Dark matter from confining SO(N)-like gauge theories with two Dirac fermions.

Joachim Pomper

Supervisor: Dr. Suchita Kulkarni

Student seminar TU Wien

16.03.2023

Der Wissenschaftsfonds.

Table of contents

SIMP dark matter

UV - models: The $SO(N)$ case

IR - description

UV - models: Other models

SIMP dark matter

Let us start with the usual story ...

Astrophysical observations point to the existence of a non-visible type of matter, that makes up 26% of universes energy budged.

Evidence on various scales:

- **Galaxy scale:**
  Rotational curves
- **Cluster scale:**
  Visible mass too little to hold together coma cluster
- **Cosmological scale:**
  CMB anisotropies

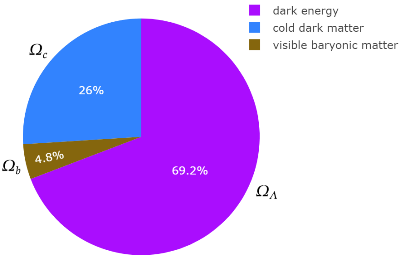


Figure: Energy budget of the universe within ΛCDM model.

Cusp vs. Core problem

Observed DM halo density profiles are more **cored** *compared to profiles found in cold DM simulations, which are rather* **cusp**.

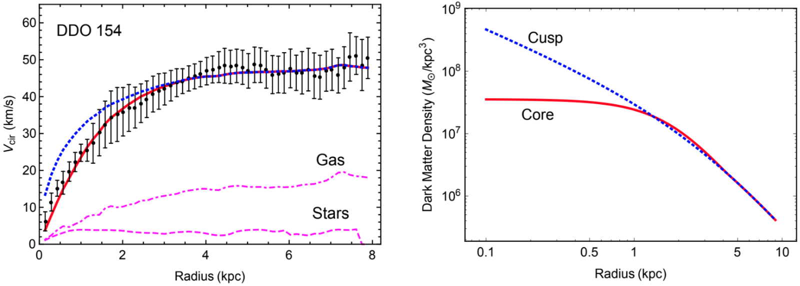


Figure: Data from the DDO 154 dwarf galaxy [Murayama (2022)].

Dark matter self-interactions

Kondo et al. (2022)

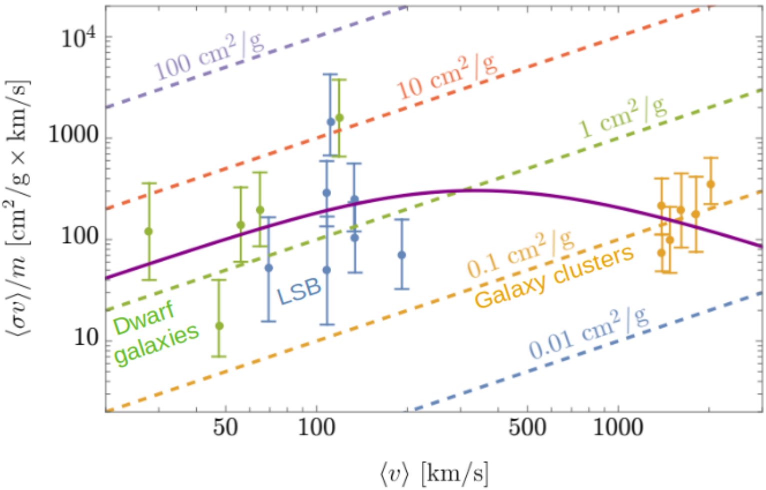
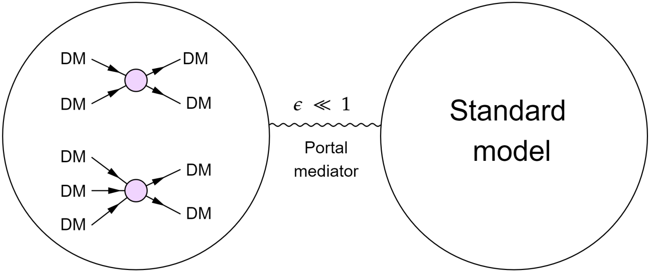


Figure: Suggested DM self-interaction crosssection in dependence of average velocity [Kaplinghat, Tulin, and Yu, American Physical Society (APS) 116 (2016)].

