Strongly interacting dark matter
with SO(N)-QCD
and
dark photon portal

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University of Stavanger

Università di Pisa
Uni Graz
TU Graz
NAWI Graz Natural Sciences

FWF Der Wissenschaftsfonds.

The dark matter problem

[P. A. R. Ade, N. Aghanim, M. Arnaud, M. Ashdown, et al. Planck2015 results. Astronomy and Astrophysics 594 (2016) A13.]

Evidence for dark matter :

- **Galaxy scale:**

  Rotational curves

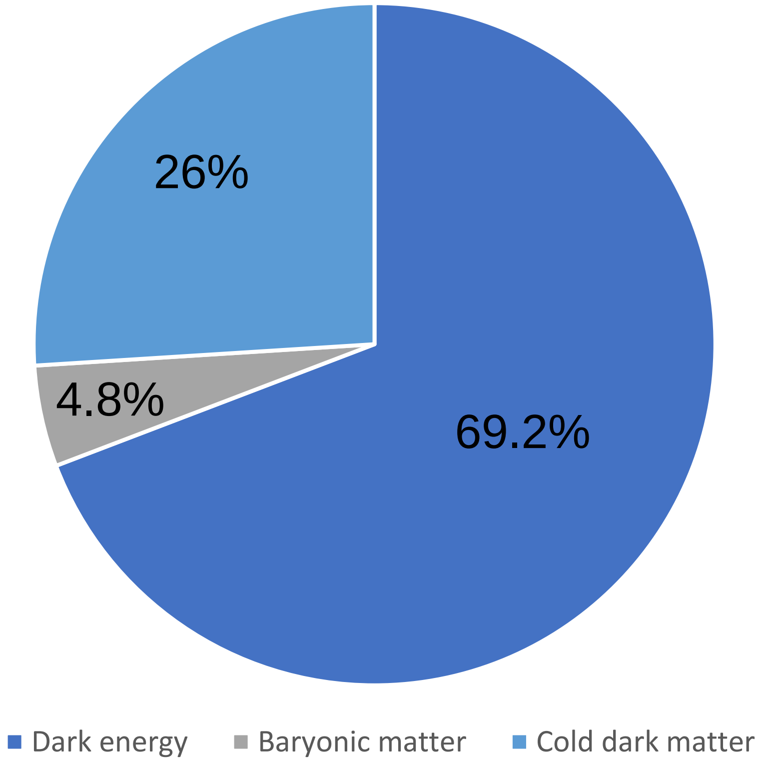
- **Cluster scale:**

  Visible mass too little to hold together Coma Cluster

- **Cosmological scale:**

  CMB anistropies

Energy budget of the universe within ΛCDM model



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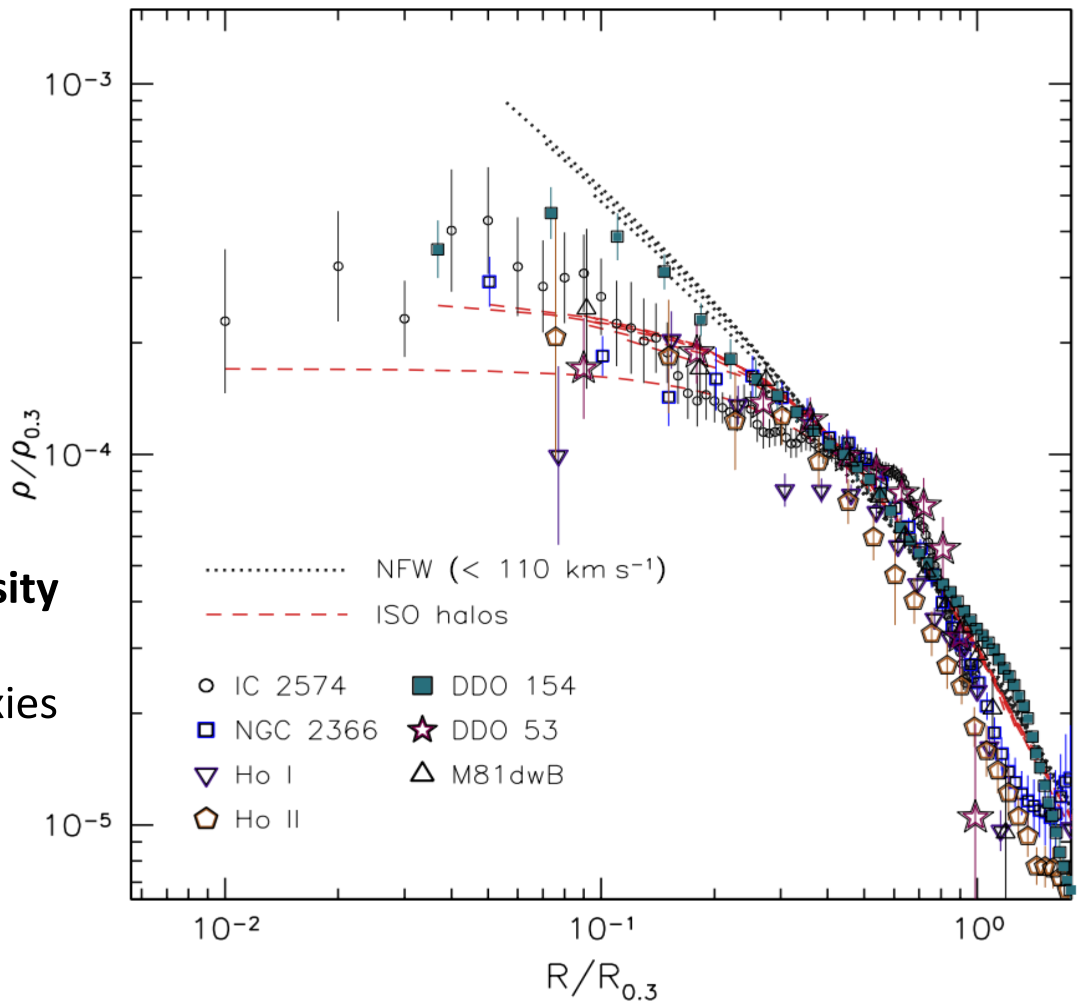
The cusp vs. core problem

[Se-Heon Oh, W. J. G. de Blok, Elias Brinks, et al. ArXiv: 1011.0899]



The **dark matter density** profiles of the **7 THINGS** dwarf galaxies

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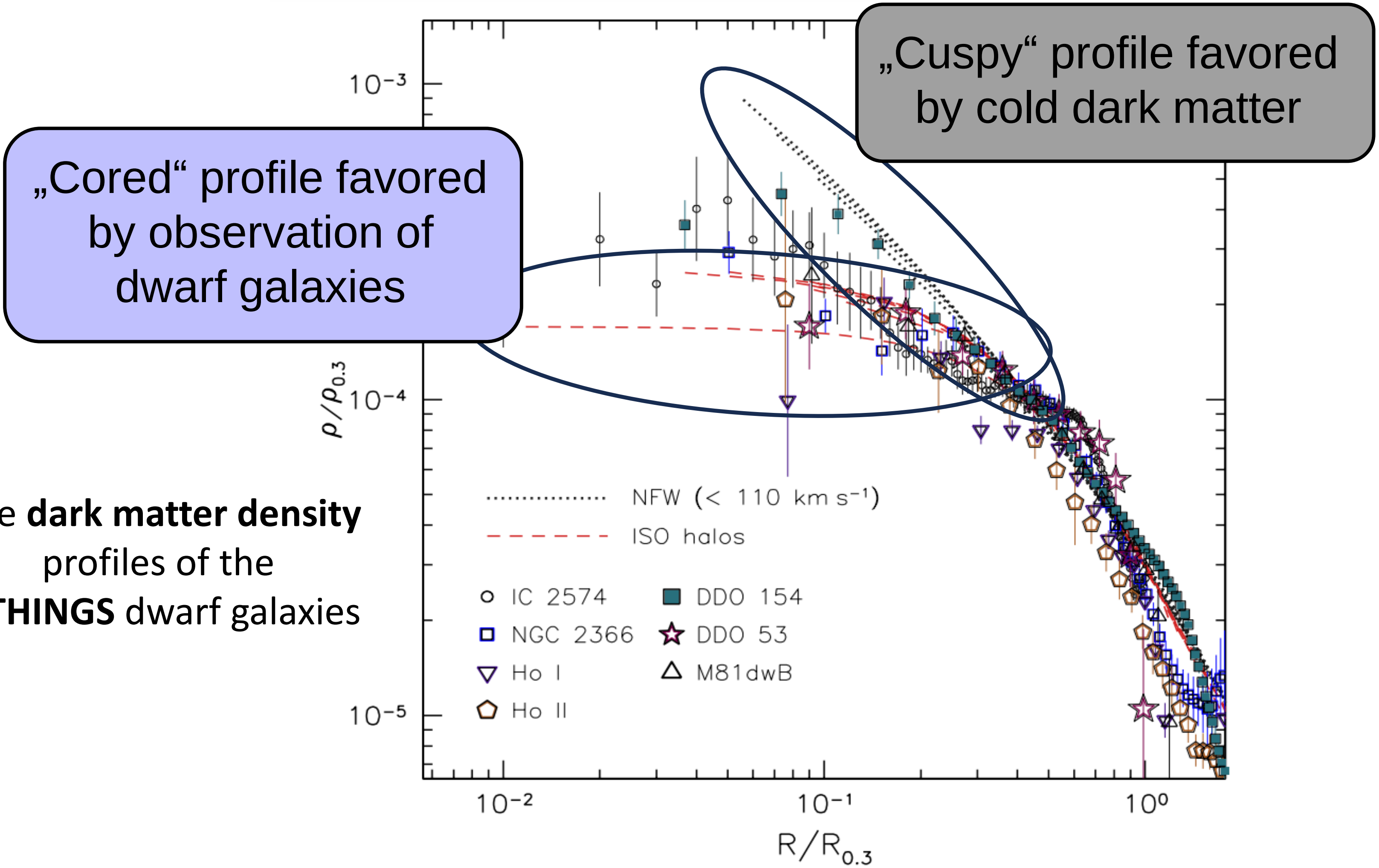

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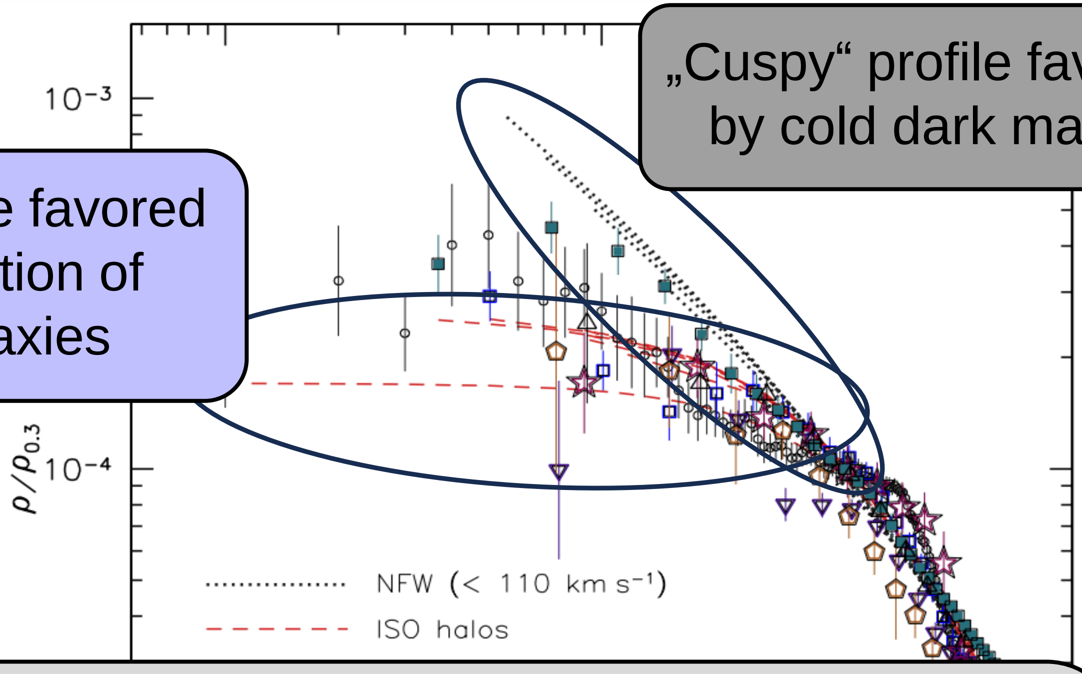

„Cuspy“ profile favored by cold dark matter

„Cored“ profile favored by observation of dwarf galaxies

Might be resolved if dark matter is self-interacting

[Dave et al., The Astrophysical Journal 547 (2001)]

$$\frac{\sigma_{2\to 2}}{m_{DM}} \approx 1 \left[\frac{\mathrm{cm}^2}{\mathrm{g}}\right]$$

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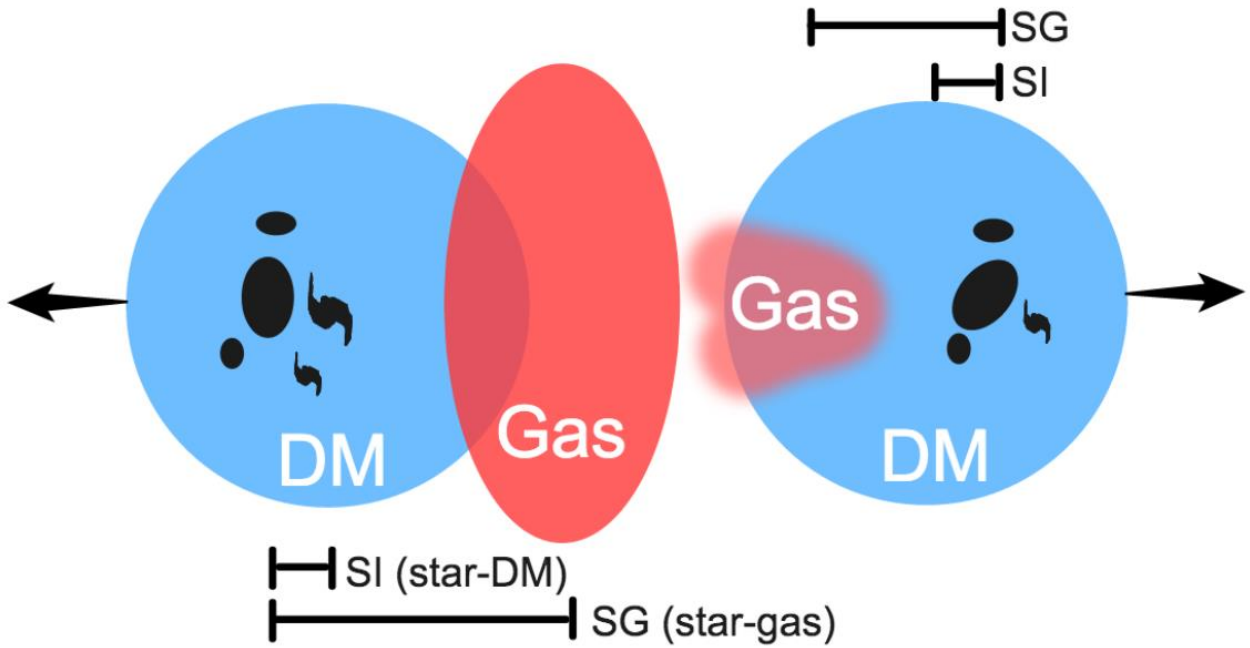
Constraining DM self-interactions

[Wittman, Golovich and Dawson; ArXiv: 1701.05877]

Observations of center of gravitational lensing of colliding galaxy cluster.



