-matching is the "tree-loop" classification scheme of Artz, Einhorn and Wudka [213, 350]. The idea is to study topologically all of the field content of weakly coupled renormalizable models that can couple at tree level to the SM states exhaustively, and to determine which $\mathcal{L}_6$ operators can be generated at tree level in matching to a basis. For the Warsaw basis, the operators classified as "tree" in this manner for $\mathcal{L}_6$ are those in Classes $\{2, 3, 5, 7, 8\}$ in Table 1. As the operators in Classes $\{1, 4, 6\}$ contain field strengths, this classification scheme could have been misunderstood as being related to a "minimal coupling" condition in matching results. Artz, Einhorn and Wudka [213] do not assert this.[58] The analysis in Ref. [213] discussed the possibility of kinetic mixing of the form $V_{\mu\nu}F^{\mu\nu}$, for F a beyond the SM U(1) vector boson only briefly. This interaction is redundant, so this does not limit the conclusions of Ref. [213].

This scheme is an assertion of UV assumptions, not a consistency condition of the EFT, which is the point of NDA. The results of Ref. [213] are limited to weakly coupled and renormalizable UV scenarios, they do not apply if the UV contains an EFT or a strongly interacting theory [34]. The classification scheme of Ref. [213] also cannot capture the low energy effects of the multi-pole expansion discussed in Section 2.2. Furthermore, $\mathcal{L}_6$ operators equated by

[58] For further discussion on minimal coupling, see Refs. [32, 34].

the EOM have Wilson coefficients that differ in this scheme, which can cause some confusion. An example is given in the EOM operator relation

$$
\begin{aligned}
\mathcal{P}_{HB} = & \frac{1}{2} g_1^2 \mathsf{y}_H Q_{H\Box} + 2 g_1^2 \mathsf{y}_H Q_{HD} - \frac{1}{2} \mathsf{y}_H g_1^2 Q_{HB} - \frac{1}{4} g_1 g_2 Q_{HWB}, \\
& + \frac{1}{2} g_1^2 \left[\mathsf{y}_l \underset{tt}{Q_{Hl}^{(1)}} + \mathsf{y}_e \underset{tt}{Q_{He}} + \mathsf{y}_q \underset{tt}{Q_{Hq}^{(1)}} + \mathsf{y}_u \underset{tt}{Q_{Hu}} + \mathsf{y}_d \underset{tt}{Q_{Hd}} \right].
\end{aligned} \tag{5.4}
$$

Here the Wilson coefficients of the operators Q_{HB}, Q_{HWB} are considered to be "loop level", and the remaining operators are considered to have Wilson coefficients that are tree level in this scheme. Recall that the operator forms are equated as a result of the freedom to redefine the SM fields by $\mathcal{O}(1/\Lambda^2)$ corrections without changing the S matrix. It is important to understand the distinction between such IR relations that are statements of consistency conditions in the SMEFT (in this case the freedom to change variables in a path integral without physical effect) and the UV assumptions that can break the corresponding relations that follow between Wilson coefficients. Unfortunately, this distinction is frequently lost in the literature where references to "tree" and "loop" operators abound, which blurs the distinction between the IR operators and the UV matching coefficients that follows from the separation of scales defining the EFT.

One scale, one coupling Meta-matching: It was observed in Ref. [351] that the NDA results of Ref. [313] could follow from the assumption of NDA being respected by QCD as an assumed high energy theory, and then considered to be matched to χPT with $g_3 \simeq 4\pi$. This point was not stressed, as it is well known that when QCD is strongly interacting a useful EFT description transitions to χPT treated as a bottom up EFT, as an essential singularity is present. In the case of QCD with $m_q \to 0$, the beta function leads to the relation

$$
\left(\frac{\Lambda_{qcd}}{\mu} \right)^{b_0} = e^{-8\pi^2/\left[\hbar\, g_3^2(\mu)\right]}, \tag{5.5}
$$

with the factors of $\hbar$ explicit [352, 353]. Λ_{qcd} is not due to summing the g_3^2 expansion to all orders with a choice $g_3 \simeq 4\pi$ due to the essential singularity at $g_3^2 \hbar = 0$ in the g_3^2 and $\hbar$ expansions. No useful perturbative matching can be performed in this limit.

Ref. [354] observed that if some of the vector resonances of χPT were considered lighter than the remaining states, an alternate normalization of the Lagrangian terms of χPT under this assumption could be constructed.

A highly influential version of Meta-matching was put forth in Ref. [218] based on arguing these two observations hold in a useful manner for general scenarios of strongly interacting physics with a light pseudo-Goldstone Higgs. The modern version of the Lagrangian normalization for a composite sector that results [355] is given by

$$
\frac{m_\star^4}{g_\star^2} \left(\frac{\partial}{m_\star} \right)^A \left(\frac{g_\star \Pi}{m_\star} \right)^B \left(\frac{g_\star \sigma}{m_\star} \right)^C \left(\frac{g_\star \Psi}{m_\star^{3/2}} \right)^D. \tag{5.6}
$$

Here the σ model scale is given by $f = m_\star/g_\star$ where $m_\star$ and $g_\star$ are the corresponding one scale and one coupling assumed in the composite sector. The Ψ, Π and σ are assumed states in the composite sector with the latter two associated with the σ model scale f. Further assuming that $g_\star < 4\pi$ one can (at least hypothetically) hope to avoid the equivalent challenge of an essential singularity being present in perturbative expansions in the strong coupling limit, and obtain a result consistent with the NDA result of Ref. [313] (with 4π replaced by $g_\star$), assuming a matching onto the SM states making up $\mathcal{L}_6$. It is however necessary to stress that the presence of an essential singularity in the strong coupling limit is the basic reason that any such approach taking the $g_\star \to 4\pi$ (for example in Eqn. 5.6) in a naive analytic continuation is not expected to be useful or informative in general, and rather unwise on very basic grounds. For a strongly coupled UV theory in general, there is no reason to expect such a simplistic counting to hold for this reason, and also for the reasons discussed in Sections 2.2, 3.4.1.

As initially formulated in Ref. [218], the approach to Meta-matching had several other essential inconsistencies, see discussion in Refs. [32, 34, 38, 313, 348, 349]. Alternative reasoning [349, 355–357] has since been advanced to justify some of the results of Ref. [218] and other assumptions of this work have been effectively abandoned [34, 38, 349].

Universal theories: An oblique, or universal theory assumption [243, 245, 358–361] is another example of Meta-matching. This idea asserts that the dominant effects of physics beyond the SM can be sequestered into modifying the two point functions of the gauge bosons $\Pi_{WW,ZZ,\gamma Z}$. The idea of oblique, or universal theories, is motivated out of past approaches to EWPD, where the global S, T EWPD fit assumes that vertex corrections due to physics beyond the SM are neglected - giving the "oblique" qualifier [362]. It is problematic that the oblique, or universal, assumption is not field redefinition invariant,[59] see Section 4.1.1.

Oblique, or universal, assumptions were tolerable in the 1990's and early 2000's as the SM Higgs couples in a dominant fashion to $\Pi_{WW,ZZ}$ when generating the mass of the W, Z bosons, and has small couplings to the light fermions, satisfying the assumption in a particular UV model that was not directly experimentally supported.

LHC results now indicate the W, Z bosons obtain their mass in a manner that is associated with the Higgs-like scalar. Corrections to $\Pi_{WW,ZZ}$ can be included for the SM, or more generally [270] due to the discovered scalar. There is no strong theoretical support to maintain an oblique, or universal, assumption to constrain the further perturbations due to new physics scenarios in the SMEFT with EWPD. It is also unclear if this idea is even a consistent theoretical concept. The reason is that a fully defined mechanism of dynamical mass generation in a UV sector has never been demonstrated to be consistent with this assumption, to define a consistent IR limit. Furthermore, assumed oblique, or universal, theories generate non-universal theories [265, 363] using the renormalization group to run the operators matched onto from $\Lambda \to \hat{m}_z$. For these reasons, this assumption has largely been abandoned in consistent SMEFT analyses in recent years.[60]

[59] A corollary to the point that Lagrangian parameters are not physical (see Section 6) is that field redefinitions cannot be used to satisfy defining physical conditions on the EFT, such as the oblique, or universal, assumption. For discussion on this point see Ref. [265].

[60] For more discussion on the limitations of a universal theories assumption, see Refs. [364, 365].

5.1.3 Meta-matching vs power counting

EFT formalizes the separation of scales in a problem so that Meta-matching assumptions are avoidable. An analogy to the SM is useful. The SM predictions depend on the numerical values of the gauge couplings $\{g_1, g_2, g_3\}$. Different values of the gauge couplings in the SM lead to different predictions of S matrix elements and relations between S matrix elements. The SM is studied without invoking as necessary a reference to a grand unified theory (GUT) embedding, even though such an embedding would form a UV boundary condition that dictates the values of $\{g_1, g_2, g_3\}$ at lower scales. The same point holds in the SMEFT and HEFT, for different values of the Wilson coefficients. It is largely a sterile and unproductive debate to dispute assumptions about the size of $\{g_1, g_2, g_3\}$ or C_i. Measuring parameters is more productive.

Making Meta-matching assumptions in an EFT framework causes confusion and is the source of much conflict in the literature. The separation of scales fundamental to the EFT construction can be violated, the assumptions are frequently assigned to the IR operator forms, not the UV dependent Wilson coefficients, and yet EFT language and concepts are used. The conclusions derived are then not model independent EFT statements, although they are usually argued to be "broadly applicable" conclusions as a subjective assertion.

On the other hand, due to the large number of parameters present in the HEFT and the SMEFT the reduction in the parameter space seems required. We stress that it is universally accepted to use the *IR assumptions* of experimentally motivated flavour symmetries being present in the operator basis, which leads directly to a sufficient reduction in Wilson coefficient parameters to enable a global constraint program.

At times extra assumptions are still employed which can lead to much stronger conclusions studying the data. Such conclusions must be defined in a consistent IR limit of a UV sector to be meaningful and, even if this is the case, are limited by model dependence. Such results can be of great value if they are properly qualified and defined.

5.2 Counting rules for the chiral Lagrangians

Power counting in HEFT has also been intensely debated recently in Refs. [347–349, 366]. In HEFT, it is not possible to define an ordering criterion among the invariants that holds in any kinematic regime. Furthermore, assumptions made on the size of the Wilson coefficients can have significant phenomenological consequences. In this HEFT discussion, LT will indicate $\mathcal{L}_0$ while NLT is $\Delta\mathcal{L}$.

5.2.1 χPT

The HEFT is a fusion of the SMEFT with χPT. These two EFTs follow different power countings. One of the main applications of χPT is the description of the octet of light QCD mesons ($\pi^{0,\pm}$, $K^{0,\pm}$, η) at $E \ll \Lambda_{\rm QCD}$. These particles are the Goldstone bosons of the spontaneous breaking of the chiral symmetry $\mathrm{SU_L}(3) \times \mathrm{SU_R}(3)$ into the diagonal $\mathrm{SU_{L+R}}(3)$ and they are collectively described by the field

$$U = \exp\left(2i\frac{\mathbf{\Phi}}{f}\right), \quad \mathbf{\Phi} = \phi^A T^A \,, \tag{5.7}$$

that transforms as $U \mapsto LUR^\dagger$ under the chiral group. Here f is the meson decay constant[61], that satisfies the relation $4\pi f \geq \Lambda$, where Λ is the cutoff of the effective description [313]. Omitting the electromagnetic interactions, and in the limit of massless quarks, the leading order Lagrangian is

$$\mathcal{L}_2 = \frac{f^2}{4}\,\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U), \tag{5.8}$$

where the presence of f^2 ensures canonical kinetic terms for the ϕ^A fields upon expanding the exponential. Because the matrix U is adimensional, this EFT is organized as an expansion in derivatives:

$$\mathcal{L}_{\chi\mathrm{PT}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots\,, \tag{5.9}$$

where $\mathcal{L}_d$ contains invariants with d derivatives. This is Weinberg's power counting approach, which is supported by a renormalization argument [71]. The Lagrangian $\mathcal{L}_2$ contains an infinite series in powers of the meson fields ϕ^A. The counterterms required to reabsorb the one-loop divergences appearing at this level correspond to interaction terms with four derivatives. This can be seen with a topological analysis: an amplitude with L loops and containing N_d vertices with d derivatives must scale with D powers of momentum, where [71]

$$D = 2L + 2 + \sum_d (d-2)N_d\,. \tag{5.10}$$

Computing at one loop ($L = 1$) with $d = 2$ vertices gives only amplitudes with $D = 4$, i.e. four derivatives. Computing at two loops with L_2 or at one loop with one insertion of a 4-derivative interaction plus an arbitrary number of $d = 2$ vertices, gives instead 4-derivative terms and so on. This is in contrast with the SMEFT case, where loops containing one insertion of a d-dimensional operators generates divergences of order $\leq d$.[62] Eqn. 5.10 implies that χPT is renormalizable at fixed order in the momentum expansion, which provides a solid iterative method for organizing the effective series: each order of the EFT must contain at least all the operators that are required as counterterms for the one-loop renormalization of the preceding order.

An alternative counting prescription for χPT is that of NDA [313]. This provides a rule for establishing the normalization of a given operator and it is constructed so that the scaling assigned to a given invariant is independent of the loop order at which it is generated in the EFT. Restricted to our simple case, it states that the overall coefficient of a generic interaction with D derivatives and S pion fields is estimated by the formula

$$f^2\Lambda^2 \left(\frac{\partial_\mu}{\Lambda}\right)^D \left(\frac{\phi}{f}\right)^S\,. \tag{5.11}$$

This rule assigns the correct coefficient f^2 to the Goldstones kinetic term (Eqn. 5.8). The suppression in powers of Λ assigned by the NDA rule is equivalent to the derivative counting.

[61] At this level all the eight mesons are degenerate and share the same decay constant. The inclusion of higher-order terms breaks the $\mathrm{SU_{L+R}}(3)$ symmetry leading to the splitting $f_\pi \simeq 93$ MeV, $f_K \simeq 113$ MeV.

[62] See Ref. [116] for the complete set of these terms.

In the NDA notation, the χPT Lagrangian can be written

$$\mathcal{L}_{\chi\mathrm{PT}} = \frac{f^2}{4}\left(\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{\Lambda^2}\mathcal{L}_4 + \frac{1}{\Lambda^4}\mathcal{L}_6 + \dots\right) . \tag{5.12}$$

Requiring that operators with 4 derivatives must be weighted by a factor at least as large as a loop suppression one obtains the constraint $f^2/\Lambda^2 \geq 1/(4\pi)^2$, i.e. $\Lambda \leq 4\pi f$.

χPT without electromagnetic interactions and in the limit of vanishing quark masses, is a simple example, in which the three counting methods (derivative, loops, NDA) all lead to the same result. The scenario becomes more complicated as soon as dimensionful quantities or fields, such as photons or mass terms, are incorporated in the EFT. The explicit chiral symmetry breaking term

$$\mathcal{L}_\chi = \frac{f^2}{4}\,\mathrm{Tr}(U^\dagger\chi + \chi^\dagger U)\,, \tag{5.13}$$

has χ as a quantity proportional to the quarks masses $\chi \sim \mathrm{diag}(M_u, M_d, M_s)$ that transforms as $\chi \to L\chi R^\dagger$. The derivative counting can be extended to this object assuming that, being dimensionful, χ shall scale as p. In this way the expansion in p/Λ is formally maintained, but upgraded to the nomenclature of "chiral dimension", which counts both the number of derivatives and χ insertions. The term $\mathcal{L}_\chi$ has chiral dimension 2 and therefore is a leading term, that should be included in $\mathcal{L}_2$.

The scaling $\chi \sim p$ is formal and represents an approximation valid only in the regime $p \simeq M_{\pi,K}$. The chiral dimension gives a correct estimate of the impact of a given operator only for processes with on-shell mesons, while it loses physical meaning as one moves outside this kinematic regime. This is because the relative importance of two given operators (such as the kinetic term and $\mathcal{L}_\chi$) can be different in S matrix elements measured at distinct energies.

5.2.2 HEFT counting

HEFT is complex as an EFT due to the simultaneous presence of a scalar sector embedded in a χPT-like construction and of fermions and longitudinal gauge bosons, whose interactions are organized according to their canonical dimension.

Following the two-step procedure of power counting presented at the beginning of this section, one first has to address the question of assigning weights to each Lagrangian term. This can be done in a self-consistent EFT approach adopting the generalized version of NDA [348], which builds upon the work in Refs. [26, 313, 345–347, 349]. For the HEFT case, this modern NDA assigns the scaling

$$\frac{\Lambda^4}{16\pi^2}\left(\frac{\partial_\mu}{\Lambda}\right)^D \left(\frac{4\pi A_\mu}{\Lambda}\right)^A \left(\frac{4\pi\psi}{\Lambda^{3/2}}\right)^F \left(\frac{4\pi\phi}{\Lambda}\right)^S \left(\frac{g}{4\pi}\right)^{N_g} \left(\frac{y}{4\pi}\right)^{N_y} , \tag{5.14}$$

to an interaction with D derivatives, A gauge fields, F fermion insertions and S scalar fields (either the Goldstones or the physical Higgs), accompanied by N_g gauge coupling constants and N_y Yukawas.

Determining the scaling of a given term does not ensure the existence of a unique ordering of effects in the EFT that holds at any energy regime and that is also independent of any UV assumptions. There are several different elements that coexist in the HEFT: one may choose to organize the expansion in loops, i.e. according to the requirement of an order by order renormalization, or rather to order the operators by their number of derivatives, chiral dimensions, inverse powers of Λ or of 4π. These rules are not generally consistent with each other and they all operate simultaneously. Such a choice can be made upon restricting to a particular kinematic regime or assuming a particular class of UV completions. It is not possible, to our knowledge, to select one of these elements as the unique dominating rationale to order terms in importance for all S matrix elements within the predictive regime of validity of this EFT, and for all possible values of the Wilson coefficients.

To see the contradictions that may emerge between different countings, consider the operator $\epsilon_{IJK} W^I_{\mu\nu} W^{J\nu\rho} W^{K\mu}_{\rho}$. This term is not required to reabsorb one-loop divergences of $\mathcal{L}_0$ and it has chiral dimension 6, so it is a NNLT according to Weinberg's counting ($\mathcal{L}_6$ in the χPT series).[63] It may be argued that, containing only transverse gauge fields, this operator should not follow the chiral counting, but rather the SMEFT classification, that sets it as a NLT. An alternative observation is that as this operator contains two derivatives more than the gauge kinetic term, which belongs to $\mathcal{L}_0$, this operator can be a NLT even in a chiral approach. Finally, NDA assigns to this term a coefficient $4\pi/\Lambda^2$, which has both a (4π) enhancement and a Λ^2 suppression. The different counting rules give contradictory estimates in this case because each rule stems from a different assumption. Depending on both the kinematic regime of interest and particular values of Wilson coefficients considered, any one of these arguments may drive the organization of the EFT: for instance in the limit $p/\Lambda \ll v/\Lambda$ the derivative expansion dominates over other criteria, but this is not the case in general.

It is necessary, whatever power counting is followed, that inconsistencies in the theory are avoided. Classifying as NLT the operators appearing as counterterms for the renormalization of $\mathcal{L}_0$ is required. Even within this category, the physical impact of the invariants can be different and dependent on the kinematic regime to which the HEFT is applied. Two such structures are for instance $\bar{Q}_L \mathbf{V}_\mu \mathbf{V}^\mu \mathbf{U} Q_R$ and $(\bar{Q}_L \mathbf{U} Q_R)^2$. The former has one fermionic current and two derivatives, that correspond to gauge bosons insertions in unitary gauge, the latter is a four-fermion operator. The NDA weight for the single-current term is $1/\Lambda$, and the one for the four-fermion term is $(4\pi)^2/\Lambda^2$. So the two can have a comparable impact in a specific kinematic region, but this isn't the case in general for all possible S matrix elements.

There are many intrinsic challenges to a systematic classification of HEFT operators into well defined orders. Some of those are present in $\mathcal{L}_0$ where one can select whether couplings of the physical Higgs to the gauge kinetic terms should be included in $\mathcal{L}_0$ or not. The choice made in Eqn. 4.45 to define these terms as NLT is phenomenologically sound, but it requires a specific assumption that the transverse gauge fields not being directly coupled to the scalar

[63] This is the case both with the classical notion of chiral dimension in χPT, defined in the previous paragraphs, and with the chiral dimension generalized to the HEFT case, defined in Ref. [347]. Note that the latter requires the definition of the operator to include a factor g_2^3.

sector in the UV, leading to a suppressed Wilson coefficient. Once the leading Lagrangian is fixed, the next order $\Delta\mathcal{L}$ can be identified following:

- All terms required for the one-loop renormalization on $\mathcal{L}_0$ must be included in $\Delta\mathcal{L}$. To this end, the one-loop renormalization of the scalar sector of the HEFT has been worked out in Refs. [230, 279, 334, 367–369]. These results build upon previous results obtained in the absence of the h degree of freedom [102, 103, 156, 370, 371].

- Structures that are not strictly required as counterterms, but that receive finite loop contributions from $\mathcal{L}_0$ and/or have a similar field composition or NDA suppression as the counterterms, should also be retained. An example corresponding to finite loops of $\mathcal{L}_0$ is dipole operators, that have the structure $\bar{\psi}_L\sigma^{\mu\nu}\mathbf{U}\psi_R X_{\mu\nu}$. An example of inclusion by similarity in the field composition are four-fermion operators with vector/axial currents (schematically $\bar{\psi}\gamma_\mu\psi\,\bar{\psi}'\gamma^\mu\psi'$), which are included by analogy with the four-fermion operators with scalar currents ($\bar{\psi}_L\mathbf{U}\psi_R\,\bar{\psi}'_L\mathbf{U}\psi'_R$) that are required for the renormalization of $\mathcal{L}_0$.

- Operator categories that are classified as NLT in at least one of the counting principles described above should be included, e.g. the term $\epsilon_{IJK}W^I_{\mu\nu}W^{J\nu\rho}W^{K\mu}_\rho$. This point ensures that all the operators that can be relevant in at least one specific scenario are included, preserving the generality of the HEFT description.

- The set of operators that form the $\Delta\mathcal{L}$ basis should close under the use of the EOMs.

These rules lead to the construction of the bases of Refs. [274, 284]. The characterization of higher orders in the HEFT expansion poses further difficulties due to the mismatch among the expansions in different parameters.

6 The S-matrix, Lagrangian parameters and measurements

Subtleties that can complicate the interpretation of experimental data in EFTs are clarified by a clear definition of S matrix elements and their distinction from Lagrangian parameters. In this section we review these two concepts for later use and emphasize the distinctions between them and their relation to measurements. The aim of this section is to discuss the physical nature of S matrix elements, the unphysical nature of Lagrangian terms, and to then build up to how the pseudo-observable concept is introduced to attempt to bridge between full fledged S matrix elements and common gauge invariant quantities that appear in many measurements.

6.1 S-matrix elements

Consider a set of interpolating spin-$\{0, 1/2, 1\}$ fields, that are representations of $\mathrm{SO}^+(3,1)$, which we denote schematically as $\theta = \{\phi, \psi, V^\mu\}$. These fields are used to construct asymptotic perturbative expansions that satisfy a set of global symmetries (G).[64] We denote the Lagrangian composed of θ, constructed to manifestly preserve G, as $\mathcal{L}(\theta)$, and the sources of the fields as J_θ. The generating functional $W[J_\theta]$ of the connected Green's functions is then defined as

$$e^{i\,W[J_\theta]} = \int \mathcal{D}\theta \exp\left[i \int d^4x \left(\mathcal{L}(\theta) + \theta\, J_\theta\right)\right]. \tag{6.1}$$

Here $\mathcal{D}$ is the measure of the functional integral. Functional differentiation of $W[J_\theta]$ with respect to n fields then defines an n-point time ordered correlation function (or Green's function) between ground states (Ω) of the interacting theory

$$\langle\Omega|T\theta(x_1)\,\theta(x_2)\cdots\theta(x_n)|\Omega\rangle = (-i)^n \frac{\delta^n W}{\delta J_\theta(x_1)\cdots\delta J_\theta(x_n)}. \tag{6.2}$$

The LSZ reduction formula [372] defines the n-point S matrix elements related to these correlation functions as

$$\begin{aligned}
&\prod_1^a \int d^4x_i\;\; e^{ip_i\cdot x_i} \prod_1^b \int d^4y_i\, e^{ik_i\cdot y_i} \langle\Omega|T\theta(x_1)\,\theta(x_2)\cdots\theta(x_a)\theta(y_1)\,\theta(y_2)\cdots\theta(y_b)|\Omega\rangle, \\
&\quad\simeq \left(\prod_{i=1}^a \frac{i\sqrt{Z_i}}{p_i^2 - m_i^2 + i\epsilon}\right)\left(\prod_{j=1}^b \frac{i\sqrt{Z_j}}{k_j^2 - m_j^2 + i\epsilon}\right)\langle p_1\cdots p_a|S|k_1\cdots k_b\rangle.
\end{aligned} \tag{6.3}$$

Here $n = a + b$, and p_i,k_j are four momenta associated via the Fourier transform of each field, which is also associated with a mass term $m_{i,j}$ and renormalization factor $Z_{i,j}$. The $\simeq$ is due to the requirement to isolate each single particle pole experimentally so that each $p_i^0 \to E_{p_i}$, $k_j^0 \to E_{k_j}$ when treating the particles as asymptotic states.

S matrix elements, unlike Lagrangian parameters, directly define the measurable quantities in the theory: the scattering and decay observables.[65] S matrix elements conserve overall four momentum and are unitary. The decomposition of an S matrix element into an Lorentz invariant amplitude $\mathcal{M}$ is given as

$$\langle p_1\cdots p_a|S|k_1\cdots k_b\rangle = I + i\,(2\pi)^4\,\delta^4(p_i - k_j)\,\frac{\mathcal{M}}{\sqrt{2E_i}\sqrt{2E_j}}, \tag{6.4}$$

with implicit sum over i, j. For most scattering observables, $2 \to n$ processes are sufficient to consider, which are given as

$$d\sigma = \frac{(2\,\pi)^4|\mathcal{M}|^2}{4\sqrt{(p_1\cdot p_2)^2 - m_1^2\,m_2^2}}\,\delta^4(p_1 + p_2 - \sum_j k_j)\prod_{j=1}^n \frac{d^3k_j}{(2\pi)^3\,E_j}. \tag{6.5}$$

[64]The global symmetry is emphasized as the local gauge redundancy does not lead to relations between S matrix elements. Gauging a symmetry does not give any additional conserved charges (and corresponding S matrix relationships) beyond those of the corresponding global symmetry, see Refs. [32, 34] for discussion.

[65]The S matrix was first introduced in Ref. [373].

Similarly, particle decays of an initial state with momentum (P) and mass (M) into $\sum_j k_j$ final states are given by

$$d\Gamma = \frac{(2\,\pi)^4}{2\,M}\,|\mathcal{M}|^2\,\delta^4(P - \sum_j k_j)\prod_{j=1}^{n}\frac{d^3k_j}{(2\pi)^3\,E_j}. \tag{6.6}$$

Measured event rates correspond to $d\sigma$ or $d\Gamma$ integrated over a measured phase space volume. Such phase space integrations can be highly non-trivial, see Refs. [374, 375] for excellent discussions on overcoming this technical hurdle. The relation between the differential scattering and decay observables and S matrix elements is direct. Intuitively, changing field variables on the path integral defining the generating functional in Eqn. 6.1 can be expected to have no physical effect on the S matrix elements derived from it, as the field variables are essentially dummy variables used to define the G preserving perturbative expansions of the correlation functions in the path integral formulation of the QFT. This intuition is correct, and not violated by quantum corrections. Such variable changes do modify the Lagrangian terms and the source terms in a correlated manner, which can be used to arrange the Lagrangian into a particular form. This understanding of field redefinitions is formalized in what is known as the Equivalence theorem [237], which is a precise formulation of the invariance of S matrix elements under G preserving field redefinitions for renormalized quantities. See Refs. [68, 223, 376–381] for related discussion. These formal developments are all focused on gauge independent, and G preserving, field redefinitions.

6.2 Lagrangian parameters

Unlike S matrix elements, individual Lagrangian terms are not invariant under field redefinitions [68, 223, 237–239, 376–381]. Lagrangian parameters are neither directly, nor trivially related to S matrix elements, or measured observables. In modern times this understanding has been advanced to a level where efforts are underway to avoid Lagrangian formulations completely in highly symmetric field theories.[66]

Several historical examples of the equivalence of Lagrangians of a distinct interaction and field variable form indicate that a particular interaction term in a Lagrangian should not be mistaken for a physical S matrix element. The famous equivalence of the Thirring [383] and sine-Gordon models under bosonization proven by Coleman [384] (see also Ref. [385]) illustrates the equivalence of a particular quantum theory with purely interacting fermions and bosons in each case. The Lagrangians are given by

$$\mathcal{L}_T = \bar{\psi}(i\not{d} - m)\psi - \frac{g}{2}(\bar{\psi}\gamma^\mu\psi)^2, \qquad \mathcal{L}_{sG} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \frac{\alpha}{\beta^2}\cos(\beta\phi). \tag{6.7}$$

The couplings are identified as $\beta^2/4\pi = 1/(1 + g/\pi)$. These Lagrangians describe the same physics despite the drastically different field content and interaction terms [384].

The practical consequences of Lagrangian terms being distinct from physical S matrix elements is drastically different in the SM and in the SMEFT/HEFT field theories. In the

[66] See Refs. [292, 382] for discussion on this approach.

case of an operator basis choice in the SMEFT or HEFT, the general non-physical nature of individual Lagrangian terms manifests itself in the freedom to pick different sets of Lagrangian terms to represent the same physics. This occurs via $(1/\Lambda^2)$ small field redefinitions in the SMEFT[67] and the EOM play an essential role in equating Lagrangian terms that are naively distinct.

In the SM, $\Lambda \to \infty$, so there is no direct equivalent of these small field redefinitions. The same point on the nature of Lagrangian terms is made by the use of auxiliary fields. When performing gauge fixing, the BRST formalism [386, 387] introduces the non-propagating Lautrup-Nakanishi [388, 389] auxiliary field B^a into a non-abelian Lagrangian as

$$\mathcal{L}_{BRST} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(i \not{D} - m)\psi + \frac{\xi}{2}(B^a)^2 + B^a \partial^\mu A^a_\mu + \bar{c}(-\partial^\mu D^{ac}_\mu) c^c. \tag{6.8}$$

Performing the Gaussian functional integral over the B^a field variable returns

$$\mathcal{L}_{FP} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(i \not{D} - m)\psi - \frac{1}{2\,\xi}\left(\partial^\mu A^a_\mu\right)^2 + \bar{c}(-\partial^\mu D^{ac}_\mu) c^c. \tag{6.9}$$

The two Lagrangians give equivalent S matrix elements despite having a distinct form and naively differing field variables. Another example is supplied by the Hubbard–Stratonovich transformation [390, 391] where a non-dynamical scalar field is "integrated in". In this case the curvature R^2 action of a gravitational theory (i.e. Starobinsky inflation [392])

$$\mathcal{L}_{inf} = \sqrt{\hat{g}}\left[-\frac{M_p^2}{2}\hat{R} + \zeta \hat{R}^2\right], \tag{6.10}$$

is equivalent to the transformed Lagrangian with the auxiliary scalar field

$$\mathcal{L}'_{inf} = \sqrt{\hat{g}}\left[-\frac{M_p^2}{2}\hat{R} - 2\,\alpha\,\Phi^2\,\hat{R} - \Phi^4\right]. \tag{6.11}$$

This can be seen by performing a Gaussian functional integral with $\zeta = \alpha^2$. Performing the Gaussian integrals modifies the field variables to a reduced (less redundant) set that still produce the same S matrix. In these cases the auxiliary field variables are non-propagating and do not have Fock spaces associated with them. So the difference in the Lagrangians is a vanishing difference for the external states labeling S matrix elements. This is the analogy to the vanishing of the difference between operators equivalent by use of the on-shell EOM (as in Eqn. 4.29) essential to choosing an operator basis in the EFT.[68]

The fact that Lagrangian parameters are not physical quantities is not changed when working with mass eigenstate fields, or unitary gauge, in any way.

[67] Again, only those that are gauge independent.

[68] We stress that the point here is on the non-physical nature of Lagrangian parameters in general not that field redefinitions leave the Lagrangian alone invariant. In general this is not the case as discussed in Section 6.1.

6.3 Scattering measurements and renormalizable/non-renormalizable theories

The distinction between Lagrangian terms and S matrix elements is crucial in the SMEFT and HEFT. It is also the case that using naive physical intuition to assign SM Lagrangian terms a naive physical meaning classically, although formally incorrect, is a reasonable rough approximation for a subset of parameters in the SM. Essentially the naive physical intuition at work is accidentally supported by the renormalizable nature of the SM Lagrangian, the small perturbative corrections in the EW sector of the SM, and the related fact that the SM unstable states have small widths compared to their masses. We discuss each of these points in turn and compare the differences that appear when extending SM studies to the SMEFT and HEFT.

6.3.1 The SM case

Extracting the SM Lagrangian parameters $\{v, g_1, g_2, g_3, Y_i\}$ from measurements of S matrix elements at tree level in the SM is generally done in the presence of significant experimental correlations and a degeneracy of parameters present in the mapping to the Lagrangian. This degeneracy is first an issue at the level of perturbative and non-perturbative corrections to the Lagrangian due to a quirk of renormalizable theories. In the renormalizable SM on-shell vertices are unique in the interaction Lagrangian, and off-shell gauge coupling vertices are related by global symmetries to on-shell coupling parameters. This is no longer trivially the case when one transitions to EFT extensions of the SM.

An unusual case of a Lagrangian parameter extraction in the SM, is the extraction of v from $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$. The physics at work is due to the SM being a weakly coupled renormalizable theory describing a Higgsing of $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1) \to \mathrm{U}(1)_{\mathrm{em}}$, in conjunction with neutrinos being weakly interacting particles. Extracting v in the SM is unusual as it can be extracted from a $\bar{\psi}_L \, \psi_L \to \bar{\psi}_L \, \psi_L$ process at low energies where an effective Lagrangian of the form

$$\mathcal{L}_{GF} = -\frac{4G_F}{\sqrt{2}} V_{ij} \, V^{\star}_{kl} (\bar{\psi}_i \gamma^\mu P_L \psi_j)(\bar{\psi}_k \gamma^\mu P_L \psi_l), \qquad \frac{4G_F}{\sqrt{2}} = \frac{g_2^2}{2 \, M_w^2} = \frac{2}{v^2}. \tag{6.12}$$

is used.[69] Here V_{ij} is the CKM or PMNS-matrix in the case of quarks or leptons. The gauge coupling g_2 cancels as a direct consequence that the SM has a Higgs mechanism generating the gauge boson masses. This cancellation is not generic in EFTs but is a rather unique result of the SM. Furthermore, as individual asymptotic eigenstates of neutrinos are not experimentally identified in this decay process, practical measurements of $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ sum over all neutrino species. Unitarity of the PMNS-matrix then leads to a direct extraction of v from this decay, which is another unique feature in extracting v from $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$.

Subsequently when using the input parameter set $\{\hat{G}_F, \hat{M}_W, \hat{M}_Z\}$ mapping these measured quantities to the Lagrangian parameters g_1, g_2 can be done with a m_Z pole scan as

[69] Remarkably, interest in Fermi theory has been acute in recent years in the literature, see Refs. [348, 357, 393]. Our discussion is most consistent with Refs. [348, 393] due to the important role that the CKM and PMNS-matrix plays, and the standard understanding of EFT adopted in this review.

performed at LEP [394], and a study of transverse W mass variables as performed at the Tevatron [375, 395, 396] or similar studies that have begun at LHC [397]. This set of Lagrangian parameter extractions is afflicted with significant experimental correlations.

- In the case of the extraction of $\hat{m}_Z$ at LEP, despite the presence of a resonance peak, the Lagrangian parameters are extracted from a fit that simultaneously defines the pseudo-observables $\{\hat{m}_Z, \Gamma_Z, \sigma^0_{had}, R^0_e, R^0_\mu, R^0_\tau\}$ [394]. See Section 7.

- In the case of $\hat{m}_W$, due the calibration of the electromagnetic calorimeter to Z decays, the experimental extraction is effectively an extraction of the ratio $\hat{m}_W/\hat{m}_Z$ [375, 395, 396].

These extractions of Lagrangian parameters from S matrix elements take place in the perturbative expansion of the EW theory. The precision of these measurements require that higher order corrections be included. It is only because the EW interactions are perturbative with typical leading loop corrections $\lesssim \mathcal{O}(1\%)$ that the resulting parameter degeneracy introduced leads to small perturbations of the highly correlated central values extracted in a naive tree level analysis of $\{v, g_1, g_2\}$.

If an input set of the form $\{\hat{G}_F, \hat{\alpha}_{ew}, \hat{M}_Z\}$ is used then the challenge of parameter degeneracy also appears due to the requirement to run the extracted Lagrangian parameter $\hat{\alpha}_{ew}$ through the hadronic resonance region. Using an input parameter $\hat{\alpha}_{ew}$ actually corresponds to a simultaneous input set of $\{\hat{\alpha}_{ew}, \nabla\alpha\}$ as extractions of $\hat{\alpha}_{ew}$ are dominated by $q^2 \to 0$ measurements determined by probing the Coulomb potential of a charged particle. The low scale measurement extracts this parameter with the mapping to the two point functions Π^{ab}

$$-i\left[\frac{4\,\pi\,\hat{\alpha}(q^2)}{q^2}\right]_{q^2\to 0} \equiv \frac{-i\,\bar{e}_0}{q^2}\left[1 + \mathrm{Re}\frac{\Sigma^{AA}(m_Z^2)}{m_Z^2} - \nabla\alpha\right]. \tag{6.13}$$

The finite terms in the low scale matching that are the largest effect are due to the vacuum polarization of the photon in the $q^2 \to 0$ limit. This unknown term is given by the last two terms on the right hand side of Eqn. 6.13, with the notation

$$\nabla\alpha = \left[\frac{\mathrm{Re}\Sigma^{AA}(m_Z^2)}{m_Z^2} - \left[\frac{\Sigma^{AA}(q^2)}{q^2}\right]_{q^2\to 0}\right]. \tag{6.14}$$

This quantity parameterizes contributions to the two point function that have to be simultaneously determined to define $\hat{\alpha}_{ew}$. For further discussion see Refs. [375, 398–402]. The uncertainty on $\nabla\alpha$ due to this parameter degeneracy completely dominates the uncertainty on $\hat{\alpha}_{ew}$ when it is used in LHC applications.

The requirement of simultaneous extractions and highly correlated Lagrangian parameters from S matrix elements in conjunction with other non-perturbative unknown parameters also returns in the cases of $\{g_3, Y_i\}$:

- Extractions of g_3 from studies of e^+e^- event shapes simultaneously extract α_s and the leading non-perturbative parameter from thrust distributions [403–405]. Similarly the extraction of g_3 from the event shape C parameter is a simultaneous extraction of α_s and a leading non-perturbative parameter [406–408].

- The extraction of m_b from inclusive $\bar{B} \to X_c \bar{\nu} \ell$ decays using HQET is a simultaneous extraction of $\hat{m}_b$ and the leading non-perturbative corrections [409].

- The extraction of light quark masses $\{\hat{m}_u, \hat{m}_d, \hat{m}_s, \hat{m}_c\}$ using Lattice QCD occurs with a simultaneous lattice cut-off parameter and is related to $\overline{\rm MS}$ masses in perturbation theory [375]. The same point holds for extractions of $\hat{m}_c$ [410]. Precise extractions of quark mass parameters $\{\hat{m}_u, \hat{m}_d, \hat{m}_s\}$ in χPT are extractions of quark mass ratios [375], not single Lagrangian terms.

Degeneracy and correlations in the space of Lagrangian parameters that are extracted from S matrix elements is essentially unavoidable. This is a reflection of the fact that Lagrangian parameters are not physical. This remains the case despite a protection of the SM against parameter degeneracy due to the nature of the Higgs mechanism in the SMEFT. As $\langle H^\dagger H \rangle = v^2/2$, interaction terms that differ by dimension $d = 2$ can lead to a degenerate interaction term of the remaining fields when expanding around the vacuum expectation value. As $\mathcal{L}_{SM}$ has $d \leq 4$ this degeneracy is avoided in the SM, as an accidental simplification due to renormalizability.

6.3.2 The EFT case

The challenge of correlations and parameter degeneracy in relating Lagrangian terms to S matrix elements is serious in the SM, and this challenge is even more acute in EFTs. The new parameters introduced in the Taylor series defining the EFTs are local (see Eqn. 3.3), and several parameters appear simultaneously in the power counting expansions. The presence of a Higgsed phase in an EFT now acts to increase parameter degeneracy.

Examining the field redefinition freedom that exists in defining a $\mathcal{L}_6$ basis further reinforces the gap between the S matrix elements and the EFT parameters. Generally, when constructing a $\mathcal{L}_6$ basis, a version of an on-shell effective field theory is constructed [222, 223]. In this approach derivative terms are systematically removed (if possible) and kinetic terms are canonically normalized. This shifts EFT corrections to vertices, which increases parameter degeneracy in how $\mathcal{L}_6$ corrections modify SM predictions. The natural expectation is a highly correlated fit space, and this is indeed frequently found [66, 233, 411–415].[70]

To expand on the example in the previous section in the context of an EFT extension of the SM, the generic conclusion is that the fit spaces of Lagrangian parameters are even more correlated. The accidental nature of the extraction of v in the SM now projects onto a three-fold Lagrangian parameter degeneracy in the $\mathrm{U}(3)^5$ limit as

$$\mathcal{L}_{G_F} \equiv -\frac{4\mathcal{G}_F}{\sqrt{2}} \, (\bar{\nu}_\mu \, \gamma^\mu P_L \mu) \, (\bar{e} \, \gamma_\mu P_L \nu_e) \,, \quad -\frac{4\mathcal{G}_F}{\sqrt{2}} = -\frac{2}{\bar{v}_T^2} - 4 \, \hat{G}_F \, \delta G_F, \tag{6.15}$$

where the leading order shift result is [185, 413]

$$\delta G_F = -\frac{1}{4\,\hat{G}_F} \left(C_{\substack{ll \\ \mu e e \mu}} + C_{\substack{ll \\ e \mu \mu e}} \right) + \frac{1}{2\,\hat{G}_F} \left(C^{(3)}_{\substack{Hl \\ ee}} + C^{(3)}_{\substack{Hl \\ \mu\mu}} \right). \tag{6.16}$$

[70] See also Refs. [416–420].

Using a $\{\hat{\alpha}_{ew}, \hat{m}_Z, \hat{G}_F\}$ input parameter set one finds the results listed in the Appendix for Z, W vertex terms in the SMEFT. The Wilson coefficients that now appear *simultaneously* in $\mathcal{L}_6$ in $\bar{\psi}\psi \to \bar{\psi}\psi$ scattering in the SMEFT (in addition to the SM couplings) are

$$\tilde{C}_i \equiv \frac{\bar{v}_T^2}{\Lambda^2}\{C_{He}, C_{Hu}, C_{Hd}, C_{Hl}^{(1)}, C_{Hl}^{(3)}, C_{Hq}^{(1)}, C_{Hq}^{(3)}, C_{HWB}, C_{HD}, C_{ll}\}. \tag{6.17}$$

Furthermore, beginning at $\mathcal{L}_6$, an EOM degeneracy appears between these parameters in $\bar{\psi}\psi \to \bar{\psi}\psi$ and a subset of $\bar{\psi}\psi \to (\bar{\psi}\psi)^n$ amplitudes due to $\langle H^\dagger H\rangle$ being dimension two, and the SMEFT having a Higgs mechanism. See Section 7.1.4 for further discussion on this resulting reparameterization invariance.[71] Extracting $\{g_3, \hat{m}_b\}$ from lower energy data introduces an even larger set of $\mathcal{L}_6$ parameters.

Highly correlated Wilson coefficient fit spaces are a central feature of the SMEFT. Even mild assumptions about parameter correlations (or lack thereof) have a significant impact on the constrained Wilson coefficient space as a result.

6.4 The idea of pseudo-observables

The previous sections can be summarized as: S matrix elements directly correspond to physical observables, and Lagrangian parameters do not. Common Lagrangian parameters do feed into many observables in a manner that is sometimes consistent with naive classical reasoning, despite classical intuition being misplaced in a QFT. A common reason that this dichotomy between formally correct field theory and naive classical reasoning persists for collider physics studies is frequently the fact that the unstable states in the SM are *narrow*, i.e have the property that $\Gamma/m \ll 1$.

The ratios of the widths (Γ_V) of the unstable $V = \{W, Z\}$ bosons to their masses (m_V) are [375]

$$\frac{\hat{\Gamma}_Z}{\hat{m}_Z} = \frac{2.4952}{91.1876} \sim 0.03, \qquad \frac{\hat{\Gamma}_W}{\hat{m}_W} = \frac{2.085}{80.385} \sim 0.03. \tag{6.19}$$

The width of the Higgs is bounded (in the SM) to be [423–427]

$$\frac{\hat{\Gamma}_h}{\hat{m}_h} \lesssim \frac{0.013}{125.09} \sim 10^{-4}. \tag{6.20}$$

The constraint on the Higgs width is model dependent [428–430] and corresponds to a bound on $\hat{\Gamma}_h$ in conjunction with κ_g and κ_t. Assuming that the width of the Higgs is only perturbed from its SM value in the SMEFT, one can expand in $\hat{\Gamma}_h/\hat{m}_h$.

[71] The first appearance of a bilinear in H outside the Higgs potential in the SM appears at dimension five. [134, 135],

$$\mathcal{L}_5 = C_5^{\beta\,\kappa}\left(\overline{\ell_L^{c,\beta}}\,\tilde{H}^\star\right)\left(\tilde{H}^\dagger\,\ell_L^\kappa\right) + h.c. \tag{6.18}$$

The notation used here is that the c superscript in Eqn. 6.18 corresponds to a charge conjugated Dirac four component spinor defined as $\psi^c = C\overline{\psi}^T$ with $C = -i\gamma_2\,\gamma_0$ in the chiral basis. ℓ_L^c denotes the doublet lepton field that is chirally projected and subsequently charge conjugated. This Lagrangian term does not have a direct degeneracy with parameters already present up to $\mathcal{L}_4$ in the SM, due to the interplay of the global symmetry structure of the SMEFT operator expansion with operator dimension [421, 422].

Narrow widths effectively factorize a full amplitude contributing to an S matrix element up into sub-blocks. Frequently, a naive classical intuition is also assigned to these sub-blocks, at least in a tree level calculation.[72] An unfortunate side effect of the highly correlated nature of the Wilson coefficient space in SMEFT studies is that such factorizing up of observables with the narrowness of the SM states, can break flat directions in the Wilson coefficient space in an inconsistent manner, and bias conclusions by orders of magnitude [233, 415].

The phase space of the scattering events is populated in a manner that usually is dominated by the impact of the narrow resonances, unless selection cuts are applied to remove all resonant contributions. This remains true studying EFT extensions of the SM in colliders. The narrowness of the SM states is actually fortunate for EFT studies at LHC, as while simultaneously avoiding the large non-resonant QCD backgrounds, EFT approaches can exploit this fact and gain a significant benefit – *so long as the exploitation of the narrow width effect occurs in a well defined manner.*

6.4.1 The naive narrow width expansion

The most naive attempt to use the narrowness of the SM states to expand is the naive narrow width approximation. Consider an s-channel scattering $\bar{\psi}\psi \rightarrow S(s)^\star \rightarrow \bar{\psi}\psi$ where $S = \{h, W, Z\}$. Expanding around the limit $\Gamma_S/m \rightarrow 0$ is subtle. It corresponds to a series around the kinematic point in the amplitude where some propagators vanish due to an intermediate state going "on-shell" at $p^2 = m^2$. It is not a trivial expansion in the Γ_S/m ratio at the amplitude level, as this limit must be taken in the sense of a distribution over the phase space.

Due to the Feynman propagator prescription [432, 433] defining the Green's function when intermediate states go on-shell corresponds to a discontinuity in the imaginary part of the Feynman diagram. Cutkosky [434] developed formally the proof that the discontinuity across a branch cut when a stable intermediate state goes on-shell in a Feynman diagram is given by the replacement of the propagator

$$\frac{dp^2}{p^2 - m^2 + i\,\epsilon} \rightarrow -2\,\pi\,i\,\delta(p^2 - m^2)\,dp^2. \tag{6.21}$$

This approach can be extended to the case where the intermediate state is unstable, leading to a generalization of the optical theorem. The most naive version of the resulting reasoning is the narrow width approximation. Define the particle mass by the condition

$$m^2 - m_0^2 - \mathrm{Re}\,\Sigma(m_0^2) = 0, \tag{6.22}$$

where m_0 is the bare mass parameter. The propagator pole is then shifted off of the real axis if the intermediate state decays, in a manner that approximates a Breit-Wigner distribution formula $\propto 1/(p^2 - m_0^2 + i\,m_0\Gamma_S(p^2))$. If $\Gamma_S/m \ll 1$, it can be approximated as a constant $\Gamma_S(p^2) \simeq \Gamma_S$ for the phase space events populated by scattering through the resonance peak.

[72] See Ref. [431] for a discussion on the explicit and implicit assumptions in the narrow width approximation.

