Gurson's model (A brief summary)

アウトライン (Outline)

- 損傷力学手法について About the damage mechanics

- 金属材料の損傷モデル【微小空孔（ボイド）の生成・成長・合体によるモデル】Damage mechanics for metallic materials (Assumption of micro voids, their existence, growth, coalescence

損傷力学手法について

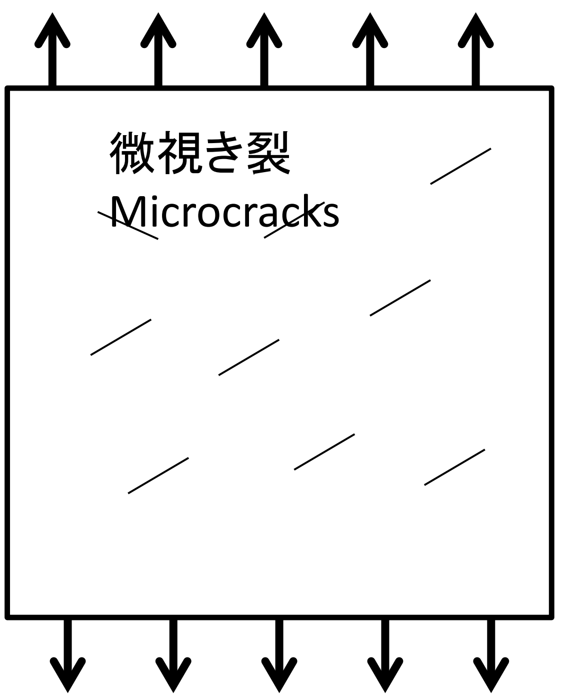
- 損傷による弾性率の低下
- 損傷による載荷能力の低下

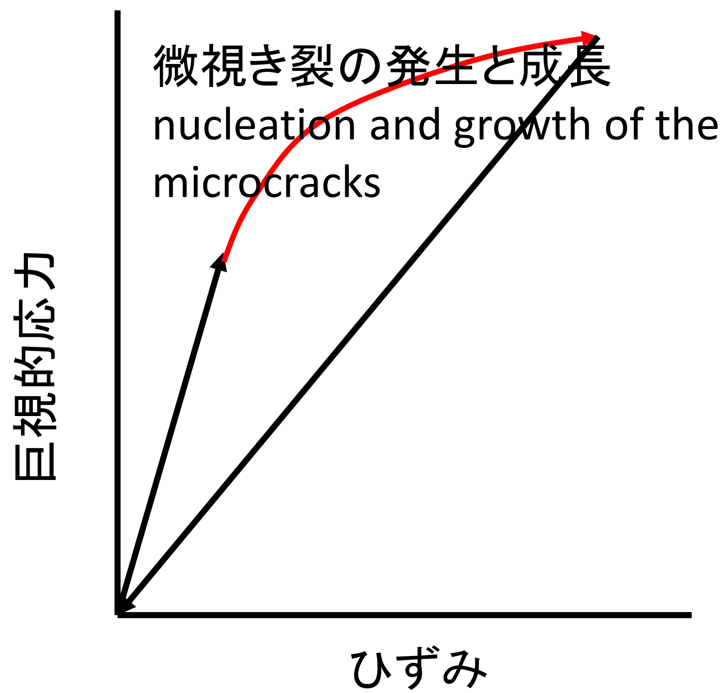
- 損傷モデル
  - 微視き裂⇒主としてぜい性材料（コンクリートなど）
  - ボイドの生成･成長･合体⇒主として金属

- 損傷モデルの考え方
  - 微視き裂⇒弾性体を仮定し、微視き裂の発生と成長による有効断面積の減少を考慮する⇒弾性率の低下
  - ボイドの生成･成長･合体⇒ボイドによる応力集中⇒塑性変形の発生⇒ボイドの成長⇒巨視的な降伏応力の低下⇒載荷能力の低下

