Low-energy effective description of pseudo-scalar mesons in SO(N)-like dark QCD

Presented by: Joachim Pomper
Co-author: Dr. Suchita Kulkarni

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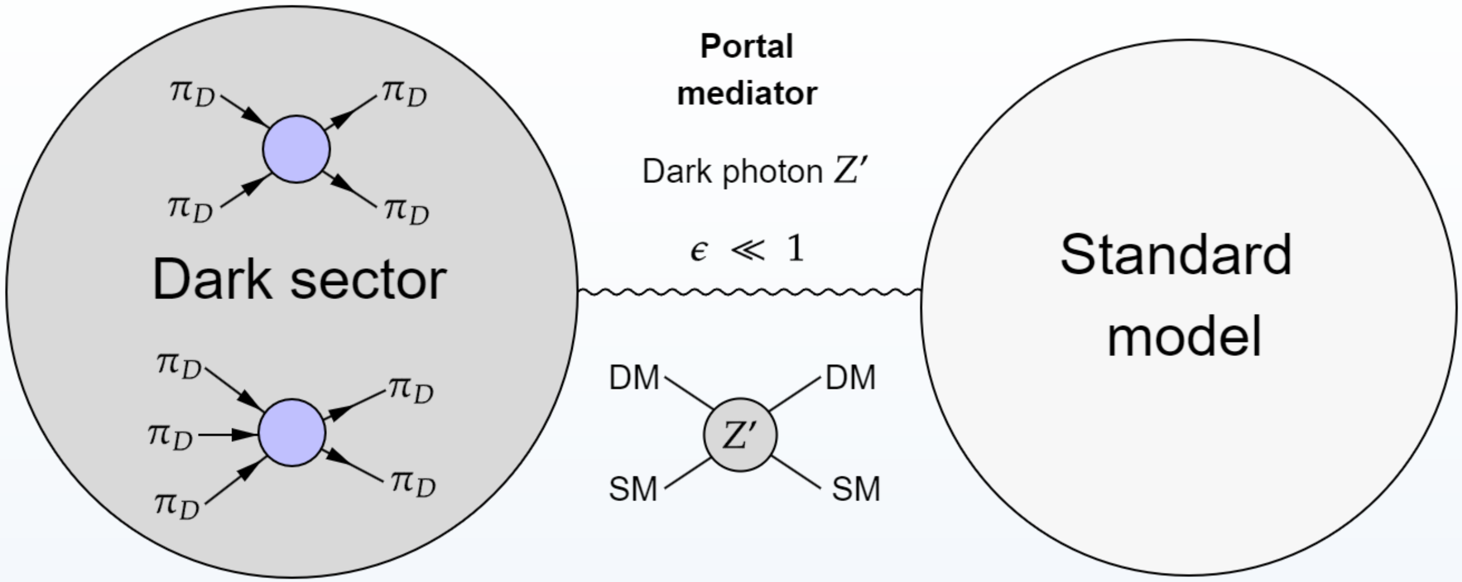
Der Wissenschaftsfonds.

SIMPs - **S**trongly **I**nteracting **M**assive **P**articles

[Hochberg et al: ArXiv: 1402.5143]



- Self-interactions resolve small scale structure formation problems.
- 3→2 cannibalization sets correct DM relic abundance.
- Dark photon Z' mediator maintains thermal equilibrium.

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SIMPs from $SO(N_C)$-like dark QCD

Strong dark sector:

- Non-abelian gauge group G_D
- N_F Dirac fermions in **real** representation $\mathcal{R}$.

Prototypical model:

$$(G_D = SO(N_C), \mathcal{R} = \boldsymbol{N_C}, N_F = 2)$$

Original motivation:

$Sp(4)$ **+** Antisymmetric fermions

$\cong$

$SO(5)$ **+** Vector fermions

(N_C, N_F) below the conformal window.

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Mediator sector: Dark photon Z'

- Gauge boson of abelian gauge-group $U(1)_D$.
- Abelian Higgs mechanism gives mass $m_{Z'}$ for Z'.
- Kinetic mixing with SM Hypercharge via $\frac{\epsilon}{2\cos(\theta_W)} Z'_{\mu\nu} B^{\mu\nu}$