SIMP dark matter

Hochberg et al., Physical Review Letters 113 (2014)



- Sufficient self-interactions resolve structure formation problems.
- $3 \to 2$ cannibalization drives freeze out.
- Mediator for thermodynamic equilibrium until freeze out.

Dark QCD

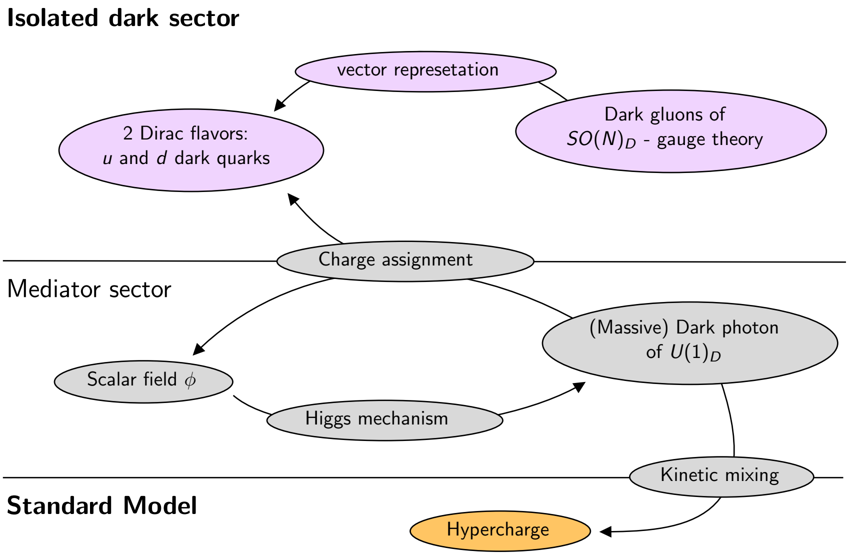
Hochberg et al., Physical Review Letters 115 (2015)

Dark matter from a SM-extension by a **confining, strongly interacting** sector based on dark $SU(N), Sp(N), SO(N)$-**gauge theory** with fermionic matter.

- Dark matter is made of pseudo Nambu-Goldstone states (dark pions) of a spontaneously broke flavor symmetry $G \to H$.
- Pion masses of $m_\pi \approx \mathcal{O}(100\mathrm{MeV})$ give required $\langle \sigma v \rangle / m_\pi$.
- Remaining flavor symmetry H may protect pions from decay.
- Pions fields correspond to maps of $\pi : \mathcal{M} \to G/H$.
- $3 \to 2$ process may be described by topological terms of Wess-Zumino type in non-linear Σ-model.

UV - models: The $SO(N)$ case

Model concept in the UV



Flavor symmetries of the theory

Isolated strong sector:

$U(4)$

↓ Axial anomaly

Only flavor transofromations with
$$\det = \pm 1$$
may be realized in quantum theory

$Z_2 \ltimes SU(4)$

↓ Chiral quark condensate $\Sigma \neq 0$
Degenerate mass $m_u = m_d \neq 0$

Mass term introduces a small perturbation, triggering correct spontanuous breaking pattern

$O(4)$

Further explicit breaking:

Coupling to $\mathrm{U}_D(1)$ → $SU(2) \times U(1)$ — Stable pions

Coupling to $\mathrm{U}_D(1)$ → $U(1) \times U(1)$

Non degenerate mass $m_u \neq m_d \neq 0$ → $U(1) \times U(1)$ — **No stable DM**

Action of $O(4)$ symmetry

Organise u- and d-quarks with charge conjugates in 2×2 matrix.

