

Dark matter from confining $SO(N)$ -like gauge theories with two Dirac fermions.

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Natural Sciences



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Outline

- SIMP dark matter
- UV-models: The $SO(N)$ case
- IR-description
- UV-models: Generalizations

SIMP dark matter

Let us start with the usual story!

Astrophysical observations point to the existence of a non-visible type of matter, that makes up 26% of universes energy budget.

Evidence on various scales:

- **Galaxy scale:**
Rotational curves
- **Cluster scale:**
Visible mass too little to hold together Coma cluster
- **Cosmological scale:**
CMB anisotropies

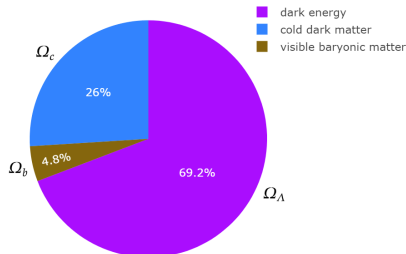


Figure: Energy budget of the universe within Λ CDM model.

Cusp vs. Core problem

*Observed DM halo density profiles are more **cored** compared to profiles found in cold DM simulations, which are rather **cusp**.*

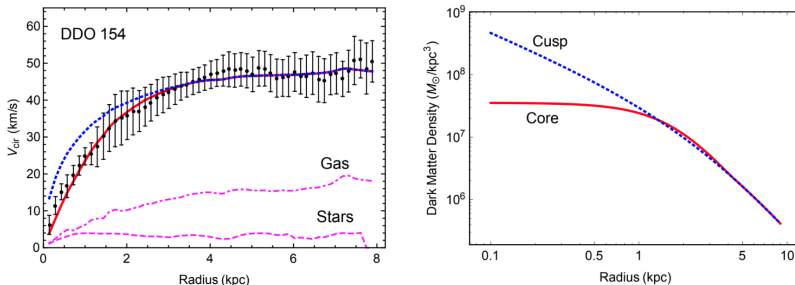


Figure: Data from the DDO 154 dwarf galaxy [Murayama (2022)].

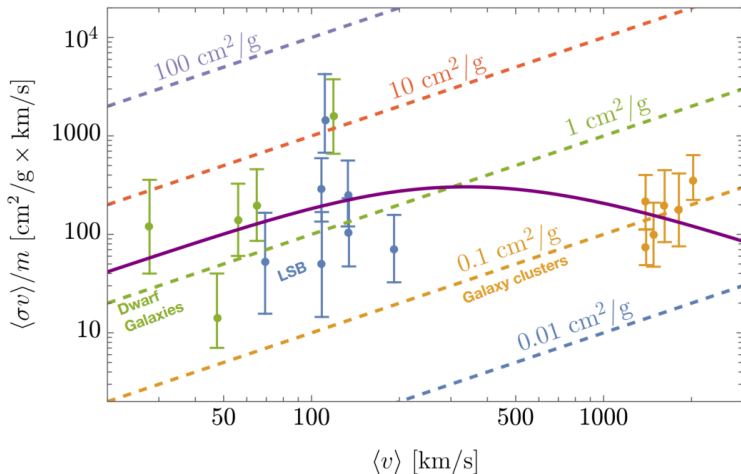
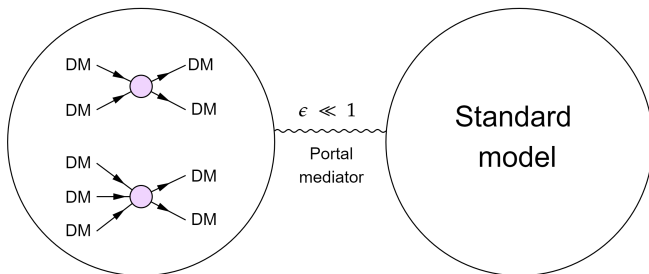