Self-interactions determine offset (SI) between center of DM halo and visible cluster

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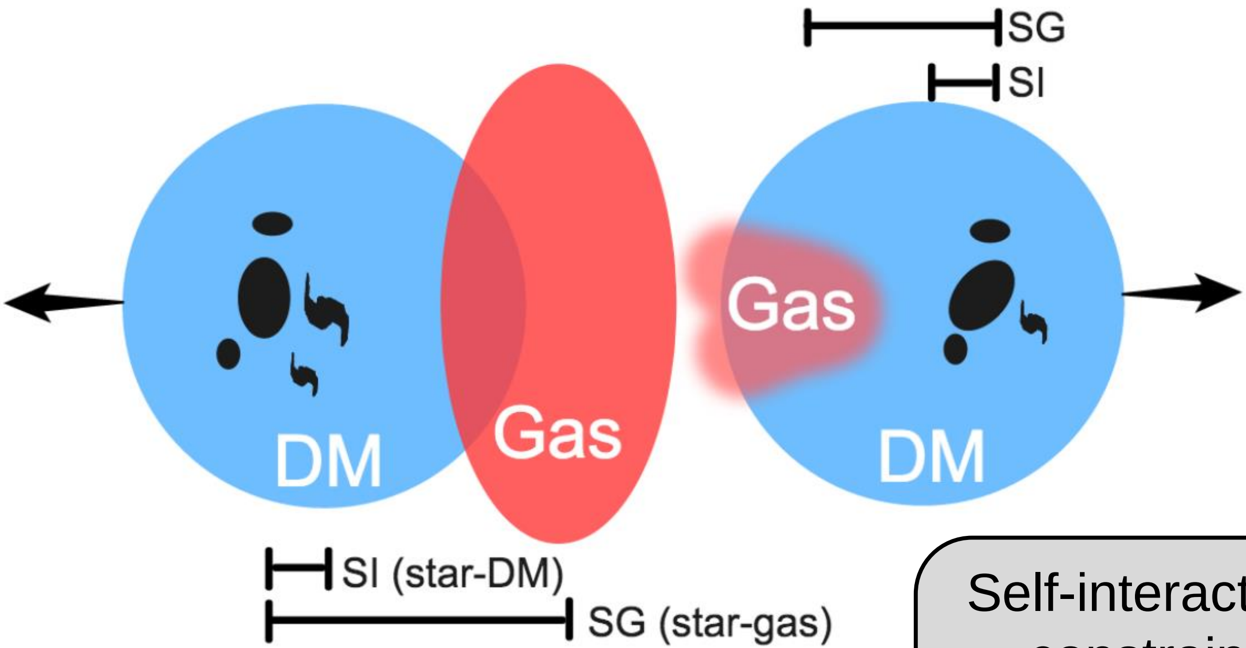


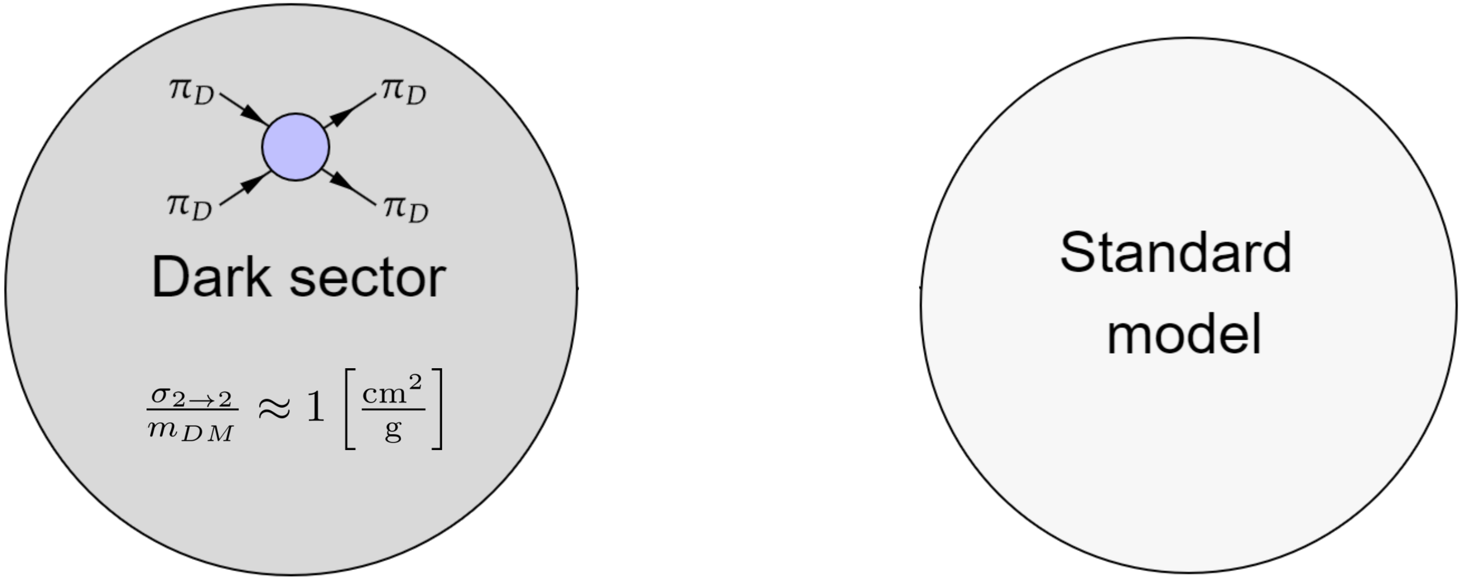
Self-interactions determine offset (SI) between center of DM halo and visible cluster

Self-interaction constraint

$$\frac{\sigma_{2\to 2}}{m_{DM}} \leq 2 \left[\frac{\mathrm{cm}^2}{\mathrm{g}}\right]$$

SIMPs - Strongly Interacting Massive Particles

[Hochberg et al.: ArXiv: 1402.5143]



- Self-interactions resolve small scale structure formation problems.

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SIMPs - **S**trongly **I**nteracting **M**assive **P**articles

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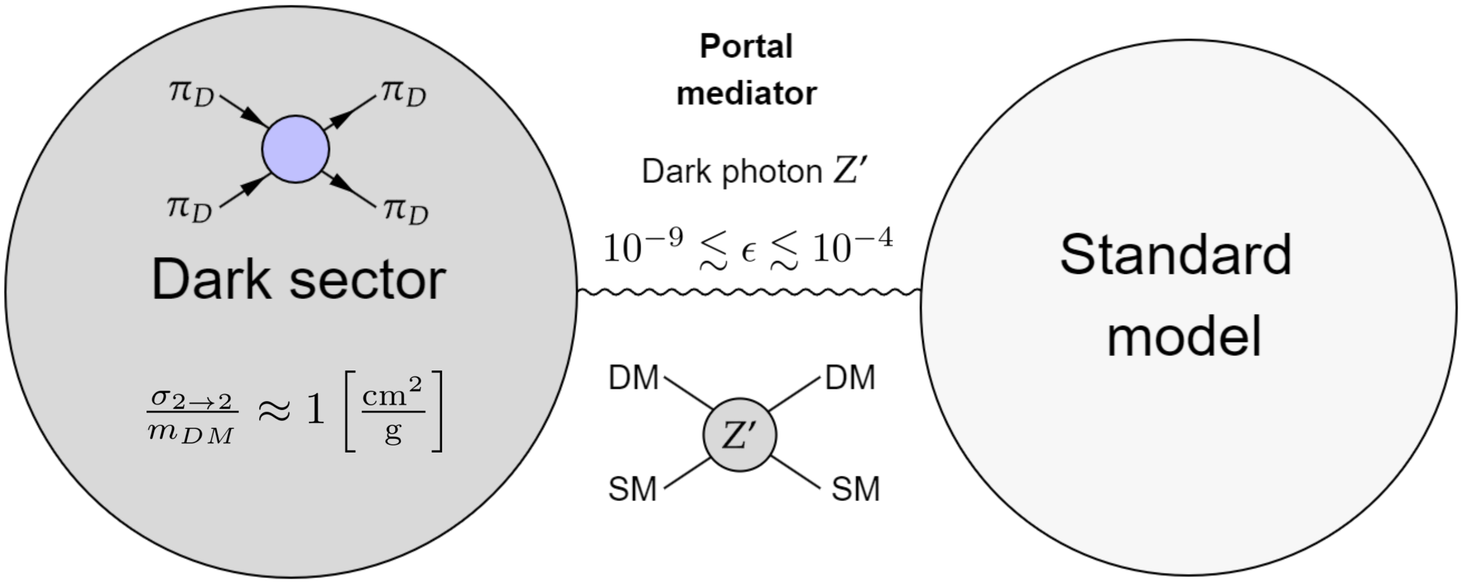

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- Dark photon Z' mediator maintains thermal equilibrium.

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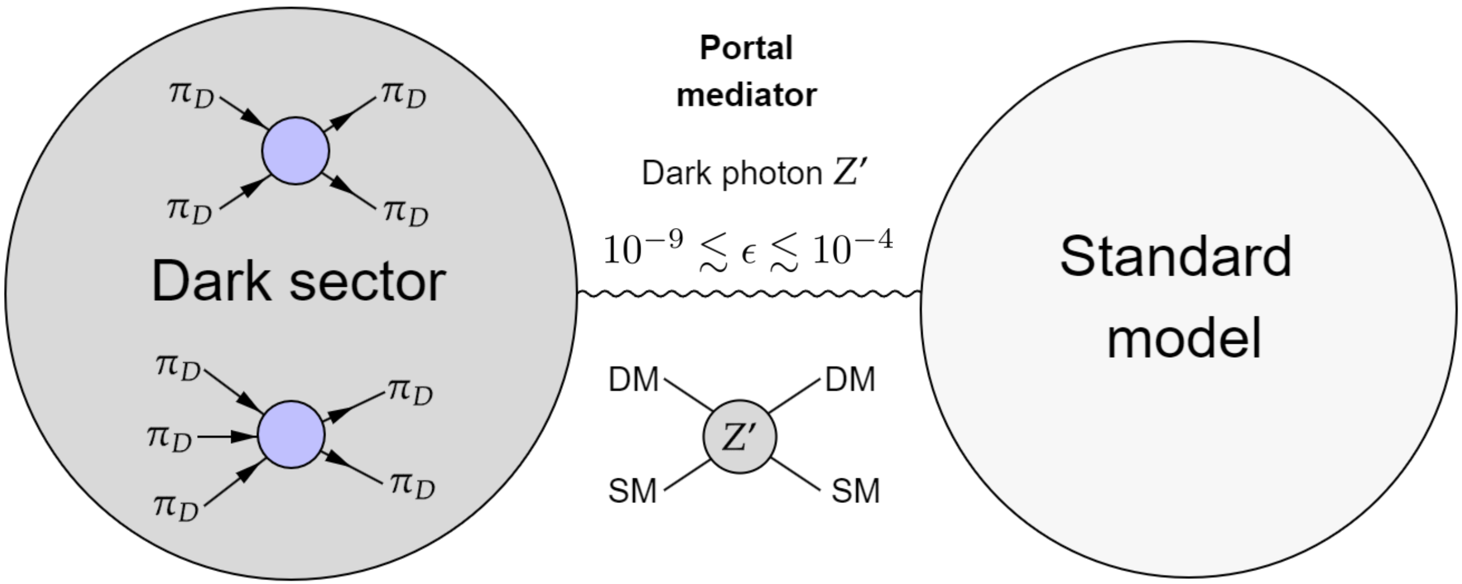

- Self-interactions resolve small scale structure formation problems.
- Dark photon Z' mediator maintains thermal equilibrium.
- 3→2 cannibalization sets correct DM relic abundance.

SIMPs from QCD-like theories.

[Hochberg et al.: ArXiv: 1402.5143]

Strong dark sector:

- **Dark gluons** of gauge group G_D .
- N_F Dirac fermions in representation $\mathcal{R}$.

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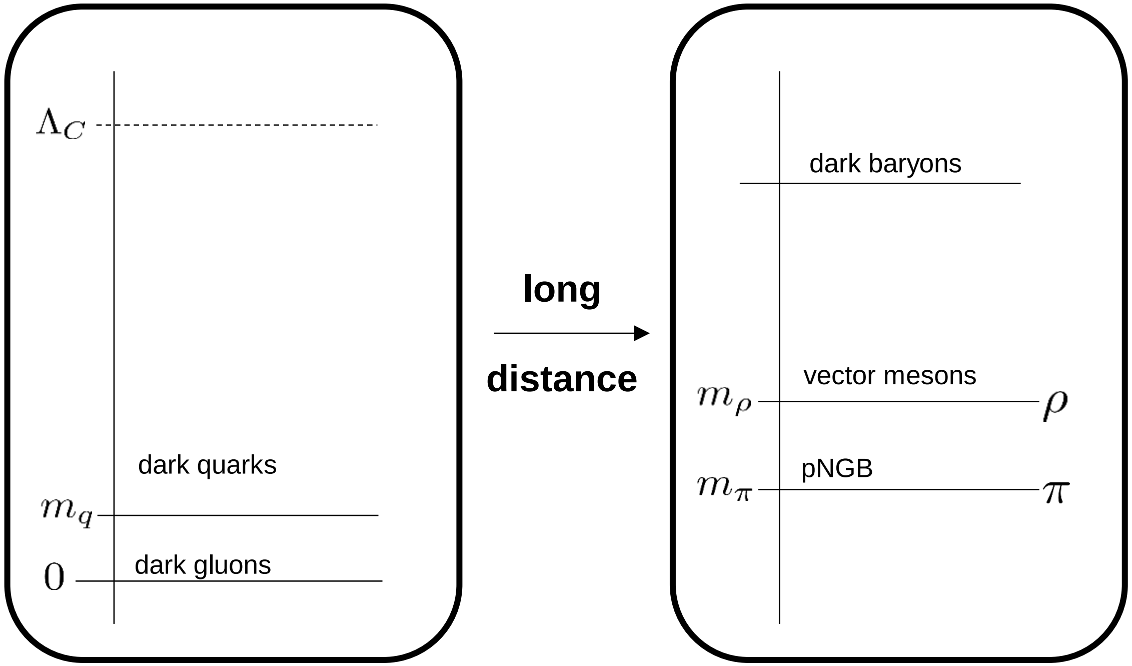
Dark matter **is composite** and made from dark fermion bound states.