Then, in a distribution sense where the phase space is integrated over a sufficiently inclusive region, the appearance of a propagator squared in a full cross section can be simplified. Shifting the zero point of the symmetric dp^2 distribution, and performing the resulting integral gives

$$\int_{-\infty}^{\infty} \frac{dp^2}{(p^2 - m_0^2)^2 + m_0^2 \, \Gamma_S^2} = \int_{-\infty}^{\infty} \frac{dp^2}{(p^2)^2 + m_0^2 \, \Gamma_S^2} = \frac{1}{m_0 \, \Gamma_S} \int_{-\infty}^{\infty} \frac{dx}{1 + x^2} = \frac{\pi}{m_0 \, \Gamma_S}. \tag{6.23}$$

To utilize this expansion for an on-shell intermediate state contributing to an S matrix element we replace

$$\frac{dp^2}{(p^2 - m_0^2)^2 + m_0^2 \, \Gamma_S^2} \to \frac{\pi}{m_0 \, \Gamma_S} \, \delta(p^2 - m^2) \, dp^2. \tag{6.24}$$

For a total $\bar{\psi}_k \psi_l \to S(s)^\star \to \bar{\psi}_i \psi_j$ cross section one obtains

$$\begin{aligned} \sigma &= \frac{1}{32 \, \pi \, s} \int \frac{d^3 k_i}{(2\pi)^3 \, E_j} \frac{d^3 k_j}{(2\pi)^3 \, E_j} |\mathcal{A}(\bar{\psi}_k \psi_l \to S)|^2 |\mathcal{A}(S \to \bar{\psi}_i \psi_j)|^2 \frac{\pi}{m \, \Gamma_S} \, \delta(s - m^2), \\ &= \sigma(\bar{\psi}_k \psi_l \to S) \mathrm{Br}(\mathrm{S} \to \bar{\psi}_\mathrm{i} \psi_\mathrm{j}). \end{aligned} \tag{6.25}$$

This is the narrow width approximation. Using this approximation for on-shell production corresponds to naive classical intuition and factorizes up phase space in a manner that is a significant numerical benefit in evaluating cross sections. The presence of narrow widths of the SM unstable states should be exploited to aid in studying the SMEFT and the HEFT at LHC. Attempting to utilize the naive narrow width approximation to achieve this end, is problematic. Narrow width approximations are subject to severe limitations, as should be evident from the derivation. The limitations include

- The derivation is limited to on-shell kinematics and a tree level exchange, and not performed in the presence of experimental cuts limiting the phase space.
- Higher order corrections and renormalization is not obviously consistent with the derivation, it is also not obvious that the approach is gauge invariant at higher orders.
- The factorization of the cross section into sub-blocks for the limited tree level exchange does not change the Hilbert space of the theory, and the excited unstable S state is still not an external particle. Technically the unstable particles do not allow a plane wave expansion as asymptotic states; their energies are imaginary and the asymptotic plane wave expansions either diverge or vanish.
- It is unclear how to formally justify using replacements such as in Eqn. 6.21 for other processes, even $\bar{\psi}\psi \to \bar{\psi}\psi \, \bar{\psi}\psi$ where one S state is on-shell while a second S state is off-shell for the same region of phase space. The formal developments of Cutkosky [434] for stable intermediate states do not directly justify replacing with Eqn. 6.21 in arbitrary Feynman diagrams for unstable intermediate states, and all points in phase space.

- In the SMEFT or HEFT cases, the corrections to the narrow width approximations are comparable to the order of the SMEFT corrections to the SM. One expects in many cases $\Gamma_{W,Z}/M_{W/Z} \sim \bar{v}_T^2/\Lambda^2, p^2/\Lambda^2$. If a narrow width prescription adopted is ambiguous or essentially an arbitrary scheme choice this can significantly impact the allowed parameter space of the Wilson coefficients in the EFT using off-shell data, biasing global fit conclusions.

- Formally, the narrow width expansion and SMEFT expansion do not commute when the $\{\hat{\alpha}_{ew}, \hat{m}_Z, \hat{G}_F\}$ input parameter scheme is used. To avoid an ambiguity, the ordering of the expansions should be defined, see Ref. [233, 435]. It is known that the ambiguity is experimentally constrained to be small, even in EFT extensions of the SM [436]. This affects the results for the top width given in the Appendix (Eq. A.167).

The challenges of unstable states in field theory are well known, see Refs. [437–450] for formal developments. The modern approach of utilizing the narrowness of the SM unstable states is still challenged by these issues, but some progress has been made. Modern efforts to decompose a cross section into gauge invariant sub-blocks exploiting the narrowness of the SM states avoid the most naive narrow width approximation.

6.4.2 Double pole expansions

Consider the process with the next level of complexity compared to $\bar{\psi}\,\psi \to S^\star \to \bar{\psi}\,\psi$. When two propagators are present one has to determine an expansion exploiting narrow width enhancements for the process

$$\bar{\psi}\,\psi \to S^\star(s_{12})\,S^\star(s_{34}) \to \bar{\psi}\,\psi\bar{\psi}\,\psi. \tag{6.26}$$

An extension of the naive narrow width approach can be developed directly by expanding the amplitude result around the two S poles, assuming the intermediate states are stable, giving the decomposition

$$\begin{aligned}\mathcal{A}(s_{12}, s_{34}) = & \frac{1}{s_{12}-\bar{m}_W^2}\frac{1}{s_{34}-\bar{m}_W^2}\mathrm{DR}[s_{12}, s_{34}, d\Omega] + \frac{1}{s_{12}-\bar{m}_W^2}\mathrm{SR}_1[s_{12}, s_{34}, d\Omega], \\ & + \frac{1}{s_{34}-\bar{m}_W^2}\mathrm{SR}_2[s_{12}, s_{34}, d\Omega] + \mathrm{NR}[s_{12}, s_{34}, d\Omega].\end{aligned} \tag{6.27}$$

Here DR, $\mathrm{SR}_{1,2}$ and NR refer to the doubly resonant, singly resonant and non-resonant contributions to the amplitude, respectively, and Ω refers to all angular dependence defined in an s_{12}, s_{34} independent manner. This expansion is defined as in Refs. [443, 451–454] and is not a trivial Feynman diagram decomposition for the off-shell phase space, but a reorganization of the full amplitude result around the physical poles[73] in a Laurent expansion. The residues

[73]One can understand that the situation is more subtle when considering off-shell production as double resonant diagrams contributing to Eqn. 6.26 are not gauge invariant as a subset of the full amplitude. The difference in axial and 't Hooft-Feynman gauge expressions for the doubly resonant diagrams generates a single-resonant diagram process [443].

of the poles are gauge invariant as they can be experimentally measured (at least in principle). This factorization of the process into gauge invariant sub-blocks can be considered an example of a pseudo-observable decomposition, with the individual terms in Eqn. 6.27 being pseudo-observables.

Adding the width of the unstable S state into these pole expressions can be performed after the residues are determined. This approach, with perturbative corrections, underlies the SM prediction of the $\bar{\psi}\,\psi \to W^\star(s_{12})\,W^\star(s_{34}) \to \bar{\psi}\,\psi\bar{\psi}\,\psi$ process in Refs. [443, 451–454]. The systematic perturbative improvement of a double-pole decomposition with higher order radiative corrections and higher order terms in the Γ/m expansion is technically challenging. The challenge arises from the need to calculate soft photon radiative corrections that are non-factorizable as well as factorizable in the sense of the pole decomposition.

In the case of $\bar{\psi}\,\psi \to W^\star(s_{12})\,W^\star(s_{34}) \to \bar{\psi}\,\psi\bar{\psi}\,\psi$ the non-factorizable corrections were small compared to the experimental uncertainty at LEP [455]. This conclusion does not directly translate to the LHC experimental environment, or future colliders where pseudo-observable approaches face significant challenges from the need to characterize radiative corrections.

6.4.3 Complex mass scheme

Probing for small SMEFT or HEFT corrections to the SM predictions argues that improvements beyond a naive narrow width or tree level double pole approximation could be required in the long term LHC program. A systematically improvable theoretical framework is required to determine such corrections. A preferred approach is to expand around unstable particle poles relying[74] on the generalization of the idea of particle mass introduced in Refs. [456] and developed for EW applications in Refs. [439–443, 450, 457, 458]. The key observation is that an unstable particle corresponds to a pole on the second Riemann sheet of an analytic continuation of the S matrix.[75] The complex mass of a state S is the solution of

$$s - m_S^2 + \Sigma_{SS}(s) = 0, \tag{6.28}$$

with renormalized mass m_S and self energy Σ_{SS} and the negative imaginary solution is taken [450].[76] Use of the Nielsen identities [450, 459, 460] establishes that the position of the pole is gauge parameter independent in the SM and the SMEFT. The decomposition of a propagator square in the cross section can be augmented from Eqn. 6.21 to be

$$\frac{dp^2}{(p^2 - m_0^2)^2 + m_0^2\,\Gamma^2} \to \frac{\pi}{m_0\,\Gamma}\,\delta(p^2 - m^2) + \mathrm{PV}\left[\frac{1}{(p^2 - m_0^2)^2}\right], \tag{6.29}$$

[74]This expansion is also known in some literature as a multi-pole expansion. This expansion is distinct from the expansion discussed in Section 2.2.

[75]For a detailed discussion on the formal development of this analytical continuation for LHC processes, see Ref. [450].

[76]A drawback when considering the SMEFT is that this scheme is tied to on-shell renormalization schemes, while EFT studies generally use $\overline{\mathrm{MS}}$ subtraction.

where PV indicates a principal value. Simultaneously factorizing up the phase space in a manner consistent with this pole decomposition allows a systematic analytical continuation of amplitudes into the complex mass scheme at tree level. In the SM this approach was also pushed to report full one loop corrections to $\bar{\psi}\,\psi \to \bar{\psi}\,\psi\bar{\psi}\,\psi$, see Ref. [461].

The complex mass scheme treats resonant and non-resonant regions of phase space in a unified manner and one loop calculations have a difficulty similar to one loop calculations in the SM. It is reasonable to expect that this approach could be extended to one loop results in the SMEFT, but such full calculations have never been carried out to date in any LHC process involving the Higgs boson. As no explicit expansion in Γ/m need be performed using the complex mass scheme, the application of this approach to SMEFT studies is favoured to systematically develop pseudo-observables [31, 262, 450, 457, 462].

An alternative approach is to use unstable particle EFT that was developed in Refs. [463–465]. It is unclear if these results can be extended to a SMEFT study. It is also unclear if the comparative (one loop) benefits of the complex mass scheme over the unstable particle EFT present in the SM will persist in SMEFT studies. For more discussion comparing these approaches in the SM, see Ref. [466].

6.5 Basics of EFT studies at colliders

Practically implementing EFT studies using data from modern colliders is challenging on several fronts. A large number of free parameters is characteristic of the EFT's. A significant parameter degeneracy (until measurements are combined) is also a fundamental feature. Utilizing a well defined naive narrow width approximation is an essential step to factorizing up S matrix elements and reducing the complexity of the analysis to a manageable level. In addition, one can simultaneously exploit the relative scaling of leading corrections in the EFT's in how phase space is populated, IR symmetry assumptions in the EFT, and the fact that parameters that violate symmetries approximately preserved in the SM interfere in a numerically suppressed fashion. The resulting reduction in parameters enables a systematic EFT program using LHC, and lower energy, data to be practically carried out.

The relative population of phase space due to resonance enhancements or suppressions is important at Hadron colliders, where an experimental measurement is always a combination of signal and background processes. Exploiting this effect in EFT studies was recently discussed more systematically in Ref. [467], and we largely reproduce this discussion here. A general scattering amplitude is depicted in Fig. 8, and shows the decomposition around the physical poles of the narrow propagating bosons B of the SM

$$\begin{aligned}
\mathcal{A} = & \frac{\mathcal{A}_a(p_1^2, \cdots p_M^2)}{(p_1^2 - m_{B_1}^2 + i\Gamma_{B_1} m_{B_1}) \cdots (p_N^2 - m_{B_N}^2 + i\Gamma_{B_N} m_{B_N})}, \\
& + \frac{\mathcal{A}_b(p_1^2, \cdots p_M^2)}{(p_1^2 - m_{B_1}^2 + i\Gamma_{B_1} m_{B_1}) \cdots (p_{N-1}^2 - m_{B_{N-1}}^2 + i\Gamma_{B_{N-1}} m_{B_{N-1}})}, \\
& + \cdots + \mathcal{A}_j(p_1^2, \cdots p_M^2).
\end{aligned} \tag{6.30}$$

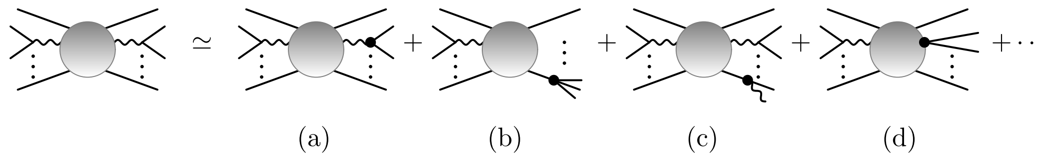


Figure 8. A general scattering amplitude expanded in a series of EFT corrections. Here the black dot indicates a possible insertion of $\mathcal{L}^{(6)}$ and shifts are only shown on the final states as an illustrative choice.

If experimental selection cuts are made so that the process is numerically dominated by a set of leading pole contributions of narrow bosons B, then this phase space volume Ω is

$$\begin{aligned}\left(\frac{d\sigma}{\delta\Omega}\right)_{pole} \simeq & \left(\frac{d\sigma_{SM}}{\delta\Omega}\right)^1 \left[1 + \mathcal{O}\left(\frac{C_i\,\bar{v}_T^2}{g_{SM}\Lambda^2}\right) + \mathcal{O}\left(\frac{C_j\,\bar{v}_T^2\,m_B}{\Lambda^2\,\Gamma_B}\right)\right], \\ & + \left(\frac{d\sigma_{SM}}{\delta\Omega}\right)^2 \left[1 + \mathcal{O}\left(\frac{C_k\,p_i^2}{g_{SM}\Lambda^2}\right)\right].\end{aligned} \tag{6.31}$$

The differential cross sections $(d\sigma_{SM}/\delta\Omega)^{1,2}$ are distinct in each case, see Ref. [467] for further details. The interference with a complex Wilson coefficient denoted C, that occurs when a resonance exchange is not present compared to the leading resonant SM signal result (shown in Fig. 8 d)) scales as

$$\begin{aligned}|\mathcal{A}|^2 &\propto \left(\frac{g_{SM}^2}{(p_i^2 - m_B^2 + i\,\Gamma(p)m_B)} + \frac{C}{\Lambda^2}\right)\left(\frac{g_{SM}^2}{(p_i^2 - m_B^2 + i\,\Gamma(p)m_B)} + \frac{C}{\Lambda^2}\right)^\star \cdots \\ &\propto \left[\frac{g_{SM}^2}{(p_i^2 - m_B^2)^2 + \Gamma_B^2\,m_B^2} + \frac{(p_i^2 - m_B^2)(C/\Lambda^2 + C^\star/\Lambda^2) - i\Gamma_B\,m_B(C^\star/\Lambda^2 - C/\Lambda^2)}{(p_i^2 - m_B^2)^2 + \Gamma_B^2\,m_B^2}\right]\cdots\end{aligned} \tag{6.32}$$

In the near on-shell region of phase space ($\sqrt{p_i^2} - m_B \sim \Gamma_B$), the SMEFT then has the additional numerically subleading corrections

$$\left(\frac{d\sigma_{SM}}{\delta\Omega}\right)^1 \mathcal{O}\left(\frac{\Gamma_B\,m_B\,\{\mathrm{Re}(C),\mathrm{Im}(C)\}}{g_{SM}^2\Lambda^2}\right) + \left(\frac{d\sigma_{SM}}{\delta\Omega}\right)^2 \mathcal{O}\left(\frac{\Gamma_B\,m_B\,\{\mathrm{Re}(C),\mathrm{Im}(C)\}}{g_{SM}^2\Lambda^2}\right) \tag{6.33}$$

For this reason, the numerical effect of the parameters not resonantly enhanced are relatively suppressed by

$$\left(\frac{\Gamma_B\,m_B}{\bar{v}_T^2}\right)\frac{\{\mathrm{Re}(C),\mathrm{Im}(C)\}}{g_{SM}\,C_i}, \qquad \left(\frac{\Gamma_B\,m_B}{p_i^2}\right)\frac{\{\mathrm{Re}(C),\mathrm{Im}(C)\}}{g_{SM}\,C_k}, \tag{6.34}$$

This relative numerical suppression occurs in addition to the power counting in the SMEFT and the combination of these two suppressions is what is experimentally relevant.

In addition to maximally exploiting resonance enhancements or suppressions of parameters other IR assumptions can be directly made. Examples of such assumptions are

- Symmetry assumptions made on the SMEFT or the HEFT operators. Typically these are global symmetry assumptions on Baryon or Lepton number conservation, $\mathrm{U}(3)^5$ flavour symmetry or a flavour subgroup. Assumed IR symmetries lead to relations between scattering amplitudes in the EFT, and hence lead to constraints even if the symmetry is spontaneously broken. Weinberg refers to such assumptions as "algebraic symmetries" [32], as they lead to algebraic relations between S matrix elements. Note that the IR limit of the full theory by definition is reproduced in the EFT. For this reason, the assumption of an algebraic symmetry in an EFT can directly enforce a UV class of theories which is required for matching. The distinction of this assumption being an IR constraint on the EFT itself, is an important conceptual point.
- Taking the $Y_{u,d,c,s} \to 0$ limit in a process.[77]
- Neglecting higher order SMEFT loops, i.e. those involving the SM fields and $\mathcal{L}_6$ that are perturbative corrections determined in the IR EFT construction.

Finally, we note that when a parameter is retained in the EFT that violates a symmetry approximately preserved in the SM, the interference term is still numerically suppressed. This fact can also be used to neglect parameters that are numerically small in contributions to measured observables. Such numerical suppressions affect a large number of parameters in Class 5, 6, 7 (in Table 1) for flavour changing neutral current contributions that interfere with small GIM suppressed processes in the SM. Numerical suppressions of this form are also present for interference between the SM and the operators of Class 5, 6, 7 that introduce CP violation, see Refs. [468, 469] for recent studies. Operators that introduce chirality flips of the light SM fermions are also suppressed when interfering with the SM, leading to the possible neglect of some parameters in dipole operators in Class 6, and contributions from right handed currents induced bu $\mathcal{Q}_{Hud}$ in Class 7 [470]. Taking all of this into account reduced sets of parameters can be well motivated in global fits in the SMEFT, see Ref. [467] for more discussion.

IR assumptions, or simplifications of this form due to kinematics in scattering processes suppressing dependence on the C_i, can be made so long as a theoretical error for the SMEFT is introduced to accommodate the neglected higher order terms that violate the assumption/simplification. In contrast, UV assumptions (setting $C_i = 0$, dropping operators and using an incomplete basis, or assuming $C_i/C_j \sim 16\pi^2$ etc.) are dangerous. The EOM makes it non-trivial and non-intuitive to determine how such assumptions modify the SMEFT framework into an alternate consistent field theory formulation.

7 The LEP example

We now turn to the interpretation of LEP data in EFT extensions of the SM, primarily focused on a SMEFT interpretation, utilizing the results of the previous sections to frame this discus-

[77]The dependence on these parameters referred to here is due to $\mathcal{L}_{SM}$ and not a normalization choice on operator Wilson coefficients.

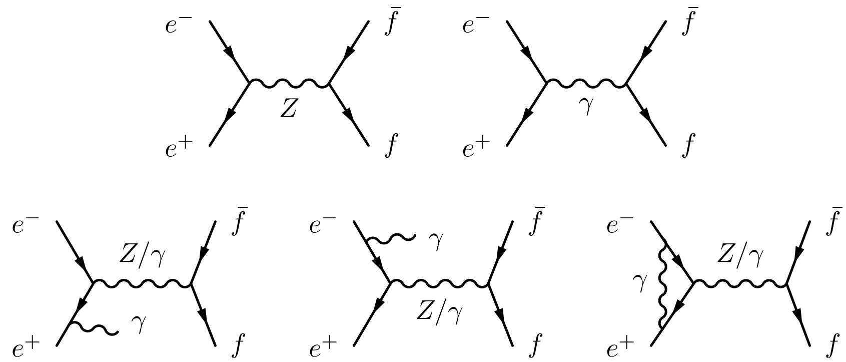


Figure 9. SM leading order contributions to H^{tot}_{QED}.

sion. Interpreting LEP data in EFT extensions of the SM is illustrative of the challenges that the LHC experimental program will face long term in enabling or disabling model independent re-interpretations of its results. It is also informative as to how a program of decomposing experimental data into pseudo-observables (PO) and mapping to EFT extensions of the SM has comparative advantages and disadvantages if LHC data reporting attempts to follow the same path as LEP, despite the very different collider environment. Here we illustrate these challenges by comparing EFT and PO interpretations of the legacy LEPI-II results.

7.1 LEPI pseudo-observables and interpretation.

The LEPI pseudo-observables are inferred from a data set that is a scan through the Z pole including $40\,\mathrm{pb}^{-1}$ of off-peak data with $155\,\mathrm{pb}^{-1}$ of on-peak data. The narrowness of the Z is directly exploited in the LEPI PO set definitions, as summarized in the Appendix.

The LEPI PO results is an ideal case of model independent reporting of experimental results with numerous consistency checks on SM assumptions used to extract and define the PO. This fortunate result was enabled by the clean LEPI collision environment, with known e^+e^- initial states scattering through a Z resonance at relatively low and fixed energies. The resulting legacy data reporting still enables EFT interpretations to be revisited and systematically improved over the years.

7.1.1 Checking the SM-like QED radiation field assumption at LEP

LEPI data is summarized in Ref. [394]. The LEPI pseudo-observables sets include flavour universal and flavour non-universal experimental results. Leading order contributions to the resonant exchanges and the photon emissions are shown in Fig. 9. We will restrict our detailed discussion to the flavour universal PO results. The raw experimental results of the LEP pole scan through the Z resonance are deconvoluted with initial and final state photon emissions being subtracted out, under a SM-like QED radiator function H^{tot}_{QED} assumption. The radiator function is calculated to third order in the SM in QED and the data is deconvoluted using this SM function. The measured pole scan is treated as a convolution of a electroweak kernel cross-section $\sigma_{ew}(s)$ with H^{tot}_{QED} as [394]

$$\sigma(s) = \int_{4m_f^2/s}^{1} dz\, H^{tot}_{QED}(z,s)\, \sigma_{ew}(z\, s). \tag{7.1}$$

Radiator functions are also applied to deconvolute the forward and backward cross sections $\sigma_{F/B}$ used to define the A_{FB} pseudo-observable extractions. The effect of this deconvolution is very dramatic. The peak is modified by 36% and its central value is shifted by 100 MeV, which is fifty times larger than the quoted error on $\hat{m}_Z$ [394]. The deconvolved cross section is then mapped to the PO set $\{\hat{m}_Z, \Gamma_Z, \sigma^0_{had}, R^0_e, R^0_\mu, R^0_\tau\}$ given in Table 2, and a subsequent mapping from PO to EFT parameters at tree or loop level is then made.

Observable	Experimental Value	Ref.	SM Theoretical Value	Ref.
$\hat{m}_Z$[GeV]	91.1875 ± 0.0021	[471]	-	-
M_W[GeV]	80.385 ± 0.015	[472]	80.365 ± 0.004	[473]
σ^0_h [nb]	41.540 ± 0.037	[471]	41.488 ± 0.006	[402]
Γ_Z[GeV]	2.4952 ± 0.0023	[471]	2.4943 ± 0.0005	[402]
R^0_ℓ	20.767 ± 0.025	[471]	20.752 ± 0.005	[402]
R^0_b	0.21629 ± 0.00066	[471]	0.21580 ± 0.00015	[402]
R^0_c	0.1721 ± 0.0030	[471]	0.17223 ± 0.00005	[402]
A^ℓ_{FB}	0.0171 ± 0.0010	[471]	0.01626 ± 0.00008	[474]
A^c_{FB}	0.0707 ± 0.0035	[471]	0.0738 ± 0.0002	[474]
A^b_{FB}	0.0992 ± 0.0016	[471]	0.1033 ± 0.0003	[474]

Table 2. Experimental and theoretical values of the LEPI pseudo-observables. The results are grouped in terms of the precision of the measurements made. The entries above the double line are measured to better than percent accuracy, the entries below the double line are measured to an accuracy of a few percent. Taken from Ref. [414].

An immediate challenge to this procedure is the possibility that the QED radiator function H^{tot}_{QED} is modified transitioning to the SMEFT in a manner that dramatically biases the results. A modification of the current

$$\mathcal{L}_{A,eff} = \sqrt{4\pi\hat{\alpha}}\left[Q_x\, J^{A,x}_\mu\right] A^\mu, \tag{7.2}$$

for $x = \{\ell, u, d\}$ due to SMEFT corrections at tree level is not present if $\hat{\alpha}_{ew}$ is used as an input parameter in a global fit. The modification of the dipole moment of the electron still leads to a potential bias to H^{tot}_{QED}. The SMEFT electron dipole moment is given by [185]

$$\mathcal{L} = \frac{ev}{\sqrt{2}}\left(\frac{1}{g_1}\underset{rs}{C_{eB}} - \frac{1}{g_2}\underset{rs}{C_{eW}}\right)\, \bar{e}_r\sigma^{\mu\nu}P_R e_s\, F_{\mu\nu} + h.c. \tag{7.3}$$

where r and s are flavour indices. Under a $\mathrm{U}(3)^5$ assumption dominantly broken by the SM Yukawa's $C_{eB}, C_{eW} \propto Y_e$, yielding an effective suppression by a small fermion mass for the LEP events. Such dipole insertions contribute to the S matrix element of $e^+e^- \to e^-e^+$ through a direct contribution to a γ exchange between the initial and final state, and also through modifying the external legs of the process in the LSZ formula in a disconnected contribution, similar to the case of photon emissions in the effective Hamiltonian for inclusive $\bar{B} \to X_s\,\gamma$ [475]. The SMEFT result of this form for has recently been calculated and is reported in Ref. [476].

Despite these facts supplying reasonable arguments that assuming H^{tot}_{QED} is SM like in the deconvolution procedure in the SMEFT introduces only a small theoretical bias, it was still checked at LEP what the impact is of large anomalous $\gamma - Z$ interference effects on the reported PO. Using a general S matrix parameterization of this interference [394, 477, 478] the possibility of anomalous $\gamma - Z$ interference does change the inferred pseudo-observable results, primarily the inferred value of $\hat{m}_Z$ by increasing the error by a factor of three compared to the quoted error in Table 2. The remaining pseudo-observables are modified by 20% of their quoted errors, when a 50% correction to the SM value is introduced that is far in excess of a SMEFT correction expected to be $\sim$ few % [394, 478] if $\hat{\alpha}_{ew}$ is not used as an input and $\Lambda \sim$ few TeV.

A benefit of the $\gamma - Z$ interference cross check being performed, is that even when $\hat{\alpha}_{ew}$ is not used as an input parameter, deviations from the SM exception in $\gamma - Z$ interference can be understood to only perturb EFT conclusions extracted from the LEP PO. The simple addition of an extra theoretical error in interpreting the PO when a SMEFT interpretation is used and $\hat{\alpha}_{ew}$ is not an input is then justified.

A secondary benefit of the S matrix parameterization check of LEP data is that it constrains another potential bias. Due to potential interference with ψ^4 operators that are present in the SMEFT and feed into LEP data due to the presence of off the Z pole scattering events [411, 413], a potential bias in the LEP PO scales as $\sim (m_Z\,\Gamma_Z/v^2)$ times a function of this ratio of off/on peak data [413]. The corresponding uncertainty does not disable using EWPD to obtain $\sim$ % level constraints on the C_i^6, as an anomalously large effect would also have shown up in the S matrix cross check reported in Refs. [394, 478].

LEPI results and cross checks establish that the assumption of a SM-like QED radiator function, and neglect of ψ^4 interference effects, is validated for the Z pole scan data set. A slight increase in the errors quoted on the pseudo-observable extractions is sufficient and appropriate if $\hat{\alpha}_{ew}$ is not used as an input to use the PO in the SMEFT. We stress that the LEP PO approach did not simply assume only SM like radiative corrections, it checked that a SM like QED radiation field was present for the processes of interest. An assumption of a SM like radiation field for pseudo-observables, is essentially an assumption that the multipole expansion that appears in the SMEFT derivative expansion directly does not lead to a significant perturbation of the SM radiation field. This is a strong condition on UV dynamics that should be avoided to maintain model independence.

An essential challenge for the PO program at LHC is to address the challenge of radiative corrections to PO's with suitable rigor to enable a precision PO program to characterize the properties of the Higgs-like scalar in a model independent fashion. To date this challenge has not been met, but some initial studies in the direction of characterizing such corrections have appeared, for example in Ref. [479]. We return to this point below.

7.1.2 LEPI interpretations

Most interpretations of EWPD that go beyond the PO level and make contact with specific models use the S, T formalism (or related approaches [362, 399, 400, 480–486]). The S, T oblique formalism parameterizes a few common corrections to the two point functions ($\Pi_{WW,ZZ,\gamma Z}$) that feed into the extracted PO in the standard form [375]

$$\frac{\hat{\alpha}(m_Z)}{4\,\hat{s}_Z^2\,\hat{c}_Z^2}\,S \equiv \frac{\Pi_{ZZ}^{new}(m_Z^2) - \Pi_{ZZ}^{new}(0)}{m_Z^2} - \frac{\hat{c}_Z^2 - \hat{s}_Z^2}{\hat{c}_Z\,\hat{s}_Z}\frac{\Pi_{Z\gamma}^{new}(m_Z^2)}{m_Z^2} - \frac{\Pi_{\gamma\gamma}^{new}(m_Z^2)}{m_Z^2}, \tag{7.4}$$

$$\hat{\alpha} T \equiv \frac{\Pi_{WW}^{new}(0)}{m_W^2} - \frac{\Pi_{ZZ}^{new}(0)}{m_Z^2}. \tag{7.5}$$

One calculates $\Pi_{WW,ZZ,\gamma Z}$ in a model and uses global fit results on EWPD with S, T corrections to constrain the model. See Ref. [487] for recent results of such fits. This can be done if the conditions on the global S, T EWPD fit are satisfied; namely that vertex corrections due to physics beyond the SM are neglected, which is the origin of the "oblique" qualifier of EWPD [362]. The SM Higgs couples in a dominant fashion to $\Pi_{WW,ZZ}$ when generating the mass of the W, Z bosons, and has small couplings to the light fermions due to the small Yukawa couplings, so it satisfies the oblique assumption. This is the reason why this assumption was usually adopted before the Higgs-like scalar discovery. In general, this assumption is very problematic as it is a UV condition, not an IR assumption in defining an EFT, as the vertex correction operators are present in general. Furthermore, the oblique assumption is not field redefinition invariant,[78] as can be seen by inspecting the EOM that result from SM field redefinitions, given in Section 4.1.1.

LHC results indicate the W, Z bosons obtain their mass in a manner that is associated with the Higgs-like scalar, see Section 8. Corrections to $\Pi_{WW,ZZ}$ can be included for the SM, or in extensions due to this scalar, see Ref. [270]. Once this is done, there is no strong theoretical support for an oblique assumption to be invoked on the remaining perturbations to EWPD. Dropping this problematic assumption leads to a SMEFT analysis which has several benefits. For example, a SMEFT analysis permits the determination of higher order corrections when interpreting EWPD, see Ref. [169]. In a SMEFT analysis of EWPD a model is mapped to the SMEFT Wilson coefficients in a tree or loop level matching calculation and model independent global fit results are used to constrain the Wilson coefficients in a global χ^2 fit.

[78] Attempts to translate the oblique condition into a requirement to use a particular operator basis by using these EOM relations are best ignored. As such attempts are based on a misconception of the nature of field redefinitions (assuming that they can satisfy physical conditions), see Ref. [265] for related discussion. It has also been argued that the oblique requirement can be interpreted as a condition defining a class of UV theories known as universal theories [358]. See Section 5.1.2 for more discussion on this idea.

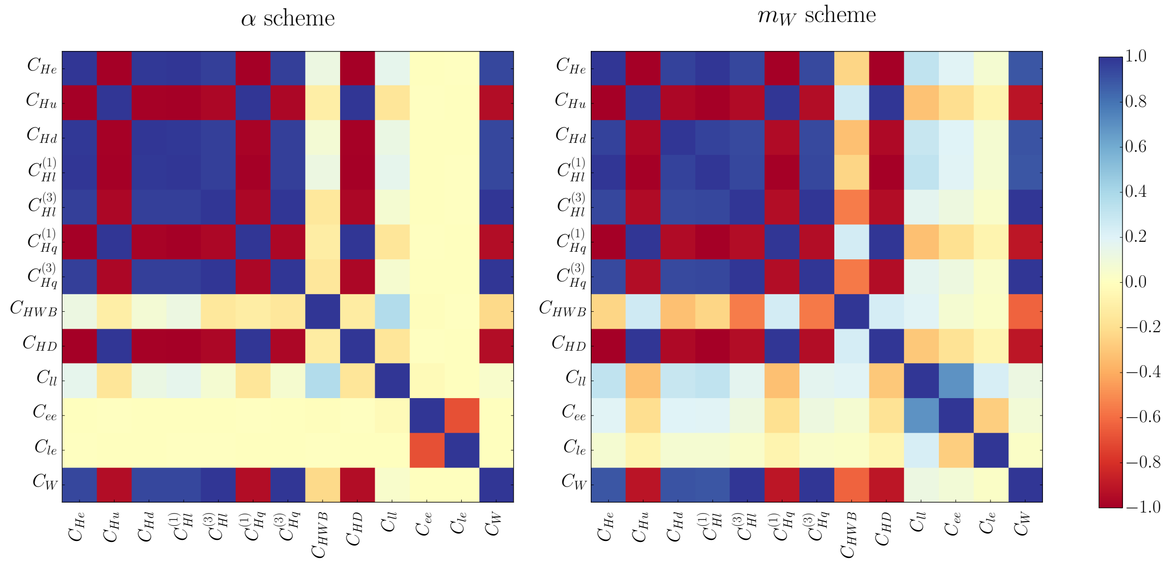


Figure 10. Color map of the correlation matrix among the Wilson coefficients [233, 415], obtained assuming zero SMEFT error, for the $\{\hat{\alpha}, \hat{m}_Z, \hat{G}_F\}$ input scheme (left) and for the $\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}$ input scheme (right). The fit space is highly correlated, as can be understood to be due to the facts that the Lagrangian parameters are unphysical in general, and there is a reparameterization invariance present in LEPI data.

Initial works pioneering this approach are Refs. [411, 488]. The analysis of Ref. [411] identified unconstrained directions in the EWPD set and correctly found a highly correlated Wilson coefficient space in the SMEFT. Recent analyses that do not break these correlations by assumption (or a chosen marginalization procedure) still find that the EWPD Wilson coefficient space is highly correlated, see Refs. [233, 364, 411–415, 489]. In determining the constraints on the Wilson coefficients of the SMEFT, one chooses an input parameter set, and predicts the EWPD PO. In the Z, W pole results of Refs. [413, 414] the mapping is

$$\{\hat{m}_Z, \hat{m}_h, \hat{m}_t, \hat{G}_F, \hat{\alpha}_{ew}, \hat{\alpha}_s, \Delta\hat{\alpha}\} \rightarrow \{m_W, \sigma_h^0, \Gamma_Z, R_\ell^0, R_b^0, R_c^0, A_{FB}^\ell, A_{FB}^c, A_{FB}^b\}, \tag{7.6}$$

through $\mathcal{L}_{SMEFT}$. Here the hat superscript indicates an input parameter. In more detail, a SMEFT fit procedure is as follows: A set of observables is denoted O_i, $\bar{O}_i^{LO}$, $\hat{O}_i$ for the SM prediction, the SMEFT prediction to first order in the $C^{(6)}$, and the experimental value of the extracted PO respectively. The measured value $\hat{O}_i$ is assumed to be a Gaussian variable centered about $\bar{O}_i$ and the likelihood function ($L(C)$) and χ^2 are defined as

$$L(C) = \frac{1}{\sqrt{(2\pi)^n |V|}} \exp\left(-\frac{1}{2}\left(\hat{O} - \bar{O}^{LO}\right)^T V^{-1}\left(\hat{O} - \bar{O}^{LO}\right)\right), \quad \chi^2 = -2\mathrm{Log}[L(C)], \tag{7.7}$$

where $V_{ij} = \Delta_i^{exp}\rho_{ij}^{exp}\Delta_j^{exp} + \Delta_i^{th}\rho_{ij}^{th}\Delta_j^{th}$ is the covariance matrix with determinant $|V|$. ρ^{exp}/ρ^{th} are the experimental/theoretical correlation matrices and Δ^{exp}/Δ^{th} the experimental/theoretical error of the observable O_i. This approach is an approximation, with neglected effects introducing a theoretical error [413, 414, 462]. The theoretical error Δ_i^{th} for an observable O_i is defined as $\Delta_i^{th} = \sqrt{\Delta_{i,SM}^2 + (\Delta_{i,SMEFT} \times O_i)^2}$, where $\Delta_{i,SM}$, $\Delta_{i,SMEFT}$ correspond to the absolute SM theoretical, and the multiplicative SMEFT theory error. The χ^2 is

$$\chi^2_{C_i^6} = \chi^2_{C_i^6,min} + \left(C_i^6 - C_{i,min}^6\right)^T \mathcal{I} \left(C_i^6 - C_{i,min}^6\right), \tag{7.8}$$

where $C_{i,min}^6$ corresponds to the Wilson coefficients vector minimizing the χ^2 and $\mathcal{I}$ is the Fisher information matrix. Recent results [415] using this methodology are shown in Fig. 10. The effect of modifying the input parameters: $\{\hat{\alpha}_{ew}, \Delta\hat{\alpha}\} \to \hat{m}_W$ was been recently examined [415], which does not change this conclusion. The Fisher matrices of the SMEFT fit space allow the construction of the SMEFT χ^2 function. These matrices were developed in a fit using 177 observables [233, 413–415] and are available upon request of the authors of Ref. [415].

7.1.3 Loop corrections to Z decay

The SMEFT and HEFT formalisms allows one to combine data sets into a global constraint picture in a consistent fashion and these EFTs also allow the systematic determination of perturbative corrections. One loop calculations in these theories are absolutely required to gain the full constraining power of the most precisely measured observables, such as the LEPI PO. This is not surprising; far more startling is that the complete one loop results of this form remain undetermined decades after the LEPI data set was reported! In the case of $\mathcal{O}(y_t^2, \lambda)$ corrections to $\{\Gamma_Z, \Gamma_{Z\to\bar{\psi}\psi}, \Gamma_Z^{had}, R_\ell^0, R_b^0\}$, about thirty loops were recently determined in Ref. [169] mapping the input parameters to these observables, while retaining the $\hat{m}_t, \hat{m}_h$ mass scales in the calculation. The renormalization of the $\mathcal{L}_6$ operators in the Warsaw basis [116, 170, 185, 227] is used in this result, which simultaneously provides a check of the terms $\propto y_t^2, \lambda$ that appear in these observables [169]. The results define a perturbative expansion for the LEPI PO

$$\bar{O}_i = \bar{O}_i^{LO}(C_i^6) + \frac{1}{16\,\pi^2}\left(F_1[C_j^6] + F_2[\lambda, C_k^6]\log\frac{\mu^2}{\hat{m}_h^2} + F_3[y_t^2, C_l^6]\log\frac{\mu^2}{\hat{m}_t^2}\right) + \cdots \tag{7.9}$$

The LO results depend on ten Wilson coefficients in the Warsaw basis, defining C_i, and $\dim(C_j) \neq \dim(C_k) \neq \dim(C_l) > \dim(C_i)$ in general. At one loop, considering $\mathcal{O}(y_t^2, \lambda)$ corrections the new SMEFT parameters that appear are [169]

$$\{C_{qq}^{(1)}, C_{qq}^{(3)}, C_{qu}^{(1)}, C_{uu}, C_{qd}^{(1)}, C_{ud}^{(1)}, C_{\ell q}^{(1)}, C_{\ell q}^{(3)}, C_{\ell u}, C_{qe}, C_{eu}, C_{Hu}, C_{HB} + C_{HW}, C_{uB}, C_{uW}, C_{uH}\}. \tag{7.10}$$

It has been shown in Ref. [169] in this manner that the number of parameters exceeds the number of precise LEPI PO measurements when one loop corrections are calculated. The LEPI PO are very important to project into the SMEFT, as for a few observables $\Delta_j^{exp} \sim 0.1\%$. When

$$\frac{1}{16\,\pi^2}\left(F_1[C_j^6] + F_2[\lambda, C_k^6]\log\frac{\mu^2}{\hat{m}_h^2} + F_3[y_t^2, C_l^6]\log\frac{\mu^2}{\hat{m}_t^2}\right) + \cdots \gtrsim \Delta_j^{exp}\hat{O}_i, \tag{7.11}$$

it is clear that these corrections can have a significant effect on the interpretation of the LEPI PO for the exact same reason. If this is the case depends on the values of the UV dependent Wilson coefficients and the global constraint picture, which is unknown. However, we note that recent global fit results indicate directly that some of the four fermion operators that feed in at one loop are very weakly constrained by lower energy data [233, 414, 490, 491]. For the near Z pole observables, one can fix $\mu = \hat{m}_Z$, but the new weakly constrained parameters are still present. Although EFT techniques can sum all of the logs that appear relating various scales, the extraction and prediction of the LEPI PO is a complex multi-scale problem with the scales

$$0 \ll \hat{m}_\mu^2 \ll \hat{m}_Z^2 < \hat{m}_h^2 < \hat{m}_t^2. \tag{7.12}$$

The required calculations to sum all the logs are not available to date.[79] These results already establish that LEPI PO data does not constrain the SMEFT parameters appearing at tree level to the per-mille level in a model independent fashion. This is very good news for hopes of the indirect techniques discussed in this review discovering evidence for physics beyond the SM at LHC.

Using the determined SMEFT constraints that result from EWPD to study LHC data, one must also run the determined constraints on $C_{i,j,k,l}(m_Z)$ to the LHC measurement scales. This also acts to reduce the power of constraints when mapping between the data sets by renormalization group equation (RGE) running. It is simply not advisable to set $C_i^6(\mu) = 0$ in LHC analyses to attempt to incorporate EWPD constraints for all these reasons. The challenge of combining LEPI PO consistently with Higgs data requires further theoretical development of the SMEFT.

7.1.4 SMEFT reparameterization invariance

A central feature of interpreting LEP data in the SMEFT is the highly correlated Wilson coefficient fit space. This results from the unphysical nature of Lagrangian parameters and the fact that several parameters in $\mathcal{L}_6$ appear at the same order in the power counting of the theory simultaneously (see Section 6.2). A further wrinkle is that unconstrained directions due to LEPI data in this Wilson coefficient space are manifest in the Warsaw basis but not in other formalisms. As a result, these unconstrained directions have caused enormous confusion over the years.[80] This has lead to some misplaced intuition that the number of SMEFT parameters is too large to do a consistent analysis of the global data set. It has also lead to claims that some operator bases are better related to experimental measurements than others. The logical extension of this thinking has lead to ad-hoc phenomenological parameterizations being promoted for the LHC experimental program, which are also argued to be better related to experimental measurements. (See Section 4.2.3 for more discussion.)

[79] Ideally these results would have been determined in the decades *before* the turn on of LHC.

[80] For discussion on these unconstrained directions in the Wilson coefficient space, see Refs. [411, 415, 492–494].

The physics of these unconstrained directions is now understood in an operator basis independent manner [415]. To understand this result the unphysical nature of Lagrangian parameters is an essential feature, and these flat directions follow from a scaling argument that is a property of $\bar{\psi}\psi \to \bar{\psi}\psi$ data.

The scaling argument underlying the reparameterization invariance is simple. A vector boson can always be transformed between canonical and non-canonical form in its kinetic term by a field redefinition without physical effect, due to a corresponding correction in the LSZ formula. Such a shift can be canceled by a corresponding shift in the $V\bar{\psi}\psi$ coupling. The same set of physical scatterings can then be parameterized by an equivalence class of fields and coupling parameters in the SMEFT as a result [415]

$$(V, g) \leftrightarrow \left(V'\,(1+\epsilon), g'\,(1-\epsilon)\right), \tag{7.13}$$

where $\epsilon \sim \mathcal{O}(\bar{v}_T^2/\Lambda^2)$. This is the SMEFT reparameterization invariance identified in Ref. [415]. Denoting $\langle \cdots \rangle_{S_R}$ as the class of $\bar{\psi}\psi \to V \to \bar{\psi}\psi$ matrix elements, the following operator relations follow from the SM EOM given in Section 4.1.1

$$\langle y_h\, g_1^2 Q_{HB}\rangle_{S_R} = \langle \sum_\psi y_k\, g_1^2\, \overline{\psi}_\kappa\, \gamma_\beta \psi_\kappa\, (H^\dagger\, i \overleftrightarrow{D}_\beta H) + 2\, g_1^2\, Q_{HD} - \frac{1}{2} g_1\, g_2\, Q_{HWB}\rangle_{S_R}, \tag{7.14}$$

$$\langle\, g_2^2 Q_{HW}\rangle_{S_R} = \langle g_2^2\, (\overline{q}\, \tau^I \gamma_\beta q + \bar{l}\, \tau^I \gamma_\beta l)\, (H^\dagger\, i \overleftrightarrow{D}^I_\beta H) - 2\, g_1\, g_2\, y_h\, Q_{HWB}\rangle_{S_R}. \tag{7.15}$$

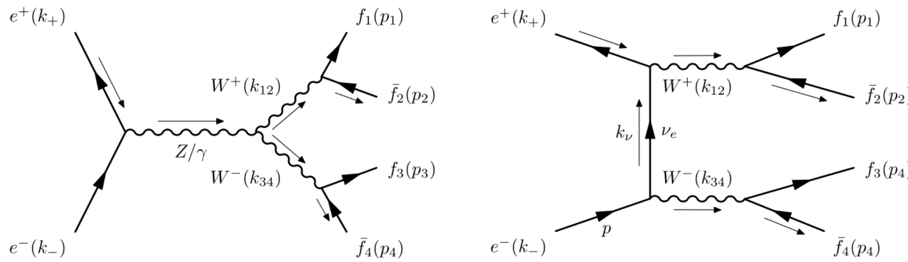
Because of the SMEFT reparameterization invariance, a Wilson coefficient multiplying the left hand side of these equations is not observable in $\bar{\psi}\psi \to \bar{\psi}\psi$ scatterings. The invariance of S matrix elements under field configurations equivalent by use of the EOM means then, that the corresponding fixed linear combinations of Wilson coefficients that appear on the right-hand sides of these equations are also not observable in the S_R matrix elements. These combinations are EOM equivalent to physical effects that cancel out due to the reparameterization invariance.

The S_R class of data is simultaneously invariant under the two independent reparameterizations (defining $w_{B,W}$) that leave the products $(g_1 B_\mu)$ and $(g_2 W^i_\mu)$ unchanged. The unconstrained directions in the global fit are found to be

$$w_1 = \frac{v^2}{\Lambda^2}\left(\frac{C_{Hd}}{3} - 2C_{HD} + C_{He} + \frac{C^{(1)}_{Hl}}{2} - \frac{C^{(1)}_{Hq}}{6} - \frac{2\,C_{Hu}}{3} - 1.29(C^{(3)}_{Hq} + C^{(3)}_{Hl}) + 1.64 C_{HWB}\right), \tag{7.16}$$

$$w_2 = \frac{v^2}{\Lambda^2}\left(\frac{C_{Hd}}{3} - 2C_{HD} + C_{He} + \frac{C^{(1)}_{Hl}}{2} - \frac{C^{(1)}_{Hq}}{6} - \frac{2C_{Hu}}{3} + 2.16(C^{(3)}_{Hq} + C^{(3)}_{Hl}) - 0.16 C_{HWB}\right). \tag{7.17}$$

These unconstrained directions can be projected into the vector space defined by $w_{B,W}$ as [415] $w_1 = -w_B - 2.59\, w_W$, and $w_2 = -w_B + 4.31\, w_W$.



(a) Amplitude $\mathcal{A}_V$ (b) Amplitude $\mathcal{A}_\nu$

Figure 11. The s-channel (a) and t-channel (b) Feynman diagrams contributing to $e^+e^- \to W^+W^- \to f_1\bar{f}_2f_3\bar{f}_4$ (from Ref. [233]).

We stress that it is important to understand that the existence of these flat directions should not be considered a sign of the SMEFT having too many parameters to interface with the data. Conversely, a correct interpretation of this physics is that a consistent EFT formalism retaining all parameters can indicate that hidden structures such as the reparameterization invariance are present.

It is required to include data from $\bar{\psi}\psi \to \bar{\psi}\psi\bar{\psi}\psi$ scattering to lift the flat directions [411]. This is understood to be the case when considering Fig. 11 (a) that contributes to $\bar{\psi}\psi \to \bar{\psi}\psi\bar{\psi}\psi$ scattering which is not invariant under Eqn. 7.13. Fig. 11 (a) contains a TGC vertex, which is the reason the reparameterization invariance is broken at an operator level. We emphasize that there is a distinction between the scaling argument in Eqn. 7.13 that applies to S matrix elements in a basis independent manner and the presence (or not) of an operator contributing to an anomalous TGC vertex. The latter depends upon the operator basis chosen and unphysical field redefinitions.