損傷力学手法について– About damage mechanics

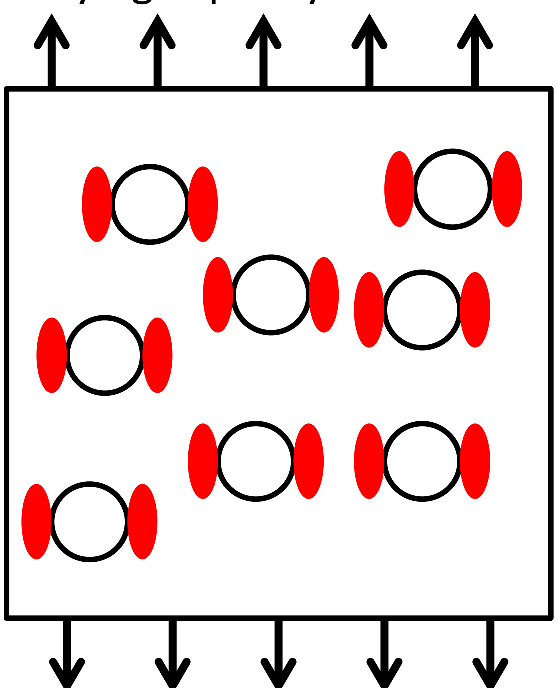
- 損傷による弾性率の低下-Reduction of stiffness (elastic constant) due to some material damage
- 損傷モデル---Damage model
  - 微視き裂⇒主としてぜい性材料（コンクリートなど）
  - Microcracks ⇒ For brittle materials such as concrete, ceramics, glasses.
- 損傷モデルの考え方—Concept of damage model
  - 微視き裂⇒弾性体を仮定し、微視き裂の発生と成長による有効断面積の減少を考慮する⇒弾性率の低下
  - Microcracks assumed ⇒ Assuming an elastic body, the reduction of effective load carrying section due to the nucleation and growth of the microcracks ⇒ Reduction of effective elastic constant (stiffness)

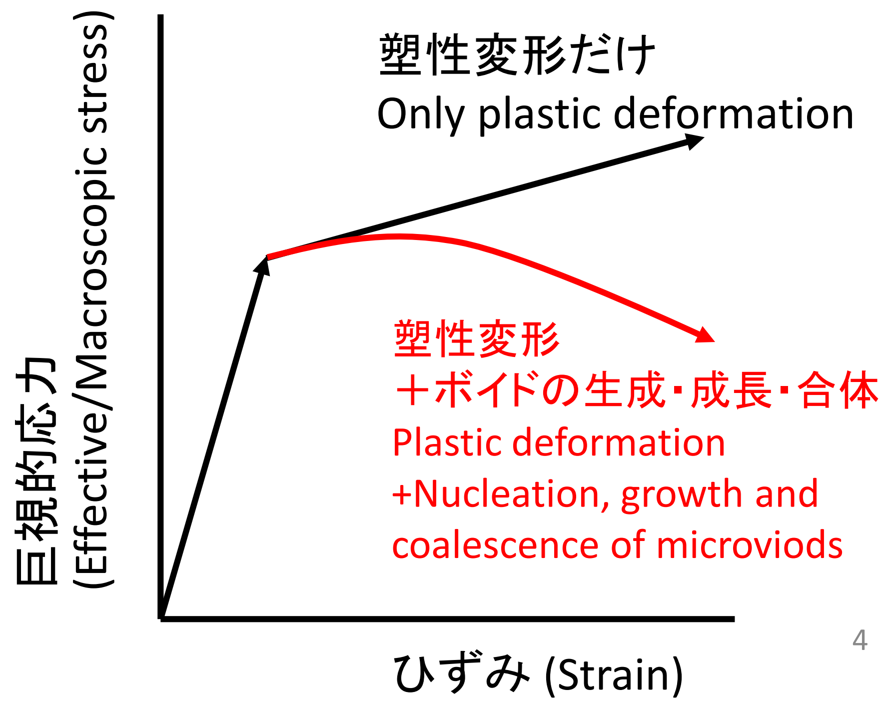




金属材料の損傷モデル—Damage model for metallic (Ductile) materials

- 損傷による載荷能力の低下—Reduction of load carrying capacity due to material damage
- 損傷モデル—Damage model
  - ボイドの生成・成長・合体⇒主として金属
  - Nucleation, growth and coalescence of microviods
- 損傷モデルの考え方 – Concept of damage model
  - ボイドの生成・成長・合体：ボイドによる応力集中⇒塑性変形の発生⇒ボイドの成長⇒巨視的な降伏応力の低下⇒載荷能力の低下
  - Nucleation, growth and coalescence of microviods: Stress concentration in the vicinity of the voids ⇒ Plastic deformation ⇒ Growths of the voids ⇒ Reduction of the effective/macroscopic yield stress ⇒ reduction of load carrying capacity





