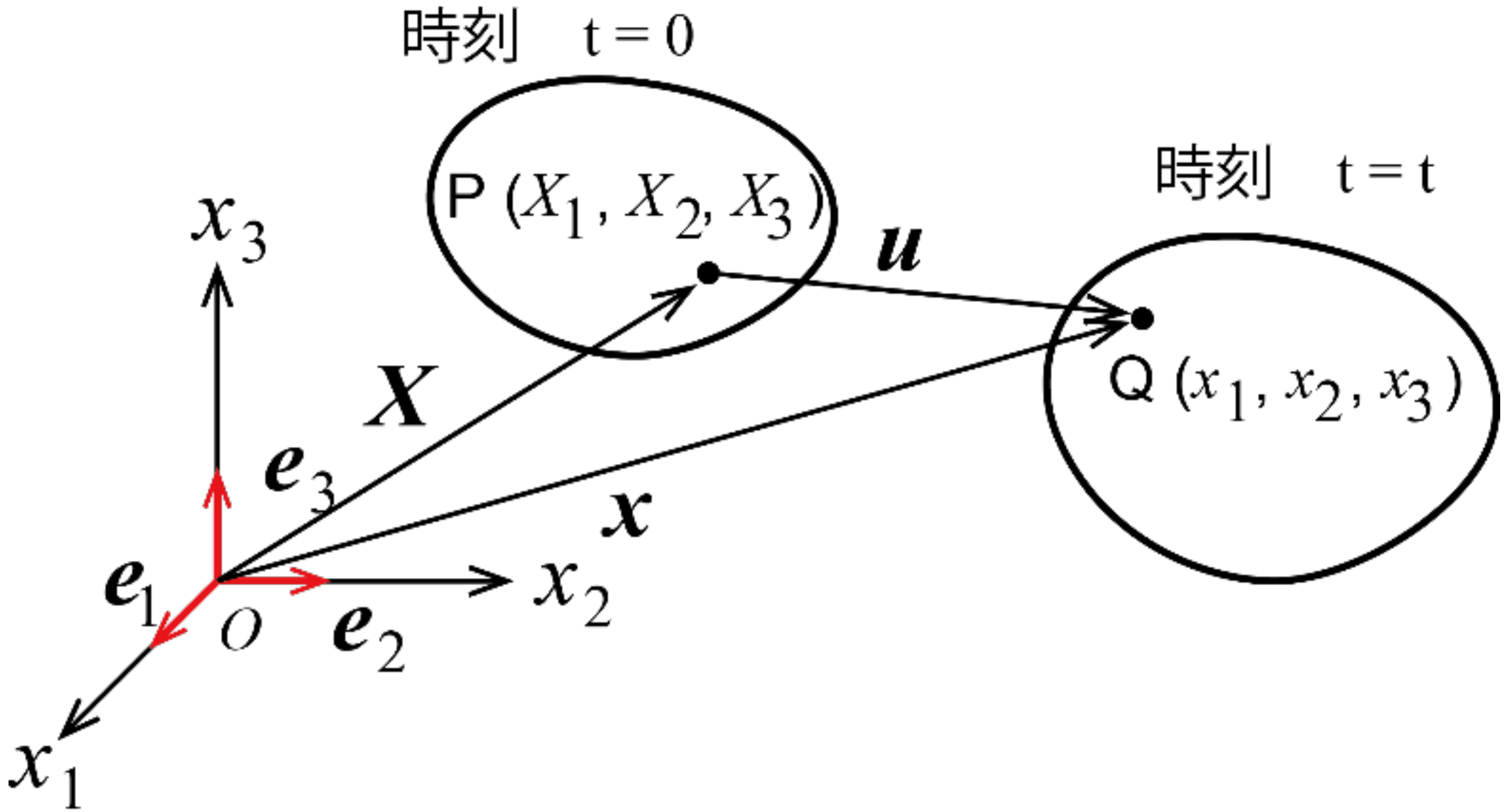
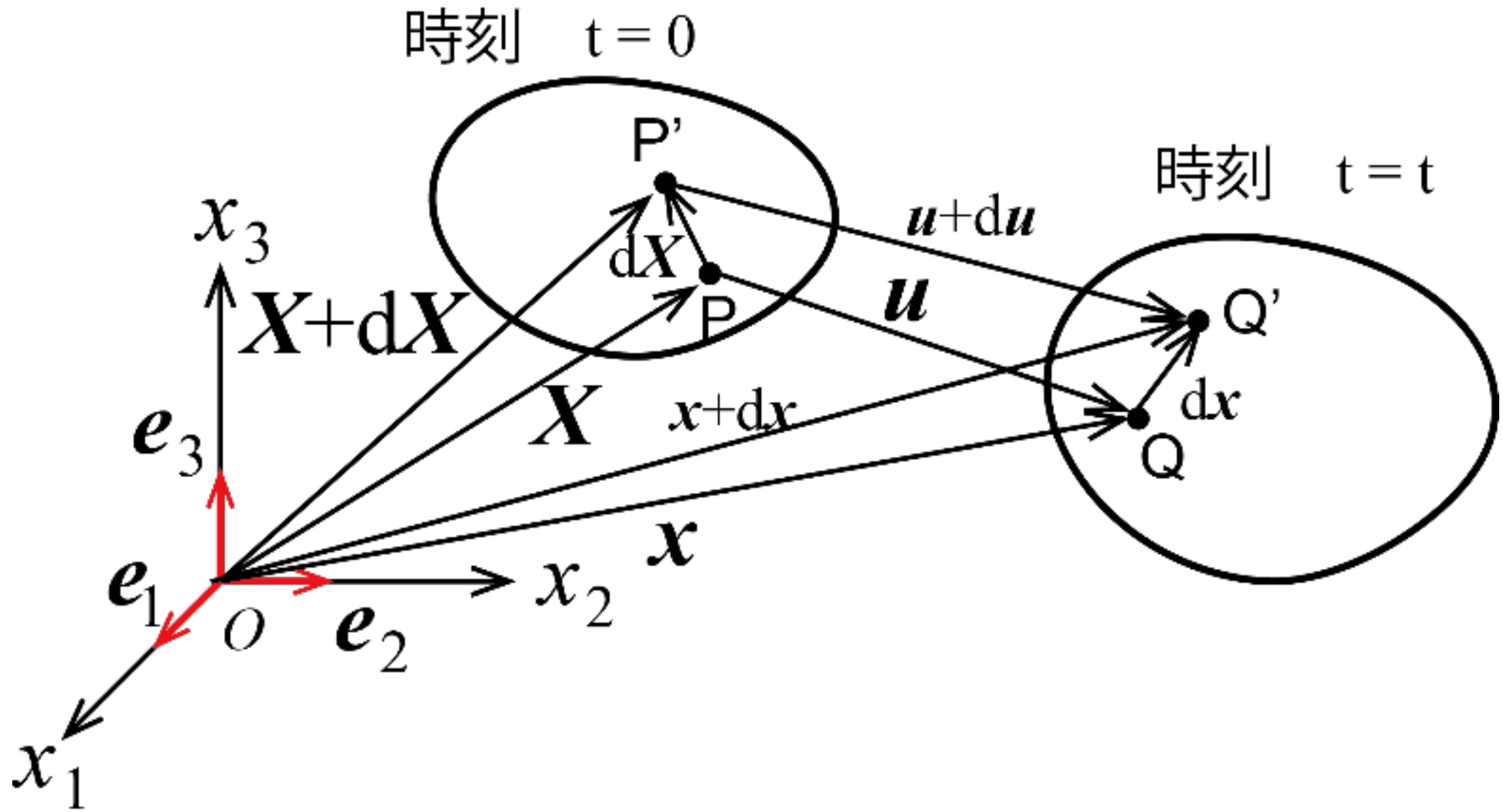
物体の運動



