6.2

T13

a) $xf(x) = \begin{cases} \sin x / x & 0 < x \le \pi \\ x & x = 0 \end{cases}$

$$xf(x) = \begin{cases} \sin x & 0 < x \le \pi \\ x^2 & x = 0 \end{cases}$$

当 $x=0$, $x^2 = 0 = \sin x$

故 $xf(x) = \begin{cases} \sin x & 0 < x \le \pi \\ x^2 & x = 0 \end{cases}$

$$= \begin{cases} \sin x & 0 < x \le \pi \\ \sin x & x = 0 \end{cases} = \sin x \ (0 \le x \le \pi)$$

b.

$$V = \int_0^\pi 2\pi x f(x) dx$$

$$= 2\pi \int_0^\pi \sin x \, dx$$

$$= -2\pi \cos x \Big|_0^\pi$$

$$= 4\pi$$

T24.

a. $V = \int_0^2 2\pi x (8 - x^3) dx$

$$= 2\pi \left(4x^2 - \frac{x^5}{5}\right)\Big|_0^2 = \frac{96\pi}{5}$$

b. $V = \int_0^2 2\pi (3-x)(8-x^3) dx$

$$= 2\pi \left(24x - \frac{3x^4}{4} - 4x^2 + \frac{x^5}{5}\right)\Big|_0^2$$

$$= \frac{264}{5}\pi$$

c. $V = \int_0^2 2\pi (x+2)(8-x^3) dx$

$$= 2\pi \left(-\frac{x^5}{5} - \frac{x^4}{2} + 4x^2 + 16x\right)\Big|_0^2$$

$$= 336\pi/5$$

d. $V = \int_0^8 2\pi y \sqrt[3]{y} \, dy$

$$= 2\pi \left(\frac{3}{7} y^{\frac{7}{3}}\right)\Big|_0^8 = \frac{768}{7}\pi$$

e. $V = \int_0^8 2\pi (8-y) \sqrt[3]{y} \, dy$

$$= 2\pi \left(6y^{4/3} - \frac{3}{7} y^{7/3}\right)\Big|_0^8 = \frac{576}{7}\pi$$

f. $V = \int_0^8 2\pi (y+1) \sqrt[3]{y} \, dy$

$$= 2\pi \left(\frac{3}{7} y^{7/3} + \frac{3}{4} y^{4/3}\right)\Big|_0^8 = \frac{936}{7}\pi$$

T30

a.

$$V=\int_0^4 \pi\left(\frac{x}{2}+2\right)^2-\pi x^2\,dx$$

$$=\pi\int_0^4 -\frac{3}{4}x^2+2x+4\,dx$$

$$=\pi\left(-\frac{1}{4}x^3+x^2+4x\right)\Big|_0^4$$

$$=16\pi$$

b.

$$V=\int_0^4 2\pi x\left(\frac{x}{2}+2-x\right)dx$$

$$=\pi\int_0^4 -\frac{3}{4}x^2+2x+4\,dx$$

$$=\pi\left(-\frac{x^3}{4}+x^2+4x\right)\Big|_0^4=\frac{32}{3}\pi$$

c.

$$V=\int_0^4 2\pi(4-x)\left(\frac{x}{2}+2-x\right)dx$$

$$=2\pi\int_0^4 \frac{x^2}{2}-4x+8\,dx$$

$$=2\pi\left(\frac{1}{6}x^3-2x^2+8x\right)\Big|_0^4$$

$$=64\pi/3$$

d.

$$V=\int_0^4 \pi(8-x)^2-\pi\left(8-\frac{x}{2}-2\right)^2dx$$

$$=\pi\int_0^4 \frac{3}{4}x^2-10x+28\,dx$$

$$=\pi\left[\frac{1}{4}x^3-5x^2+28x\right)\Big|_0^4$$

$$=48\pi$$

T42

$$V=2\pi\int_1^{\sqrt{\pi+1}} x\sin(x^2-1)\,dx$$

$$=\frac{\pi}{2}\int_1^{\sqrt{\pi+1}}\sin(x^2-1)\,d(x^2-1)$$

$$=\frac{\pi}{2}\cos(x^2-1)\Big|_{\sqrt{\pi+1}}^{1}=2\pi$$

6.3

T11

$$L=\int_{-\pi/4}^{\pi/4}\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$=\int_{-\pi/4}^{\pi/4}\sec^2 y$$

$$=\tan y\Big|_{-\pi/4}^{\pi/4}=2$$

T21

a.

$$L=\int_1^4\sqrt{1+\frac{1}{4x}}\,dx$$

$$=\int_1^4\sqrt{1+f'(x)^2}\,dx$$

已知$f'(x)>0$

$$\Rightarrow f'(x)=\frac{1}{2\sqrt{x}}$$

$$f(x)=\int\frac{1}{2\sqrt{x}}\,dx=\frac{1}{2}\frac{x^{1/2}}{1/2}+C=\sqrt{x}+C.$$

由$f(1)=1$得 $1=\sqrt{1}+C\Rightarrow C=0$. 故$f(x)=\sqrt{x}$, $1\le x\le 4$.

b.