金属材料の損傷モデル—Damage model for metallic (Ductile) materials

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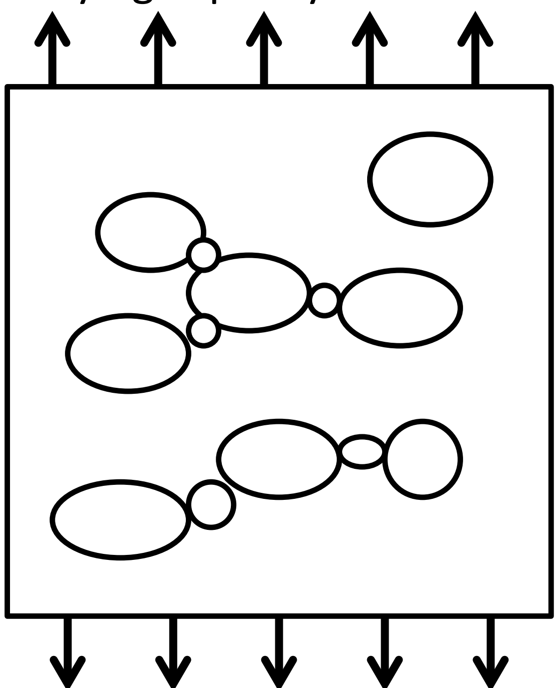

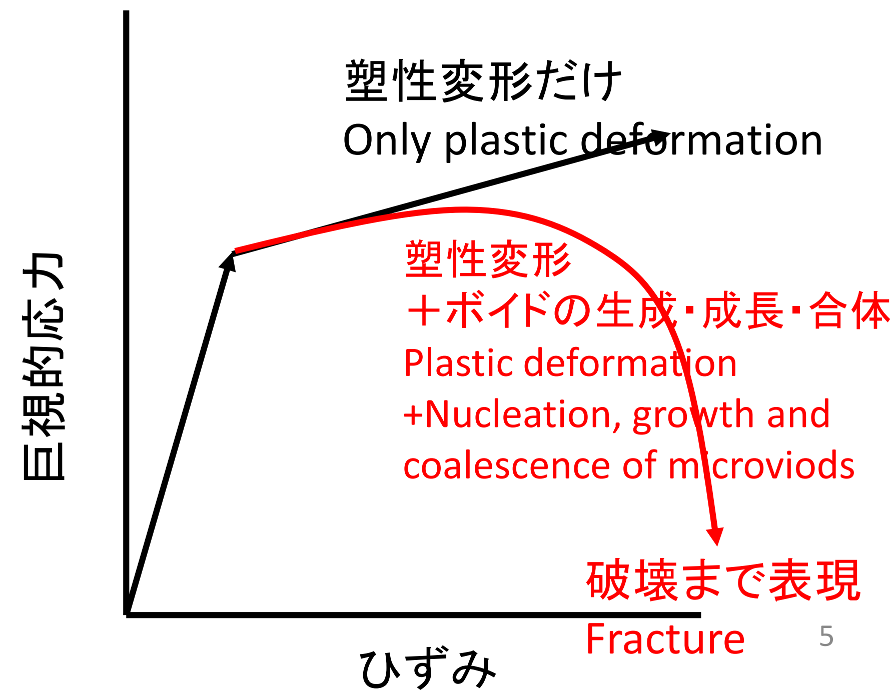

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- 損傷による載荷能力の低下—Reduction of load carrying capacity due to material damage
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田中啓介著、材料強度学、丸善、2008　より
From, K. Tanaka, Strength of materials, Maruzen 2008