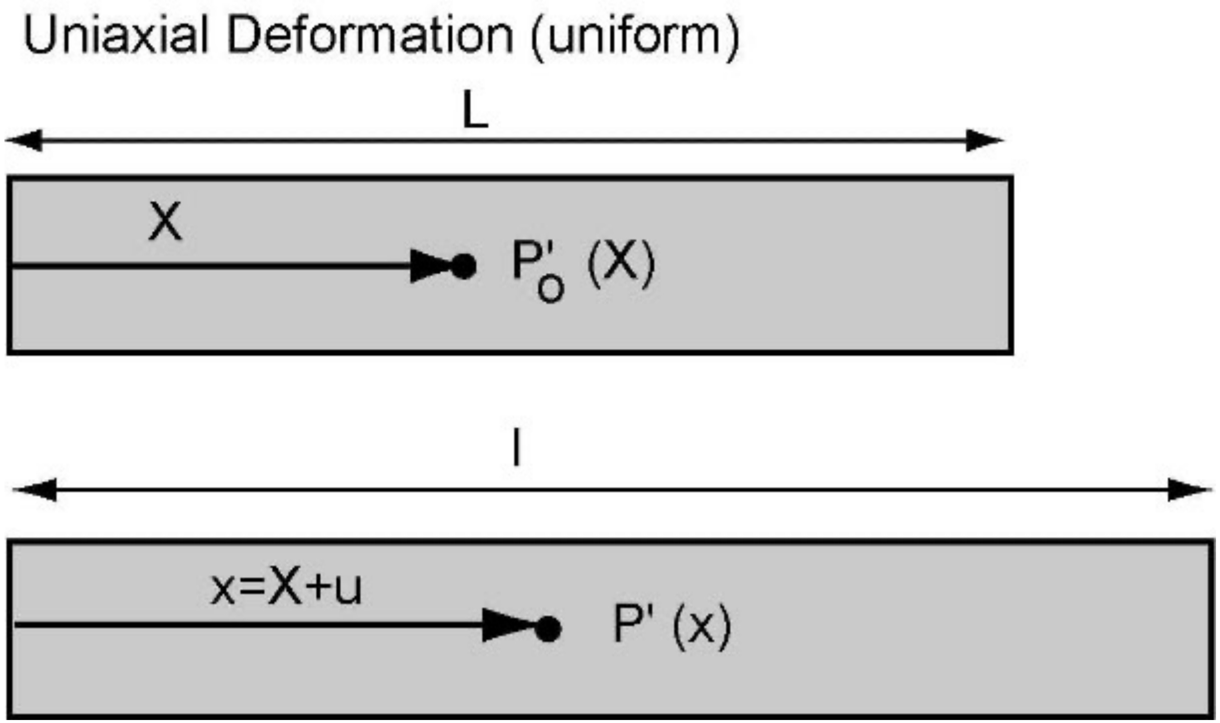
時刻　t = 0
P (X_1, X_2, X_3)
時刻　t = t
$\boldsymbol{u}$
Q (x_1, x_2, x_3)
x_3
$\boldsymbol{X}$
$\boldsymbol{e}_3$
$\boldsymbol{x}$
$\boldsymbol{e}_1$
O
$\boldsymbol{e}_2$
x_2
x_1