A recent approach of using mass eigenstate (unitary gauge) coupling parameters to characterize deviations from the Standard Model makes the presence of these unconstrained directions even harder to uncover in data analyses. The reason is that EOM relations key to understanding the reparameterization invariance do not have a (manifest) equivalent in the parameterization chosen, although the fact that there remain un-probed aspects of the Z boson phenomenology in $\bar{\psi}\psi \to \bar{\psi}\psi$ scatterings is directly acknowledged in Refs. [235, 412]. Defining correlations for mass eigenstate parameters in a form that manifestly preserves the consequences of the reparameterization invariance remains an unsolved problem, and assuming no correlations between these parameters can bias results by breaking the reparameterization invariance.

As $\bar{\psi}\psi \to \bar{\psi}\psi\bar{\psi}\psi$ occurs through narrow $W^\pm$ states, the requirement to utilize the narrowness of the $W^\pm$ boson in a consistent theoretical approach is now front and center.

7.2 LEPII pseudo-observables and interpretation.

The LEPII data and analysis summary is reported in Ref. [478]. A major goal of the LEPII run was to exploit the sensitivity of scattering $\sigma_{2\psi}^{4\psi} \equiv \sigma(e^+e^- \to W^+W^- \to f_1\bar{f}_2f_3\bar{f}_4)$ to CP even Triple Gauge Couplings (TGCs) vertices to test the non-abelian structure of the SM. For this reason, measurements of $\sigma^{4\psi}_{2\psi}$ were stepped up to a running energy of $\sqrt{s} = 206.5\,\mathrm{GeV}$ through the $W^{\pm}$ pair production threshold.

The SM prediction of this process was developed in Refs. [443, 495–500]. The direct calculation of the process $\sigma^{4\psi}_{2\psi}$, and related differential distributions, is sufficiently complex that it benefits from using spinor-helicity results developed in Refs. [212, 500]. See the Appendix for the results broken into Helicity eigenstates in the SMEFT. To define radiative corrections to this process in the SM, the narrowness of the $W^{\pm}$ bosons is exploited in a double pole expansion [443, 451–454]. Functionally the program RacoonWW is used [454] which utilizes this expansion.

Due to the reparameterization invariance present in $e^+e^- \to f_1\bar{f}_2$ scattering, this data set is critical to lifting the flat directions in the Wilson coefficient space, but this was not the motivation for extending the LEPI data set to measure $\sigma^{4\psi}_{2\psi}$. Only retrospectively was it realized how this extension of the LEP data is critical to globally constrain the SMEFT Wilson coefficient space in a consistent SMEFT analysis [411]. Unfortunately, the pseudo-observables reported for LEPII were subject to less cross checks on SM-like assumptions used in extracting the PO, and are less directly connected to S matrix elements. The LEPI PO set is focused on the observable cross sections given in Table 2 while LEPII PO generally refer to the reported values of some Lagrangian parameters treated as extracted pseudo-observable quantities.[81] This distinction is important for the LHC program, where proposals exist that are closely related to the LEPII approach to PO as opposed to the LEPI PO. For this reason, we review what has been understood about the LEPII case using the SMEFT in recent years.

7.2.1 TGC pseudo-nonobservables and effective pseudo-observables

Several studies of $\sigma^{4\psi}_{2\psi}$ in an EFT context developed the formalism used in LEPII results [212, 497, 500, 501]. The most general C, P conserving TGCs is introduced as [212, 500]

$$\frac{-\mathcal{L}_{TGC}}{g_{VWW}} = i\bar{g}_1^V \left(W^+_{\mu\nu}W^{-\mu} - W^-_{\mu\nu}W^{+\mu}\right)V^\nu + i\bar{\kappa}_V W^+_\mu W^-_\nu V^{\mu\nu} + i\frac{\bar{\lambda}_V}{\hat{m}_W^2}V^{\mu\nu}W^{+\rho}_\nu W^-_{\rho\mu}, \tag{7.18}$$

where $V = \{Z, A\}$, $W^{\pm}_{\mu\nu} = \partial_\mu W^{\pm}_\nu - \partial_\nu W^{\pm}_\mu$ and similarly $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. At tree level in the SM

$$g_{AWW} = e, \qquad g_{ZWW} = e\cot\theta, \qquad g_1^V = \kappa_V = 1, \qquad \lambda_V = 0. \tag{7.19}$$

As it stands, this parameterization of SM deviations is not based on a linearly realized operator formalism [501, 502]. It can be considered to be an example of a non-linearly realized $\mathrm{SU_L}(2)\times \mathrm{U_Y}(1)$ theory; in modern EFT Language this requires an embedding in the HEFT. A complete

[81] In this latter case a label of pseudo-nonobservable is perhaps more appropriate, as the distinction between Lagrangian parameters and S matrix elements has been obscured. We use this tongue-in-cheek nomenclature in what follows to emphasize this point. Note that the neutral gauge boson parameters $h_i^{Z,\gamma}, f_i^{Z,\gamma}$ also are in this class of pseudo-nonobservable.

HEFT description contains more interaction terms, see Ref. [276] for more discussion. This ad-hoc construction can also be embedded into the SMEFT by relating it to gauge invariant operators as shown in Ref. [503]. An up-to-date embedding of this form in the Warsaw basis is given in the Appendix A.1. This embedding is straightforward as this parameterization does not have any gauge dependent defining conditions.

Traditional interpretations of LEPII pseudo-nonobservables, reported as effective bounds on the TGC parameters shifting the SM predictions $\delta g_1^V, \delta\kappa_V, \delta\lambda_V$ are problematic. In many studies, including Refs. [327, 411], the constraints on $\delta g_1^V, \delta\lambda_V$ reported by the LEPII experiments are used as observables. These parameters cannot be treated directly as physical observables to constrain the SMEFT parameter space consistently [265].[82] Losing this distinction in SMEFT studies of $\sigma_{2\psi}^{4\psi}$ related observables leads to spurious results. The reason is that constraints on the non-physical TGC parameters are generally developed using Monte-Carlo tools where vertex corrections of the massive gauge boson couplings are assumed to be "SM-like" for all fermion species. This assumption when interpreted as an operator basis independent constraint on the SMEFT, implies a "SM-like" TGC vertex is also required in the SMEFT [265] due to the EOM relations linking the relevant Lagrangian parameters.[83] Attempts to then develop "effective TGC parameters" were subsequently reported in Ref. [412].[84] Ref. [233] showed that to define a PO set of "effective TGC parameters" for the SMEFT also requires that the shift to the $W^\pm$ mass (δm_W^2) and width ($\delta\Gamma_W^2$) must be taken into account in addition to the gauge boson vertex corrections, to accommodate using a double pole expansion to define the observables. Taking these corrections into account gives

$$(\delta g_1^V)^{eff} = \delta g_1^V - 2\,\delta D^W(m_W^2) - \delta D^V(m_V^2), \tag{7.20}$$

$$(\delta\kappa_V)^{eff} = \delta\kappa_V - 2\,\delta D^W(m_W^2) - \delta D^V(m_V^2). \tag{7.21}$$

We have approximated the dependence in the residue in the effective TGC as $s_{ij} = m_W^2$, $s = m_V^2$, consistent with a double pole expansion. δD^W, δD^V are not relatively suppressed by an extra factor of Γ_W/m_W or Γ_V/m_V as might be expected to result from the narrow width expansions of the SM gauge bosons. Results of this form were reported in Ref. [233], as detailed in the Appendix.

7.2.2 LEP II bounds

The LEP experiments during the LEPII run extracted limits on the effective parameters in Eqn. 7.18, both in the individual experiments and in combination. This was a significant focus

[82] See Section 6 for a general discussion on the observable/Lagrangian parameter distinction.

[83] Ref. [359] pointed out that the conclusion does not hold in the case of universal theories. On the other hand, see Section 5.1.2 for a discussion on universal theories.

[84] Ref. [412] does not specify the treatment of the expansion used due to the narrowness of the SM states to define the observables. It seems apparent that a narrow width approximation is implicitly used to define the $\sigma_{2\psi}^{4\psi}$ observable. This result is then combined with SM predictions defined using the double pole expansion results of Ref. [478]. The inconsistencies so introduced in such a procedure are not suppressed by Γ_W/m_W.

Parameter	Cons.	Ref.	Cons.	Ref.	Cons.	Ref
g_1^Z	$0.984^{+0.018}_{-0.020}$	[478]	$0.975^{+0.033}_{-0.030}$	[504]	$0.95^{+0.05}_{-0.07}$	[412]
κ_γ	0.982 ± 0.042	[478]	$1.024^{+0.077}_{-0.081}$	[504]	$1.05^{+0.04}_{-0.04}$	[412]
λ_γ	-0.022 ± 0.019	[478]	0.002 ± 0.035	[504]	0.00 ± 0.07	[412]

Table 3. Results for the LEPII pseudo-nonobservables produced while varying the TGC Lagrangian parameters one at a time under the simultaneous assumption of a "SM-like" coupling of the massive gauge bosons to fermions.

of experimental efforts. The focus of the theoretical community leading up to the LEPII data reporting was to move beyond the naive narrow width approximation in $\sigma^{4\psi}_{2\psi}$ and to define SM radiative corrections to hypothesis test the SM at LEPII. A good summary of the theoretical issues that were priorities going into LEPII is reported in Ref. [505]. A self-consistent SMEFT approach was not a theoretical priority as the SM had not yet been validated in a test of the non-abelian nature of the massive SM gauge boson interactions, and no Higgs-like scalar was known to be found experimentally. The situation has now changed due to LHC and LEP results.

LEPII data has been reexamined in recent years to develop a consistent SMEFT interpretation in Refs. [233, 327, 336–338, 341, 411, 412, 506–510]. The conclusions found are generally consistent but the detailed numerical results are subject to significant uncertainties. This is illustrated in Table 3 where the quoted results for bounds on the parameters $\delta g_1^V, \delta\kappa_V, \delta\lambda_V$ that have been produced from the experiments, and an external group, are listed. These LEPII bounds were reported by varying one parameter at a time while assuming the other TGC parameters vanish in Ref. [478, 504].[85] All of these results are produced under the assumption of a "SM-like" coupling of the massive gauge bosons to all fermions in the Monte-Carlo modeling, despite the fact that imposing such a constraint in a basis independent manner in the general SMEFT renders this approach functionally redundant [265].

Comparing the quoted results in Ref, [412] shows the bounds on anomalous TGC parameters are sensitive to the inclusion of quartic terms in the likelihood. Expanding a general χ^2 function as defined in Eqn. 7.8 in the correction to the observables one obtains [414]

$$+2\sum_{i=1}^{n}\sum_{k,l=1}^{q}\sum \frac{1}{\Delta_i^2}\;\left[\zeta_{i,k,l}\,C_k^6\,C_l^6\right]\left(\hat{O}-O\right)_i + 2\sum_{i=1}^{n}\sum_{k=1}^{r}\frac{1}{\Delta_i^2}\gamma_{i,k}C_k^8\left(\hat{O}-O\right)_i, \tag{7.22}$$

when neglecting correlations between the different observables. These effects are numerically suppressed relative to the terms

$$\sim \sum_{i=1}^{n}\sum_{k=1}^{q_i}\sum_{l=1}^{q_i}\frac{C^6_{i,k}\,C^6_{i,l}}{(\Delta_i)^2}. \tag{7.23}$$

[85] Ref [412] does not specify this is the procedure it follows, but we assume this is the practice as no correlation matrix is reported for the results quoted.

This is due to the fact that when $\left(\hat{O} - O\right)_i \sim \Delta_i$ a relative suppression of C_k^8 terms by Δ_i is numerically present.[86] Including $C_{i,k}^6 \, C_{i,l}^6$ effects and neglecting C_k^8 corrections in the χ^2 function is most justified if $\Delta_i << 1$. In the case of TGC parameters treated as observables, the results in Table 3 have $\Delta_i \sim 1 - 10\%$ at one sigma, while $\sigma_{2\psi}^{4\psi}$ based results, that only use actual observables, have $\Delta_i \sim 10 - 50\%$ [233]. Retaining $C_{i,k}^6 \, C_{i,l}^6$ terms while neglecting C_k^8 corrections relies on other numerical effects not overwhelming this relative numerical enhancement in either case. Unfortunately, it is known that

- The number of operators dramatically increases order by order in the SMEFT expansion. The increase is exponential [207], leading to the expectation of a large multiplicity of C_k^8 parameters compared to the number of $C_{i,k}^6$ parameters.
- There are numerical suppressions of the linear interference terms due to $C_{i,l}^6$ following from the Helicity arguments of Refs. [340–344], introducing more numerical sensitivity to C_k^8 corrections.

$\sigma_{2\psi}^{4\psi}$ results projected into the SMEFT retaining $C_{i,k}^6 \, C_{i,l}^6$ terms in the likelihood are subject to substantial theoretical uncertainties for all these reasons. The approach of Ref. [233, 414] is to assign and vary a theoretical error due to neglected higher order terms in the SMEFT when using LEPII data, and to only use the total and differential $\sigma_{2\psi}^{4\psi}$ results with identified final states, avoiding the use of reported LEPII pseudo-nonobservables. It was noted in Ref. [414] that when numerical behavior indicates that the neglect or inclusion of quartic terms have a small effect on the likelihood, without simultaneously changing the theory error in the fit, can lead to the wrong conclusion on the sensitivity of the fit to higher order effects. The substantial theoretical uncertainties present when projecting LEPII results into the SMEFT are acknowledged in Ref. [506]. This work also argued that a likelihood that combines Higgs data and TGC data has numerical behavior that indicates that these higher order terms have a small effect on the likelihood.

7.3 LEP SMEFT summary

The SMEFT interpretation of LEP data demonstrates the challenge of consistently combining the data sets in this EFT can be overcome. This requires a careful separation of IR and UV assumptions and a consistent SMEFT analysis.

Inconsistent treatments of the LEPII results treat Lagrangian parameters as directly observable, even though many of the corresponding vertices are off-shell, and formalisms used have been subject to gauge dependence. LEPII quantities are extracted with other SMEFT parameters being set to zero in reported results. This introduces non-intuitive consequences in the SMEFT, and inconsistencies due to the EOM relations between $\mathcal{L}_6$ operators. A consistent approach to LEPII results can be developed using the double pole expansion to exploit the

[86]This does not correspond to a power counting suppression as there is no evidence of beyond the SM (BSM) physics.

narrowness of the SM massive gauge bosons, and only using the experimental $\sigma_{2\psi}^{4\psi}$ total and differential observables.

LEPI PO and interpretations are on a much firmer footing. They are more directly related to measured quantities and extractions involved cross checks of assumptions of a QED like radiation field with S matrix techniques. As a direct result, the LEPI PO are not subject to the degree of misinterpretation that has plagued LEPII interpretations.

The numerical differences between results developed using inconsistent methodology and more consistent SMEFT interpretations is small, as can be seen comparing results in Refs. [233, 327, 336–338, 341, 411, 412, 506–510]. The experimental uncertainties at LEPII are significant, and no evidence of physics beyond the SM emerged from the LEP data sets. This fact does not validate, and should not encourage, using inconsistent results and methods to interpret LHC data. The inconsistencies that can be introduced in SMEFT studies illustrated with LEP data here can tragically obscure the meaning of a real deviation being discovered using EFT techniques, if the inconsistency is numerically larger than the experimental errors. In the presence of unknown UV dependent Wilson coefficients, numerically estimating the size of the inconsistencies introduced is a severe challenge. This point holds for LHC studies of the Higgs-like boson using EFT methods.

8 The Higgs-like scalar

The experimental determination of the couplings of the newly discovered $J^P = 0^+$ scalar is essential. It is important to study these couplings in a consistent framework and to upgrade this approach to a full EFT interpretation. The SMEFT and HEFT approach are now fully defined at leading order and available to be used, but transitioning from the currently used formalism known as the "κ formalism" to these consistent field theory frameworks is still a work in progress for the LHC experiments. The challenges to performing this task in the LHC collider environment are profound, but it is important to emphasize that these challenges can be overcome in a consistent program of applying EFT methods to collider studies, including the discovered scalar's couplings, benefiting from the lessons learned interpreting LEP data. In this section, we review the current dominant paradigm for reporting constraints on Higgs properties, known as the κ formalism, due to the notation of Ref. [511].

8.1 The κ formalism

The κ formalism is not an EFT approach to Higgs data as it was set up in Ref. [511], but is the fusion of two approaches. The idea to reweigh the SM couplings and extract and limit deviations in the partial and total widths of a discovered scalar was laid out in Ref. [512–515]. This approach is an ad-hoc rescaling of couplings in the SM without a field theory embedding. See Fig. 12 for Feynman diagrams of some of the production and decay modes rescaled. Some of the rescaled couplings appear in loop diagrams, which mediate the leading production $gg \to h$ and decay mode $h \to \gamma\gamma$ used to probe the properties of this state. That these modes appear first at the one loop level in the SM is due to the fact that the SM is

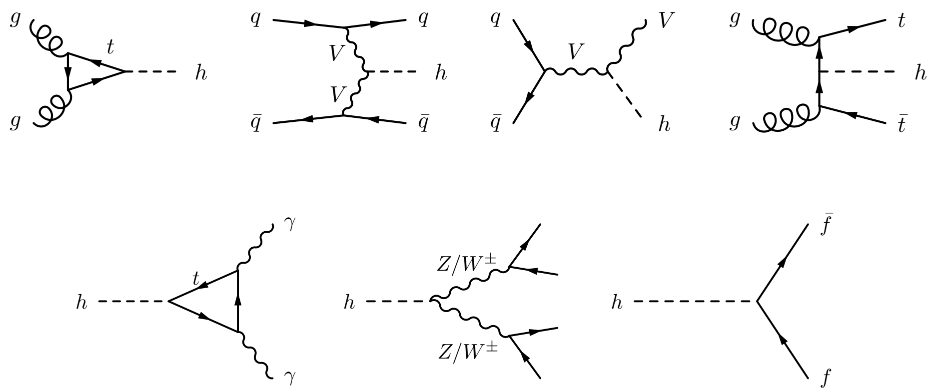


Figure 12. Higgs production (top) and decay modes (bottom) that are rescaled in the κ formalism.

renormalizable. Introducing such ad-hoc rescalings into the SM parameters is not a small perturbation for such one loop processes, as the counterterm structure is changed and the theory being used is no longer the SM. The κ formalism can thus only make sense when it is embedded into a consistent field theory framework that can be renormalized. Refs. [512–515] did not perform this embedding, although this challenge was understood to be present.

A rescaling approach to the parameters of the SM can be interpreted in principle in the SMEFT or the HEFT. Systematic field theory embeddings of a rescaling of the Higgs-like scalar couplings, to interpret the emergence of a signal for this state, appeared in the influential Refs. [296, 516, 517].[87] Ref. [511] closely follows in its proposed methodology these works, as it directly notes in its introduction. The κ formalism made a series of assumptions in its detailed implementation that are not IR assumptions consistent with either field theory embedding. As a result, in both the HEFT and the SMEFT, a direct relation between the κ approach and a general EFT framework is not present. In both cases, when further UV assumptions are made a mapping can be performed. For example, Refs. [296, 517] are formulated with a general HEFT approach in mind, while Ref. [516] is constructed in a linear SMEFT formalism in unitary gauge. The κ formalism can accommodate large corrections to the properties of the Higgs-like scalar, and relations due to $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1)$ linear $\mathcal{L}_6$ operators are not directly imposed. The HEFT also contains parameters whose behavior is analogous to that of the κ's, for example a_C plays the same role as κ_V. For these reasons, the rescaling approach, of which the κ formalism is a particular example, has been widely considered to be a restricted version of the HEFT.[88]

The κ formalism assumes the existence of a CP even scalar resonance with $\hat{m}_h \sim 125$ GeV, whose couplings have the same Lorentz structure as those of the SM Higgs boson and whose

[87] These works themselves were also influenced by Ref. [215, 217–219, 242, 267, 518–520].

[88] Works based on this understanding include Refs. [296, 296–298, 517, 521–525]. Perhaps the most comprehensive discussion of this embedding is given in Refs. [281, 283].

width is $\Gamma \sim \Gamma_h^{SM}$, i.e. narrower than the energy resolution of the LHC experiments. The assumption of a narrow resonance even in the presence of SM perturbations is used in Ref. [511] to factorize the total event rate into a production cross section and a partial decay width. Although the exact expansion used for the narrow width is not specified in Ref. [511], a naive narrow width assumption is functionally present (and explicitly mentioned in Ref. [526] in this context). A set of scale factors κ_i are defined, such that each Higgs production cross-section and decay channel is formally rescaled by a corresponding κ_i^2. For instance, in the case of the process $gg \to H \to \gamma\gamma$ one has the parameterization

$$\sigma(gg \to H) \cdot \mathrm{BR}(H \to \gamma\gamma) = \frac{\kappa_g^2\, \kappa_\gamma^2}{\kappa_H^2}\, \sigma(gg \to H)_{\mathrm{SM}} \cdot \mathrm{BR}(H \to \gamma\gamma)_{\mathrm{SM}}\,, \tag{8.1}$$

where κ_H rescales the Higgs total width. In the limit $\kappa_i \equiv 1$ the SM is recovered, while values $\kappa_i \neq 1$ indicate deviations from the SM. A list of relevant κ factors is defined as [511, 526]:

$$\begin{array}{lllll}
\dfrac{\sigma_{WH}}{\sigma_{WH}^{\mathrm{SM}}} = \kappa_W^2 & \dfrac{\sigma_{ZH}}{\sigma_{ZH}^{\mathrm{SM}}} = \kappa_Z^2 & \dfrac{\sigma_{VBF}}{\sigma_{VBF}^{\mathrm{SM}}} = \kappa_{VBF}^2 & \dfrac{\sigma_{ggH}}{\sigma_{ggH}^{\mathrm{SM}}} = \kappa_g^2 & \dfrac{\sigma_{ttH}}{\sigma_{ttH}^{\mathrm{SM}}} = \kappa_t^2 \\
\dfrac{\Gamma_{WW^*}}{\Gamma_{WW^*}^{\mathrm{SM}}} = \kappa_W^2 & \dfrac{\Gamma_{ZZ^*}}{\Gamma_{ZZ^*}^{\mathrm{SM}}} = \kappa_Z^2 & \dfrac{\Gamma_{\gamma\gamma}}{\Gamma_{\gamma\gamma}^{\mathrm{SM}}} = \kappa_\gamma^2 & \dfrac{\Gamma_{Z\gamma}}{\Gamma_{Z\gamma}^{\mathrm{SM}}} = \kappa_{Z\gamma}^2 & \dfrac{\Gamma_{ff}}{\Gamma_{ff}^{\mathrm{SM}}} = \kappa_f^2\,.
\end{array} \tag{8.2}$$

The κ's are in a sense pseudo-observables, as they are defined as a rescaling of SM observables in many cases, or inferred quantities while using the narrowness of the SM gauge bosons to factorize a scattering amplitude. The assumption of SM like soft radiation effects is present, and no detailed procedure for a correction factor is introduced to remove these corrections. This pseudo-observable understanding is not developed in great detail in the specific κ proposal, and projecting constraints on these parameters onto the Higgs couplings requires further information and assumptions.[89] It deserves to be emphasized that many of these limitations and subtleties are well understood and very clearly stated in Ref. [511].

An example of an assumption adopted in a κ fit is the case where the scaling factors for VH production and $H \to VV^*$ decay have been defined to be the same. This approximation holds only for tree level computations[90] and under the assumption that deviations in these observables can be interpreted in terms of an anomalous hZZ coupling only, with corrections to e.g. Zqq vertices being neglected.[91] Within these assumptions, the prescriptions of the κ formalism are equivalent to the use of the phenomenological ad-hoc Lagrangian

$$\begin{aligned}
\mathcal{L}_\kappa = &- \sum_\psi \kappa_\psi \frac{\sqrt{2} M_\psi}{\hat{v}} \bar{\psi}\psi h + \kappa_Z \frac{M_Z^2}{\hat{v}} Z_\mu Z^\mu h + \kappa_W \frac{2M_W^2}{\hat{v}} W_\mu^+ W^{-\mu} h, \\
&+ \kappa_{g,c} \frac{g_3^2}{16\pi^2 \hat{v}} G_{\mu\nu} G^{\mu\nu} h + \kappa_{\gamma,c} \frac{e^2}{16\pi^2 \hat{v}} F_{\mu\nu} F^{\mu\nu} h + \kappa_{Z\gamma,c} \frac{e^2}{16\pi^2 c_{\hat{\theta}} \hat{v}} Z_{\mu\nu} F^{\mu\nu} h,
\end{aligned} \tag{8.3}$$

[89] More explicit PO interpretations of these parameters have since been advanced in Refs. [31, 262, 329, 479, 527, 528].

[90] In this section, any reference to perturbative orders is implicitly referred to the EW sector. Radiative QCD corrections can be factorized due to the narrow SM widths, with some exception, in the κ's definitions (see Eqn 8.2).

[91] This procedure is beset with inconsistencies that are discussed in Section 4.2.5.

where h denotes the physical Higgs boson and $F_{\mu\nu}$ is the photon field strength. For this reason, the κ's are often referred to as "coupling modifiers". Those in the second line of Eqn. 8.3 have been defined with an additional "c" to underline that they are associated to effective contact interactions. To match the κ's definitions above (with in particular $\kappa_{i,c} = \kappa_i$ for $i = g, \gamma, Z\gamma$), the Lagrangian $\mathcal{L}_\kappa$ must be strictly used for tree-level computations only. We stress that this parameterization of Lagrangian terms should not be interpreted to give a naive physical meaning to the κ_i. The distinction between observables and Lagrangian parameters discussed in Section 6 applies, and formulating the κ_i in terms of mass eigenstate shifts does not change this fact.

For processes that are produced at one-loop already in the SM, it is possible to employ the Lagrangian above at NLO, with the caveat that in this case the parameters $\kappa_{g,c}$, $\kappa_{\gamma,c}$ and $\kappa_{Z\gamma,c}$ appearing in Eqn. 8.3 do not coincide with the κ's defined in Eqn. 8.2 and in particular their SM value is $\kappa_{i,c} = 0$. Consider the one loop SM process $\sigma(gg \to h)$, the leading contribution is generated radiatively with a top or bottom quark running in the loop. In this case, the amplitude can be formally split separating the various contributions as

$$\mathcal{A}_{ggH} = \kappa_t \mathcal{A}^t_{ggH} + \kappa_b \mathcal{A}^b_{ggH} + \kappa_{g,c}, \tag{8.4}$$

where $\mathcal{A}^{t(b)}_{ggH}$ represents the contribution from the top (bottom) loop.

Although the most general approach is that of retaining $\kappa_{g,c}$ as an independent parameter that can capture contributions from BSM diagrams, it is possible to consider the approximation $\kappa_{g,c} = 0$. This corresponds to the implicit assumption that the ggH vertex does not receive direct new physics contributions from e.g. a heavy state running in the loop. As a consequence the factor κ_g (see Eqn. 8.2) can be expressed as a function of κ_t and κ_b according to

$$\kappa_g^2(\kappa_t, \kappa_b) = \frac{\kappa_t^2\, \sigma^{tt}_{ggH} + \kappa_b^2\, \sigma^{bb}_{ggH} + \kappa_t \kappa_b\, \sigma^{tb}_{ggH}}{\sigma^{tt}_{ggH} + \sigma^{bb}_{ggH} + \sigma^{tb}_{ggH}}\,. \tag{8.5}$$

Here $\sigma^{tt(bb)}_{ggH}$ denotes the contribution of the top (bottom) loop to the ggH cross section, while σ^{tb}_{ggH} stands for the interference term. Similar considerations hold for the $\gamma\gamma$ and $Z\gamma$ decays, for the tree-level case of vector boson fusion (VBF) production, where the SM diagrams are modified with $\kappa_{Z,W}$, and for the total Γ_h, whose corresponding modifier κ_H receives contributions from $\kappa_{f,W,Z,\gamma,g}$ in addition to a BSM component that can be denoted $\kappa_{H,\mathrm{BSM}}$.

The experimental collaborations have provided limits on the κ parameters, that are extracted from a global fit to Higgs production and decay measurements [288, 529–531]. Both benchmarks described above in the ggH example have been considered: Figure 13 shows the most recent constraints [288] obtained including $\kappa_{\gamma,c}$, $\kappa_{g,c}$, $\kappa_{H,\mathrm{BSM}}$ (the latter is denoted $\mathrm{B}_{\mathrm{BSM}}$ in the plot) as free parameters, while Figure 14 shows the results for $\kappa_{\gamma,c} = \kappa_{g,c} = \kappa_{H,\mathrm{BSM}} = 0$. In the first case (Figure 13) the system is under-constrained, and therefore one additional condition is required to constrain all the parameters. Two alternative choices were considered: either imposing $|\kappa_V| = |\kappa_Z|, |\kappa_W| \leq 1$ while allowing $\kappa_{H,\mathrm{BSM}} \neq 0$ (left panel), or fixing $\kappa_{H,\mathrm{BSM}} = 0$ (right panel).

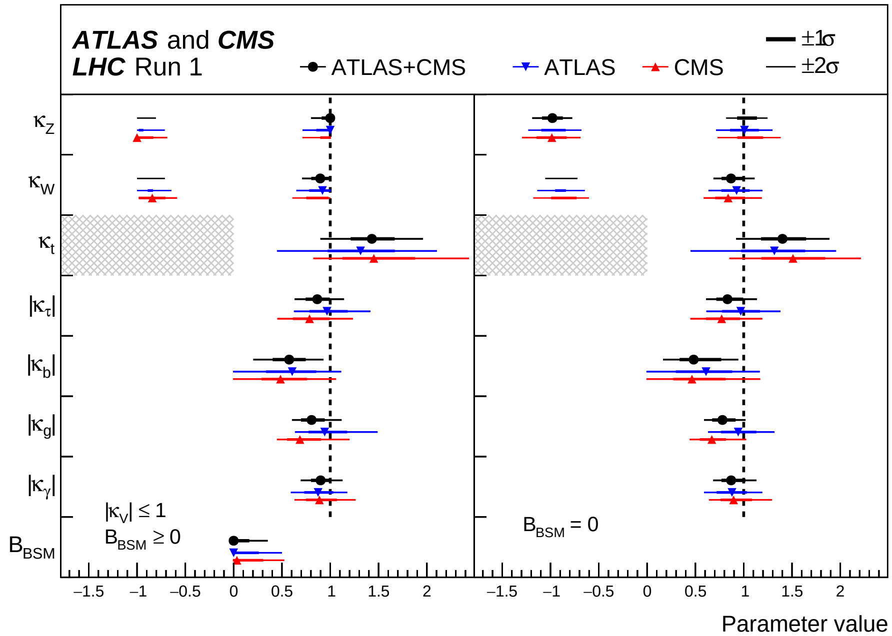


Figure 13. Fit results from Ref [288] obtained including $\kappa_{g,c}$, $\kappa_{\gamma,c}$ as free parameters. In the left panel the BSM Higgs width $\kappa_{H,\mathrm{BSM}} = \mathrm{B}_{\mathrm{BSM}}$ has also been treated as independent, while requiring $|\kappa_V| \leq 1$ for $V = Z, W$. The right panel assumes instead $\kappa_{H,\mathrm{BSM}} = 0$. The hatched area for κ_t has been forbidden in the fit, to break the degeneracy due to the absence of sensitivity to the sign of this parameter in the data used.

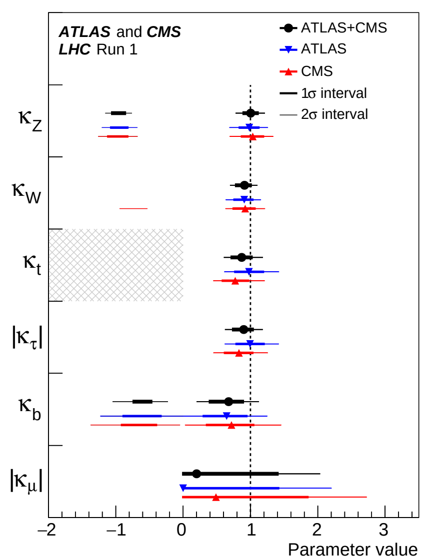


Figure 14. Fit results from Ref. [288] obtained imposing $\kappa_{\gamma,c} = \kappa_{g,c} = \kappa_{H,\mathrm{BSM}} = 0$, i.e. assuming the absence of BSM contributions to the hGG and $h\gamma\gamma$ interactions, as well as in the Higgs width, beyond those stemming from modified Higgs couplings. The hatched area for κ_t has also been forbidden.

The results generically show that κ_t is sensitive to which of the two scenarios is assumed, as expected considering that it gives the dominant contribution to the radiative decay and production channels. The other parameters do not show a dramatic variation among the different setups. It also worth noting that, as anticipated in Section 4.3 the current uncertainty in the determination of the Higgs couplings is $10 - 20\%$ on average.[92]

Notably, as the experimental accuracy drops below the 10%, EW radiative corrections become significant. As a consequence, it is not appropriate to use the κ framework to project directly the signal-strength measurements into constraints on the Higgs couplings except in some limited applications.

The κ-formalism was constructed as a first probe of the Higgs boson's properties and constitutes a reasonable framework for the interpretation of the Higgs dataset collected so far at the LHC. The key strength of this approach is not that it is an EFT, but that it allows a series of hypothesis tests addressing the question on the consistency of the properties of the discovered scalar with the SM Higgs. Perhaps the most elegant of these tests is the two dimensional test with only a universal (κ_F, κ_V). A comparison of results of this form produced at the time of discovery in 2012 in Ref. [298] in Fig.15 (left) and the combined ATLAS+CMS results in Fig.15 (right) demonstrates the degree to which the RunI data set increasingly supported the hypothesis that the discovered scalar is the Higgs boson.

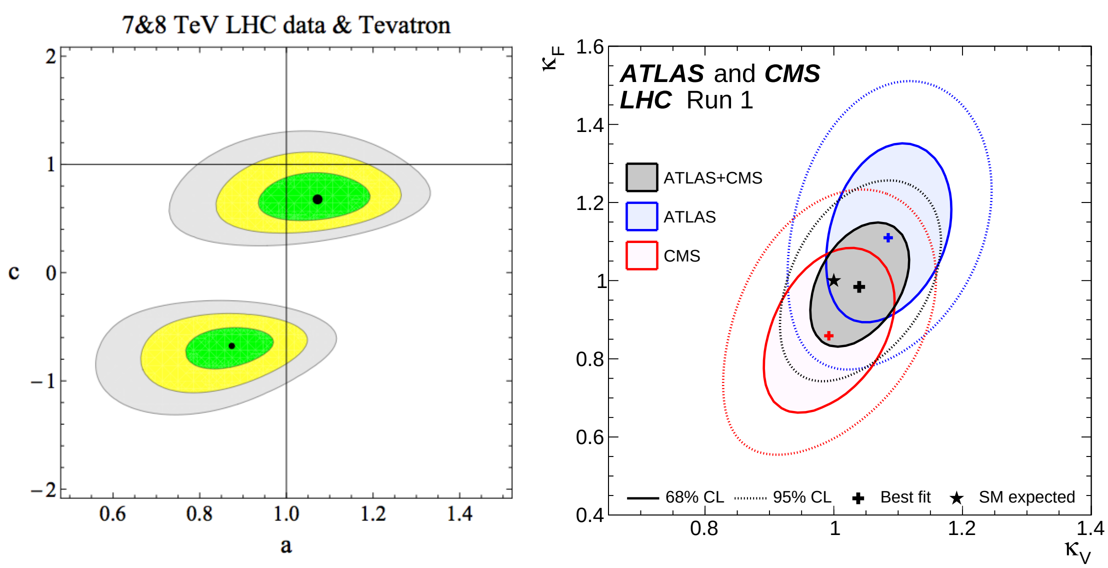


Figure 15. Hypothesis test of the discovered scalar as a Higgs-like boson with a universal rescaling parameter set for the fermions and vector bosons (κ_F, κ_V) (right) Ref. [288] and using the alternate notation (c, a) (left) around the time of discovery in Ref. [298]. Note the different scales of the plots.

The κ formalism does not constitute a suitable tool for a consistent analysis of the Higgs properties going forward, as the experimental data improves. Figure 16 shows the projections for these measurements at the CMS experiment at 14 TeV and for integrated luminosities of 300 fb^{-1} (left panel) and 3000 fb^{-1} (right panel) [534]. Similar results are expected at

[92] Note that the degeneracy of the κ_t sign can be lifted using the method discussed in Refs. [525, 532, 533].

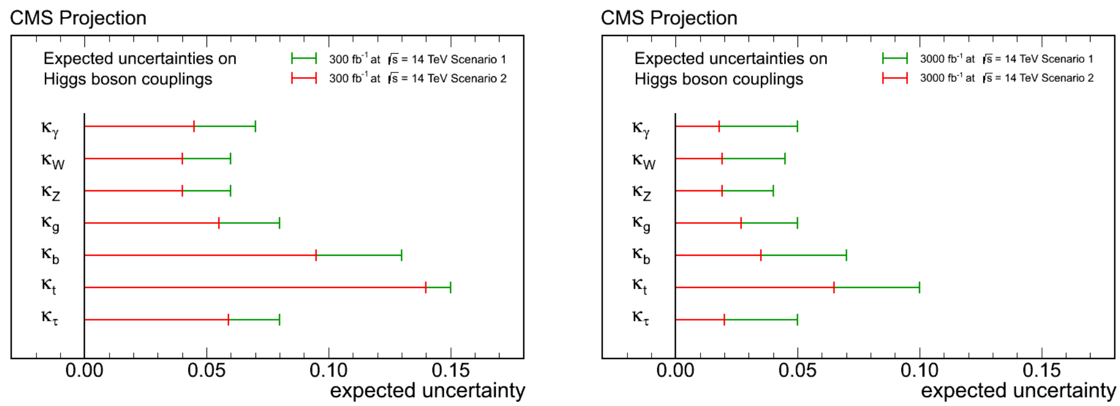


Figure 16. Prospects for the measurement of the κ parameters at the CMS detector [534] at $\sqrt{s}$ = 14 TeV and with an integrated luminosity of 300 fb^{-1} (left) and 3000 fb^{-1} (right). The red and green lines are described in the text.

ATLAS [535]. The plot shows that the sensitivity will approximately reach the 5 – 10 % level, with a significant dependence on the scaling of experimental errors assumed. The green lines correspond to a quite conservative case (Scenario 1) in which all the systematics are assumed to be the same as in the 2012 performance. The red lines (Scenario 2), instead, are derived rescaling the theoretical uncertainties by a factor 1/2 and the other systematics by the square root of the luminosity. Although the projections are extremely uncertain, and subject to a number of unverified assumptions it is clear that the properties of the Higgs-like scalar will be increasingly resolved experimentally in RunII and beyond. The κ formalism is not the right tool for this era of increasing experimental precision. Some of its main limitations are

- The κ formalism is not an EFT as formulated in Ref. [511]. As the κ formalism can only be related to EFT constructions with a set of further UV assumptions, it is not guaranteed that it captures a consistent IR limit of an underlying new physics sector. If κ fits show deviations from the SM constructing a consistent inverse map to the UV sector is not guaranteed to be possible as a result.

- The κ formalism is not systematically improvable with perturbative corrections. This is a fatal flaw that introduces a multitude of difficulties. These difficulties always appear in any ad-hoc construction. Any replacement formalism must be able to systematically determine perturbative corrections without assuming the SM to address this central flaw. The only known way to accomplish this is with a well defined effective field theory embedding. As the accuracy of the data descends below the $\sim$ 10% range, higher order calculations simply become indispensable in important channels sensitive to Higgs properties.

- The rescalings of parameters that are off-shell vertices, in particular $h \to V V^\star$ is ambiguous as the off-shell massive gauge boson is not an external state and has no precise definition without a field theory embedding.

- The κ formalism cannot be consistently used to interface Higgs data with LEP data, due to the different scales involved in the measurements. Again this requires a field theory embedding to relate the Wilson coefficient constraints. Similarly, the κ formalism is not a useful tool to interface with even lower energy measurements.

- The formulation is intrinsically non-gauge invariant, and at best an example of a non-linear realization of the gauge symmetry of the SM, i.e. a restricted version of the HEFT. The couplings of the scalar to fermions and gauge bosons are left arbitrary. It is also the case that amplitudes computed with the Lagrangian given in Eqn. 8.3 generically lead to non-unitary S-matrix elements. This is not a concern if the κ formalism is embedded in an EFT extension of the SM, as such a theory need only be unitary to the cut off scale. As the κ formalism is not so embedded in an EFT without further assumptions, its lack of unitarity at high energies renders it obviously inconsistent, and unsuitable for studies of differential distributions at high energies in particular.

- The construction of the κ formalism is not a truly general set of deviations from the SM, but a biased construction informed by experimental constraints in a fairly haphazard fashion. For example, having two independent parameters $\kappa_Z \neq \kappa_W$ introduces a hard breaking of the custodial symmetry, but this is avoided in many κ fits due to experimental constraints (see the discussion in Ref. [524]). On the other hand, these constraints are not consistently determined in the κ formalism itself, due to its inability to relate LEP and LHC data which requires a field theory embedding.

- The κ formalism includes only couplings with standard Lorentz structures in a renormalizable field theory. As such, it can only capture deviations in total production/decay rates, and this is another reason it cannot be consistently used for the analysis of kinematic distributions.

Despite all of these flaws, and the clear need to go beyond the κ framework, we wish to emphasize that the κ framework and RunI results reported in it were a profound and important achievement. The theoretical framework to interface with LHC data in a consistent EFT extension of the SM was simply not available in RunI. As such, the κ framework, despite all its flaws, was a sensible and insightful choice to project the raw experimental data into a useful and informative form. It is clear that the use of the κ framework to hypothesis test the SM was an informative application. This data reporting formalism should be maintained into RunII and beyond despite all its limitations for this application. Nevertheless, it is time to go beyond the κ framework. The HEFT and the SMEFT have now been developed to a sufficient degree that they can be systematically used for this task going forward.

8.2 Relation of the κ formalism to SMEFT and HEFT

In order to overcome the κ framework limitations listed above, it is necessary to switch from the κ-parameterization to one given in terms of Wilson coefficients of a non-redundant EFT basis (see e.g.[238, 462]). Once such a basis has been chosen, it is possible to identify a one way mapping between the $\{\kappa_i\}$ and $\{C_i\}$.[93] In general, each κ_i can be decomposed as

$$\kappa_i^2 = 1 + \Delta\kappa_i, \tag{8.6}$$

with $\Delta\kappa_i$ a linear combination of EFT parameters, whose numerical coefficients are computed calculating the relevant σ_i or Γ_i at a given order in the EFT. We stress that translating the κ framework into an EFT form does not just represent a bare reparameterization, but actually improves the theoretical consistency of the description. The EFT embedding makes manifest the presence of correlations among different observables that are required by gauge invariance or other imposed symmetries and structures such as the SMEFT reparameterization invariance due to the EOM. The generic result, once again, is a correlated fit space of unphysical Lagrangian parameters, as in the LEP case represented in Fig. 10.

To illustrate how the procedure is carried out in the SMEFT, consider for instance the decay $h \to \bar{b}b$. A partial NLO calculation of this process in the SMEFT has been presented in Refs. [536, 537]. For illustrative purposes, here we report only the tree-level result computed in the Warsaw basis, which gives

$$\begin{aligned}\kappa_b^2 &= \frac{\mathcal{A}^2(h \to \bar{b}b)_{\rm SMEFT}}{\mathcal{A}^2(h \to \bar{b}b)_{\rm SM}} = 1 + \Delta\kappa_b\,,\\ \Delta\kappa_b &= 2\,\bar{v}_T^2\left(C_{H\Box} - \frac{C_{HD}}{4} - C_{Hl}^{(3)} + \frac{C_{ll}}{2} - \frac{C_{dH}}{[Y_d]_{33}}\right).\end{aligned} \tag{8.7}$$

The terms $C_{H\Box}, C_{HD}$ come from normalizing the Higgs' kinetic term to the canonical form, while $C_{Hl}^{(3)}, C_{ll}$ appear due to the shift between the true vev $\bar{v}_T$ and the value inferred from the measurement of G_F. Finally, C_{dH} represents the only direct $d = 6$ contribution, which is due to the operator $\mathcal{Q}_{dH}$, that perturbs the Yukawa coupling. See Appendix A for details on shift parameters.

Another important example is that of $h \to \gamma\gamma$. In this case it is necessary to carry out the computation at one-loop, which gives

$$\kappa_\gamma^2 = \frac{\mathcal{A}^2(h \to \gamma\gamma)_{\rm SMEFT}}{\mathcal{A}^2(h \to \gamma\gamma)_{\rm SM}} = 1 + \Delta\kappa_\gamma^{\rm LO} + \Delta\kappa_\gamma^{\rm NLO} \tag{8.8}$$

where $\Delta\kappa_\gamma^{\rm LO}$ and $\Delta\kappa_\gamma^{\rm NLO}$ include the contributions computed respectively at tree-level and at one-loop in the SMEFT, both normalized to the SM amplitude calculated at a specific order in perturbation theory. Here we normalize by the one loop SM amplitude. The tree-level term is easily derived in the Warsaw basis. The SMEFT contributions from CP even operators at LO reads [215]

$$\Delta\kappa_\gamma^{\rm LO} = -\frac{16\pi^2}{e^2\mathcal{I}^\gamma} v_T^2 \left(C_{HWB}\, s_\theta\, c_{\hat\theta} - C_{HW} s_\theta^2 - C_{HB}\, c_{\hat\theta}^2\right), \tag{8.9}$$

where $\mathcal{I}^\gamma$ encodes the adimensional SM amplitude, whose expression can be found in Ref. [538]. It is interesting to notice that $\Delta\kappa_\gamma^{\rm LO}$ carries an enhancement of $8\pi^2/e^2$ compared e.g. to

[93] This does not turn the κ's into a basis, as no gauge invariant field redefinitions result in the mapping.

$\Delta\kappa_b$. This could give $\mathcal{O}(1)$ deviations for $C_i \sim 1$ and Λ as low as about 4 TeV (barring cancellations among the coefficients), which simply reflects the fact that processes that are radiatively suppressed in the SM are *a priori* more sensitive to the presence of new physics as the SMEFT has a multi-pole expansion in general. This was emphasized long ago in Ref. [215]. The one-loop term $\Delta\kappa_\gamma^{\rm NLO}$ has a much more complex structure and is not so easy to derive. It contains a large number of $\mathcal{L}_6$ Wilson coefficients, feeding in relatively suppressed by $16\pi^2$ compared to the LO results. Many of these Wilson coefficients do not appear at tree level in the SMEFT, see the results in Refs. [232, 262, 539]. Note also that an explicit matching of the κ's into the Warsaw basis, extended to all the Higgs two-body decay channels, has been partially given in Ref. [262].

The mapping procedure illustrated above for the SMEFT case can be done with the HEFT. In this case, the κ's are mapped to combinations of parameters belonging both to the leading and next-to-leading order Lagrangian. Consider the tree-level expression of $\Delta\kappa_b$. Using the basis of Ref. [284] one has

$$\Delta\kappa_b = \frac{32\pi^2 v_T^2}{\Lambda^2}(r_2^l - r_5^l) + \frac{[Y_d^{(1)}]_{33}}{[Y_d]_{33}}\,. \tag{8.10}$$

The coefficients $r^l_{2,5}$ are associated to four-fermion operators that enter through $\delta v_T \sim \delta G_F$ (analog to C_{ll} in 8.7) and belong to the NLO Lagrangian[94] $\Delta\mathcal{L}$. In the last term, instead, $Y_d^{(1)}$ belongs to $\mathcal{L}_0$: it is the Yukawa matrix appearing in the second term of the expansion of the functional $\mathcal{Y}_Q(h)$, defined in Eqn. 4.45. The impact of this term is analogous to that of the operator $\mathcal{Q}_{dH}$ in the SMEFT, with the difference that in the HEFT case anomalous Higgs couplings appear already at LO due to the singlet nature of the h field. This is true for the interaction terms with $d \leq 4$, while it is still necessary to include $\Delta\mathcal{L}$ terms to match $H\gamma\gamma$, HGG, $HZ\gamma$ couplings. Namely, $\kappa_{W,Z}$ and κ_f receive leading contributions respectively from a_C and $Y_f^{(1)}$, while κ_γ, κ_g, $\kappa_{Z\gamma}$ are mapped to a combination of higher order Wilson coefficients. For instance,[95]

$$\Delta\kappa_\gamma^{\rm LO} = \frac{8\pi^2}{e^2\mathcal{I}^\gamma}\left(-4 s_\theta\, c_{\hat\theta}\tilde a_1 + c_{\hat\theta}^2 \tilde a_B + s_\theta^2(\tilde a_W - 4\tilde a_{12})\right)\,, \tag{8.11}$$

where the shorthand notation $\tilde a_i = C_i a_i$ stands for the product of C_i with the coefficient of the linear term in the function $\mathcal{F}_i(h) = 1 + 2a_i h/v + \ldots$

Restricted versions of the SMEFT and the HEFT can be used in this manner to develop one way mappings to the κ's. This can be done as no defining conditions in the κ approach are fundamentally gauge dependent. As the assumptions of the κ formalism itself starts to fail

[94] Compared to Eqn. 8.7, here there is no equivalent of the terms $C_{H\Box}$ and C_{HD} because there is no operator in the basis of Ref. [284] that modifies the Higgs kinetic term. The operator corresponding to $\mathcal{Q}_{Hl}^{(3)}$ was not retained either.