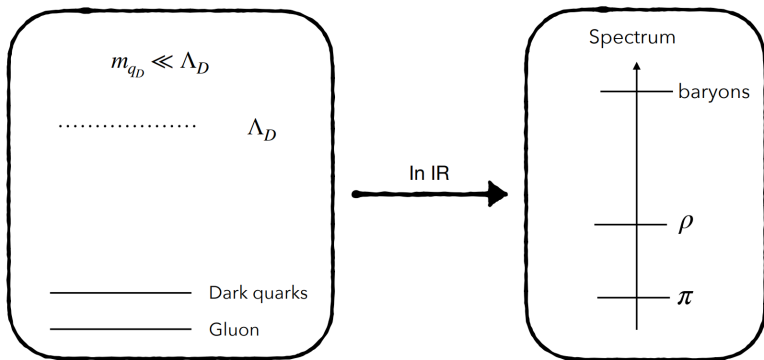
Figure: Suggested DM self-interaction crosssection in dependence of average velocity [Kaplinghat, Tulin, and Yu, American Physical Society (APS) 116 (2016)].



- Sufficient self-interactions resolve structure formation problems.
- $3 \rightarrow 2$ cannibalization drives freeze out.
- Mediator for thermal equilibrium until freeze out.

Dark QCD

Dark matter from a SM-extension by a confining, strongly interacting sector based on dark $SU(N)$, $Sp(N)$, $SO(N)$ -gauge theory with fermionic matter.



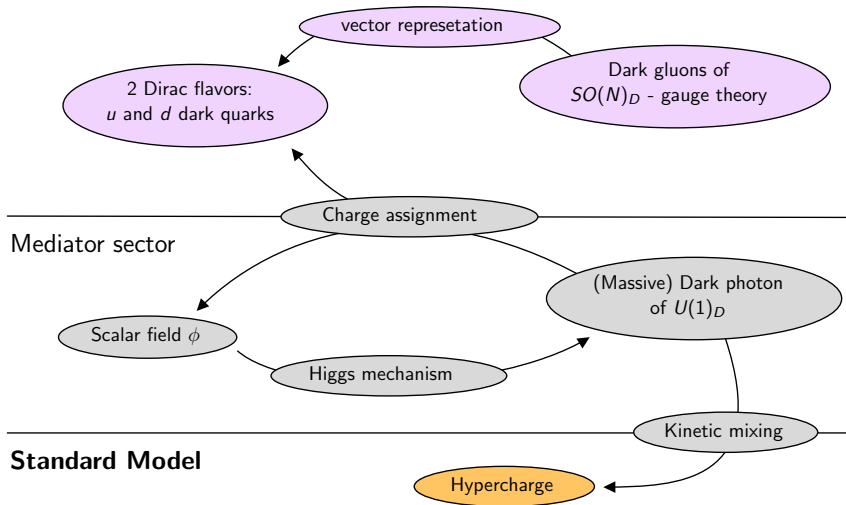
Dark matter from a SM-extension by a **confining, strongly interacting** sector based on dark $SU(N)$, $Sp(N)$, $SO(N)$ -**gauge theory** with fermionic matter.

- Dark matter is made of pseudo Nambu-Goldstone states (dark pions) of a spontaneously broke flavor symmetry $G \rightarrow H$.
- Pion masses of $m_\pi \approx \mathcal{O}(100\text{MeV})$ give required $\langle\sigma v\rangle/m_\pi$.
- Remaining flavor symmetry H may protect pions from decay.
- Pions fields correspond to maps of $\pi : \mathcal{M} \rightarrow G/H$.
- $3 \rightarrow 2$ process may be described by topological terms of Wess-Zumino type in non-linear Σ -model.

UV - models: The $SO(N)$ case

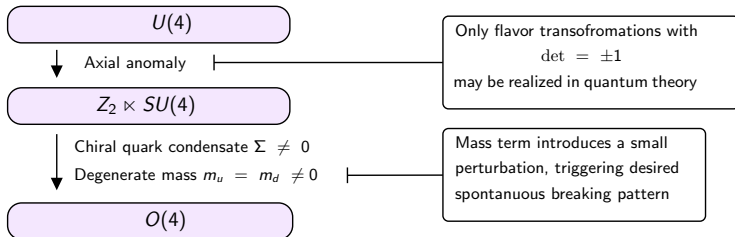
Model concept in the UV

Isolated dark sector



Flavor symmetries of the theory

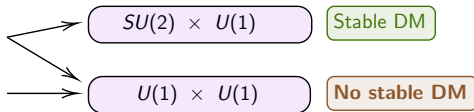
Isolated strong sector:



Further explicit breaking:

Coupling to $U_D(1)$

Non degenerate mass
 $m_u \neq m_d \neq 0$



Action of $O(4)$ symmetry

Organise u - and d -quarks with charge conjugates in 2×2 matrix.