ひずみの定義

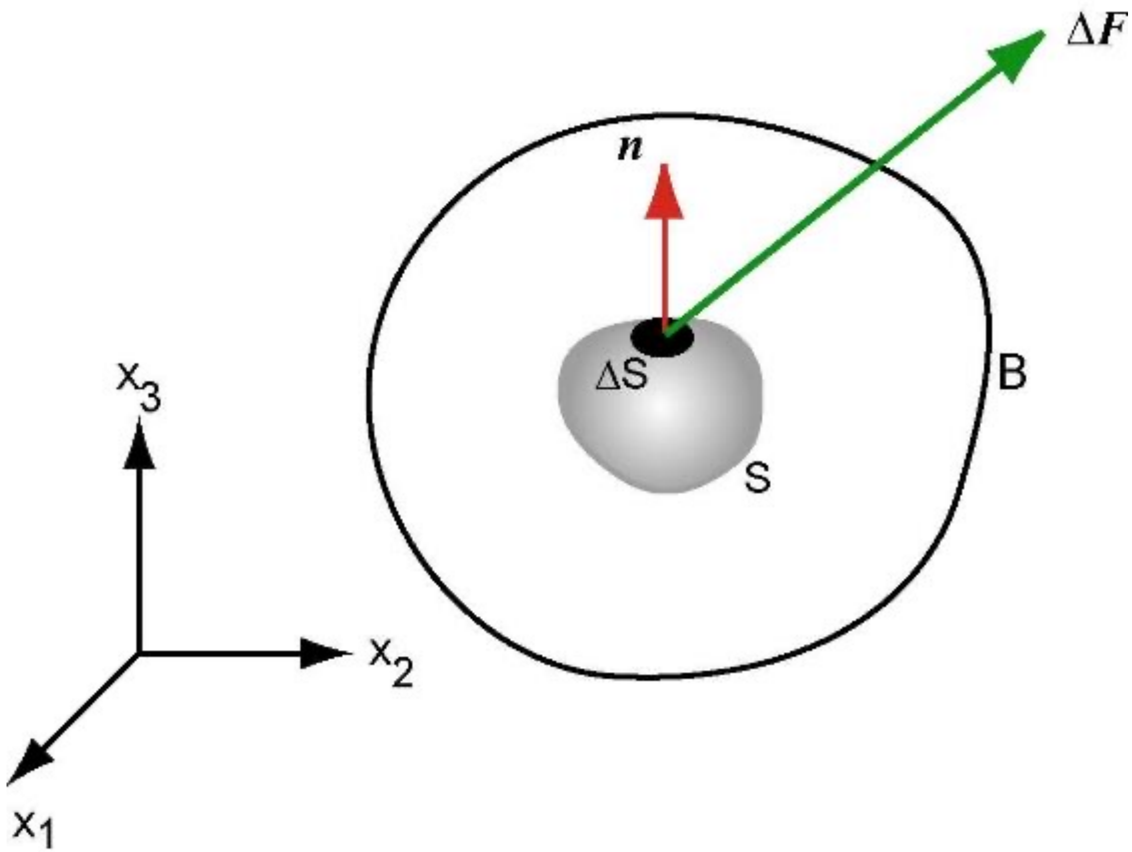


線分　PP'と 線分 QQ' の長さを比較する

何故　1／2　？？？？？？

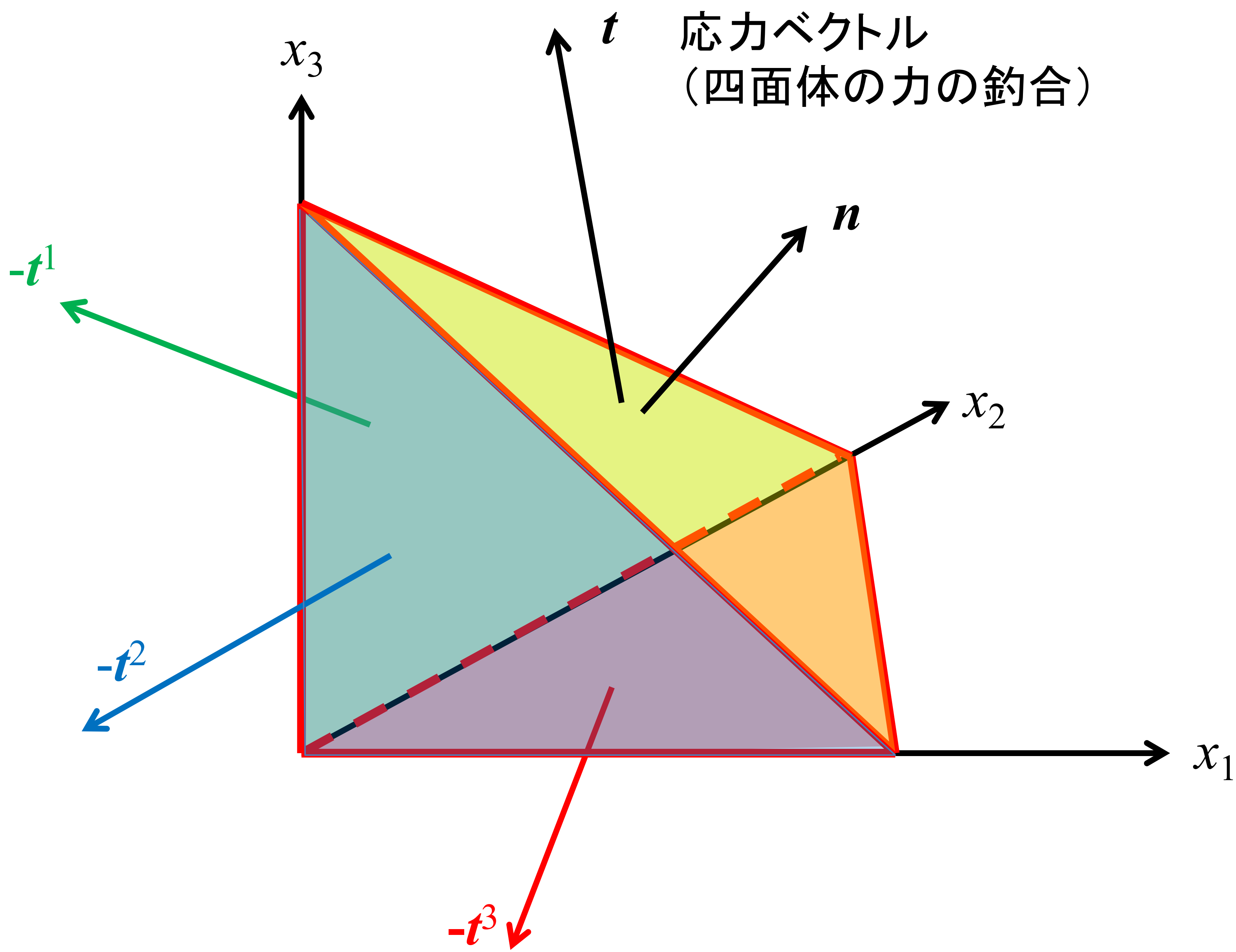


応力ベクトル、トラクション（Stress Vector, Traction Vector）



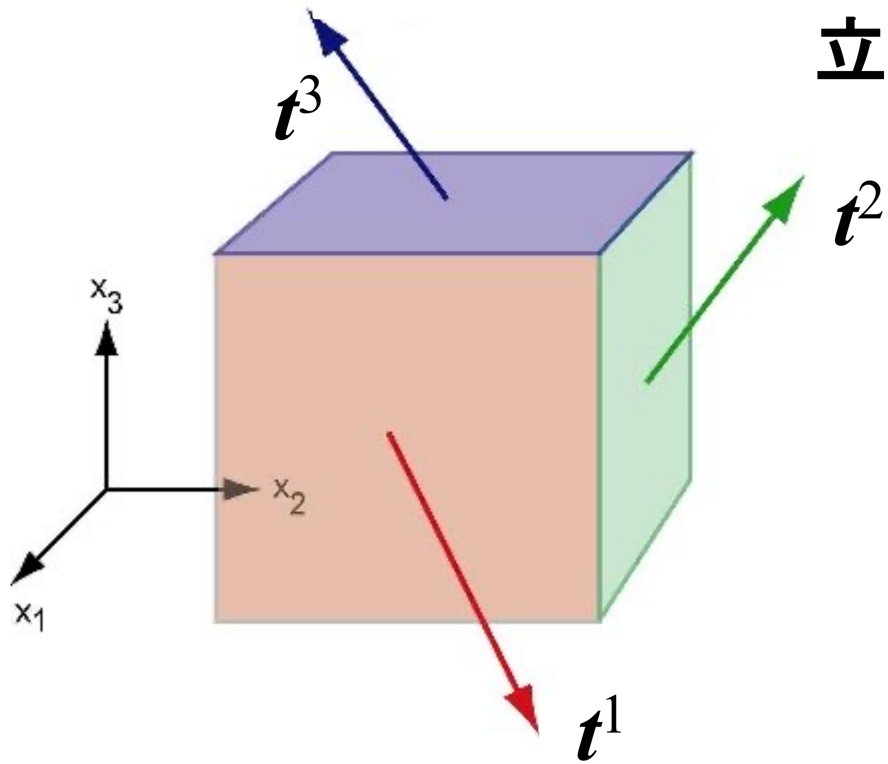
応力ベクトル：　$$\boldsymbol{T} = \frac{\mathrm{d}\boldsymbol{F}}{\mathrm{d}S} = \lim_{\Delta S \to 0} \frac{\Delta \boldsymbol{F}}{\Delta S}$$