Symmetries and lightest stable particles

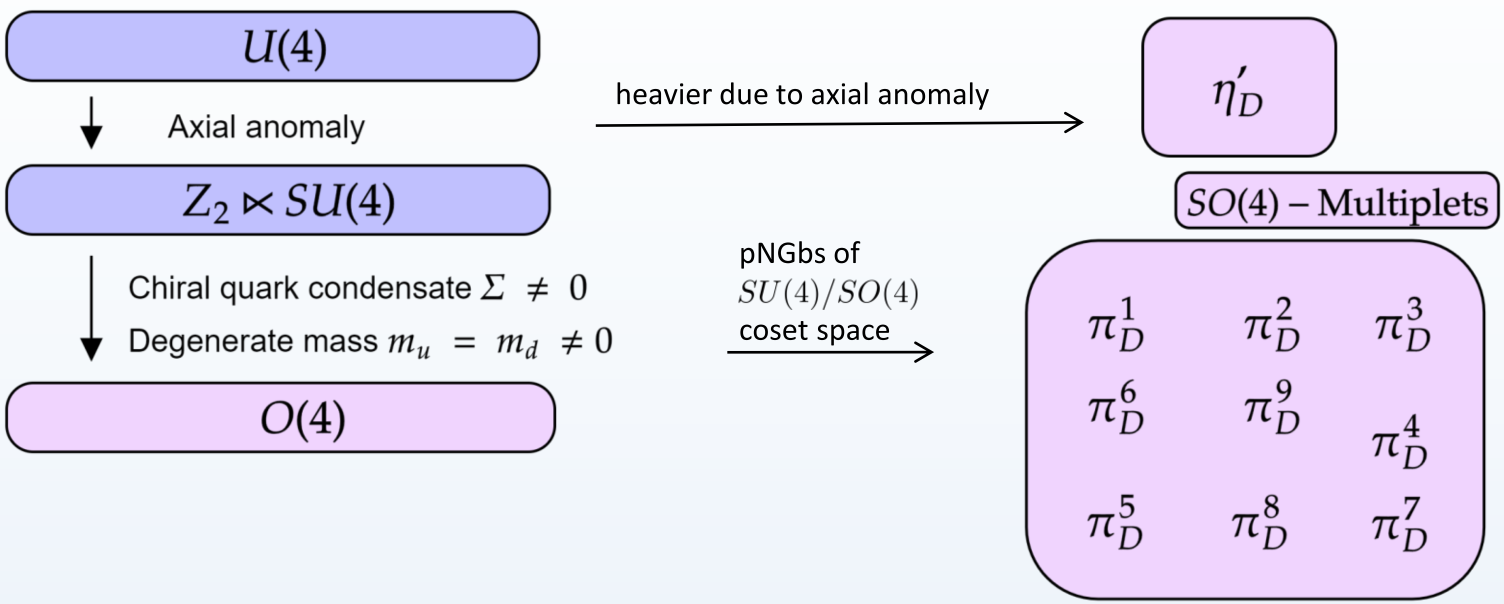
Breaking pattern

$$
\begin{array}{c}
U(4) \\
\Big\downarrow \ \text{Axial anomaly} \\
Z_2 \ltimes SU(4) \\
\Big\downarrow \ \begin{array}{l}\text{Chiral quark condensate } \Sigma \neq 0 \\ \text{Degenerate mass } m_u = m_d \neq 0\end{array} \\
O(4)
\end{array}
$$

Symmetries and lightest stable particles

Breaking pattern

Pseudo-scalar particles

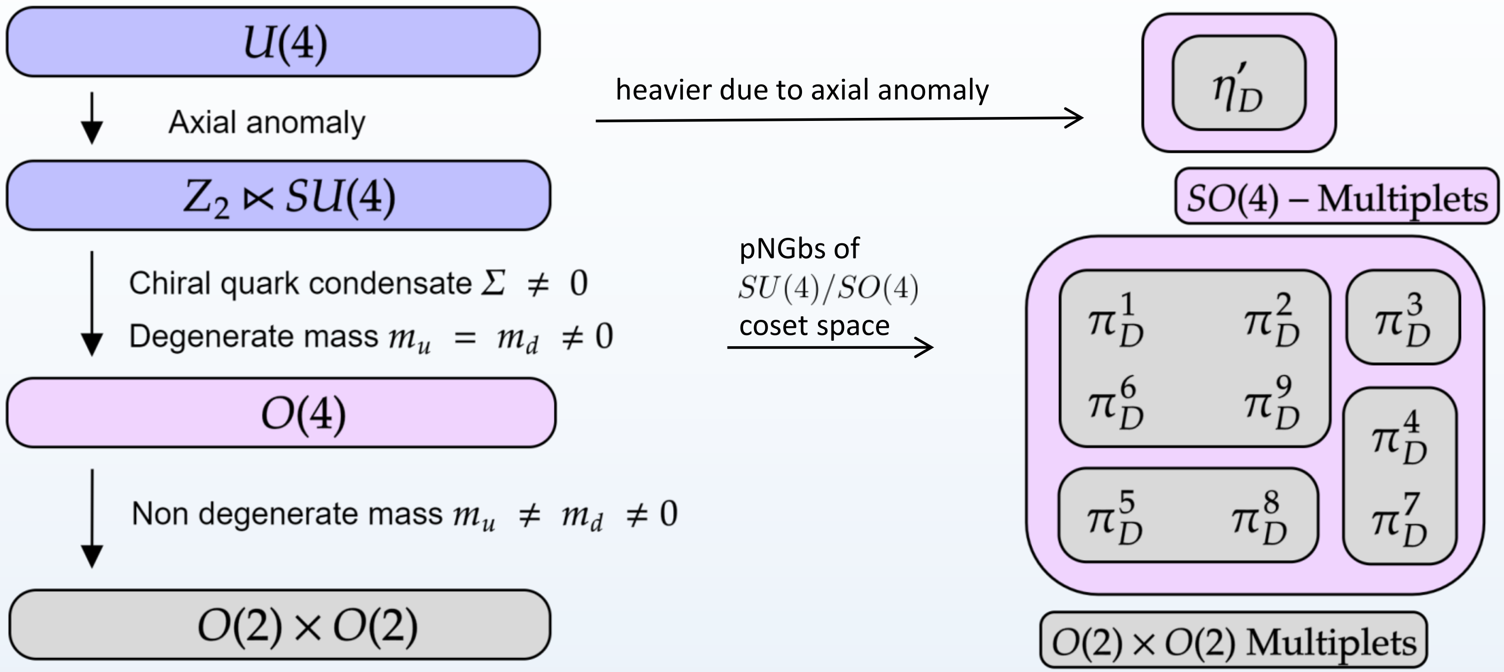


$$\dim SU(4)/SO(4) = 9$$

Symmetries and lightest stable particles

Breaking pattern

Pseudo-scalar particles



Light flavor singlets are not desirable since they may **decay in the presence of a mediator** to the SM !