Action of SO(4) subgroup :

$$\begin{pmatrix} u & -d_C \\ d & u_C \end{pmatrix} \xmapsto{SO(4)\cong SU(2)\times SU(2)} U_l \begin{pmatrix} u & -d_C \\ d & u_C \end{pmatrix} U_R^\dagger$$

Action of Z_2 subgroup :

$$\begin{pmatrix} u & -d_C \\ d & u_C \end{pmatrix} \xmapsto{Z_2} \begin{pmatrix} u_C & -d_C \\ d & u \end{pmatrix}$$

Charge assignments for dark photon

- $U(1)_D$ gauge theory is consistent
- Pion currents are non-anomalous
- Non-abelian flavor symmetry remains

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} \Rightarrow \quad \begin{array}{c}\text{Anomaly cancellation} \\ \mathcal{Q}^2 = \mathbb{1}\end{array}$$

Charge assignment: $\mathcal{Q} \propto \sigma_{3,R}$

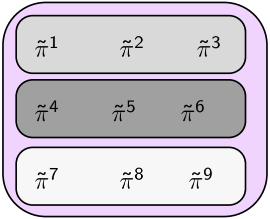
Explict breaking: $O(4) \to SU(2) \times U(1)$

$U(1)_D$ gauge transformation:

$$\begin{pmatrix} u & -d_C \\ d & u_C \end{pmatrix} \xmapsto{\quad U(1)_D \quad} \begin{pmatrix} u & -d_C \\ d & u_C \end{pmatrix} e^{-i\alpha\mathcal{Q}}$$

Particles relevant for DM phenomenology

pseudo scalar mesons

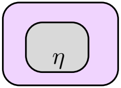


(Pseudo) Nambu-Goldstone bosons:

One for each local coordinate in the coset space

$$G/\,H \;=\; Z_2 \ltimes SU(4)/O(4) \;\cong\; SU(4)/SO(4)$$

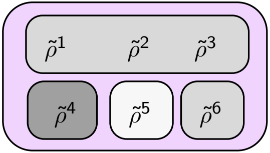
These are the lightes states !

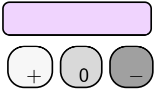


Flavor singlet meson:

Heavier than the pions

vector mesons





$O(4)$ Classification

$SU(2) \times U(1)$ Classification

Large N arguments

Using a 't Hoof large N-limit comparing $SO(N)$-vector theories.

Scaling of gauge-coupling:

$$g \xrightarrow[N\to\infty]{} 0 \quad \text{with} \quad Ng^2 = \lambda = \textit{fixed}$$

Axial anomaly in chiral limit:

$$\partial_\mu j^\mu_\eta = -g^2 \frac{\epsilon^{\mu\nu\rho\sigma}}{8\pi^2} G_{\mu\nu} G_{\rho\sigma} \xrightarrow[N\to\infty]{} 0$$

Conclusion

For large N, the η may be important for phenomenology.

IR - description

(Pseudo) Nambu-Goldstone fields

Nambu-Goldstone fields $\xi = (\pi, \eta)$ parametrize local deviations from the vacuum configuration, e.g.

$$\xi : \mathcal{M} \to G_F/H = \begin{cases} SU(4)/SO(4) & \\ \quad U(4)/O(4) & \text{in Large N limit} \end{cases}$$

Since G_F/H is connected, compact and symmetric we may choose

$$U[\xi](x) = \exp\left(-2i\, \frac{\xi^a(x)}{f_\pi} T_\alpha^{F,\mathrm{broken}}\right)$$

which transforms linearly under G_F flavor transformations.