[95] This result can be compared with Eqn. 8.9. There is a close correspondence between the coefficients $C_{HWB} \to a_1$, $C_{HB} \to a_B$, $C_{HW} \to a_W$, while the term a_{12} in the HEFT comes from the custodial breaking operator ${\rm Tr}(\mathbf{T}W_{\mu\nu})^2$ that has an equivalent only at $d = 8$ in the SMEFT.

at the experimental precision where this mapping becomes of interest, this task is not a high priority. Directly developing the corresponding results in the SMEFT and HEFT to interface with past data and future LHC results at leading, and next to leading order, is ongoing in a manner that is essentially bypassing the κ formalism.

9 SMEFT developments in the top sector

Being the heaviest known particle, and the one with the largest Yukawa coupling, the top quark is possibly the SM state that is closest to new physics sectors. In particular, it represents a sensitive probe of new physics driving the EW symmetry breaking (EWSB): for instance, its mass plays a fundamental role in determining the RG evolution and stability of the Higgs potential in the UV. At the same time, the top is typically expected to exhibit the largest mixing with exotic states in scenarios with non-linear EWSB sectors, such as composite Higgs models or models with warped extra dimensions.

Studying the properties and couplings of the top quark can thus give a unique insight into new physics, which is complementary to that offered by the Higgs boson. Top physics also benefits from a significantly larger dataset compared to Higgs physics, as top quarks are abundantly produced at high energy hadron colliders such as Tevatron and LHC. This has motivated several analyses of the top sector based on the SMEFT approach.

Early studies explored the possibility of constraining its interactions at e^+e^- colliders [540–553] (for recent analyses at future lepton colliders see e.g. [554–559]) and in $\gamma\gamma$ collisions [547, 553, 560–564], which constitute a particularly suitable environment to probe CP violating couplings. Here we give an overview of SMEFT studies of the top sector, focusing on the processes relevant for top physics at the Tevatron and LHC.

In the Warsaw basis there are 28 operators that directly involve the top quark at $\mathcal{L}^{(6)}$ in unitary gauge (see Table 1 for the operators definitions)[96]:

$$
\begin{aligned}
&\mathcal{Q}_{uH},\, \mathcal{Q}_{Hu},\, \mathcal{Q}_{Hq}^{(1),(3)},\, \mathcal{Q}_{Hud},\, \mathcal{Q}_{uW},\, \mathcal{Q}_{uB},\, \mathcal{Q}_{uG},\, \mathcal{Q}_{dW},\\
&\mathcal{Q}_{qq}^{(1),(3)},\, \mathcal{Q}_{lq}^{(1),(3)},\, \mathcal{Q}_{uu},\, \mathcal{Q}_{ud}^{(1),(8)},\, \mathcal{Q}_{eu},\, \mathcal{Q}_{lu},\, \mathcal{Q}_{qe},\, \mathcal{Q}_{qu}^{(1),(8)},\, \mathcal{Q}_{qd}^{(1),(8)},\\
&\mathcal{Q}_{ledq},\, \mathcal{Q}_{quqd}^{(1),(8)},\, \mathcal{Q}_{lequ}^{(1),(3)}.
\end{aligned}
\tag{9.1}
$$

In addition to these, other operators can be relevant for a global analysis of the top sector, either because they enter the top couplings due to input parameter definitions (see Appendix A) or because they modify other interactions entering top production processes. The first class includes

$$\mathcal{Q}_{H\Box},\, \mathcal{Q}_{HD},\, \mathcal{Q}_{HWB},\, \mathcal{Q}_{ll},\, \mathcal{Q}_{Hl}^{(3)}, \tag{9.2}$$

while the operators fulfilling the latter condition are

$$\mathcal{Q}_{G},\, \mathcal{Q}_{\tilde{G}},\, \mathcal{Q}_{HG},\, \mathcal{Q}_{H\tilde{G}}. \tag{9.3}$$

[96] Prior to the construction of the Warsaw basis, a systematic parameterization of $d = 6$ effects in the top sector was proposed in [565, 566].

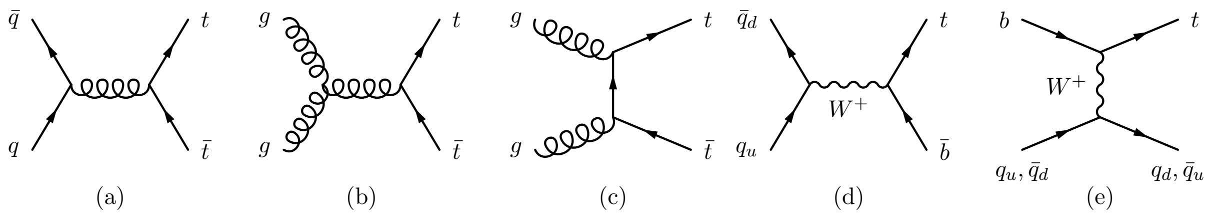


Figure 17. Main diagrams for top pair and single top production in pp collisions in the SM.

Considering a general flavor scenario and retaining only the index contractions that select a top quark, the overall number of independent parameters is 1179 (622 absolute values + 557 complex phases). The picture is remarkably simplified in the approximation in which the quarks of the first two generations respect a $\mathrm{U}(2)^3$ symmetry (a U(2) for each field q, u, d) and, simultaneously, flavor universality is assumed in the lepton sector. In this case, that represents the customary set of assumptions for global top analyses, the number of independent parameters is reduced to 85, corresponding to 68 absolute values and 17 phases[97].

This number is still too large for a complete analysis to be performed, but a rich variety of studies has been carried out in the literature, focusing on specific subsets of the parameter space. In particular, the operators in Eq. 9.2 are usually neglected in top physics analyses, under the assumption that they can be constrained in other classes of measurements. An exception are fits to EWPD, where C_{WB} and C_{HD} are typically retained. These studies allow to constrain top operators of classes 6 and 7, via loop contributions to the gauge bosons self energies [567, 568] and four fermion operators arising either at tree level or at 1-loop due to RG mixing [569]. CP violating contributions are also negligible for a large number of top measurements at the LHC, because the spin-averaged SM amplitudes for these processes are dominantly CP-conserving, implying that the interference term $\mathcal{A}_{SM}\mathcal{A}^*_{d=6}$ is typically suppressed. Bounds on the CP-odd parameters are rather inferred from measurements of top polarizations and t-$\bar{t}$ spin correlations [570–577] or from lower energy experiments, such as EDM measurements [578–583] and B meson decays [584–586].

The first global fit to top results from Tevatron and LHC has been presented by the TopFitter collaboration in [587, 588] (see also [589, 590]), where 12 independent combinations of 14 Wilson coefficients were constrained using both differential and inclusive measurements.

The relevant processes for top physics measurements at the LHC are the following:

(i) Top pair production $pp \to t\bar{t}$.

In the SM, this process is dominated by QCD contributions: both $q\bar{q}$ and gg initiated diagrams contribute, as shown in Figure 17 (a)-(c). In the SMEFT, diagram (a) is

[97] The number of independent parameters is further reduced in the presence of a full $\mathrm{U}(3)^5$ flavor symmetry, that gives 46 independent quantities (38 absolute values + 8 phases). However, this option is rarely considered in top analyses, as they implicitly assume that new physics effects may impact top physics more significantly compared to processes involving the first two generations.

corrected only by contributions proportional to $(C_{uG})_{33}$, modifying the $Gt\bar{t}$ coupling. This is the coefficient with the largest impact on top pair production. Diagrams obtained inserting $(C_{uG})_{ii}$ with $i = \{1, 2\}$ in the initial $Gq\bar{q}$ vertex are negligible because their interference with the SM amplitude is proportional to m_u or m_d, see Section 6.5. In addition, $q\bar{q}$ initiated top pair production receives tree-level SMEFT contributions from the 6 four-quark operators $\mathcal{Q}_{qq}^{(1)}$, $\mathcal{Q}_{qq}^{(3)}$, $\mathcal{Q}_{uu}$, $\mathcal{Q}_{ud}^{(8)}$, $\mathcal{Q}_{qu}^{(8)}$, $\mathcal{Q}_{qd}^{(8)}$ [211, 575], whose impact can be expressed in terms of only four combinations of Wilson coefficients [575]:

$$\begin{aligned} C_u^1 &= (C_{qq}^{(1)})_{1331} + (C_{qq}^{(3)})_{1331} + (C_{uu})_{1331}, & C_d^1 &= 4(C_{qq}^{(3)})_{1133} + (C_{ud}^{(8)})_{3311}, \\ C_u^2 &= (C_{qu}^{(8)})_{1133} + (C_{qu}^{(8)})_{3311}, & C_d^2 &= (C_{qu}^{(8)})_{1133} + (C_{qd}^{(8)})_{3311}. \end{aligned} \tag{9.4}$$

Diagrams induced by $\mathcal{Q}_{qu}^{(1)}$, $\mathcal{Q}_{qd}^{(1)}$, having color-singlet contractions in the fermion currents, do not interfere with the QCD SM diagrams, but only with the EW production. Their contributions to the total cross section are roughly an order of magnitude smaller that those of the six four-quark operators listed above[98].

For the gg initiated channel, that dominates at high energy, the SM diagrams in Fig. 17 (b) and (c) can be dressed with $d = 6$ contributions from $\mathcal{Q}_{uG}$ in the $Gt\bar{t}$ vertices. This operator contributes, in addition, through a $GGt\bar{t}$ four-point interaction. $\mathcal{Q}_{uG}$ is the operator with the most significant impact on $t\bar{t}$ production, and it has been extensively studied in the literature, see e.g. [591–597]. Diagram (b) can also be corrected with an insertion of $\mathcal{Q}_G$ (or $\mathcal{Q}_{\tilde{G}}$) in the GGG vertex. Due to the helicity structure of the external gluons, this term interferes only with the diagram in Figure 17 (c) proportionally to m_t^2, and thus yields a smaller correction compared to $\mathcal{Q}_{uG}$. The operators $\mathcal{Q}_{HG}$, $\mathcal{Q}_{H\tilde{G}}$ can also contribute inducing a tree $gg \to h \to t\bar{t}$ diagram. However, this term is suppressed by the Higgs propagator being always largely off-shell.

The main observables in $pp \to t\bar{t}$ is the total cross section, that is currently measured at the LHC with an uncertainty $\lesssim 5\%$ [598–603]. Differential cross sections are also important for constraining the relevant Wilson coefficients. In particular, the analysis of the $m_{t\bar{t}}$ spectrum is useful to target and disentangle four-fermion operators [604].

Further, charge asymmetries received much attention in the past, mainly due to a discrepancy with the SM expectation registered by the CDF experiment at Tevatron [605], in the forward-backward asymmetry

$$A_{FB} = \frac{N(\Delta y > 0) - N(\Delta y < 0)}{N(\Delta y > 0) + N(\Delta y < 0)}, \quad \Delta y = y_t - y_{\bar{t}}, \tag{9.5}$$

[98] Note that the interference of $\mathcal{Q}_{qq}^{(1),(3)}$ and $\mathcal{Q}_{uu}$ contributions with SM QCD production is non-vanishing, as particular linear combinations of these operators are equivalent to the color octet contractions $(\bar{q}\gamma_\mu T^A q)^2$, $(\bar{u}\gamma_\mu T^A u)^2$ via Fierz transformations and the SU(3) completeness relation $T^A_{BC} T^A_{DE} = 2\delta_{BE}\delta_{DC} - 2/3\delta_{BC}\delta_{DE}$. Among the 3 remaining operators admitting interactions with two top quarks, $\mathcal{Q}_{ud}^{(1)}$ gives contributions that only interfere with the EW SM diagrams. $Q_{quqd}^{(1),(8)}$ contain currents with a $(\bar{L}R)$ chiral structure, so they give only diagrams whose interference with the SM is suppressed by the mass of the initial state quarks.

where y_f is the rapidity of the fermion f in the laboratory frame. The excess was subsequently reduced in analyses with higher statistics, and the most recent combination of the Tevatron measurements finds agreement with the SM expectation at NNLO QCD + NLO EW within 1.6σ [606]. Besides refined experimental techniques, an important role in reducing the anomaly has been played by higher order calculations in the SM [607–614] showing that A_{FB} receives large radiative corrections.
At the LHC, the forward-backward asymmetry is washed out by the huge symmetric $gg \to t\bar{t}$ contributions and by the fact that the initial pp state is forward-backward symmetric. A better observable for LHC measurements is the central charge asymmetry A_C, which is correlated with A_{FB} and defined as [615]

$$A_C = \frac{N(\Delta|y| > 0) - N(\Delta|y| < 0)}{N(\Delta|y| > 0) + N(\Delta|y| < 0)}. \tag{9.6}$$

The A_C asymmetry has been measured by the ATLAS and CMS collaborations in pp collisions at 7 and 8 TeV (see [616] and references therein), finding agreement with the SM expectation. Interestingly, the two asymmetries A_{FB} and A_C are different linear functions of the parameters in Eq. 9.4. Therefore the four combinations of Wilson coefficients can be constrained combining Tevatron measurements of A_{FB} and LHC measurements of A_C [617].

The first systematic EFT studies of $t\bar{t}$ production were carried out in Refs. [211, 575, 604], including a phenomenological analysis considering all the observables mentioned above. The role of angular observables in the top decay products has been explored, for instance, in [577, 618].

Higher order results are also available: the SMEFT amplitude has been computed at NLO QCD including the contribution of a real $(C_{uG})_{33}$ [596]; effects induced via RG mixing were also considered within the class of four-fermion operators [619].

(ii) Single top production $pp \to qt$ $(q \neq t)$, $pp \to tW$.

Single top production is usually classified into s-channel, t-channel and tW production.

Focusing on the $pp \to qt$ modes, the s-channel is characterized by a b in the final state (Fig. 17 (d)) while the t-channel has typically a light quark in the final state (Fig. 17 (e)). The SM diagrams receive SMEFT corrections from operators entering the Wtb vertex [620], namely $\mathcal{Q}_{uW}$, $\mathcal{Q}_{Hq}^{(3)}$ and the operators in Eq. 9.2 (all but $\mathcal{Q}_{H\Box}$, that only corrects the top Yukawa coupling), or modifying the W coupling to a light quark pair. Among the latter, only $\mathcal{Q}_{Hq}^{(3)}$ and the terms in Eq. 9.2 give relevant contributions, while the interference terms for $\mathcal{Q}_{uW}$, $\mathcal{Q}_{dW}$, $\mathcal{Q}_{Hud}$ are always suppressed by the mass of one of the quarks entering the vertex. Among four-fermion operators, $\mathcal{Q}_{qq}^{(1),(3)}$ have the largest impact: due to the chiral structure of the SM diagrams, the interference terms of invariants with at least one right-handed fermion are proportional to the mass of one of the external quarks [621].

The $pp \to tW$ mode is mostly produced from the gb partonic initial state and and is sensitive to $(C_{uG})_{33}$ in addition to the coefficients of the operators affecting $pp \to qt$ [622, 623].

The total cross section for mono-top production is dominated by the t-channel contribution, that has been measured at the LHC with a precision of $\sim 10\%$ [624–629]. The cross-section for the s-channel is ~ 20 times smaller at the LHC, due to the presence of an anti-quark in the initial state [630]. ATLAS and CMS have reported evidence for this process with LHC-Run I data, although with a quite low significance of 3.2 and $2.5\,\sigma$ respectively [631, 632] (see [633] for a recent experimental review).

The SM cross-section of $pp \to tW$ is also sizable at the LHC (although about 3–4 times smaller than the t-channel cross section). Evidence for this process at the LHC has been found already at $\sqrt{s} = 7$ TeV [634, 635]. Recently, the ATLAS and CMS collaborations reported measurements of the total and differential rates at $\sqrt{s} = 13$ TeV [636–638], with a maximum precision of $\sim 10\%$ reached by CMS using the full 35.9 fb^{-1} dataset collected in 2016. Despite the accessible cross sections, measurements of tW production are challenging at the LHC, mainly because the process interferes with $t\bar{t}$ production beyond LO in QCD. Disentangling the two signals is a quite complex task [639–642].

Measurements of the top polarizations in mono-top processes can play an important role in this context, helping to constrain anomalous Wtb interactions [643] and four-fermion operators [644].

Comprehensive EFT analyses of mono-top can be found in [211, 575, 645]. NLO QCD corrections to the SMEFT amplitude have been computed for all three channels [646] finding, in particular, that they have a non-trivial impact on differential distributions.

(iii) Top pair production in association with a neutral gauge boson $pp \to t\bar{t}V$, $V = \{Z, \gamma\}$.

The production of a top in association with a neutral gauge boson Z/γ can be both $q\bar{q}$ and gg-initiated: the relevant SM diagrams have the same structure as those in Figure 17 (a)-(c), with a gauge boson emitted from any of the fermion lines. In addition to the set of operators that modify $t\bar{t}$ production, these channels give a unique access to the Ztt and γtt couplings [554, 647], that are corrected by $\mathcal{Q}_{Hq}^{(1),(3)}$, $\mathcal{Q}_{Hu}$, $\mathcal{Q}_{uW}$, $\mathcal{Q}_{uB}$, $\mathcal{Q}_{dW}$, $\mathcal{Q}_{dB}$ and the operators in Eq. 9.2 (except $\mathcal{Q}_{H\Box}$). Four-fermion operators also contribute to the $q\bar{q}$-initiated channel. The corresponding corrections behave as in the $pp \to t\bar{t}$ case.

The total cross section for both $t\bar{t}Z$, $t\bar{t}\gamma$ have been measured at the LHC with an accuracy of $\sim 15 - 20\%$ [648–652]. Their constraining power is quite low at the moment [588], but shall become significant with higher statistics. It has been pointed out in Ref. [653] that cross section ratios of $t\bar{t}V/t\bar{t}$ are also convenient observables, that allow to isolate and constrain the top dipole moments induced by $(C_{uW})_{33}$, $(C_{dW})_{33}$, $(C_{uB})_{33}$.

The SMEFT contributions from class 6 and 7 operators to the total cross section and differential distributions of $pp \to t\bar{t}Z/\gamma$ have been computed at NLO QCD accuracy [554, 647, 654].

(iv) Top pair production in association with a Higgs boson $pp \to t\bar{t}h$.

In the SM $pp \to t\bar{t}h$ takes place mainly through diagrams analogous to Figs. 17 (a)-(c), with a Higgs radiated from one of the top propagators. SMEFT corrections to these diagrams are therefore analogous to those for $pp \to t\bar{t}$, with the addition of possible insertions of the coefficients $C_{H\Box}$, C_{HD} and $(C_{uH})_{33}$ in the tth vertex. The operator $\mathcal{Q}_{HG}$ ($\mathcal{Q}_{H\tilde{G}}$) also contributes, inducing new diagrams in which the Higgs is radiated from a gluon line rather than from a top, and a gg-initiated diagram containing a $GGGh$ four-point interaction.

This process represents therefore an interesting bridge between top and Higgs global analyses, providing information complementary to that extracted from Higgs production and decay [533, 655, 656]. In particular, the interplay of the $t\bar{t}h$ channel with h, hh and $h + j$ production at the LHC has been explored with the inclusion of NLO QCD corrections to contributions from $(C_{uH})_{33}$, $(C_{uG})_{33}$, C_{HG} [656]. The analysis of angular observables of the decay products of the tops is also interesting in this context, as it would allow to probe the imaginary part of $(C_{uH})_{33}$, testing the CP nature of the top Yukawa vertex [657–660].

Due to the low cross section and the presence of large irreducible backgrounds, significant constraints from $pp \to t\bar{t}h$ are likely to be extracted only at the high luminosity phase of the LHC. At present, the ATLAS and CMS collaborations have reported evidence for this process at $\sqrt{s} = 13$ TeV with an integrated luminosity of 36 pb^{-1}, with still large uncertainties on the measured cross sections [661, 662].

(v) Top decays.

In the SM, top quarks decay nearly 100% of the time to bW. As such, a study of the properties of the top decay products can probe the Wtb interaction to a good accuracy. In the Warsaw basis, this vertex receives tree-level corrections from $\mathcal{Q}_{Hq}^{(3)}$ and the operators in Eq. 9.2 (except $\mathcal{Q}_{H\Box}$), that preserve the Lorentz structure of the SM interaction[99], from $\mathcal{Q}_{Hud}$ that introduces a right-handed coupling, and from $(C_{uW})_{33}$ and $(C_{dW})_{33}$, with a dipole contraction. The presence of $\mathcal{Q}_{Hq}^{(3)}$, $\mathcal{Q}_{ll}$, $\mathcal{Q}_{Hl}^{(3)}$, $\mathcal{Q}_{HD}$ or $\mathcal{Q}_{HWB}$ therefore determines a rescaling of the total decay rate, while $\mathcal{Q}_{Hud}$, $\mathcal{Q}_{uW}$, $\mathcal{Q}_{dW}$ impact the kinematic properties of the decay products. The four fermion operators $\mathcal{Q}_{qq}^{(3)}$, $\mathcal{Q}_{lq}^{(3)}$ also contribute to this decay for the hadronic and leptonic final states of the W respectively [211].

[99] The operators in Eq. 9.2 enter the vertex due to the parameter shifts determined with the choice of the input quantities. In particular, $(C_{ll})_{1221}$ and $(C_{Hl}^{(3)})_{11,22}$ are present due to $\hat{G}_F$ being chosen as an input. On the other hand C_{HD} and C_{HWB} enter when choosing the set $\{\hat{\alpha}_{ew}, \hat{m}_Z, \hat{G}_F\}$ as inputs but do not contribute if $\hat{\alpha}_{ew}$ is replaced with $\hat{m}_W$.

The total width of the top has been measured both at the Tevatron [663, 664] and at the LHC [665–667] with quite large uncertainties. More precise measurements (with a 3–4% accuracy) are available for the helicity fractions of the W boson [668, 669], that are among the most promising observables. They are predicted to the permille accuracy in the SM [670] and they are modified only by[100] $(C_{Hud})_{33}$, $(C_{uW})_{33}$ and $(C_{dW})_{33}$. Measurements of these quantities (possibly in combination with other angular observables) allow therefore to derive significant constraints on these coefficients, see e.g. [573, 671–676].

The decay $t \to Wb$ in the presence of anomalous couplings has been computed analytically up to NLO QCD [677, 678]. In particular, Ref. [678] also explored the impact of four-fermion operators.

Finally, upper limits on the observation of exotic top decays through FCNC can be used to set bounds on the flavor off-diagonal entries of some Wilson coefficients. Relevant processes in this sense include $t \to u_i\, X$, with $u_i = \{u, c\}$ and $X = \{Z, \gamma, g, h\}$ [679–683]. A full list of the operators contributing (up to NLO QCD accuracy) can be found in [678]. Contributions from class 6 and 7 operators to $t \to u_i\, Z/\gamma$ have been computed to NLO QCD [684–686]. More recently, the calculation has been extended to the $t \to u_i\, h$ channel [244, 678] and with the inclusion of parton shower effects [687]. Ref. [688] included these observables in a global analysis of flavor-changing top couplings. Operators giving flavor-changing charged currents and flavor-changing four-fermion interactions were considered in Ref. [689], that also performed a global analysis including constraints from single top production and B and Z decays (see also [211]).

In a complementary approach, Ref. [690], considered lepton flavor violating top decays $t \to q\ell^+\ell'^-$, $\ell \neq \ell'$ induced by four-fermion interactions, showing that their measurement at the LHC can give bounds comparable to those obtained from flavor physics at HERA.

(v) Other processes.

Other processes can be relevant for constraining the couplings of the top at the LHC.

For instance, $pp \to t\bar{t}t\bar{t}$ and $pp \to t\bar{t}b\bar{b}$ would give access to the (3333) flavor contraction of the operators with four quarks, that do not affect significantly any other process above [604, 691, 692].

Single top production in association with a neutral gauge boson $pp \to t\, Z/\gamma$ is also of interest, as it can give relevant constraints on FCNC top interactions, complementary to those from decays [688, 693–698].

Single top production in association with a Higgs boson $pp \to th$ can instead help setting constraints on $(C_{uH})_{33}$, as this process is sensitive to the sign (an therefore the

[100] There are in principle corrections proportional to $(C_{Hud,uW,dW})_{ii}$, $i = 1, 2$ and $(C_{eW})_{jj}$ that enter the W decay vertex. These, however, interfere with the SM amplitude proportionally to the mass of one of the W decay products, which is always negligible.

complex phase) of the top Yukawa coupling [525, 532, 699–702], and on $(C_{uH})_{3i}$, as to the presence of flavor-changing twh couplings [683, 703, 704].

10 Progress and challenges for LHC pseudo-observables, HEFT and SMEFT

The key problem of the κ formalism is that it is not a systematically improvable framework. It does not have well defined perturbative corrections, and its results cannot be combined in a predictive fashion with different data sets. In short, the κ framework is not an EFT. To go beyond the κ framework there are currently two main approaches being developed for LHC applications, the pseudo-observables (PO) approach, and the systematic development of EFT extensions to the SM. Both of these approaches face significant challenges and are underdeveloped. In the previous sections we have reviewed the extensive development of EFT extensions to the SM studying LEPI, LEPII and top physics. In this section we summarize the overall state of affairs partway through LHC RunII.

10.1 Challenges for pseudo-observables

Recently the paradigm of pseudo-observables (PO) has been reinvigorated at the LHC. Initial work in this direction was reported in Ref. [450], and further precursor studies [325, 326] have been developed into a theoretical paradigm in Refs. [329, 479, 527, 528]. These developments are a welcome advance over the κ formalism. They are theoretically grounded in formal expansions [450] around the poles of the narrow SM states, factorizing observables into gauge invariant sub-blocks (see Section 6.4). This transitions a κ approach to a firmer theoretical footing. The evolution of the κ approach to the results of Ref. [527] for inclusive Higgs decays is rather direct. A key strength of the PO approach is that it is defined as a gauge invariant decomposition around the physical poles in the process that is disconnected from an underlying assumed Lagrangian. This approach directly exploits the narrowness of the unstable massive states known to exist, and can be mapped to both the HEFT and the SMEFT in principle as the underlying Lagrangian field theory is not fixed. This approach is also based on the correct understanding of the distinction between observable S matrix elements and unphysical Lagrangian parameters (see Section 6). For this reason, a PO decomposition, at least in inclusive Higgs decays, can form a sensible bridge (at tree level) to the underlying EFT's in data reporting. PO decompositions also have some predictive power. By exploiting crossing symmetry, relations between different classes of observables can be determined in the PO approach, such as $h \to V\mathcal{F}$ and $\mathcal{F} \to h\,V$ [325, 326] or between $h \to f_1\,f_2\,f_3\,f_4$ and $f_1\,f_2 \to h\,f_3\,f_4$ [528].

The challenges to the successful development of a model independent PO program at LHC are also clear. Decomposing all observable amplitudes into a PO set is challenging at the LHC compared to LEP for a simple reason, the LEP initial state was well defined, while the LHC initial state is an overlap of various partonic processes convoluted with parton distribution functions. This challenge can be avoided if a narrow width expansion is employed to factorize up an observable, and this issue is not present for characterizing inclusive Higgs decays, where PO approaches have been well developed [527] and are clearly applicable.

A further important issue is related to the core model independent strength of the pseudo-observable approach, namely the lack of a fixed embedding in an explicit EFT Lagrangian that extends the SM. As a direct result, some of the limitations of the κ approach remain. Without mapping to a particular field theory, relations between observables (unrelated by crossing symmetry) are absent, and radiative emission is ill-defined in general. The idea to accommodate soft radiation has been to use universal radiator functions, mimicking the approach at LEP, to dress amplitudes. However, unlike at LEP there is currently no feasible proposal to check the SM-like radiator functions assumption in the LHC environment. Explicit assumptions of no effects of physics beyond the SM in radiative emissions dressing the PO have been invoked, but these are UV assumptions, not IR assumptions, and they have no known interpretation as a precise condition on UV dynamics. It is possible these challenges can be overcome to enable a precision pseudo-observable program at LHC that extends beyond characterizing inclusive Higgs decays. Assuming away these pressing issues with UV assumptions reduces the model independence of the PO approach, and should be avoided if at all possible.

10.2 Challenges for SMEFT/HEFT

Any formalism that seeks to improve upon the κ framework must address its core defects comprehensively, consistently and without invoking UV assumptions if it seeks to maintain model independence. The new framework must be able to capture the IR limit of physics beyond the SM, without assuming that the physics beyond the SM is already known, and allow an inverse map to the underlying theory if deviations are discovered at LHC, or in future facilities where LHC data is also used as legacy information. It is fortunate that EFT is constructed and defined to exactly meet these demands, that have essentially resulted from the LHC data set not indicating non-SM resonances around the EW scale. The powerful constraints of

- Lorentz invariance and the global symmetry constraints due to the Higgsing of $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1) \to \mathrm{U_{em}}(1)$ in the case of the SMEFT,
- local analytic operators extending the SM due to the assumption of a degree of decoupling $\bar{v}_T \ll \Lambda$, (see Section 3),

leads to a predictive and well defined extension of the SM, that can be systematically constrained experimentally. The EFT approach allows a well defined characterization of higher order perturbative and non-perturbative neglected effects defining the *approximate theoretical precision* in an analysis, as is a fundamental part of the EFT description. Such an approximate precision can be characterized as a theoretical error in global data analysis that is varied to represent various cases of the size of the neglected higher order terms. This error can be continually and appropriately reduced with further development of this theoretical paradigm. This approach stops one from overinterpreting the data set and being too aggressive on the constraints found in the EFT framework, considering the limited theoretical precision of the EFT description. The HEFT has less constraints due to the presence of a singlet scalar in the spectrum, but still carries powerful constraints due to a local analytic operator expansion and can accommodate assumed global symmetries as *IR assumptions* reducing its complexity.

A core challenge to SMEFT/HEFT is – which EFT should be used? The existence of two self-consistent constructions must be seriously considered. It is not appropriate to casually dismiss the HEFT construction just because the Higgs-like scalar has converged on the properties of the SM Higgs to date. The key distinction between the HEFT and the SMEFT is an IR assumption about the states in the spectrum in the presence of $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1) \to \mathrm{U_{em}}(1)$. Considering the viewpoint laid out in Section 2, on the Higgs potential being an effective parameterization of the true dynamical mechanism underlying $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1) \to \mathrm{U_{em}}(1)$, this EFT choice essentially corresponds to the assumption of one low energy parameterization of such physics being preferred over the other. $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1) \to \mathrm{U_{em}}(1)$ can occur due to weakly coupled or strongly coupled dynamics in the UV sector. The core problem with making this choice outright is that no good understanding of the low energy limit of all possible strongly interacting sectors exists, due to the difficulties in calculating non-perturbative physics. On the other hand, the fact that the HEFT and the SMEFT seem to be functionally indistinguishable on SM pole processes requiring studies of tails of distributions to seek out differences (see Section 4.4.4) indicates that a dedicated pole constraint program formulated in one of these theories can be mapped to the alternate EFT construction directly.

The efficient way to develop the constraint picture for each EFT is fairly clear and is undergoing a rapid development in the community. First, a consolidation/data mining phase that distills past LEPI/LEPII/Tevatron/LHC RunI and lower energy results into constraints on the Wilson coefficients is developed. The first priority for this effort is to map the constraints on pole processes (scattering events where $p^2 \sim m^2$ for a SM intermediate state) to the SMEFT. This restriction to single pole resonantly enhanced processes allows the narrow width factorization of the dependence on Wilson coefficients in $\mathcal{L}_6$ into those that are resonantly enhanced, and those suppressed by an additional factor of Γ/m. As the data set of such pole processes is limited, it is appropriate to invoke flavour symmetries when pursuing such bounds, at least initially.

This initial analysis phase is extended with data from the tails of distributions and low energy observables. The much larger data sets present in these cases allow the flavour symmetries to be relaxed. Tails of distributions are an important source of information on Wilson coefficients, but it cannot be avoided that when the EFT expansion is breaking down, predictivity is lost. This can be the case in the tails of distributions. It is also clear that when examining tails of distributions the choice between the HEFT and the SMEFT as a field theory approach is more pressing. Furthermore, in either EFT, the number of parameters grows dramatically as the IR effect of class 8 ψ^4 operators being further suppressed is absent. These challenges can all be overcome without invoking UV assumptions so long as an appropriate theoretical error is assigned to EFT studies in such tails of distributions.

To combine these data sets measured at disparate energies, it is required to develop the

SMEFT and HEFT to the order of one loop calculations for the most precise observables. It is also clear that the challenge of statistical estimates of constraints in multi-dimensional Wilson coefficient spaces are underdeveloped. One loop results are not available in almost every process of phenomenological interest, although it has been shown they can have a remarkable impact on standard interpretations of precise measurements such as the LEP EWPD PO [169, 262, 705]. It is unclear if a systematic one loop SMEFT and HEFT paradigm can be developed in time to have maximum impact on LHC studies.

A non-physics challenge to the HEFT and the SMEFT is the literature is manifestly conflicted. There is little agreement on EFT conventions, basis choice (the definition of a basis), the meaning of power counting, the degree of constraint on Wilson coefficients, the possibility of doing model independent EFT studies or not, and other issues. The great interest in developing EFT extensions of the SM, in response to the discovery of a Higgs-like scalar and no other resonances in the LHC data set, has resulted in a significant disarray in the rapidly advancing literature. This makes this fascinating and important area of research incomprehensible when comparing various parts of the literature, and effectively unapproachable for the next generation of students. We hope this review will have a positive and clarifying impact by removing some of these barriers and resolving some of these issues. We hope it will encourage students to work in this area.[101] There is an enormous amount of important work to do to develop and use the SMEFT and the HEFT to gain the most out of the unprecedented LHC data set, that is soon to arrive.

Acknowledgments

MT and IB thank the Villum Foundation, NBIA and the Discovery Centre at Copenhagen University for support. MT thanks the many experts in EFT that he has learned from over the years, and acknowledges informative and revealing conversations on this subject with Cliff Burgess, Ben Grinstein, Gino Isidori, Mike Luke, Aneesh Manohar, Giampiero Passarino, Maxim Pospelov and Mark Wise. MT also thanks past collaborators and members of the Higgs Cross Section Working Group, in particular André Mendes and Michael Dührssen, for discussions and motivation. IB acknowledges instructive conversations with past collaborators and thanks in particular Belén Gavela and Luca Merlo for many teachings and formative discussions. We thank Poul Damgaard, Andreas Helset, Yun Jiang, André Mendes, Subodh Patil, Duccio Pappadopulo, William Shepherd, Michael Spira, Anagha Vasudeven and Jordy de Vries for comments on the review. We welcome comments on the review and encourage feedback and corrections.

[101] To this end, in the Appendix we have also included some results on LO corrections to a number of processes with a unified and common notation, in the Warsaw basis, for the SMEFT.

A Cross sections/decay widths of selected processes in the SMEFT

In this Appendix we report the analytic expression of the cross-sections for selected EW processes in the SMEFT including leading order shifts due to $\mathcal{L}_6$. All the observables are computed at tree-level in the Warsaw basis with the operators defined as in Ref. [203]. We choose the set $\hat{\alpha}_{ew}, \hat{m}_Z, \hat{G}_F, \hat{m}_h\}$ as input parameters[102] and adopt the notation and assumptions of Ref. [413]. Many of these results were already reported in the literature in various bases. A unified presentation with common notational conventions is lacking, so we have included this summary of known results here, and simultaneously extended the known literature.

We adopt the *IR assumption* that light quark masses are neglected both in the SM predictions and in the SMEFT corrections. Unless otherwise specified, the SMEFT Lagrangian is assumed to respect an approximate $\mathrm{U}(3)^5$ flavour symmetry as a further *IR assumption*, which is only violated through insertions of the Yukawa couplings. In this scenario, the Wilson coefficients of operators containing chirality-flipping fermion currents have the[103] flavour structure

$$C_{\underset{rs}{fH}}, C_{\underset{rs}{fW}}, C_{\underset{rs}{fB}}, C_{\underset{rs}{fG}} \propto \left[Y_f^\dagger\right]_{rs},$$

$$C_{\underset{rs}{Hud}} \propto \left[Y_u\, Y_d^\dagger\right]_{rs}, \qquad C_{\underset{rs}{ledq}} \propto \left[Y_e^\dagger Y_d\right]_{rs}, \qquad C^{(1),(8)}_{\underset{rs}{quqd}} \propto \left[Y_u^\dagger Y_d^\dagger\right]_{rs}, \qquad C^{(1),(3)}_{\underset{rs}{lequ}} \propto \left[Y_e^\dagger Y_u^\dagger\right]_{rs}. \tag{A.1}$$

The terms that give very suppressed contributions to interactions involving light fermions proportional to the light quark masses are neglected here. Furthermore, the Wilson coefficients have been redefined so as to absorb the factor Λ^{-2}: we use the dimensionful parameters $C_i' = C_i/\Lambda^2$ with the prime implicitly dropped. We use the indices p, r, $s\ldots$ for the flavour space and I, J, $K\ldots$ for $\mathrm{SU_L}(2)$ consistent with the notational conventions in Section 4.1.

A.1 Core shifts in the $\{\hat{\alpha}, \hat{G}_F, \hat{m}_Z\}$ input scheme

The SM Lagrangian is written in terms of several internal parameters whose values can be determined via the measurement of a few input quantities. For the EW sector it is customary to adopt the inputs set $\{\hat{\alpha}, \hat{G}_F, \hat{m}_Z, \hat{m}_h, \cdots\}$, where $\hat{\alpha}$ is the electromagnetic structure constant extracted from Thomson scattering, $\hat{G}_F$ is the Fermi constant extracted from muon decay, and $\hat{m}_Z$, $\hat{m}_h$ are respectively the measured masses of the Z and Higgs bosons.[104] The numerical values of some of the inputs are reported in Table 4. The other relevant Lagrangian parameters

[102] The subscript in $\hat{\alpha}_{ew}$ will be dropped in the following.

[103] At lowest order in the linear MFV expansion [706–711].

[104] Arguably a transition to the $\{\hat{m}_W, \hat{G}_F, \hat{m}_Z, \hat{m}_h, \cdots\}$ scheme is favoured for several reasons as discussed in Ref. [415]. But we do not fight the tide of historical convention here.

Parameter	Input Value	Ref.
$\hat{\alpha}$	$1/137.035999139(31)$	[375, 712]
$\hat{G}_F$	$1.1663787(6)\times 10^{-5}$ GeV^{-2}	[375, 712]
$\hat{m}_Z$	91.1876 ± 0.0021 GeV	[375, 471, 712]
$\hat{m}_h$	125.09 ± 0.22 GeV	[375]

Table 4. Current best fit values of the core input parameters.

are fixed by the following definitions:

$$\begin{aligned}
s_{\hat{\theta}}^2 &= \frac{1}{2}\left[1-\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}\right], & \hat{e} &= \sqrt{4\pi\hat{\alpha}}, \\
\hat{g}_1 &= \frac{\hat{e}}{c_{\hat{\theta}}}, & \hat{g}_2 &= \frac{\hat{e}}{s_{\hat{\theta}}}, \\
\hat{v}_T &= \frac{1}{2^{1/4}\sqrt{\hat{G}_F}}, & \hat{m}_W^2 &= \hat{m}_Z^2 c_{\hat{\theta}}^2.
\end{aligned} \tag{A.2}$$

When applied in the SMEFT, this procedure introduces a mismatch between the quantities determined from the input measurements and the parameters defined in the canonically normalized Lagrangian consistent with an on-shell EFT construction. Denoting the former quantities with a hat and the latter with a bar, a generic parameter κ receives a shift from its SM value given by

$$\delta\kappa = \bar{\kappa} - \hat{\kappa}\,. \tag{A.3}$$

In the SM limit ($C_i \to 0$) hatted and bar quantities coincide. It is convenient to define the quantities

$$\delta G_F = \frac{1}{\sqrt{2}\,\hat{G}_F}\left(\sqrt{2}\,C_{Hl}^{(3)} - \frac{C_{ll}}{\sqrt{2}}\right), \tag{A.4}$$

$$\delta m_Z^2 = \frac{1}{2\sqrt{2}}\frac{\hat{m}_Z^2}{\hat{G}_F}C_{HD} + \frac{2^{1/4}\sqrt{\pi\hat{\alpha}}\,\hat{m}_Z}{\hat{G}_F^{3/2}}C_{HWB}, \tag{A.5}$$

$$\delta m_h^2 = \frac{\hat{m}_h^2}{\sqrt{2}\hat{G}_F}\left(-\frac{3C_H}{2\lambda} + 2C_{H\Box} - \frac{C_{HD}}{2}\right), \tag{A.6}$$

$$\delta m_W^2 = \hat{m}_W^2\left(\sqrt{2}\delta G_F + 2\frac{\delta g_2}{\hat{g}_2}\right), \tag{A.7}$$

$$= -\frac{s_{2\hat{\theta}}\bar{v}_T^2}{4c_{2\hat{\theta}}}\left(\frac{c_{\hat{\theta}}}{s_{\hat{\theta}}}C_{HD} + \frac{s_{\hat{\theta}}}{c_{\hat{\theta}}}(4C_{Hl}^{(3)} - 2C_{ll}) + 4C_{HWB}\right), \tag{A.8}$$

Using this notation, related results are[105]

$$\delta v_T^2 = \bar{v}_T^2 - \hat{v}_T^2 = \frac{\delta G_F}{\hat{G}_F} \tag{A.9}$$

$$\begin{aligned}\delta g_1 = \bar{g}_1 - \hat{g}_1 &= \frac{\hat{g}_1}{2c_{2\hat{\theta}}}\left[s_{\hat{\theta}}^2\left(\sqrt{2}\delta G_F + \frac{\delta m_Z^2}{\hat{m}_Z^2}\right) + c_{\hat{\theta}}^2\, s_{2\hat{\theta}}\bar{v}_T^2 C_{HWB}\right] \\ &= \frac{\hat{g}_1}{c_{2\hat{\theta}}}\frac{\bar{v}_T^2}{4}\left[s_{\hat{\theta}}^2\left(C_{HD} + 4C_{Hl}^{(3)} - 2C_{ll}\right) + 2s_{2\hat{\theta}}C_{HWB}\right],\end{aligned} \tag{A.10}$$

$$\begin{aligned}\delta g_2 = \bar{g}_2 - \hat{g}_2 &= -\frac{\hat{g}_2}{2c_{2\hat{\theta}}}\left[c_{\hat{\theta}}\left(\sqrt{2}\delta G_F + \frac{\delta m_Z^2}{\hat{m}_Z^2}\right) + s_{\hat{\theta}}^2\, s_{2\hat{\theta}}\bar{v}_T^2 C_{HWB}\right] \\ &= -\frac{\hat{g}_2}{c_{2\hat{\theta}}}\frac{\bar{v}_T^2}{4}\left[c_{\hat{\theta}}^2\left(C_{HD} + 4C_{Hl}^{(3)} - 2C_{ll}\right) + 2s_{2\hat{\theta}}C_{HWB}\right]\end{aligned} \tag{A.11}$$

$$\begin{aligned}\delta s_\theta^2 = s_{\bar{\theta}}^2 - s_{\hat{\theta}}^2 &= 2c_{\hat{\theta}}^2 s_{\hat{\theta}}^2\left(\frac{\delta g_1}{\hat{g}_1} - \frac{\delta g_2}{\hat{g}_2}\right) + \bar{v}_T^2\frac{s_{2\hat{\theta}}c_{2\hat{\theta}}}{2}C_{HWB} \\ &= \frac{s_{2\hat{\theta}}}{8c_{2\hat{\theta}}\sqrt{2}\hat{G}_F}\left[s_{2\hat{\theta}}\left(C_{HD} + 4C_{Hl}^{(3)} - 2C_{ll}\right) + 4C_{HWB}\right]\end{aligned} \tag{A.12}$$

Couplings of the gauge bosons to fermions

The photon couplings do not receive corrections in this input parameter set, due to the fact that α is an input and, at the same time, the $U(1)_{\rm em}$ gauge symmetry is preserved in the SM. The relevant Lagrangian term is

$$\mathcal{L}_{A,\rm eff} = -2\sqrt{\pi\hat{\alpha}}\, Q_\psi\, J_\mu^{\psi,em}\, A^\mu\,, \tag{A.13}$$

where $J_\nu^{\psi,em}$ is the electromagnetic current with the fermion $\psi = \{\ell, u, d\}$. On the other hand, the Z and W couplings to fermions are modified. Using the notation of Refs [413, 414], the former can be parameterized as

$$\mathcal{L}_{Z,\rm eff} = \hat{g}_Z\,\left(J_\mu^{Z\ell}Z^\mu + J_\mu^{Z\nu}Z^\mu + J_\mu^{Zu}Z^\mu + J_\mu^{Zd}Z^\mu\right), \tag{A.14}$$

where $\hat{g}_Z = -\hat{g}_2/c_{\hat{\theta}} = -2\,2^{1/4}\sqrt{\hat{G}_F}\,\hat{m}_Z$, $(J_\mu^{Z\psi})^{pr} = \bar{\psi}_p\,\gamma_\mu\left[(\bar{g}_V^\psi)_{pr} - (\bar{g}_A^\psi)_{pr}\,\gamma_5\right]\psi_r$ for $\psi = \{u, d, \ell, \nu\}$. The couplings' normalization is such that

$$g_V^{\psi,SM} = T_3/2 - Q_\psi s_{\hat{\theta}}^2, \qquad g_A^{\psi,SM} = T_3/2$$

with $T_3 = \pm 1/2$ and $Q_\psi = \{-1, 2/3, -1/3\}$ for $\psi = \{\ell, u, d\}$. The couplings deviate from the SM expressions as

$$\delta(g_{V,A}^\psi)_{pr} = (\bar{g}_{V,A}^\psi)_{pr} - (g_{V,A}^{\psi,SM})_{pr}, \tag{A.15}$$

[105] To define the SMEFT in R^ξ gauge the approach used here to define the diagonalization of the mass eigenstate fields is advantageous, see Refs. [539, 713].

with $F[C_1, C_2, C_3 + \cdots]_{pr} = (\underset{pr}{C_1} + \underset{pr}{C_2} + \underset{pr}{C_3} + \cdots)/(4\sqrt{2}\hat{G}_F)$

$$\delta(g_V^\ell)_{pr} = \delta\bar{g}_Z\,(g_V^{\ell,SM})_{pr} - F[C_{He}, C_{H\ell}^{(1)}, C_{H\ell}^{(3)}]_{pr} + \delta s_\theta^2, \tag{A.16}$$

$$\delta(g_A^\ell)_{pr} = \delta\bar{g}_Z\,(g_A^{\ell,SM})_{pr} + F[C_{He}, -C_{H\ell}^{(1)}, -C_{H\ell}^{(3)}]_{pr}, \tag{A.17}$$

$$\delta(g_{A/V}^\nu)_{pr} = \delta\bar{g}_Z\,(g_{A/V}^{\nu,SM})_{pr} - F[C_{H\ell}^{(1)}, -C_{H\ell}^{(3)}]_{pr}, \tag{A.18}$$

$$\delta(g_V^u)_{pr} = \delta\bar{g}_Z\,(g_V^{u,SM})_{pr} + F[-C_{Hq}, C_{Hq}^{(3)}, -C_{Hu}]_{pr} - \frac{2}{3}\delta s_{\hat{\theta}}^2, \tag{A.19}$$

$$\delta(g_A^u)_{pr} = \delta\bar{g}_Z\,(g_A^{u,SM})_{pr} - F[C_{Hq}^{(1)}, -C_{Hq}^{(3)}, -C_{Hu}]_{pr}, \tag{A.20}$$

$$\delta(g_V^d)_{pr} = \delta\bar{g}_Z\,(g_V^{d,SM})_{pr} - F[C_{Hq}^{(1)}, C_{Hq}^{(3)}, C_{Hd}]_{pr} + \frac{1}{3}\delta s_{\hat{\theta}}^2, \tag{A.21}$$

$$\delta(g_A^d)_{pr} = \delta\bar{g}_Z\,(g_A^{d,SM})_{pr} + F[-C_{Hq}^{(1)}, -C_{Hq}^{(3)}, C_{Hd}]_{pr}, \tag{A.22}$$

and

$$\delta\bar{g}_Z = -\frac{\delta G_F}{\sqrt{2}} - \frac{\delta m_Z^2}{2\hat{m}_Z^2} + \frac{s_{\hat{\theta}}\,c_{\hat{\theta}}}{\sqrt{2}\hat{G}_F}\,C_{HWB}. \tag{A.23}$$

For the charged currents

$$\mathcal{L}_{W,eff} = -\frac{\sqrt{2\,\pi\,\hat{\alpha}}}{s_{\hat{\theta}}}\left[(J_\mu^{W_\pm,\ell})_{pr}W_\pm^\mu + (J_\mu^{W_\pm,q})_{pr}W_\pm^\mu\right], \tag{A.24}$$

with

$$(J_\mu^{W_+,\ell})_{pr} = \bar{\nu}_p\,\gamma^\mu\,\left(\bar{g}_V^{W_+,\ell} - \bar{g}_A^{W_+,\ell}\gamma_5\right)\,\ell_r, \tag{A.25}$$

$$(J_\mu^{W_-,\ell})_{pr} = \bar{\ell}_p\,\gamma^\mu\,\left(\bar{g}_V^{W_-,\ell} - \bar{g}_A^{W_-,\ell}\gamma_5\right)\,\nu_r, \tag{A.26}$$

and analogously for quarks. In the SM

$$(\bar{g}_V^{W_\pm,\ell})_{pr}^{SM} = (\bar{g}_A^{W_\pm,\ell})_{pr}^{SM} = \frac{(U_{PMNS}^\dagger)_{pr}}{2}, \qquad (\bar{g}_V^{W_\pm,q})_{pr}^{SM} = (\bar{g}_A^{W_\pm,q})_{pr}^{SM} = \frac{V_{pr}}{2}, \tag{A.27}$$

where V is the CKM matrix. In the SMEFT $\bar{g}_{V/A}^{W_\pm,\psi} = (\bar{g}_{V/A}^{W_\pm,\psi})^{SM} + \delta\bar{g}_{V/A}^{W_\pm,\psi}$ where, for the flavour diagonal component:

$$\delta(g_V^{W_\pm,\ell})_{rr} = \delta(g_A^{W_\pm,\ell})_{rr} = \frac{1}{2\sqrt{2}\hat{G}_F}\left(\underset{rr}{C_{H\ell}^{(3)}} + \frac{1}{2}\frac{c_{\hat{\theta}}}{s_{\hat{\theta}}}\,C_{HWB}\right) - \frac{1}{4}\frac{\delta s_\theta^2}{s_{\hat{\theta}}^2}, \tag{A.28}$$

$$\delta(g_V^{W_\pm,q})_{rr} = \delta(g_A^{W_\pm,q})_{rr} = \frac{1}{2\sqrt{2}\hat{G}_F}\left(\underset{rr}{C_{Hq}^{(3)}} + \frac{1}{2}\frac{c_{\hat{\theta}}}{s_{\hat{\theta}}}\,C_{HWB}\right) - \frac{1}{4}\frac{\delta s_\theta^2}{s_{\hat{\theta}}^2}. \tag{A.29}$$

the off diagonal components are the obvious generalization of this result.