単位断面積あたりに働く力をベクトルで表現

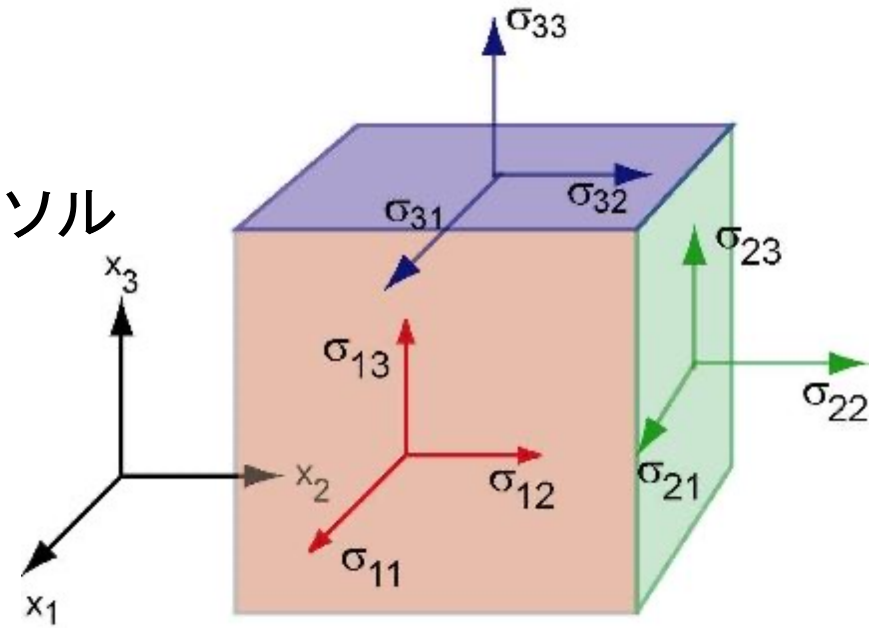


応力ベクトル
（四面体の力の釣合）
t
n
x_3
x_2
x_1
$-t^1$
$-t^2$
$-t^3$

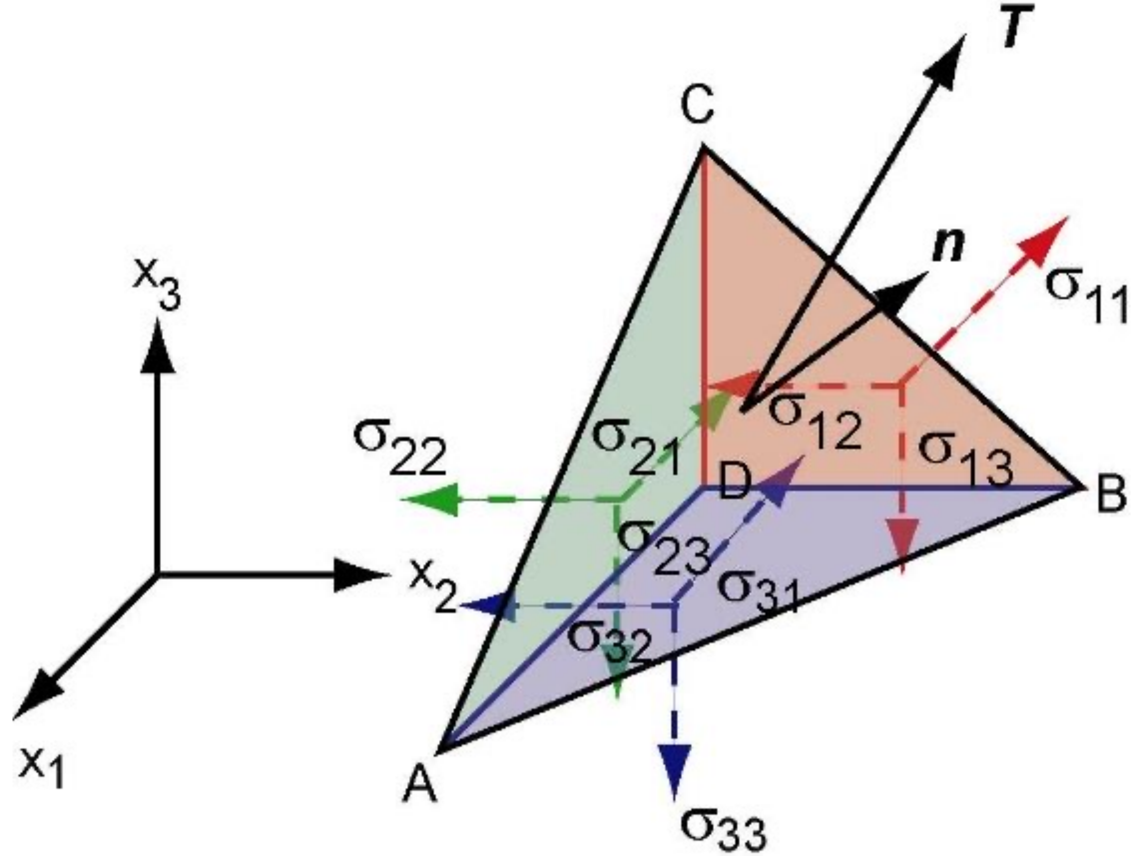
立方体各面の応力ベクトル



応力ベクトルの成分が応力テンソル

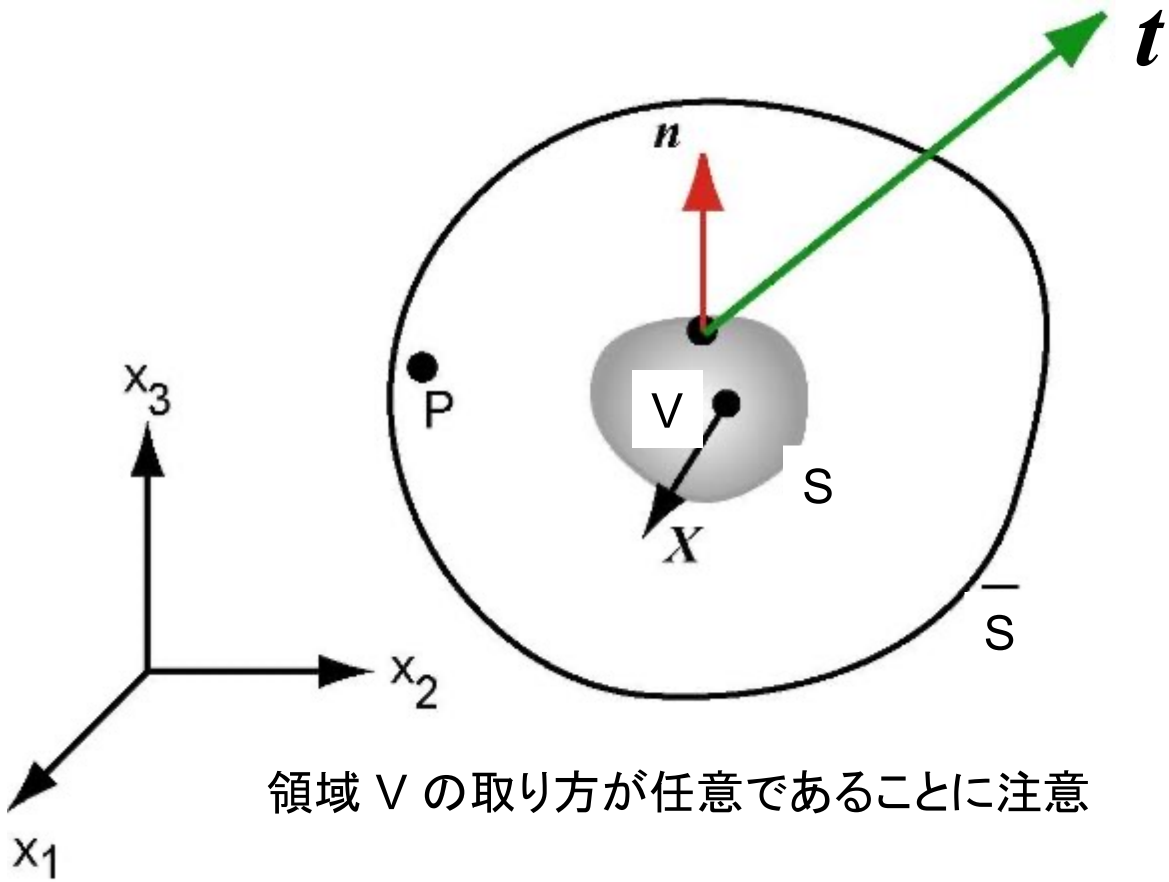