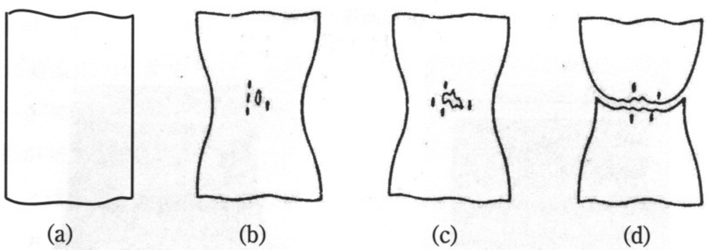


図 9.6　カップアンドコーン型破壊における延性破壊過程

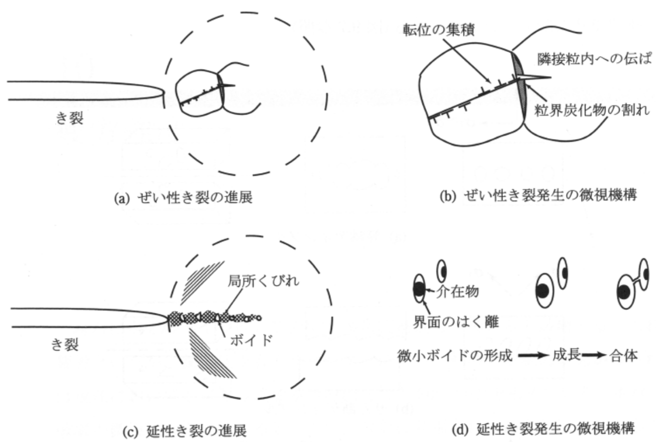


図 9.8　破壊じん性試験におけるぜい性き裂および延性き裂の発生

Cup and corn type failure of a tensile bar (Mild steel)

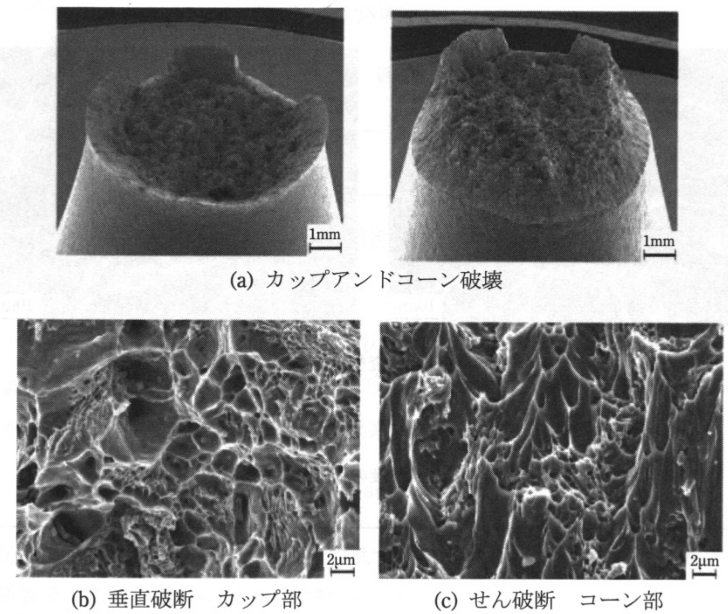


図 9.7　機械構造用炭素鋼 (S45C) の引張り破面

←延性破壊の模式図
Model of crack growth (Nucleation, growth and coalescence of microviods)

Gurson モデルに基づく 弾塑性損傷モデル 応力ーひずみ関係の導出
Stress-strain relationship based on the Gurson’s elastic-plastic damage model

Gursonモデルの降伏関数: Yield funciton

$$F = \left(\frac{\bar{\sigma}}{\sigma_M} \right)^2 + 2q_1 f \cosh\left(\frac{q_2 \sigma_{kk}}{2\sigma_M} \right) - \left[1 + q_3 f^2 \right] = 0$$

f :空孔（ボイド）体積率【損傷（ダメージ）パラメータ】
Void volume fraction (Damage parameter)

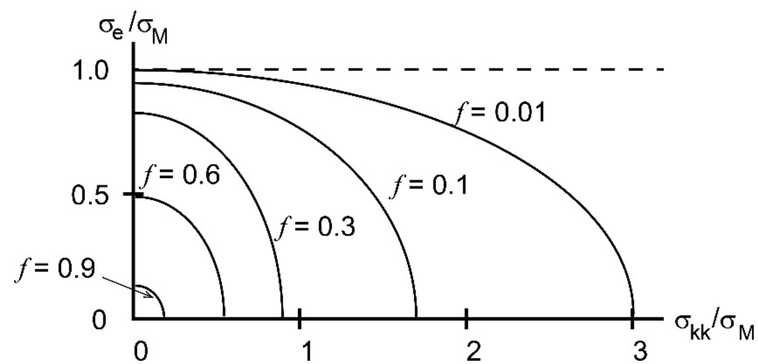
σ_M :母材の降伏応力【塑性ひずみの関数⇒硬化則に従う】
Yield stress of the mother material (follows some hardening rule)

