Low-energy effective description of light pseudo-scalar mesons in $SO(N)$-like dark QCD

Joachim Pomper

Supervisor: Dr. Suchita Kulkarni

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The tale of dark matter

Astrophysical observations point to the existence of a non-visible type of matter, that makes up 26% of universes energy budget.

Evidence on various scales:

- **Galaxy scale:**
  Rotational curves
- **Cluster scale:**
  Visible mass too little to hold together coma cluster
- **Cosmological scale:**
  CMB anisotropies

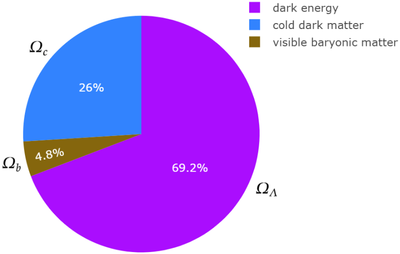
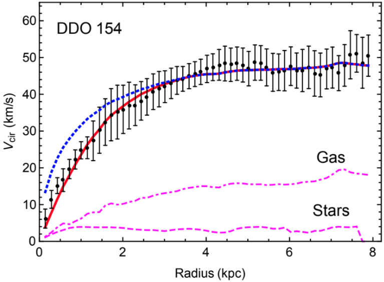


Figure: Energy budget of the universe within ΛCDM model.

Cusp vs. Core problem

Observed DM halo density profiles are more **cored** *compared to profiles found in cold DM simulations, which are rather* **cusp**.



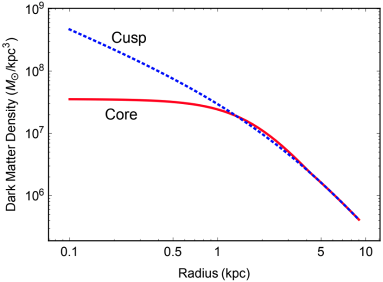
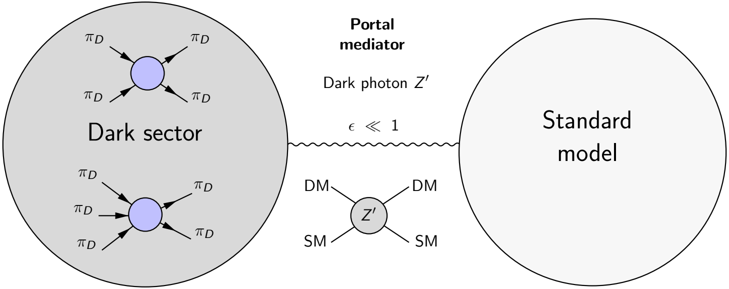


Figure: Data from the DDO 154 dwarf galaxy [Murayama (2022)].

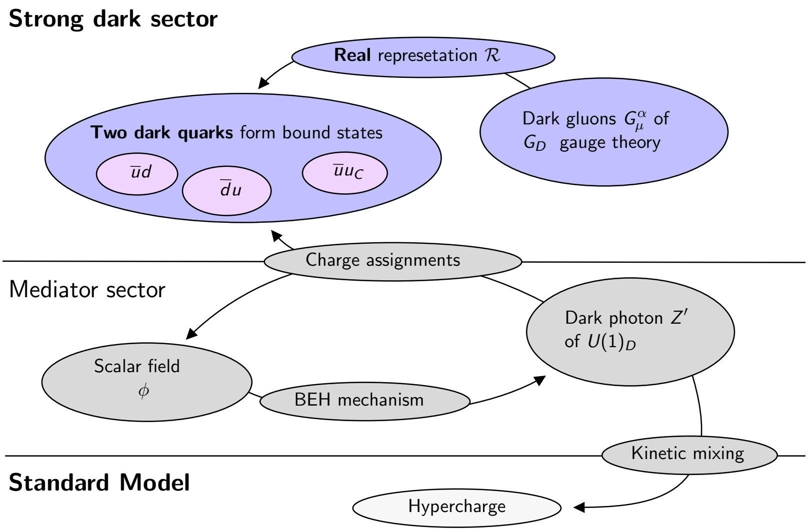
SIMP dark matter

Hochberg et al., Physical Review Letters 113 (2014)



- Sufficient self-interactions resolve structure formation problems.
- Dark Photon maintains kinetic equilibrium until freeze-out.
- $3 \rightarrow 2$ cannibalization drives freeze-out.

