Numerical Analysis – Part II: COURSEWORK

1. The matrix A is given by

$$A = \begin{bmatrix} -\sigma & \sigma & 0 \\ \rho & -1 & 0 \\ 0 & 0 & -\beta \end{bmatrix}$$

2(a) The maximum step size (computed by considering the linearised system) is

$$h_{max} = 0.190$$

2(b),(c) Add the first figure (time histories of x, y, z):

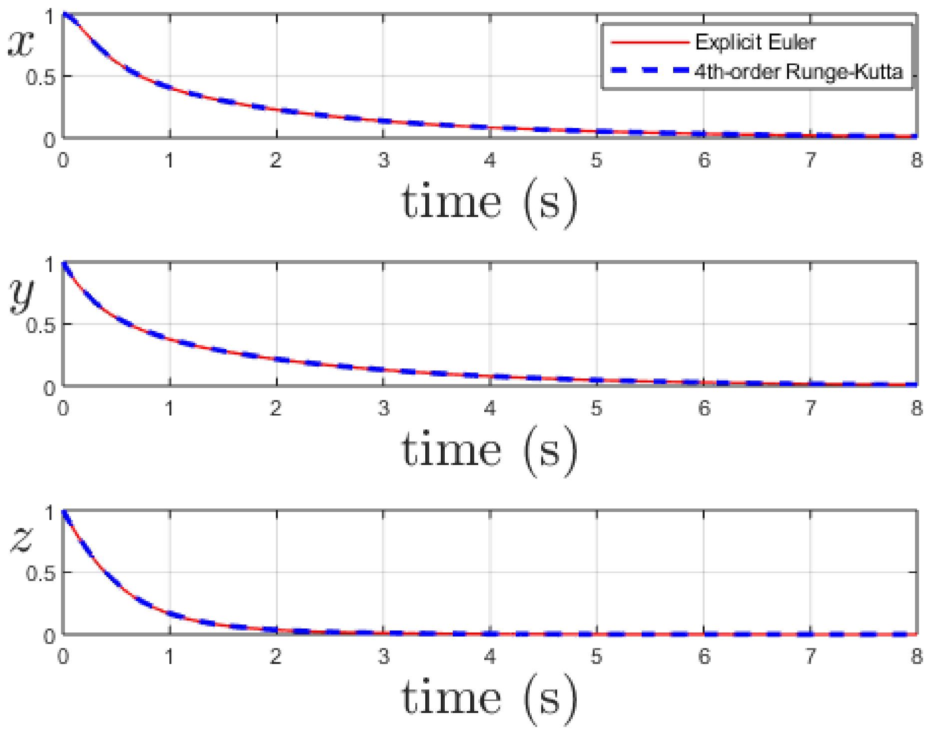

Figure 1: Time histories of x, y and z. Numerical solution for the (nonlinear) ODE using explicit Euler and 4th-order Runge-Kutta with step size h = 0.01.

2(b), (c) Add the second figure (a 3D plot of x vs. y vs. z for the entire interval of t)