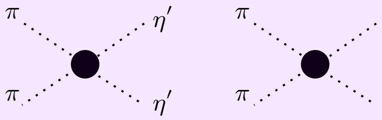
$$U[\xi](x) \xrightarrow[g \in G_F]{} gU[\xi](x)g^\dagger$$

(Non-anomalous) Low energy effective Lagrangian

$$\mathcal{L}_{eff} \;=\; \frac{f_\pi^2}{4}\; Tr\left\{(\partial_\mu U)^\dagger\,\partial^\mu U\right\} \;-\; \frac{f_\eta^2 - f_\pi^2}{2 f_\eta^2}\partial_\mu \eta'\;\partial^\mu \eta'$$

$$+\;\frac{\mu^3 m}{2}\, Tr\left\{U \;+\; (U)^\dagger\right\} \;-\; \frac{\Delta m^2_{\eta'}}{2}\;\eta'\eta'$$

Contact terms:



Gell-Mann-Oakes-Renner:

$$m_\pi^2 \;=\; \frac{m\mu^3}{2\; f_\pi^2}$$

$$m^2_{\eta'} \;=\; m_\pi^2 + \Delta m^2_{\eta'}$$

Topological terms and coset homotopy

The coset space G/H has non vanishing 4th homotopy group:

$$\pi_4(U(4)/O(4)) = \pi_4(SU(4)/SO(4)) \neq 0$$

Problem:

- The standard construction/classification of topological terms in non-linear sigma model by Witten, Weinberg and d'Hoker requires $\pi_4(G/H) = 0$.

  [Witten, Nucl. Phys. B 223 (1983)]

  [D'Hoker and Weinberg, Physical Review D 50 (1994)]

  [Brauner and Kolešová, Nuclear Physics B 945 (2019)]

- Modern approaches give more general classification but no practical construction.

  [Davighi and Gripaios, Journal of High Energy Physics 2018 (2018)]

  [Lee, Ohmori, and Tachikawa, SciPost Physics 10 (2021)]

't Hooft anomaly matching argument

Using the gauge principle to detect anomalous terms in the effective field theory.

Anomalous WTI: $\delta S_{eff}^{gauged} = \mathcal{A}_{UV}$ and $S_{eff} = \lim_{A\to 0} S_{eff}^{gauged}$

Step 1

$$S_{UV} \xrightarrow{\text{Gauge } G_F} S_{UV}^{gauged} \xrightarrow{\delta_{G_F}} \mathcal{A}_{UV} \neq 0$$

Step 2

$$S_{UV} + S_{Free} \longrightarrow S_{UV}^{gauged} + S_{Free}^{gauged} \longrightarrow \mathcal{A}_{UV} = 0$$

Step 3

$$S_{eff} + S_{Free} \longrightarrow S_{eff}^{gauged} + S_{Free}^{gauged} \longrightarrow \mathcal{A}_{IR} = 0$$

Step 4

$$S_{eff} \longrightarrow S_{eff}^{gauged} \longrightarrow \mathcal{A}_{IR} = \mathcal{A}_{UV} \neq 0$$

The Wess-Zumino term

Chu, Ho, and Zumino, Nuclear Physics B 475 (1996)

Wess and Zumino derived an effective action that solves the anomaly equation. [Wess and Zumino, Physics Letters B 37 (1971)]

Wess-Zumino effective action

$$S_{WZ}[\xi = (\eta', \pi)] = \frac{D_C}{48\pi^2 f_\pi} \int_0^1 \mathrm{d}t \int_{S^4} \mathrm{Tr}\left\{\xi \left((U[t\xi])^{-1} \mathrm{d}U[t\xi]\right)^4\right\}$$

$$\approx \frac{D_C}{250 f_\pi \pi^2} \epsilon^{\mu\nu\sigma\rho} \int_{S^4} \mathrm{d}^4x \ \mathrm{Tr}\left\{\pi \partial_\mu \pi \partial_\nu \pi \partial_\sigma \pi \partial_\rho \pi\right\}$$

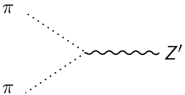
- Does not depend on η' in lowest order χPT.
- Removes superfluous $\pi(t,x) \mapsto -\pi(t,x)$ symmetry in S_{eff}.
- Incorporates $3 \to 2$ process.