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Model parameter

Discrete:

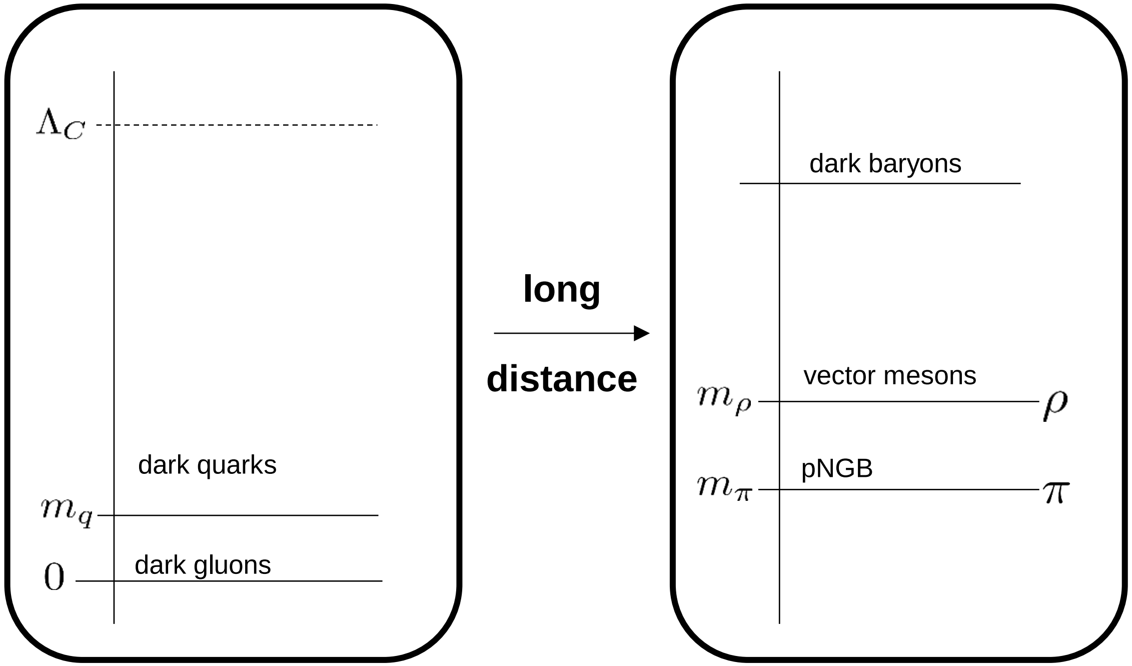
$G_D(N_C), \mathcal{R}$

N_F

Continuous:

$g_D(\Lambda_{UV})$

$m_q^{(1)}, m_q^{(2)}, \ldots$



$SO(N_C)$- like QCD

Real fermion representation:

Dark quark $\boxed{q} \cong \boxed{q_c}$ Dark anti-quark

$SO(N_C)$ - like QCD

Real fermion representation:

$$\begin{tikzcd}
\text{Dark quark} & \boxed{q} \arrow[r, phantom, "\cong"] \arrow[r, bend right=80, "\mathcal{C}"'] \arrow[d, "\text{Charge conjugation}"'] & \boxed{q_c} \arrow[r, phantom, "="] & \boxed{SC\overline{q}^{\top}} & \text{Dark anti-quark} \\
 & \boxed{C\overline{q}^{\top}} \arrow[urr, "\text{Linear transforamtion S}"'] & & &
\end{tikzcd}$$

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Decompose Dirac fermions into Majorana fermions

$$\mathcal{C}\psi_i = \psi_i$$

$$\boxed{u} = \boxed{\psi_1} + i\,\boxed{\psi_3}$$

$$\boxed{d} = \boxed{\psi_2} + i\,\boxed{\psi_4}$$

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$$\xrightarrow[\textbf{distance}]{\textbf{long}}$$

Meson states

$\boxed{\overline{u}u + \overline{d}d}$

$\boxed{\overline{u}d}$ $\boxed{\overline{d}u}$ $\boxed{\overline{u}u - \overline{d}d}$

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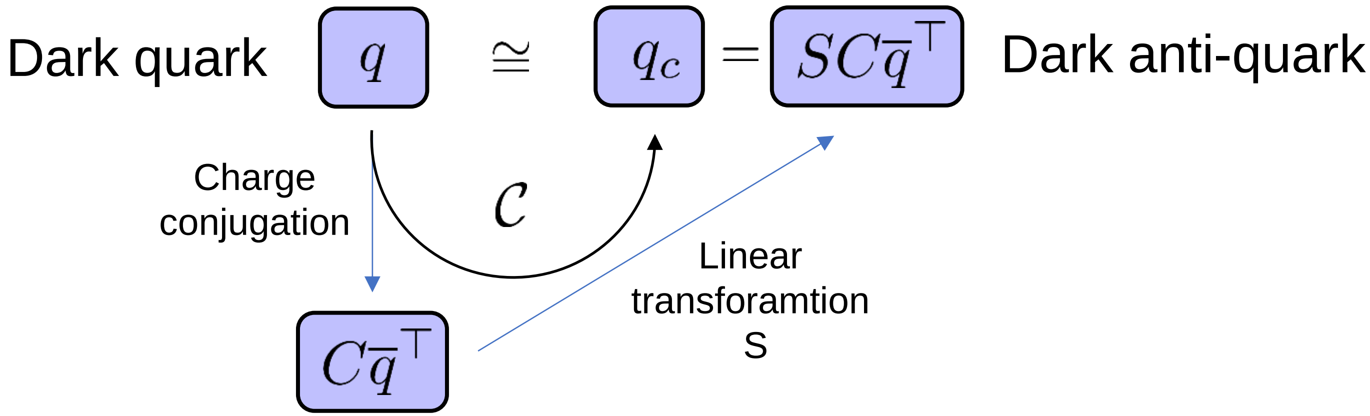

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$\bar{u}u + \bar{d}d$		
$\bar{u}d$	$\bar{d}u$	$\bar{u}u - \bar{d}d$
$\bar{u}u_c$	$\bar{u}d_c$	$\bar{d}d_c$
$\bar{u}_c u$	$\bar{u}_c d$	$\bar{d}_c d$

Symmetries and lightest stable particles

We focus on $SO(N_C)$ with $N_F = 2$ fermions.

Breaking pattern

$$\boxed{U(4)}$$

$$\Big\downarrow \text{Axial anomaly}$$

$$\boxed{\mathbb{Z}_2 \ltimes SU(4)}$$

$$\Big\downarrow \begin{array}{l} \langle \overline{q}q \rangle \neq 0 \\ m_q \neq 0 \end{array}$$

$$\boxed{O(4)}$$

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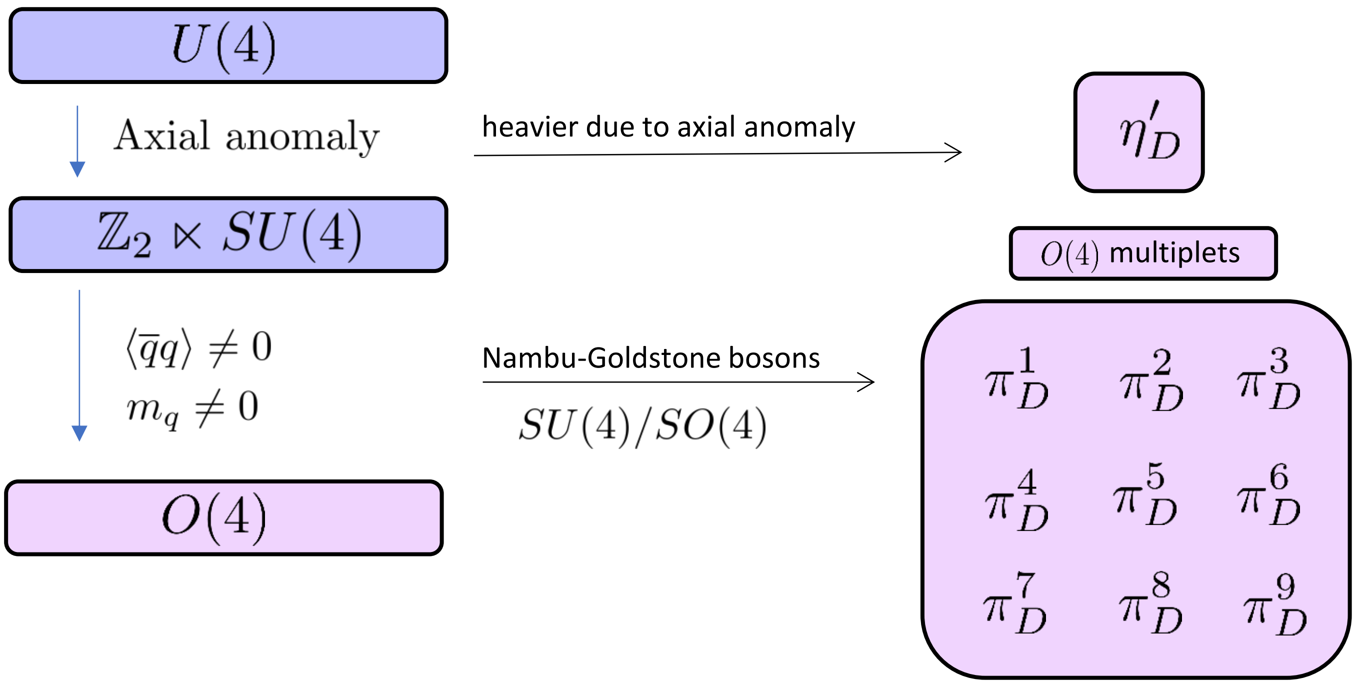

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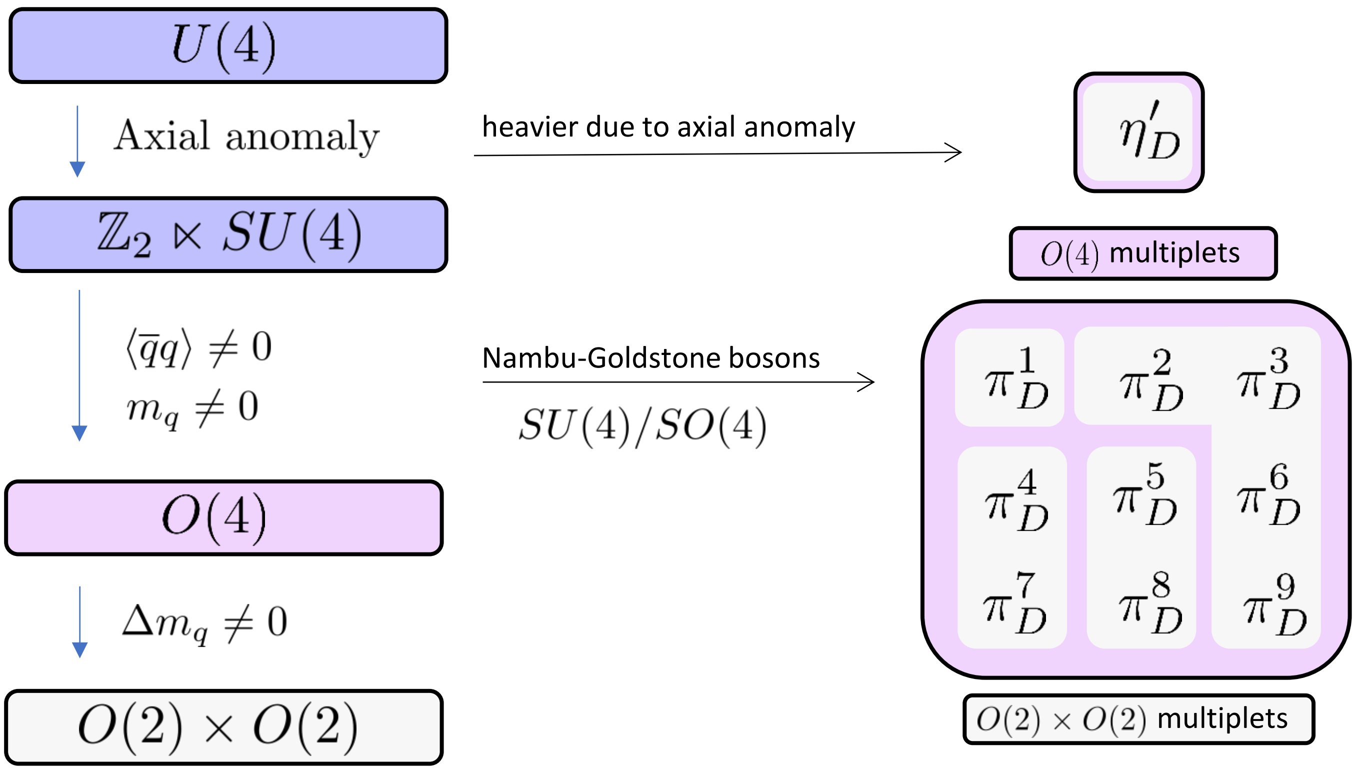

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Can η'_D be close in mass to the π_D ?

A large N_C argument analog to real world QCD :

$$\partial_\mu j^\mu_{\eta'_D} = \text{const.} \times \frac{T_\mathcal{R}}{C_\text{adj}} \tilde{F}^{\mu\nu}_a F^a_{\mu\nu}$$

Gluonic sources for the flavor singlet state

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- Satisfied for fermions in the fundamental or vector representation.
  $\Rightarrow \eta'_D$ becomes light in large N_C limit.

- Not satisfied for example for higher tensor or adjoint representations.
  $\Rightarrow \eta'_D$ expected to remain heavy.