The model in the UV



Symmetries and particles

Flavor symmetry breaking pattern

$U(4)$

↓ Axial anomaly → $\tilde{\eta}$ is heavier than π

$SU(4)$

↓ Chiral quark condensate $\Sigma \neq 0$

Degenerate mass $m_u = m_d \neq 0$ → Pseudo NGBs of $SU(4)/SO(4)$ coset space

$SO(4)$

↓ Introduction of dark photon charge assignment $\mathcal{Q}$

$SU(2) \times U(1)$

Pseudo scalar mesons

$\eta_D^{'}$

$SO(4) - \mathrm{Multiplets}$

π_D^1	π_D^2	π_D^3
π_D^4	π_D^5	π_D^6
π_D^7	π_D^8	π_D^9

$\dim SU(4)/SO(4) = 9$

When might η'_D become light?

't Hooft large N limit:

$\frac{T(R)}{C_{\mathrm{adj}}} \xrightarrow[N\to\infty]{} 0$ $\Rightarrow$ $\lim_{N\to\infty} \partial_\mu J^\mu_{\eta'_D} = 0$ $\Rightarrow$ Like in QCD, axial anomaly contribution to η'_D vanishes and $m_{\eta'_D} \xrightarrow[N\to\infty]{} m_{\pi_D}$

$T(R)$ ... Dynkin Index of representation R of G_D

C_{adj} ... Quadratic Casimir of adjoint of G_D

For which theories might η'_D be relevant ?

i) This argument works for $SO(N)$-vector theories

$\Rightarrow \eta'_D$ might be important

ii) Does not work for other real theories we investigated

(Non-anomalous) Low energy effective Lagrangian

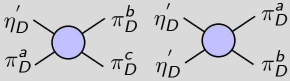
Coset representative

$$\Sigma = \exp\left(-i\xi^a T_a\right)$$

$$\xi^a = \begin{cases} \eta_D' \,/\, F_{\eta_D'} & \text{if} \quad a = 0 \\ \pi_D^a \,/\, F_\pi & \text{else} \end{cases}$$

$$\mathcal{L}^{IR} = \frac{F_\pi^2}{4}\,\mathrm{tr}\left\{\partial_\mu \Sigma^\dagger\, \partial^\mu \Sigma\right\} + \frac{F_\pi^2 - F_{\eta_D'}^2}{4}\,\mathrm{tr}\left\{\Sigma \partial_\mu \Sigma^\dagger\right\}\mathrm{tr}\left\{\Sigma^\dagger \partial^\mu \Sigma\right\}$$
$$+ \frac{m_\pi^2\, F_\pi^2}{4}\,\mathrm{tr}\left\{\omega \Sigma^\dagger + \omega^\dagger \Sigma\right\} + \frac{\Delta m_{\eta_D'}^2\, F_{\eta_D'}^2}{4}(\ln(\det(\Sigma))^2$$

Contact terms:



GMOR - relation:

$$m_\pi^2 = \frac{m_q\ \langle \bar{q} q \rangle}{2\ F_\pi^2}$$

η_D' - mass:

$$m_{\eta_D'}^2 = m_\pi^2 + \Delta m_{\eta_D'}^2$$

Topological terms and coset homotopy

The coset space G/H has non vanishing 4th homotopy group:

$$\pi_4(SU(4)/SO(4)) \neq 0$$

Problem:

- The standard construction/classification of topological terms in non-linear sigma model by Witten, Weinberg and d'Hoker requires $\pi_4(G/H) = 0$.

  [Witten, Nucl. Phys. B 223 (1983)]

  [D'Hoker and Weinberg, Physical Review D 50 (1994)]

  [Brauner and Kolešová, Nuclear Physics B 945 (2019)]

- Modern approaches give more general classification but no practical construction.