Action of $SO(4)$ subgroup :

$$\begin{pmatrix} u & -d_C \\ d & u_C \end{pmatrix} \xrightarrow{SO(4) \cong SU(2) \times SU(2)} U_L \begin{pmatrix} u & -d_C \\ d & u_C \end{pmatrix} U_R^\dagger$$

Action of Z_2 subgroup :

$$\begin{pmatrix} u & -d_C \\ d & u_C \end{pmatrix} \xrightarrow{Z_2} \begin{pmatrix} u_C & -d_C \\ d & u \end{pmatrix}$$

Charge assignments for dark photon

- $U(1)_D$ gauge theory is consistent
 - Pion currents are non-anomalous
 - Non-abelian flavor symmetry remains
- $$\left. \vphantom{\begin{matrix} \bullet \\ \bullet \\ \bullet \end{matrix}} \right\} \Rightarrow \begin{matrix} \text{Anomaly cancellation} \\ Q^2 = \mathbb{1} \end{matrix}$$

Charge assignment: $Q \propto \sigma_{3,R}$

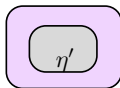
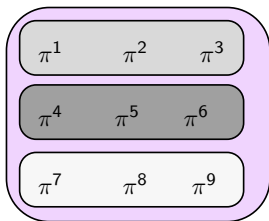
Explicit breaking: $O(4) \rightarrow SU(2) \times U(1)$

$U(1)_D$ gauge transformation:

$$\begin{pmatrix} u & -d_C \\ d & u_C \end{pmatrix} \xrightarrow{U(1)_D} \begin{pmatrix} u & -d_C \\ d & u_C \end{pmatrix} e^{-i\alpha Q}$$

Particles relevant for DM phenomenology

pseudo scalar mesons



(Pseudo) Nambu-Goldstone bosons:

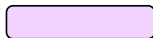
One for each local coordinate in the coset space

$$G/H = Z_2 \ltimes SU(4)/O(4) \cong SU(4)/SO(4)$$

These are the lightest states !

Flavor singlet meson:

Heavier than the pions

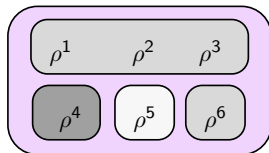


$O(4)$ Classification



$SU(2) \times U(1)$ Classification

vector mesons



Large N arguments

Using a 't Hooft large N -limit comparing $SO(N)$ -vector theories.

Scaling of gauge-coupling:

$$g \xrightarrow{N \rightarrow \infty} 0 \quad \text{with} \quad Ng^2 = \lambda = \text{fixed}$$

Axial anomaly in chiral limit:

$$\partial_\mu j_{\eta'}^\mu = -g^2 \frac{\epsilon^{\mu\nu\rho\sigma}}{8\pi^2} G_{\mu\nu} G_{\rho\sigma} \xrightarrow{N \rightarrow \infty} 0$$

Conclusion

For large N , the η' may be important for phenomenology.

IR - description

(Pseudo) Nambu-Goldstone fields

Nambu-Goldstone fields $\xi = (\pi, \eta')$ parametrize local deviations from the vacuum configuration, e.g.

$$\xi : \mathcal{M} \rightarrow G_F/H = \begin{cases} SU(4)/SO(4) \\ U(4)/O(4) \end{cases} \quad \text{in Large N limit}$$

Since G_F/H is connected, compact and symmetric we may choose

$$\Sigma[\xi](x) = \exp\left(-2i \frac{\xi^a(x)}{f_\pi} T_a^{F,\text{broken}}\right)$$

which transforms linearly under G_F flavor transformations.