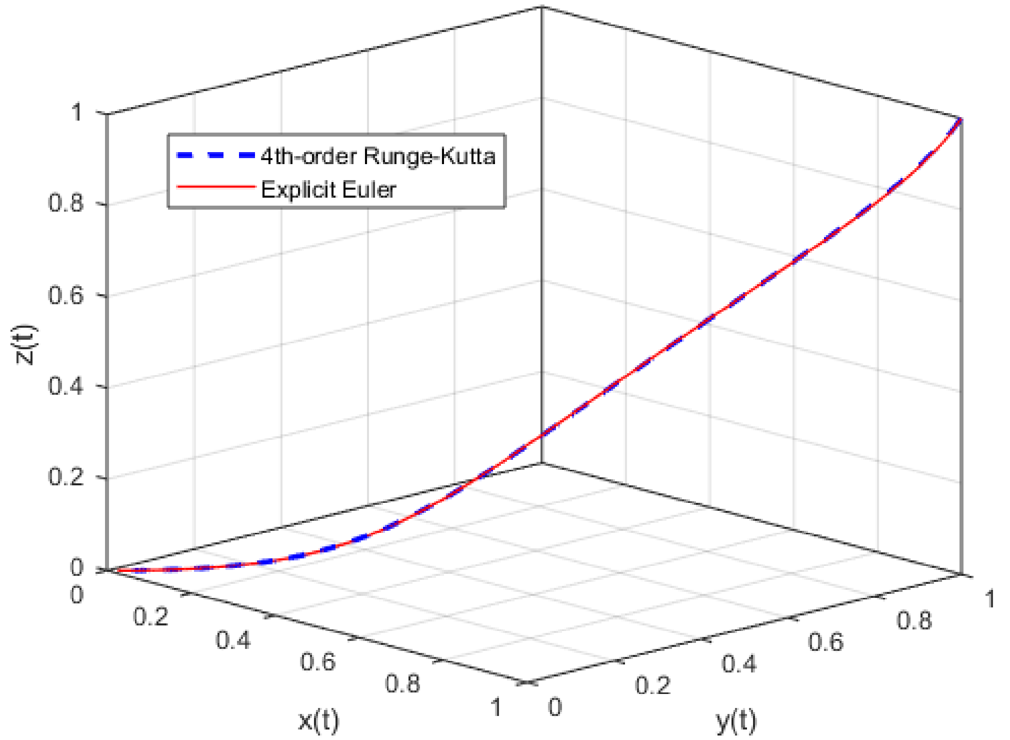

Figure 2: 3D plot of the trajectory (x(t), y(t), z(t)) for the entire interval of t, time.

2(d) Your answer must not exceed 70 words

The numerical solution using explicit Euler calculates one slope at an interval, and is globally first order accurate, O(h), and locally second order accurate, O(h^2). The solution using 4th-order Runge-Kutta is “most correct” because it calculates four different slopes, using them as weighted averages, and is globally third order accurate, O(h^3), and locally fourth order accurate, O(h^4). Therefore, it is more accurate considering the timestep used is the same.

3(a),(b) Add the first figure (time histories of x, y, z):

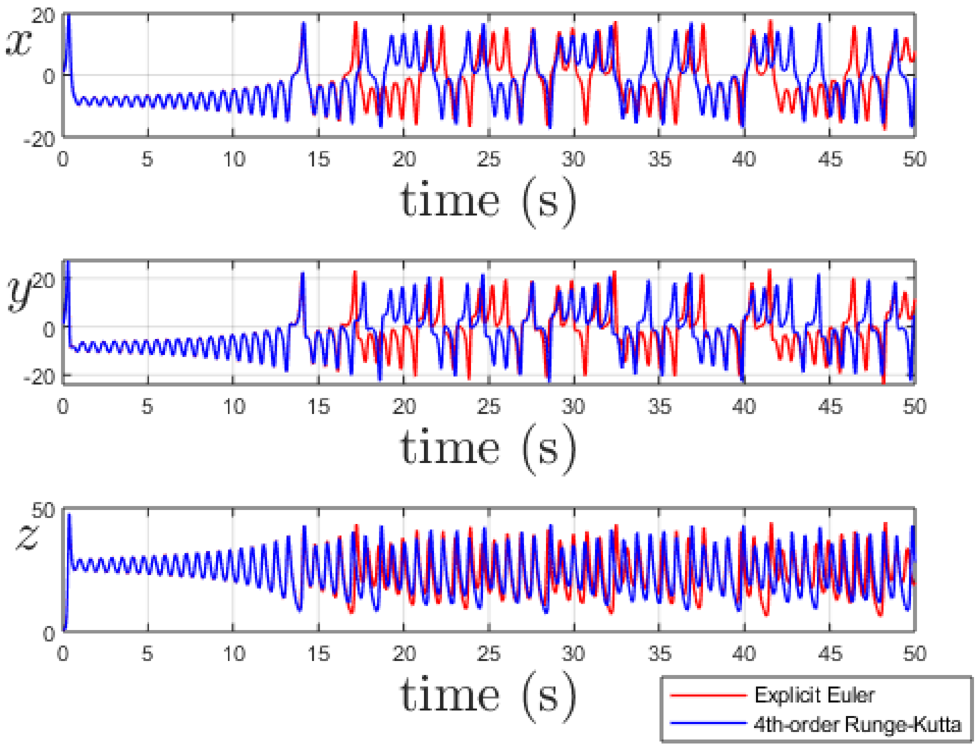

Figure 3: Time histories of x, y and z. Numerical solution for the (nonlinear) ODE using explicit Euler with step size h = 0.00001 and 4th-order Runge-Kutta with step size h = 0.01.

