Analytic answers for Assignment1

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January 31, 2022

Problem 1

The three trajectories of linear pendulum are saved in three dat files ("problem1a1.dat", "problem1a2.dat", "problem1a3.dat"). The 2D plot of (x,v) is saved as "problem1a.jpg".
The three trajectories of non-linear pendulum are saved in three dat files ("problem1b1.dat", "problem1b2.dat", "problem1b3.dat"). The 2D plot of (x,v) is saved as "problem1b.jpg". The python codes for generating the 2D plot are later adapted to plot planet orbits.
The animation of the nonlinear trajectory is saved in "problem1c.gif".
3D plots are generated by "3DPlot.py". The non-linear is saved as "3Dplot2.png". The linear one is saved as "3Dplot1.png".

Problem 2

All data files of the planets are generated from "problem2.c". The dat files are saved in directory planetorbitdat/. The plot is made from "Problem2.py". The plot is saved as "planetorbitplot.png".

Problem 3

"problem3.c" is used to generate the analytic orbits of Mars and Earth. "Earthanalytic.dat" and "Marsanalytic.dat" are the data files with two columns (x,y). "problem3.py" is used to plot the two orbits together. "animation2.py" is used to generate animation of earth's orbit ("earthanimation.gif")
a.
Based on the graphs provided ("Problem3Earth.png", "Problem3Mars.png"), we can see that the leapfrog orbit fluctuates quite a good deal after several repetitions of cycles. Moreover, the fluctuations worsen as the number of revolutions grow.

Problem 4

$$\vec{L} = L_z \hat{z} L_z = m(xv_y - yv_x) = m(x(t)v_y(t) - y(t)v_x(t))$$

$$v_y(t) = v_y(t - \frac{\delta t}{2}) + a_y(t) \times 0.5 \cdot \delta t = v_y(t - \delta t) + a_y(t - \delta) \times 0.5 \cdot \delta t + a_y(t) \times 0.5 \cdot \delta t$$

$$y(t) = y(t - \delta t) + v_y(t - \frac{\delta t}{2})\delta t = y(t - \delta t) + v_y(t - \delta t)\delta t + a_y(t - \delta t) \times 0.5 \cdot \delta t^2$$

Similarly,

$$v_x(t) = v_x(t - \frac{\delta t}{2}) + a_x(t) \times 0.5 \cdot \delta t = v_x(t - \delta t) + a_x(t - \delta t) \times 0.5 \cdot \delta t + a_x(t) \times 0.5 \cdot \delta t$$

$$x(t) = x(t - \delta t) + v_x(t - \frac{\delta t}{2})\delta t = x(t - \delta t) + v_x(t - \delta t)\delta t + a_x(t - \delta t) \times 0.5 \cdot \delta t^2$$

Rearrange the equations, we can discover that some terms cancel, and thus,

$$x(t)v_y(t) - y(t)v_x(t) = x(t - \delta t)v_y(t - \delta t) - y(t - \delta t)v_x(t - \delta t)$$

We thus prove the conservation of angular momentum in leapfrog-verlet integration.