Can η'_D be close in mass to the π_D ?

A large N_C argument analog to real world QCD :

$$\partial_\mu j^\mu_{\eta'_D} \propto \frac{T_\mathcal{R}}{C_\mathcal{R}} \tilde{F}^{\mu\nu} F_{\mu\nu} \xrightarrow[N_C \to \infty]{!} 0$$

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Gives a sufficient criterion :

$$\frac{T_\mathcal{R}}{C_\mathcal{R}} \xrightarrow[N_C \to \infty]{} 0$$

- Only satisfied for fermions in the fundamental or vector representation.

  $\Rightarrow \eta'_D$ becomes light in large N_C limit.

- Not satisfied for example for higher tensor or adjoint representations.

  $\Rightarrow \eta'_D$ expected to remain heavy.

Low energy effective Lagrangian

Chiral coset representative:

$$\Sigma = \exp\left(i\xi^a T_a\right) \qquad \xi^a = \begin{cases} \eta'_D/f_{\eta'_D} & \text{if } a = 0 \\ \pi_D/f_{\pi_D} & \text{else} \end{cases}$$

Low energy effective Lagrangian

$$\mathcal{L}_{\mathrm{IR}} = \frac{f_\pi^2}{4}\mathrm{tr}\left\{\partial_\mu\Sigma^\dagger\partial^\mu\Sigma\right\} + \frac{f_\pi^2 - f_{\eta'_D}^2}{4}\mathrm{tr}\left\{\Sigma\partial_\mu\Sigma^\dagger\right\}\mathrm{tr}\left\{\Sigma^\dagger\partial^\mu\Sigma\right\}$$
$$+\frac{m_\pi^2 f_\pi^2}{4}\mathrm{tr}\left\{\omega\Sigma^\dagger + \omega^\dagger\Sigma\right\} + \frac{\Delta m_{\eta'_D}^2 f_{\eta'_D}^2}{4}\left(\ln\left(\det\left(\Sigma\right)\right)\right)^2$$

GMOR relation: $m_\pi^2 = \frac{m_q\langle\overline{q}q\rangle}{2f_\pi^2}$

Decay constants: $f_{\eta'_D} \xrightarrow[N_C\to\infty]{} f_\pi$

η'_D - mass: $m_{\eta'_D}^2 = m_\pi^2 + \frac{f_{\eta'_D}^2}{f_\pi^2}\Delta m_{\eta'_D}^2$

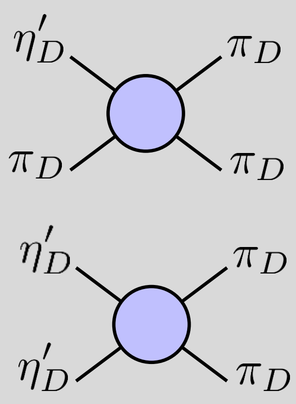
$\Delta m_{\eta'_D}^2 \xrightarrow[N_C\to\infty]{} 0$

Low energy effective Lagrangian

$$\mathcal{L}_{\mathrm{IR}} = \frac{f_\pi^2}{4}\mathrm{tr}\left\{\partial_\mu \Sigma^\dagger \partial^\mu \Sigma\right\} + \frac{f_\pi^2 - f_{\eta'_D}^2}{4}\mathrm{tr}\left\{\Sigma \partial_\mu \Sigma^\dagger\right\}\mathrm{tr}\left\{\Sigma^\dagger \partial^\mu \Sigma\right\}$$

$$\boxed{+\frac{m_\pi^2 f_\pi^2}{4}\mathrm{tr}\left\{\omega \Sigma^\dagger + \omega^\dagger \Sigma\right\}} + \frac{\Delta m_{\eta'_D}^2 f_{\eta'_D}^2}{4}\left(\ln\left(\det\left(\Sigma\right)\right)\right)^2$$

Contact terms:



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Anomalous action

[Wess, Zumino: 1971, Physics Letters B]
[Witten: 1983, Nuclear Physics B]
[Chu, Ho, Zumino: 1996, Nuclear Physics B]

Wess-Zumino-Witten term :

$$
\begin{aligned}
S_{\mathrm{WZW}} &= \frac{\Gamma_{\mathrm{WZW}}}{48\pi^2 f_\pi} \int_{S^4} \mathrm{d}^4x \int_0^1 \mathrm{d}\tau \ \mathrm{tr}\left\{\xi\left(\Sigma[\tau\xi]^{-1}\mathrm{d}\Sigma[\tau\xi]\right)^4\right\} \\
&\approx \frac{\Gamma_{\mathrm{WZW}}}{15\pi^2 f_\pi^5} \int_{S^4} \mathrm{tr}\left\{\pi_D \mathrm{d}\pi_D \wedge \mathrm{d}\pi_D \wedge \mathrm{d}\pi_D \wedge \mathrm{d}\pi_D\right\}
\end{aligned}
$$

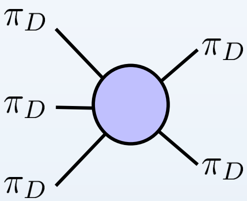
Non-standard form due to coset geometry:

$$\pi_4(SU(4)/SO(4)) \neq 0$$

Anomaly-matching:

$$\Gamma_{\mathrm{WZW}} = \dim\mathcal{R}$$

Five point vertex between π_D :



No participation of η'_D in 3→2 DM freeze-out.