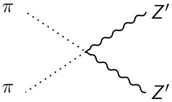
Inclusion of the dark photon Z'

Non-Anomalous contributions:

Exchange derivatives:

$$\partial_\mu U^2 \longmapsto \mathrm{D}_\mu U^2 = \partial_\mu U^2 + ie_D A_\mu \left(\mathcal{Q} U^2 + U^2 \Sigma_0^\dagger \mathcal{Q}^\top \Sigma_0 \right)$$

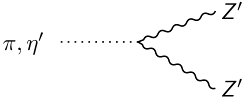




Anomalous contributions:

Anomaly equation:

$$S_{eff} = \lim_{A \to Z'} S_{eff}^{gauged}$$

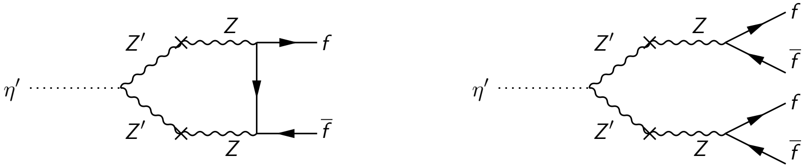


$$\propto \begin{cases} Tr\left\{\pi \;\, Q^2\right\} & = 0 \\ Tr\left\{\eta' \;\, Q^2\right\} & \neq 0 \end{cases}$$

η'-decay to Standard Model

Hochberg, Kuflik, and Murayama, Physical Review D 99 (2019)

Depending on lifetime $\tau_{\eta'}$ (and mass m'_η) of the η' the dark matter scenario might be significantly altered or spoiled.



η'-decay may lower the calculated relic abundance of dark matter significantly.

Limiting cases:

- η' decays almost instantly
  $\Rightarrow$ NO DARK MATTER
- $\tau_{\eta'} \approx$ age of universe.
  $\Rightarrow$ Pion abundance not affected.

UV - models: Other models

Generalizations

Almost all the results so far depend on

- The presence of a chirally broken phase.
- The fact that fermions transform under a real representation.

So can we get the same for different gauge-theories than $SO(N)$ with vector-representation fermions?

For example :

- Adjoint representations
- SO(N) tensor representations
- Sp(2N) two index anti-symmetric representation

Conformal window considerations

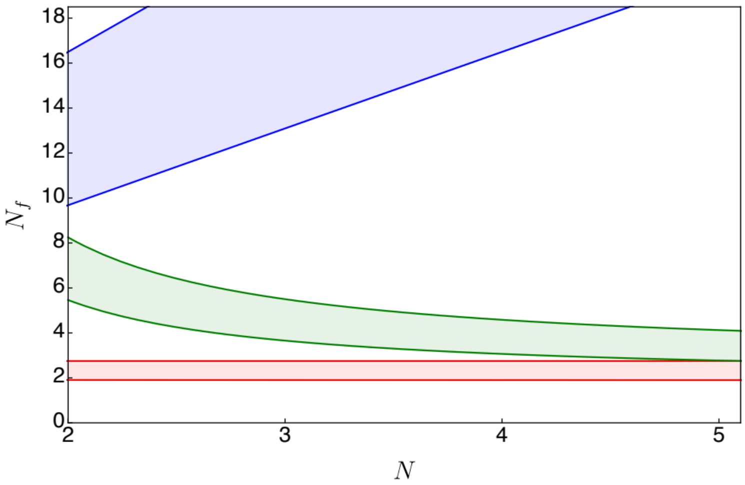


Figure: Conformal window for Sp(2N) with fundamental (blue), adjoint (red) and two index anti-symmetric (green). [Lee, Ohmori, and Tachikawa, SciPost Physics 10 (2021)]

Anomalous symmetry breaking

The chiral anomaly functional depends on the gauge-group representation.

1) General symmetry breaking pattern

$$U(4) \to Z_K \ltimes SU(4) \to O(4)$$

	$SO(N)$-Vector	$Sp(2N)$-A2T	$SO(N)$-Adj.	$Sp(2N)$-Adj.
K	2	2(N-1)	2(N-2)	2(N+1)

2) In naive 't Hooft large N considerations, the anomalous contribution does not vanish.

$\Rightarrow \eta$ might not be light and thus not too relevant.