$$\Sigma[\xi] \xrightarrow{g \in G_F} g \Sigma[\xi] g^\dagger$$

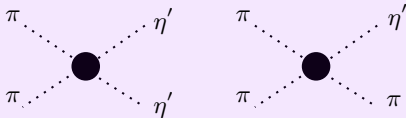
(Non-anomalous) Low energy effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{f_\pi^2}{4} \text{Tr} \left\{ (\partial_\mu \Sigma)^\dagger \partial^\mu \Sigma \right\} - \frac{f_\eta^2 - f_\pi^2}{2f_\eta^2} \partial_\mu \eta' \partial^\mu \eta'$$

$$+ \frac{\mu^3 m}{2} \text{Tr} \left\{ \Sigma + (\Sigma)^\dagger \right\} - \frac{\Delta m_{\eta'}^2}{2} \eta' \eta'$$



Contact terms:



Gell-Mann-Oakes-Renner:

$$m_\pi^2 = \frac{m\mu^3}{2f_\pi^2}$$

$$m_{\eta'}^2 = m_\pi^2 + \Delta m_{\eta'}^2$$

Topological terms and coset homotopy

The coset space G/H has non vanishing 4th homotopy group:

$$\pi_4(U(4)/O(4)) = \pi_4(SU(4)/SO(4)) \neq 0$$

Problem:

- The standard construction/classification of topological terms in non-linear sigma model by Witten, Weinberg and d'Hoker requires $\pi_4(G/H) = 0$.

[Witten, Nucl. Phys. B 223 (1983)]

[D'Hoker and Weinberg, Physical Review D 50 (1994)]

[Brauner and Kolečová, Nuclear Physics B 945 (2019)]

- Modern approaches give more general classification but no practical construction.

[Davighi and Gripaos, Journal of High Energy Physics 2018 (2018)]

[Lee, Ohmori, and Tachikawa, SciPost Physics 10 (2021)]

't Hooft anomaly matching argument

Using the gauge principle to detect anomalous terms in the effective field theory.

Anomalous WTI: $\delta S_{\text{eff}}^{\text{gauged}} = \mathcal{A}_{UV}$ and $S_{\text{eff}} = \lim_{A \rightarrow 0} S_{\text{eff}}^{\text{gauged}}$

Step 1

$$S_{UV} \xrightarrow{\text{Gauge } G_F} S_{UV}^{\text{gauged}} \xrightarrow{\delta_{G_F}} \mathcal{A}_{UV} \neq 0$$

Step 2

$$S_{UV} + S_{\text{Free}} \xrightarrow{\quad} S_{UV}^{\text{gauged}} + S_{\text{Free}}^{\text{gauged}} \xrightarrow{\quad} \mathcal{A}_{UV} = 0$$

Step 3

$$S_{\text{eff}} + S_{\text{Free}} \xrightarrow{\quad} S_{\text{eff}}^{\text{gauged}} + S_{\text{Free}}^{\text{gauged}} \xrightarrow{\quad} \mathcal{A}_{IR} = 0$$

Step 4

$$S_{\text{eff}} \xrightarrow{\quad} S_{\text{eff}}^{\text{gauged}} \xrightarrow{\quad} \mathcal{A}_{IR} = \mathcal{A}_{UV} \neq 0$$

Wess and Zumino derived an effective action that solves the anomaly equation. [Wess and Zumino, Physics Letters B 37 (1971)]

Wess-Zumino effective action

$$S_{WZ}[\xi = (\eta', \pi)] = \frac{D_C}{48\pi^2 f_\pi} \int_0^1 dt \int_{S^4} \text{Tr} \left\{ \xi \left((U[t\xi])^{-1} dU[t\xi] \right)^4 \right\}$$
$$\approx \frac{D_C}{250 f_\pi \pi^2} \epsilon^{\mu\nu\sigma\rho} \int_{S^4} d^4x \text{Tr} \{ \pi \partial_\mu \pi \partial_\nu \pi \partial_\sigma \pi \partial_\rho \pi \}$$

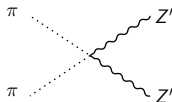
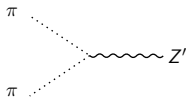
- Does not depend on η' in lowest order χ PT.
- Removes superfluous $\pi(t, x) \mapsto -\pi(t, x)$ symmetry in S_{eff} .
- Incorporates $3 \rightarrow 2$ process.