  [Davighi and Gripaios, Journal of High Energy Physics 2018 (2018)]

  [Lee, Ohmori, and Tachikawa, SciPost Physics 10 (2021)]

Derivation of WZW term

Chu, Ho, and Zumino, Nuclear Physics B 475 (1996)

Wess and Zumino, Physics Letters B 37 (1971)

Step 1: Calculate chiral anomaly in the UV

$\mathrm{Tr}\left\{F^3\right\} \xrightarrow{\mathbf{P}}$ Stora-Zumino descent equations $\xrightarrow{\triangle\text{- diagrams}}$ Consistent anomaly $\mathcal{A}[\mathfrak{g},\ A] \neq\ 0$ $\xrightarrow{\text{Bardeen counter terms}}$ Chiral anomaly $\mathcal{A}[\mathfrak{h},\ A] = 0$

Step 2: t' Hooft anomaly matching

Anomaly equation: $\delta_\epsilon S^{IR}_{\mathrm{cov.}}[\xi, A]\ =\ \mathcal{A}[\epsilon,\ A]$

Step 3: Solve anomaly equation in the IR

$$S^{IR}_{\mathrm{cov.}}[\xi, A]\ = \int_0^1 d\tau\ \int \mathcal{A}[\xi,\ A_\tau(\xi)]\ \text{ with } A_\tau(\xi) = \exp\left(-\tau\ \int dy\ \xi^a \mathcal{D}_a\right) A$$

The Wess-Zumino-Witten term

Wess-Zumino effective action

$$\lim_{A\to 0} S^{WZ}_{\mathrm{cov.}} = \frac{D_C}{48\pi^2 f_\pi} \int_0^1 \mathrm{d}\tau \int_{S^4} \mathrm{Tr}\left\{\xi \left((\Sigma[\tau\xi])^{-1}\mathrm{d}\Sigma[\tau\xi]\right)^4\right\}$$

$$\approx \frac{D_C}{15 f_\pi^5 \pi^2} \epsilon^{\mu\nu\sigma\rho} \int_{S^4} \mathrm{d}^4 x \; \mathrm{Tr}\left\{\pi\partial_\mu\pi\partial_\nu\pi\partial_\sigma\pi\partial_\rho\pi\right\}$$

- Incorporates $3 \to 2$ process.
- Low energy coefficient fixed by construction.
- Does not depend on η' in lowest order χPT.

Charge assignments dark photon Z'

Double covering map

$SO(4) \cong SU(2)_I \times SU(2)_C$

Remaining flavor symmetry

$SO(4) \cong SU(2)_I \times SU(2)_C \xrightarrow{Z'} SU(2)_I \times U(1)_C$

explicit breaking by
coupling to dark photon Z'

Gauge U(1) subgroup
of SU(2) factor

Charge assignments $\mathcal{Q} = e_D\, \sigma^3_C$

$\rightarrow$ consistency (no gauge anomalies)

$\rightarrow$ Maintain a non-abelian flavor symmetry
$\rightarrow$ No anomalous pion decays occur
$\Rightarrow$ π_D states are stable

This choice of $\mathcal{Q}$ is physically unique !

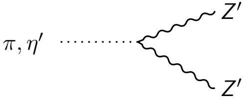
$SU(2)_I \times U(1)_C$ Multiplets			Charges
	η'_D		0
π^1_D	π^2_D	π^3_D	0
π^4_D	π^5_D	π^6_D	$-$
π^7_D	π^8_D	π^9_D	$+$

Dark photon anomalous vertices

Anomalous contributions:

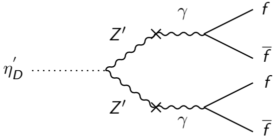
Anomaly equation:

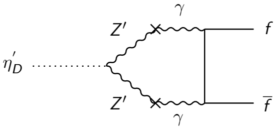
$$S^{IR}[\xi] = \lim_{A \to Z'} S^{IR}_{\mathrm{cov.}}[\xi, A]$$



$$\propto \quad \begin{cases} Tr\left\{\pi \ Q^2\right\} & = 0 \\ Tr\left\{\eta' \ Q^2\right\} & \neq 0 \end{cases}$$

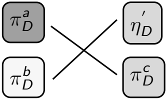
Allows for decay of η'_D to standard model





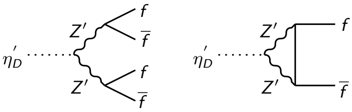
η'_D affects freeze-out

$\pi^a_D \pi^b_D \rightarrow \pi^c_D \eta'_D$



followed by

η'_D decays into the standard model



Suppression in the late universe

Stable DM is made of neutral π_D states

Depending on $m_{\eta'_D}$ and $\tau_{\eta'_D}$ the reaction chain may alter the freeze-out of the DM:

Limiting cases:

1) η'_D decays almost instantly

Correct Ω_{DM} can not be produced

$\Rightarrow$ **not a viable DM model**

2) $\tau_{\eta'_D} \approx$ Age of the universe

No effects Ω_{DM} on relic density

Summary

- Delivered an extensive description of a model for SIMP DM.
  - Case of real representations of gauge group G_D
  - Problem and alternative construction of WZW
  - Charge assignments of Z'
  - Symmetries and particle classification
  - Low energy effective description
- Discussed role of η'_D for phenomenology
  - Phenomenological limits where η'_D becomes important
  - η'_D might affect DM parameters

Bonus content

Outlook and future projects

- Evaluate DM parameter space taking into account η'_D (WIP)
- Study $Sp(4)$ with antisymmetric fermions on the lattice.
- Study discrete symmetries related to the axial anomaly for higher tensor representations.
  - Semi-direct product structure $Z_K \ltimes SU(4)$
  - Spontaneous breaking of these symmetries (Domain walls ?)
- Inclusion of ρ-mesons via local hidden symmetry

Symmetries and particles

Flavor symmetry breaking pattern

$U(4)$

↓ Axial anomaly → $\tilde{\eta}$ is heavier than π

$Z_{2T_R} \ltimes SU(4)$

↓ Chiral quark condensate $\Sigma \neq 0$
Degenerate mass $m_u = m_d \neq 0$ → Pseudo NGBs of $SU(4)/SO(4)$ coset space

$O(4)$

↓ Non degenerate mass $m_u \neq m_d \neq 0$

$O(2) \times O(2)$

Pseudo scalar mesons

η'_D

$SO(4) - \text{Multiplets}$

π^1_D π^2_D π^6_D π^9_D

π^3_D

π^4_D π^7_D

π^5_D π^8_D

$SO(2) \times SO(2)$ Multiplets

The axial anomaly and discrete symmetries

General form of Axial Anomaly

$$\mathcal{A}_{\mathrm{Axial}}[\epsilon, A] \; = \; -2i \; T_{\mathcal{R}} \; \mathrm{tr}\{\epsilon\} \mathcal{Q}_{\mathrm{Topo}}[A]$$

Quantum chiral transfomrmations

$$\begin{tikzcd}
\boxed{U(4)} \ni U = \exp(-\epsilon) \arrow[d] \arrow[rr] & & D\psi D\psi \overset{U}{\longmapsto} e^{-i\mathcal{A}[\epsilon,A]} \, D\psi D\psi \arrow[d] \\
\boxed{Z_{2T_{\mathcal{R}}} \ltimes SU(4)} & & \det(U) = \exp\left(-i\frac{\pi k}{T_R}\right) \Leftrightarrow \exp(-i\mathcal{A}[\epsilon, A]) = 1 \arrow[ll]
\end{tikzcd}$$

$k \in \{0, \ldots, 2T_R - 1\}$

Dynkin Index $T_{\mathcal{R}}$

SU(N) - Fund.	SO(N) - Vec.	Sp(2N) - Fund	Sp(2N) - AT2T
$T_R = 1/2$	$T_R = 1$	$T_R = 1/2$	$T_R = N - 1$

't Hooft large N considerations of η'_D

Idea: Compare for example $SO(N)$-vector theories for N very large.