Summary

What I talked about today:

- Dark QCD-like models based on $SO(N)$-vector might realize SIMP dark matter.
- One has to be careful about the role of η.
- Problems and solution concerning Wess-Zumino terms.
- Situation for other real representations not so clear.

Summary

What I talked about today:

- Dark QCD-like models based on $SO(N)$-vector might realize SIMP dark matter.
- One has to be careful about the role of η.
- Problems and solution concerning Wess-Zumino terms.
- Situation for other real representations not so clear.

What I left out:

- Construction of gauge-invariant operators.
- Details on discrete symmetries
- Inclusion of vector mesons via local hidden symmetry.

Bonus content

Discrete symmetries

Drach, Janowski, and Pica, EPJ Web of Conferences 175 (2018)

Charge conjugation C:

UV $q \mapsto \Omega C \bar{q}^\top$

IR $\pi \mapsto \pi^\top$

Spatial parity P:

UV $q(t, \vec{x}) \mapsto \eta_P \gamma^0 q(t, -\vec{x})$

IR $\pi(t, \vec{x}) \mapsto -\pi(t, -\vec{x})$

The choice $\eta_P = \pm i$ is adopted:

- Parity and flavor symmetries commute.
- All (pseudo) Nambu-Goldstone bosons (pNGB) are pseudo scalars.

Interpolating operators for pions

Bennett et al., Phys. Rev. D 101 (2020)

	$\overline{\Psi_{\mathcal{C}}}\ T_n^{\Psi}\Psi + \left(\overline{\Psi_{\mathcal{C}}}\ T_n^{\Psi}\Psi\right)^*$	J^D		$\overline{\Psi_{\mathcal{C}}}\ T_N^{\pi}\Psi + \left(\overline{\Psi_{\mathcal{C}}}\ T_N^{\pi\top}\Psi\right)^*$	I_3	e_D
π_1	$\frac{1}{\sqrt{2}}\left(\overline{u}\gamma^5 d + \overline{d}\gamma^5 u\right)$	1^-	π^A	$\overline{u}\gamma^5 d$	1	0
π_2	$\frac{1}{\sqrt{2}}\left(\overline{u}\gamma^5 d - \overline{d}\gamma^5 u\right)$	1^-	π^B	$\overline{d}\gamma^5 u$	-1	0
π_3	$\frac{1}{\sqrt{2}}\left(\overline{u}\gamma^5 u - \overline{d}\gamma^5 d\right)$	1^-	π^C	$\frac{1}{\sqrt{2}}\left(\overline{u}\gamma^5 u - \overline{d}\gamma^5 d\right)$	0	0
π_4	$\frac{1}{2}\left(\overline{u_{\mathcal{C}}}\gamma^5 u + \overline{u}\gamma^5 u_{\mathcal{C}}\right)$	1^-	π^D	$\frac{1}{\sqrt{2}}\overline{u_{\mathcal{C}}}\gamma^5 u$	-1	-1
π_5	$\frac{1}{2}\left(\overline{d_{\mathcal{C}}}\gamma^5 d + \overline{d}\gamma^5 d_{\mathcal{C}}\right)$	1^-	π^E	$\frac{1}{\sqrt{2}}\overline{d}\gamma^5 d_{\mathcal{C}}$	1	-1
π_6	$\frac{1}{\sqrt{2}}\left(\overline{u}\gamma^5 d_{\mathcal{C}} + \overline{u_{\mathcal{C}}}\gamma^5 d\right)$	1^-	π^F	$\overline{u}\gamma^5 d_{\mathcal{C}}$	0	-1
π_7	$\frac{1}{2}\left(\overline{u_{\mathcal{C}}}\gamma^5 u - \overline{u}\gamma^5 u_{\mathcal{C}}\right)$	1^-	π^G	$\frac{1}{\sqrt{2}}\overline{u}\gamma^5 u_{\mathcal{C}}$	1	1
π_8	$\frac{1}{2}\left(\overline{d}\gamma^5 d_{\mathcal{C}} - \overline{d_{\mathcal{C}}}\gamma^5 d\right)$	1^-	π^H	$\frac{1}{\sqrt{2}}\overline{d}_{\mathcal{C}}\gamma^5 d$	-1	1
π_9	$\frac{1}{\sqrt{2}}\left(\overline{u}\gamma^5 d_{\mathcal{C}} - \overline{u_{\mathcal{C}}}\gamma^5 d\right)$	1^-	π^I	$\overline{u_{\mathcal{C}}}\gamma^5 d$	0	1