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Low energy effective Lagrangian

Chiral coset representative:

$$\Sigma = \exp\left(i2\xi^a T_a\right)\omega \qquad \xi^a = \begin{cases} \eta'_D/f_{\eta'_D} & \text{if } a = 0 \\ \pi_D/f_{\pi_D} & \text{else} \end{cases}$$

Symmetries:

- $\mathbb{Z} \ltimes SU(4)$ flavor symmetry $\quad \Sigma \mapsto U\Sigma U^\top \quad \pi \mapsto F(\pi)$
  - $O(4)$ symmetry linearily realized $\quad \Sigma \mapsto O\Sigma O^\top \quad \pi \mapsto O\pi O^\dagger$
- Spatial parity $\quad \Sigma(x) \mapsto \Sigma^\dagger(Px) \quad \pi(x) \mapsto -\pi(Px)$

Low energy effective Lagrangian

$$\mathcal{L}_{\mathrm{IR}} = \frac{f_\pi^2}{4} \mathrm{tr}\left\{\partial_\mu \Sigma^\dagger \partial^\mu \Sigma\right\}$$

EFT parameter:

f_π

GMOR relation: $m_\pi^2 = 0$

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Low energy effective Lagrangian

$$\mathcal{L}_{\mathrm{IR}} = \frac{f_\pi^2}{4}\mathrm{tr}\left\{\partial_\mu \Sigma^\dagger \partial^\mu \Sigma\right\}$$
$$+\frac{m_\pi^2 f_\pi^2}{4}\mathrm{tr}\left\{\omega\Sigma^\dagger + \omega^\dagger \Sigma\right\}$$

EFT parameter:

f_π, m_π

GMOR relation: $$m_\pi^2 = \frac{m_q\langle\overline{q}q\rangle}{2f_\pi^2}$$

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Low energy effective Lagrangian

$$\mathcal{L}_{\mathrm{IR}} = \frac{f_\pi^2}{4}\mathrm{tr}\left\{\partial_\mu\Sigma^\dagger\partial^\mu\Sigma\right\} + \frac{f_\pi^2 - f_{\eta'_D}^2}{4}\mathrm{tr}\left\{\Sigma\partial_\mu\Sigma^\dagger\right\}\mathrm{tr}\left\{\Sigma^\dagger\partial^\mu\Sigma\right\}$$

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$$+ \frac{m_\pi^2 f_\pi^2}{4}\mathrm{tr}\left\{\omega \Sigma^\dagger + \omega^\dagger \Sigma\right\} + \frac{\Delta m_{\eta'_D}^2 f_{\eta'_D}^2}{4}\left(\ln\left(\det\left(\Sigma\right)\right)\right)^2$$

EFT parameter:

$f_\pi, m_\pi, f_\eta, \Delta m_\eta$

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Can **not** be independent !

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Decay constants: $$f_{\eta'_D} \xrightarrow[N_C\to\infty]{} f_\pi$$

η'_D - mass: $$m_{\eta'_D}^2 = m_\pi^2 + \frac{f_{\eta'_D}^2}{f_\pi^2}\Delta m_{\eta'_D}^2$$
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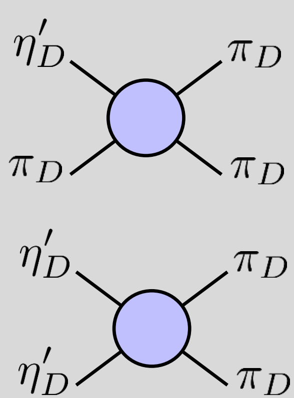
Contact terms:



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$$\Delta m_{\eta'_D}^2 \xrightarrow[N_C \to \infty]{} 0$$

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The problem of naïve parity

Actual symmetries of the non-anomalous Lagrangian :

- $\mathbb{Z} \ltimes SU(4)$ flavor symmetry $\quad \Sigma \mapsto U\Sigma U^\top \quad \pi \mapsto F(\pi)$
  - $O(4)$ symmetry linearily realized $\quad \Sigma \mapsto O\Sigma O^\top \quad \pi \mapsto O\pi O^\dagger$
- Spatial parity $\quad \Sigma(x) \mapsto \Sigma^\dagger(Px) \quad \pi(x) \mapsto -\pi(Px)$
- Naive parity $\quad \Sigma(x) \mapsto \Sigma^\dagger(x) \quad \pi(x) \mapsto -\pi(x)$

There are more symmetries
than in the UV theory!

Did we míssed a part of the action ?

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Anomalous action – the idea

What happens if we gauge the $SU(4)$ in the UV Lagrangian ?

Calculation: We produce an anomaly !

$$\delta_\epsilon^{\text{gauge}} S_{\text{cov.}}^{\text{UV}}[\psi, A] = \boxed{\mathcal{A}[A]} \longleftarrow$$

Anomaly is a functional of the $SU(4)$ gauge fields A_μ.

What happens if we gauge the $SU(4)$ in the IR Lagrangian ?

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Anomaly is a functional of the $SU(4)$ gauge fields A_μ.

What happens if we gauge the $SU(4)$ in the IR Lagrangian ?

`t Hooft: We must produce the same anomaly !

$$\boxed{\delta_\epsilon^{\text{gauge}} S_{\text{cov.}}^{\text{IR}}[\psi, A] \stackrel{!}{=} \mathcal{A}[A]}$$

Problem !

The action constructed so far gives something that is gauge-invariant !

We míssed a part of the action !

Anomalous action

[Wess, Zumino: 1971, Physics Letters B]
[Witten: 1983, Nuclear Physics B]
[Chu, Ho, Zumino: 1996, Nuclear Physics B]

Wess-Zumino-Witten term :

$$S_{\rm WZW} = \frac{\Gamma_{\rm WZW}}{48\pi^2 f_\pi} \int_{S^4} \mathrm{d}^4x \int_0^1 \mathrm{d}\tau \ \mathrm{tr}\left\{\xi \left(\Sigma[\tau\xi]^{-1}\mathrm{d}\Sigma[\tau\xi]\right)^4\right\}$$

$$\approx \frac{\Gamma_{\rm WZW}}{15\pi^2 f_\pi^5} \int_{S^4} \mathrm{tr}\left\{\pi_D \mathrm{d}\pi_D \wedge \mathrm{d}\pi_D \wedge \mathrm{d}\pi_D \wedge \mathrm{d}\pi_D\right\}$$

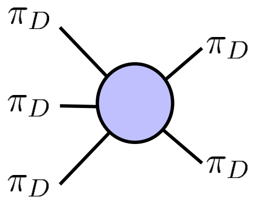
Non-standard form due to coset geometry:

$$\pi_4(SU(4)/SO(4)) \neq 0$$

Anomaly-matching:

$$\Gamma_{\rm WZW} = \mathrm{dim}\mathcal{R}$$

Five point vertex between π_D :



No participation of η'_D in 3→2 DM freeze-out.

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Mediator between DM and SM

[Hochberg et al.: ArXiv: 1512.07917]

Dark photon:

- Z' as gauge boson of a dark $U(1)_D \subset SO(4)$
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Coupling to dark sector:

Covariantize:

$$\partial_\mu \to D_\mu[Z']$$

$$S_{\mathrm{WZW}}[\pi] \to S_{\mathrm{WZW}}[\pi, Z']$$

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Coupling to SM:

Mass term

$$-\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{\epsilon}{\cos(\theta_W)}F'_{\mu\nu}B^{\mu\nu} + \frac{m_{Z'}}{2}Z'_\mu Z'^\mu$$

Kinetic mixing

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Model parameter

Discrete:

$\mathcal{Q}$

Continuous:

$e_D,\ m_{Z'}$

ϵ

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Kinetic mixing

$U(1)_D$ charge assignments

[Hochberg et al.: ArXiv: 1512.07917]

Charge assignment: $\mathcal{Q}$

Consistency:

- No gauge anomalies

Pion stability:

- Maintain non-abelian global symmetry
- No anomalous π_D - decays occur

Charge assignment $\mathcal{Q}$ is physically unique!

[Hochberg et al.: ArXiv: 1512.07917]

$U(1)_D$ charge assignments

Double covering map

$SO(4) \cong SU(2)_I \times SU(2)_B$ $\xrightarrow{Z'}$ $SU(2)_I \times U(1)_B$

Remaining flavor symmetry

Gauge U(1) subgroup of SU(2) factor

explicit breaking by coupling to dark photon Z'

Charge assignment: $\mathcal{Q}$

Consistency:

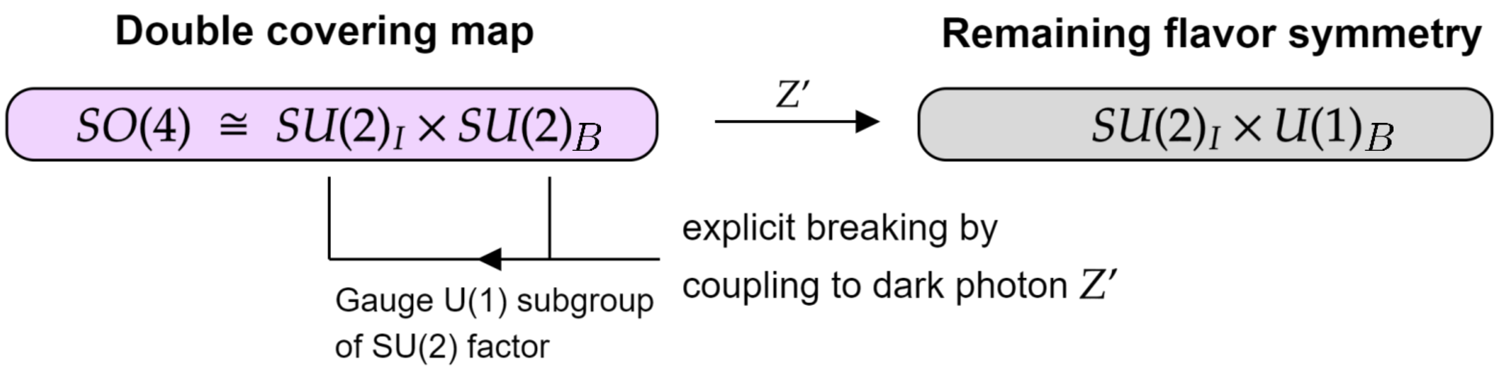
- No gauge anomalies

Pion stability:

- Maintain non-abelian global symmetry
- No anomalous π_D - decays occur

Charge assignment $\mathcal{Q}$ is physically unique!

$U(1)_D$ charge assignments



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Charge assignment $\mathcal{Q}$ is physically unique!

$SU(2)_I \times U(1)_B$ Multiplets			Charges
	η'_D		0
π_D^1	π_D^2	π_D^3	0
π_D^4	π_D^5	π_D^6	−
π_D^7	π_D^8	π_D^9	+

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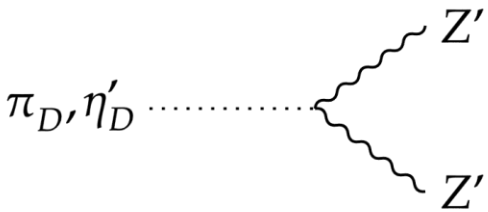
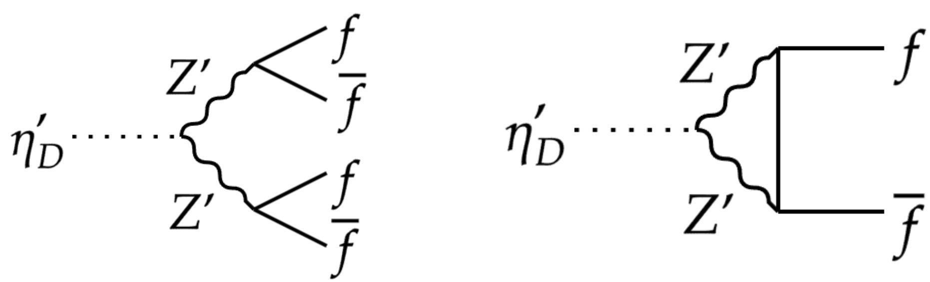
Anomalous η'_D decay

Gauged WZW term introduces anomalous vertices:

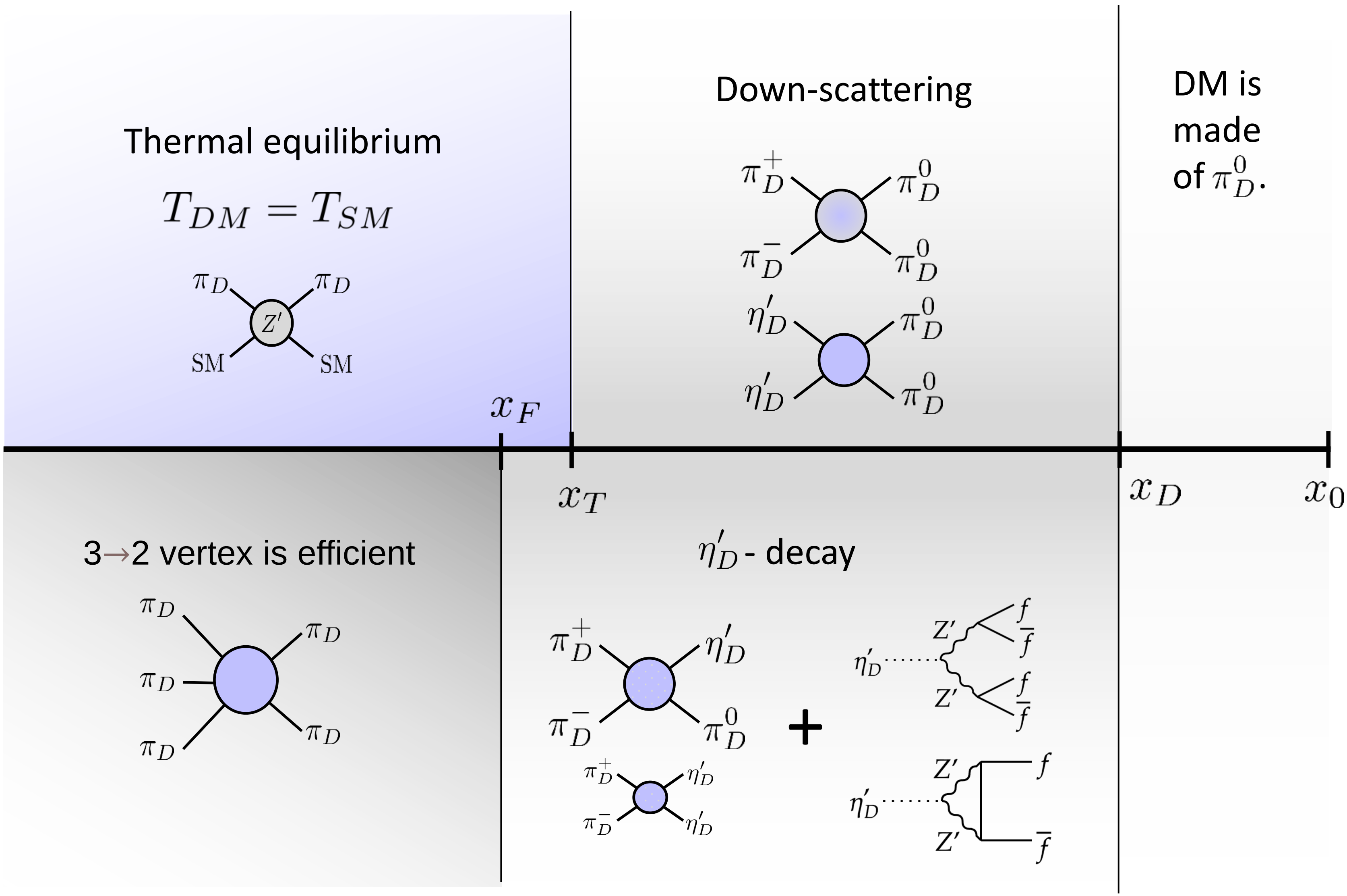


$$\propto \begin{cases} \mathrm{tr}\left\{\pi_D \mathcal{Q}^2\right\} = 0 \\ \mathrm{tr}\left\{\eta'_D \mathcal{Q}^2\right\} \neq 0 \end{cases}$$

