A set of proofs that definitely deserve a 6

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Axioms

1 The removal of parenthesis

We assume this equality:

$$(a) = a$$

with a being unknown or known.

2 The power distribution

We suppose the following equality:

$$b^x b^y = b^{(x+y)}$$

with b, x and y being unknown (or known).

3 The identity exponent

We suppose this proposition:

$$a^1 = a$$

with a being unknown (or known).

4 The commutativity of the product

We assume this proposition to be true:

$$ab = ba$$

with a and b being unknown (or known).

5 The left distributivity of the product

We take the following affirmation as an axiom:

$$a\,(b + c) = ab + ac$$

with a, b and c being unknown (or known).

6 The left multiplication by the identity

We suppose that this proposition is true:

$$1a = a$$

with a being unknown (or known).

7 The commutativity of the addition

We take the following proposition as an axiom:

$$a + b = b + a$$

with a and b being unknown (or known).

Part II
Theorems

8 The addition

8.1 Theorem

We are trying to prove this affirmation:

$$a + b = c$$

with a and b being unknown (or known), and with $c = a + b$ getting simplified (as a number).

8.2 Proof

Let a and b be unknown (or known). Let $c = a + b$ be a simplification (as a number). Starting with the following expression:

$$a + b$$

We have let $a + b = c$, so

$$a + b = c$$

This proof thus allows us to be convinced that:

$$a + b = c$$

QED

9 The product

9.1 Theorem

We are aiming to prove the following equality:

$$ab = c$$

with a and b being unknown (or known), and with $c = ab$ getting simplified (as a number).

9.2 Proof

Let a and b be unknown (or known). Let $c = ab$ be a simplification (as a number). Let us begin with:

$$ab$$

We have let $ab = c$, so

$$ab = c$$

Which thus makes us sure that:

$$ab = c$$

QED

10 The power

10.1 Theorem

We are aiming to show that:

$$a^b = c$$

with a and b being unknown (or known), and with $c = a^b$ getting simplified (as a number).

10.2 Proof

Let a and b be unknown (or known). Let $c = a^b$ be a simplification (as a number). Starting with the following expression:

$$a^b$$

We have let $a^b = c$, so

$$a^b = c$$

Which thus makes us sure:

$$a^b = c$$

QED

11 The distribution of the square

11.1 Theorem

We aim to show this proposition:

$$a^2 = aa$$

with a being unknown (or known).

11.2 Proof

Let a be unknown (or known). The following expression can be our starting point:

$$a^2$$

According to the removal of parenthesis (section 1), we know that $2 = (2)$. Therefore,

$$a^2 = a^{(2)}$$

From the addition (section 8), we get that $2 = 1 + 1$. This means that:

$$a^{(2)} = a^{(1+1)}$$

From the power distribution (section 2), we have that

$$a^{(1+1)} = a^1 a^1$$

By the identity exponent (section 3), we have that $a^1 = a$. Therefore,

$$a^1 a^1 = aa^1$$

From the identity exponent (section 3), we know that $a^1 = a$. This means that:

$$aa^1 = aa$$

This proof thus makes us sure:

$$a^2 = aa$$

QED

12 The right distributivity of the product

12.1 Theorem

We aim to prove this equality:

$$(a + b)\, c = ac + bc$$

with a, b and c being unknown (or known).

12.2 Proof

Let a, b and c be unknown (or known). Let us start with:

$$(a+b)\,c$$

Using the commutativity of the product (section 4), we have

$$(a+b)\,c = c\,(a+b)$$

Using the left distributivity of the product (section 5), we get

$$c\,(a+b) = ca + cb$$

From the commutativity of the product (section 4), we know that $ca = ac$. This allows us to infer that:

$$ca + cb = ac + cb$$

Using the commutativity of the product (section 4), we have $cb = bc$. Thus,

$$ac + cb = ac + bc$$

This proof makes us sure:

$$(a+b)\,c = ac + bc$$

QED

13 The litteral addition

13.1 Theorem

We are trying to prove the following affirmation:

$$ax + bx = cx$$

with a, b and x being unknown (or known), and with $c = a + b$ getting simplified (as a number).