4th Homotopy group of SU(4)/SO(4)

	π_3	π_4	π_5
$SO(4)$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$SU(4)$	$\mathbb{Z}$	0	$\mathbb{Z}$

Fibration:

$$SO(4) \rightarrow SU(4) \rightarrow SU(4)/SO(4)$$

Long exact sequence:

$$\begin{array}{ccccccc} \pi_4(SU(4)) & \xrightarrow{h_1} & \pi_4(SU(4)/SO(4)) & \xrightarrow{h_2} & \pi_3(SO(4)) & \xrightarrow{h_3} & \pi_3(SU(4)) \\ 0 & \xrightarrow{h_1} & ? & \xrightarrow{h_2} & \mathbb{Z} \oplus \mathbb{Z} & \xrightarrow{h_3} & \mathbb{Z} \end{array}$$

- $Ker(h_2) = Img(h_1) = 0 \rightarrow h_2$ is injective
- $\pi_4(SU(4)/SO(4)) \cong Img(h_2) = Ker(h_3)$
- $Ker(h_3) \neq 0$

$\Rightarrow$ $\pi_4(SU(4)/SO(4))$ cannot be trivial

Full UV Lagrangian

Hochberg, Kuflik, and Murayama, Journal of High Energy Physics (2016)

$-\frac{1}{4}\ G^{\alpha}_{\mu\nu}\ G^{\mu\nu}_{\alpha}$

Yang-Mills term for dark gluons : Based on $G_D\ =\ Sp(4)$

$-\frac{1}{4}\ F'_{\mu\nu}\ F'^{\mu\nu}$

Yang-Mills term for dark photon : Based on $U_D(1)$

$+\ \overline{u}\ i\ \gamma^{\mu} D_{\mu}\ u\ +\ m\ \overline{u} u$

$+\ \overline{d}\ i\ \gamma^{\mu} D_{\mu}\ d\ +\ m\ \overline{d} d$

Dirac term of dark quarks
2 flavors: u and d quarks
Charged under $G_D\ \times\ U_D(1)$

$+\ \left(D'_{\mu}\phi\right)^{\dagger} D'^{\mu}\phi\ +\ V\left[\phi^{\dagger}\phi\right]$

Dark scalar charged under $U_D(1)$

$+\ \frac{\epsilon}{2\cos(\theta_W)}\ F'_{\mu\nu}\ B^{\mu\nu}$

Kinetic mixing of $U_D(1)$ with SM hypercharge

Meson spectrum in 2 flavor Sp(4)-fund. gauge theory

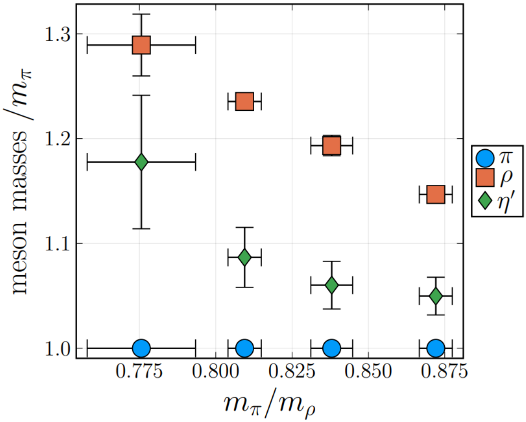


Figure: Meson spectrum in $Sp(4)$-fundamental with 2 flavors.
Taken from [Zierler et al. (2022), arXiv:hep-lat/2210.11187]