the thermodynamic state variables (say, $p$ and $\theta$) or the dependence of $\nabla\theta$ on spatial position.

4.  When there is no temperature gradient, the surface heat flux is identically zero.

It should be emphasized that all the preceding points are simply assumptions underlying the assumed constitutive form (2–61). We can make no claim *on the basis of continuum mechanics alone* that these assumptions or the basic linearity of (2–61) will necessarily be satisfied by any real fluid.

Fortunately, in view of the simplicity of (2–61), comparison between predicted and measured data for the heat flux and temperature gradient shows that the general linear form does work extremely well for many common gases and liquids. However, the majority of these materials exhibit one additional characteristic that leads to further simplification of the constitutive form (2–61)—they are *isotropic*. This means that the magnitude of the heat flux at any point is dependent only on the *magnitude* of the temperature gradient, not on its *orientation* relative to axes fixed in the material. A common material that is not isotropic in this sense is wood, because a temperature gradient of given magnitude in wood generally produces a larger heat flux if it is oriented along the grain than it does if it is oriented across the grain. In the absence of motion, almost all common fluids will be isotropic (an exception is a liquid crystalline material). If the fluid is made up of molecules and/or particles that are not spherical (or spherically symmetric), the orientations of these structural elements will generally be random as a consequence of random (Brownian) motions. Hence, when seen from the spatially averaged continuum viewpoint, such a fluid will be isotropic.

A mathematical statement of the *property of isotropy* is that the constitutive equation must be completely invariant to rotations of the coordinate reference system. In other words, in the present case, if we consider the components of $\mathbf{q}$ and $\nabla\theta$ defined relative to two coordinate systems that differ by rigid body rotation through some arbitrary angle (or angles) and then if

$$\mathbf{q} = \mathbf{F}(\nabla\theta),$$

it follows from the assumption of isotropy that

$$\bar{\mathbf{q}} = \bar{\mathbf{F}}(\overline{\nabla\theta}),$$

where $\mathbf{F}$ and $\bar{\mathbf{F}}$ are *identical* functions.

We have already seen that the constitutive form (2–61) transforms under a general orthogonal transformation to the form

$$\bar{\mathbf{q}} = \bar{\mathbf{K}} \cdot \nabla\theta,$$

where

$$\bar{\mathbf{K}} = \mathbf{L}^T \cdot \mathbf{K} \cdot \mathbf{L}.$$

Hence, the condition of isotropy, in this case, requires that

$$\boxed{\bar{\mathbf{K}} \equiv \mathbf{K}.} \qquad (2\text{–}68)$$

This condition on **K** will be satisfied for an arbitrary rotation tensor, **L**, if and only if **K** is a product of a scalar and the unit tensor **I**, that is,

$$\boxed{\mathbf{K} = k\mathbf{I},}$$

(2–69)

where $k$ is a *scalar* property of the fluid that is known as the *thermal conductivity*.

Thus, for an *isotropic* fluid that exhibits a linear, instantaneous relationship between the heat flux and temperature gradient, the most general constitutive form for **q** is

$$\boxed{\mathbf{q} = -k\nabla\theta.}$$

(2–70)

This is known as *Fourier's law of heat conduction*. We may note that the inequality (2–54) imposes a restriction on the sign of the thermal conductivity $k$. In particular, in the absence of fluid motion, (2–54) reduces to the simple form

$$-\frac{\mathbf{q}\cdot\nabla\theta}{\theta^2} \geq 0.$$

It follows from this inequality and the constitutive form (2–70) that

$$k\left(\frac{\nabla\theta}{\theta}\right)^2 \geq 0.$$

Hence, assuming $k \neq 0$, we see that the thermal conductivity must be positive, $k > 0$.

Although this simplified version of Fourier's heat conduction law is well known to be an accurate constitutive model for many real gases, liquids, and solids, it is important to keep in mind that, in the absence of empirical data, it is no more than an educated guess, based upon a series of assumptions about material behavior that one cannot guarantee ahead of time to be satisfied by any real material. This status is typical of all constitutive equations in continuum mechanics, except for the relatively few that have been derived via a molecular theory.

## G. Constitutive Equations for a Flowing Fluid— General Considerations

In the previous section, we discussed constitutive approximations for stress and surface heat flux in a stationary fluid, where $\mathbf{u} \equiv 0$. In view of the molecular origins of **q**, there is no reason to expect that the basic linear form for its constitutive behavior should be modified by the presence of mean motion, at least for materials that are not too complicated in structure. Of course this situation may be changed for materials such as polymeric liquids or suspensions, because in these cases the presence of motion may cause the structure to become anisotropic or changed in other ways that will affect the heat transfer process. We will return to this question in Section I.

The constitutive equation (2–58) for the stress, on the other hand, will be modified for all fluids in the presence of a mean motion in which the velocity gradient $\nabla\mathbf{u}$ is nonzero. To see that this must be true, we can again consider the simplest possi-