$\bar{\sigma}$:ミーゼス（相当）応力
von Mises stress (Effective, equivalent stress)

$q_1 = 1.5$:定数
Constants

$q_2 = 1$

$q_3 = q_1^2$



空孔（ボイド）体積率 f がゼロのとき通常の弾塑性理論と一致
When the void volume fraction f is zero, the model is exactly the same as an ordinary J2-plasticity model

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Gursonモデルの降伏関数: Yield function

$$F\left(\sigma_{ij},\sigma_M,f\right)=\left(\frac{\bar{\sigma}}{\sigma_M}\right)^2+2q_1 f\cosh\left(\frac{q_2\sigma_{kk}}{2\sigma_M}\right)-\left[1+q_3^2 f^2\right]=0$$

空孔(ボイド)率の増加: $\dot{f} = \dot{f}^{\text{growth}} + \dot{f}^{\text{nucleation}} + \dot{f}^{\text{failure}}$

Increase of void volume fraction

変形による成長 Growth due to plastic deformation	応力／ひずみによる発生 Nucleation due to stress or strain	ボイド／微小き裂合体による破壊 Fracture due to coalescence of voids

Gurson モデルに基づく　弾塑性損傷モデル　応力ーひずみ関係の導出
Stress-strain relationship based on the Gurson's elastic-plastic damage model

□空孔率の発展方程式 は，その成長（ $\dot{f}_{\mathrm{growth}}$ ），発生（ $\dot{f}_{\mathrm{nucleation}}$ ），局所破壊（$\dot{f}_{\mathrm{failure}}$ ）に関するものからなる．そして，空孔率の速度はそれらの和で表現される．

Evolution equation is consisting of three difference parts: Growth ($\dot{f}_{\mathrm{growth}}$) , Nucleation ($\dot{f}_{\mathrm{nucleation}}$) and Failure due to coalescence ($\dot{f}_{\mathrm{failure}}$)

$$\dot{f} = \dot{f}_{\mathrm{growth}} + \dot{f}_{\mathrm{nucleation}} + \dot{f}_{\mathrm{failure}}$$

□空孔の塑性変形--- Growth (due to plastic deformation)　空孔の塑性変形による成長

$$\dot{f}_{\mathrm{growth}} = (1-f)\dot{\varepsilon}^{P}_{kk}$$

$$(1-f)V = V_o \rightarrow V = \frac{V_o}{1-f}$$

$$\dot{V} = \frac{V_o \dot{f}}{(1-f)^2} = \frac{V\dot{f}}{1-f}$$

$$\dot{V} = V\dot{\varepsilon}^{P}_{kk}$$

$$\frac{V\dot{f}}{1-f} = V\dot{\varepsilon}^{P}_{kk} \rightarrow \dot{f} = (1-f)\dot{\varepsilon}^{P}_{kk}$$

□発生に関するもの -- Nucleation

$$\dot{f}_{\mathrm{nucleation}} = A\dot{\varepsilon}^{P}_{M} + \frac{1}{3}B\dot{\sigma}_{kk}$$

A と B は材料定数である -- A, B: Material constants.

□局所破壊に関するもの -- Failure due to coalescence

$$\dot{f}_{\mathrm{failure}} = \frac{f_U - f_C}{\Delta\varepsilon}\dot{\varepsilon}^{P}_{M}$$

f_C は局所破壊が開始する空孔率，f_U は材料が完全に局所破壊したと判定する空孔率である．$\Delta\varepsilon$ は材料定数である．Failure due to coalescence initiates at f_C and failure occurs completely at f_U . $\Delta\varepsilon$ Is a material constant

Gurson モデルに基づく 弾塑性損傷モデル 応力ーひずみ関係の導出

Stress-strain relationship based on the Gurson’s elastic-plastic damage model

Flow rule:

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda}\frac{\partial F}{\partial \sigma_{ij}} = \dot{\lambda}\frac{\partial}{\partial \sigma_{ij}}\left[\left(\frac{\bar{\sigma}}{\sigma_M}\right)^2 + 2q_1 f\cosh\left(\frac{q_2\sigma_{kk}}{2\sigma_M}\right) - \left[1+q_3^2 f^2\right]\right]$$

$$= \dot{\lambda}\frac{\partial}{\partial \sigma_{ij}}\left[\frac{1}{\sigma_M^2}\frac{3}{2}\left(\sigma'_{kl}\sigma'_{kl}\right) + 2q_1 f\cosh\left(\frac{q_2\sigma_{kk}}{2\sigma_M}\right) - \left[1+q_3^2 f^2\right]\right]$$

$$= \dot{\lambda}\left[\frac{1}{\sigma_M^2}\frac{\partial}{\partial \sigma_{ij}}\left(\frac{3}{2}\left(\sigma'_{kl}\sigma'_{kl}\right)\right) + 2q_1 f\frac{\partial}{\partial \sigma_{ij}}\left(\cosh\left(\frac{q_2\sigma_{kk}}{2\sigma_M}\right)\right)\right]$$

$$= \dot{\lambda}\left[\frac{1}{\sigma_M^2}3\sigma'_{ij} + 2q_1 f\sinh\left(\frac{q_2\sigma_{kk}}{2\sigma_M}\right)\frac{\partial}{\partial \sigma_{ij}}\left(\frac{q_2\sigma_{ll}}{2\sigma_M}\right)\right]$$

$$= \dot{\lambda}\left[\frac{3\sigma'_{ij}}{\sigma_M^2} + q_1 q_2\frac{\delta_{ij}}{\sigma_M} f\sinh\left(\frac{q_2\sigma_{kk}}{2\sigma_M}\right)\right] = \frac{\dot{\lambda}}{\sigma_M}\left[\frac{3\sigma'_{ij}}{\sigma_M} + \delta_{ij}q_1 q_2 f\sinh\left(\frac{q_2\sigma_{kk}}{2\sigma_M}\right)\right]$$

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Consistency condition

$$0 = \frac{\partial F}{\partial \sigma_{ij}}\dot{\sigma}_{ij} + \frac{\partial F}{\partial \sigma_M}\dot{\sigma}_M + \frac{\partial F}{\partial f}\dot{f}$$

Here,

$$\frac{\partial F}{\partial \sigma_{ij}} = \frac{1}{\sigma_M}\left[\frac{3\sigma'_{ij}}{\sigma_M} + \delta_{ij} q_1 q_2 f \sinh\left(\frac{q_2 \sigma_{kk}}{2\sigma_M}\right)\right]$$

$$\frac{\partial F}{\partial \sigma_M} = \frac{\partial}{\partial \sigma_M}\left[\left(\frac{\bar{\sigma}}{\sigma_M}\right)^2 + 2q_1 f \cosh\left(\frac{q_2\sigma_{kk}}{2\sigma_M}\right) - \left[1 + q_3^2 f^2\right]\right]$$

$$= -2\frac{\bar{\sigma}^2}{\sigma_M^3} - \frac{q_2\sigma_{kk}}{2\sigma_M^2} 2q_1 f \sinh\left(\frac{q_2\sigma_{kk}}{2\sigma_M}\right) = -\frac{1}{\sigma_M}\left[2\frac{\bar{\sigma}^2}{\sigma_M^2} - \frac{2q_1 f q_2 \sigma_{kk}}{\sigma_M}\sinh\left(\frac{q_2\sigma_{kk}}{2\sigma_M}\right)\right]$$

$$\frac{\partial F}{\partial f} = \frac{\partial}{\partial f}\left[\left(\frac{\bar{\sigma}}{\sigma_M}\right)^2 + 2q_1 f \cosh\left(\frac{q_2\sigma_{kk}}{2\sigma_M}\right) - \left[1 + q_3^2 f^2\right]\right] = 2q_1 \cosh\left(\frac{q_2\sigma_{kk}}{2\sigma_M}\right) - 2 f q_3^2$$

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Consistency condition

$$0 = \frac{\partial F}{\partial \sigma_{ij}} \dot{\sigma}_{ij} + \frac{\partial F}{\partial \sigma_M} \dot{\sigma}_M + \frac{\partial F}{\partial f} \dot{f}$$

