Numerical Analysis – Part II: COURSEWORK

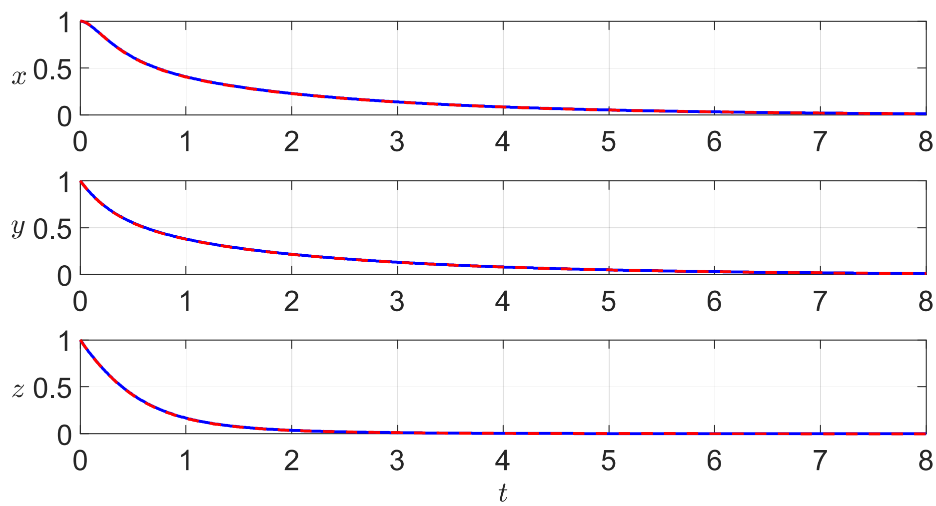
1. The matrix A is given by

$$A = \begin{bmatrix} -\sigma & \sigma & 0 \\ \rho & -1 & 0 \\ 0 & 0 & -\beta \end{bmatrix}$$

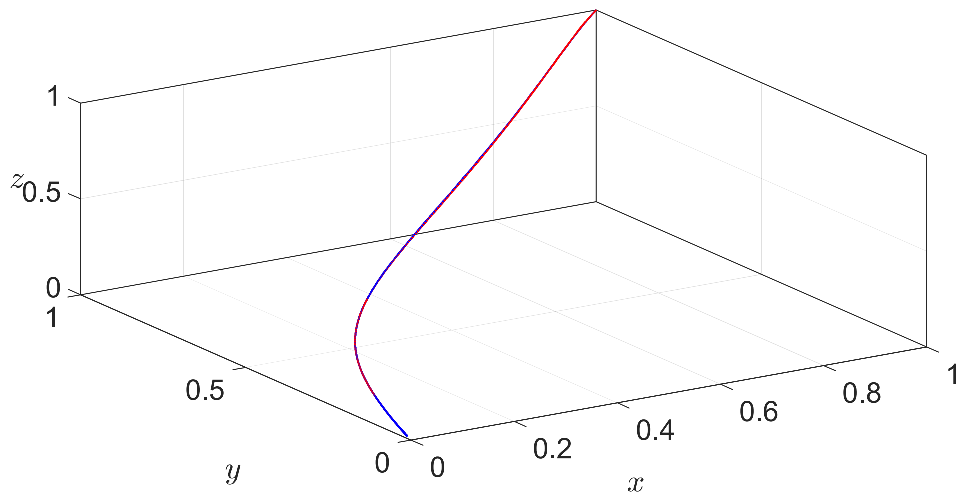
2(a) The maximum step size (computed by considering the linearised system) is

$$h_{max} = 0.19$$

2(b), (c) Add the first figure (time histories of x, y, z):