$U(1)_D$ charge assignments

Charge assignment: $\mathcal{Q}$

Consistency:

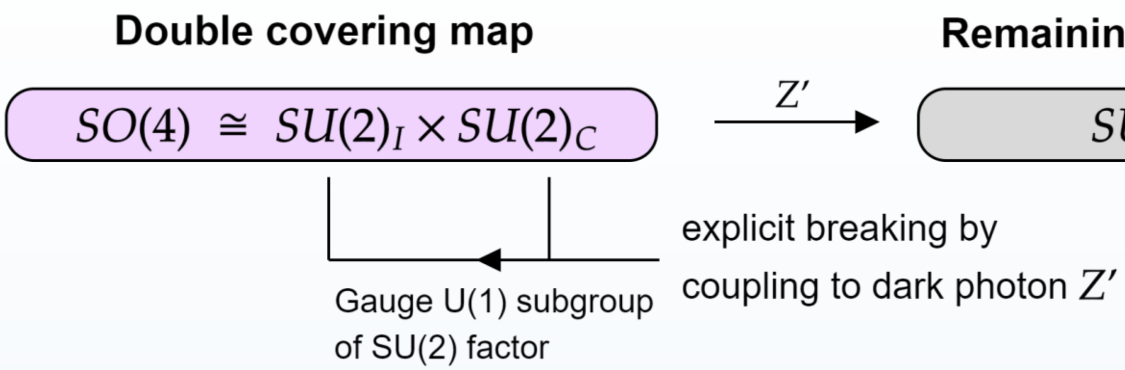
- No gauge anomalies

Pion stability:

- Maintain non-abelian global symmetry
- No anomalous π_D - decays occur

Charge assignment $\mathcal{Q}$ is physically unique!

$U(1)_D$ charge assignments



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$U(1)_D$ charge assignments

Double covering map

$SO(4) \cong SU(2)_I \times SU(2)_C \xrightarrow{Z'} SU(2)_I \times U(1)_C$

Remaining flavor symmetry

Gauge U(1) subgroup of SU(2) factor

explicit breaking by coupling to dark photon Z'

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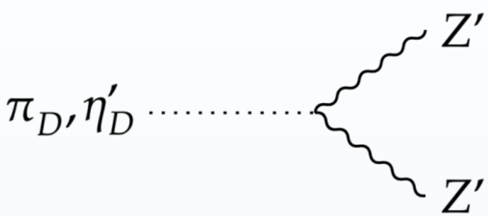
Charge assignment $\mathcal{Q}$ is physically unique!

$SU(2)_I \times U(1)_C$ Multiplets			Charges
	η'_D		0
π_D^1	π_D^2	π_D^3	0
π_D^4	π_D^5	π_D^6	$-$
π_D^7	π_D^8	π_D^9	$+$

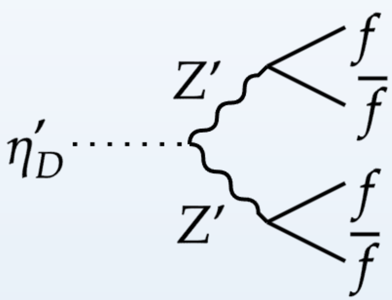
Anomalous η'_D decay

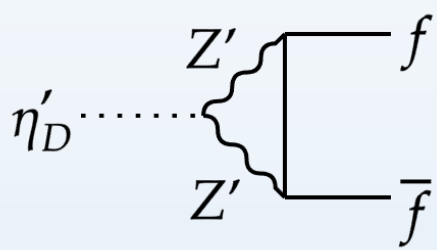
Gauged WZW term introduces anomalous vertices:



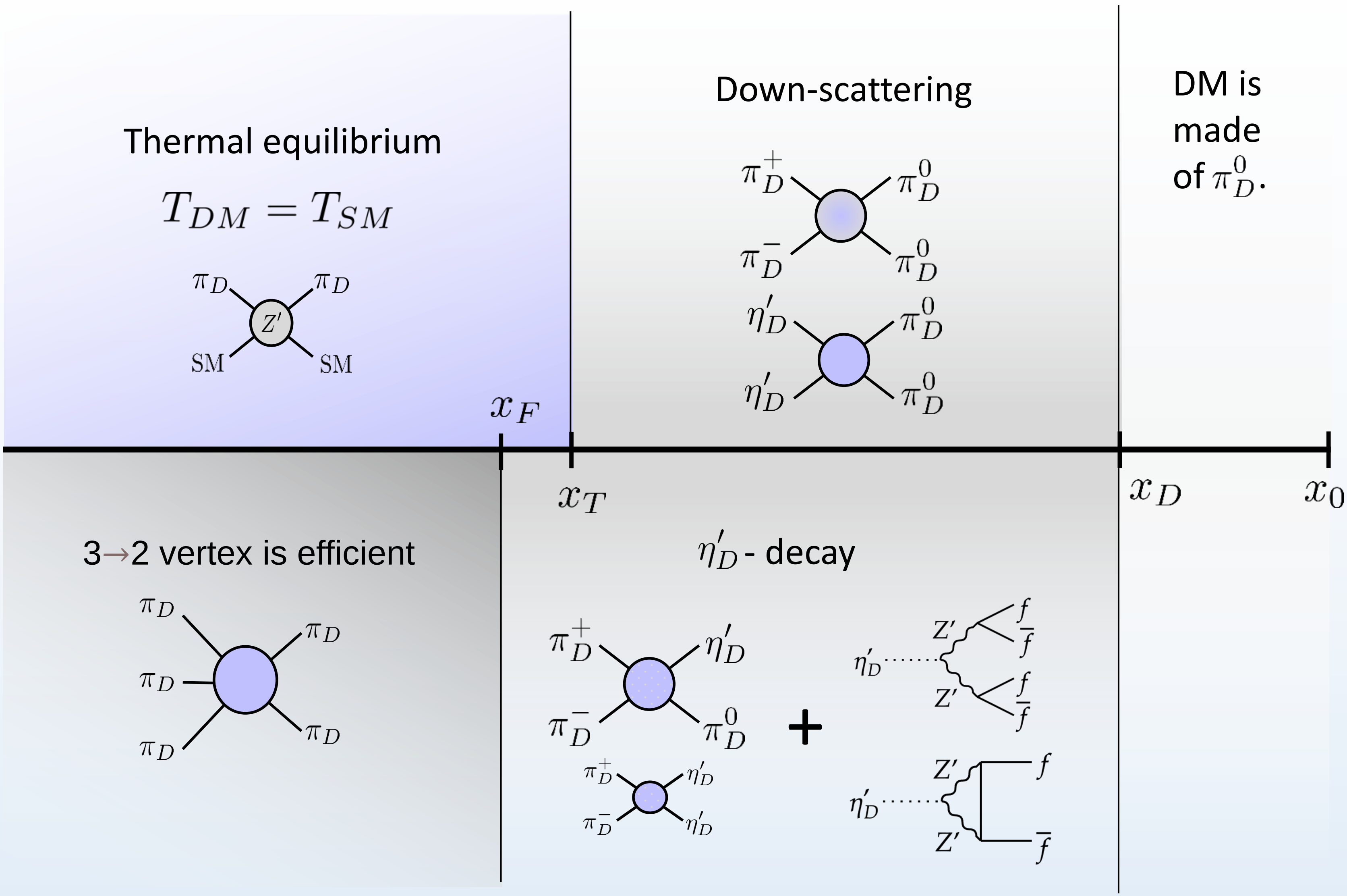
$$\propto \begin{cases} \mathrm{tr}\left\{\pi_D \mathcal{Q}^2\right\} = 0 \\ \mathrm{tr}\left\{\eta'_D \mathcal{Q}^2\right\} \neq 0 \end{cases}$$

Allows for decay of η'_D to SM :





Freeze-out timeline



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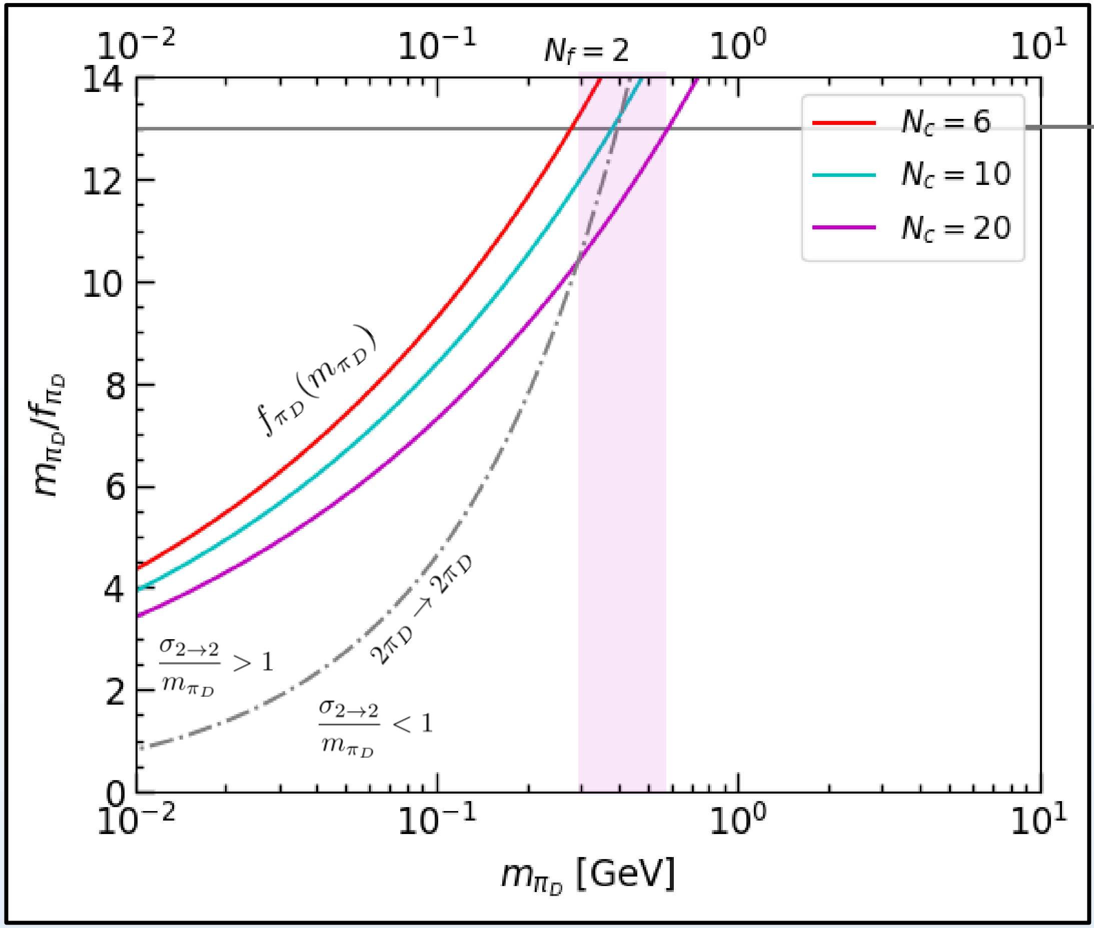
[Hochberg et al: ArXiv: 1411.3727]