13.2 Proof

Let a, b and x be unknown (or known). Let $c = a + b$ be a simplification (as a number). We will begin our proof with the following expression:

$$ax + bx$$

By the right distributivity of the product (section 12), we have that

$$ax + bx = (a+b)\,x$$

We have let $a + b = c$, so

$$(a+b)\,x = (c)\,x$$

From the removal of parenthesis (section 1), we get $(c) = c$. Therefore,

$$(c)\,x = cx$$

This proof therefore allows us to conclude that:

$$ax + bx = cx$$

QED

14 The first remarkable identity

14.1 Theorem

We are trying to prove the following equality:

$$(a+b)^2 = a^2 + 2ab + b^2$$

with a and b being unknown (or known).

14.2 Proof

Let a and b be unknown (or known). Let us take this as a starting point:

$$(a+b)^2$$

Using the distribution of the square (section 11), we have

$$(a+b)^2 = (a+b)(a+b)$$

According to the right distributivity of the product (section 12), we get that

$$(a+b)(a+b) = a(a+b) + b(a+b)$$

Using the left distributivity of the product (section 5), we get that $a(a+b) = aa + ab$. This allows us to infer that:

$$a(a+b) + b(a+b) = aa + ab + b(a+b)$$

By the left distributivity of the product (section 5), we have $b(a+b) = ba + bb$. This allows us to infer that:

$$aa + ab + b(a+b) = aa + ab + ba + bb$$

From the commutativity of the product (section 4), we get $ba = ab$. Thus,

$$aa + ab + ba + bb = aa + ab + ab + bb$$

By the left multiplication by the identity (section 6), we have $ab = 1ab$. This means that:

$$aa + ab + ab + bb = aa + 1ab + ab + bb$$

From the left multiplication by the identity (section 6), we know that $ab = 1ab$. Thus,

$$aa + 1ab + ab + bb = aa + 1ab + 1ab + bb$$

From the litteral addition (section 13), we get $1ab + 1ab = 2ab$. Thus,

$$aa + 1ab + 1ab + bb = aa + 2ab + bb$$

Using the distribution of the square (section 11), we get that $aa = a^2$. This means that:

$$aa + 2ab + bb = a^2 + 2ab + bb$$

By the distribution of the square (section 11), we know that $bb = b^2$. This means that:

$$a^2 + 2ab + bb = a^2 + 2ab + b^2$$

This proof therefore makes us sure:

$$(a+b)^2 = a^2 + 2ab + b^2$$

QED

15 The right multiplication by the identity

15.1 Theorem

We aim to prove this affirmation:

$$a \cdot 1 = a$$

with a being unknown (or known).

15.2 Proof

Let a be unknown (or known). Let us take this as a starting point:

$$a \cdot 1$$

Using the commutativity of the product (section 4), we know that

$$a \cdot 1 = 1a$$

Using the left multiplication by the identity (section 6), we know that

$$1a = a$$

Which allows us to conclude:

$$a \cdot 1 = a$$

QED

16 The first identity with a twist

16.1 Theorem

We are trying to prove the following equality:

$$(x+1)^2 = x^2 + 2x + 1$$

with x being unknown (or known).

16.2 Proof

Let x be unknown (or known). We can begin with:

$$(x+1)^2$$

According to the first remarkable identity (section 14), we get

$$(x+1)^2 = x^2 + 2x \cdot 1 + 1^2$$

Using the right multiplication by the identity (section 15), we have that $2x \cdot 1 = 2x$. Thus,

$$x^2 + 2x \cdot 1 + 1^2 = x^2 + 2x + 1^2$$

From the power (section 10), we have $1^2 = 1$. This means that:

$$x^2 + 2x + 1^2 = x^2 + 2x + 1$$

This proof allows us to conclude:

$$(x+1)^2 = x^2 + 2x + 1$$

QED

17 The triple left distributivity of the product

17.1 Theorem

We are aiming to prove this affirmation:

$$a\,(b + c + d) = ab + ac + ad$$

with a, b, c and d being unknown (or known).