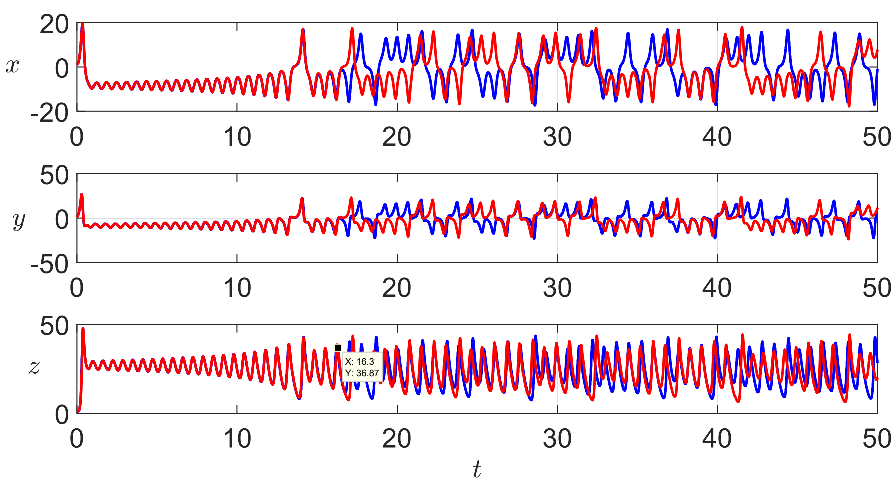
2(b), (c) Add the second figure (a 3D plot of x vs. y vs. z for the entire interval of t)



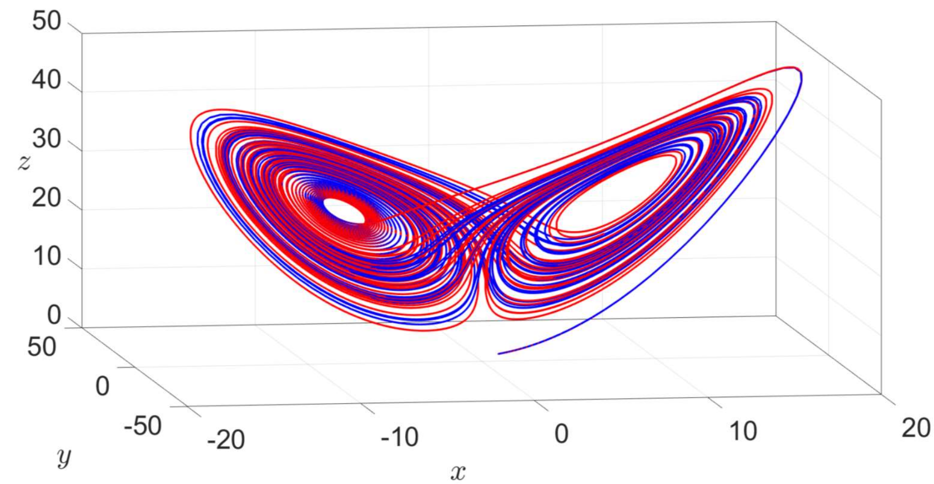
2(d) Your answer must not exceed 70 words

The numerical solutions are not identical, because the solutions have been obtained using two different numerical schemes. These will provide different approximations of the exact solution. Since explicit Euler is a first-order method and 4th-order Runge-Kutta is a fourth-order method and we are using the same stepsize, the numerical solution obtained using 4th-order Runge-Kutta is the most accurate.

3(a), (b) Add the first figure (time histories of x, y, z):



3(a),(b) Add the second figure (a 3D plot of x vs. y vs. z for the entire interval of t)



2. (c) Your answer must not exceed 120 words

The two numerical solutions are very close for the first 16 seconds or so. After around 16 seconds, the solutions become very different! Thus, the two numerical solutions are not similar. This is due to the fact that any numerical scheme yields an approximate solution to the problem. Due to limited accuracy and round-off errors, the two schemes (even though we use a very small step size for Euler) will not give identical solutions. Note that the local truncation error for both schemes are the same. Since the system is chaotic, these “numerical discrepancies” have a similar effect to perturbations in the initial condition – they lead to completely different outcomes in the long run.