Inclusion of the dark photon Z'

Non-Anomalous contributions:

Exchange derivatives:

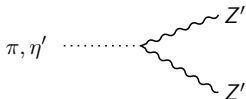
$$\partial_\mu U^2 \longmapsto D_\mu U^2 = \partial_\mu U^2 + ie_D A_\mu \left(Q U^2 + U^2 \Sigma_0^\dagger Q^\top \Sigma_0 \right)$$



Anomalous contributions:

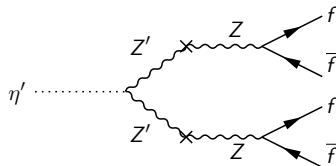
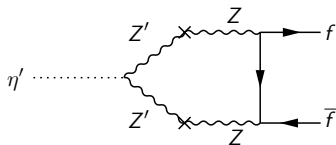
Anomaly equation:

$$S_{\text{eff}} = \lim_{A \rightarrow Z'} S_{\text{eff}}^{\text{gauged}}$$



$$\propto \begin{cases} \text{Tr} \{ \pi Q^2 \} & = 0 \\ \text{Tr} \{ \eta' Q^2 \} & \neq 0 \end{cases}$$

Depending on lifetime $\tau_{\eta'}$ (and mass $m_{\eta'}$) of the η' the dark matter scenario might be significantly altered or spoiled.



η' -decay may lower the calculated relic abundance of dark matter significantly.

Limiting cases:

- η' decays almost instantly
 \Rightarrow **NO DARK MATTER**
- $\tau_{\eta'} \approx$ age of universe.
 \Rightarrow Pion abundance not affected.

UV - models: Generalizations

Generalizations

Almost all the results so far depend on

- The presence of a chirally broken phase.
- The fact that fermions transform under a real representation.

So can we get the same for different gauge-theories than $SO(N)$ with vector-representation fermions?

For example :

- Adjoint representations
- $SO(N)$ tensor representations
- $Sp(2N)$ two index anti-symmetric representation

Conformal window considerations

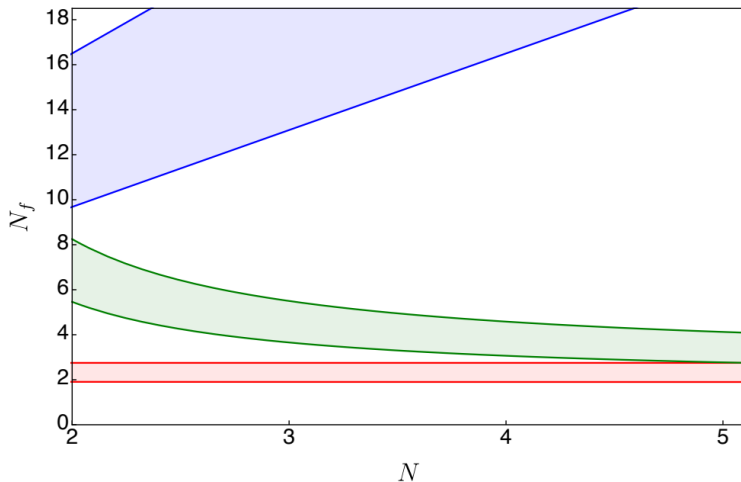


Figure: Conformal window for $Sp(2N)$ with fundamental (blue), adjoint (red) and two index anti-symmetric (green). [Lee, Ohmori, and Tachikawa, SciPost Physics 10 (2021)]

Anomalous symmetry breaking

The chiral anomaly functional depends on the gauge-group representation.

1) General symmetry breaking pattern

$$U(4) \rightarrow Z_K \ltimes SU(4) \rightarrow O(4)$$

	$SO(N)$ -Vector	$Sp(2N)$ -A2T	$SO(N)$ -Adj.	$Sp(2N)$ -Adj.
K	2	$2(N-1)$	$2(N-2)$	$2(N+1)$

2) In naive 't Hooft large N considerations, the anomalous contribution does not vanish.

$\Rightarrow \eta'$ might not be light and thus not too relevant.