Triple Gauge Couplings

Going from the SM to the SMEFT, the TGC couplings get redefined by a subset of $\mathcal{L}_6$ operators, so that $\bar{g}_1^V = g_1^V + \delta g_1^V$, $\bar{\kappa}_V = \kappa_V + \delta\kappa_V$, $\bar{\lambda}_V = \lambda_V + \delta\lambda_V$ with

$$\delta g_1^A = 0, \qquad \delta g_1^Z = \frac{1}{2\sqrt{2}\hat{G}_F}\left(\frac{s_{\hat{\theta}}}{c_{\hat{\theta}}} + \frac{c_{\hat{\theta}}}{s_{\hat{\theta}}}\right) C_{HWB} - \frac{1}{2}\delta s_{\theta}^2 \left(\frac{1}{s_{\hat{\theta}}^2} + \frac{1}{c_{\hat{\theta}}^2}\right), \tag{A.30}$$

$$\delta\kappa_A = \frac{1}{\sqrt{2}\hat{G}_F}\frac{c_{\hat{\theta}}}{s_{\hat{\theta}}} C_{HWB}, \quad \delta\kappa_Z = \frac{1}{2\sqrt{2}\hat{G}_F}\left(-\frac{s_{\hat{\theta}}}{c_{\hat{\theta}}} + \frac{c_{\hat{\theta}}}{s_{\hat{\theta}}}\right) C_{HWB} - \frac{1}{2}\delta s_{\theta}^2 \left(\frac{1}{s_{\hat{\theta}}^2} + \frac{1}{c_{\hat{\theta}}^2}\right), \tag{A.31}$$

$$\delta\lambda_A = 6 s_{\hat{\theta}} \frac{\hat{m}_W^2}{g_{AWW}} C_W, \qquad \delta\lambda_Z = 6 c_{\hat{\theta}} \frac{\hat{m}_W^2}{g_{ZWW}} C_W. \tag{A.32}$$

Notice that three relations between parameters in this input scheme (at the level of $\mathcal{L}_6$ [212]) hold in the SMEFT: $\delta\kappa_Z = \delta g_1^Z - t_{\hat{\theta}}^2 \delta\kappa_A$, $\delta\lambda_A = \delta\lambda_Z$ and $\delta g_1^A = 0$.

Fermion masses and Yukawa couplings

If the assumption of massless fermions is relaxed, the measured masses of quarks and leptons can be incorporated in the set of input parameters and they allow to determine the Yukawa couplings through the definition

$$\hat{Y}_f = 2^{3/4} \hat{m}_f \sqrt{\hat{G}_F}\,. \tag{A.33}$$

In the SM the coupling of the Higgs boson to fermions is then $g_{h\bar{f}f}^{SM} = \hat{Y}_f/\sqrt{2}$. In the SMEFT it is shifted as $\bar{g}_{h\bar{f}f} = g_{h\bar{f}f}^{SM} + \delta g_{h\bar{f}f}$ where [185]

$$\delta g_{h\bar{f}f} = \frac{\hat{Y}_f}{\sqrt{2}}\left[\bar{v}_T^2\left(C_{H\Box} - \frac{C_{HD}}{4}\right) - \frac{\delta G_F}{\sqrt{2}}\right] - \frac{\bar{v}_T^2}{\sqrt{2}} C_{fH}^*\,. \tag{A.34}$$

A.2 Generic $2 \to 2$ scattering processes via gauge boson exchange

Scattering $\ell^+\ell^- \to \bar{f}f\,, \quad f = \{\ell' \neq \ell, u, c, b, d, s\}$

The general s channel differential cross section $d\sigma(\ell^+\ell^- \to f\,\bar{f})/dc_\theta$, valid on and off resonance scattering, has been computed in the SMEFT in Ref. [413]. The result includes the contributions from Z and γ exchange, the effect of ψ^4 operators and the interference of all of these terms, up to leading order in the interference of the ψ^4 operators with the SM amplitude. Initial and final state radiation (including possible α_s corrections to final state fermions) have been neglected, together with fermion masses. Consistently with the other results reported here, a U(3)^5 flavour symmetry is assumed, which allows to neglect interference effects with operators of the form $LRRL$, $LRLR$, that are proportional to SM Yukawas. Finally, the initial

e^+, e^- are taken to be unpolarized. The final expression, in Feynman gauge, reads [106]

$$
\begin{aligned}
\frac{1}{N_c}\frac{d\sigma}{dc_\theta} = & \frac{\hat{G}_F^2 \hat{m}_Z^4}{\pi}\,\bar{\chi}(s)\left[\left(|\bar{g}_V^\ell|^2+|\bar{g}_A^\ell|^2\right)\,\left(|\bar{g}_V^f|^2+|\bar{g}_A^f|^2\right)\left(1+c_\theta^2\right)-8\,\mathrm{Re}\left[\bar{g}_A^\ell \bar{g}_V^{\ell,\star}\right]\,\mathrm{Re}\left[\bar{g}_A^f \bar{g}_V^{f,\star}\right]c_\theta\right],\\
& + \frac{|\hat{\alpha}|^2\,|Q_\ell|^2\,|Q_f|^2\,\pi}{2\,s}\left(1+c_\theta^2\right)+\frac{\hat{G}_F \hat{m}_Z^2 Q_\ell\,Q_f}{\sqrt{2}}\left[\alpha^\star\frac{\bar{g}_V^\ell\,\bar{g}_V^f\left(1+c_\theta^2\right)+2\,c_\theta\,\bar{g}_A^\ell\,\bar{g}_A^f}{s-\bar{m}_Z^2+i\,\bar{w}(s)}+\mathrm{h.c.}\right],\\
& + \frac{Q_\ell\,Q_f}{32}\left[\alpha^\star\,C^{\ell,f}_{LL,RR}\,(1+c_\theta)^2+\mathrm{h.c.}\right]+\frac{Q_\ell\,Q_f}{32}\left[\alpha^\star\,C^{\ell,f}_{LR}\,(1-c_\theta)^2+\mathrm{h.c.}\right],\\
& + \left(\frac{\hat{G}_F \hat{m}_Z^2}{16\sqrt{2}\,\pi}\right)\left[\left(\frac{s}{s-\bar{m}_Z^2+i\,\bar{w}(s)}\right)C^{\ell,f,\star}_{LL,RR,LR}(\bar{g}_V^\ell\pm\bar{g}_A^\ell)(\bar{g}_V^f\pm\bar{g}_A^f)\left(1+c_\theta^2\right)+\mathrm{h.c.}\right],\\
& + \left(\frac{\hat{G}_F \hat{m}_Z^2}{16\sqrt{2}\,\pi}\right)\left[\left(\frac{s}{s-\bar{m}_Z^2+i\,\bar{w}(s)}\right)C^{\ell,f,\star}_{LL,RR,LR}\,(\bar{g}_A^\ell\pm\bar{g}_V^\ell)(\bar{g}_A^f\pm\bar{g}_V^f)\,2\,c_\theta+\mathrm{h.c.}\right]. \qquad \text{(A.35)}
\end{aligned}
$$

where in the last two lines the couplings combinations with signs $(++,\ --,\ +-)$ are associated to the LL, RR and LR operators respectively. We have also defined

$$
\bar{\chi}(s) = \frac{s}{(s-\bar{m}_Z^2)^2+|\bar{w}(s)|^2}\,, \qquad \text{(A.36)}
$$

where $\bar{w}(s)$ represents the Breit-Wigner distribution [714], that can be either expressed as an s dependent width ($\bar{w}(s) = s\,\bar{\Gamma}_Z/\bar{m}_Z$, which is the approach used at LEP) or alternatively using directly the real part of the complex pole $\bar{w}(s) = \bar{\Gamma}_Z\,\bar{m}_Z$. The parameter c_θ is the cosine of the angle between the incoming ℓ^- and the outgoing $\bar{f}$, and $s = (p_{\ell^+}+p_{\ell^-})^2$. N_C is the dimension of the SU(3) group of the produced fermion f.

Flavour indicies on the ψ^4 operator Wilson coefficients and the effective gauge couplings have been suppressed. Reintroducing them, one has $C^\star \to C^\star_{\ell\ell ff}, C^\star_{\ell ff\ell}, C^\star_{f\ell\ell f}$ for $C^\star_{LL,RR}$. For the LR operators $C^\star \to C^\star_{\ell\ell ff}$ is as in the previous chirality cases, while the cases $C^\star_{\ell ff\ell}, C^\star_{f\ell\ell f}$ vanish.

Expanding linearly in the Wilson coefficient, the differential cross section is shifted com-

[106] Here we correct a factor of $1/\pi$ in the first line compared to Ref. [413], which has a typo.

pared to the SM prediction by[107]

$$
\begin{aligned}
&\frac{1}{N_c}\delta\left(\frac{d\sigma}{dc_\theta}\right) = \\
&\frac{\hat{G}_F^2\hat{m}_Z^4}{\pi}\chi(s)\left[2\mathrm{Re}\left[(g_V^{\ell,SM})^*\delta g_V^\ell + (g_A^{\ell,SM})^*\delta g_A^\ell\right]\left(|g_V^{f,SM}|^2+|g_A^{f,SM}|^2\right)\left(1+c_\theta^2\right)+(\ell\leftrightarrow f)\right], \\
&-\frac{8\hat{G}_F^2\hat{m}_Z^4}{\pi}\chi(s)\left[\mathrm{Re}\left[\delta g_A^\ell(g_V^{\ell,SM})^* + (g_A^{\ell,SM})^*\delta g_V^{\ell,\star}\right]\mathrm{Re}\left[g_A^{f,SM}(g_V^{f,SM})^*\right]c_\theta+(\ell\leftrightarrow f)\right], \\
&+\frac{\hat{G}_F^2\hat{m}_Z^4}{\pi}\delta\chi(s)\Bigg[\left(|g_V^{\ell,SM}|^2+|g_A^{\ell,SM}|^2\right)\left(|g_V^{f,SM}|^2+|g_A^{f,SM}|^2\right)\left(1+c_\theta^2\right) \\
&\qquad\qquad -8\mathrm{Re}\left[g_A^{\ell,SM}(g_V^{\ell,SM})^*\right]\mathrm{Re}\left[g_A^{f,SM}(g_V^{f,SM})^*\right]c_\theta\Bigg], \\
&+\frac{\hat{G}_F\hat{m}_Z^2 Q_\ell Q_f}{\sqrt{2}}\left[\alpha^\star\chi_2(s)\frac{(\delta g_V^\ell g_V^{f,SM}+g_V^{\ell,SM}\delta g_V^f)\left(1+c_\theta^2\right)+2c_\theta\left(\delta g_A^\ell g_A^{f,SM}+g_A^{\ell,SM}\delta g_A^f\right)}{s}+\mathrm{h.c.}\right], \\
&+\frac{\hat{G}_F\hat{m}_Z^2 Q_\ell Q_f}{\sqrt{2}}\left[\alpha^\star\delta\chi_2(s)\frac{g_V^{\ell,SM}g_V^{f,SM}\left(1+c_\theta^2\right)+2c_\theta g_A^{\ell,SM}g_A^{f,SM}}{s}+\mathrm{h.c.}\right], \\
&+\frac{Q_\ell Q_f}{32}\left[\alpha^\star C_{LL,RR}^{\ell,f}(1+c_\theta)^2+\mathrm{h.c.}\right]+\frac{Q_\ell Q_f}{32}\left[\alpha^\star C_{LR}^{\ell,f}(1-c_\theta)^2+\mathrm{h.c.}\right], \\
&+\left(\frac{\hat{G}_F\hat{m}_Z^2}{16\sqrt{2}\pi}\right)\left[\chi_2(s)C_{LL,RR,LR}^{\ell,f,\star}(g_V^{\ell,SM}\pm g_A^{\ell,SM})(g_V^{f,SM}\pm g_A^{f,SM})\left(1+c_\theta^2\right)+\mathrm{h.c.}\right], \\
&+\left(\frac{\hat{G}_F\hat{m}_Z^2}{16\sqrt{2}\pi}\right)\left[\chi_2(s)C_{LL,RR,LR}^{\ell,f,\star}(g_A^{\ell,SM}\pm g_V^{\ell,SM})(g_A^{f,SM}\pm g_V^{f,SM})2c_\theta+\mathrm{h.c.}\right].
\end{aligned}
$$

Here

$$\chi(s)=|\Xi(s)|^2/s, \qquad \delta\chi(s)=\frac{1}{s}\left[\Xi(s)\,\delta\Xi^\star(s)+\delta\Xi(s)\,\Xi^\star(s)\right], \tag{A.37}$$

$$\chi_2(s)=\Xi(s), \qquad \delta\chi_2(s)=\delta\,\Xi(s), \tag{A.38}$$

with

$$\Xi(s)=\frac{s}{s-\hat{m}_Z^2+i(w(s))_{SM}}, \tag{A.39}$$

$$\delta\Xi(s)=\frac{s}{[s-\hat{m}_Z^2+i(w(s))_{SM}]^2}\left[-i\delta w(s)\right]. \tag{A.40}$$

The shift in the Breit-Wigner distribution depends on the specific form assumed for $w(s)$:

$$\bar{w}(s)=s\frac{\bar{\Gamma}_Z}{\bar{m}_Z} \qquad \to \qquad \delta w(s)=s\frac{\delta\Gamma_Z}{\hat{m}_Z}, \tag{A.41}$$

$$\bar{w}(s)=\bar{\Gamma}_Z\bar{m}_Z \qquad \to \qquad \delta w(s)=\hat{m}_Z\,\delta\Gamma_Z. \tag{A.42}$$

The expression for the Z width correction $\delta\Gamma_Z$ is given in Eqn. A.54.

[107] Here we correct a $1/\pi$ compared to the result in the first three lines that is derived from Ref. [413].

Scattering $\bar{f}\,f \to \bar{f}\,f$

Here we consider the particular case where the initial and final states fermion f are identical. Two kinematic channels, s and t, are present in this process. Adopting the same set of approximations and assumptions as above, the differential cross section for Bhabha scattering ($e^+\,e^- \to e^+\,e^-$) in the SMEFT is given by [413]

$$
\begin{aligned}
\frac{d\sigma}{dc_\theta} = \frac{2\,\hat{G}_F^2 \hat{m}_Z^4}{\pi s} & \left[(|\bar{g}_V^\ell|^2 + |\bar{g}_A^\ell|^2)^2 \left(\frac{u^2+s^2}{(t-\bar{m}_Z^2)^2} + \frac{\bar{\chi}(s)}{s}\left(u^2+t^2\right) + 2\,\bar{\chi}(s)\frac{u^2(1-\bar{m}_Z^2/s)}{t-\bar{m}_Z^2} \right), \right. \\
& \left. -4\,\mathrm{Re}\left[\bar{g}_V^{\ell *}\bar{g}_A^\ell\right]^2 \left(\frac{s^2-u^2}{(t-\bar{m}_Z^2)^2} + \frac{\bar{\chi}(s)}{s}\left(u^2-t^2\right) - 2\,\bar{\chi}(s)\frac{u^2(1-\bar{m}_Z^2/s)}{t-\bar{m}_Z^2} \right) \right], \\
& + \frac{\sqrt{2}\,\hat{G}_F \hat{m}_Z^2}{s}\left[\hat{\alpha}^* \frac{(\bar{g}_V^\ell)^2(u^2+t^2) + (\bar{g}_A^\ell)^2(u^2-t^2)}{s\left(s-\bar{m}_Z^2+i\bar{w}(s)\right)} + \hat{\alpha}^* \frac{(\bar{g}_V^\ell)^2(u^2+s^2) + (\bar{g}_A^\ell)^2(u^2-s^2)}{t\left(t-\bar{m}_Z^2\right)} + h.c. \right], \\
& + \frac{\sqrt{2}\,\hat{G}_F \hat{m}_Z^2\,u^2}{s}\left[\frac{\hat{\alpha}^*}{t}\frac{(\bar{g}_V^\ell)^2+(\bar{g}_A^\ell)^2}{\left(s-\bar{m}_Z^2+i\bar{w}(s)\right)} + \frac{\hat{\alpha}}{s}\frac{(\bar{g}_V^{\ell,\star})^2+(\bar{g}_A^{\ell,\star})^2}{\left(t-\bar{m}_Z^2\right)} \right], \\
& + \frac{2\,\pi\,\hat{\alpha}^2}{s}\left[\frac{u^2+s^2}{t^2} + \frac{u^2+t^2}{s^2} + \frac{2u^2}{ts} \right] + \frac{\hat{\alpha}}{4s}\left[2\left(\frac{u^2}{s}+\frac{u^2}{t}\right) C^\star_{LL,RR} + \left(\frac{t^2}{s}+\frac{s^2}{t}\right) C^\star_{LR} + h.c \right], \\
& + \frac{\hat{G}_F \hat{m}_Z^2}{4\sqrt{2}\pi s}\left[\frac{4u^2\left(\bar{g}_A^\ell \pm \bar{g}_V^\ell\right)^2 C^\star_{LL,RR} + 2t^2\left((\bar{g}_V^\ell)^2 - (\bar{g}_A^\ell)^2\right) C^\star_{LR}}{s-\bar{m}_Z^2+iw(s)} + h.c \right], \\
& + \frac{\hat{G}_F \hat{m}_Z^2}{4\sqrt{2}\pi s}\left[\frac{4u^2\left(\bar{g}_A^\ell \pm \bar{g}_V^\ell\right)^2 C^\star_{LL,RR} + 2s^2\left((\bar{g}_V^\ell)^2 - (\bar{g}_A^\ell)^2\right) C^\star_{LR}}{t-\bar{m}_Z^2} + h.c \right].
\end{aligned}
$$

The shift from the SM result is [414]

$$
\begin{aligned}
\delta\left(\frac{d\sigma_{e^+e^-\to e^+e^-}}{d\cos(\theta)}\right) = & \frac{2\,\hat{G}_F^2}{\pi s}\left[\frac{u^2\,F_3^+ + s^2\,F_3^-}{P(t)^2} + \frac{u^2\,F_3^- + t^2\,F_3^+}{P(s)^2} + \frac{2\,u^2\,F_3^+}{P(s)P(t)} \right], \\
& + \frac{2\sqrt{2}\hat{G}_F\hat{\alpha}}{s}\left[\frac{u^2F_7^+ + t^2F_7^-}{sP(s)} + \frac{u^2F_7^+ + s^2F_7^-}{tP(t)} + \frac{u^2F_7^+}{tP(s)} + \frac{u^2F_7^+}{sP(t)} \right], \\
& + \frac{2\hat{G}_F}{\pi s}\left[F_4 u^2\left(\frac{1}{P(s)}+\frac{1}{P(t)}\right) + F_5\left(\frac{t^2}{P(s)}+\frac{s^2}{P(t)}\right) \right], \\
& + \frac{\hat{\alpha}}{2s}\left[2\left(\frac{u^2}{s}+\frac{u^2}{t}\right) C_{LL/RR} + \left(\frac{t^2}{s}+\frac{s^2}{t}\right) C_{LR} \right].
\end{aligned}
\tag{A.43}
$$

where $P(x) = x/\hat{m}_Z^2 - 1$. The factors F_i are defined as follows:

$$
\begin{aligned}
F_3^\pm &= 4(N_{VA}^\ell)^3 G_{VA}^\ell \delta G_{VAAV}^\ell \pm 8(N_{VA}^\ell)^2 \delta G_{VVAA}^\ell, & F_4 &= \frac{(g_V^{\ell,SM} \pm g_A^{\ell,SM})^2}{\sqrt{2}} C_{LL/RR}, \\
F_5 &= \frac{(g_V^{\ell,SM})^2 - (g_A^{\ell,SM})^2}{2\sqrt{2}} C_{LR}, & F_7^\pm &= 2g_V^{\ell,SM}\delta g_V^\ell \pm 2g_A^{\ell,SM}\delta g_A^\ell,
\end{aligned}
\tag{A.44}
$$

where

$$N_{VA}^{\ell} = g_V^{\ell,SM} g_A^{\ell,SM}, \qquad G_{VA}^{\ell} = \frac{(g_V^{\ell,SM})^2 + (g_A^{\ell,SM})^2}{(g_V^{\ell,SM} g_A^{\ell,SM})^2}, \qquad \delta G_{ijkl}^{\ell} = \frac{\delta g_i^{\ell}}{g_j^{\ell,SM}} + \frac{\delta g_k^{\ell}}{g_l^{\ell,SM}}. \tag{A.45}$$

A.3 Electroweak observables near the Z pole

Analytic expressions for the electroweak precision observables in the SMEFT can be extracted from the general parameterization of $2 \to 2$ scattering given in the previous section. This section summarizes the results, using the notation of Ref [413].

Partial and total Z widths

In the SMEFT, at tree level, one has

$$\bar{\Gamma}\left(Z \to f\bar{f}\right) = \frac{\sqrt{2}\,\hat{G}_F \hat{m}_Z^3\, N_c}{3\pi}\left(|\bar{g}_V^f|^2 + |\bar{g}_A^f|^2\right), \tag{A.46}$$

$$\bar{\Gamma}\left(Z \to \text{Had}\right) = 2\,\bar{\Gamma}\left(Z \to u\bar{u}\right) + 3\,\bar{\Gamma}\left(Z \to d\bar{d}\right). \tag{A.47}$$

These expressions can be written down separating the SM contribution from the SMEFT correction:

$$\bar{\Gamma}\left(Z \to f\bar{f}\right) = \Gamma_{Z\to f\bar{f}}^{SM} + \delta\Gamma_{Z\to f\bar{f}} \tag{A.48}$$

for each fermion f. The same kind of relation holds for the total width $\bar{\Gamma}_Z$. Specifically, the shifts in each channel read:

$$\delta\Gamma_{Z\to\ell\bar{\ell}} = \frac{\sqrt{2}\hat{G}_F \hat{m}_Z^3}{6\pi}\left[-\delta g_A^{\ell} + \left(-1 + 4s_{\hat{\theta}}^2\right)\delta g_V^{\ell}\right] + \delta\Gamma_{Z\to\bar{\ell}\ell,\psi^4}, \tag{A.49}$$

$$\delta\Gamma_{Z\to\nu\bar{\nu}} = \frac{\sqrt{2}\hat{G}_F \hat{m}_Z^3}{6\pi}\left[\delta g_A^{\nu} + \delta g_V^{\nu}\right] + \delta\Gamma_{Z\to\nu\bar{\nu},\psi^4}, \tag{A.50}$$

$$\delta\Gamma_{Z\to\text{Had}} = 2\delta\Gamma_{Z\bar{u}u} + 3\delta\Gamma_{Z\bar{d}d}, \tag{A.51}$$

$$= \frac{\sqrt{2}\hat{G}_F \hat{m}_Z^3}{\pi}\left[\delta g_A^{u} - \frac{1}{3}\left(-3 + 8s_{\hat{\theta}}^2\right)\delta g_V^{u} - \frac{3}{2}\delta g_A^{d} + \frac{1}{2}\left(-3 + 4s_{\hat{\theta}}^2\right)\delta g_V^{d}\right] + \delta\Gamma_{Z\to\text{Had},\psi^4}, \tag{A.52}$$

$$\delta\Gamma_Z = 3\delta\Gamma_{Z\to\ell\bar{\ell}} + 3\delta\Gamma_{Z\to\nu\bar{\nu}} + \delta\Gamma_{Z\to\text{Had}}, \tag{A.53}$$

$$\begin{aligned} =&\frac{\sqrt{2}\hat{G}_F \hat{m}_Z^3}{2\pi}\Big[\delta g_A^{\nu} + \delta g_V^{\nu} - \delta g_A^{\ell} + \left(-1 + 4s_{\hat{\theta}}^2\right)\delta g_V^{\ell} \\ &+ 2\delta g_A^{u} - \frac{2}{3}\left(-3 + 8s_{\hat{\theta}}^2\right)\delta g_V^{u} - 3\delta g_A^{d} + \left(-3 + 4s_{\hat{\theta}}^2\right)\delta g_V^{d}\Big] \\ &+ \delta\Gamma_{Z\to\text{Had},\psi^4} + 3\delta\Gamma_{Z\to\ell\bar{\ell},\psi^4} + 3\delta\Gamma_{Z\to\nu\bar{\nu},\psi^4}. \end{aligned} \tag{A.54}$$

The corrections due to four-fermion operators have been generically denoted by $\delta\Gamma_{Z\to f\bar{f},\psi^4}$ and can be derived directly from Eqn. A.35. Their expressions are not given here as they are suppressed by $\bar{v}_T\,\Gamma_Z/m_Z^2$ beyond the power counting suppression, see Ref [413] for details.

The ratios of decay rates are defined in the SM as $R_f^0 = \Gamma_{Z\to \mathrm{Had}}/\Gamma_{Z\to \bar{f}f}$ where f can be a charged lepton ℓ or a neutrino. These are shifted by $\bar{R}_f^0 = R_f^0 + \delta R_f^0$ with

$$\delta R_f^0 = \frac{1}{(\Gamma^{SM}_{Z\to f\bar{f}})^2}\left[\delta\Gamma_{Z\to \mathrm{Had}}\Gamma^{SM}_{Z\to f\bar{f}} - \delta\Gamma_{Z\to f\bar{f}}\Gamma^{SM}_{Z\to \mathrm{Had}}\right]. \tag{A.55}$$

When f is an identified quark, the ratio R_q^0 is defined as the inverse of the lepton case.

Forward-backward asymmetries

The forward backward asymmetry for the scattering $\ell^+\ell^- \to f\bar{f}$ is defined as

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}, \tag{A.56}$$

where σ_F is defined by $\theta \in [0, \pi/2]$ and σ_B by $\theta \in [\pi/2, \pi]$ with θ the angle between the incoming ℓ^- and the outgoing $\bar{f}$. In the SM:

$$A_{FB}^{0,f} = \frac{3}{4}A_\ell A_f, \quad \text{with} \quad A_\ell = 2\frac{g_V^\ell g_A^\ell}{(g_V^\ell)^2 + (g_A^\ell)^2}, \quad A_f = 2\frac{g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}. \tag{A.57}$$

In the SMEFT $\bar{A}_f$ can be written as

$$\bar{A}_f = \frac{2\bar{r}_f}{1+\bar{r}_f^2}\ , \qquad \bar{r}_f = \frac{\bar{g}_V^f}{\bar{g}_A^f} \tag{A.58}$$

and it is shifted due to modifications of the Z couplings as $\bar{A}_f = (A_f)^{SM} + \delta A_f$ with

$$\delta A_f = (A_f)^{SM}\left(1 - \frac{2(r_f^2)^{SM}}{1+(r_f^2)^{SM}}\right)\delta r_f \tag{A.59}$$

and

$$r_f = (r_f)^{SM}\left(1+\delta r_f\right)\ , \qquad \delta r_f = \frac{\delta g_V^f}{g_V^{f,SM}} - \frac{\delta g_A^f}{g_A^{f,SM}}\ . \tag{A.60}$$

The corresponding correction to $A_{FB}^{0,f}$ is

$$\delta A_{FB}^{0,f} = \frac{3}{4}\left[\delta A_\ell\, (A_f)^{SM} + (A_\ell)^{SM}\,\delta A_f\right]. \tag{A.61}$$

Corrections due to four-fermion operators are negligible, as the forward backward asymmetry measurements are direct cross section measurements extracted near the Z pole.

A.4 Properties of the $W^{\pm}$ boson

W width

The partial $W^{\pm}$ widths in the SMEFT read [233]

$$\begin{aligned}
\bar{\Gamma}_{W\to\bar{f}_p f_r} =& \Gamma^{SM}_{W\to\bar{f}_p f_r} + \delta\Gamma_{W\to\bar{f}_p f_r}, && \text{(A.62)}\\
\Gamma^{SM}_{W\to\bar{f}_p f_r} =& \frac{N_C\,|V^f_{pr}|^2\sqrt{2}\hat{G}_F\,\hat{m}^3_W}{12\pi}, && \text{(A.63)}\\
\delta\Gamma_{W\to\bar{f}_p f_r} =& \frac{N_C\,|V^f_{pr}|^2\sqrt{2}\hat{G}_F\,\hat{m}^3_W}{12\pi}\left(4\delta g^{W,f}_{V/A} + \frac{1}{2}\frac{\delta m^2_W}{\hat{m}^2_W}\right). && \text{(A.64)}
\end{aligned}$$

As above, N_C depends on the colour representation of final state fermions and V^f corresponds to the CKM ($f = q$) or PMNS ($f = \ell$) matrix. In the lepton case, as the neutrino flavour of the decay of a $W^{\pm}$ boson is not identified, the sum over the neutrino species gives $\sum_r |V^\ell_{pr}|^2 = 1$. As a result, the total width is $\bar{\Gamma}_W = \Gamma^{SM}_W + \delta\Gamma_W$ with

$$\Gamma^{SM}_W = \frac{3\sqrt{2}\hat{G}_F\hat{m}^3_W}{4\pi}, \qquad \delta\Gamma_W = \Gamma^{SM}_W\left(\frac{4}{3}\delta g^\ell_W + \frac{8}{3}\delta g^q_W + \frac{\delta m^2_W}{2\hat{m}^2_W}\right). \tag{A.65}$$

Here $\hat{m}_W$ is the standard model value of the W-mass at tree level in terms of the input parameters, $\hat{m}_W = c_{\hat{\theta}}\,\hat{m}_Z$.

A.5 Scattering $\ell^+\ell^- \to 4f$ through $W^{\pm}$ currents

The doubly resonant contribution to the $\ell^+\ell^- \to f_1\bar{f}_2f_3\bar{f}_4$ scattering via $W^{\pm}$ currents was computed in the SMEFT in Ref. [233]. There are two relevant diagrams contributing for each fixed final state, as illustrated in Figure 11. The total amplitude can be written as $\mathcal{A} = \mathcal{A}_V + \mathcal{A}_\nu$ with

$$\begin{aligned}
\mathcal{A}_V &= \mathcal{A}^{\lambda_{12}\lambda_{23}\lambda_+\lambda_-}_{\ell\ell\to WW,V}\,D^W(s_{12})D^W(s_{23})\mathcal{A}^{\lambda_{12}}_{W^+\to f_1\bar{f}_2}\mathcal{A}^{\lambda_{34}}_{W^-\to f_3\bar{f}_4}\,, && \text{(A.66)}\\
\mathcal{A}_\nu &= \mathcal{A}^{\lambda_{12}\lambda_{23}\lambda_+\lambda_-}_{\ell\ell\to WW,\nu}\,D^W(s_{12})D^W(s_{23})\mathcal{A}^{\lambda_{12}}_{W^+\to f_1\bar{f}_2}\mathcal{A}^{\lambda_{34}}_{W^-\to f_3\bar{f}_4}\,. && \text{(A.67)}
\end{aligned}$$

Here $\lambda_\pm$ is the helicity of the initial $\ell^\pm$ and λ_{12}, $\lambda_{34} = \{0, +, -, L\}$ are the helicities of the W^+ and W^- boson respectively (see Refs [233, 500, 715] for further details on this spinor helicity formalism). The contribution of the longitudinal helicity (L) vanishes in the limit of massless fermions and therefore it can be neglected. Each amplitude has been decomposed as the product of three helicity-dependent sub-amplitudes (for WW production via $V = \{Z, \gamma\}$ or ν exchange and for the decay of each W) and of the $W^{\pm}$ propagators, parameterized as:

$$D^W(s_{ij}) = \frac{1}{s_{ij} - \bar{m}^2_W + i\bar{\Gamma}_W\bar{m}_W + i\epsilon}\,, \tag{A.68}$$

with $s_{ij} = s_{12}$ for the W^+ and $s_{ij} = s_{34}$ for the W^-. In the SMEFT and with the $\{\hat{\alpha}, \hat{G}_F, \hat{m}_Z\}$ input scheme both the W pole mass and width are shifted compared to the SM prediction.

The propagators the need to be expanded up to linear order in the Wilson coefficients as [233]

$$D^W(s_{ij}) = \frac{1}{s_{ij} - \hat{m}_W^2 + i\hat{\Gamma}_W\hat{m}_W + i\epsilon}\left[1 + \delta D^W(s_{ij})\right], \tag{A.69}$$

$$\delta D^W(s_{ij}) = \frac{1}{s_{ij} - \hat{m}_W^2 + i\hat{\Gamma}_W\hat{m}_W}\left[\left(1 - \frac{i\hat{\Gamma}_W}{2\hat{m}_W}\right)\delta m_W^2 - i\hat{m}_W\delta\Gamma_W\right]. \tag{A.70}$$

The expressions of $\mathcal{A}^{\lambda_{12}\lambda_{23}\lambda_+\lambda_-}_{\ell\ell\to WW,V}$, $\mathcal{A}^{\lambda_{12}\lambda_{23}\lambda_+\lambda_-}_{\ell\ell\to WW,\nu}$ in the SMEFT for each λ_{ij} assignment are listed in Tables 5, 6 and 7. Those for $\mathcal{A}^{\lambda_{12}}_{W^+\to f_1\bar{f}_2}$, $\mathcal{A}^{\lambda_{34}}_{W^-\to f_3\bar{f}_4}$ are instead in Table 8. The tables use the notation

$$D^V(s) = \frac{1}{s - \hat{m}_V^2 + i\hat{\Gamma}_V\hat{m}_V + i\epsilon},$$

$$F_1^Z = -\hat{g}_{Z,eff}\, g_{ZWW}\, \bar{g}_L^e, \qquad F_2^Z = -\hat{g}_Z\, g_{ZWW}\, \bar{g}_R^e, \qquad F_1^\gamma = \bar{F}_2^\gamma = \sqrt{4\pi\hat{\alpha}}\, g_{AWW}, \tag{A.71}$$

for the V couplings and propagators, and $\lambda^{1/2}(x,y,z)$ is the square root of the Källén function

$$\lambda(x,y,z) = \left[x^2 + y^2 + z^2 - 2xy - 2xz - 2yz\right]. \tag{A.72}$$

The quantities $\bar{g}_1^V$, $\bar{\kappa}_V$, $\bar{\lambda}_V$ are the triple gauge couplings in the parameterization of Eqn. 7.18. The phase space is parameterized as follows: θ is the angle between the momenta of the incoming e^- and the outgoing W^+ in the center-of-mass frame; $\tilde{\phi}_{12(34)}$ and $\tilde{\theta}_{12(34)}$ are the azimuthal and polar angle of the momentum of the final state fermion $f_{1(3)}$ in the rest frame of the $W^{+(-)}$ boson.

The total spin averaged cross section is

$$\bar{\sigma}(s) = \int \frac{\sum|\mathcal{A}|^2}{8s}\frac{ds_{12}ds_{34}}{(2\pi)^2}\left[\frac{\bar{\beta}_{12}}{8\pi}\frac{d\cos\tilde{\theta}_{12}}{2}\frac{d\tilde{\phi}_{12}}{2\pi}\right]\left[\frac{\bar{\beta}_{34}}{8\pi}\frac{d\cos\tilde{\theta}_{34}}{2}\frac{d\tilde{\phi}_{34}}{2\pi}\right]\left[\frac{\bar{\beta}}{8\pi}\frac{d\cos\theta}{2}\frac{d\phi}{2\pi}\right], \tag{A.73}$$

where, for $\ell = e$

$$\begin{aligned}\sum|\mathcal{A}|^2 = {} & |D^W(s_{12})D^W(s_{34})|^2 \sum_{\lambda_{12},\lambda'_{12}}\sum_{\lambda_{34},\lambda'_{34}}\left(\mathcal{A}^{\lambda_{12}}_{W^+\to f_1\bar{f}_2}\right)\left(\mathcal{A}^{\lambda'_{12}}_{W^+\to f_1\bar{f}_2}\right)^*\left(\mathcal{A}^{\lambda_{34}}_{W^-\to f_3\bar{f}_4}\right)\left(\mathcal{A}^{\lambda'_{34}}_{W^-\to f_3\bar{f}_4}\right)^* \\ & \times\sum_{\lambda_+}\sum_{\lambda_-}\left(\mathcal{A}^{\lambda_{12}\lambda_{34},\lambda_+,\lambda_-}_{ee\to WW}\right)\left(\mathcal{A}^{\lambda'_{12}\lambda'_{34},\lambda_+,\lambda_-}_{ee\to WW}\right)^*,\end{aligned} \tag{A.74}$$

and the WW production amplitudes contain both the V and ν exchange contributions.

The β-factors are

$$\bar{\beta} = \sqrt{1 - \frac{2(s_{12}+s_{34})}{s} + \frac{(s_{12}-s_{34})^2}{s^2}}, \qquad \bar{\beta}_{ij} = 1 \tag{A.75}$$

and the integration over the parameters can be performed numerically in the regions

$$\begin{aligned}\tilde{\phi}_{ij} &\in [0,2\pi], & s_{34} &\in [0,(\sqrt{s}-\sqrt{s}_{12})^2], \\ \cos\theta,\cos\tilde{\theta}_{ij} &\in [-1,1], & s_{12} &\in [0,s].\end{aligned} \tag{A.76}$$

λ_{12}	λ_{34}	$\mathcal{A}^{\lambda_{12}\lambda_{34}+-}_{e^+e^-\to W^+W^-,V}$
0	0	$-F_1^V\left(\bar{g}_1^V\left(s_{12}+s_{34}\right)+\bar{\kappa}_V s\right)\lambda^{1/2}\sin\theta D^V\left(s\right)\,/\left(2\sqrt{s_{12}}\sqrt{s_{34}}\right)$
+	+	$F_1^V\left(2\bar{g}_1^V\bar{m}_W^2+\bar{\lambda}_V s\right)\lambda^{1/2}\sin\theta D^V\left(s\right)\,/\left(2\bar{m}_W^2\right)$
−	−	$F_1^V\left(2\bar{g}_1^V\bar{m}_W^2+\bar{\lambda}_V s\right)\lambda^{1/2}\sin\theta D^V\left(s\right)\,/\left(2\bar{m}_W^2\right)$
0	−	$-\sqrt{s}\lambda^{1/2}\left(F_1^V\cos\theta-F_1^V\right)\left(\bar{g}_1^V\bar{m}_W^2+\bar{\kappa}_V\bar{m}_W^2+\bar{\lambda}_V s_{12}\right)D^V\left(s\right)\,/\left(2\sqrt{2}\sqrt{s_{12}}\bar{m}_W^2\right)$
0	+	$\sqrt{s}\lambda^{1/2}\left(F_1^V\cos\theta+F_1^V\right)\left(\bar{g}_1^V\bar{m}_W^2+\bar{\kappa}_V\bar{m}_W^2+\bar{\lambda}_V s_{12}\right)D^V\left(s\right)\,/\left(2\sqrt{2}\sqrt{s_{12}}\bar{m}_W^2\right)$
+	0	$\sqrt{s}\lambda^{1/2}\left(F_1^V\cos\theta-F_1^V\right)\left(\bar{g}_1^V\bar{m}_W^2+\bar{\kappa}_V\bar{m}_W^2+\bar{\lambda}_V s_{34}\right)D^V\left(s\right)\,/\left(2\sqrt{2}\sqrt{s_{34}}\bar{m}_W^2\right)$
−	0	$-\sqrt{s}\lambda^{1/2}\left(F_1^V\cos\theta+F_1^V\right)\left(\bar{g}_1^V\bar{m}_W^2+\bar{\kappa}_V\bar{m}_W^2+\bar{\lambda}_V s_{34}\right)D^V\left(s\right)\,/\left(2\sqrt{2}\sqrt{s_{34}}\bar{m}_W^2\right)$
+	−	0
−	+	0

Table 5. The W-production matrix elements $\mathcal{A}^{\lambda_{12}\lambda_{34}+-}_{e^+e^-\to W^+W^-,V}$ for $\lambda_{12},\lambda_{34}=\{0,+,-\}$, $V=\{Z,\gamma\}$ and $\lambda^{1/2}=\lambda^{1/2}\left(s,s_{12},s_{34}\right)$.

λ_{12}	λ_{34}	$\mathcal{A}^{\lambda_{12}\lambda_{34}-+}_{e^+e^-\to W^+W^-,V}$
0	0	$-F_2^V\left(\bar{g}_1^V\left(s_{12}+s_{34}\right)+\bar{\kappa}_V s\right)\lambda^{1/2}\sin\theta D^V\left(s\right)\,/\left(2\sqrt{s_{12}}\sqrt{s_{34}}\right)$
+	+	$F_2^V\left(2\bar{g}_1^V\bar{m}_W^2+\bar{\lambda}_V s\right)\lambda^{1/2}\sin\theta D^V\left(s\right)\,/\left(2\bar{m}_W^2\right)$
−	−	$F_2^V\left(2\bar{g}_1^V\bar{m}_W^2+\bar{\lambda}_V s\right)\lambda^{1/2}\sin\theta D^V\left(s\right)\,/\left(2\bar{m}_W^2\right)$
0	−	$-\sqrt{s}\lambda^{1/2}\left(F_2^V\cos\theta+F_2^V\right)\left(\bar{g}_1^V\bar{m}_W^2+\bar{\kappa}_V\bar{m}_W^2+\bar{\lambda}_V s_{12}\right)D^V\left(s\right)\,/\left(2\sqrt{2}\sqrt{s_{12}}\bar{m}_W^2\right)$
0	+	$\sqrt{s}\lambda^{1/2}\left(F_2^V\cos\theta-F_2^V\right)\left(\bar{g}_1^V\bar{m}_W^2+\bar{\kappa}_V\bar{m}_W^2+\bar{\lambda}_V s_{12}\right)D^V\left(s\right)\,/\left(2\sqrt{2}\sqrt{s_{12}}\bar{m}_W^2\right)$
+	0	$\sqrt{s}\lambda^{1/2}\left(F_2^V\cos\theta+F_2^V\right)\left(\bar{g}_1^V\bar{m}_W^2+\bar{\kappa}_V\bar{m}_W^2+\bar{\lambda}_V s_{34}\right)D^V\left(s\right)\,/\left(2\sqrt{2}\sqrt{s_{34}}\bar{m}_W^2\right)$
−	0	$-\sqrt{s}\lambda^{1/2}\left(F_2^V\cos\theta-F_2^V\right)\left(\bar{g}_1^V\bar{m}_W^2+\bar{\kappa}_V\bar{m}_W^2+\bar{\lambda}_V s_{34}\right)D^V\left(s\right)\,/\left(2\sqrt{2}\sqrt{s_{34}}\bar{m}_W^2\right)$
+	−	0
−	+	0

Table 6. The W-production matrix elements $\mathcal{A}^{\lambda_{12}\lambda_{34}-+}_{e^+e^-\to W^+W^-,V}$ for $\lambda_{12},\lambda_{34}=\{0,+,-\}$, $V=\{Z,\gamma\}$ and $\lambda^{1/2}=\lambda^{1/2}\left(s,s_{12},s_{34}\right)$.

A.6 Low energy precision measurements

ν–lepton scattering

The scattering process $\nu\, e^{\pm}\to\nu\, e^{\pm}$ can be described by the following Effective Lagrangian [414]

$$\mathcal{L}_{\nu e}=-\frac{\hat{G}_F}{\sqrt{2}}\left[\bar{e}\gamma^\mu\left((\bar{g}_V^{\nu e})-(\bar{g}_A^{\nu e})\gamma^5\right)e\right]\left[\bar{\nu}\gamma_\mu\left(1-\gamma^5\right)\nu\right]. \tag{A.77}$$

the shifts are then $\bar{g}_V^{\nu e}=g_V^{\nu e}+\delta g_V^{\nu e}$, $\bar{g}_A^{\nu e}=g_A^{\nu e}+\delta g_A^{\nu e}$ where

$$\delta(g_V^{\nu e})=2\left(\delta g_V^{\ell}+2\delta g_V^{\ell,W_\pm}\right)+4\delta g_V^{\nu}\left(-\frac{1}{2}+2s_{\hat{\theta}}^2\right)-\frac{1}{2\sqrt{2}\hat{G}_F}\left(2C_{ll}+C_{le}\right)-\frac{\delta m_W^2}{m_W^2}, \tag{A.78}$$

$$\delta(g_A^{\nu e})=2\left(\delta g_A^{\ell}+2\delta g_A^{\ell,W_\pm}\right)-2\delta g_V^{\nu}-\frac{1}{2\sqrt{2}\hat{G}_F}\left(2C_{ll}-C_{le}\right)-\frac{\delta m_W^2}{m_W^2}. \tag{A.79}$$

λ_{12}	λ_{34}	$\left(\mathcal{A}^{\lambda_{12}\lambda_{34}+-}_{e^+e^-\to W^+W^-,\nu}\right) / \left(2\pi\hat{\alpha}\left(\bar{g}_V^{W\pm,\ell}\right)^2\right)$
0	0	$\frac{2\sin\theta}{s_{\hat{\theta}}^2\sqrt{s_{12}}\sqrt{s_{34}}\lambda^{1/2}}\left(\left(s^2-(s_{12}-s_{34})^2\right)-\frac{8ss_{12}s_{34}}{s-s_{12}-s_{34}+\lambda^{1/2}\cos\theta}\right)$
+	+	$-\frac{4\sin\theta}{s_{\hat{\theta}}^2\lambda^{1/2}}\left(s+\frac{-s\,(s_{12}+s_{34})-(s_{12}-s_{34})\left(-s_{12}+s_{34}+\lambda^{1/2}\right)}{s-s_{12}-s_{34}+\lambda^{1/2}\cos\theta}\right)$
−	−	$-\frac{4\sin\theta}{s_{\hat{\theta}}^2\lambda^{1/2}}\left(s+\frac{-s\,(s_{12}+s_{34})+(s_{12}-s_{34})\left(s_{12}-s_{34}+\lambda^{1/2}\right)}{s-s_{12}-s_{34}+\lambda^{1/2}\cos\theta}\right)$
0	−	$-\frac{4(1-\cos\theta)\sqrt{s}}{s_{\hat{\theta}}^2\sqrt{2}\sqrt{s_{12}}\lambda^{1/2}}\left((s+s_{12}-s_{34})-\frac{2s_{12}\left(s-s_{12}+s_{34}-\lambda^{1/2}\right)}{s-s_{12}-s_{34}+\lambda^{1/2}\cos\theta}\right)$
0	+	$-\frac{4(1+\cos\theta)\sqrt{s}}{s_{\hat{\theta}}^2\sqrt{2}\sqrt{s_{12}}\lambda^{1/2}}\left((s+s_{12}-s_{34})-\frac{2s_{12}\left(s-s_{12}+s_{34}+\lambda^{1/2}\right)}{s-s_{12}-s_{34}+\lambda^{1/2}\cos\theta}\right)$
+	0	$\frac{4(1-\cos\theta)\sqrt{s}}{s_{\hat{\theta}}^2\sqrt{2}\sqrt{s_{34}}\lambda^{1/2}}\left((s-s_{12}+s_{34})-\frac{2s_{34}\left(s+s_{12}-s_{34}-\lambda^{1/2}\right)}{s-s_{12}-s_{34}+\lambda^{1/2}\cos\theta}\right)$
−	0	$\frac{4(1+\cos\theta)\sqrt{s}}{s_{\hat{\theta}}^2\sqrt{2}\sqrt{s_{34}}\lambda^{1/2}}\left((s-s_{12}+s_{34})-\frac{2s_{34}\left(s+s_{12}-s_{34}+\lambda^{1/2}\right)}{s-s_{12}-s_{34}+\lambda^{1/2}\cos\theta}\right)$
+	−	$-\frac{4}{s_{\hat{\theta}}^2}\frac{s\sin\theta\,(1-\cos\theta)}{s-s_{12}-s_{34}+\lambda^{1/2}\cos\theta}$
−	+	$\frac{4}{s_{\hat{\theta}}^2}\frac{s\sin\theta\,(1+\cos\theta)}{s-s_{12}-s_{34}+\lambda^{1/2}\cos\theta}$

Table 7. The $W^\pm$ pair production matrix elements $\mathcal{A}^{\lambda_{12}\lambda_{34}+-}_{e^+e^-\to W^+W^-,\nu}$ for helicities $\lambda_{12},\lambda_{34}=\{0,+,-\}$ and we have used the notation $\lambda^{1/2}=\lambda^{1/2}\,(s,s_{12},s_{34})$.