Here,

$$\dot{\sigma}_{ij} = E_{ijkl}\left(\dot{\varepsilon}_{kl} - \dot{\varepsilon}^{p}_{kl}\right) = E_{ijkl}\left(\dot{\varepsilon}_{kl} - \frac{\dot{\lambda}}{\sigma_M}\left[\frac{3\sigma'_{kl}}{\sigma_M} + \delta_{kl} q_1 q_2 f \sinh\left(\frac{q_2 \sigma_{kk}}{2\sigma_M}\right)\right]\right)$$

Work due to the plastic deformation of matrix material

$$\dot{W}^{p} = (1-f)\sigma_M \dot{\varepsilon}^{p}_{M} = (1-f)\sigma_M \frac{\dot{\sigma}_M}{\bar{h}}$$

and

$$\dot{W}^{p} = \sigma_{ij}\dot{\varepsilon}^{p}_{ij}$$

$\bar{h}$: Hardening modulus of matrix material

We have: $$\dot{\sigma}_M = \frac{\bar{h}\sigma_{ij}\dot{\varepsilon}^{p}_{ij}}{(1-f)\sigma_M}$$

Void volume fraction:

$$\dot{f} = \dot{f}_{\text{growth}} + \dot{f}_{\text{nucleation}} + \dot{f}_{\text{failure}} = (1-f)\dot{\varepsilon}^{P}_{kk} + A\dot{\varepsilon}^{P}_{M} + \frac{1}{3}B\dot{\sigma}_{kk} + \frac{f_U - f_C}{\Delta\varepsilon}\dot{\varepsilon}^{P}_{M}$$

and, $$\dot{\varepsilon}^{p}_{M} = \frac{\dot{\sigma}_M}{\bar{h}}$$

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Rate from stress-strain relationship:

$$\dot{\sigma}_{uv} = E_{uvij}\dot{\varepsilon}_{ij}^{e} = E_{uvij}\left(\dot{\varepsilon}_{ij} - \dot{\varepsilon}_{ij}^{P}\right) = E_{uvij}\dot{\varepsilon}_{ij} - \gamma \frac{E_{uvij} M_{ij}^{G} M_{mn}^{F} E_{mnk\ell}}{M_{pq}^{F} E_{pqrs} M_{rs}^{G} - \dfrac{\sigma_M}{2}\bar{G}} \dot{\varepsilon}_{k\ell}$$

$$\gamma = \gamma\left(\sigma_{ij}, \varepsilon_M^{P}, \dot{\varepsilon}_{ij}\right) = \begin{cases} 1 & (\text{Plastic loading}) \\ 0 & (\text{Elastic}) \end{cases}$$

Here,

$$M_{ij}^{G} = \frac{\sigma_M}{2}\frac{\partial F}{\partial \sigma_{ij}} = \frac{\sigma_M}{2}\left\{\frac{3\sigma'_{ij}}{\sigma_M^2} + \delta_{ij}\frac{f q_1 q_2}{\sigma_M}\sinh\left(\frac{q_2 \sigma_{kk}}{2\sigma_M}\right)\right\}$$

$$M_{ij}^{F} = \frac{\sigma_M}{2}\left(\frac{\partial F}{\partial \sigma_{ij}} + \frac{1}{3}\frac{\partial F}{\partial f}B\delta_{ij}\right) = \frac{\sigma_M}{2}\left\{\frac{3\sigma'_{ij}}{\sigma_M^2} + \delta_{ij}\frac{f q_1 q_2}{\sigma_M}\sinh\left(\frac{q_2 \sigma_{kk}}{2\sigma_M}\right) + \frac{1}{3}\frac{\partial F}{\partial f}B\delta_{ij}\right\}$$

$$\bar{G} = -\left\{\frac{\partial F}{\partial \sigma_M}\frac{\partial \sigma_M}{\partial \varepsilon_M^{P}} + \frac{\partial F}{\partial f}\left(A + \frac{f_U - f_C}{\Delta\varepsilon}\right)\right\}\frac{\sigma_{ij} M_{ij}^{G}}{(1-f)\sigma_M} - \frac{\partial F}{\partial f}(1-f) M_{kk}^{G}$$

Plastic strain rate:

$$\dot{\varepsilon}_{uv}^{p} = \frac{E_{uvij} M_{ij}^{G} M_{mn}^{F} E_{mnk\ell}}{M_{pq}^{F} E_{pqrs} M_{rs}^{G} - \dfrac{\sigma_M}{2}\bar{G}} \dot{\varepsilon}_{k\ell}$$

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