Estimating m_{π_D}

Match pion abundance with DM relic density today: $f_{\pi_D} = f_{\pi_D}(m_{\pi_D})$



Perturbative limit

$$\frac{m_{\pi_D}}{f_{\pi_D}} < 4\pi$$

Bullet cluster constraint

[Randall et al.: ArXiv: 0704.0261]

$$\Rightarrow \frac{\sigma_{2\to2}}{m_{\pi_D}} < 1 \left[\frac{\mathrm{cm}^2}{\mathrm{g}}\right]$$

Pion mass :

$$m_{\pi_D} \approx 400\mathrm{MeV}$$

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[Hochberg et al: ArXiv: 1411.3727]

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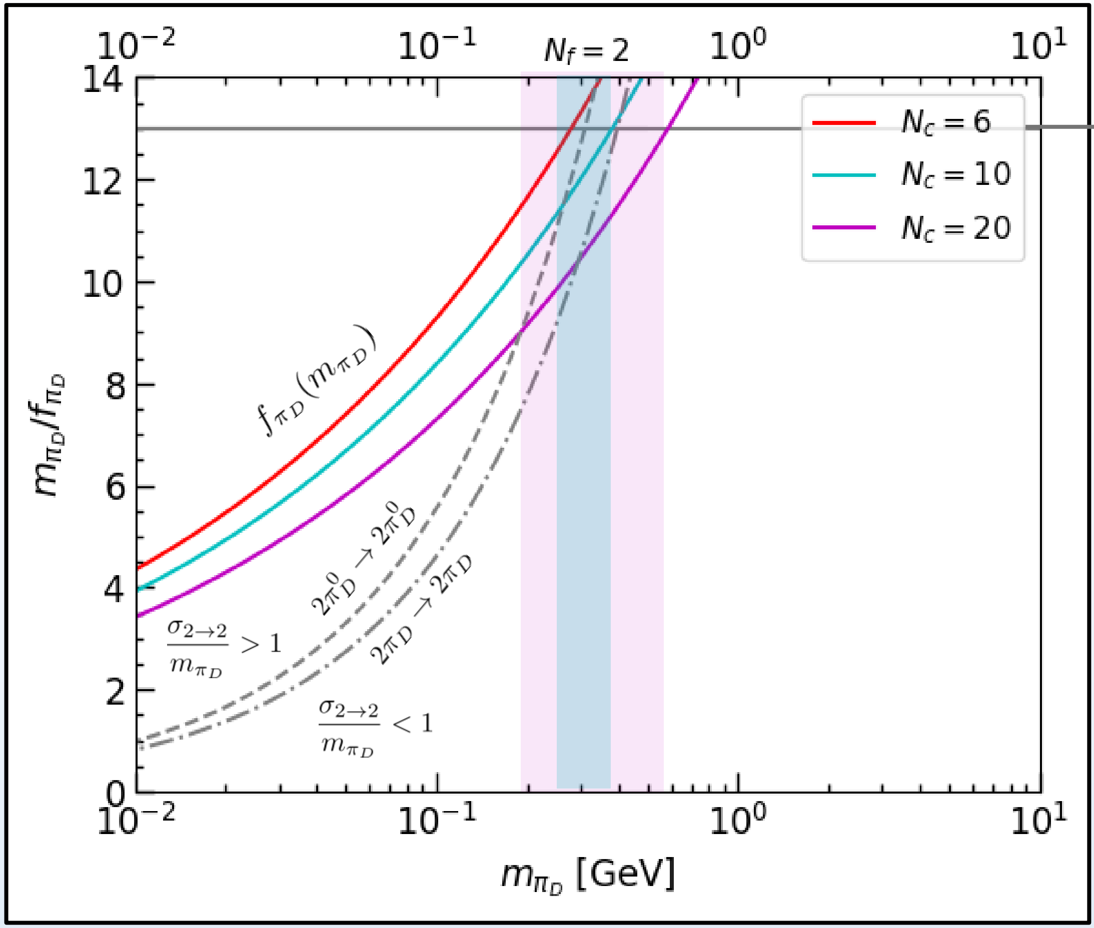
Match pion abundance with DM relic density today: $f_{\pi_D} = f_{\pi_D}(m_{\pi_D})$