λ_{12}	$\mathcal{M}^{\lambda_{12}}_{W^+\to f_1\bar{f}_2}/C\sqrt{2\pi\hat{\alpha}}$
0	$\frac{-2\bar{g}_V^{W_+,f_1}}{s_{\hat{\theta}}}\sqrt{s_{12}}\sin\tilde{\theta}_{12}$
+	$\frac{\bar{g}_V^{W_+,f_1}}{s_{\hat{\theta}}}\sqrt{s_{12}}\sqrt{2}\left(1-\cos\tilde{\theta}_{12}\right)\,e^{i\tilde{\phi}_{12}}$
−	$\frac{\bar{g}_V^{W_+,f_1}}{s_{\hat{\theta}}}\sqrt{s_{12}}\sqrt{2}\left(1+\cos\tilde{\theta}_{12}\right)\,e^{-i\tilde{\phi}_{12}}$

λ_{34}	$\mathcal{M}^{\lambda_{34}}_{W^-\to f_3\bar{f}_4}/C'\sqrt{2\pi\hat{\alpha}}$
0	$\frac{2\,\bar{g}_V^{W_-,f_3}}{s_{\hat{\theta}}}\sqrt{s_{34}}\sin\tilde{\theta}_{34}$
+	$\frac{-\bar{g}_V^{W_-,f_3}}{s_{\hat{\theta}}}\sqrt{s_{34}}\sqrt{2}\left(1-\cos\tilde{\theta}_{34}\right)\,e^{-i\tilde{\phi}_{34}}$
−	$\frac{-\bar{g}_V^{W_-,f_3}}{s_{\hat{\theta}}}\sqrt{s_{34}}\sqrt{2}\left(1+\cos\tilde{\theta}_{34}\right)\,e^{+i\tilde{\phi}_{34}}$

Table 8. Decay amplitudes $\mathcal{A}^{\lambda_{ij}}_{W^\pm\to f_i\bar{f}_j}$. $C,C'=\{1,\sqrt{3}\}$ for final state leptons and quarks respectively.

these shifts add the contributions of W and Z exchange. Depending on the neutrino flavour some terms are absent. The shift that is relevant for $g_{A,V}^{\nu_\mu e}$ does not have a δM_W^2 or $\delta g_{V,A}^{\ell,W_\pm}$ contribution, whereas a shift for $g_{A,V}^{\nu_\mu\mu}$ has both contributions.

ν–Nucleon scattering

The scattering process $\nu\,N\to\nu\,X$ via Z exchange can be described by the following Effective Lagrangian [414]

$$\mathcal{L}^{NC}_{\nu\,q}=-\frac{\hat{G}_F}{\sqrt{2}}\left[\bar{\nu}\gamma^\mu\left(1-\gamma^5\right)\nu\right]\left[\bar{\epsilon}^q_L\bar{q}\gamma_\mu\left(1-\gamma^5\right)q+\bar{\epsilon}^q_R\bar{q}\gamma_\mu\left(1+\gamma^5\right)q\right]. \tag{A.80}$$

where $q=\{u,d\}$. At tree level in the SM: $(\epsilon^q_L)^{SM}=g_V^{q,SM}+g_A^{q,SM}$ and $(\epsilon^q_R)_{SM}=g_V^{q,SM}-g_A^{q,SM}$. The redefinition of the Z couplings and the corrections due to ψ^4 operators lead to a shift in ϵ_L^q and ϵ_R^q of the form $\bar{\epsilon}^q_{L/R} = \epsilon^q_{L/R} + \delta\epsilon^q_{L/R}$ with

$$\delta\epsilon_L^u = -\frac{1}{2\sqrt{2}\hat{G}_F}\left(C_{lq}^{(1)} + C_{lq}^{(3)}\right) + \delta g_V^u + \delta g_A^u + 4\delta g_V^\nu (\epsilon_L^u)^{SM}, \tag{A.81}$$

$$\delta\epsilon_L^d = -\frac{1}{2\sqrt{2}\hat{G}_F}\left(C_{lq}^{(1)} - C_{lq}^{(3)}\right) + \delta g_V^d + \delta g_A^d + 4\delta g_V^\nu (\epsilon_L^d)^{SM}, \tag{A.82}$$

$$\delta\epsilon_R^u = -\frac{1}{2\sqrt{2}\hat{G}_F} C_{lu} + \delta g_V^u - \delta g_A^u + 4\delta g_V^\nu (\epsilon_R^u)^{SM}, \tag{A.83}$$

$$\delta\epsilon_R^d = -\frac{1}{2\sqrt{2}\hat{G}_F} C_{lu} + \delta g_V^u - \delta g_A^u + 4\delta g_V^\nu (\epsilon_R^u)^{SM}. \tag{A.84}$$

The scattering $\nu N \to \ell X$ and the inverse process take place through W exchange and can be described by

$$\mathcal{L} = -\frac{\hat{G}_F}{\sqrt{2}}\left[\bar{\ell}\gamma^\mu\left(1-\gamma^5\right)\nu\right]\left[\bar{\Sigma}_L^{ij}\bar{u}_i\gamma_\mu\left(1-\gamma^5\right)d_j\right] + \text{h.c.}, \tag{A.85}$$

where at tree level in the SM $(\Sigma_L^{ij})_{SM} = V_{CKM}^{ij}$. This coupling receives corrections from redefinitions of the W mass and couplings, so that $\bar{\Sigma}_L^{ij} = (\Sigma_L^{ij})_{SM} + \delta\Sigma_L^{ij}$ with

$$\delta\Sigma_L^{ij} = \left[-\frac{\delta m_W^2}{\hat{m}_W^2} + 2\,\delta g_V^{q,W} + 2\,\delta g_V^{\ell,W} - \frac{1}{\sqrt{2}\hat{G}_F}C_{lq}^{(3)}\right]V_{CKM}^{ij}. \tag{A.86}$$

The inclusion of a right-handed coupling $\bar{\Sigma}_R^{ij}$ is not forbidden in principle in the SMEFT, it can be generated by the operator $\mathcal{Q}_{\ell edq}$. This contribution has been neglected here because such a correction is proportional to the Yukawa matrices in the U(3)^5 limit assumed, and therefore vanishes in the limit of massless fermions.

Charged and neutral current process are related by [716]

$$\frac{d^2\,\sigma(\nu N \to \nu X)}{d\,x\,d\,y} = g_{L,eff}^2\,\frac{d^2\,\sigma(\nu N \to \mu^- X)}{d\,x\,d\,y} + g_{R,eff}^2\,\frac{d^2\,\sigma(\bar{\nu} N \to \mu^+ X)}{d\,x\,d\,y}. \tag{A.87}$$

for the scattering variables $x = -q^2/(2p_N\cdot q)$, $y = (p_N\cdot q)/(p_N\cdot p_\nu)$, where q^2 is the momentum transfer and p_N, p_ν are respectively the nucleon and neutrino momenta. The effective couplings $g_{L/R,eff}$ receive corrections in the SMEFT so that $\bar{g}^2_{L/R,eff} = g^2_{L/R,eff} + \delta g^2_{L/R,eff}$ and they can be expressed in terms of the $\epsilon^q_{L/R}$ parameters as

$$\bar{g}^2_{L/R,eff} = \sum_{i,j}\left[\left|\bar{\epsilon}^{u^i}_{L/R}\right|^2 + \left|\bar{\epsilon}^{d^j}_{L/R}\right|^2\right]\left|(\bar{\Sigma}_L^{ij})\right|^{-2}. \tag{A.88}$$

Relevant quantities for these processes are the ratios of cross sections

$$R^\nu = \frac{\sigma\left(\nu N \to \nu X\right)}{\sigma\left(\nu N \to \ell^- X\right)} = g_{L,eff}^2 + r\,g_{R,eff}^2, \quad R^{\bar{\nu}} = \frac{\sigma\left(\bar{\nu} N \to \bar{\nu} X\right)}{\sigma\left(\bar{\nu} N \to \ell^+ X\right)} = g_{L,eff}^2 + \frac{g_{R,eff}^2}{r}, \tag{A.89}$$

where the factor r in an ideal experiment with full acceptance (in the absence of sea quarks) is given by $r = 1/3$. SMEFT contributions to the r parameter can be neglected as long as dimension-6 operators have a negligible impact on the parton distributions of nucleons. This condition is plausibly verified, as such corrections are expected to scale as $\Lambda^2_{\rm QCD}/\Lambda^2$.

Finally, the parameter κ defined by

$$\kappa = 1.7897\, g^2_{L,eff} + 1.1479 g^2_{R,eff} - 0.0916\, h^2_{L,eff} - 0.0782\, h^2_{R,eff} \tag{A.90}$$

has been used to report data, e.g. by the CCFR collaboration [717]. Here $g_{L/R,eff}$ are the couplings introduced in Eqn. A.88 and

$$\bar{h}^2_{L/R,eff} = \sum_{i,j} \left[\left| \bar{\epsilon}^{u^i}_{L/R} \right|^2 - \left| \bar{\epsilon}^{d^j}_{L/R} \right|^2 \right] \left| (\bar{\Sigma}^{ij}_L) \right|^{-2}. \tag{A.91}$$

Neutrino Trident Production

Neutrino trident production is the pair production of leptons from the scattering of a neutrino off the Coulomb field of a nucleus, $\nu\, N \to \nu\, N\, \ell^+\, \ell^-$. The SM calculation of this process is well known, see Refs.[718–720]. Using the notation of the effective Lagrangian in Eqn.A.77, the constraint on the SMEFT is through the ratio of the partonic cross sections

$$\frac{\bar{\sigma}_{SMEFT}}{\sigma_{SM}} = \frac{(\bar{g}^{\nu_e e}_V)^2 + (\bar{g}^{\nu_e e}_A)^2}{(g^{\nu_e e,SM}_V)^2 + (g^{\nu_e e,SM}_A)^2}. \tag{A.92}$$

Atomic Parity Violation

For Atomic Parity Violation (APV) the standard Effective Lagrangian is given by [414]

$$\mathcal{L}_{eq} = \frac{\hat{G}_F}{\sqrt{2}} \left[\sum_q \bar{g}^{eq}_{AV} \left(\bar{e}\gamma_\mu \gamma^5 e \right) \left(\bar{q}\gamma^\mu q \right) + \bar{g}^{eq}_{VA} \left(\bar{e}\gamma_\mu e \right) \left(\bar{q}\gamma^\mu \gamma^5 q \right) \right]. \tag{A.93}$$

In the SM: $g^{eq,SM}_{AV} = 8\, g^{q,SM}_V\, g^{\ell,SM}_A$ and $g^{eq,SM}_{VA} = 8\, g^{q,SM}_A\, g^{\ell,SM}_V$ and the relevant couplings are for $q = u, d$. The effective shifts are

$$\delta g^{eu}_{AV} = \frac{1}{2\sqrt{2}\hat{G}_F} \left(-C^{(1)}_{lq} + C^{(3)}_{lq} - C_{lu} + C_{eu} + C_{qe} \right) + 2 \left(1 - \frac{8}{3} s^2_{\hat{\theta}} \right) \delta g^\ell_A - 2\delta g^u_V, \tag{A.94}$$

$$\delta g^{eu}_{VA} = \frac{1}{2\sqrt{2}\hat{G}_F} \left(-C^{(1)}_{lq} + C^{(3)}_{lq} + C_{lu} + C_{eu} - C_{qe} \right) + 2\delta g^u_A \left(-1 + 4 s^2_{\hat{\theta}} \right) + 2\delta g^\ell_V, \tag{A.95}$$

$$\delta g^{ed}_{AV} = \frac{1}{2\sqrt{2}\hat{G}_F} \left(-C^{(1)}_{lq} - C^{(3)}_{lq} - C_{ld} + C_{ed} + C_{qe} \right) + 2 \left(-1 + \frac{4}{3} s^2_{\hat{\theta}} \right) \delta g^\ell_A - 2\delta g^d_V, \tag{A.96}$$

$$\delta g^{ed}_{VA} = \frac{1}{2\sqrt{2}\hat{G}_F} \left(-C^{(1)}_{lq} - C^{(3)}_{lq} + C_{ld} + C_{ed} - C_{qe} \right) + 2\delta g^d_A \left(-1 + 4 s^2_{\hat{\theta}} \right) - 2\delta g^\ell_V. \tag{A.97}$$

It is convenient to define the four combinations

$$\bar{g}^{ep}_{AV/VA} = 2\bar{g}^{eu}_{AV/VA} + \bar{g}^{ed}_{AV/VA}, \qquad \bar{g}^{en}_{AV} = \bar{g}^{eu}_{AV/VA} + 2\bar{g}^{ed}_{AV/VA},$$

that are shifted from their SM values by

$$\delta g^{ep}_{AV/VA} = 2\delta g^{eu}_{AV/VA} + \delta g^{ed}_{AV/VA}, \tag{A.98}$$

$$\delta g^{en}_{AV/VA} = \delta g^{eu}_{AV/VA} + 2\delta g^{ed}_{AV/VA}. \tag{A.99}$$

The weak charge $Q_W^{Z,N}$ of an element X_Z^A defined by [721–723]

$$Q_W^{Z,N} = -2\left[Z\left(g^{ep}_{AV} + 0.00005\right) + N\left(g^{en}_{AV} + 0.00006\right)\right]\left(1 - \frac{\bar{\alpha}}{2\pi}\right), \tag{A.100}$$

is measured very precisely for thallium [724] and cesium [725] and in the SMEFT it is shifted by

$$\delta Q_W^{Z,N} = -2\left[Z\delta g^{ep}_{AV} + N\delta g^{en}_{AV}\right]\left(1 - \frac{\hat{\alpha}}{2\pi}\right) \tag{A.101}$$

compared to the SM value.

Parity Violating Asymmetry in eDIS

For inelastic polarized electron scattering $e_{L,R}\,N \to e\,X$ the right-left asymmetry A is defined as [721]:

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}, \tag{A.102}$$

where

$$\frac{A}{Q^2} = a_1 + a_2\frac{1-(1-y)^2}{1+(1-y)^2} \tag{A.103}$$

$$a_1 = \frac{3\hat{G}_F}{5\sqrt{2}\pi\hat{\alpha}}\left(g^{eu}_{AV} - \frac{1}{2}g^{ed}_{AV}\right), \tag{A.104}$$

$$a_2 = \frac{3\hat{G}_F}{5\sqrt{2}\pi\hat{\alpha}}\left(g^{eu}_{VA} - \frac{1}{2}g^{ed}_{VA}\right). \tag{A.105}$$

Here $Q^2 \geq 0$ is the momentum transfer and y is the fractional energy transfer in the scattering $y \simeq Q^2/s$. In the SMEFT $\bar{g}^{eq}_{AV/VA} = g^{eq}_{AV/VA} + \delta g^{eq}_{AV/VA}$ so that a_1 and a_2 receive the corrections [414]

$$\delta a_1 = \frac{3\hat{G}_F}{5\sqrt{2}\pi\hat{\alpha}}\left(\delta g^{eu}_{AV} - \frac{1}{2}\delta g^{ed}_{AV}\right), \tag{A.106}$$

$$\delta a_2 = \frac{3\hat{G}_F}{5\sqrt{2}\pi\hat{\alpha}}\left(\delta g^{eu}_{VA} - \frac{1}{2}\delta g^{ed}_{VA}\right). \tag{A.107}$$

Møller scattering

Parity Violation Asymmetry (A_{PV}) in Møller scattering can be parameterized with the standard Effective Lagrangian

$$\mathcal{L}_{ee} = \frac{\hat{G}_F}{\sqrt{2}}g^{ee}_{AV}\left(\bar{e}\gamma^\mu\gamma^5 e\right)\left(\bar{e}\gamma_\mu e\right). \tag{A.108}$$

In the SM $g_{AV}^{ee} = 8 g_V^{\ell,SM} g_A^{\ell,SM} = \frac{1}{2}\left(1 - 4 s_{\hat\theta}^2\right)$. In the SMEFT we have the correction [414]

$$\delta g_{AV}^{ee} = \frac{1}{\sqrt{2}\hat{G}_F}\left(-C_{ll} + C_{ee}\right) - 2\delta g_V^\ell - 2\left(1 - 4 s_{\hat\theta}^2\right)\delta g_A^\ell, \tag{A.109}$$

so that the parity violating asymmetry A_{PV} is expressed as

$$\frac{A_{PV}}{Q^2} = -2 g_{AV}^{ee}\frac{\hat{G}_F}{\sqrt{2}\pi\hat\alpha}\frac{1-y}{1+y^4+(1-y)^4}. \tag{A.110}$$

Q^2 and y are defined as above.

Universality in β decays

It is possible to place bounds on combinations of four fermion operators and $W^\pm$ vertex corrections by comparing the extraction of G_F from $\mu^- \to e^- + \bar\nu_e + \nu_\mu$ decays to the value determined in semileptonic β decays [726]. Assuming $\mathrm{U}(3)^5$ universality in the SMEFT, this represents a constraint on the unitarity of the CKM matrix and it translates into a bound on the following combination of operators

$$\delta|V_{CKM}|^2 = \frac{\sqrt{2}}{\hat{G}_F}\left(-C_{lq}^{(3)} + C_{ll} + C_{Hq}^{(3)} - C_{Hl}^{(3)}\right). \tag{A.111}$$

A.7 Higgs production and decay

In the following we list the expressions of the partonic cross sections for the main Higgs production processes and the partial width for the relevant Higgs decay channels, computed at tree level in the SMEFT.

$gg \to h$ and $h \to gg$

The dominant Higgs production mechanism at the LHC is via gluon fusion. In the SM this process is generated at one loop, as the SM is renormalizable. The leading contribution comes from a top quark loop and it can be computed in the $m_t \to \infty$ approximation[108], where the contact interaction $h G_{\mu\nu}^A G^{A\mu\nu}$ is present. In the SMEFT, this coupling receives a contribution from the operator $\mathcal{Q}_G$. In addition, the operator $\mathcal{Q}_{\tilde G}$ introduces a CP violating Lorentz structure that does not interfere with the SM amplitude.

The leading SM contribution (at tree-level in the EFT obtained integrating out the top quark) gives the partonic cross section [728–730]

$$\sigma^{SM}(gg \to h) = \frac{G_F \alpha_s^2}{32\sqrt{2}\pi}\left|I^g\right|^2 \tag{A.112}$$

where I^g is a Feynman integral that accounts for the top-quark loop contribution and the result is understood in a distribution sense multiplying a suppressed delta function. Including

[108]In this approximation, the gluon-fusion cross section is now known at N3LO in QCD corrections [727].

QCD corrections up to NLO[109] [215, 538, 729, 730]:

$$I^g = \left(1 + \frac{11}{4}\frac{\alpha_s}{\pi}\right) \int_0^1 dx \int_0^{1-x} dy \, \frac{1 - 4xy}{1 - (m_h^2/m_t^2)xy} \simeq 0.375. \tag{A.113}$$

Compared to the SM, the total cross section in the SMEFT is rescaled by [185, 215]

$$\frac{\sigma(gg \to h)}{\sigma^{SM}(gg \to h)} \simeq \left|1 + \frac{16\pi^2 \bar{v}_T^2}{\bar{g}_3^2 I^g} C_{HG}\right|^2 + \left|\frac{16\pi^2 \bar{v}_T^2}{\bar{g}_3^2 I^g} C_{H\tilde{G}}\right|^2 . \tag{A.114}$$

The decay $h \to gg$ (which is not observable at the LHC) proceeds through the same diagrams if initial and final state gluon emission is neglected and therefore (for this limited case) it is modified by the same relative correction:

$$\frac{\Gamma(h \to gg)}{\Gamma^{SM}(h \to gg)} \simeq \frac{\sigma(gg \to h)}{\sigma^{SM}(gg \to h)}. \tag{A.115}$$

hV associated production

The amplitude for Vh associated production can be decomposed as [275, 325][110]

$$\mathcal{A}_{hV} = \frac{i N_V g_V^2 \hat{m}_V}{q^2 - \hat{m}_V^2 + i\hat{\Gamma}_V \hat{m}_V} \, J_\nu^{V,\psi} \, \epsilon_\mu^* \, T_V^{\mu\nu} \tag{A.116}$$

where $g_V = \{\bar{g}_2, \bar{g}_2/c_{\hat{\theta}}\}$ and $N_V = \{1/\sqrt{2}, 1\}$ for $V = \{W, Z\}$ respectively and ϵ_μ^* denotes the polarization vector of the V boson.[111] The fermionic currents are defined as in Eqs. A.14, A.24. The tensor $T_V^{\mu\nu}$ can be decomposed in the sum of four independent Lorentz structures and form factors [325]

$$T_V^{\mu\nu} = f_1^V(q^2) g^{\mu\nu} + f_2^V(q^2) q^\mu q^\nu + f_3^V(q^2)(p \cdot q \, g^{\mu\nu} - q^\mu p^\nu) + f_4^V(q^2) \epsilon^{\mu\nu\rho\sigma} p_\rho q_\sigma \,, \tag{A.117}$$

where q denotes the four-momentum of the fermion pair ($q^2 = s$) and p is the four-momentum of the outgoing V boson. At tree level in the SM: $f_1^{V,SM}(q^2) = -m_V^2 f_2^{V,SM}(q^2) \equiv 1$, $f_{3,4}^{V,SM}(q^2) \equiv 0$.

In the SMEFT, the amplitude receives corrections that can be decomposed as follows

$$\delta\mathcal{A}_{hV} = \frac{i N_V g_V^2 \hat{m}_V}{q^2 - \hat{m}_V^2 + i\hat{\Gamma}_V \hat{m}_V} \, \epsilon_\mu^* \left[\delta J_\nu^{V,\psi} (T_V^{\mu\nu})^{SM} + (J_\nu^{V,\psi})^{SM} \delta T_V^{\mu\nu}\right] + \delta\mathcal{A}_{hV}, \tag{A.118}$$

where the first term contains corrections to the SM diagram, while $\delta\mathcal{A}_{hV}$ stands for the corrections from extra diagrams that are present only in the SMEFT case. In particular $\delta J_\nu^{V,\psi}$

[109] The normalization is such that $I^g = 1/3$ if QCD corrections are omitted and in the limit $m_h/m_t \to 0$.

[110] See also Ref. [731].

[111] The massive vector boson is not an external state and does not appear in the Hilbert space of the SMEFT. Here we are considering the approximate experimental reconstruction of the massive vector boson V with kinematics being dominated by the approximate on-shell region of phase space.

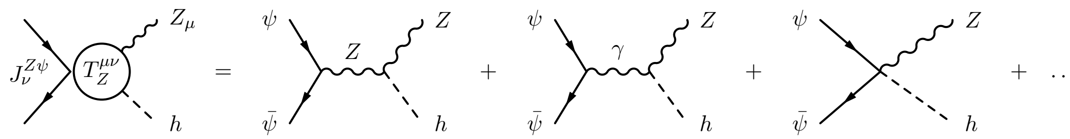


Figure 18. Diagrams for hZ associated production in the SMEFT. The dots stand for diagrams in which the Higgs is coupled to the fermion current, that are suppressed by small Yukawa couplings in the U(3)^5 flavour symmetric limit and can be neglected. In the case of hW production there are only two diagrams: with a W in s-channel and the 4-point interaction.

contains the shift in the fermion couplings to the V boson that can be inferred from the results in Section A.1 and

$$\delta T_V^{\mu\nu} = \delta f_1^V(q^2) g^{\mu\nu} + \delta f_2^V(q^2) q^\mu q^\nu + \delta f_3^V(q^2)(p\cdot q\, g^{\mu\nu} - q^\mu p^\nu) + \delta f_4^V(q^2)\epsilon^{\mu\nu\rho\sigma} p_\rho q_\sigma$$

accounts for modifications to the VVh interaction and the V propagator (in the W case). For hW production, $\delta\mathcal{A}_{hW}$ corresponds to a contribution from the 4-point interaction $udhW$, while for hZ production $\delta\mathcal{A}_{hZ}$ contains both the contribution from the 4-point interaction $\bar\psi\psi hZ$ and that stemming from the diagram with photon exchange in the s-channel (see Figure 18).

In the SMEFT the corrections $\delta f_i^V(q^2)$ and the amplitude shifts $\delta\mathcal{A}_{hV}$ can be expressed as linear functions in the Wilson coefficients. For the case $V = Z$, with the Warsaw basis and in the U(3)^5 limit [275][112]

$$\delta f_1^Z(q^2) = \delta D^Z(q^2) + \bar v_T^2 \left(C_{H\Box} + \frac{C_{HD}}{4} - C_{Hl}^{(3)} + \frac{C_{ll}}{2} \right), \tag{A.119}$$

$$\delta f_2^Z(q^2) = -\frac{1}{\hat m_Z^2}\delta f_1^Z(q^2), \tag{A.120}$$

$$\delta f_3^Z(q^2) = \frac{2\bar v_T^2}{\hat m_Z^2}\left[c_{\hat\theta}^2 C_{HW} + s_{\hat\theta}^2 C_{HB} + s_{\hat\theta} c_{\hat\theta} C_{HWB}\right], \tag{A.121}$$

$$\delta f_4^Z(q^2) = -\frac{2\bar v_T^2}{\hat m_Z^2}\left(c_{\hat\theta}^2 C_{H\tilde W} + s_{\hat\theta}^2 C_{H\tilde B} + s_{\hat\theta} c_{\hat\theta} C_{H\tilde W B}\right), \tag{A.122}$$

$$\begin{aligned} \delta\mathcal{A}_{hZ} = \frac{2i\bar e \bar v_T}{q^2} Q_\psi\, J_\nu^{\psi,em}\, \epsilon_\mu^* \Big[&\left(s_{2\hat\theta}(C_{HW} - C_{HB}) - c_{2\hat\theta} C_{HWB}\right)(p\cdot q\, g^{\mu\nu} - q^\mu p^\nu) + \\ &+ \left(s_{2\hat\theta}(C_{H\tilde B} - C_{H\tilde W}) + c_{2\hat\theta} C_{H\tilde W B}\right)\epsilon^{\mu\nu\rho\sigma} p_\rho q_\sigma \Big] + \\ &+ 2i\hat m_Z\, \epsilon_\mu^*\ \bar\psi_s \gamma^\mu \left(\underset{sr}{C_{H\psi}} P_R + (C_{Hq}^{(1)} \pm C_{Hq}^{(3)})_{sr} P_L \right)\psi_r. \end{aligned} \tag{A.123}$$

[112] Here we correct factors of 2 compared to results quoted in Ref. [325] for only a subset of SMEFT operators.

In A.119, $\delta D^Z(q^2)$ denotes the correction due to the modified Z-width in the propagator: choosing the Breit-Wigner distribution to be $\bar{\Gamma}_Z \bar{m}_Z$, it is given by

$$\delta D^Z(q^2) = \frac{-i\hat{m}_Z \delta\Gamma_Z}{q^2 - \hat{m}_Z^2 + i\hat{\Gamma}_Z \hat{m}_Z} \,. \tag{A.124}$$

The first two lines of A.123 contain the contribution from the s-channel photon exchange and $J_\nu^{em,\psi} = \bar{\psi}\gamma_\nu\psi$ is the electromagnetic current. The last line accounts for the 4-point interaction $\bar{\psi}\psi h Z$ with ψ a quark (the expression for a lepton current is analogous). Here $P_{L,R} = (1 \mp \gamma_5)/2$ are the left and right chirality projectors and s, r are flavour indices. The left-handed couplings is $\sim (C_{Hq}^{(1)} - C_{Hq}^{(3)})$ for $\psi = u$ and $(C_{Hq}^{(1)} + C_{Hq}^{(3)})$ for $\psi = d$.

For the charged current case $V = W^+$:

$$\delta f_1^W(q^2) = \delta D^W(q^2) + \bar{v}_T^2 \left(C_{H\Box} - \left(2 + \frac{1}{c_{2\hat{\theta}}}\right) \frac{C_{HD}}{4} + \frac{C_{ll} - 2C_{Hl}^{(3)}}{2c_{2\hat{\theta}}} - \frac{s_{2\hat{\theta}}}{c_{2\hat{\theta}}} C_{HWB} \right) , \tag{A.125}$$

$$\delta f_2^W(q^2) = -\frac{1}{\hat{m}_Z^2} \delta f_1^W(q^2) \,, \tag{A.126}$$

$$\delta f_3^W(q^2) = \frac{2\bar{v}_T^2}{\hat{m}_W^2} C_{HW} \,, \tag{A.127}$$

$$\delta f_4^W(q^2) = -\frac{2\bar{v}_T^2}{\hat{m}_W^2} C_{H\tilde{W}} \,, \tag{A.128}$$

$$\delta\mathcal{A}_{hW} = -2\sqrt{2} i\, \hat{m}_W\, \epsilon_\mu^* \;\bar{u}_{L,a} \gamma^\mu (C_{Hq}^{(3)} V_{CKM})_{sr}\, d_{L,r}. \tag{A.129}$$

Here $\delta D^W(s)$ is the correction to the W propagator due to the shift in the W pole mass and width defined in A.70.

The partonic cross sections are completely determined by the amplitude structure given above. Their analytic expressions are simple in the case in which only the $V = \{W, Z\}$ mediated diagrams are retained. This approximation is justified as these diagrams are assumed to largely dominate the process in the kinematic region selected for the experimental measurement. With this simplification, the SM partonic cross sections at fixed q^2 are [732]

$$\sigma(\bar{\psi}\psi \to Zh)^{\rm SM} = \frac{2\pi\alpha^2[(g_V^{\psi,SM})^2 + (g_A^{\psi,SM})^2]}{3N_c s_{\hat{\theta}}^4 c_{\hat{\theta}}^4} \frac{|\vec{p}_h|}{\sqrt{q^2}} \frac{|\vec{p}_h|^2 + 3\hat{m}_Z^2}{(q^2 - \hat{m}_Z^2 + i\hat{\Gamma}_Z \hat{m}_Z)^2} \,, \tag{A.130}$$

$$\sigma(\bar{\psi}_s\psi_r \to Wh)^{\rm SM} = \frac{\pi\alpha^2 |V_{rs}|^2}{18 s_{\hat{\theta}}^4} \frac{|\vec{p}_h|}{\sqrt{q^2}} \frac{|\vec{p}_h|^2 + 3\hat{m}_W^2}{(q^2 - \hat{m}_W^2 + i\hat{\Gamma}_W \hat{m}_W)^2} \,, \tag{A.131}$$

where $|\vec{p}_h| = [\lambda(m_h^2, m_V^2, q^2)/(4q^2)]^{1/2}$ is the center of mass momentum of the Higgs-like boson and V_{rs} denotes CKM matrix elements. The function $\lambda(x, y, z)$ was defined in A.72. In the

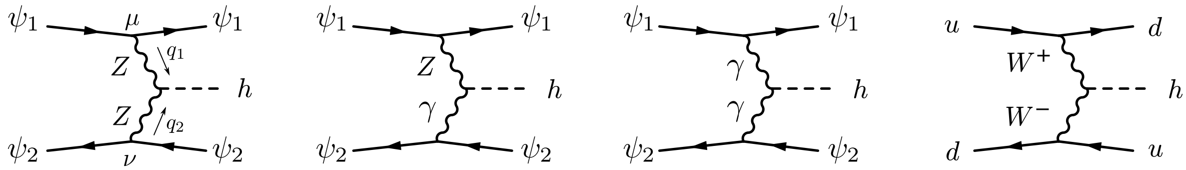


Figure 19. Diagrams contributing to VBF Higgs production in the SMEFT.

SMEFT these expressions are modified according to[113]

$$\begin{aligned}\frac{\sigma^{\rm BSM}(\psi\bar{\psi}\to Vh)}{\sigma^{\rm SM}(\psi\bar{\psi}\to Vh)} = &\left|f_1^V(q^2)\right|^2 + 3\ \mathrm{Re}\left[f_1^V(q^2)f_3^{V*}(q^2)\right]\frac{\hat{m}_V^2(q^2+\hat{m}_V^2-\hat{m}_h^2)}{|\vec{p}_h|^2+3\hat{m}_V^2}\\ &+\frac{\hat{m}_V^2 q^2}{|\vec{p}_h|^2+3\hat{m}_V^2}\left[\left|f_3^V(q^2)\right|^2(3\hat{m}_V^2+2|\vec{p}_h|^2)+2|\vec{p}_h|^2\left|f_4^V(q^2)\right|\right]\\ &+\frac{2}{(g_{V,SM}^{V,\psi})^2+(g_{A,SM}^{V,\psi})^2}\left[g_{V,SM}^{V,\psi}\,\delta g_V^{V,\psi}+g_{A,SM}^{V,\psi}\,\delta g_A^{V,\psi}\right],\end{aligned} \tag{A.132}$$

where $f_i^V(q^2) = f_i^{V,SM}(q^2) + \delta f_i^V(q^2)$ and the notation $g_{\chi,SM}^{V,\psi}$ denotes the SM coupling of the fermion ψ with chiral structure $\chi = \{V, A\}$ to the V boson.

VBF production

Adopting a formalism similar to that employed for Vh associated production, the generic amplitude for Higgs production via $VV = \{ZZ, W^+W^-\}$ fusion can be written as

$$\mathcal{A}_{VBF,VV} = \frac{iN_V^2 g_V^3 \hat{m}_V}{(q_1^2-\hat{m}_V^2+i\hat{\Gamma}_V\hat{m}_V)(q_2^2-\hat{m}_V^2+i\hat{\Gamma}_V\hat{m}_V)} J_\mu^{V,\psi_1} J_\nu^{V,\psi_2} T_V^{\mu\nu}, \tag{A.133}$$

where the currents and momenta are labeled as in Figure 19 and, in the limit of massless fermions, the tensor $T_V^{\mu\nu}$ can be decomposed into three Lorentz structures[114]:

$$T_V^{\mu\nu} = f_1^V(q_1^2,q_2^2)g^{\mu\nu} + f_3^V(q_1^2,q_2^2)(q_1\cdot q_2\, g^{\mu\nu} - q_1^\nu q_2^\mu) + f_4^V(q_1^2,q_2^2)\epsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}\,. \tag{A.134}$$

For the ZZ fusion case $f_1^{Z,SM}(q_1^2,q_2^2)\equiv 1$, $f_{3,4}^{Z,SM}(q_1^2,q_2^2)\equiv 0$, while in the SMEFT

$$f_1^Z(q_1^2,q_2^2) = 1+\delta D^Z(q_1^2)+\delta D^Z(q_2^2)+\bar{v}_T^2\left(C_{H\Box}+\frac{C_{HD}}{4}-C_{Hl}^{(3)}+\frac{C_{ll}}{2}\right), \tag{A.135}$$

$$f_3^Z(q_1^2,q_2^2) = -\frac{2\bar{v}_T^2}{\hat{m}_Z^2}\left[c_{\hat{\theta}}^2 C_{HW}+s_{\hat{\theta}}^2 C_{HB}+s_{\hat{\theta}}c_{\hat{\theta}}C_{HWB}\right], \tag{A.136}$$

$$f_4^Z(q_1^2,q_2^2) = \frac{2\bar{v}_T^2}{\hat{m}_Z^2}\left(c_{\hat{\theta}}^2 C_{H\tilde{W}}+s_{\hat{\theta}}^2 C_{H\tilde{B}}+s_{\hat{\theta}}c_{\hat{\theta}}C_{H\tilde{W}B}\right). \tag{A.137}$$

[113] Partial results for this expression were reported in Ref. [325].
[114] An alternative, equivalent decomposition was used in [528].

In the SMEFT the neutral current process receives additional contributions from diagrams with $Z\gamma$ and $\gamma\gamma$ fusion, whose amplitudes are

$$
\begin{aligned}
\mathcal{A}_{VBF,Z\gamma} =& 2i\bar{e}g_Z\bar{v}_T \left(\frac{Q_{\psi_2}J_\mu^{Z,\psi_1}J_\nu^{\psi_2,em}}{(q_1^2-\hat{m}_Z^2+i\hat{\Gamma}_Z\hat{m}_Z)q_2^2}+\frac{Q_{\psi_1}J_\mu^{\psi_1,em}J_\nu^{V,\psi_2,}}{(q_2^2-\hat{m}_Z^2+i\hat{\Gamma}_Z\hat{m}_Z)q_1^2}\right)\times \\
& \left[\left(s_{2\hat{\theta}}(C_{HB}-C_{HW})+c_{2\hat{\theta}}C_{HWB}\right)(q_1\cdot q_2\, g^{\mu\nu}-q_2^\mu q_1^\nu)+\right. \\
& \left.+\left(s_{2\hat{\theta}}(C_{H\tilde{W}}-C_{H\tilde{B}})-c_{2\hat{\theta}}C_{H\tilde{W}B}\right)\epsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}\right], \qquad (A.138)\\
\mathcal{A}_{VBF,\gamma\gamma} =& \frac{4i\bar{e}^2\bar{v}_T}{q_1^2q_2^2}Q_{\psi_1}Q_{\psi_2}\,J_\mu^{\psi_1,em}J_\nu^{\psi_2,em}\left[\left(s_{\hat{\theta}}^2C_{HW}+c_{\hat{\theta}}^2C_{HB}-c_{\hat{\theta}}s_{\hat{\theta}}C_{HWB}\right)(q_1\cdot q_2\, g^{\mu\nu}-q_2^\mu q_1^\nu)+\right. \\
& \left.+\left(s_{\hat{\theta}}^2C_{H\tilde{W}}+c_{\hat{\theta}}^2C_{H\tilde{B}}-c_{\hat{\theta}}s_{\hat{\theta}}C_{H\tilde{W}B}\right)\epsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}\right]. \qquad (A.139)
\end{aligned}
$$

The total shift in the neutral-current VBF production can then be written as

$$
\begin{aligned}
\delta\mathcal{A}_{VBF,\text{n.c.}} =& \frac{i\bar{g}_2^3\hat{m}_Z}{c_{\hat{\theta}}^3(q_1^2-\hat{m}_Z^2+i\hat{\Gamma}_Z\hat{m}_Z)(q_2^2-\hat{m}_Z^2+i\hat{\Gamma}_Z\hat{m}_Z)}\cdot \\
& \left[\delta J_\mu^{Z,\psi_1}(J_\nu^{Z,\psi_2}T_Z^{\mu\nu})^{SM}+\delta J_\nu^{Z,\psi_2}(J_\mu^{Z,\psi_1}T_Z^{\mu\nu})^{SM}+(J_\mu^{Z,\psi_1}J_\nu^{Z,\psi_2})^{SM}\delta T_Z^{\mu\nu}\right]+ \\
& +\mathcal{A}_{VBF,Z\gamma}+\mathcal{A}_{VBF,\gamma\gamma}\,.
\end{aligned}
\qquad (A.140)
$$

For the charged current case $f_1^{W,SM}(q_1^2,q_2^2)\equiv 1$, $f_{3,4}^{W,SM}(q_1^2,q_2^2)\equiv 0$ and in the SMEFT

$$
\begin{aligned}
f_1^W(q_1^2,q_2^2) =& 1+\delta D^W(q_1^2)+\delta D^W(q_2^2)+ \\
& +\bar{v}_T^2\left(C_{H\Box}-\left(2+\frac{1}{c_{2\hat{\theta}}}\right)\frac{C_{HD}}{4}+\frac{C_{ll}-2C_{Hl}^{(3)}}{2c_{2\hat{\theta}}}-\frac{s_{2\hat{\theta}}}{c_{2\hat{\theta}}}C_{HWB}\right), \qquad (A.141)\\
f_3^W(q_1^2,q_2^2) =& -\frac{2\bar{v}_T^2}{\hat{m}_W^2}C_{HW}\,, \qquad (A.142)\\
f_4^W(q_1^2,q_2^2) =& \frac{2\bar{v}_T^2}{\hat{m}_W^2}C_{H\tilde{W}}\,. \qquad (A.143)
\end{aligned}
$$

with $\delta D^W(s)$ defined in Eqn. A.70. Since there are no additional contributions from different diagrams, in this case the overall shift in the amplitude, compared to the SM is

$$
\begin{aligned}
\delta\mathcal{A}_{VBF,\text{c.c.}} =& \frac{i\bar{g}_2^3m_W}{2(q_1^2-m_W^2)(q_2^2-m_W^2)}\left[\delta J_\mu^{W+,q}(J_\nu^{W-,q}T_W^{\mu\nu})^{SM}+\delta J_\nu^{W-,q}(J_\mu^{W+,q}T_W^{\mu\nu})^{SM}+\right. \\
& \left.+(J_\mu^{W+,q}J_\nu^{W-,q})^{SM}\delta T_W^{\mu\nu}\right],
\end{aligned}
\qquad (A.144)
$$

where the expression has been specialized to the quark current case.

$\boldsymbol{h \to \bar{f} f}$

The decay width of the Higgs boson into a fermion pair is given by

$$\Gamma(h \to \bar{f}\, f) = \frac{(\bar{g}_{hf\bar{f}})^2\, \hat{m}_h\, N_c}{8\,\pi} \left(1 - 4\frac{m_f^2}{\hat{m}_h^2}\right)^{3/2}, \tag{A.145}$$

where N_c is the number of colors of the final state fermion f. The generic coupling is [185]

$$\left[\bar{g}_{hf\bar{f}}\right]_{rs} = \frac{1}{\sqrt{2}} \left[\hat{Y}_f\right]_{rs} \left[1 + \bar{v}_T^2 \left(C_{H\Box} - \frac{C_{HD}}{4}\right) - \frac{\delta G_F}{\sqrt{2}}\right] - \frac{\bar{v}_T^2}{\sqrt{2}} C^*_{\substack{fH \\ rs}}, \qquad f = u, d, e \tag{A.146}$$

with $[\hat{Y}_f]_{rs} = 2^{3/4}\sqrt{\hat{G}_F}\,[m_f]_{rr}\,\delta_{rs}$ as defined in Eqn. A.33.

$\boldsymbol{h \to V \bar{f} f'}$

The decay of the Higgs into an experimentally reconstructed nearly on-shell vector boson $V = \{W^\pm, Z\}$ and a fermion pair $\bar{f} f'$ proceeds through the same diagrams that give Vh associated production. The amplitude of this process can then be decomposed in the same way [326] exploiting the narrow width of the intermediate vector boson.

$$\mathcal{A}_{h \to V \bar{\psi}\psi'} = \frac{i N_V g_V^2 m_V}{q^2 - m_V^2}\, J_\nu^{V,\psi}\, \epsilon_\mu^*\, T_V^{\mu\nu} \tag{A.147}$$

where q is the four-momentum of the fermion pair in the final state and the momentum p appearing in the decomposition of $T_V^{\mu\nu}$ (Eqn. A.117) is again that of the outgoing V boson.

Because the two processes have the same diagram forms, the relative correction to the partial width is also analogous to that of A.132. The contributions from the 4-point contact interactions and from a γ-mediated diagram in $Zf\bar{f}$ production:

$$\begin{aligned} \frac{d\Gamma(h \to V\psi\bar{\psi})/dq^2}{d\Gamma(h \to V\psi\bar{\psi})^{SM}/dq^2} = & \left|f_1^V(q^2)\right|^2 + 3\ \mathrm{Re}\left[f_1^V(q^2) f_3^{V*}(q^2)\right] \frac{m_V^2(q^2 + m_V^2 - m_h^2)}{|\vec{p}|^2 + 3m_V^2} \\ & + \frac{m_V^2 q^2}{|\vec{p}|^2 + 3m_V^2} \left[\left|f_3^V(q^2)\right|^2 (3m_V^2 + 2|\vec{p}|^2) + 2|\vec{p}|^2 \left|f_4^V(q^2)\right|\right] \\ & + \frac{2}{(g_{V,SM}^{V,\psi})^2 + (g_{A,SM}^{V,\psi})^2} \left[g_{V,SM}^{V,\psi}\, \delta g_V^{V,\psi} + g_{A,SM}^{V,\psi}\, \delta g_A^{V,\psi}\right], \end{aligned} \tag{A.148}$$

where $|\vec{p}| = [\lambda(m_h^2, m_V^2, q^2)/(4q^2)]^{1/2}$ is the momentum of the V boson in the final state and the differential SM width is [275][115]

$$\begin{aligned} \frac{d\Gamma(h \to V\psi\bar{\psi})^{SM}}{dq^2} = & \frac{N_V^2 g_V^4\, [(g_{V,SM}^{V,\psi})^2 + g_{A,SM}^{V,\psi})^2]}{96\pi^3 m_h^3}\ \frac{3m_V^2 + |\vec{p}|^2}{(q^2 - m_V^2)^2}\, (q^2)^{3/2}\, |\vec{p}| \\ = & \frac{N_V^2 g_V^4\, [(g_{V,SM}^{V,\psi})^2 + g_{A,SM}^{V,\psi})^2]}{768\pi^3 m_h^3}\ \frac{12q^2 m_V^2 + \lambda(m_h^2, m_V^2, q^2)}{(q^2 - m_V^2)^2}\ \lambda^{1/2}(m_h^2, m_V^2, q^2). \end{aligned} \tag{A.149}$$

[115] Ref. [275] corrects an overall factor of 1/2 compared to the result in Ref. [326].

The form factors read[116]

$$f_1^Z(q^2) = 1 + \delta D^Z(q^2) + \bar{v}_T^2 \left(C_{H\Box} + \frac{C_{HD}}{4} - C_{Hl}^{(3)} + \frac{C_{ll}}{2} \right) , \tag{A.150}$$

$$f_2^Z(q^2) = -\frac{1}{\hat{m}_Z^2} \delta f_1^Z(q^2) , \tag{A.151}$$

$$f_3^Z(q^2) = -\frac{2\bar{v}_T^2}{\hat{m}_Z^2} \left[c_{\hat{\theta}}^2 C_{HW} + s_{\hat{\theta}}^2 C_{HB} + s_{\hat{\theta}} c_{\hat{\theta}} C_{HWB} \right] , \tag{A.152}$$

$$f_4^Z(q^2) = \frac{2\bar{v}_T^2}{\hat{m}_Z^2} \left(c_{\hat{\theta}}^2 C_{H\tilde{W}} + s_{\hat{\theta}}^2 C_{H\tilde{B}} + s_{\hat{\theta}} c_{\hat{\theta}} C_{H\tilde{W}B} \right) , \tag{A.153}$$

$$f_1^W(q^2) = 1 + \delta D^W(q^2) + \bar{v}_T^2 \left(C_{H\Box} - \left(2 + \frac{1}{c_{2\hat{\theta}}} \right) \frac{C_{HD}}{4} + \frac{C_{ll} - 2C_{Hl}^{(3)}}{2c_{2\hat{\theta}}} - \frac{s_{2\hat{\theta}}}{c_{2\hat{\theta}}} C_{HWB} \right) , \tag{A.154}$$

$$f_2^W(q^2) = -\frac{1}{\hat{m}_Z^2} \delta f_1^W(q^2) , \tag{A.155}$$

$$f_3^W(q^2) = -\frac{2\bar{v}_T^2}{\hat{m}_W^2} C_{HW} , \tag{A.156}$$

$$f_4^W(q^2) = \frac{2\bar{v}_T^2}{\hat{m}_W^2} C_{H\tilde{W}} . \tag{A.157}$$

The contributions from the diagram $h \to Z\gamma^* \to Zf\bar{f}$ and from the 4-point contact interactions $\bar{\psi}\psi hZ$, $\bar{\psi}\psi hW$ have the same form, with an opposite sign, as those given for hV associated production in Eqs. A.123, A.129.