Allows for decay of η'_D to SM :



Freeze-out timeline



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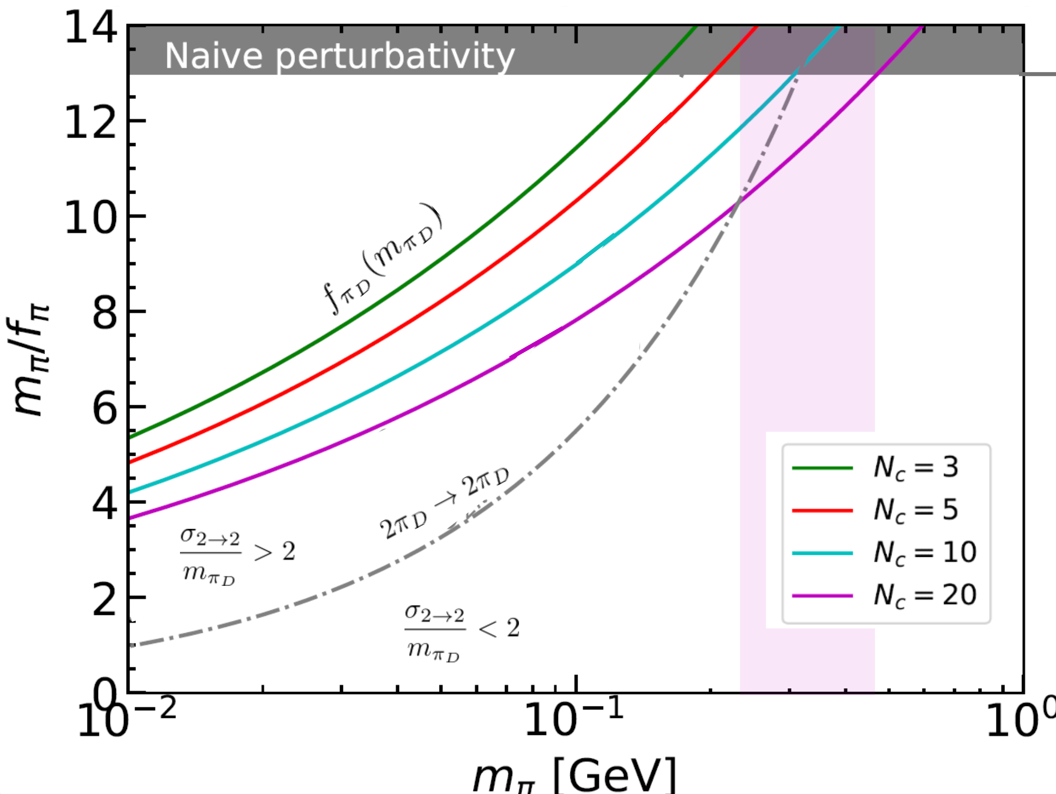
[Hochberg et al: ArXiv: 1411.3727]

Estimating m_{π_D}

Match pion abundance with DM relic density today: $f_{\pi_D} = f_{\pi_D}(m_{\pi_D})$



Perturbative limit

$$\frac{m_{\pi_D}}{f_{\pi_D}} < 4\pi$$

Bullet cluster constraint

[Randall et al.: ArXiv: 0704.0261]

$$\Rightarrow \frac{\sigma_{2\to2}}{m_{\pi_D}} < 2 \left[\frac{\mathrm{cm}^2}{\mathrm{g}}\right]$$

Pion mass :

$$m_{\pi_D} \approx 350\mathrm{MeV}$$

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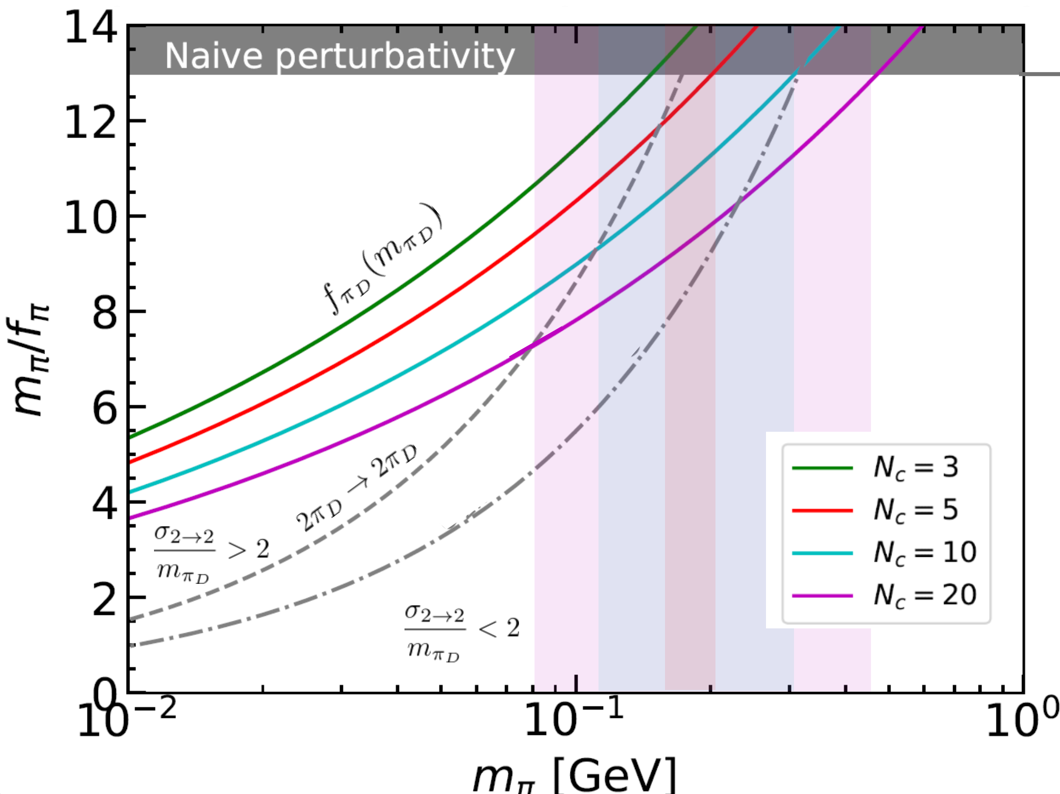

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Bullet cluster constraint

[Randall et al.: ArXiv: 0704.0261]

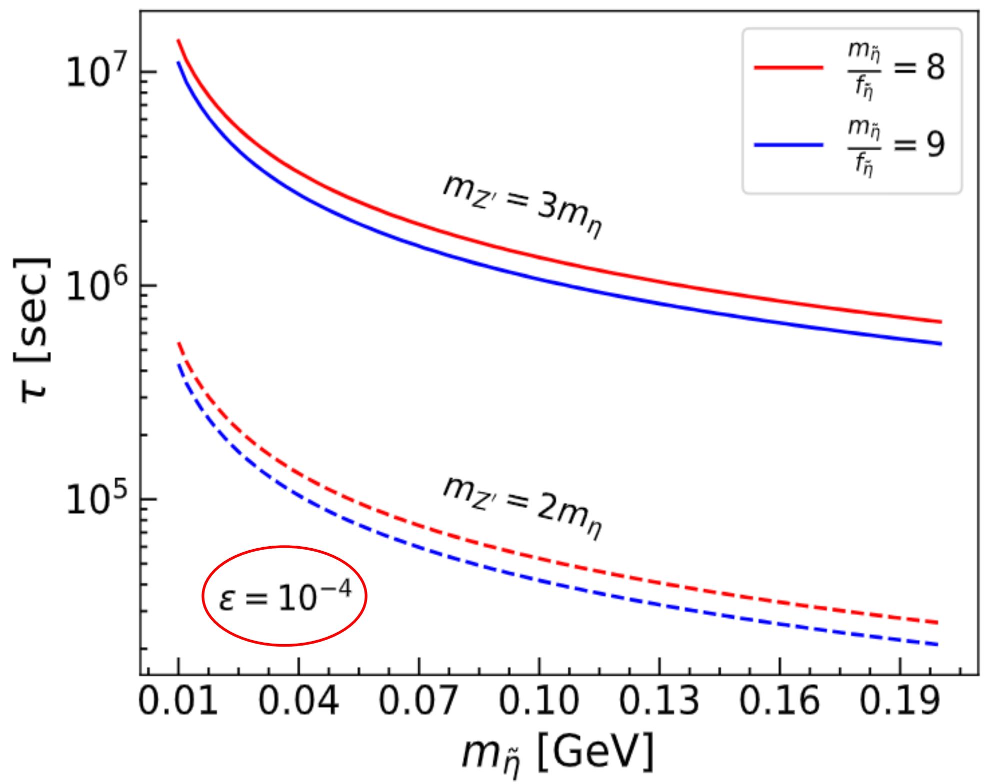
$$\Rightarrow \frac{\sigma_{2\to2}}{m_{\pi_D}} < 2\left[\frac{\mathrm{cm}^2}{\mathrm{g}}\right]$$

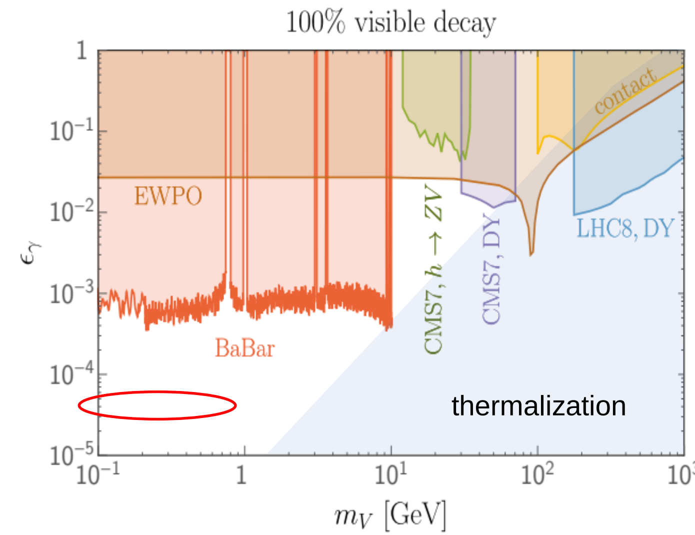
Pion mass :

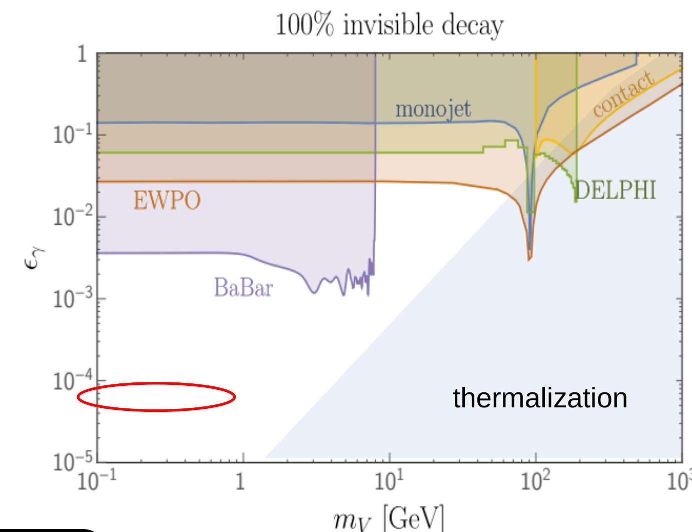
$$m_{\pi_D} \approx 200\mathrm{MeV}$$

Estimates of η'_D lifetime

[Hochberg et al.: ArXiv: 1512.07917]







On the timescale of freeze-out η'_D is long lived.

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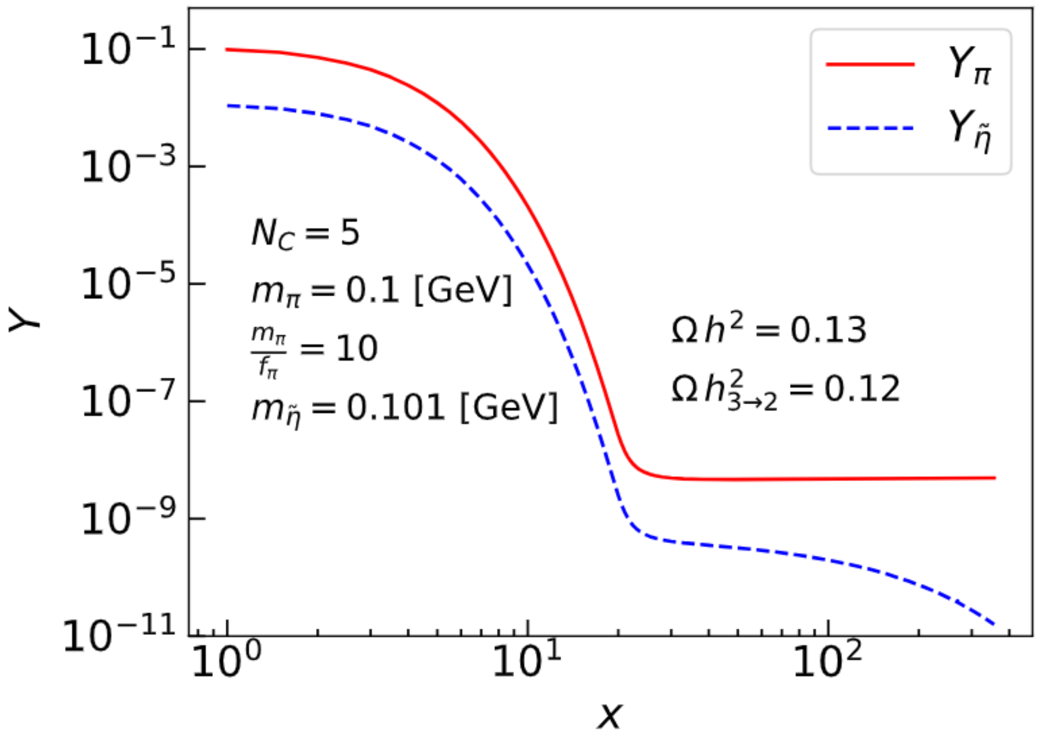
Effects of light η'_D on the relic density estimate

Relic abundance is slightly over estimated:

- η'_D is relatively long lived.
- Down-scattering $2\eta'_D \to 2\pi^0_D$.