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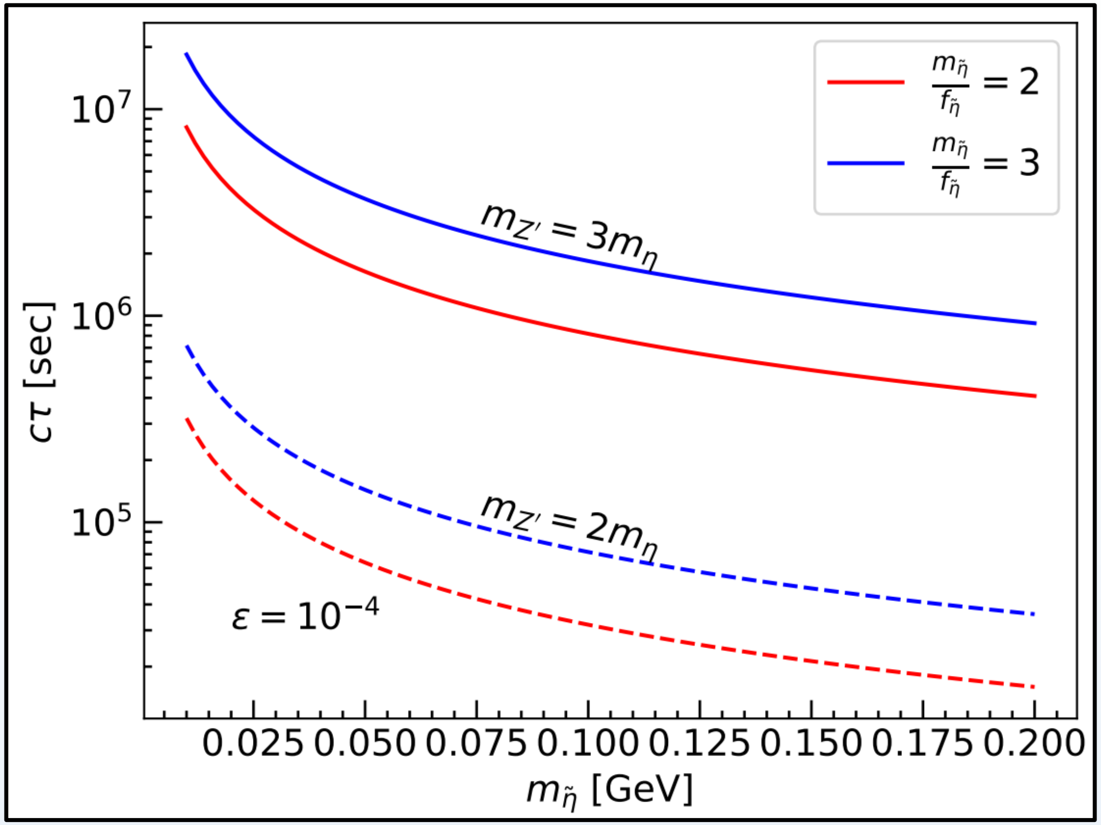
$$\Rightarrow \frac{\sigma_{2\to2}}{m_{\pi_D}} < 1\left[\frac{\mathrm{cm}^2}{\mathrm{g}}\right]$$

Pion mass :

$$m_{\pi_D} \approx 300\mathrm{MeV}$$

Estimates of η'_D lifetime



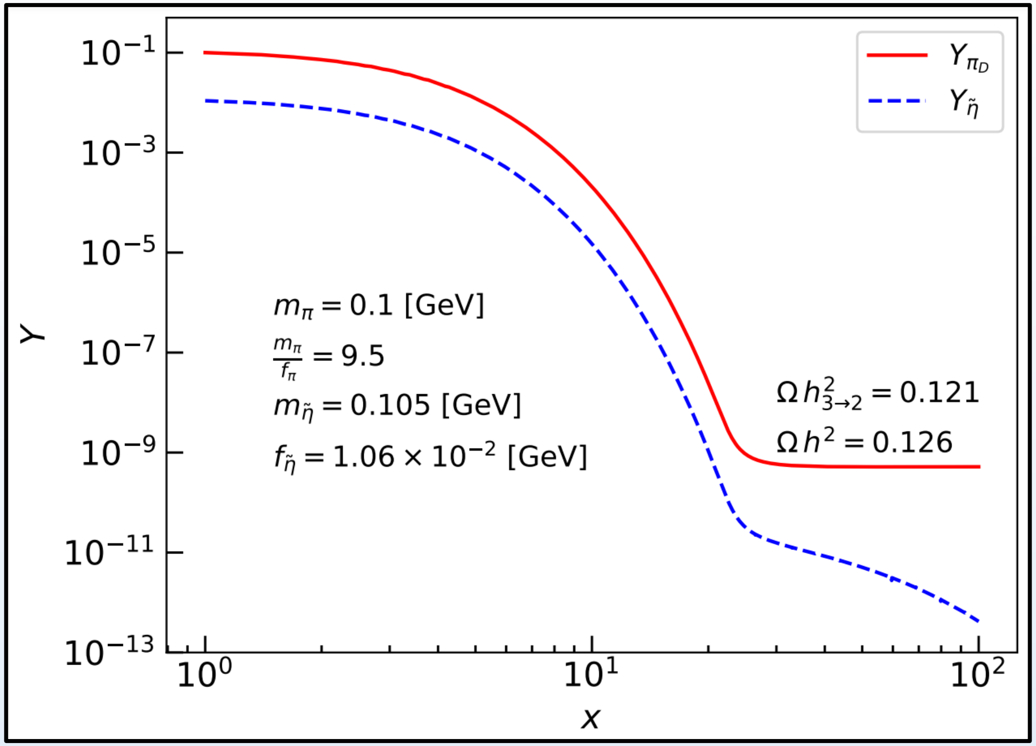
On the timescale of freeze-out η'_D is long lived.