Technicality: Define 't Hooft coupling λ

$$\lambda \;:=\; C_{adj}(N)\, g^2 \qquad\qquad \lambda(\mu_{UV}) \;=\; \text{fixed}$$

$\rightarrow$ Running of λ is independent of N up to $1/N$ corrections.
$\rightarrow$ A controlable perturbative scale $1/N$ is introduced into the theory.

Axial anomaly in the chiral limit:

$$\partial_\mu J^\mu_{\eta'_D} = -\, \frac{T(R)}{C_{\mathrm{adj}}}\, \frac{\lambda N_F}{32\pi^2}\, \epsilon^{\mu\nu\rho\sigma}\, G^\alpha_{\mu\nu} G_{\rho\sigma\ \beta}$$

Gives potential large N suppression

$\frac{T(R)}{C_{\mathrm{adj}}} \xrightarrow[N\to\infty]{!} 0$ must hold for the anomaly to vanish in large N limit

Example:
$SU(N)$-Fund.

$$\lambda \;:=\; N\, g^2$$

$$g^2 \xrightarrow[N\to\infty]{} 0$$

$$\frac{T(R)}{C_{\mathrm{adj}}} = \frac{1}{2N}$$

4th Homotopy group of SU(4)/SO(4)

	π_3	π_4	π_5
$SO(4)$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$SU(4)$	$\mathbb{Z}$	0	$\mathbb{Z}$

Fibration:

$$SO(4) \to SU(4) \to SU(4)/SO(4)$$

Long exact sequence:

$$\begin{array}{ccccccc}
\pi_4(SU(4)) & \xrightarrow{h_1} & \pi_4(SU(4)/SO(4)) & \xrightarrow{h_2} & \pi_3(SO(4)) & \xrightarrow{h_3} & \pi_3(SU(4)) \\
0 & \xrightarrow{h_1} & ? & \xrightarrow{h_2} & \mathbb{Z} \oplus \mathbb{Z} & \xrightarrow{h_3} & \mathbb{Z}
\end{array}$$

- $Ker(h_2) = Img(h_1) = 0 \to h_2$ is injective
- $\pi_4(SU(4)/SO(4)) \cong Img(h_2) = Ker(h_3)$
- $Ker(h_3) \neq 0$

$\Rightarrow$ $\pi_4(SU(4)/SO(4))$ cannot be trivial

Full UV Lagrangian

Hochberg, Kuflik, and Murayama, Journal of High Energy Physics (2016)

$-\frac{1}{4}\ G^{\alpha}_{\mu\nu}\ G^{\mu\nu}_{\alpha}$

Yang-Mills term for dark gluons : Based on $G_D = Sp(4)$

$-\frac{1}{4}\ F'_{\mu\nu}\ F'^{\mu\nu}$

Yang-Mills term for dark photon : Based on $U_D(1)$

$+\ \overline{u}\ i\ \gamma^{\mu} D_{\mu}\ u\ +\ m\ \overline{u}u$

$+\ \overline{d}\ i\ \gamma^{\mu} D_{\mu}\ d\ +\ m\ \overline{d}d$

Dirac term of dark quarks
2 flavors: u and d quarks
Charged under $G_D \times U_D(1)$

$+\ \left(D'_{\mu}\phi\right)^{\dagger} D'^{\mu}\phi\ +\ V\left[\phi^{\dagger}\phi\right]$

Dark scalar charged under $U_D(1)$

$+\ \frac{\epsilon}{2\cos(\theta_W)}\ F'_{\mu\nu}\ B^{\mu\nu}$

Kinetic mixing of $U_D(1)$ with SM hypercharge

Dark matter self-interactions

Kondo et al. (2022)