四面体に働く表面力（力の釣合を考える）



例えば、三角形ABCの x_1 軸に垂直な面（x_2-x_3 平面）への投影の面積に注意して考える

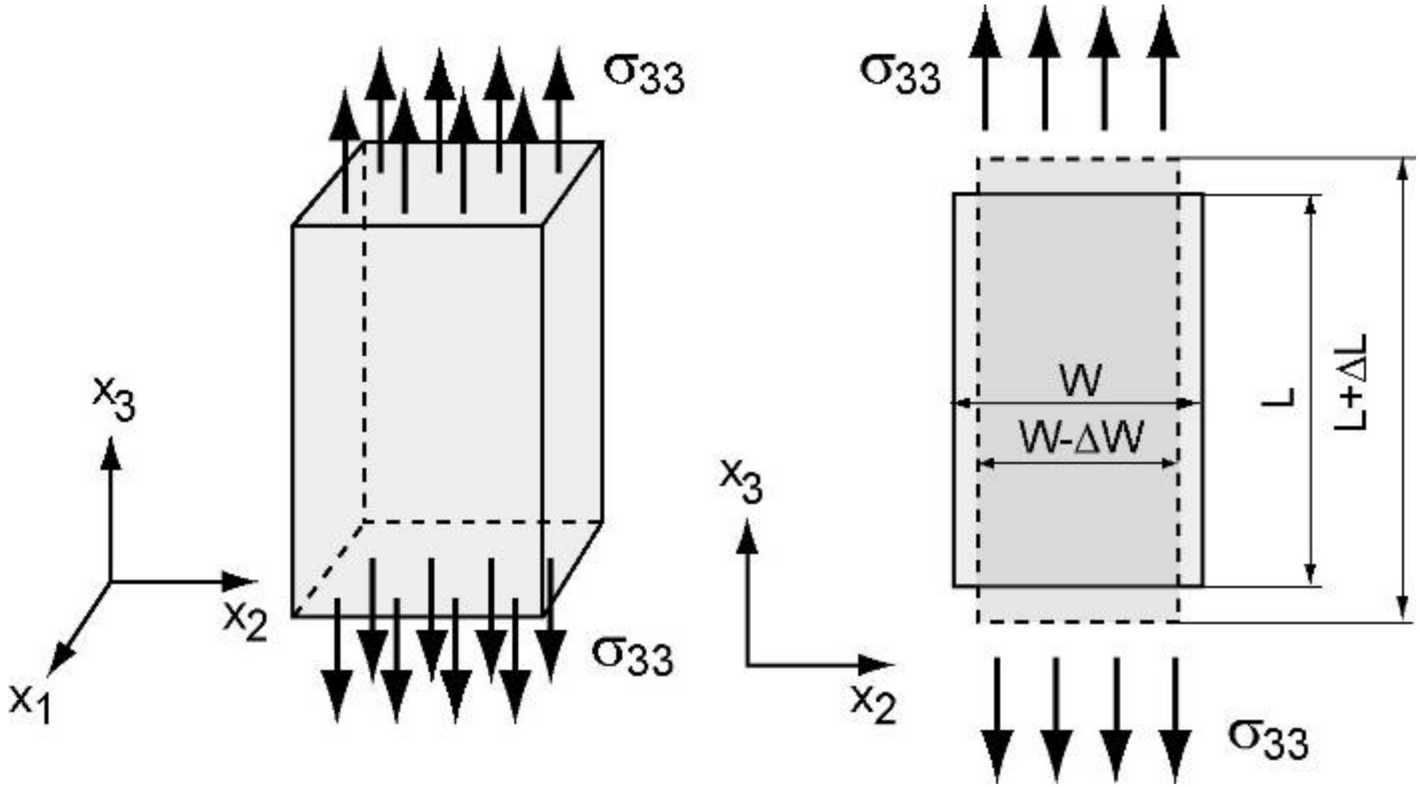
釣合方程式、平衡方程式（Equation of equilibrium, linear momentum balance law）
応力テンソルの対称性（Angular momentum balance law）