Max overestimation is about 8% .

Over estimation effects vanish quickly with increasing $\Delta m^2_{\eta'_D}$.

Effects of light η'_D on the relic density estimate

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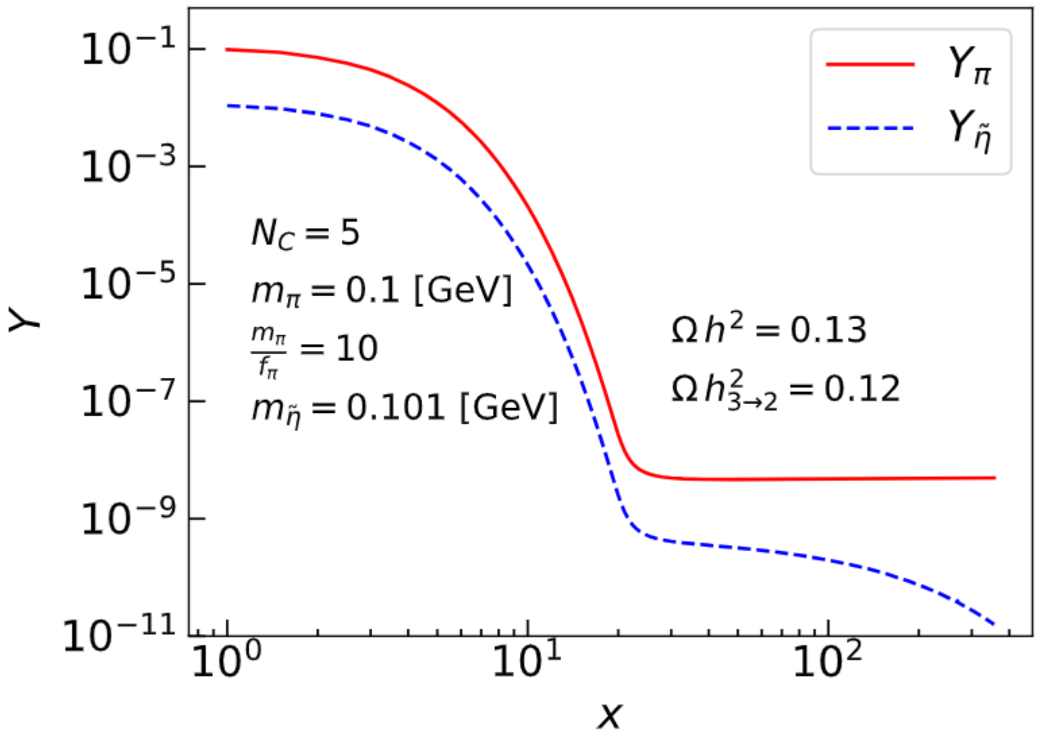
- η'_D is relatively long lived.
- Down-scattering $2\eta'_D \to 2\pi^0_D$.

Lower values of f_{π_D} possible.



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Dark vector mesons

Why include them ?

- Important if $m_\rho < 2m_{\pi_D}$

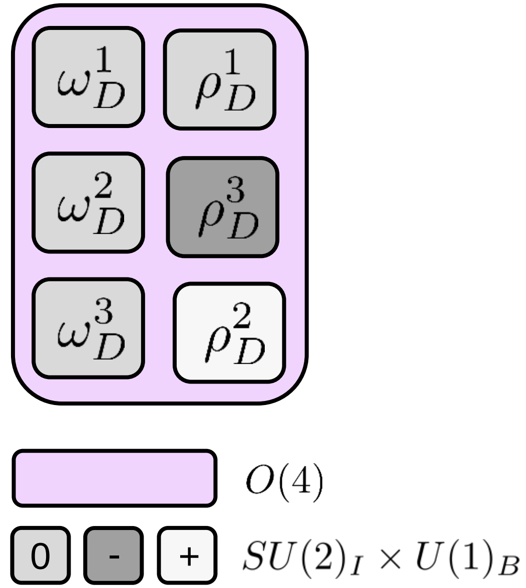
  [Asher Berlin, Nikita Blinov et al.: ArXiv: 801.05805v2]

- Improvement of „perturbativity“

  [Choi, Lee, Ko, Natale : ArXiv: 1801.07726]

- Collider signals

  [Hochberg et al.: ArXiv: 1512.07917]



How to include in EFT ?

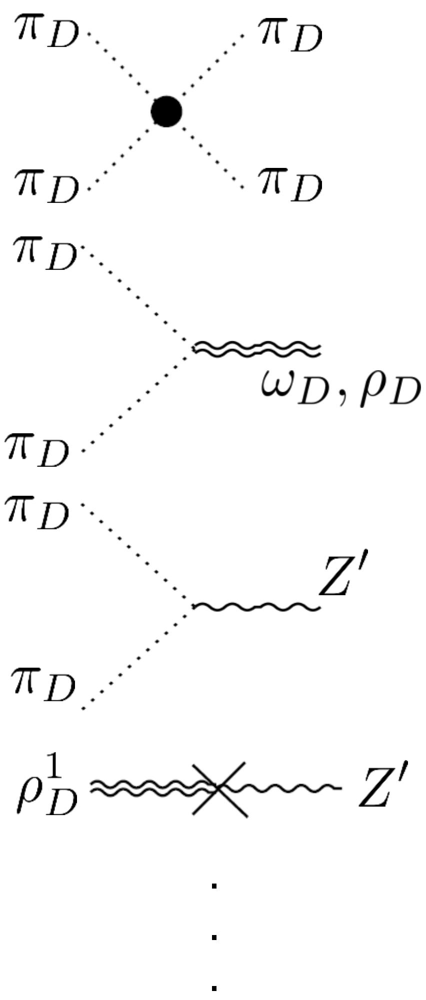
- Hidden local symmetry (HLS)

  [Bando et al.: ArXiv: 1504.07263]

Idea: Introdce vectors mesons as gauge bosons of auxiliary symmetry and then break it to obtain masses.

[Bando et al.: ArXiv: 1504.07263]

Non – anomalous EFT Part



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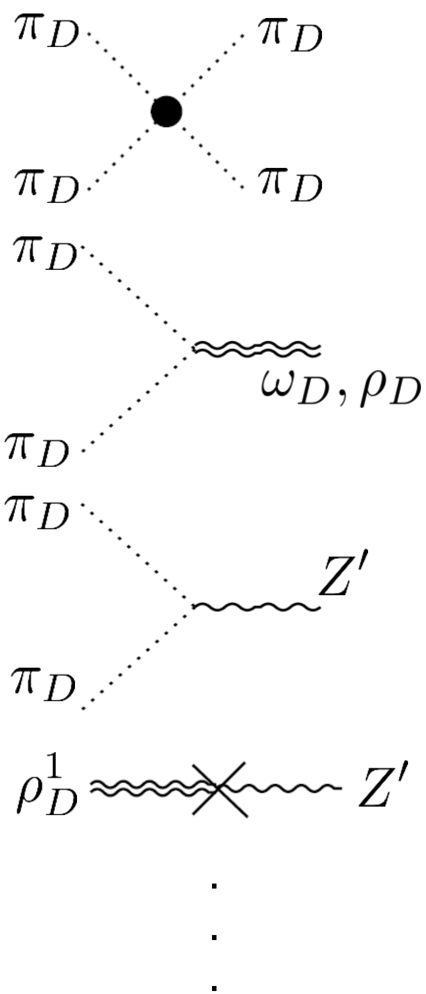

EFT parameter:

f_π, m_π	$f_\eta, \Delta m_\eta$
$e_D, m_{Z'}$	
ϵ	

independent

LEC

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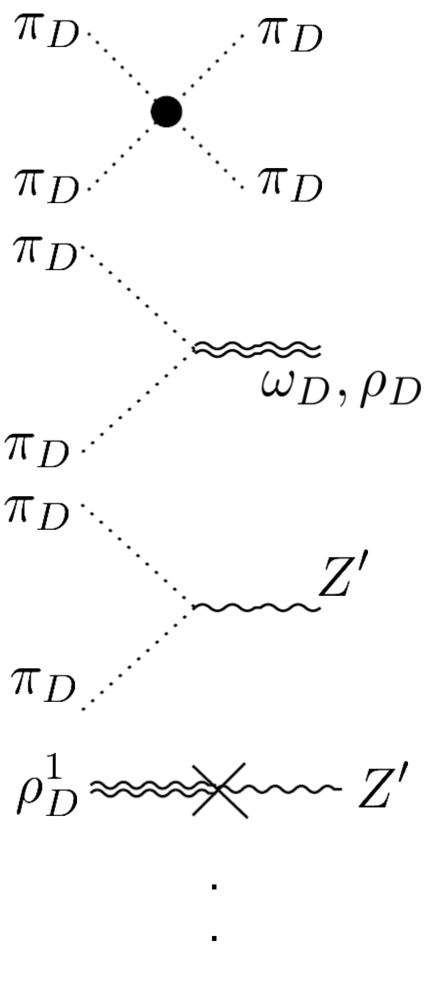
Non – anomalous EFT Part



$$\propto \frac{4-3a}{24 f_\pi^2}$$

$$\propto \frac{a g_V}{2}$$

$$\propto e_D \frac{2-a}{2}$$

$$\propto \frac{e_D}{g_V}$$

EFT parameter:

f_π, m_π	$f_\eta, \Delta m_\eta$
$e_D, m_{Z'}$	
ϵ	
	$g_V, \; a$

independent

LEC

$g_V \ldots$ HLS gauge coupling

$a \ldots$ HLS parameter

Estimates for the HLS parameters

Vector meson mass:

$$\boxed{m_V^2 = a f_\pi^2 g_V^2}$$

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$$\frac{m_V^2}{g_{\pi VV}} = 2 f_\pi^2 g_V$$

Accessible via lattice !

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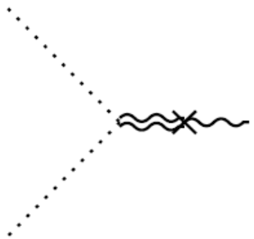
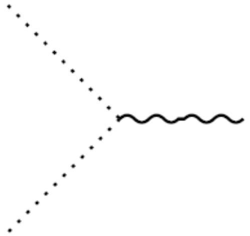
Vector meson dominance of dark π form factor:



$$\propto g_{\pi VV} \frac{e_D^2}{g_V^2}$$



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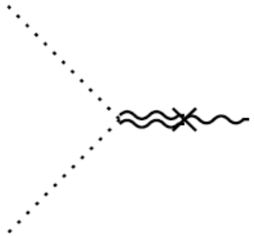

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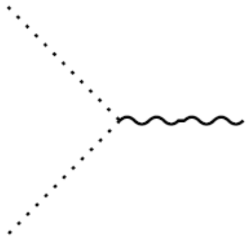

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Idea: Pions interacts with vector mesons which then mixes into a dark photon.

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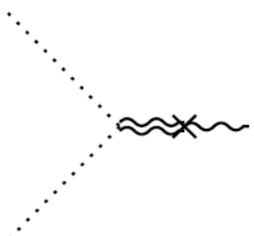
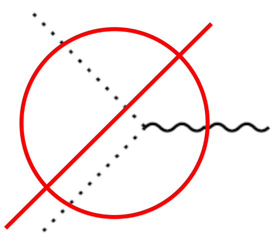


$$\propto g_{\pi VV} \frac{e_D^2}{g_V^2}$$



$$\propto e_D \frac{2-a}{2}$$

$$\boxed{\Rightarrow a = 2}$$

Idea: Pions interacts with vector mesons which then mixes into a dark photon.

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Anomalous EFT part

[Fujiwara, Kugo, Terao, Uehara ; (1985), Progress of Theoretical Physics]
[Chu, Ho, Zumino: 1996, Nuclear Physics B]

WZW breaks naive parity

$$\pi_D \mapsto -\pi_D$$

+

WZW as solution of anomaly equation

$$\delta_\epsilon S_{\mathrm{WZW}}[\pi, A] = \mathcal{A}[\epsilon, A]$$

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Additional terms

$$\mathcal{L}^{NA}_{1-4}[\pi_D, \rho_D, \omega_D, Z']$$

They are non-anomalous

$$\delta_\epsilon \int \sum_i c_i \mathcal{L}^{\mathrm{NA}}_i = 0$$

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They are non-anomalous

$$\delta_\epsilon \int \sum_i c_i \mathcal{L}_i^{\mathrm{NA}} = 0$$

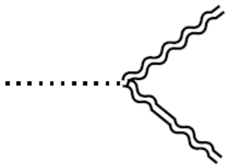
General solution to anomaly equation

$$\tilde{S}_{\mathrm{WZW}}[\pi_D, Z', \rho_D, \omega_D] = S_{\mathrm{WZW}}[\pi_D, Z'] + \int \mathrm{d}^4x \; c_1\mathcal{L}_1 + c_2\mathcal{L}_2 + c_3\mathcal{L}_3 + c_4\mathcal{L}_4$$

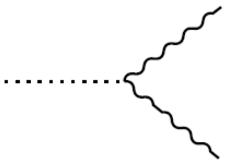
How to fix the new parameters ?