3(a),(b) Add the second figure (a 3D plot of x vs. y vs. z for the entire interval of t)

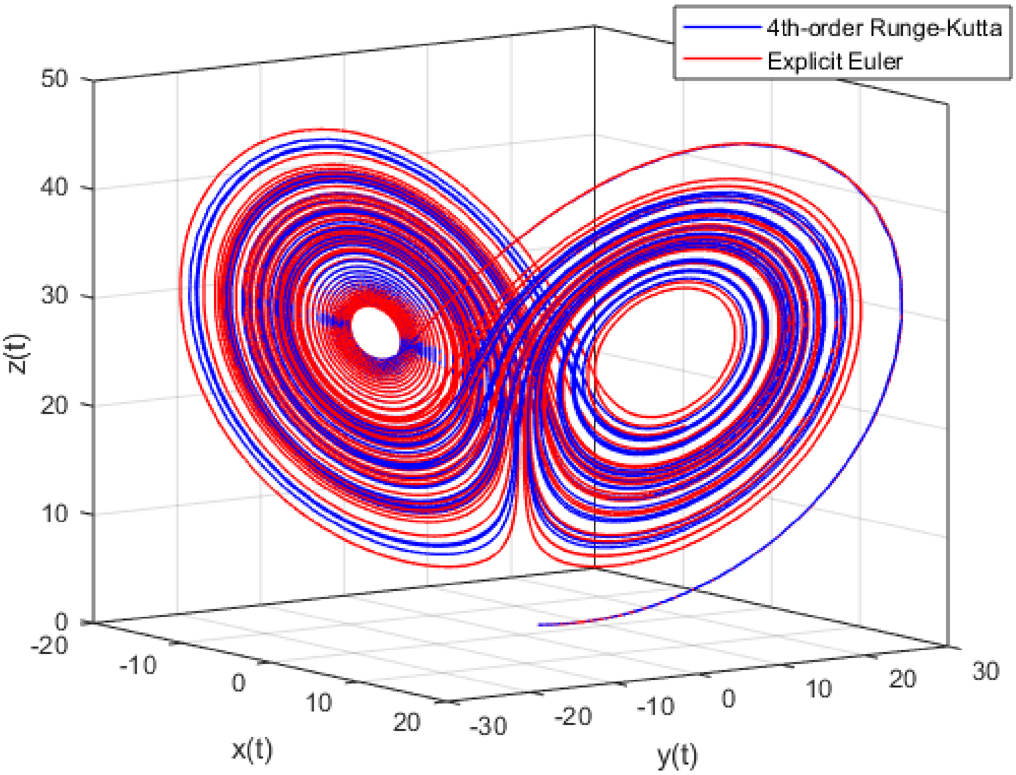

Figure 4: 3D plot of the trajectory (x(t), y(t), z(t)) for the entire interval of t, time.

3(c) Your answer must not exceed 120 words

The two solutions obtained in 3(a), (b) => Time histories of x and y are identical, both solutions being similar at the beginning and becomes visibly different after time, t=15s. The two solutions for time history of z are almost identical, differs slightly after t=15s.

As with the Lorenz system this chaotic system is sensitive to the initial conditions, in our case the x, y, and z values. Two initial states no matter how close will diverge, usually sooner than later, giving a butterfly effect.

As stated in 2(d), the solution using Runge-Kutta is more accurate, hence less sensitive. Even though the step size for explicit Euler is 1000 times smaller, it is still less accurate, hence more sensitive.