Summary

What I talked about today:

- Dark QCD-like models based on $SO(N)$ -vector might realize SIMP dark matter.
- One has to be careful about the role of η' .
- Problems and solution concerning Wess-Zumino terms.
- Situation for other real representations not so clear.

Summary

What I talked about today:

- Dark QCD-like models based on $SO(N)$ -vector might realize SIMP dark matter.
- One has to be careful about the role of η' .
- Problems and solution concerning Wess-Zumino terms.
- Situation for other real representations not so clear.

What I left out:

- Construction of gauge-invariant operators.
- Details on discrete symmetries
- Inclusion of vector mesons via local hidden symmetry.

Bonus content

Charge conjugation C :

$$\text{UV } q \mapsto \Omega C \bar{q}^T$$

$$\text{IR } \pi \mapsto \pi^T$$

Spatial parity P :

$$\text{UV } q(t, \vec{x}) \mapsto \eta_P \gamma^0 q(t, -\vec{x})$$

$$\text{IR } \pi(t, \vec{x}) \mapsto -\pi(t, -\vec{x})$$

The choice $\eta_P = \pm i$ is adopted:

- Parity and flavor symmetries commute.
- All (pseudo) Nambu-Goldstone bosons (pNGB) are pseudo scalars.

Dark pion mass

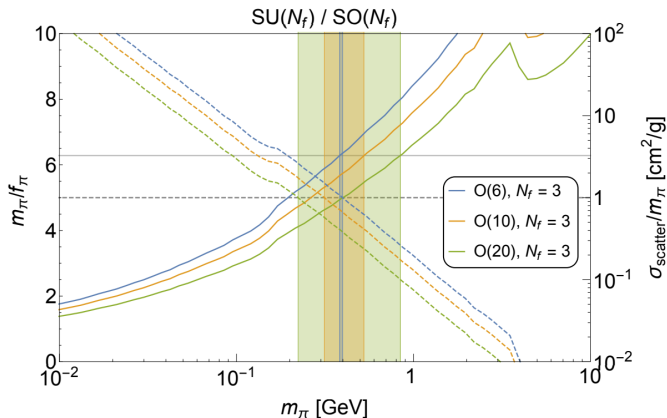


Figure: The solid horizontal line depicts the perturbative limit of $m_\pi/f_\pi \approx < 2\pi$, providing a rough upper limit on the pion mass; the dashed horizontal line depicts the bullet-cluster and halo shape constraints on the self-scattering cross section, placing a lower limit on the pion mass. Each shaded region depicts the resulting approximate range for m_π [Hochberg et al., Physical Review Letters 115 (2015)]

4th Homotopy group of $SU(4)/SO(4)$

	π_3	π_4	π_5
$SO(4)$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$SU(4)$	\mathbb{Z}	0	\mathbb{Z}

Fibration:

$$SO(4) \rightarrow SU(4) \rightarrow SU(4)/SO(4)$$

Long exact sequence:

$$\begin{array}{ccccccc}
 \pi_4(SU(4)) & \xrightarrow{h_1} & \pi_4(SU(4)/SO(4)) & \xrightarrow{h_2} & \pi_3(SO(4)) & \xrightarrow{h_3} & \pi_3(SU(4)) \\
 0 & \xrightarrow{h_1} & ? & \xrightarrow{h_2} & \mathbb{Z} \oplus \mathbb{Z} & \xrightarrow{h_3} & \mathbb{Z}
 \end{array}$$

- $\text{Ker}(h_2) = \text{Im}(h_1) = 0 \rightarrow h_2$ is injective
- $\pi_4(SU(4)/SO(4)) \cong \text{Im}(h_2) = \text{Ker}(h_3)$
- $\text{Ker}(h_3) \neq 0$

$\Rightarrow \pi_4(SU(4)/SO(4))$
cannot be trivial

$$-\frac{1}{4} G_{\mu\nu}^{\alpha} G_{\alpha}^{\mu\nu}$$

Yang-Mills term for dark gluons : Based on $G_D = Sp(4)$

$$-\frac{1}{4} F'_{\mu\nu} F'^{\mu\nu}$$

Yang-Mills term for dark photon : Based on $U_D(1)$

$$+ \bar{u} i \gamma^{\mu} D_{\mu} u + m \bar{u} u$$

Dirac term of dark quarks

2 flavors: u and d quarks

$$+ \bar{d} i \gamma^{\mu} D_{\mu} d + m \bar{d} d$$

Charged under $G_D \times U_D(1)$

$$+ \left(D'_{\mu} \phi \right)^{\dagger} D'^{\mu} \phi + V [\phi^{\dagger} \phi]$$