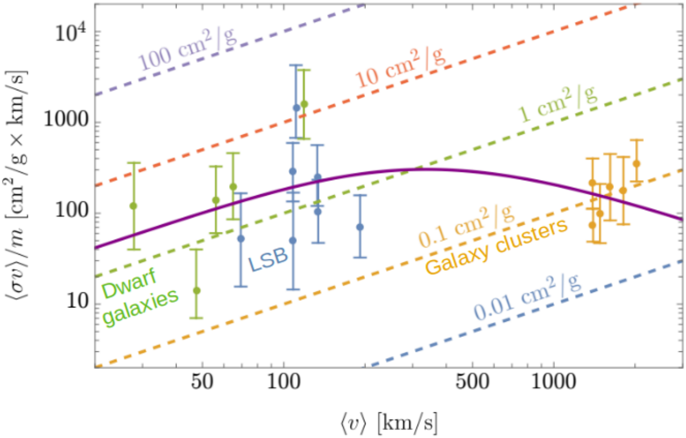


Figure: Suggested DM self-interaction crosssection in dependence of average velocity [Kaplinghat, Tulin, and Yu, American Physical Society (APS) 116 (2016)].

Dark pion mass

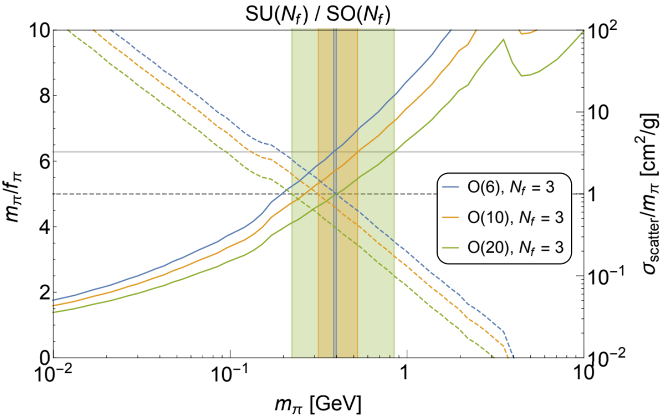


Figure: The solid horizontal line depicts the perturbative limit of $m_\pi/f_\pi \approx< 2\pi$, providing a rough upper limit on the pion mass; the dashed horizontal line depicts the bullet-cluster and halo shape constraints on the self-scattering cross section, placing a lower limit on the pion mass. Each shaded region depicts the resulting approximate range for m_π [Hochberg et al., Physical Review Letters 115 (2015)]

Discrete symmetries

Drach, Janowski, and Pica, EPJ Web of Conferences 175 (2018)

Charge conjugation C:

UV $q \mapsto \Omega C \bar{q}^\top$

IR $\pi \mapsto \pi^\top$

Spatial parity P:

UV $q(t, \vec{x}) \mapsto \eta_P \gamma^0 q(t, -\vec{x})$

IR $\pi(t, \vec{x}) \mapsto -\pi(t, -\vec{x})$

The choice $\eta_P = \pm i$ is adopted:

- Parity and flavor symmetries commute.
- All (pseudo) Nambu-Goldstone bosons (pNGB) are pseudo scalars.

Meson spectrum in 2 flavor Sp(4)-fund. gauge theory

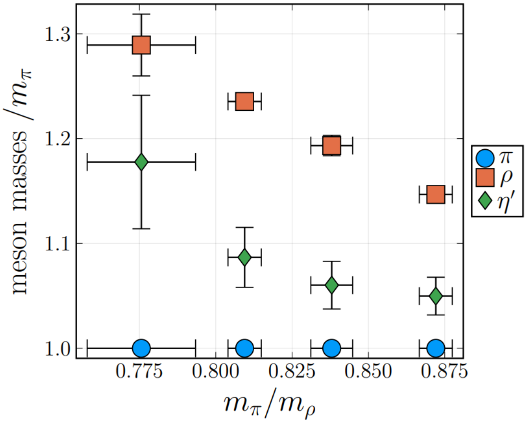


Figure: Meson spectrum in $Sp(4)$-fundamental with 2 flavors.
Taken from [Zierler et al. (2022), arXiv:hep-lat/2210.11187]