of uncertainty in the use of the continuum hypothesis to achieve a tractable mathematical description of fluid motions and heat transfer processes.

A second similar consequence of the continuum hypothesis is an uncertainty in the boundary conditions to be used in conjunction with the resulting equations for motion and heat transfer. With the continuum hypothesis adopted, the conservation principles of classical physics, listed earlier, will be shown to provide a set of so-called field equations for molecular average variables such as the continuum point velocity **u**. To solve these equations, however, the values of these variables or their derivatives must be specified at the boundaries of the fluid domain. These boundaries may be solid surfaces, or the phase boundary between a liquid and a gas, or the phase boundary between two liquids. In any case, when viewed on the *molecular* scale, the "boundaries" are seen to be regions of rapid but continuous variation in fluid properties such as number density. Thus, in a molecular theory, boundary conditions would not be necessary. When viewed with the much coarser resolution of the macroscopic or continuum description, on the other hand, these local variations of density (and other molecular variables) can only be distinguished as discontinuities, and the continuum (or molecular average) variables such as **u** appear to vary smoothly on the scale L, right up to the boundary where some boundary condition is applied.

One consequence of the inability of the continuum description to resolve the region nearest the boundary is that the continuum variables extrapolated toward the boundary from the two sides may experience jumps or discontinuities. This is definitely the case at a fluid interface, as we shall see. Even at solid boundaries, the fluid velocity **u** may appear to "slip" when the fluid is a high molecular weight material or a particulate suspension.[2]

Here the situation is very similar to that encountered in connection with the need for continuum (constitutive) models for the molecular transport processes in that a derivation of appropriate boundary conditions from the more fundamental, molecular description has not been accomplished to date. In both cases, the knowledge that we have of constitutive models and boundary conditions that are appropriate for the continuum level description is largely empirical in nature. In effect, we make an educated guess for both constitutive equations and boundary conditions, and then normally judge the success of our choices by the resulting comparison between predicted and measured continuum velocity or temperature fields. Models derived from molecular theories are generally not available for comparison with the empirically proposed models. We shall discuss some of these matters in more detail later in this chapter, where specific choices will be proposed for both the constitutive equations and boundary conditions.

B. Conservation of Mass; The Continuity Equation

Once we adopt the continuum hypothesis and choose to describe fluid motions and heat transfer processes from a macroscopic point of view, the governing equations are derived by invoking the usual conservation principles of classical continuum physics. These are conservation of mass, conservation of linear and angular momentum, and conservation of energy. As usual in macroscopic mechanics, we apply these

principles to a specific mass of the material that moves along through space. The only real conceptual difficulty is the fact that the chosen mass of material changes shape as a consequence of spatial gradients in the continuum point velocity field.

The simplest of the various conservation principles to apply is conservation of mass. It is instructive to consider its application relative to two different, but equivalent, descriptions of our fluid system. In both cases, we begin by identifying a specific macroscopic body of fluid that lies within an arbitrarily chosen volume element at some initial instant of time. Since we have adopted the continuum mechanics point of view, this volume element will be large enough that any flux of mass across its surface due to random molecular motions can be neglected completely. Indeed, in this continuum description of our system, we can resolve only the molecular average (or continuum point) velocities, and it is convenient to drop any reference to the averaging symbol $< >$. The continuum point velocity vector is denoted as $\mathbf{u}$.

In the first description of mass conservation for our system, we consider the volume element (here called a *control volume*) to be fixed in position and shape as illustrated in Figure 2–2. Thus, at each point on its surface, there is a mass flux of fluid $\rho\mathbf{u}\cdot\mathbf{n}$. With $\mathbf{n}$ chosen as the outer unit normal to the surface, this mass flux will be negative at points where fluid enters the volume element and positive where it exits. There is no reason, at this point, to assume that the fluid density ρ is necessarily constant. Indeed, conservation of mass requires the density inside the volume element to change with time in such a way that any imbalance in the mass flux in and out of the volume element is compensated by an accumulation of mass inside. Expressing this statement in mathematical terms

$$\boxed{\int_V \frac{\partial \rho}{\partial t} dV = -\int_S \rho \mathbf{u}\cdot\mathbf{n} dS,} \tag{2–3}$$

where V denotes the arbitrarily chosen volume element of fixed position and shape, and S denotes its (closed) surface. Equation (2–3) is an integral constraint on the velocity and density fields within a given closed volume element of fluid. Since this volume element was chosen arbitrarily, however, an equivalent differential constraint at each point in the fluid can be derived easily. First, the well-known divergence theorem[3] is applied to the right-hand side of (2–3), which thus becomes

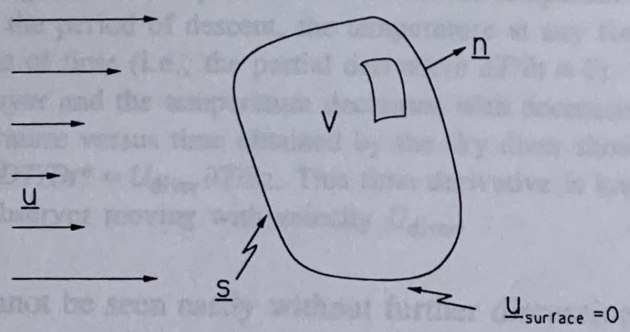

Figure 2–2 An arbitrarily chosen control volume of *fixed* position and shape, immersed in a fluid with velocity $\mathbf{u}$. The velocity of the surface of the control volume is zero, and thus, there is a net flux of fluid through its surface. Equation 2–3 represents a mass balance on the volume V, with the left hand side giving the rate of mass accumulation and the right hand side the net flux of mass *into* V due to the motion $\mathbf{u}$.