[Harada, Yamawaki; (2003); Arxiv:0302103]

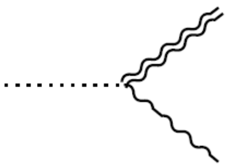
Complete vector meson dominance:



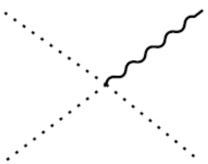
$$\propto \quad c_3$$



$$\propto \quad 1 - c_4$$



$$\propto \quad c_4 - c_3$$



$$\propto \quad c_1 - c_4 + \frac{4}{3}$$

$$\propto \quad -c_1 - c_3$$

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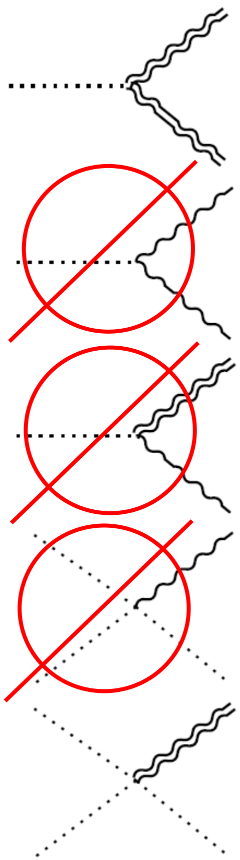

$$\propto \quad c_3$$

$$\propto \quad 1 - c_4$$

$$\propto \quad c_4 - c_3$$

$$\propto \quad c_1 - c_4 + \frac{4}{3}$$

$$\propto \quad -c_1 - c_3$$

$$\Rightarrow \quad \boxed{\begin{aligned} c_1 &= -\frac{1}{3} \\ c_3 &= 1 \\ c_4 &= 1 \end{aligned}}$$

Idea: Pions interacts with vector mesons which then mixes into a dark photon.

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Summary

These dark sector theories are **elegant but involved** !

- Discussed low energy effective theory of composite dark matter, including more states than just dark pions e.g. $\pi_D, \rho_D, \omega_D, \eta'_D$
  - Generalized solution of anomaly equation and derivation of WZW.
- Discussed various tools to gain intuition about the additional parameters introduced e.g. vector meson dominance and large N.
- First phenomenological results:
  - Estimate of lifetime of η'_D.
  - Relevance of η'_D for DM parameter estimation e.g. overestimation of relic abbundance.

Ready for your

Questions

The axial anomaly and discrete symmetries

General form of Axial Anomaly

$$\mathcal{A}_{\mathrm{Axial}}[\epsilon, A] \; = \; -2i \; T_{\mathcal{R}} \, \mathrm{tr}\{\epsilon\} \mathcal{Q}_{\mathrm{Topo}}[A]$$

Quantum chiral transfomrmations

$$\begin{array}{ccccc} U(4) \ni U = \exp(-\epsilon) & \longrightarrow & D\psi D\psi & \xrightarrow{U} & e^{-i\mathcal{A}[\epsilon,A]} \, D\psi D\psi \\ \downarrow & & & & \downarrow \\ Z_{2T_{\mathcal{R}}} \ltimes SU(4) & \longleftarrow & \det(U) = \exp\left(-i\frac{\pi k}{T_R}\right) & \Leftrightarrow & \exp(-i\mathcal{A}[\epsilon,A]) = 1 \end{array}$$

$k \in \{0, \ldots, 2T_R - 1\}$

Dynkin Index $T_{\mathcal{R}}$

SO(N) - Vec.	Sp(2N) - Fund	Sp(2N) - AT2T
$T_R = 1$	$T_R = 1/2$	$T_R = N - 1$

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't Hooft large N considerations of η'_D

Idea: Compare for example $SO(N)$-vector theories for N very large.

Technicality: Define 't Hooft coupling λ

$$\lambda \;:=\; C_{adj}(N)\, g^2 \qquad\qquad \lambda(\mu_{UV}) \;=\; \text{fixed}$$

$\rightarrow$ Running of λ is independent of N up to $1/N$ corrections.
$\rightarrow$ A controlable perturbative scale $1/N$ is introduced into the theory.

Axial anomaly in the chiral limit:

$$\partial_\mu J^\mu_{\eta'_D} \;=\; -\; \frac{T(R)}{C_{\mathrm{adj}}} \; \frac{\lambda N_F}{32\pi^2}\; \epsilon^{\mu\nu\rho\sigma}\; G^{\alpha}_{\mu\nu}\, G_{\rho\sigma\;\beta}$$

Gives potential large N suppression

$\frac{T(R)}{C_{\mathrm{adj}}} \xrightarrow[N\to\infty]{!} 0$ must hold for the anomaly to vanish in large N limit

Example:
$SU(N)$-Fund.

$$\lambda \;:=\; N\, g^2$$

$$g^2 \xrightarrow[N\to\infty]{} 0$$

$$\frac{T(R)}{C_{\mathrm{adj}}} \;=\; \frac{1}{2N}$$

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4th Homotopy group of SU(4)/SO(4)

	π_3	π_4	π_5
$SO(4)$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$SU(4)$	$\mathbb{Z}$	0	$\mathbb{Z}$

Fibration:

$$SO(4) \to SU(4) \to SU(4)/SO(4)$$

Long exact sequence:

$$\begin{array}{ccccccc}
\pi_4(SU(4)) & \xrightarrow{h_1} & \pi_4(SU(4)/SO(4)) & \xrightarrow{h_2} & \pi_3(SO(4)) & \xrightarrow{h_3} & \pi_3(SU(4)) \\
0 & \xrightarrow{h_1} & ? & \xrightarrow{h_2} & \mathbb{Z} \oplus \mathbb{Z} & \xrightarrow{h_3} & \mathbb{Z}
\end{array}$$

- $Ker(h_2) = Img(h_1) = 0 \to h_2$ is injective
- $\pi_4(SU(4)/SO(4)) \cong Img(h_2) = Ker(h_3)$
- $Ker(h_3) \neq 0$

$\Rightarrow$ $\pi_4(SU(4)/SO(4))$ cannot be trivial

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WZW-term as solution to t'Hooft anomaly equation

[Wess, Zumino: 1971, Physics Letters B]
[Witten: 1983, Nuclear Physics B]
[Chu, Ho, Zumino: 1996, Nuclear Physics B]

Step 1: Calculate chiral anomaly in the UV

$$\mathrm{Tr}\left\{F^3\right\} \xrightarrow{\ \mathbf{P}\ } \begin{array}{c}\text{Stora-Zumino}\\ \text{descent equations}\end{array} \xrightarrow{\ \Delta\text{- diagrams}\ } \begin{array}{c}\text{Consistent anomaly}\\ \mathcal{A}[\mathfrak{g},\ A] \neq\ 0\end{array} \xrightarrow{\ \text{Bardeen counter terms}\ } \begin{array}{c}\text{Chiral anomaly}\\ \mathcal{A}[\mathfrak{h},\ A] = 0\end{array}$$

Step 2: t' Hooft anomaly matching

Anomaly equation: $\delta_\epsilon S^{IR}_{\mathrm{cov.}}[\xi, A] \ = \ \mathcal{A}[\epsilon,\ A]$

Step 3: Solve anomaly equation in the IR

$$S^{IR}_{\mathrm{cov.}}[\xi, A] \ = \int_0^1 d\tau \int \mathcal{A}[\xi,\ A_\tau(\xi)] \ \text{ with } A_\tau(\xi) = \exp\left(-\tau \int dy\ \xi^a \mathcal{D}_a\right) A$$

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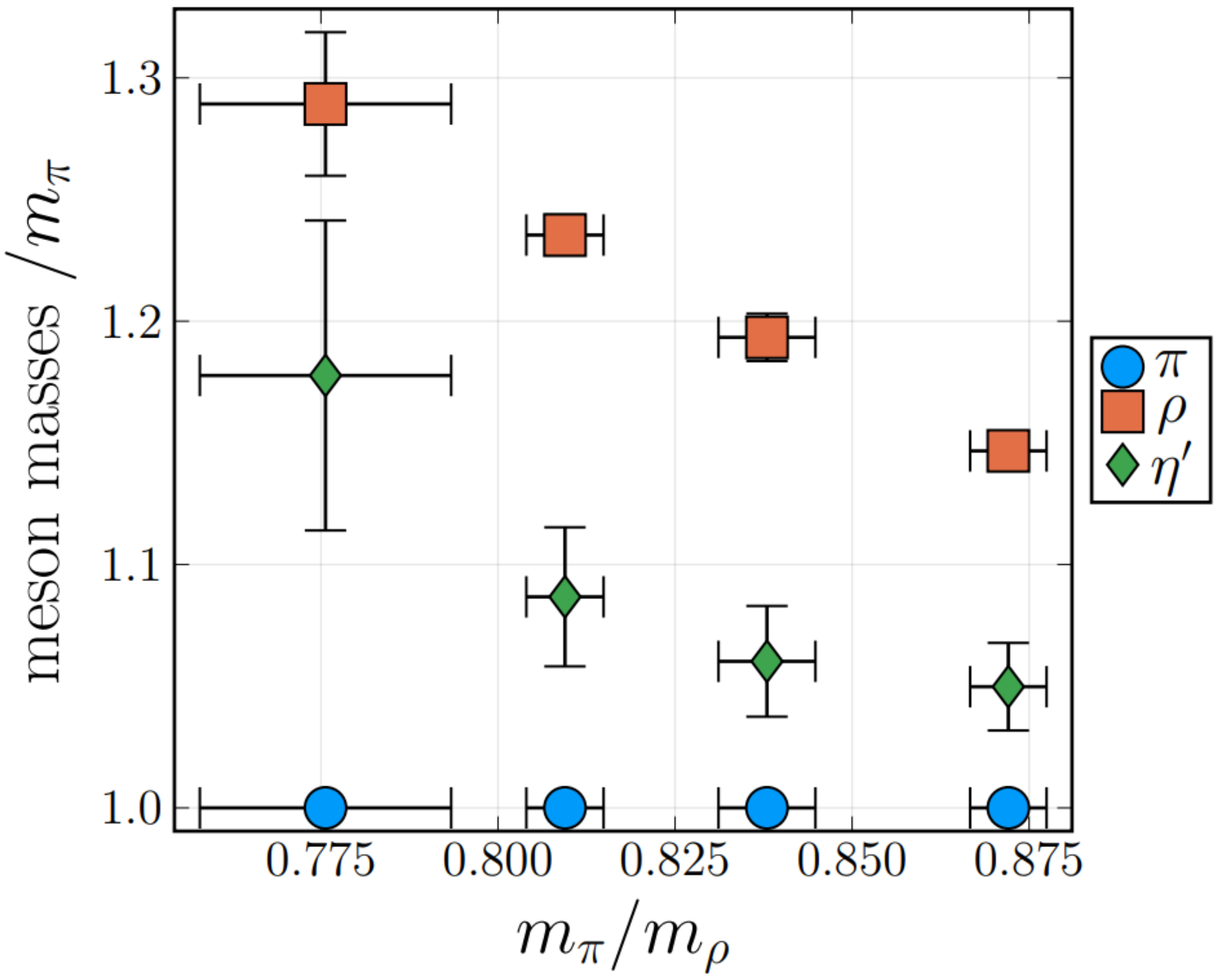
Low-energy spectrum for Sp(4)-Fund. dark matter

[Zierler, Lee, Maas, Pressler: ArXiv: 2210.11187]



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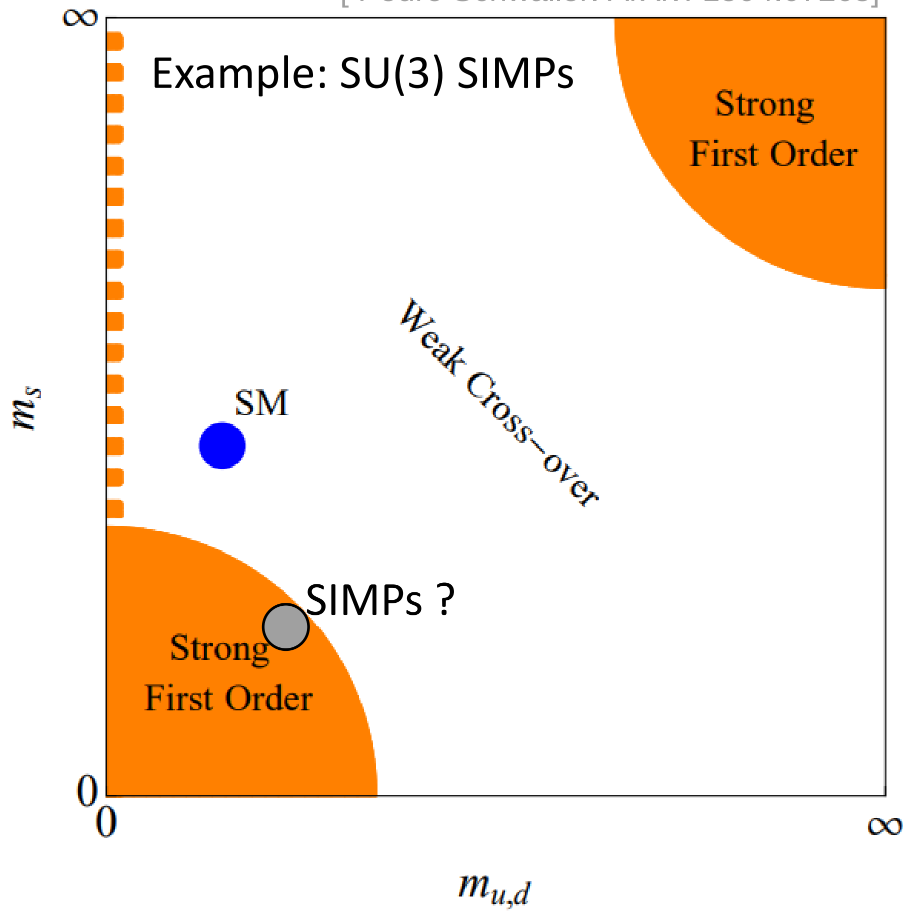
Outlook: Gravitational waves from 1st order phase transition in hidden dark sectors

.

First order phase transitions in dark sectos migh be viable.