$h \to \gamma\gamma$

At one loop in the SM, the partial width of the Higgs decay into photons is [215, 538]

$$\Gamma^{SM}(h \to \gamma\gamma) = \frac{\sqrt{2}\hat{G}_F \hat{\alpha}^2 \hat{m}_h^3}{16\pi^3} |I^\gamma|^2 \tag{A.158}$$

where I^γ is a Feynman parameter integral including both contributions from the top quark (with leading QCD corrections) and W-boson loops. Its analytic expression is given in Ref [215] and numerically it is $I^\gamma \approx -1.65$ [215]. At tree level in the SMEFT, the decay rate is modified as [185, 215]

$$\frac{\Gamma(h \to \gamma\gamma)}{\Gamma^{SM}(h \to \gamma\gamma)} \simeq \left| 1 + \frac{8\pi^2 \bar{v}_T^2}{I^\gamma} \mathscr{C}_{\gamma\gamma} \right|^2 + \left| \frac{8\pi^2 \bar{v}_T^2}{I^\gamma} \widetilde{\mathscr{C}}_{\gamma\gamma} \right|^2 \tag{A.159}$$

[116] Note the difference in sign in $f_3(q^2)$ and $f_4(q^2)$ w.r.t. the Vh production case, which is due to the fact that here p and q are both outgoing momenta, while in the production q is incoming and p is outgoing. The form factors for $h \to Vf\bar{f}$ decay coincide with those for VBF production in the limit of massless fermions.

where

$$\mathscr{C}_{\gamma\gamma} = \frac{1}{\bar{g}_2^2} C_{HW} + \frac{1}{\bar{g}_1^2} C_{HB} - \frac{1}{\bar{g}_1 \bar{g}_2} C_{HWB} \tag{A.160}$$

$$\tilde{\mathscr{C}}_{\gamma\gamma} = \frac{1}{\bar{g}_2^2} C_{H\tilde{W}} + \frac{1}{\bar{g}_1^2} C_{H\tilde{B}} - \frac{1}{\bar{g}_1 \bar{g}_2} C_{H\tilde{W}B}. \tag{A.161}$$

$h \to Z\gamma$

The one-loop rate for the Higgs decay into $Z\gamma$ in the SM reads [538]

$$\Gamma^{SM}(h \to Z\gamma) = \frac{\sqrt{2} G_F \hat{\alpha}^2 \hat{m}_h^3}{8\pi^3} \left(1 - \frac{\hat{m}_Z^2}{\hat{m}_h^2}\right)^3 \left|I^{Z\gamma}\right|^2, \tag{A.162}$$

where, including both top and W loop contributions, $I^{Z\gamma} \approx -2.87$ [215, 538]. The modification of the decay rate in the SMEFT has the form [185, 215]

$$\frac{\Gamma(h \to \gamma Z)}{\Gamma^{SM}(h \to \gamma Z)} \simeq \left|1 + \frac{8\pi^2 \bar{v}_T^2}{I^{Z\gamma}} \mathscr{C}_{\gamma Z}\right|^2 + \left|\frac{8\pi^2 \bar{v}_T^2}{I^{Z\gamma}} \widetilde{\mathscr{C}}_{\gamma Z}\right|^2 \tag{A.163}$$

with

$$\mathscr{C}_{\gamma Z} = \frac{1}{\bar{g}_1 \bar{g}_2} C_{HW} - \frac{1}{\bar{g}_1 \bar{g}_2} C_{HB} - \left(\frac{1}{2\bar{g}_1^2} - \frac{1}{2\bar{g}_2^2}\right) C_{HWB} \tag{A.164}$$

$$\tilde{\mathscr{C}}_{\gamma Z} = \frac{1}{\bar{g}_1 \bar{g}_2} C_{H\tilde{W}} - \frac{1}{\bar{g}_1 \bar{g}_2} C_{H\tilde{B}} - \left(\frac{1}{2\bar{g}_1^2} - \frac{1}{2\bar{g}_2^2}\right) C_{H\tilde{W}B}. \tag{A.165}$$

Note the inverse gauge coupling dependence that follows from the choice to not scale the Wilson coefficients by a gauge coupling.

A.8 Top quark properties

Top width

The width of the top quark can be computed at tree-level in the SMEFT using the narrow width approximation for the W boson:

$$\bar{\Gamma}(t \to b \bar{f}_p f_r) = \bar{\Gamma}(t \to b W^+) \overline{\mathrm{Br}}(W^+ \to \bar{f}_p f_r)\,. \tag{A.166}$$

The narrow width approximation and the SMEFT approximation do not commute. We perform first the narrow width approximation and then the SMEFT expansion, finding, in the notation of Sec. A.4

$$\begin{aligned}\bar{\Gamma}(t \to b \bar{f}_p f_r) = {} & \Gamma(t \to b W^+)^{SM} \frac{\Gamma^{SM}_{W \to \bar{f}_p f_r}}{\Gamma^{SM}_W} + \delta\Gamma(t \to b W^+) \frac{\Gamma^{SM}_{W \to \bar{f}_p f_r}}{\Gamma^{SM}_W} \\ & + \Gamma(t \to b W^+)^{SM} \frac{\delta\Gamma_{W \to \bar{f}_p f_r}}{\Gamma^{SM}_W} - \Gamma(t \to b W^+)^{SM} \frac{\Gamma^{SM}_{W \to \bar{f}_p f_r}}{\Gamma^{SM}_W} \frac{\delta\Gamma_W}{\Gamma^{SM}_W}\,.\end{aligned} \tag{A.167}$$

Summing over the polarizations of the W boson one has

$$\Gamma(t \to bW^+)^{SM} = \frac{\bar{g}_2^2}{64\pi} \frac{\hat{m}_t}{\hat{m}_W^2} \lambda^{1/2}(\hat{m}_t^2, \hat{m}_b^2, \hat{m}_W^2) |V_{tb}|^2 \left(1 + x_W^2 - 2x_b^2 - 2x_W^4 + x_W^2 x_b^2 + x_b^4\right), \tag{A.168}$$

$$\begin{aligned}\delta\Gamma(t \to bW^+) =& \frac{\bar{g}_2^2}{64\pi} \frac{\hat{m}_t}{\hat{m}_W^2} \lambda^{1/2}(\hat{m}_t^2, \hat{m}_b^2, \hat{m}_W^2) \Bigg[\\ & 2\mathrm{Re}\left[V_{tb}^* \left(\delta(g_V^{W,q})_{33} + \delta(g_A^{W,q})_{33}\right)\right] (1 + x_W^2 - 2x_b^2 - 2x_W^4 + x_W^2 x_b^2 + x_b^4) \\ & - 12 x_W^2 x_b \, \mathrm{Re}\left[V_{tb}^* \left(\delta(g_V^{W,q})_{33} - \delta(g_A^{W,q})_{33}\right)\right] \\ & + \frac{6}{\hat{G}_F} x_W (1 - x_W^2 - x_b^2) \left(\mathrm{Re}\left(V_{tb} C_{\substack{uW^* \\ 33}}\right) - \mathrm{Re}\left(V_{tb} C_{\substack{dW \\ 33}}\right) x_b\right)\Bigg],\end{aligned} \tag{A.169}$$

where $x_i = \hat{m}_i/\hat{m}_t$, $\lambda(x, y, z)$ is defined in Eq. A.72 and $\delta g_{V,A}^{W,q}$ are the shifts for the W couplings to quarks defined in Eq. A.29. These results are in agreement with previous calculations, see e.g. [573, 575]. Note that if $\mathcal{Q}_{Hud}$ is included, it also contributes to the latter as

$$\delta(g_V^{W,q})_{rr} = -\frac{\bar{v}_T^2}{4} C^*_{\substack{Hud \\ rr}} + \dots, \qquad \delta(g_A^{W,q})_{rr} = \frac{\bar{v}_T^2}{4} C_{\substack{Hud \\ rr}} + \dots, \tag{A.170}$$

making these quantities complex.

References

[1] T. Appelquist, J. Carazzone, *Phys. Rev.* **D11**, 2856 (1975).

[2] J. M. Cornwall, D. N. Levin, G. Tiktopoulos, *Phys. Rev.* **D10**, 1145 (1974). [Erratum: Phys. Rev. D11 972 (1975)].

[3] C. E. Vayonakis, *Lett. Nuovo Cim.* **17**, 383 (1976).

[4] B. W. Lee, C. Quigg, H. B. Thacker, *Phys. Rev. Lett.* **38**, 883 (1977).

[5] B. W. Lee, C. Quigg, H. B. Thacker, *Phys. Rev.* **D16**, 1519 (1977).

[6] M. S. Chanowitz, M. A. Furman, I. Hinchliffe, *Phys. Lett.* **B78**, 285 (1978).

[7] M. S. Chanowitz, M. A. Furman, I. Hinchliffe, *Nucl. Phys.* **B153**, 402 (1979).

[8] E. Eichten, I. Hinchliffe, K. D. Lane, C. Quigg, *Rev. Mod. Phys.* **56**, 579 (1984). [Addendum: Rev. Mod. Phys. 58 1065 (1986)].

[9] CMS collaboration, S. Chatrchyan, *et al.*, *Phys. Rev. Lett.* **110**, 081803 (2013), 1212.6639.

[10] ATLAS collaboration, G. Aad, *et al.*, *Phys. Lett.* **B726**, 120 (2013), 1307.1432.

[11] CMS collaboration, S. Chatrchyan, *et al.*, *Phys. Rev.* **D89**, 092007 (2014), 1312.5353.

[12] ATLAS collaboration, G. Aad, *et al.*, *Phys. Lett.* **B716**, 1 (2012), 1207.7214.

[13] CMS collaboration, S. Chatrchyan, *et al.*, *Phys. Lett.* **B716**, 30 (2012), 1207.7235.

[14] CDF collaboration, F. Abe, *et al.*, *Phys. Rev. Lett.* **74**, 2626 (1995), hep-ex/9503002.

[15] D0 collaboration, S. Abachi, *et al.*, *Phys. Rev. Lett.* **74**, 2632 (1995), hep-ex/9503003.

[16] DONUT collaboration, K. Kodama, *et al.*, *Phys. Lett.* **B504**, 218 (2001), hep-ex/0012035.

[17] S. Weinberg, *The quantum theory of fields. Vol. 2: Modern applications* (Cambridge University Press, 2013).

[18] E. Witten, *Bull. Amer. Math. Soc.* **44**, 361 (2007).

[19] V. L. Ginzburg, L. D. Landau, *Zh. Eksp. Teor. Fiz.* **20**, 1064 (1950).

[20] P. W. Higgs, *Phys. Rev. Lett.* **13**, 508 (1964).

[21] F. Englert, R. Brout, *Phys. Rev. Lett.* **13**, 321 (1964).

[22] G. S. Guralnik, C. R. Hagen, T. W. B. Kibble, *Phys. Rev. Lett.* **13**, 585 (1964).

[23] J. Bardeen, L. N. Cooper, J. R. Schrieffer, *Phys. Rev.* **108**, 1175 (1957).

[24] P. W. Anderson, *Phys. Rev.* **130**, 439 (1963).

[25] Y. Jiang, M. Trott, 1612.02040.

[26] E. E. Jenkins, A. V. Manohar, M. Trott, *Phys.Lett.* **B726**, 697 (2013), 1309.0819.

[27] R. Alonso, E. E. Jenkins, A. V. Manohar, *Phys. Lett.* **B739**, 95 (2014), 1409.0868.

[28] C. Cheung, C.-H. Shen, *Phys. Rev. Lett.* **115**, 071601 (2015), 1505.01844.

[29] R. S. Gupta, H. Rzehak, J. D. Wells, *Phys. Rev.* **D86**, 095001 (2012), 1206.3560.

[30] S. Dawson, *et al.* (2013). 1310.8361.

[31] LHC Higgs Cross Section Working Group collaboration, D. de Florian, *et al.*, 1610.07922.

[32] S. Weinberg, *Proceedings of the Summer Institute* **1**, 285 (1970).

[33] H. Georgi, *Ann. Rev. Nucl. Part. Sci.* **43**, 209 (1993).

[34] E. E. Jenkins, A. V. Manohar, M. Trott, *JHEP* **1309**, 063 (2013), 1305.0017.

[35] M. Trott, https://sites.google.com/a/umich.edu/the-4th-mctp-spring-symposium/home. 2015 Talk, pages 39-40.

[36] M. Le Gendre, *Tome X, pp. 411–435 (Paris, 1785)* .

[37] J. D. Jackson, *Classical Electrodynamics* (Wiley, 1998).

[38] D. Liu, A. Pomarol, R. Rattazzi, F. Riva, *JHEP* **11**, 141 (2016), 1603.03064. (journal version only).

[39] S. Dimopoulos, J. Preskill, *Nucl. Phys.* **B199**, 206 (1982).

[40] D. B. Kaplan, H. Georgi, *Phys. Lett.* **B136**, 183 (1984).

[41] D. B. Kaplan, H. Georgi, S. Dimopoulos, *Phys. Lett.* **B136**, 187 (1984).

[42] H. Georgi, D. B. Kaplan, P. Galison, *Phys. Lett.* **B143**, 152 (1984).

[43] H. Georgi, D. B. Kaplan, *Phys. Lett.* **B145**, 216 (1984).

[44] M. J. Dugan, H. Georgi, D. B. Kaplan, *Nucl. Phys.* **B254**, 299 (1985).

[45] Y. Hosotani, *Phys. Lett.* **B126**, 309 (1983).

[46] N. S. Manton, *Nucl. Phys.* **B158**, 141 (1979).

[47] N. Arkani-Hamed, *et al.*, *JHEP* **08**, 021 (2002), hep-ph/0206020.

[48] N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, *JHEP* **07**, 034 (2002), hep-ph/0206021.

[49] I. Antoniadis, K. Benakli, M. Quiros, *New J. Phys.* **3**, 20 (2001), hep-th/0108005.

[50] L. J. Hall, Y. Nomura, D. Tucker-Smith, *Nucl. Phys.* **B639**, 307 (2002), hep-ph/0107331.

[51] C. A. Scrucca, M. Serone, L. Silvestrini, *Nucl. Phys.* **B669**, 128 (2003), hep-ph/0304220.

[52] C. A. Scrucca, M. Serone, L. Silvestrini, A. Wulzer, *JHEP* **02**, 049 (2004), hep-th/0312267.

[53] R. Contino, Y. Nomura, A. Pomarol, *Nucl. Phys.* **B671**, 148 (2003), hep-ph/0306259.

[54] K. Agashe, A. Delgado, M. J. May, R. Sundrum, *JHEP* **08**, 050 (2003), hep-ph/0308036.

[55] B. A. Lippmann, J. Schwinger, *Phys. Rev.* **79**, 469 (1950).

[56] W. E. Caswell, G. P. Lepage, *Phys. Lett.* **167B**, 437 (1986).

[57] M. E. Luke, A. V. Manohar, *Phys. Rev.* **D55**, 4129 (1997), hep-ph/9610534.

[58] S. Weinberg, *Phys. Lett.* **B251**, 288 (1990).

[59] D. B. Kaplan, M. J. Savage, M. B. Wise, *Phys. Lett.* **B424**, 390 (1998), nucl-th/9801034.

[60] D. B. Kaplan, M. J. Savage, M. B. Wise, *Nucl. Phys.* **B534**, 329 (1998), nucl-th/9802075.

[61] D. B. Kaplan, *Beyond the standard model 5. Proceedings, 5th Conference, Balholm, Norway, April 29-May 4, 1997* (1995). nucl-th/9506035.

[62] A. V. Manohar, *Lect. Notes Phys.* **479**, 311 (1997), hep-ph/9606222.

[63] J. Polchinski (1992). hep-th/9210046.

[64] A. Pich (1998). hep-ph/9806303.

[65] I. Z. Rothstein (2003). hep-ph/0308266.

[66] W. Skiba (2011). 1006.2142.

[67] C. P. Burgess, *Ann. Rev. Nucl. Part. Sci.* **57**, 329 (2007), hep-th/0701053.

[68] S. R. Coleman, J. Wess, B. Zumino, *Phys. Rev.* **177**, 2239 (1969).

[69] C. G. Callan, Jr., S. R. Coleman, J. Wess, B. Zumino, *Phys. Rev.* **177**, 2247 (1969).

[70] E. Witten, *Nucl. Phys.* **B122**, 109 (1977).

[71] S. Weinberg, *Physica* **A96**, 327 (1979).

[72] F. J. Gilman, M. B. Wise, *Phys. Rev.* **D20**, 2392 (1979).

[73] S. Weinberg, *Phys. Lett.* **B91**, 51 (1980).

[74] G. 't Hooft, M. J. G. Veltman, *Nucl. Phys.* **B44**, 189 (1972).

[75] N. N. Bogoliubov, O. S. Parasiuk, *Acta Math.* **97**, 227 (1957).

[76] J. Polchinski, *Nucl. Phys.* **B231**, 269 (1984).

[77] S. Weinberg, *Phys. Rev.* **118**, 838 (1960).

[78] K. Costello, *Renormalization and Effective Field Theory*, Mathematical surveys and monographs (American Mathematical Soc.).

[79] G. Kallen, *Helv. Phys. Acta* **25**, 417 (1952).

[80] H. Lehmann, *Il Nuovo Cimento (1943-1954)* **11**, 342 (1954).

[81] N. Isgur, D. Scora, B. Grinstein, M. B. Wise, *Phys. Rev.* **D39**, 799 (1989).

[82] N. Isgur, M. B. Wise, *Phys. Lett.* **B232**, 113 (1989).

[83] T. Mannel, M. Neubert, *Phys. Rev.* **D50**, 2037 (1994), hep-ph/9402288.

[84] C. W. Bauer, S. Fleming, M. E. Luke, *Phys. Rev.* **D63**, 014006 (2000), hep-ph/0005275.

[85] C. W. Bauer, S. Fleming, D. Pirjol, I. W. Stewart, *Phys. Rev.* **D63**, 114020 (2001), hep-ph/0011336.

[86] K. Symanzik, *Commun. Math. Phys.* **34**, 7 (1973).

[87] J. C. Collins, *Renormalization: An Introduction to Renormalization, the Renormalization Group and the Operator-Product Expansion*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, 1984).

[88] T. Kinoshita, *J. Math. Phys.* **3**, 650 (1962).

[89] T. D. Lee, M. Nauenberg, *Phys. Rev.* **133**, B1549 (1964).

[90] S. L. Adler, *Phys. Rev.* **177**, 2426 (1969).

[91] J. C. Collins, F. Wilczek, A. Zee, *Phys. Rev.* **D18**, 242 (1978).

[92] W. J. Marciano, *Nucl. Phys.* **B84**, 132 (1975).

[93] D. Toussaint, *Phys. Rev.* **D18**, 1626 (1978).

[94] M. J. G. Veltman, *Acta Phys. Polon.* **B8**, 475 (1977).

[95] M. J. G. Veltman, *Nucl. Phys.* **B123**, 89 (1977).

[96] D. A. Ross, M. J. G. Veltman, *Nucl. Phys.* **B95**, 135 (1975).

[97] T. Appelquist, C. W. Bernard, *Phys. Rev.* **D22**, 200 (1980).

[98] A. C. Longhitano, *Phys. Rev.* **D22**, 1166 (1980).

[99] A. C. Longhitano, *Nucl. Phys.* **B188**, 118 (1981).

[100] T. Appelquist, G.-H. Wu, *Phys. Rev.* **D48**, 3235 (1993), hep-ph/9304240.

[101] F. Feruglio, *Int.J.Mod.Phys.* **A8**, 4937 (1993), hep-ph/9301281.

[102] M. J. Herrero, E. Ruiz Morales, *Nucl. Phys.* **B437**, 319 (1995), hep-ph/9411207.

[103] S. Dittmaier, C. Grosse-Knetter, *Nucl. Phys.* **B459**, 497 (1996), hep-ph/9505266.

[104] T. Inami, C. S. Lim, *Prog. Theor. Phys.* **65**, 297 (1981). [Erratum: Prog. Theor. Phys. 65 1772 (1981)].

[105] G. Buchalla, A. J. Buras, *Nucl. Phys.* **B398**, 285 (1993).

[106] G. Buchalla, A. J. Buras, *Nucl. Phys.* **B400**, 225 (1993).

[107] G. Buchalla, A. J. Buras, *Nucl. Phys.* **B412**, 106 (1994), hep-ph/9308272.

[108] G. Buchalla, A. J. Buras, M. E. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996), hep-ph/9512380.

[109] S. L. Glashow, J. Iliopoulos, L. Maiani, *Phys. Rev.* **D2**, 1285 (1970).

[110] F. J. Gilman, M. B. Wise, *Phys. Lett.* **83B**, 83 (1979).

[111] M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, *Nucl. Phys.* **B120**, 316 (1977).

[112] A. I. Vainshtein, V. I. Zakharov, M. A. Shifman, *JETP Lett.* **22**, 55 (1975). [Pisma Zh. Eksp. Teor. Fiz. 22 123 (1975)].

[113] M. Bando, T. Kugo, N. Maekawa, H. Nakano, *Phys. Lett.* **B301**, 83 (1993), hep-ph/9210228.

[114] M. Bando, T. Kugo, N. Maekawa, H. Nakano, *Prog. Theor. Phys.* **90**, 405 (1993), hep-ph/9210229.

[115] J. A. Casas, V. Di Clemente, M. Quiros, *Nucl. Phys.* **B553**, 511 (1999), hep-ph/9809275.

[116] E. E. Jenkins, A. V. Manohar, M. Trott, *JHEP* **1310**, 087 (2013), 1308.2627.

[117] K. G. Wilson, *Phys. Rev.* **B4**, 3174 (1971).

[118] K. G. Wilson, *Phys. Rev.* **B4**, 3184 (1971).

[119] P. Minkowski, *Phys. Lett.* **67B**, 421 (1977).

[120] M. Gell-Mann, P. Ramond, R. Slansky, *Conf. Proc.* **C790927**, 315 (1979), 1306.4669.

[121] T. Yanagida, *Conf. Proc.* **C7902131**, 95 (1979).

[122] R. N. Mohapatra, G. Senjanovic, *Phys. Rev. Lett.* **44**, 912 (1980).

[123] A. Broncano, M. B. Gavela, E. E. Jenkins, *Phys. Lett.* **B552**, 177 (2003), hep-ph/0210271. [Erratum: Phys. Lett. B636 332 (2006)].

[124] M. B. Gavela, T. Hambye, D. Hernandez, P. Hernandez, *JHEP* **09**, 038 (2009), 0906.1461.

[125] M. B. Gavela, D. Hernandez, T. Ota, W. Winter, *Phys. Rev.* **D79**, 013007 (2009), 0809.3451.

[126] A. Abada, C. Biggio, F. Bonnet, M. B. Gavela, T. Hambye, *JHEP* **12**, 061 (2007), 0707.4058.

[127] A. Broncano, M. B. Gavela, E. E. Jenkins, *Nucl. Phys.* **B672**, 163 (2003), hep-ph/0307058.

[128] F. Bonnet, M. Hirsch, T. Ota, W. Winter, *JHEP* **07**, 153 (2012), 1204.5862.

[129] F. Bonnet, D. Hernandez, T. Ota, W. Winter, *JHEP* **10**, 076 (2009), 0907.3143.

[130] F. del Aguila, S. Bar-Shalom, A. Soni, J. Wudka, *Phys. Lett.* **B670**, 399 (2009), 0806.0876.

[131] F. del Aguila, A. Aparici, S. Bhattacharya, A. Santamaria, J. Wudka, *JHEP* **06**, 146 (2012), 1204.5986.

[132] S. Bhattacharya, J. Wudka, *Phys. Rev.* **D94**, 055022 (2016), 1505.05264. [Erratum: Phys. Rev. D95, no.3 039904 (2017)].

[133] P. W. Angel, N. L. Rodd, R. R. Volkas, *Phys. Rev.* **D87**, 073007 (2013), 1212.6111.

[134] S. Weinberg, *Phys.Rev.Lett.* **43**, 1566 (1979).

[135] F. Wilczek, A. Zee, *Phys. Rev. Lett.* **43**, 1571 (1979).

[136] G. Elgaard-Clausen, M. Trott, *JHEP* **11**, 088 (2017), 1703.04415.

[137] A. V. Manohar, *Phys. Rev.* **D68**, 114019 (2003), hep-ph/0309176.

[138] S. Dawson, E. Furlan, *Phys. Rev.* **D86**, 015021 (2012), 1205.4733.

[139] B. Henning, X. Lu, H. Murayama, *JHEP* **01**, 023 (2016), 1412.1837.

[140] M. Gorbahn, J. M. No, V. Sanz, *JHEP* **10**, 036 (2015), 1502.07352.

[141] J. Brehmer, A. Freitas, D. Lopez-Val, T. Plehn, *Phys. Rev.* **D93**, 075014 (2016), 1510.03443.

[142] A. Freitas, D. López-Val, T. Plehn, *Phys. Rev.* **D94**, 095007 (2016), 1607.08251.

[143] S. Dawson, A. Ismail, I. Low, *Phys. Rev.* **D91**, 115008 (2015), 1504.05596.

[144] C.-Y. Chen, S. Dawson, I. M. Lewis, *Phys. Rev.* **D90**, 035016 (2014), 1406.3349.

[145] C.-Y. Chen, S. Dawson, E. Furlan, 1703.06134.

[146] S. Dawson, C. W. Murphy, 1704.07851.

[147] J. de Blas, J. C. Criado, M. Perez-Victoria, J. Santiago, 1711.10391.

[148] M. K. Gaillard, *Nucl. Phys.* **B268**, 669 (1986).

[149] O. Cheyette, *Nucl. Phys.* **B297**, 183 (1988).

[150] F. del Aguila, Z. Kunszt, J. Santiago, *Eur. Phys. J.* **C76**, 244 (2016), 1602.00126.

[151] M. Boggia, R. Gomez-Ambrosio, G. Passarino, *JHEP* **05**, 162 (2016), 1603.03660.

[152] A. Drozd, J. Ellis, J. Quevillon, T. You, *JHEP* **03**, 180 (2016), 1512.03003.

[153] B. Henning, X. Lu, H. Murayama, 1604.01019.

[154] S. A. R. Ellis, J. Quevillon, T. You, Z. Zhang, *Phys. Lett.* **B762**, 166 (2016), 1604.02445.

[155] J. Fuentes-Martin, J. Portoles, P. Ruiz-Femenia, *JHEP* **09**, 156 (2016), 1607.02142.

[156] S. Dittmaier, C. Grosse-Knetter, *Phys. Rev.* **D52**, 7276 (1995), hep-ph/9501285.

[157] M. Beneke, V. A. Smirnov, *Nucl. Phys.* **B522**, 321 (1998), hep-ph/9711391.

[158] V. A. Smirnov, *Springer Tracts Mod. Phys.* **177**, 1 (2002).

[159] B. Jantzen, *JHEP* **12**, 076 (2011), 1111.2589.

[160] A. V. Manohar, I. W. Stewart, *Phys. Rev.* **D76**, 074002 (2007), hep-ph/0605001.

[161] Z. Zhang, *JHEP* **05**, 152 (2017), 1610.00710.

[162] S. A. R. Ellis, J. Quevillon, T. You, Z. Zhang, *JHEP* **08**, 054 (2017), 1706.07765.

[163] D. M. Pierce, J. A. Bagger, K. T. Matchev, R.-j. Zhang, *Nucl. Phys.* **B491**, 3 (1997), hep-ph/9606211.

[164] J. D. Wells, Z. Zhang, 1711.04774.

[165] G. 't Hooft, *Prog. Theor. Phys. Suppl.* **170**, 56 (2007).

[166] A. Petermann, *Helv. Phys. Acta* **26**, 499 (1953).

[167] L. P. Kadanoff, *Physics* **2**, 263 (1966).

[168] K. G. Wilson, M. E. Fisher, *Phys. Rev. Lett.* **28**, 240 (1972).

[169] C. Hartmann, W. Shepherd, M. Trott, *JHEP* **03**, 060 (2017), 1611.09879.

[170] E. E. Jenkins, A. V. Manohar, M. Trott, *JHEP* **1401**, 035 (2014), 1310.4838.

[171] B. S. DeWitt, *Phys. Rev.* **162**, 1195 (1967).

[172] G. 't Hooft, *Nucl. Phys.* **B62**, 444 (1973).

[173] L. F. Abbott, *Acta Phys. Polon.* **B13**, 33 (1982).

[174] W. A. Bardeen (1995). http://lss.fnal.gov/cgi-bin/find_paper.pl?conf-95-391.

[175] K. A. Meissner, H. Nicolai, *Phys. Lett.* **B648**, 312 (2007), hep-th/0612165.

[176] R. Foot, A. Kobakhidze, K. L. McDonald, R. R. Volkas, *Phys. Rev.* **D77**, 035006 (2008), 0709.2750.

[177] M. Shaposhnikov, D. Zenhausern, *Phys. Lett.* **B671**, 162 (2009), 0809.3406.

[178] M. Lindner, S. Schmidt, J. Smirnov, *JHEP* **10**, 177 (2014), 1405.6204.

[179] A. J. Helmboldt, P. Humbert, M. Lindner, J. Smirnov, 1603.03603.

[180] H. Davoudiasl, I. M. Lewis, *Phys. Rev.* **D90**, 033003 (2014), 1404.6260.

[181] l. Brivio, M. Trott, 1703.10924.

[182] S. L. Glashow, *Nucl. Phys.* **22**, 579 (1961).

[183] S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967).

[184] A. Salam, *Conf. Proc.* **C680519**, 367 (1968).

[185] R. Alonso, E. E. Jenkins, A. V. Manohar, M. Trott, *JHEP* **1404**, 159 (2014), 1312.2014.

[186] G. 't Hooft, *Nucl. Phys.* **B79**, 276 (1974).

[187] A. M. Polyakov, *JETP Lett.* **20**, 194 (1974). [Pisma Zh. Eksp. Teor. Fiz. 20 430 (1974)].

[188] A. A. Belavin, A. M. Polyakov, A. S. Schwartz, Y. S. Tyupkin, *Phys. Lett.* **B59**, 85 (1975).

[189] G. 't Hooft, *Phys. Rev. Lett.* **37**, 8 (1976).

[190] R. D. Peccei, H. R. Quinn, *Phys. Rev.* **D16**, 1791 (1977).

[191] C. G. Callan, Jr., R. F. Dashen, D. J. Gross, *Phys. Lett.* **B63**, 334 (1976).

[192] F. Zwicky, *Helv. Phys. Acta* **6**, 110 (1933). [Gen. Rel. Grav. 41 207 (2009)].

[193] V. C. Rubin, W. K. Ford, Jr., *Astrophys. J.* **159**, 379 (1970).

[194] Planck collaboration, P. A. R. Ade, *et al.*, *Astron. Astrophys.* **594**, A13 (2016), 1502.01589.

[195] V. A. Kuzmin, V. A. Rubakov, M. E. Shaposhnikov, *Phys. Lett.* **155B**, 36 (1985).

[196] A. D. Sakharov, *Pisma Zh. Eksp. Teor. Fiz.* **5**, 32 (1967). [Usp. Fiz. Nauk 161 61 (1991)].

[197] J. N. Bahcall, *Phys. Rev. Lett.* **12**, 300 (1964).

[198] R. Davis, *Phys. Rev. Lett.* **12**, 303 (1964).

[199] Super-Kamiokande collaboration, Y. Fukuda, *et al.*, *Phys. Rev. Lett.* **81**, 1562 (1998), hep-ex/9807003.

[200] SNO collaboration, Q. R. Ahmad, *et al.*, *Phys. Rev. Lett.* **87**, 071301 (2001), nucl-ex/0106015.

[201] SNO collaboration, Q. R. Ahmad, *et al.*, *Phys. Rev. Lett.* **89**, 011301 (2002), nucl-ex/0204008.

[202] W. Buchmuller, D. Wyler, *Nucl.Phys.* **B268**, 621 (1986).

[203] B. Grzadkowski, M. Iskrzynski, M. Misiak, J. Rosiek, *JHEP* **1010**, 085 (2010), 1008.4884.

[204] L. Abbott, M. B. Wise, *Phys.Rev.* **D22**, 2208 (1980).

[205] L. Lehman, *Phys.Rev.* **D90**, 125023 (2014), 1410.4193.

[206] L. Lehman, A. Martin, *JHEP* **02**, 081 (2016), 1510.00372.

[207] B. Henning, X. Lu, T. Melia, H. Murayama, 1512.03433.

[208] L. Lehman, A. Martin, *Phys. Rev.* **D91**, 105014 (2015), 1503.07537.

[209] B. Henning, X. Lu, T. Melia, H. Murayama, 1507.07240.

[210] C. N. Leung, S. T. Love, S. Rao, *Z. Phys.* **C31**, 433 (1986).

[211] J. A. Aguilar-Saavedra, *Nucl. Phys.* **B843**, 638 (2011), 1008.3562. [Erratum: Nucl. Phys. B851 443 (2011)].

[212] K. Hagiwara, S. Ishihara, R. Szalapski, D. Zeppenfeld, *Phys. Rev.* **D48**, 2182 (1993).

[213] C. Arzt, M. B. Einhorn, J. Wudka, *Nucl. Phys.* **B433**, 41 (1995), hep-ph/9405214.

[214] G. J. Gounaris, J. Layssac, F. M. Renard, *Phys. Rev.* **D58**, 075006 (1998), hep-ph/9803422.

[215] A. V. Manohar, M. B. Wise, *Phys. Lett.* **B636**, 107 (2006), hep-ph/0601212.

[216] A. V. Manohar, M. B. Wise, *Phys. Rev.* **D74**, 035009 (2006), hep-ph/0606172.

[217] V. Barger, T. Han, P. Langacker, B. McElrath, P. Zerwas, *Phys. Rev.* **D67**, 115001 (2003), hep-ph/0301097.

[218] G. F. Giudice, C. Grojean, A. Pomarol, R. Rattazzi, *JHEP* **06**, 045 (2007), hep-ph/0703164.

[219] B. Grinstein, M. Trott, *Phys.Rev.* **D76**, 073002 (2007), 0704.1505.

[220] R. Contino, M. Ghezzi, C. Grojean, M. Muhlleitner, M. Spira, *JHEP* **07**, 035 (2013), 1303.3876.

[221] J. Elias-Miró, C. Grojean, R. S. Gupta, D. Marzocca, *JHEP* **05**, 019 (2014), 1312.2928.

[222] J. Gasser, H. Leutwyler, *Annals Phys.* **158**, 142 (1984).

[223] H. Georgi, *Nucl. Phys.* **B361**, 339 (1991).

[224] E. E. Jenkins, A. V. Manohar, *JHEP* **10**, 094 (2009), 0907.4763.

[225] A. Hanany, E. E. Jenkins, A. V. Manohar, G. Torri, *JHEP* **03**, 096 (2011), 1010.3161.

[226] P. Pouliot, *JHEP* **01**, 021 (1999), hep-th/9812015.

[227] C. Grojean, E. E. Jenkins, A. V. Manohar, M. Trott, *JHEP* **04**, 016 (2013), 1301.2588.

[228] R. Alonso, H.-M. Chang, E. E. Jenkins, A. V. Manohar, B. Shotwell, *Phys. Lett.* **B734**, 302 (2014), 1405.0486.

[229] C. P. Burgess, H. M. Lee, M. Trott, *JHEP* **07**, 007 (2010), 1002.2730.

[230] R. Alonso, E. E. Jenkins, A. V. Manohar, *Phys. Lett.* **B754**, 335 (2016), 1511.00724.

[231] R. Alonso, E. E. Jenkins, A. V. Manohar, *JHEP* **08**, 101 (2016), 1605.03602.

[232] C. Hartmann, M. Trott, *Phys. Rev. Lett.* **115**, 191801 (2015), 1507.03568.

[233] L. Berthier, M. Bj ørn, M. Trott, *JHEP* **09**, 157 (2016), 1606.06693.

[234] E. Masso, *JHEP* **10**, 128 (2014), 1406.6376.

[235] R. S. Gupta, A. Pomarol, F. Riva, *Phys. Rev.* **D91**, 035001 (2015), 1405.0181.

[236] A. Falkowski, *Pramana* **87**, 39 (2016), 1505.00046.

[237] R. E. Kallosh, I. V. Tyutin, *Yad. Fiz.* **17**, 190 (1973). [Sov. J. Nucl. Phys. 17 98 (1973)].

[238] G. Passarino, M. Trott, 1610.08356.

[239] G. Passarino, *Eur. Phys. J. Plus* **132**, 16 (2017), 1610.09618.

[240] A. Yu. Morozov, *Sov. J. Nucl. Phys.* **40**, 505 (1984). [Yad. Fiz. 40 788 (1984)].

[241] E. Braaten, C.-S. Li, T.-C. Yuan, *Phys. Rev. Lett.* **64**, 1709 (1990).

[242] K. Hagiwara, R. Szalapski, D. Zeppenfeld, *Phys. Lett.* **B318**, 155 (1993), hep-ph/9308347.

[243] J. Elias-Miró, J. R. Espinosa, E. Masso, A. Pomarol, *JHEP* **08**, 033 (2013), 1302.5661.

[244] C. Zhang, F. Maltoni, *Phys. Rev.* **D88**, 054005 (2013), 1305.7386.

[245] J. Elias-Miro, J. R. Espinosa, E. Masso, A. Pomarol, *JHEP* **11**, 066 (2013), 1308.1879.

[246] G. Altarelli, L. Maiani, *Phys. Lett.* **B52**, 351 (1974).

[247] M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, *Sov. Phys. JETP* **45**, 670 (1977). [Zh. Eksp. Teor. Fiz. 72 1275 (1977)].

[248] E. G. Floratos, D. A. Ross, C. T. Sachrajda, *Nucl. Phys.* **B152**, 493 (1979).

[249] F. J. Gilman, M. B. Wise, *Phys. Rev.* **D21**, 3150 (1980).

[250] F. J. Gilman, M. B. Wise, *Phys. Rev.* **D27**, 1128 (1983).

[251] B. Grinstein, R. P. Springer, M. B. Wise, *Nucl. Phys.* **B339**, 269 (1990).

[252] W. A. Bardeen, A. J. Buras, D. W. Duke, T. Muta, *Phys. Rev.* **D18**, 3998 (1978).

[253] A. J. Buras, M. Jamin, P. H. Weisz, *Nucl. Phys.* **B347**, 491 (1990).

[254] G. Buchalla, A. J. Buras, M. K. Harlander, *Nucl. Phys.* **B337**, 313 (1990).

[255] M. Ciuchini, E. Franco, G. Martinelli, L. Reina, *Nucl. Phys.* **B415**, 403 (1994), hep-ph/9304257.

[256] C. Arzt, M. B. Einhorn, J. Wudka, *Phys. Rev.* **D49**, 1370 (1994), hep-ph/9304206.

[257] G. Degrassi, E. Franco, S. Marchetti, L. Silvestrini, *JHEP* **11**, 044 (2005), hep-ph/0510137.

[258] J. Gao, C. S. Li, C. P. Yuan, *JHEP* **07**, 037 (2012), 1204.4773.

[259] H. Mebane, N. Greiner, C. Zhang, S. Willenbrock, *Phys. Lett.* **B724**, 259 (2013), 1304.1789.

[260] H. Mebane, N. Greiner, C. Zhang, S. Willenbrock, *Phys. Rev.* **D88**, 015028 (2013), 1306.3380.

[261] A. Celis, J. Fuentes-Martin, A. Vicente, J. Virto, *Eur. Phys. J.* **C77**, 405 (2017), 1704.04504.

[262] M. Ghezzi, R. Gomez-Ambrosio, G. Passarino, S. Uccirati, *JHEP* **07**, 175 (2015), 1505.03706.

[263] C. W. Bauer, A. V. Manohar, *Phys. Rev.* **D57**, 337 (1998), hep-ph/9708306.

[264] A. Pineda, *Phys. Rev.* **D65**, 074007 (2002), hep-ph/0109117.

[265] M. Trott, *JHEP* **02**, 046 (2015), 1409.7605.

[266] R. Contino (2011). 1005.4269.

[267] R. Contino, C. Grojean, M. Moretti, F. Piccinini, R. Rattazzi, *JHEP* **05**, 089 (2010), 1002.1011.

[268] R. Contino, C. Grojean, D. Pappadopulo, R. Rattazzi, A. Thamm, *JHEP* **02**, 006 (2014), 1309.7038.

[269] C. Burgess, J. Matias, M. Pospelov, *Int.J.Mod.Phys.* **A17**, 1841 (2002), hep-ph/9912459.

[270] R. Barbieri, B. Bellazzini, V. S. Rychkov, A. Varagnolo, *Phys. Rev.* **D76**, 115008 (2007), 0706.0432.

[271] G. Buchalla, O. Catà, *JHEP* **1207**, 101 (2012), 1203.6510.

[272] R. Alonso, M. Gavela, L. Merlo, S. Rigolin, J. Yepes, *Phys.Lett.* **B722**, 330 (2013), 1212.3305.

[273] R. Alonso, M. B. Gavela, L. Merlo, S. Rigolin, J. Yepes, *Phys. Rev.* **D87**, 055019 (2013), 1212.3307.

[274] G. Buchalla, O. Catà, C. Krause, *Nucl. Phys.* **B880**, 552 (2014), 1307.5017. [Erratum: Nucl. Phys. B913 475 (2016)].

[275] G. Buchalla, O. Catà, G. D'Ambrosio, *Eur. Phys. J.* **C74**, 2798 (2014), 1310.2574.

[276] I. Brivio, *et al.*, *JHEP* **03**, 024 (2014), 1311.1823.

[277] I. Brivio, *et al.*, *JHEP* **12**, 004 (2014), 1405.5412.

[278] M. B. Gavela, *et al.*, *JHEP* **10**, 44 (2014), 1406.6367.

[279] M. Gavela, K. Kanshin, P. Machado, S. Saa, 1409.1571.

[280] R. Alonso, I. Brivio, B. Gavela, L. Merlo, S. Rigolin, *JHEP* **1412**, 034 (2014), 1409.1589.

[281] G. Buchalla, O. Catà, A. Celis, C. Krause, *Phys. Lett.* **B750**, 298 (2015), 1504.01707.

[282] I. M. Hierro, L. Merlo, S. Rigolin, 1510.07899.

[283] G. Buchalla, O. Catà, A. Celis, C. Krause, *Eur. Phys. J.* **C76**, 233 (2016), 1511.00988.

[284] I. Brivio, J. Gonzalez-Fraile, M. C. Gonzalez-Garcia, L. Merlo, *Eur. Phys. J.* **C76**, 416 (2016), 1604.06801.

[285] M. B. Gavela, K. Kanshin, P. A. N. Machado, S. Saa, *Eur. Phys. J.* **C76**, 690 (2016), 1610.08083.

[286] L. Merlo, S. Saa, M. Sacristán-Barbero, *Eur. Phys. J.* **C77**, 185 (2017), 1612.04832.

[287] P. Hernandez-Leon, L. Merlo, 1703.02064.

[288] ATLAS, CMS collaboration, G. Aad, *et al.*, *JHEP* **08**, 045 (2016), 1606.02266.

[289] E. Halyo, *Mod. Phys. Lett.* **A8**, 275 (1993).

[290] W. D. Goldberger, B. Grinstein, W. Skiba, *Phys. Rev. Lett.* **100**, 111802 (2008), 0708.1463.

[291] S. Weinberg, *Phys. Rev. Lett.* **17**, 616 (1966).

[292] N. Arkani-Hamed, F. Cachazo, J. Kaplan, *JHEP* **09**, 016 (2010), 0808.1446.

[293] L. Susskind, *Phys. Rev.* **D20**, 2619 (1979).

[294] S. Weinberg, *Phys. Rev.* **D19**, 1277 (1979).

[295] P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov, *Nucl. Phys.* **B173**, 189 (1980).

[296] J. Espinosa, C. Grojean, M. Muhlleitner, M. Trott, *JHEP* **1205**, 097 (2012), 1202.3697.

[297] J. R. Espinosa, M. Muhlleitner, C. Grojean, M. Trott, *JHEP* **09**, 126 (2012), 1205.6790.

[298] J. R. Espinosa, C. Grojean, M. Muhlleitner, M. Trott, *JHEP* **12**, 045 (2012), 1207.1717.

[299] C. Grojean, *Nucl. Part. Phys. Proc.* **267-269**, 15 (2015).

[300] T. Banks, *Nucl. Phys.* **B243**, 125 (1984).

[301] K. Agashe, R. Contino, A. Pomarol, *Nucl. Phys.* **B719**, 165 (2005), hep-ph/0412089.

[302] B. Gripaios, A. Pomarol, F. Riva, J. Serra, *JHEP* **04**, 070 (2009), 0902.1483.

[303] D. Marzocca, M. Serone, J. Shu, *JHEP* **08**, 013 (2012), 1205.0770.

[304] F. Feruglio, *et al.*, *JHEP* **06**, 038 (2016), 1603.05668.

[305] F. L. Bezrukov, M. Shaposhnikov, *Phys. Lett.* **B659**, 703 (2008), 0710.3755.

[306] F. Bezrukov, A. Magnin, M. Shaposhnikov, S. Sibiryakov, *JHEP* **01**, 016 (2011), 1008.5157.

[307] C. P. Burgess, H. M. Lee, M. Trott, *JHEP* **09**, 103 (2009), 0902.4465.

[308] J. L. F. Barbon, J. R. Espinosa, *Phys. Rev.* **D79**, 081302 (2009), 0903.0355.

[309] M. P. Hertzberg, *JHEP* **11**, 023 (2010), 1002.2995.

[310] C. P. Burgess, S. P. Patil, M. Trott, *JHEP* **06**, 010 (2014), 1402.1476.

[311] R. Alonso, E. E. Jenkins, A. V. Manohar, *Phys. Lett.* **B756**, 358 (2016), 1602.00706.

[312] G. Buchalla, O. Catà, A. Celis, C. Krause, *Nucl. Phys.* **B917**, 209 (2017), 1608.03564.

[313] A. Manohar, H. Georgi, *Nucl. Phys.* **B234**, 189 (1984).

[314] I. Brivio, *et al.*, *JHEP* **04**, 141 (2016), 1511.01099.

[315] I. Brivio, *et al.*, 1701.05379.

[316] R. Contino, *et al.*, *JHEP* **08**, 154 (2012), 1205.5444.

[317] R. Grober, M. Muhlleitner, *JHEP* **06**, 020 (2011), 1012.1562.

[318] M. J. Dolan, C. Englert, M. Spannowsky, *Phys. Rev.* **D87**, 055002 (2013), 1210.8166.

[319] M. Gouzevitch, *et al.*, *JHEP* **07**, 148 (2013), 1303.6636.

[320] A. Azatov, R. Contino, G. Panico, M. Son, *Phys. Rev.* **D92**, 035001 (2015), 1502.00539.

[321] F. Kling, T. Plehn, P. Schichtel, *Phys. Rev.* **D95**, 035026 (2017), 1607.07441.

[322] F. Bishara, R. Contino, J. Rojo, 1611.03860.

[323] D. de Florian, I. Fabre, J. Mazzitelli, 1704.05700.

[324] R. Grober, M. Muhlleitner, M. Spira, 1705.05314.

[325] G. Isidori, M. Trott, *JHEP* **02**, 082 (2014), 1307.4051.

[326] G. Isidori, A. V. Manohar, M. Trott, *Phys. Lett.* **B728**, 131 (2014), 1305.0663.

[327] A. Pomarol, F. Riva, *JHEP* **01**, 151 (2014), 1308.2803.

[328] B. Grinstein, C. W. Murphy, D. Pirtskhalava, *JHEP* **10**, 077 (2013), 1305.6938.

[329] M. Gonzalez-Alonso, A. Greljo, G. Isidori, D. Marzocca, 1504.04018.

[330] S. Boselli, *et al.*, 1703.06667.

[331] R. L. Delgado, A. Dobado, F. J. Llanes-Estrada, *JHEP* **02**, 121 (2014), 1311.5993.

[332] R. L. Delgado, A. Dobado, F. J. Llanes-Estrada, *Phys. Rev.* **D91**, 075017 (2015), 1502.04841.

[333] A. Castillo, R. L. Delgado, A. Dobado, F. J. Llanes-Estrada, 1607.01158.

[334] R. L. Delgado, A. Dobado, M. J. Herrero, J. J. Sanz-Cillero, *JHEP* **07**, 149 (2014), 1404.2866.

[335] CMS collaboration, *Tech. Rep. CMS-PAS-FSQ-13-008*, CERN, Geneva (2015), https://cds.cern.ch/record/2025577.

[336] T. Corbett, O. J. P. Eboli, J. Gonzalez-Fraile, M. C. Gonzalez-Garcia, *Phys. Rev.* **D86**, 075013 (2012), 1207.1344.

[337] T. Corbett, O. J. P. Eboli, J. Gonzalez-Fraile, M. C. Gonzalez-Garcia, *Phys. Rev.* **D87**, 015022 (2013), 1211.4580.

[338] T. Corbett, O. J. P. Éboli, J. Gonzalez-Fraile, M. C. Gonzalez-Garcia, *Phys. Rev. Lett.* **111**, 011801 (2013), 1304.1151.

[339] C. Englert, M. Spannowsky, *Phys. Lett.* **B740**, 8 (2015), 1408.5147.

[340] A. Azatov, R. Contino, C. S. Machado, F. Riva, 1607.05236.

[341] A. Falkowski, M. Gonzalez-Alonso, A. Greljo, D. Marzocca, M. Son, 1609.06312.

[342] L. J. Dixon (1996). hep-ph/9601359.

[343] M. L. Mangano, S. J. Parke, *Phys. Rept.* **200**, 301 (1991), hep-th/0509223.

[344] L. J. Dixon, Y. Shadmi, *Nucl. Phys.* **B423**, 3 (1994), hep-ph/9312363. [Erratum: Nucl. Phys. B452 724 (1995)].

[345] A. G. Cohen, D. B. Kaplan, A. E. Nelson, *Phys. Lett.* **B412**, 301 (1997), hep-ph/9706275.

[346] M. A. Luty, *Phys. Rev.* **D57**, 1531 (1998), hep-ph/9706235.

[347] G. Buchalla, O. Catà, C. Krause, *Phys. Lett.* **B731**, 80 (2014), 1312.5624.

[348] B. M. Gavela, E. E. Jenkins, A. V. Manohar, L. Merlo, *Eur. Phys. J.* **C76**, 485 (2016), 1601.07551.

[349] G. Buchalla, O. Catà, C. Krause, *Nucl. Phys.* **B894**, 602 (2015), 1412.6356.

[350] M. B. Einhorn, J. Wudka, *Nucl. Phys.* **B876**, 556 (2013), 1307.0478.

[351] H. Georgi, L. Randall, *Nucl. Phys.* **B276**, 241 (1986).