17.2 Proof

Let a, b, c and d be unknown (or known). Let us take this as a starting point:

$$a\,(b+c+d)$$

Using the removal of parenthesis (section 1), we know that $c+d=(c+d)$. This allows us to infer that:

$$a\,(b+c+d)=a\,(b+(c+d))$$

From the left distributivity of the product (section 5), we have

$$a\,(b+(c+d))=ab+a\,(c+d)$$

From the left distributivity of the product (section 5), we get that $a\,(c+d)=ac+ad$. Thus,

$$ab+a\,(c+d)=ab+ac+ad$$

Which thus allows us to conclude:

$$a\,(b+c+d)=ab+ac+ad$$

QED

18 The triple right distributivity of the product

18.1 Theorem

We want to show this equality:

$$(a+b+c)\,d=ad+bd+cd$$

with a, b, c and d being unknown (or known).

18.2 Proof

Let a, b, c and d be unknown (or known). Let us take this as a starting point:

$$(a+b+c)\,d$$

According to the commutativity of the product (section 4), we get

$$(a+b+c)\,d=d\,(a+b+c)$$

According to the triple left distributivity of the product (section 17), we get

$$d\,(a+b+c)=da+db+dc$$

Using the commutativity of the product (section 4), we get $da=ad$. This means that:

$$da+db+dc=ad+db+dc$$

By the commutativity of the product (section 4), we get $db=bd$. Therefore,

$$ad+db+dc=ad+bd+dc$$

By the commutativity of the product (section 4), we get that $dc=cd$. Therefore,

$$ad+bd+dc=ad+bd+cd$$

This proof allows us to conclude that:

$$(a+b+c)\,d=ad+bd+cd$$

QED

19 The left distribution of the cube

19.1 Theorem

We want to show this proposition:

$$a^3 = a^2 a$$

with a being unknown (or known).

19.2 Proof

Let a be unknown (or known). Starting with the following expression:

$$a^3$$

According to the removal of parenthesis (section 1), we get $3 = (3)$. This means that:

$$a^3 = a^{(3)}$$

According to the addition (section 8), we have that $3 = 2 + 1$. This allows us to infer that:

$$a^{(3)} = a^{(2+1)}$$

From the power distribution (section 2), we know that

$$a^{(2+1)} = a^2 a^1$$

From the identity exponent (section 3), we have that $a^1 = a$. Therefore,

$$a^2 a^1 = a^2 a$$

Which makes us sure:

$$a^3 = a^2 a$$

QED

20 The right distribution of the cube

20.1 Theorem

We are trying to prove the following proposition:

$$a^3 = a a^2$$

with a being unknown (or known).

20.2 Proof

Let a be unknown (or known). A friend of mine told me to start with:

$$a^3$$

According to the left distribution of the cube (section 19), we get that

$$a^3 = a^2 a$$

According to the commutativity of the product (section 4), we have

$$a^2 a = a a^2$$

Which therefore makes us sure:

$$a^3 = a a^2$$

QED

21 The cube remarkable identity

21.1 Theorem

We are trying to prove the following affirmation:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

with a and b being unknown (or known).