領域 V の取り方が任意であることに注意

領域Vに関する<u>力の釣合</u>と<u>モーメントの釣合</u>を計算する

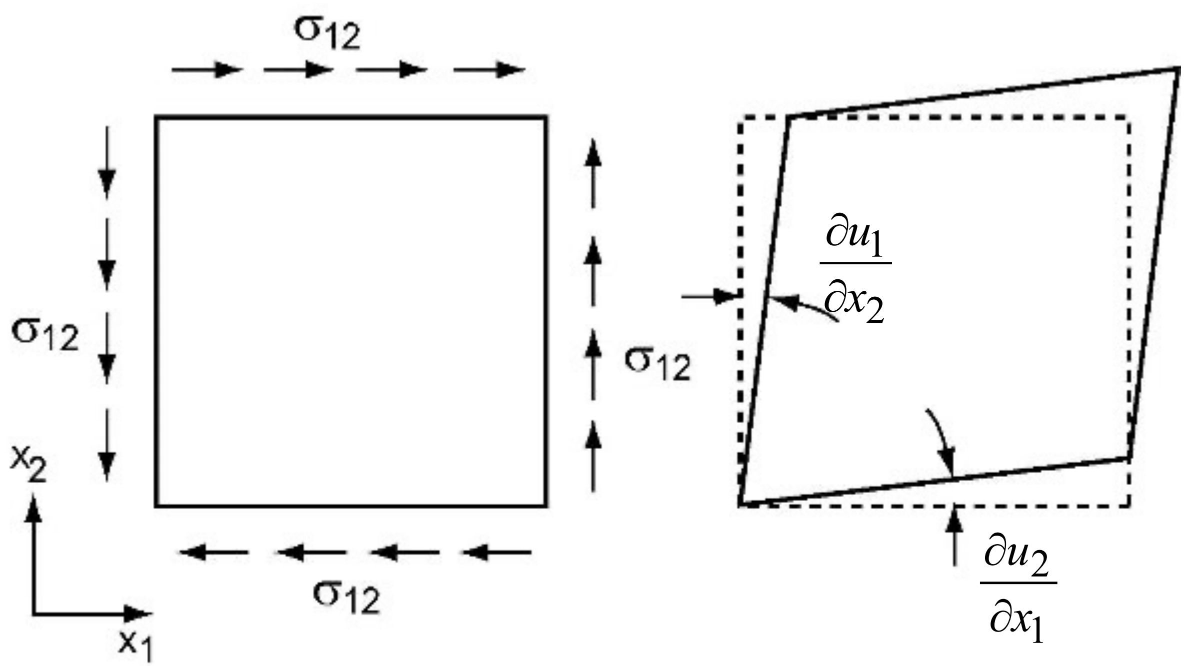
ヤング率とポアソン比



$$\sigma_{11} = \sigma_{22} = 0$$
$$\sigma_{33} \neq 0$$
$$\sigma_{12} = \sigma_{23} = \sigma_{31} = 0$$

$$\varepsilon_{11} = \varepsilon_{22} = -\frac{\Delta W}{W}$$
$$\varepsilon_{33} = \frac{\Delta L}{L}$$