Effects of light η'_D on the relic density estimate

Relic abundance is slightly over estimated:

- η'_D is relatively long lived.
- Down-scattering $2\eta'_D \to 2\pi^0_D$.

Lower values of f_{π_D} possible.



Over estimation effects vanish quickly with increasing $\Delta m^2_{\eta'_D}$.

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Summary

- Dark QCD with real representations is a viable dark matter model for „large“ values of N_C .

- Considering down-scattering in the cosmic evolution has effects on the model.

- For large N_C the η'_D might become light.
    - η'_D is stable and leads to over estimation of DM relic density.
    - The inclusion of η'_D improves the model.
    - Minor quantitive / no qualitative changes of the original SIMP model.

Ready for your questions

The axial anomaly and discrete symmetries

General form of Axial Anomaly

$$\mathcal{A}_{\mathrm{Axial}}[\epsilon, A] \;=\; -2i\;T_{\mathcal{R}}\;\mathrm{tr}\{\epsilon\}\mathcal{Q}_{\mathrm{Topo}}[A]$$

Quantum chiral transfomrmations

$U(4)$ $\ni U \;=\; \mathsf{exp}(-\epsilon) \longrightarrow D\psi D\psi \;\overset{U}{\longmapsto}\; e^{-i\mathcal{A}[\epsilon, A]}\;D\psi D\psi$

$$\mathrm{det}(U) \;= \exp\left(-i\frac{\pi k}{T_R}\right) \;\Leftrightarrow\; \exp(-i\mathcal{A}[\epsilon, A]) = \;1$$

$$k \;\in\; \{0, ..., 2T_R - 1\}$$

$\longleftarrow$ $Z_{2T_{\mathcal{R}}} \ltimes SU(4)$

Dynkin Index $T_{\mathcal{R}}$

SU(N) - Fund.	SO(N) - Vec.	Sp(2N) - Fund	Sp(2N) - AT2T
$T_R \;=\; 1/2$	$T_R \;=\; 1$	$T_R \;=\; 1/2$	$T_R \;=\; N-1$

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't Hooft large N considerations of η'_D

Idea: Compare for example $SO(N)$-vector theories for N very large.

Technicality: Define 't Hooft coupling λ

$$\lambda := C_{adj}(N)\, g^2 \qquad\qquad \lambda(\mu_{UV}) = \text{fixed}$$

$\rightarrow$ Running of λ is independent of N up to $1/N$ corrections.

$\rightarrow$ A controlable perturbative scale $1/N$ is introduced into the theory.

Axial anomaly in the chiral limit:

$$\partial_\mu J^\mu_{\eta'_D} = - \frac{T(R)}{C_{\mathrm{adj}}} \frac{\lambda N_F}{32\pi^2}\, \epsilon^{\mu\nu\rho\sigma}\, G^{\alpha}_{\mu\nu}\, G_{\rho\sigma\ \beta}$$

Gives potential large N suppression

$\frac{T(R)}{C_{\mathrm{adj}}} \xrightarrow[N\to\infty]{!} 0$ must hold for the anomaly to vanish in large N limit

Example:
$SU(N)$-Fund.

$$\lambda := N\, g^2$$

$$g^2 \xrightarrow[N\to\infty]{} 0$$

$$\frac{T(R)}{C_{\mathrm{adj}}} = \frac{1}{2N}$$

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4th Homotopy group of SU(4)/SO(4)

	π_3	π_4	π_5
$SO(4)$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$SU(4)$	$\mathbb{Z}$	0	$\mathbb{Z}$

Fibration:

$$SO(4) \to SU(4) \to SU(4)/SO(4)$$

Long exact sequence:

$$\begin{array}{ccccccc} \pi_4(SU(4)) & \xrightarrow{h_1} & \pi_4(SU(4)/SO(4)) & \xrightarrow{h_2} & \pi_3(SO(4)) & \xrightarrow{h_3} & \pi_3(SU(4)) \\ 0 & \xrightarrow{h_1} & ? & \xrightarrow{h_2} & \mathbb{Z} \oplus \mathbb{Z} & \xrightarrow{h_3} & \mathbb{Z} \end{array}$$

- $Ker(h_2) = Img(h_1) = 0 \to h_2$ is injective
- $\pi_4(SU(4)/SO(4)) \cong Img(h_2) = Ker(h_3)$
- $Ker(h_3) \neq 0$

$\Rightarrow$ $\pi_4(SU(4)/SO(4))$ cannot be trivial

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