1条，导函数可确定一族原函数，函数过 $(1,1)$ 确定唯一一条曲线。

T24

$$L = 8\int_{\sqrt{2}/4}^{1} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$= 8\int_{\sqrt{2}/4}^{1} \sqrt{1+x^{-\frac{2}{3}}(1-x^{\frac{2}{3}})}\,dx$$

$$= 8\int_{\sqrt{2}/4}^{1} x^{-1/3}\,dx$$

$$= 8\left.\frac{x^{2/3}}{2/3}\right|_{\sqrt{2}/4}^{1} = 6$$

6.4

T20

$$S = \int_{5/8}^{1} 2\pi f(y)\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$= 2\pi\int_{5/8}^{1}\sqrt{2y}\,dy$$

$$= 2\pi\sqrt{2}\left.\frac{y^{3/2}}{3/2}\right|_{5/8}^{1}$$

$$= \frac{16\sqrt{2}-5\sqrt{5}}{12}\pi$$

T31

m_k处切线为

$$y - f(m_k) = f'(m_k)(x-m_k).$$

取 $x = x_{k-1}$，得

$$r_1 = f(m_k) + f'(m_k)(x_{k-1}-m_k)$$

$$= f(m_k) - f'(m_k)\frac{\Delta x_k}{2}$$

取 $x = x_k$，得

$$r_2 = f(m_k) + f'(m_k)(x_k - m_k)$$

$$= f(m_k) + f'(m_k)\frac{\Delta x_k}{2}$$

b.

$$L_k = \sqrt{(\Delta x_k)^2 + (r_2-r_1)^2}$$

$$= \sqrt{(\Delta x_k)^2 + (f'(m_k)\Delta x_k)^2}$$

c.

$$S = \frac{(2\pi r_1 + 2\pi r_2)\cdot L}{2}$$

将 r_1, r_2 表达式代入

$$= 2\pi f(m_k)\sqrt{1+f'(m_k)^2}\,\Delta x_k$$

d.

$$\lim_{n\to\infty}\sum_{k=1}^{n} 2\pi f(m_k)\sqrt{1+f'(m_k)^2}\,\Delta x_k$$

$$= \int_a^b 2\pi f(x)\sqrt{1+f'(x)^2}\,dx$$

6.5

T11

$$W \int_0^6 \left(78-\frac{78}{6}x\right) dx$$

$$= 78x - \frac{78}{6}\times\frac{x^2}{2}\Big|_0^6 = 234\,J$$

T16.

$$\begin{cases} V = \frac{\pi}{3}\times 25\times 10 = \frac{250}{3}\pi \\ \frac{V}{2} = \frac{1}{3}\pi y\cdot\left(\frac{y}{2}\right)^2 \end{cases}$$

$$\Rightarrow y = 500^{1/3}.$$

$$F(y) = 8820\,\pi\left(\tfrac{1}{2}y\right)^2\Delta y$$

$$= \frac{8820\pi}{4}y^2\Delta y$$

$$W = \int_0^{\sqrt[3]{500}} 8820\,\pi\left(\tfrac{1}{2}y\right)^2(11-y)\,dy$$

$$\simeq 5824270$$

T21

$$W = \int_{-5}^{0} -9800\pi(25-y^2)(4-y)\,dy$$

$$\approx 15073099.75$$

T29. $g = 10\,m/s^2 = 10^3\,cm/s^2$

$$W = \int_0^{18}(21-y)\,\pi\left(\frac{y+36}{12}\right)^2 0.8\times 10^3\,dy$$

$$= \frac{800\pi}{144}\int_0^{18}(21-y)(y^2+1296+72y)\,dy$$

$$= \frac{800\pi}{144}\int_0^{18} 21y^2+27216+1512y-y^3 - 1296y - 72y^2\,dy$$

$$= \frac{50}{9}\pi\int_0^{18} -51y^2 - y^3 + 216y + 27216\,dy$$

$$= \frac{50}{9}\pi\left(-\frac{51}{3}y^3 - \frac{y^4}{4} + \frac{216}{2}y^2 + 27216y\right)\Big|_0^{18}$$

$$\approx 6972450 \quad g\cdot cm^2/s^2$$

$$\approx 0.6972\,J$$

$$(J = kg\cdot m/s^2)$$