- Complementary signal to direct searches
- The only signal in case of hidden dark sectors

[Pedro Schwaller. ArXiv: 1504.07263]



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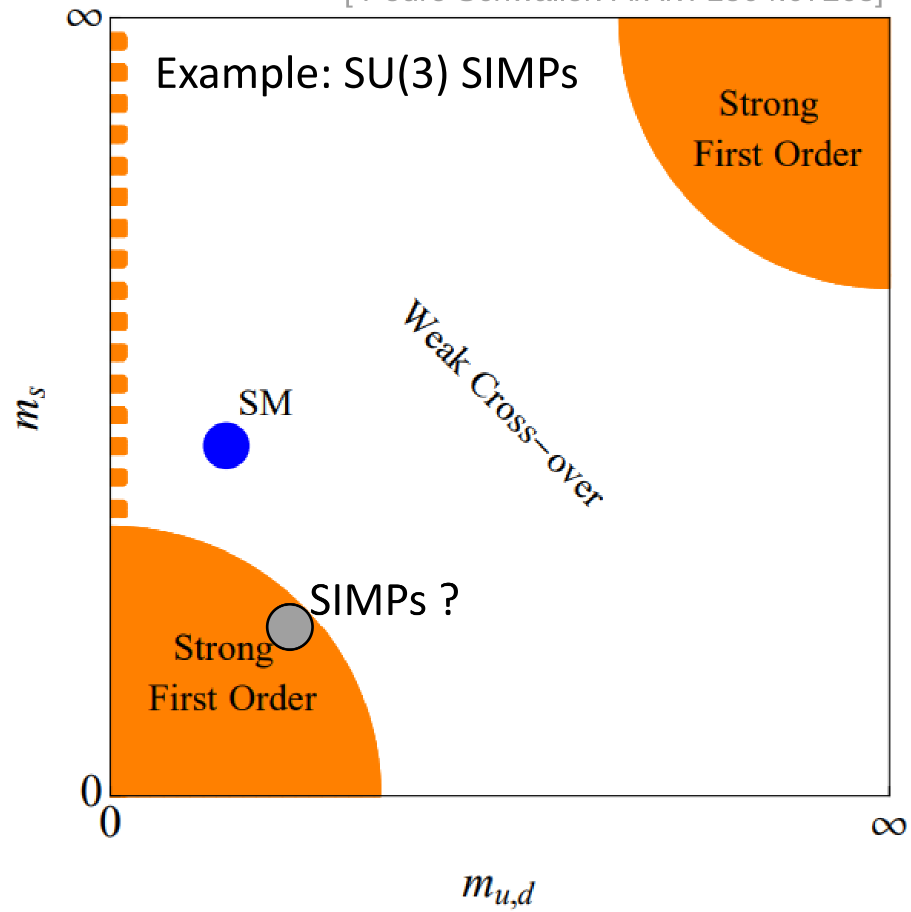
Its not yet clear if such phase transitions can be realized in SIMPs.

[Pedro Schwaller. ArXiv: 1504.07263]



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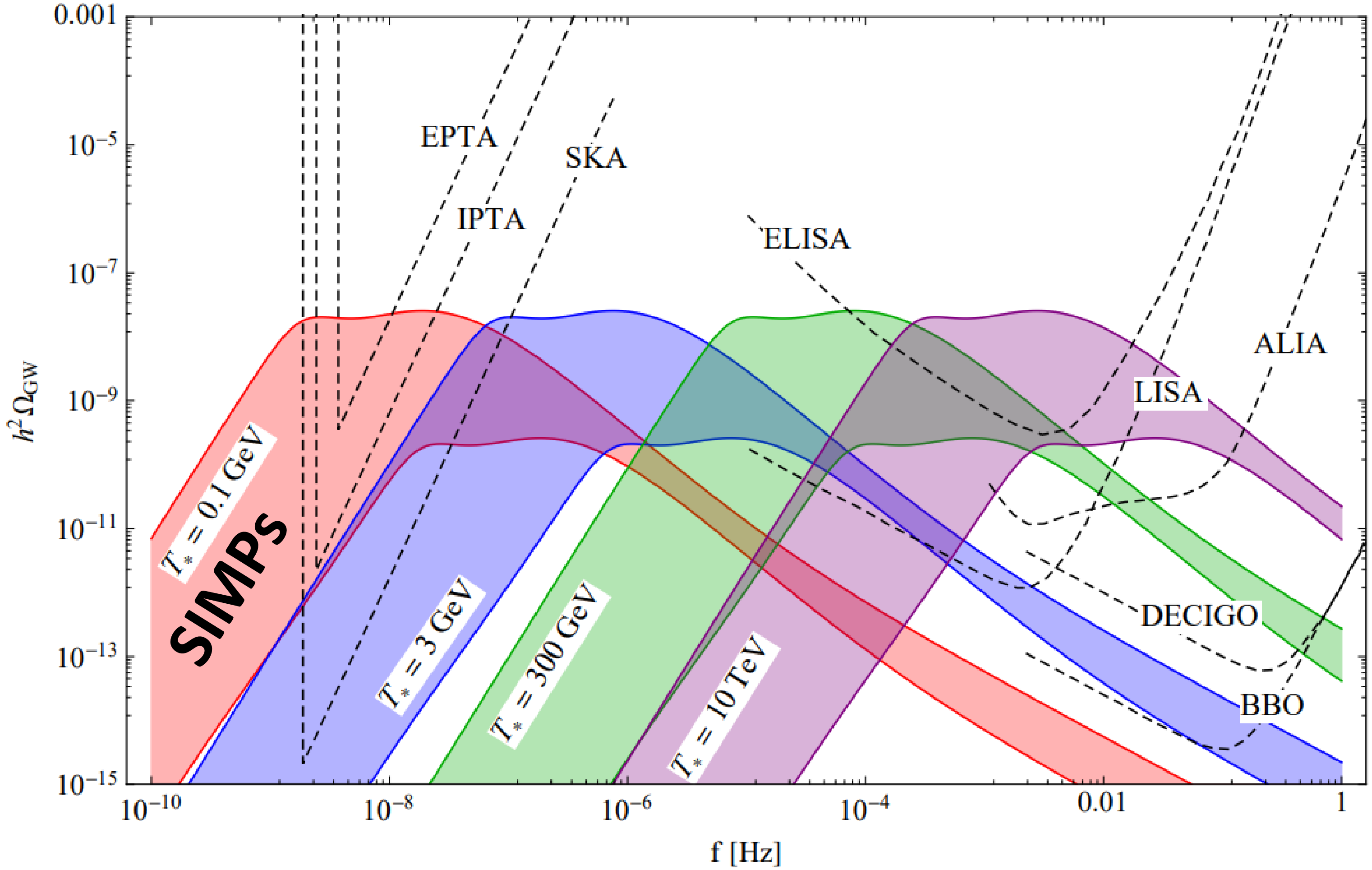
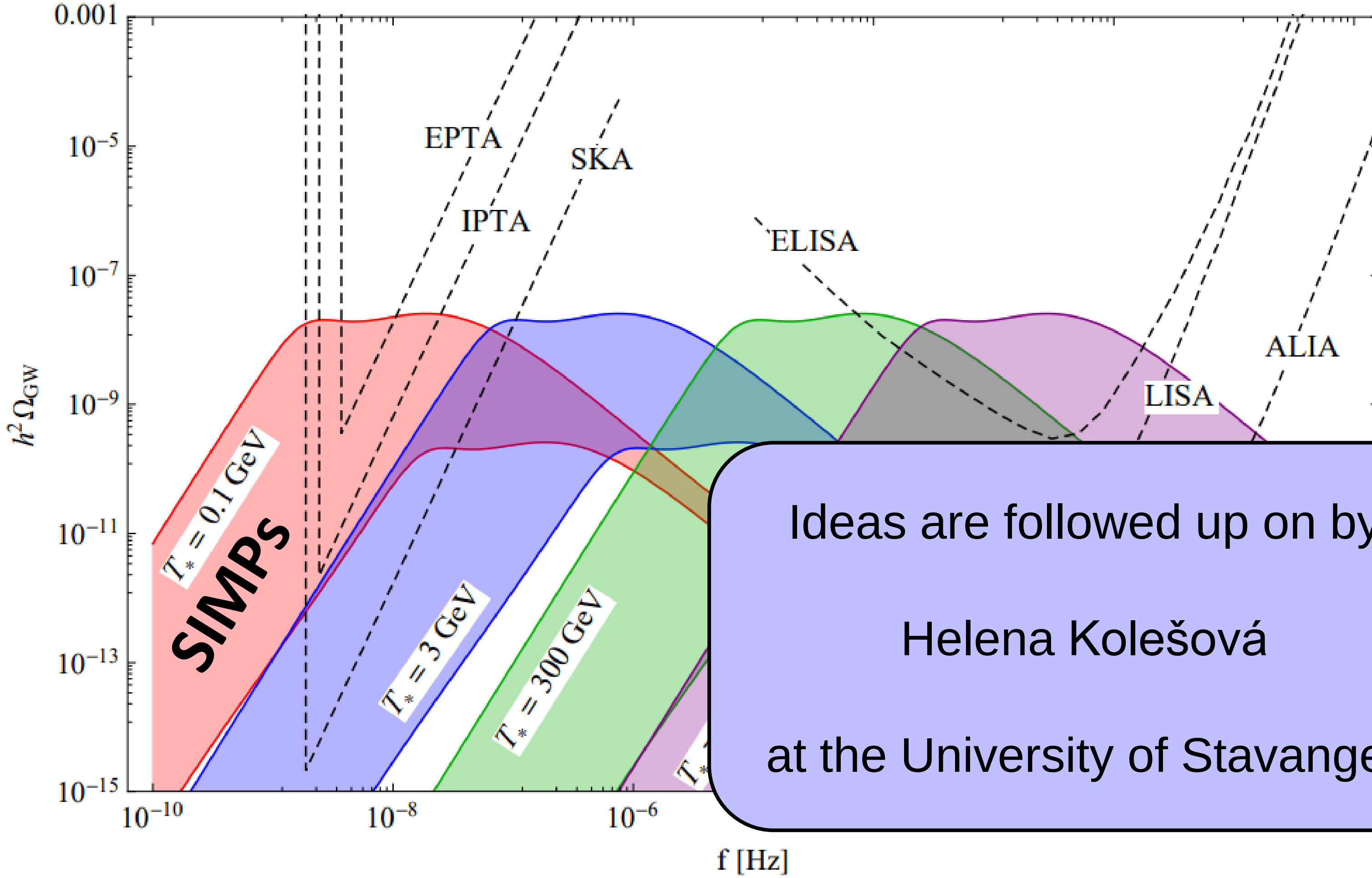


Gravitational waves from various dark phase transitions

Outlook: Gravitational waves from 1st order phase transition in hidden dark sectors

[Pedro Schwaller. ArXiv: 1504.07263]



Ideas are followed up on by

Helena Kolešová

at the University of Stavanger

Gravitational waves from various dark phase transitions

Outlook: Gravitational waves from domain wall collapse in hidden dark sectors

- For non-fundamental fermion representations $\mathcal{R}$ of G_D additional discrete „chiral“ symmetries pop up.
- Those are spontanously broken by the chiral condensate
  $\Rightarrow$ **Cosmic domain walls. (Excluded - overclosing of universe)**
- They are only approximate due to the mass-term
  $\Rightarrow$ **Domain walls collapse and produce GW signal.**

[Ken'ichi Saikawa. ArXiv: 1703.02576]

G_D	$\mathcal{R}$	Breaking Pattern	
$Sp(2N_C)$	2 Index antisym.	$\mathbb{Z}_{2N_C-2} \to \mathbb{Z}_2$	$N_C > 2$
$Sp(2N_C)$	Adjoint	$\mathbb{Z}_{2N_C+2} \to \mathbb{Z}_2$	$N_C > 1$
$SO(N_C)$	2 Index sym.	$\mathbb{Z}_{N_C+2} \to \mathbb{Z}_2$	$N_C > 5$

Might be used to exclude or constraint these models!

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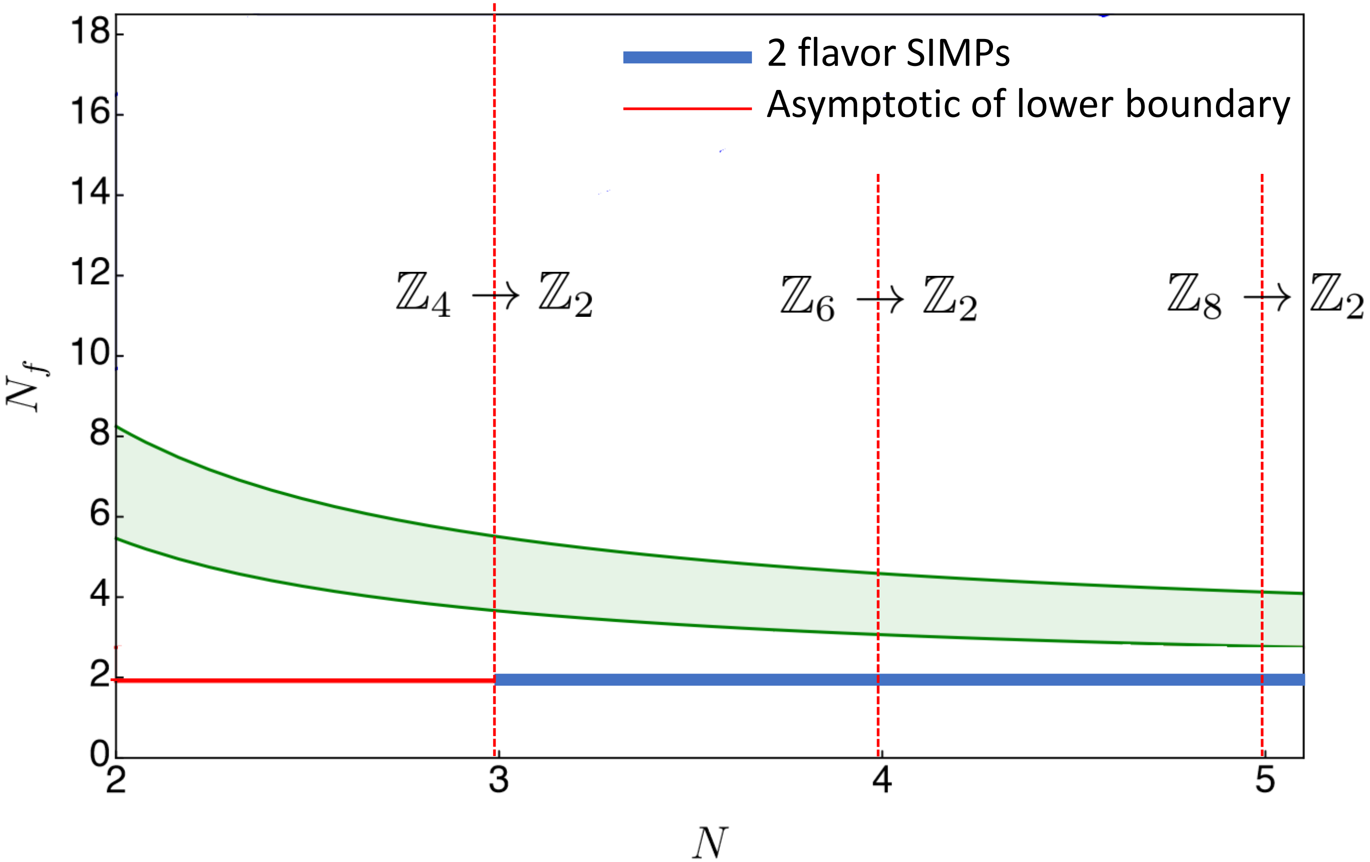
Outlook: Gravitational waves from domain wall collapse in hidden dark sectors

Conformal window for N_f Dirac fermions in $Sp(2N)$ -2 index antisymmetric representation. [Jong-Wan Lee, ArXiv: 2008.12223]



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