せん断弾性定数

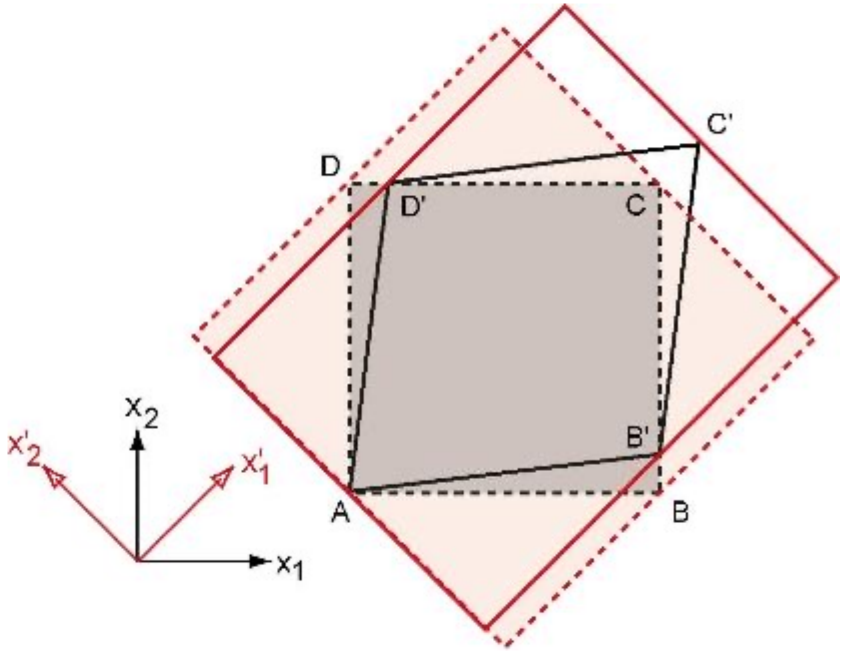


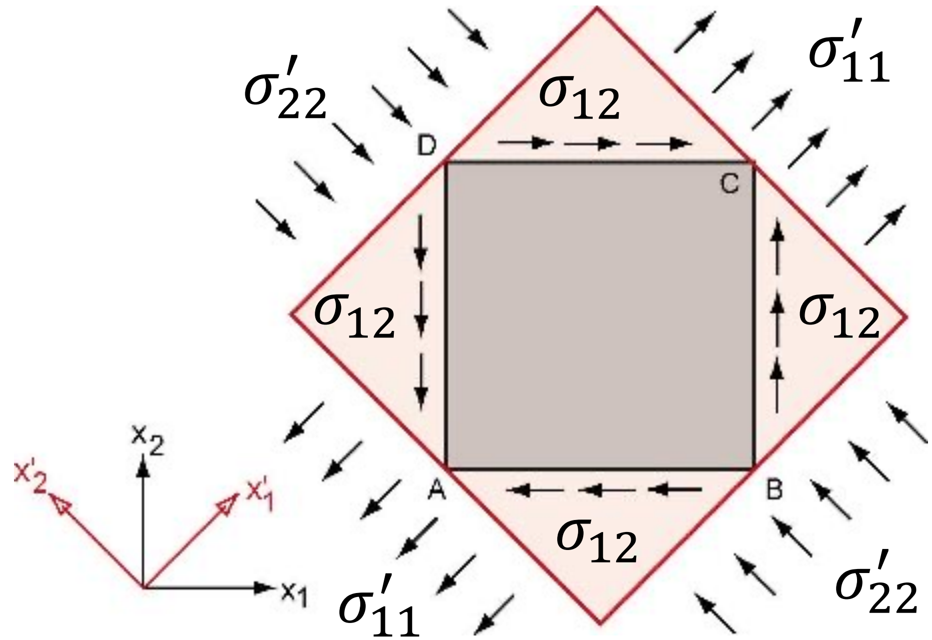
$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$$

$$\varepsilon_{12} = \frac{\sigma_{12}}{2\mu}$$

μ :せん断弾性定数
（shear modulus）

E と μ の関係ー単純せん断





x_1-x_2 座標系：せん断変形

x'_1: 引張、x'_2: 圧縮

応力-ひずみ関係

$$\sigma_{ij} = D^e_{ijkl}\varepsilon_{kl}$$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{21} \\ \sigma_{22} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{32} \\ \sigma_{33} \end{Bmatrix} = \begin{bmatrix} D^e_{1111} & D^e_{1112} & D^e_{1113} & D^e_{1121} & D^e_{1122} & D^e_{1123} & D^e_{1131} & D^e_{1132} & D^e_{1133} \\ D^e_{1211} & D^e_{1212} & D^e_{1213} & D^e_{1221} & D^e_{1222} & D^e_{1223} & D^e_{1231} & D^e_{1232} & D^e_{1233} \\ D^e_{1311} & D^e_{1312} & D^e_{1313} & D^e_{1321} & D^e_{1322} & D^e_{1323} & D^e_{1331} & D^e_{1332} & D^e_{1333} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{21} \\ \varepsilon_{22} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{32} \\ \varepsilon_{33} \end{Bmatrix}$$

計81個の比例定数（弾性定数）

応力-ひずみ関係
（応力とひずみの独立な成分はそれぞれ6つ）

$$
\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix}
=
\begin{bmatrix}
D^e_{1111} & D^e_{1122} & D^e_{1133} & D^e_{1112} & D^e_{1123} & D^e_{1131} \\
D^e_{2211} & D^e_{2222} & D^e_{2222} & D^e_{2212} & D^e_{2223} & D^e_{2231} \\
D^e_{3311} & D^e_{3322} & D^e_{3333} & D^e_{3312} & D^e_{3323} & D^e_{3331} \\
D^e_{1211} & D^e_{1222} & D^e_{1233} & D^e_{1212} & D^e_{1223} & D^e_{1231} \\
D^e_{2311} & D^e_{2322} & D^e_{2333} & D^e_{2312} & D^e_{2323} & D^e_{2331} \\
D^e_{3111} & D^e_{3122} & D^e_{3133} & D^e_{3112} & D^e_{3123} & D^e_{3131}
\end{bmatrix}
\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix}
$$

計36個の比例定数（弾性定数）

$$
\frac{1}{2}\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix}^T
\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix}
=
\frac{1}{2}\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix}^T
\begin{bmatrix}
D^e_{1111} & D^e_{1122} & D^e_{1133} & D^e_{1112} & D^e_{1123} & D^e_{1131} \\
D^e_{2211} & D^e_{2222} & D^e_{2222} & D^e_{2212} & D^e_{2223} & D^e_{2231} \\
D^e_{3311} & D^e_{3322} & D^e_{3333} & D^e_{3312} & D^e_{3323} & D^e_{3331} \\
D^e_{1211} & D^e_{1222} & D^e_{1233} & D^e_{1212} & D^e_{1223} & D^e_{1231} \\
D^e_{2311} & D^e_{2322} & D^e_{2333} & D^e_{2312} & D^e_{2323} & D^e_{2331} \\
D^e_{3111} & D^e_{3122} & D^e_{3133} & D^e_{3112} & D^e_{3123} & D^e_{3131}
\end{bmatrix}
\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix}
\geq 0
$$

ひずみエネルギ密度は負にならない

応力-ひずみ関係

$$\frac{1}{2}\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix}^T \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} = \frac{1}{2}\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix}^T \begin{bmatrix} D^e_{1111} & D^e_{1122} & D^e_{1133} & D^e_{1112} & D^e_{1123} & D^e_{1131} \\ & D^e_{2222} & D^e_{2222} & D^e_{2212} & D^e_{2223} & D^e_{2231} \\ & & D^e_{3333} & D^e_{3312} & D^e_{3323} & D^e_{3331} \\ & & & D^e_{1212} & D^e_{1223} & D^e_{1231} \\ & \text{Symm.} & & & D^e_{2323} & D^e_{2331} \\ & & & & & D^e_{3131} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix} \geq 0$$