[352] S. R. Coleman, *Subnucl. Ser.* **13**, 297 (1977).

[353] S. R. Coleman, *Subnucl. Ser.* **15**, 805 (1979).

[354] H. Georgi, *Nucl. Phys.* **B331**, 311 (1990).

[355] G. Panico, A. Wulzer, *Lect. Notes Phys.* **913**, pp.1 (2016), 1506.01961.

[356] A. Pomarol (2016). 1412.4410.

[357] R. Contino, A. Falkowski, F. Goertz, C. Grojean, F. Riva, *JHEP* **07**, 144 (2016), 1604.06444.

[358] R. Barbieri, A. Pomarol, R. Rattazzi, A. Strumia, *Nucl. Phys.* **B703**, 127 (2004), hep-ph/0405040.

[359] J. D. Wells, Z. Zhang, *JHEP* **01**, 123 (2016), 1510.08462.

[360] M. Farina, *et al.*, 1609.08157.

[361] S. Alioli, M. Farina, D. Pappadopulo, J. T. Ruderman, *JHEP* **07**, 097 (2017), 1706.03068.

[362] M. E. Peskin, T. Takeuchi, *Phys.Rev.* **D46**, 381 (1992).

[363] J. D. Wells, Z. Zhang, *JHEP* **06**, 122 (2016), 1512.03056.

[364] G. Cacciapaglia, C. Csaki, G. Marandella, A. Strumia, *Phys. Rev.* **D74**, 033011 (2006), hep-ph/0604111.

[365] M. Trott (2017). 1705.05652.

[366] G. Buchalla, O. Catà, A. Celis, C. Krause, 1603.03062.

[367] D. Espriu, F. Mescia, B. Yencho, *Phys. Rev.* **D88**, 055002 (2013), 1307.2400.

[368] R. L. Delgado, A. Dobado, F. J. Llanes-Estrada, *J. Phys.* **G41**, 025002 (2014), 1308.1629.

[369] F.-K. Guo, P. Ruiz-Femenía, J. J. Sanz-Cillero, *Phys. Rev.* **D92**, 074005 (2015), 1506.04204.

[370] M. Herrero, E. Ruiz-Morales, *Phys. Lett.* **B296**, 397 (1992), hep-ph/9208220.

[371] M. J. Herrero, E. Ruiz Morales, *Nucl. Phys.* **B418**, 431 (1994), hep-ph/9308276.

[372] H. Lehmann, K. Symanzik, W. Zimmermann, *Nuovo Cim.* **1**, 205 (1955).

[373] J. A. Wheeler, *Phys. Rev.* **52**, 1107 (1937).

[374] E. Byckling, K. Kajantie, *Particle Kinematics* (University of Jyvaskyla, Jyvaskyla, Finland, 1971).

[375] Particle Data Group collaboration, C. Patrignani, *et al.*, *Chin. Phys.* **C40**, 100001 (2016).

[376] J. S. R. Chisholm, *Nucl. Phys.* **26**, 469 (1961).

[377] S. Kamefuchi, L. O'Raifeartaigh, A. Salam, *Nucl. Phys.* **28**, 529 (1961).

[378] G. 't Hooft, M. J. G. Veltman, *Nucl. Phys.* **B50**, 318 (1972).

[379] M. C. Bergere, Y.-M. P. Lam, *Phys. Rev.* **D13**, 3247 (1976).

[380] H. D. Politzer, *Nucl. Phys.* **B172**, 349 (1980).

[381] C. Arzt, *Phys. Lett.* **B342**, 189 (1995), hep-ph/9304230.

[382] H. Elvang, Y.-t. Huang, 1308.1697.

[383] W. E. Thirring, *Annals Phys.* **3**, 91 (1958).

[384] S. Coleman, *Phys. Rev. D* **11**, 2088 (1975).

[385] S. Mandelstam, *Phys. Rev.* **D11**, 3026 (1975).

[386] C. Becchi, A. Rouet, R. Stora, *Annals Phys.* **98**, 287 (1976).

[387] M. Z. Iofa, I. V. Tyutin, *Teor. Mat. Fiz.* **27**, 38 (1976). [Theor. Math. Phys. 27 316 (1976)].

[388] B. Lautrup, *Mat. Fys. Medd. Dan. Vid. Selsk.* **35**, 29 (1967).

[389] N. Nakanishi, *Prog. Theor. Phys.* **49**, 640 (1973).

[390] R. L. Stratonovich, *Soviet Physics Doklady* **2**, 416 (1957).

[391] J. Hubbard, *Physical Review Letters* **3**, 77 (1959).

[392] A. A. Starobinsky, *JETP Lett.* **30**, 682 (1979). [Pisma Zh. Eksp. Teor. Fiz. 30 719 (1979)].

[393] M. Trott, https://indico.cern.ch/event/399452/contributions/1840100/attachments/800095/1096568/NLOSMEFT.pd Monday 15 Jun 2015.

[394] SLD Electroweak Group, DELPHI, ALEPH, SLD, SLD Heavy Flavour Group, OPAL, LEP Electroweak Working Group, L3 collaboration, S. Schael, *et al.*, *Phys. Rept.* **427**, 257 (2006), hep-ex/0509008.

[395] D0 collaboration, V. M. Abazov, *et al.*, *Phys. Rev. Lett.* **108**, 151804 (2012), 1203.0293.

[396] CDF collaboration, T. Aaltonen, *et al.*, *Phys. Rev. Lett.* **108**, 151803 (2012), 1203.0275.

[397] ATLAS collaboration, M. Aaboud, *et al.*, 1701.07240.

[398] W. F. L. Hollik, *Fortsch. Phys.* **38**, 165 (1990).

[399] D. C. Kennedy, B. W. Lynn, *Nucl. Phys.* **B322**, 1 (1989).

[400] I. Maksymyk, C. P. Burgess, D. London, *Phys. Rev.* **D50**, 529 (1994), hep-ph/9306267.

[401] J. D. Wells (2005). hep-ph/0512342.

[402] A. Freitas, *JHEP* **1404**, 070 (2014), 1401.2447.

[403] R. A. Davison, B. R. Webber, *Eur. Phys. J.* **C59**, 13 (2009), 0809.3326.

[404] R. Abbate, M. Fickinger, A. H. Hoang, V. Mateu, I. W. Stewart, *Phys. Rev.* **D86**, 094002 (2012), 1204.5746.

[405] T. Gehrmann, G. Luisoni, P. F. Monni, *Eur. Phys. J.* **C73**, 2265 (2013), 1210.6945.

[406] G. Parisi, *Phys. Lett.* **74B**, 65 (1978).

[407] J. F. Donoghue, F. E. Low, S.-Y. Pi, *Phys. Rev.* **D20**, 2759 (1979).

[408] A. H. Hoang, D. W. Kolodrubetz, V. Mateu, I. W. Stewart, *Phys. Rev.* **D91**, 094018 (2015), 1501.04111.

[409] C. W. Bauer, Z. Ligeti, M. Luke, A. V. Manohar, M. Trott, *Phys. Rev.* **D70**, 094017 (2004), hep-ph/0408002.

[410] HPQCD collaboration, I. Allison, *et al.*, *Phys. Rev.* **D78**, 054513 (2008), 0805.2999.

[411] Z. Han, W. Skiba, *Phys.Rev.* **D71**, 075009 (2005), hep-ph/0412166.

[412] A. Falkowski, F. Riva, *JHEP* **02**, 039 (2015), 1411.0669.

[413] L. Berthier, M. Trott, *JHEP* **05**, 024 (2015), 1502.02570.

[414] L. Berthier, M. Trott, *JHEP* **02**, 069 (2016), 1508.05060.

[415] I. Brivio, M. Trott, 1701.06424.

[416] J. Ellis, V. Sanz, T. You, *JHEP* **1407**, 036 (2014), 1404.3667.

[417] F. Ferreira, B. Fuks, V. Sanz, D. Sengupta, 1612.01808.

[418] T. Corbett, *et al.*, *JHEP* **08**, 156 (2015), 1505.05516.

[419] T. Corbett, *et al.*, 1511.08188.

[420] J. Brehmer, K. Cranmer, F. Kling, T. Plehn, *Phys. Rev.* **D95**, 073002 (2017), 1612.05261.

[421] A. de Gouvea, J. Herrero-Garcia, A. Kobach, *Phys. Rev.* **D90**, 016011 (2014), 1404.4057.

[422] A. Kobach, *Phys. Lett.* **B758**, 455 (2016), 1604.05726.

[423] N. Kauer, G. Passarino, *JHEP* **08**, 116 (2012), 1206.4803.

[424] F. Caola, K. Melnikov, *Phys. Rev.* **D88**, 054024 (2013), 1307.4935.

[425] J. M. Campbell, R. K. Ellis, C. Williams, *JHEP* **04**, 060 (2014), 1311.3589.

[426] CMS collaboration, V. Khachatryan, *et al.*, *Phys. Lett.* **B736**, 64 (2014), 1405.3455.

[427] ATLAS collaboration, G. Aad, *et al.*, *Eur. Phys. J.* **C75**, 335 (2015), 1503.01060.

[428] C. Englert, M. Spannowsky, *Phys. Rev.* **D90**, 053003 (2014), 1405.0285.

[429] G. Cacciapaglia, A. Deandrea, G. Drieu La Rochelle, J.-B. Flament, *Phys. Rev. Lett.* **113**, 201802 (2014), 1406.1757.

[430] A. Azatov, C. Grojean, A. Paul, E. Salvioni, *Zh. Eksp. Teor. Fiz.* **147**, 410 (2015), 1406.6338. [J. Exp. Theor. Phys. 120 354 (2015)].

[431] D. Berdine, N. Kauer, D. Rainwater, *Phys. Rev. Lett.* **99**, 111601 (2007), hep-ph/0703058.

[432] R. P. Feynman, *Phys. Rev.* **76**, 749 (1949).

[433] R. P. Feynman, *Phys. Rev.* **80**, 440 (1950).

[434] R. E. Cutkosky, *J. Math. Phys.* **1**, 429 (1960).

[435] A. Helset, M. Trott, 1711.07954.

[436] M. Bjørn, M. Trott, 1606.06502.

[437] R. Jacob, R. G. Sachs, *Phys. Rev.* **121**, 350 (1961).

[438] M. J. G. Veltman, *Physica* **29**, 186 (1963).

[439] H. A. Weldon, *Phys. Rev.* **D14**, 2030 (1976).

[440] R. G. Stuart, *Phys. Lett.* **B262**, 113 (1991).

[441] R. G. Stuart, *Phys. Lett.* **B272**, 353 (1991).

[442] A. Aeppli, G. J. van Oldenborgh, D. Wyler, *Nucl. Phys.* **B428**, 126 (1994), hep-ph/9312212.

[443] W. Beenakker, A. Denner, *Int. J. Mod. Phys.* **A9**, 4837 (1994).

[444] G. Passarino, *Nucl. Phys.* **B574**, 451 (2000), hep-ph/9911482.

[445] M. W. Grunewald, *et al.*, hep-ph/0005309.

[446] P. A. Grassi, B. A. Kniehl, A. Sirlin, *Phys. Rev. Lett.* **86**, 389 (2001), hep-th/0005149.

[447] S. Actis, G. Passarino, *Nucl. Phys.* **B777**, 100 (2007), hep-ph/0612124.

[448] B. A. Kniehl, A. Sirlin, *Phys. Rev.* **D77**, 116012 (2008), 0801.0669.

[449] B. A. Kniehl, A. Sirlin, *Phys. Rev. Lett.* **81**, 1373 (1998), hep-ph/9805390.

[450] G. Passarino, C. Sturm, S. Uccirati, *Nucl. Phys.* **B834**, 77 (2010), 1001.3360.

[451] W. Beenakker, *et al.* (1996). hep-ph/9602351.

[452] D. Yu. Bardin, *et al.* (1997). hep-ph/9709270.

[453] A. Denner, S. Dittmaier, M. Roth, D. Wackeroth, *Nucl. Phys.* **B587**, 67 (2000), hep-ph/0006307.

[454] A. Denner, S. Dittmaier, M. Roth, D. Wackeroth, *Comput. Phys. Commun.* **153**, 462 (2003), hep-ph/0209330.

[455] A. Denner, S. Dittmaier, M. Roth, *Phys. Lett.* **B429**, 145 (1998), hep-ph/9803306.

[456] C. Moeller, *K. Danske. Vid. Selsk.* **B19** (1946).

[457] G. Passarino, *Nucl. Phys.* **B868**, 416 (2013), 1209.5538.

[458] A. Denner, J.-N. Lang, *Eur. Phys. J.* **C75**, 377 (2015), 1406.6280.

[459] P. A. Grassi, B. A. Kniehl, A. Sirlin, *Phys. Rev.* **D65**, 085001 (2002), hep-ph/0109228.

[460] P. Gambino, P. A. Grassi, *Phys. Rev.* **D62**, 076002 (2000), hep-ph/9907254.

[461] A. Denner, S. Dittmaier, M. Roth, L. H. Wieders, *Nucl. Phys.* **B724**, 247 (2005), hep-ph/0505042. [Erratum: Nucl. Phys. B854 504 (2012)].

[462] A. David, G. Passarino, *Rev. Phys.* **1**, 13 (2016), 1510.00414.

[463] M. Beneke, A. P. Chapovsky, A. Signer, G. Zanderighi, *Phys. Rev. Lett.* **93**, 011602 (2004), hep-ph/0312331.

[464] M. Beneke, A. P. Chapovsky, A. Signer, G. Zanderighi, *Nucl. Phys.* **B686**, 205 (2004), hep-ph/0401002.

[465] M. Beneke, P. Falgari, C. Schwinn, A. Signer, G. Zanderighi, *Nucl. Phys.* **B792**, 89 (2008), 0707.0773.

[466] M. Beneke, *Nucl. Part. Phys. Proc.* **261-262**, 218 (2015), 1501.07370.

[467] I. Brivio, Y. Jiang, M. Trott, *JHEP* **12**, 070 (2017), 1709.06492.

[468] Y. T. Chien, V. Cirigliano, W. Dekens, J. de Vries, E. Mereghetti, *JHEP* **02**, 011 (2016), 1510.00725.

[469] W. Dekens, J. de Vries, *JHEP* **05**, 149 (2013), 1303.3156.

[470] S. Alioli, V. Cirigliano, W. Dekens, J. de Vries, E. Mereghetti, *JHEP* **05**, 086 (2017), 1703.04751.

[471] T. ALEPH, *et al.*, *Phys. Rept.* **427**, 257 (2006), hep-ex/0509008.

[472] CDF, D0 collaboration, T. E. W. Group, 1204.0042.

[473] M. Awramik, M. Czakon, A. Freitas, G. Weiglein, *Phys.Rev.* **D69**, 053006 (2004), hep-ph/0311148.

[474] M. Awramik, M. Czakon, A. Freitas, *JHEP* **11**, 048 (2006), hep-ph/0608099.

[475] B. Grinstein, R. P. Springer, M. B. Wise, *Phys. Lett.* **B202**, 138 (1988).

[476] M. Boggia, *et al.*, 1711.09875.

[477] A. Leike, T. Riemann, J. Rose, *Phys. Lett.* **B273**, 513 (1991), hep-ph/9508390.

[478] The ALEPH, DELPHI, L3, OPAL Collaborations, the LEP Electroweak Working Group, *Phys. Rept.* **532**, 119 (2013), 1302.3415.

[479] M. Bordone, A. Greljo, G. Isidori, D. Marzocca, A. Pattori, *Eur. Phys. J.* **C75**, 385 (2015), 1507.02555.

[480] G. Altarelli, R. Barbieri, *Phys.Lett.* **B253**, 161 (1991).

[481] M. Golden, L. Randall, *Nucl.Phys.* **B361**, 3 (1991).

[482] B. Holdom, J. Terning, *Phys.Lett.* **B247**, 88 (1990).

[483] M. E. Peskin, T. Takeuchi, *Phys.Rev.Lett.* **65**, 964 (1990).

[484] C. P. Burgess, S. Godfrey, H. Konig, D. London, I. Maksymyk, *Phys. Lett.* **B326**, 276 (1994), hep-ph/9307337.

[485] C. P. Burgess, S. Godfrey, H. Konig, D. London, I. Maksymyk, *Phys. Rev.* **D49**, 6115 (1994), hep-ph/9312291.

[486] P. Bamert, C. P. Burgess, J. M. Cline, D. London, E. Nardi, *Phys. Rev.* **D54**, 4275 (1996), hep-ph/9602438.

[487] Gfitter Group collaboration, M. Baak, *et al.*, *Eur.Phys.J.* **C74**, 3046 (2014), 1407.3792.

[488] B. Grinstein, M. B. Wise, *Phys.Lett.* **B265**, 326 (1991).

[489] G. Buchalla, O. Cata, R. Rahn, M. Schlaffer, *Eur. Phys. J.* **C73**, 2589 (2013), 1302.6481.

[490] A. Falkowski, M. González-Alonso, K. Mimouni, 1706.03783.

[491] A. Falkowski, K. Mimouni, *JHEP* **02**, 086 (2016), 1511.07434.

[492] G. Sanchez-Colon, J. Wudka, *Phys. Lett.* **B432**, 383 (1998), hep-ph/9805366.

[493] W. Kilian, J. Reuter, *Phys. Rev.* **D70**, 015004 (2004), hep-ph/0311095.

[494] C. Grojean, W. Skiba, J. Terning, *Phys. Rev.* **D73**, 075008 (2006), hep-ph/0602154.

[495] Y.-S. Tsai, A. C. Hearn, *Phys. Rev.* **140**, B721 (1965).

[496] V. V. Flambaum, I. B. Khriplovich, O. P. Sushkov, *Sov. J. Nucl. Phys.* **20**, 537 (1975). [Yad. Fiz. 20 1016 (1974)].

[497] K. J. F. Gaemers, G. J. Gounaris, *Z. Phys.* **C1**, 259 (1979).

[498] C. L. Bilchak, J. D. Stroughair, *Phys. Rev.* **D30**, 1881 (1984).

[499] J. F. Gunion, Z. Kunszt, *Phys. Rev.* **D33**, 665 (1986).

[500] K. Hagiwara, R. D. Peccei, D. Zeppenfeld, K. Hikasa, *Nucl. Phys.* **B282**, 253 (1987).

[501] A. De Rujula, M. B. Gavela, P. Hernandez, E. Masso, *Nucl. Phys.* **B384**, 3 (1992).

[502] C. P. Burgess, D. London, *Phys. Rev. Lett.* **69**, 3428 (1992).

[503] G. Gounaris, *et al.* (1996). hep-ph/9601233.

[504] DELPHI collaboration, J. Abdallah, *et al.*, *Eur. Phys. J.* **C66**, 35 (2010), 1002.0752.

[505] G. Altarelli, T. Sjöstrand, F. Zwirner, CERN (CERN, Geneva, 1996), https://cds.cern.ch/record/300671.

[506] A. Falkowski, M. Gonzalez-Alonso, A. Greljo, D. Marzocca, *Phys. Rev. Lett.* **116**, 011801 (2016), 1508.00581.

[507] A. Butter, *et al.*, *JHEP* **07**, 152 (2016), 1604.03105.

[508] C. Englert, *et al.*, *J. Phys.* **G41**, 113001 (2014), 1403.7191.

[509] J. Ellis, V. Sanz, T. You, *JHEP* **03**, 157 (2015), 1410.7703.

[510] C. Englert, R. Kogler, H. Schulz, M. Spannowsky, *Eur. Phys. J.* **C76**, 393 (2016), 1511.05170.

[511] LHC Higgs Cross Section Working Group collaboration, 1209.0040.

[512] D. Zeppenfeld, R. Kinnunen, A. Nikitenko, E. Richter-Was, *Phys. Rev.* **D62**, 013009 (2000), hep-ph/0002036.

[513] M. Dührssen.

[514] M. Dührssen, *et al.*, *Phys. Rev.* **D70**, 113009 (2004), hep-ph/0406323.

[515] R. Lafaye, T. Plehn, M. Rauch, D. Zerwas, M. Dührssen, *JHEP* **08**, 009 (2009), 0904.3866.

[516] D. Carmi, A. Falkowski, E. Kuflik, T. Volansky, *JHEP* **07**, 136 (2012), 1202.3144.

[517] A. Azatov, R. Contino, J. Galloway, *JHEP* **1204**, 127 (2012), 1202.3415.

[518] I. Low, R. Rattazzi, A. Vichi, *JHEP* **04**, 126 (2010), 0907.5413.

[519] J. R. Espinosa, C. Grojean, M. Muhlleitner, *JHEP* **05**, 065 (2010), 1003.3251.

[520] M. C. Gonzalez-Garcia, *Int. J. Mod. Phys.* **A14**, 3121 (1999), hep-ph/9902321.

[521] A. Azatov, *et al.*, *JHEP* **06**, 134 (2012), 1204.4817.

[522] A. Azatov, J. Galloway, *Int. J. Mod. Phys.* **A28**, 1330004 (2013), 1212.1380.

[523] A. Azatov, R. Contino, A. Di Iura, J. Galloway, *Phys. Rev.* **D88**, 075019 (2013), 1308.2676.

[524] M. Farina, C. Grojean, E. Salvioni, *JHEP* **07**, 012 (2012), 1205.0011.

[525] M. Farina, C. Grojean, F. Maltoni, E. Salvioni, A. Thamm, *JHEP* **05**, 022 (2013), 1211.3736.

[526] LHC Higgs Cross Section Working Group collaboration, J. R. Andersen, *et al.*, 1307.1347.

[527] M. Gonzalez-Alonso, A. Greljo, G. Isidori, D. Marzocca, *Eur.Phys.J.* **C75**, 128 (2015), 1412.6038.

[528] A. Greljo, G. Isidori, J. M. Lindert, D. Marzocca, *Eur. Phys. J.* **C76**, 158 (2016), 1512.06135.

[529] CMS collaboration, V. Khachatryan, *et al.*, *Eur. Phys. J.* **C75**, 212 (2015), 1412.8662.

[530] ATLAS collaboration, G. Aad, *et al.*, *JHEP* **11**, 206 (2015), 1509.00672.

[531] ATLAS collaboration, G. Aad, *et al.*, *Eur. Phys. J.* **C76**, 6 (2016), 1507.04548.

[532] S. Biswas, E. Gabrielli, B. Mele, *JHEP* **01**, 088 (2013), 1211.0499.

[533] C. Degrande, J. M. Gerard, C. Grojean, F. Maltoni, G. Servant, *JHEP* **07**, 036 (2012), 1205.1065. [Erratum: JHEP03 032 (2013)].

[534] CMS collaboration, *Tech. Rep. CMS-NOTE-2013-002*, CERN, Geneva (2013), http://cds.cern.ch/record/1565454.

[535] ATLAS collaboration, *Tech. Rep. ATL-PHYS-PUB-2014-016*, CERN, Geneva (2014), http://cds.cern.ch/record/1956710.

[536] R. Gauld, B. D. Pecjak, D. J. Scott, *JHEP* **05**, 080 (2016), 1512.02508.

[537] R. Gauld, B. D. Pecjak, D. J. Scott, 1607.06354.

[538] L. Bergstrom, G. Hulth, *Nucl. Phys.* **B259**, 137 (1985). [Erratum: Nucl. Phys. B276 744 (1986)].

[539] C. Hartmann, M. Trott, *JHEP* **07**, 151 (2015), 1505.02646.

[540] G. A. Ladinsky, C. P. Yuan, *Phys. Rev.* **D49**, 4415 (1994), hep-ph/9211272.

[541] D. Chang, W.-Y. Keung, I. Phillips, *Nucl. Phys.* **B408**, 286 (1993), hep-ph/9301259. [Erratum: Nucl. Phys. B429 255 (1994)].

[542] B. Grzadkowski, *Acta Phys. Polon.* **B27**, 921 (1996), hep-ph/9511279.

[543] B. Grzadkowski, Z. Hioki, *Nucl. Phys.* **B484**, 17 (1997), hep-ph/9604301.

[544] B. Grzadkowski, Z. Hioki, *Phys. Lett.* **B391**, 172 (1997), hep-ph/9608306.

[545] K. Whisnant, J.-M. Yang, B.-L. Young, X. Zhang, *Phys. Rev.* **D56**, 467 (1997), hep-ph/9702305.

[546] J. M. Yang, B.-L. Young, *Phys. Rev.* **D56**, 5907 (1997), hep-ph/9703463.

[547] M. S. Baek, S. Y. Choi, C. S. Kim, *Phys. Rev.* **D56**, 6835 (1997), hep-ph/9704312.

[548] L. Brzezinski, B. Grzadkowski, Z. Hioki, *Int. J. Mod. Phys.* **A14**, 1261 (1999), hep-ph/9710358.

[549] B. Grzadkowski, Z. Hioki, M. Szafranski, *Phys. Rev.* **D58**, 035002 (1998), hep-ph/9712357.

[550] A. Bartl, E. Christova, T. Gajdosik, W. Majerotto, *Phys. Rev.* **D58**, 074007 (1998), hep-ph/9802352.

[551] B. Grzadkowski, Z. Hioki, *Phys. Rev.* **D61**, 014013 (2000), hep-ph/9805318.

[552] B. Grzadkowski, Z. Hioki, *Phys. Lett.* **B476**, 87 (2000), hep-ph/9911505.

[553] E. Boos, M. Dubinin, A. Pukhov, M. Sachwitz, H. J. Schreiber, *Eur. Phys. J.* **C21**, 81 (2001), hep-ph/0104279.

[554] R. Röntsch, M. Schulze, *JHEP* **08**, 044 (2015), 1501.05939.

[555] M. S. Amjad, *et al.*, *Eur. Phys. J.* **C75**, 512 (2015), 1505.06020.

[556] Q.-H. Cao, B. Yan, *Phys. Rev.* **D92**, 094018 (2015), 1507.06204.

[557] M. Vos, *et al.*, 1604.08122.

[558] C. Englert, M. Russell, *Eur. Phys. J.* **C77**, 535 (2017), 1704.01782.

[559] W. Bernreuther, *et al.*, 1710.06737.

[560] S. Y. Choi, K. Hagiwara, *Phys. Lett.* **B359**, 369 (1995), hep-ph/9506430.

[561] P. Poulose, S. D. Rindani, *Phys. Rev.* **D57**, 5444 (1998), hep-ph/9709225. [Erratum: Phys. Rev. D61 119902 (2000)].

[562] B. Grzadkowski, Z. Hioki, K. Ohkuma, J. Wudka, *Nucl. Phys.* **B689**, 108 (2004), hep-ph/0310159.

[563] B. Grzadkowski, Z. Hioki, K. Ohkuma, J. Wudka, *Phys. Lett.* **B593**, 189 (2004), hep-ph/0403174.

[564] B. Grzadkowski, Z. Hioki, K. Ohkuma, J. Wudka, *JHEP* **11**, 029 (2005), hep-ph/0508183.

[565] J. A. Aguilar-Saavedra, *Nucl. Phys.* **B812**, 181 (2009), 0811.3842.

[566] J. A. Aguilar-Saavedra, *Nucl. Phys.* **B821**, 215 (2009), 0904.2387.

[567] N. Greiner, S. Willenbrock, C. Zhang, *Phys. Lett.* **B704**, 218 (2011), 1104.3122.

[568] C. Zhang, N. Greiner, S. Willenbrock, *Phys. Rev.* **D86**, 014024 (2012), 1201.6670.

[569] J. de Blas, M. Chala, J. Santiago, *JHEP* **09**, 189 (2015), 1507.00757.

[570] G. L. Kane, G. A. Ladinsky, C. P. Yuan, *Phys. Rev.* **D45**, 124 (1992).

[571] D. Atwood, A. Aeppli, A. Soni, *Phys. Rev. Lett.* **69**, 2754 (1992).

[572] D. Atwood, S. Bar-Shalom, G. Eilam, A. Soni, *Phys. Rept.* **347**, 1 (2001), hep-ph/0006032.

[573] J. A. Aguilar-Saavedra, J. Carvalho, N. F. Castro, F. Veloso, A. Onofre, *Eur. Phys. J.* **C50**, 519 (2007), hep-ph/0605190.

[574] T. Han, Y. Li, *Phys. Lett.* **B683**, 278 (2010), 0911.2933.

[575] C. Zhang, S. Willenbrock, *Phys. Rev.* **D83**, 034006 (2011), 1008.3869.

[576] S. S. Biswal, S. D. Rindani, P. Sharma, *Phys. Rev.* **D88**, 074018 (2013), 1211.4075.

[577] W. Bernreuther, Z.-G. Si, *Phys. Lett.* **B725**, 115 (2013), 1305.2066. [Erratum: Phys. Lett. B744 413 (2015)].

[578] A. Cordero-Cid, J. M. Hernandez, G. Tavares-Velasco, J. J. Toscano, *J. Phys.* **G35**, 025004 (2008), 0712.0154.

[579] J. F. Kamenik, M. Papucci, A. Weiler, *Phys. Rev.* **D85**, 071501 (2012), 1107.3143. [Erratum: Phys. Rev. D88, no.3 039903 (2013)].

[580] J. Brod, U. Haisch, J. Zupan, *JHEP* **11**, 180 (2013), 1310.1385.

[581] M. Gorbahn, U. Haisch, *JHEP* **06**, 033 (2014), 1404.4873.

[582] V. Cirigliano, W. Dekens, J. de Vries, E. Mereghetti, *Phys. Rev.* **D94**, 016002 (2016), 1603.03049.

[583] V. Cirigliano, W. Dekens, J. de Vries, E. Mereghetti, *Phys. Rev.* **D94**, 034031 (2016), 1605.04311.

[584] G. Burdman, M. C. Gonzalez-Garcia, S. F. Novaes, *Phys. Rev.* **D61**, 114016 (2000), hep-ph/9906329.

[585] B. Grzadkowski, M. Misiak, *Phys. Rev.* **D78**, 077501 (2008), 0802.1413. [Erratum: Phys. Rev. D84 059903 (2011)].

[586] J. Drobnak, S. Fajfer, J. F. Kamenik, *Nucl. Phys.* **B855**, 82 (2012), 1109.2357.

[587] A. Buckley, *et al.*, *Phys. Rev.* **D92**, 091501 (2015), 1506.08845.

[588] A. Buckley, *et al.*, *JHEP* **04**, 015 (2016), 1512.03360.

[589] L. Moore, Top quark physics in the standard model effective field theory, Ph.D. thesis, Glasgow U. (2016-12). http://theses.gla.ac.uk/7841/.

[590] M. Russell, Top quark physics in the Large Hadron Collider era, Ph.D. thesis, Glasgow U. (2017). 1709.10508.

[591] D. Atwood, A. Kagan, T. G. Rizzo, *Phys. Rev.* **D52**, 6264 (1995), hep-ph/9407408.

[592] Z. Hioki, K. Ohkuma, *Eur. Phys. J.* **C65**, 127 (2010), 0910.3049.

[593] D. Choudhury, P. Saha, *Pramana* **77**, 1079 (2011), 0911.5016.

[594] Z. Hioki, K. Ohkuma, *Phys. Rev.* **D88**, 017503 (2013), 1306.5387.

[595] J. A. Aguilar-Saavedra, B. Fuks, M. L. Mangano, *Phys. Rev.* **D91**, 094021 (2015), 1412.6654.

[596] D. Buarque Franzosi, C. Zhang, *Phys. Rev.* **D91**, 114010 (2015), 1503.08841.

[597] D. Barducci, M. Fabbrichesi, A. Tonero, *Phys. Rev.* **D96**, 075022 (2017), 1704.05478.

[598] ATLAS collaboration, G. Aad, *et al.*, *Eur. Phys. J.* **C74**, 3109 (2014), 1406.5375. [Addendum: Eur. Phys. J.C76, no.11 642 (2016)].

[599] CMS collaboration, V. Khachatryan, *et al.*, *Eur. Phys. J.* **C77**, 15 (2017), 1602.09024.

[600] ATLAS collaboration, M. Aaboud, *et al.*, *Phys. Lett.* **B761**, 136 (2016), 1606.02699. [Erratum: Phys. Lett B772 879 (2017)].

[601] CMS collaboration, V. Khachatryan, *et al.*, *JHEP* **08**, 029 (2016), 1603.02303.

[602] CMS collaboration, V. Khachatryan, *et al.*, *Eur. Phys. J.* **C77**, 172 (2017), 1611.04040.

[603] CMS collaboration, A. M. Sirunyan, *et al.*, *JHEP* **09**, 051 (2017), 1701.06228.

[604] C. Degrande, J.-M. Gerard, C. Grojean, F. Maltoni, G. Servant, *JHEP* **03**, 125 (2011), 1010.6304.

[605] CDF collaboration, T. Aaltonen, *et al.*, *Phys. Rev.* **D83**, 112003 (2011), 1101.0034.

[606] CDF, D0 collaboration, T. A. Aaltonen, *et al.*, *Submitted to: Phys. Rev. Lett.* (2017), 1709.04894.

[607] N. Kidonakis, *Phys. Rev.* **D84**, 011504 (2011), 1105.5167.

[608] W. Hollik, D. Pagani, *Phys. Rev.* **D84**, 093003 (2011), 1107.2606.

[609] J. H. Kuhn, G. Rodrigo, *JHEP* **01**, 063 (2012), 1109.6830.

[610] A. V. Manohar, M. Trott, *Phys.Lett.* **B711**, 313 (2012), 1201.3926.

[611] M. Czakon, P. Fiedler, A. Mitov, *Phys. Rev. Lett.* **115**, 052001 (2015), 1411.3007.

[612] N. Kidonakis, *Phys. Rev.* **D91**, 071502 (2015), 1501.01581.

[613] M. Czakon, P. Fiedler, D. Heymes, A. Mitov, *JHEP* **05**, 034 (2016), 1601.05375.

[614] M. Czakon, *et al.*, 1711.03945.

[615] R. Diener, S. Godfrey, T. A. W. Martin, *Phys. Rev.* **D80**, 075014 (2009), 0909.2022.

[616] ATLAS, CMS collaboration, M. Aaboud, *et al.*, *Submitted to: JHEP* (2017), 1709.05327.

[617] M. P. Rosello, M. Vos, *Eur. Phys. J.* **C76**, 200 (2016), 1512.07542.

[618] W. Bernreuther, D. Heisler, Z.-G. Si, *JHEP* **12**, 026 (2015), 1508.05271.

[619] S. Jung, P. Ko, Y. W. Yoon, C. Yu, *JHEP* **08**, 120 (2014), 1406.4570.

[620] J. A. Aguilar-Saavedra, *Nucl. Phys.* **B804**, 160 (2008), 0803.3810.

[621] F. Bach, T. Ohl, *Phys. Rev.* **D90**, 074022 (2014), 1407.2546.

[622] S. Yaser Ayazi, H. Hesari, M. Mohammadi Najafabadi, *Phys. Lett.* **B727**, 199 (2013), 1307.1846.

[623] S. D. Rindani, P. Sharma, A. W. Thomas, *JHEP* **10**, 180 (2015), 1507.08385.

[624] CMS collaboration, S. Chatrchyan, *et al.*, *JHEP* **12**, 035 (2012), 1209.4533.

[625] ATLAS collaboration, G. Aad, *et al.*, *Phys. Rev.* **D90**, 112006 (2014), 1406.7844.

[626] CMS collaboration, V. Khachatryan, *et al.*, *JHEP* **06**, 090 (2014), 1403.7366.

[627] ATLAS collaboration, M. Aaboud, *et al.*, *Eur. Phys. J.* **C77**, 531 (2017), 1702.02859.

[628] ATLAS collaboration, M. Aaboud, *et al.*, *JHEP* **04**, 086 (2017), 1609.03920.

[629] CMS collaboration, A. M. Sirunyan, *et al.*, *Phys. Lett.* **B772**, 752 (2017), 1610.00678.

[630] N. Kidonakis, *Phys. Rev.* **D81**, 054028 (2010), 1001.5034.

[631] ATLAS collaboration, G. Aad, *et al.*, *Phys. Lett.* **B756**, 228 (2016), 1511.05980.

[632] CMS collaboration, V. Khachatryan, *et al.*, *JHEP* **09**, 027 (2016), 1603.02555.

[633] A. Giammanco, R. Schwienhorst, 1710.10699.

[634] CMS collaboration, S. Chatrchyan, *et al.*, *Phys. Rev. Lett.* **112**, 231802 (2014), 1401.2942.

[635] ATLAS collaboration, G. Aad, *et al.*, *JHEP* **01**, 064 (2016), 1510.03752.

[636] ATLAS collaboration, M. Aaboud, *et al.*, 1612.07231.

[637] CMS collaboration, *Tech. Rep. CMS-PAS-TOP-17-018*, CERN, Geneva (2017), https://cds.cern.ch/record/2284831.

[638] ATLAS collaboration, M. Aaboud, *et al.*, 1712.01602.

[639] B. P. Kersevan, I. Hinchliffe, *JHEP* **09**, 033 (2006), hep-ph/0603068.

[640] S. Frixione, E. Laenen, P. Motylinski, B. R. Webber, C. D. White, *JHEP* **07**, 029 (2008), 0805.3067.

[641] C. D. White, S. Frixione, E. Laenen, F. Maltoni, *JHEP* **11**, 074 (2009), 0908.0631.

[642] T. Ježo, J. M. Lindert, P. Nason, C. Oleari, S. Pozzorini, *Eur. Phys. J.* **C76**, 691 (2016), 1607.04538.

[643] J. A. Aguilar-Saavedra, S. Amor Dos Santos, *Phys. Rev.* **D89**, 114009 (2014), 1404.1585.

[644] J. A. Aguilar-Saavedra, C. Degrande, S. Khatibi, *Phys. Lett.* **B769**, 498 (2017), 1701.05900.

[645] Q.-H. Cao, J. Wudka, C. P. Yuan, *Phys. Lett.* **B658**, 50 (2007), 0704.2809.

[646] C. Zhang, *Phys. Rev. Lett.* **116**, 162002 (2016), 1601.06163.

[647] R. Röntsch, M. Schulze, *JHEP* **07**, 091 (2014), 1404.1005. [Erratum: JHEP09 132 (2015)].

[648] ATLAS collaboration, G. Aad, *et al.*, *Phys. Rev.* **D91**, 072007 (2015), 1502.00586.

[649] ATLAS collaboration, G. Aad, *et al.*, *JHEP* **11**, 172 (2015), 1509.05276.

[650] ATLAS collaboration, M. Aaboud, *et al.*, *JHEP* **11**, 086 (2017), 1706.03046.

[651] CMS collaboration, A. M. Sirunyan, *et al.*, *JHEP* **10**, 006 (2017), 1706.08128.

[652] CMS collaboration, A. M. Sirunyan, *et al.*, 1711.02547.

[653] M. Schulze, Y. Soreq, *Eur. Phys. J.* **C76**, 466 (2016), 1603.08911.

[654] O. Bessidskaia Bylund, F. Maltoni, I. Tsinikos, E. Vryonidou, C. Zhang, *JHEP* **05**, 052 (2016), 1601.08193.

[655] J. Bramante, A. Delgado, A. Martin, *Phys. Rev.* **D89**, 093006 (2014), 1402.5985.

[656] F. Maltoni, E. Vryonidou, C. Zhang, *JHEP* **10**, 123 (2016), 1607.05330.

[657] F. Demartin, F. Maltoni, K. Mawatari, B. Page, M. Zaro, *Eur. Phys. J.* **C74**, 3065 (2014), 1407.5089.

[658] S. Khatibi, M. Mohammadi Najafabadi, *Phys. Rev.* **D90**, 074014 (2014), 1409.6553.

[659] M. R. Buckley, D. Goncalves, *Phys. Rev. Lett.* **116**, 091801 (2016), 1507.07926.

[660] N. Mileo, K. Kiers, A. Szynkman, D. Crane, E. Gegner, *JHEP* **07**, 056 (2016), 1603.03632.

[661] CMS collaboration, *Tech. Rep. CMS-PAS-HIG-17-004*, CERN, Geneva (2017), https://cds.cern.ch/record/2256103.

[662] ATLAS collaboration, *Tech. Rep. ATLAS-CONF-2017-077*, CERN, Geneva (2017), https://cds.cern.ch/record/2291405.

[663] D0 collaboration, V. M. Abazov, *et al.*, *Phys. Rev.* **D85**, 091104 (2012), 1201.4156.

[664] CDF collaboration, T. A. Aaltonen, *et al.*, *Phys. Rev. Lett.* **111**, 202001 (2013), 1308.4050.

[665] CMS collaboration, V. Khachatryan, *et al.*, *Phys. Lett.* **B736**, 33 (2014), 1404.2292.

[666] CMS collaboration, *Tech. Rep. CMS-PAS-TOP-16-019*, CERN, Geneva (2016), https://cds.cern.ch/record/2218019.

[667] ATLAS collaboration, M. Aaboud, *et al.*, 1709.04207.

[668] ATLAS collaboration, M. Aaboud, *et al.*, *Eur. Phys. J.* **C77**, 264 (2017), 1612.02577.

[669] CMS collaboration, V. Khachatryan, *et al.*, *Phys. Lett.* **B762**, 512 (2016), 1605.09047.

[670] A. Czarnecki, J. G. Korner, J. H. Piclum, *Phys. Rev.* **D81**, 111503 (2010), 1005.2625.

[671] C.-R. Chen, F. Larios, C. P. Yuan, *Phys. Lett.* **B631**, 126 (2005), hep-ph/0503040.

[672] J. A. Aguilar-Saavedra, J. Carvalho, N. F. Castro, A. Onofre, F. Veloso, *Eur. Phys. J.* **C53**, 689 (2008), 0705.3041.

[673] J. A. Aguilar-Saavedra, J. Bernabeu, *Nucl. Phys.* **B840**, 349 (2010), 1005.5382.

[674] C. Bernardo, *et al.*, *Phys. Rev.* **D90**, 113007 (2014), 1408.7063.

[675] Q.-H. Cao, B. Yan, J.-H. Yu, C. Zhang, *Chin. Phys.* **C41**, 063101 (2017), 1504.03785.

[676] Z. Hioki, K. Ohkuma, *Phys. Lett.* **B752**, 128 (2016), 1511.03437.

[677] J. Drobnak, S. Fajfer, J. F. Kamenik, *Phys. Rev.* **D82**, 114008 (2010), 1010.2402.

[678] C. Zhang, *Phys. Rev.* **D90**, 014008 (2014), 1404.1264.

[679] A. Cordero-Cid, M. A. Perez, G. Tavares-Velasco, J. J. Toscano, *Phys. Rev.* **D70**, 074003 (2004), hep-ph/0407127.

[680] P. M. Ferreira, O. Oliveira, R. Santos, *Phys. Rev.* **D73**, 034011 (2006), hep-ph/0510087.

[681] R. A. Coimbra, *et al.*, *Phys. Rev.* **D79**, 014006 (2009), 0811.1743.

[682] K.-F. Chen, W.-S. Hou, C. Kao, M. Kohda, *Phys. Lett.* **B725**, 378 (2013), 1304.8037.

[683] A. Greljo, J. F. Kamenik, J. Kopp, *JHEP* **07**, 046 (2014), 1404.1278.

[684] J. Drobnak, S. Fajfer, J. F. Kamenik, *Phys. Rev. Lett.* **104**, 252001 (2010), 1004.0620.

[685] J. Drobnak, S. Fajfer, J. F. Kamenik, *Phys. Rev.* **D82**, 073016 (2010), 1007.2551.

[686] J. J. Zhang, *et al.*, *Phys. Rev.* **D82**, 073005 (2010), 1004.0898.

[687] C. Degrande, F. Maltoni, J. Wang, C. Zhang, *Phys. Rev.* **D91**, 034024 (2015), 1412.5594.

[688] G. Durieux, F. Maltoni, C. Zhang, *Phys. Rev.* **D91**, 074017 (2015), 1412.7166.

[689] R. Romero Aguilar, A. O. Bouzas, F. Larios, *Phys. Rev.* **D92**, 114009 (2015), 1509.06431.

[690] S. Davidson, M. L. Mangano, S. Perries, V. Sordini, *Eur. Phys. J.* **C75**, 450 (2015), 1507.07163.

[691] K. Kumar, T. M. P. Tait, R. Vega-Morales, *JHEP* **05**, 022 (2009), 0901.3808.

[692] C. Zhang, 1708.05928.

[693] F. del Aguila, J. A. Aguilar-Saavedra, L. Ametller, *Phys. Lett.* **B462**, 310 (1999), hep-ph/9906462.

[694] F. del Aguila, J. A. Aguilar-Saavedra, *Nucl. Phys.* **B576**, 56 (2000), hep-ph/9909222.

[695] P. M. Ferreira, R. B. Guedes, R. Santos, *Phys. Rev.* **D77**, 114008 (2008), 0802.2075.

[696] J. A. Aguilar-Saavedra, *Nucl. Phys.* **B837**, 122 (2010), 1003.3173.

[697] J.-L. Agram, *et al.*, *Phys. Lett.* **B725**, 123 (2013), 1304.5551.

[698] J. A. Aguilar-Saavedra, *Eur. Phys. J.* **C77**, 769 (2017), 1709.03975.

[699] C. Englert, E. Re, *Phys. Rev.* **D89**, 073020 (2014), 1402.0445.

[700] J. Chang, K. Cheung, J. S. Lee, C.-T. Lu, *JHEP* **05**, 062 (2014), 1403.2053.

[701] A. Kobakhidze, L. Wu, J. Yue, *JHEP* **10**, 100 (2014), 1406.1961.

[702] F. Demartin, F. Maltoni, K. Mawatari, M. Zaro, *Eur. Phys. J.* **C75**, 267 (2015), 1504.00611.

[703] J. A. Aguilar-Saavedra, G. C. Branco, *Phys. Lett.* **B495**, 347 (2000), hep-ph/0004190.

[704] Y. Wang, *et al.*, *Phys. Rev.* **D86**, 094014 (2012), 1208.2902.

[705] G. Passarino, *PoS* **LL2016**, 003 (2016), 1607.01236.

[706] R. S. Chivukula, H. Georgi, *Phys. Lett.* **B188**, 99 (1987).

[707] L. J. Hall, L. Randall, *Phys. Rev. Lett.* **65**, 2939 (1990).

[708] G. D'Ambrosio, G. F. Giudice, G. Isidori, A. Strumia, *Nucl. Phys.* **B645**, 155 (2002), hep-ph/0207036.

[709] T. Feldmann, T. Mannel, *Phys. Rev. Lett.* **100**, 171601 (2008), 0801.1802.

[710] A. L. Kagan, G. Perez, T. Volansky, J. Zupan, *Phys. Rev.* **D80**, 076002 (2009), 0903.1794.

[711] T. Feldmann, M. Jung, T. Mannel, *Phys. Rev.* **D80**, 033003 (2009), 0906.1523.

[712] P. J. Mohr, B. N. Taylor, D. B. Newell, *Rev. Mod. Phys.* **84**, 1527 (2012), 1203.5425.

[713] A. Dedes, W. Materkowska, M. Paraskevas, J. Rosiek, K. Suxho, 1704.03888.

[714] G. Breit, E. Wigner, *Phys. Rev.* **49**, 519 (1936).

[715] K. Hagiwara, D. Zeppenfeld, *Nucl. Phys.* **B274**, 1 (1986).

[716] C. H. Llewellyn Smith, *Nucl. Phys.* **B228**, 205 (1983).

[717] CCFR, E744, E770 collaboration, K. S. McFarland, *et al.*, *Eur.Phys.J.* **C1**, 509 (1998), hep-ex/9701010.

[718] R. Belusevic, J. Smith, *Phys. Rev.* **D37**, 2419 (1988).

[719] R. W. Brown, R. H. Hobbs, J. Smith, N. Stanko, *Phys. Rev.* **D6**, 3273 (1972).

[720] W. Altmannshofer, S. Gori, M. Pospelov, I. Yavin, *Phys. Rev. Lett.* **113**, 091801 (2014), 1406.2332.

[721] Particle Data Group collaboration, K. Olive, *et al.*, *Chin.Phys.* **C38**, 090001 (2014).

[722] J. Erler, S. Su, *Prog.Part.Nucl.Phys.* **71**, 119 (2013), 1303.5522.

[723] P. G. Blunden, W. Melnitchouk, A. W. Thomas, *Phys. Rev. Lett.* **109**, 262301 (2012), 1208.4310.

[724] P. Vetter, D. Meekhof, P. Majumder, S. Lamoreaux, E. Fortson, *Phys.Rev.Lett.* **74**, 2658 (1995).

[725] C. Wood, *et al.*, *Science* **275**, 1759 (1997).

[726] V. Cirigliano, J. Jenkins, M. Gonzalez-Alonso, *Nucl. Phys.* **B830**, 95 (2010), 0908.1754.

[727] C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, B. Mistlberger, *Phys. Rev. Lett.* **114**, 212001 (2015), 1503.06056.

[728] F. Wilczek, *Phys. Rev. Lett.* **39**, 1304 (1977).

[729] S. Dawson, *Nucl. Phys.* **B359**, 283 (1991).

[730] A. Djouadi, M. Spira, P. M. Zerwas, *Phys. Lett.* **B264**, 440 (1991).

[731] C. Degrande, B. Fuks, K. Mawatari, K. Mimasu, V. Sanz, *Eur. Phys. J.* **C77**, 262 (2017), 1609.04833.

[732] J. R. Ellis, M. K. Gaillard, D. V. Nanopoulos, *Nucl. Phys.* **B106**, 292 (1976).