PHYS 139 Final Project - Individual Part

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Part A.

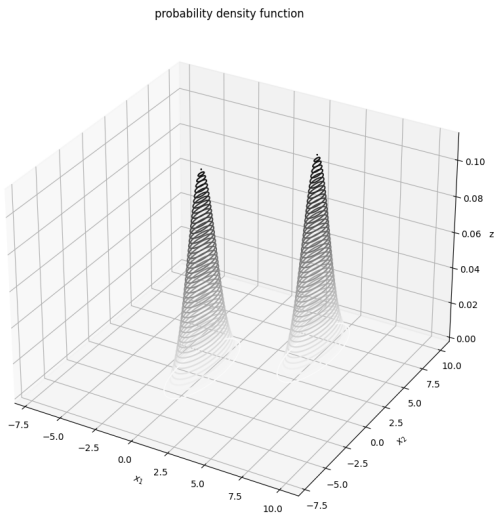

Figure 1: Analytical plot of the probability density function

See *problem1.py* for plotting.

Metropolis Sampling

The Metropolis algorithm is implemented by the *metropolis* function in *problem1.py*. A sample output is shown below. The starting value is (3,3) for this trial and the number of sampled (accepted) points is 200,000.

```
yuntongzhou@Yuntongs-MacBook-Air Final_individual % python3 problem1.py
The mean value of (x1,x2) is  (2.5043507086016783, 2.502325666550363)
```

Figure 2: Sample mean output

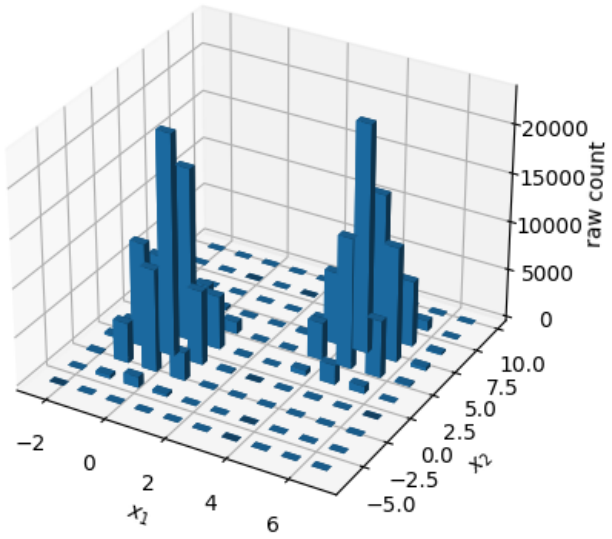

Figure 3: Histogramed results of (x1,x2)

The transition probability is chosen as Gaussian normal distribution centered around the current point with a standard deviation of 2.

Comparison of MC results with analytic expectations

We know that a multivariate gaussian with two random variables has the analytical form,

$$f(x) = \frac{1}{(2\pi)^2|\Sigma|} exp(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu))$$

Therefore, the mixture of two multivariate gaussians, in this case, is,

$$p(x) = 0.5 \times \frac{1}{(2\pi)^2|\Sigma|}[exp(-\frac{1}{2}(x-\mu_1)^T\Sigma^{-1}(x-\mu_1)) + exp(-\frac{1}{2}(x-\mu_2)^T\Sigma^{-1}(x-\mu_2))]$$

From Figure 1, we can see that the result is basically the superposition of two wavefunctions. The mean values of (x1,x2) fluctuate with different choices of initial values and the transition probability functions.

```
yuntongzhou@Yuntongs-MacBook-Air Final_individual % python3 problem1.py
The mean value of (x1,x2) with starting value  (0, 0)  is  (2.532618014104902, 2.5382250154199104)
The mean value of (x1,x2) with starting value  (1, 1)  is  (2.4951404533651957, 2.4946489565944936)
The mean value of (x1,x2) with starting value  (2, 2)  is  (2.4197135136620904, 2.42748935033825)
The mean value of (x1,x2) with starting value  (3, 3)  is  (2.4564278649720688, 2.457300996203891)
The mean value of (x1,x2) with starting value  (4, 4)  is  (2.571057884043884, 2.5674714085078314)
```

Figure 4: Effects of starting values on the final means

The metropolis sampling is not precise but does reflect the general shape of the wavefunctions.