21.2 Proof

Let a and b be unknown (or known). We can begin with:

$$(a+b)^3$$

According to the left distribution of the cube (section 19), we get that

$$(a+b)^3 = (a+b)^2 (a+b)$$

By the removal of parenthesis (section 1), we get $(a+b)^2 = \left((a+b)^2\right)$. This allows us to infer that:

$$(a+b)^2 (a+b) = \left((a+b)^2\right)(a+b)$$

Using the first remarkable identity (section 14), we know that $(a+b)^2 = a^2 + 2ab + b^2$. This means that:

$$\left((a+b)^2\right)(a+b) = \left(a^2 + 2ab + b^2\right)(a+b)$$

From the left distributivity of the product (section 5), we have

$$\left(a^2 + 2ab + b^2\right)(a+b) = \left(a^2 + 2ab + b^2\right)a + \left(a^2 + 2ab + b^2\right)b$$

From the triple right distributivity of the product (section 18), we get $\left(a^2 + 2ab + b^2\right)a = a^2a + 2aba + b^2a$. Therefore,

$$\left(a^2 + 2ab + b^2\right)a + \left(a^2 + 2ab + b^2\right)b = a^2a + 2aba + b^2a + \left(a^2 + 2ab + b^2\right)b$$

From the triple right distributivity of the product (section 18), we get that $\left(a^2 + 2ab + b^2\right)b = a^2b + 2abb + b^2b$. This allows us to infer that:

$$a^2a + 2aba + b^2a + \left(a^2 + 2ab + b^2\right)b = a^2a + 2aba + b^2a + a^2b + 2abb + b^2b$$

Using the left distribution of the cube (section 19), we get $a^2a = a^3$. Therefore,

$$a^2a + 2aba + b^2a + a^2b + 2abb + b^2b = a^3 + 2aba + b^2a + a^2b + 2abb + b^2b$$

Using the left distribution of the cube (section 19), we have $b^2b = b^3$. Thus,

$$a^3 + 2aba + b^2a + a^2b + 2abb + b^2b = a^3 + 2aba + b^2a + a^2b + 2abb + b^3$$

By the commutativity of the addition (section 7), we get that $b^2a + a^2b = a^2b + b^2a$. This means that:

$$a^3 + 2aba + b^2a + a^2b + 2abb + b^3 = a^3 + 2aba + a^2b + b^2a + 2abb + b^3$$

Using the commutativity of the product (section 4), we have $ba = ab$. This allows us to infer that:

$$a^3 + 2aba + a^2b + b^2a + 2abb + b^3 = a^3 + 2aab + a^2b + b^2a + 2abb + b^3$$

From the distribution of the square (section 11), we have $aa = a^2$. This means that:

$$a^3 + 2aab + a^2b + b^2a + 2abb + b^3 = a^3 + 2a^2b + a^2b + b^2a + 2abb + b^3$$

By the left multiplication by the identity (section 6), we know that $a^2b = 1a^2b$. This means that:

$$a^3 + 2a^2b + a^2b + b^2a + 2abb + b^3 = a^3 + 2a^2b + 1a^2b + b^2a + 2abb + b^3$$

According to the litteral addition (section 13), we know that $2a^2b + 1a^2b = 3a^2b$. Thus,

$$a^3 + 2a^2b + 1a^2b + b^2a + 2abb + b^3 = a^3 + 3a^2b + b^2a + 2abb + b^3$$

By the commutativity of the product (section 4), we know that $b^2a = ab^2$. This allows us to infer that:

$$a^3 + 3a^2b + b^2a + 2abb + b^3 = a^3 + 3a^2b + ab^2 + 2abb + b^3$$

By the distribution of the square (section 11), we know that $bb = b^2$. This allows us to infer that:

$$a^3 + 3a^2b + ab^2 + 2abb + b^3 = a^3 + 3a^2b + ab^2 + 2ab^2 + b^3$$

According to the left multiplication by the identity (section 6), we have $ab^2 = 1ab^2$. This means that:

$$a^3 + 3a^2b + ab^2 + 2ab^2 + b^3 = a^3 + 3a^2b + 1ab^2 + 2ab^2 + b^3$$

From the litteral addition (section 13), we get $1ab^2 + 2ab^2 = 3ab^2$. Thus,

$$a^3 + 3a^2b + 1ab^2 + 2ab^2 + b^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Which therefore allows us to conclude:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

QED