マトリックスは対称（21個の定数）

応力-ひずみ関係
軸を交換(3つの直交する座標方向に対して性質が同じ)

$$
\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix}
=
\begin{bmatrix}
D^e_{1111} & D^e_{1122} & D^e_{1133} & D^e_{1112} & D^e_{1123} & D^e_{1131} \\
 & D^e_{2222} & D^e_{2233} & D^e_{2212} & D^e_{2223} & D^e_{2231} \\
 & & D^e_{3333} & D^e_{3312} & D^e_{3323} & D^e_{3331} \\
 & & & D^e_{1212} & D^e_{1223} & D^e_{1231} \\
 & \text{Symm.} & & & D^e_{2323} & D^e_{2331} \\
 & & & & & D^e_{3131}
\end{bmatrix}
\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix}
$$

$$
\begin{Bmatrix} \sigma_{11} \\ \sigma_{33} \\ \sigma_{22} \\ \sigma_{12} \\ \sigma_{31} \\ \sigma_{23} \end{Bmatrix}
=
\begin{bmatrix}
D^e_{1111} & D^e_{1122} & D^e_{1133} & D^e_{1112} & D^e_{1123} & D^e_{1131} \\
 & D^e_{2222} & D^e_{2233} & D^e_{2212} & D^e_{2223} & D^e_{2231} \\
 & & D^e_{3333} & D^e_{3312} & D^e_{3323} & D^e_{3331} \\
 & & & D^e_{1212} & D^e_{1223} & D^e_{1231} \\
 & \text{Symm.} & & & D^e_{2323} & D^e_{2331} \\
 & & & & & D^e_{3131}
\end{bmatrix}
\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{33} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \\ 2\varepsilon_{31} \\ 2\varepsilon_{23} \end{Bmatrix}
$$

応力-ひずみ関係
対称性の議論

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} = \begin{bmatrix} D^e_{1111} & D^e_{1122} & D^e_{1133} & D^e_{1112} & D^e_{1123} & D^e_{1131} \\ & D^e_{2222} & D^e_{2233} & D^e_{2212} & D^e_{2223} & D^e_{2231} \\ & & D^e_{3333} & D^e_{3312} & D^e_{3323} & D^e_{3331} \\ & & & D^e_{1212} & D^e_{1223} & D^e_{1231} \\ & \text{Symm.} & & & D^e_{2323} & D^e_{2331} \\ & & & & & D^e_{3131} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ -\sigma_{12} \\ -\sigma_{23} \\ \sigma_{31} \end{Bmatrix} = \begin{bmatrix} D^e_{1111} & D^e_{1122} & D^e_{1133} & D^e_{1112} & D^e_{1123} & D^e_{1131} \\ D^e_{1122} & D^e_{2222} & D^e_{2233} & D^e_{2212} & D^e_{2223} & D^e_{2231} \\ D^e_{1133} & D^e_{2233} & D^e_{3333} & D^e_{3312} & D^e_{3323} & D^e_{3331} \\ D^e_{1112} & D^e_{2212} & D^e_{3312} & D^e_{1212} & D^e_{1223} & D^e_{1231} \\ D^e_{1123} & D^e_{2223} & D^e_{3323} & D^e_{1223} & D^e_{2323} & D^e_{2331} \\ D^e_{1131} & D^e_{2231} & D^e_{3331} & D^e_{1231} & D^e_{2331} & D^e_{3131} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ -2\varepsilon_{12} \\ -2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix}$$

応力-ひずみ関係
対称性の議論

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ -\sigma_{23} \\ -\sigma_{31} \end{Bmatrix} = \begin{bmatrix} D^e_{1111} & D^e_{1122} & D^e_{1133} & D^e_{1112} & D^e_{1123} & D^e_{1131} \\ D^e_{1122} & D^e_{2222} & D^e_{2233} & D^e_{2212} & D^e_{2223} & D^e_{2231} \\ D^e_{1133} & D^e_{2233} & D^e_{3333} & D^e_{3312} & D^e_{3323} & D^e_{3331} \\ D^e_{1112} & D^e_{2212} & D^e_{3312} & D^e_{1212} & D^e_{1223} & D^e_{1231} \\ D^e_{1123} & D^e_{2223} & D^e_{3323} & D^e_{1223} & D^e_{2323} & D^e_{2331} \\ D^e_{1131} & D^e_{2231} & D^e_{3331} & D^e_{1231} & D^e_{2331} & D^e_{3131} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ -2\varepsilon_{23} \\ -2\varepsilon_{31} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ -\sigma_{12} \\ \sigma_{23} \\ -\sigma_{31} \end{Bmatrix} = \begin{bmatrix} D^e_{1111} & D^e_{1122} & D^e_{1133} & D^e_{1112} & D^e_{1123} & D^e_{1131} \\ D^e_{1122} & D^e_{2222} & D^e_{2233} & D^e_{2212} & D^e_{2223} & D^e_{2231} \\ D^e_{1133} & D^e_{2233} & D^e_{3333} & D^e_{3312} & D^e_{3323} & D^e_{3331} \\ D^e_{1112} & D^e_{2212} & D^e_{3312} & D^e_{1212} & D^e_{1223} & D^e_{1231} \\ D^e_{1123} & D^e_{2223} & D^e_{3323} & D^e_{1223} & D^e_{2323} & D^e_{2331} \\ D^e_{1131} & D^e_{2231} & D^e_{3331} & D^e_{1231} & D^e_{2331} & D^e_{3131} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ -2\varepsilon_{12} \\ 2\varepsilon_{23} \\ -2\varepsilon_{31} \end{Bmatrix}$$

３つの独立な成分

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} = \begin{bmatrix} D^e_{1111} & D^e_{1122} & D^e_{1122} & 0 & 0 & 0 \\ & D^e_{1111} & D^e_{1122} & 0 & 0 & 0 \\ & & D^e_{1111} & 0 & 0 & 0 \\ & & & D^e_{1212} & 0 & 0 \\ & \text{Symm.} & & & D^e_{1212} & 0 \\ & & & & & D^e_{1212} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix}$$