Dark scalar charged under $U_D(1)$

$$+ \frac{\epsilon}{2 \cos(\theta_W)} F'_{\mu\nu} B^{\mu\nu}$$

Kinetic mixing of $U_D(1)$ with SM hypercharge

Meson spectrum in 2 flavor $Sp(4)$ -fund. gauge theory

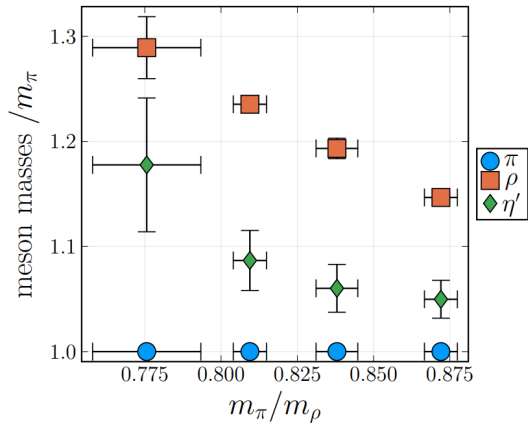


Figure: Meson spectrum in $Sp(4)$ -fundamental with 2 flavors.
Taken from [Zierler et al. (2022), arXiv:hep-lat/2210.11187]

	$\overline{\Psi}_C T_n^\Psi \Psi + (\overline{\Psi}_C T_n^\Psi \Psi)^*$	J^D		$\overline{\Psi}_C T_N^\pi \Psi + (\overline{\Psi}_C T_N^\pi \Psi)^*$	I_3	e_D
π_1	$\frac{1}{\sqrt{2}} (\bar{u}\gamma^5 d + \bar{d}\gamma^5 u)$	1^-	π^A	$\bar{u}\gamma^5 d$	1	0
π_2	$\frac{1}{\sqrt{2}} (\bar{u}\gamma^5 d - \bar{d}\gamma^5 u)$	1^-	π^B	$\bar{d}\gamma^5 u$	-1	0
π_3	$\frac{1}{\sqrt{2}} (\bar{u}\gamma^5 u - \bar{d}\gamma^5 d)$	1^-	π^C	$\frac{1}{\sqrt{2}} (\bar{u}\gamma^5 u - \bar{d}\gamma^5 d)$	0	0
π_4	$\frac{1}{2} (\bar{u}_C\gamma^5 u + \bar{u}\gamma^5 u_C)$	1^-	π^D	$\frac{1}{\sqrt{2}} \bar{u}_C\gamma^5 u$	-1	-1
π_5	$\frac{1}{2} (\bar{d}_C\gamma^5 d + \bar{d}\gamma^5 d_C)$	1^-	π^E	$\frac{1}{\sqrt{2}} \bar{d}\gamma^5 d_C$	1	-1
π_6	$\frac{1}{\sqrt{2}} (\bar{u}\gamma^5 d_C + \bar{u}_C\gamma^5 d)$	1^-	π^F	$\bar{u}\gamma^5 d_C$	0	-1
π_7	$\frac{1}{2} (\bar{u}_C\gamma^5 u - \bar{u}\gamma^5 u_C)$	1^-	π^G	$\frac{1}{\sqrt{2}} \bar{u}\gamma^5 u_C$	1	1
π_8	$\frac{1}{2} (\bar{d}\gamma^5 d_C - \bar{d}_C\gamma^5 d)$	1^-	π^H	$\frac{1}{\sqrt{2}} \bar{d}_C\gamma^5 d$	-1	1
π_9	$\frac{1}{\sqrt{2}} (\bar{u}\gamma^5 d_C - \bar{u}_C\gamma^5 d)$	1^-	π^I	$\bar{u}_C\gamma^5 d$	0	1