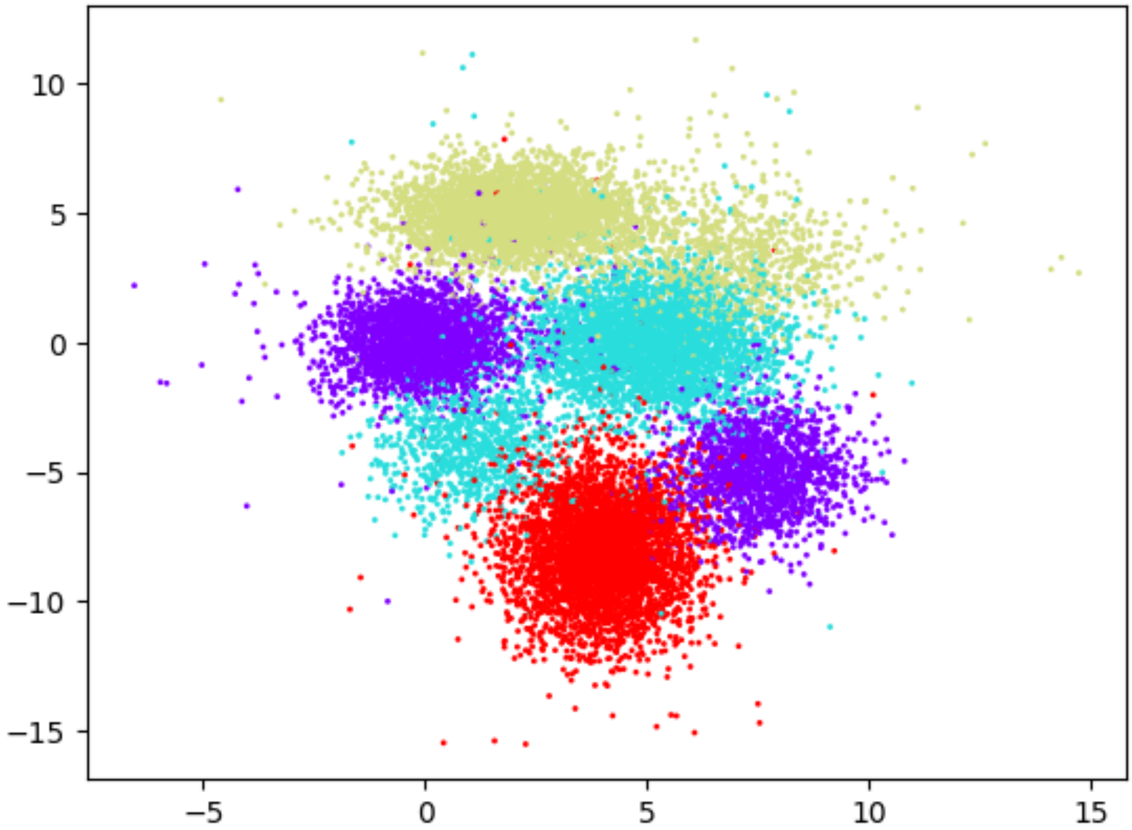
TP1 - Forêt d'arbres aléatoires & compromis Biais / Variance

In [1]:
```
import sys
import os
import numpy as np
import matplotlib.pyplot as plt
sys.path.append(os.getcwd())

# la configuration ci-dessous évite de recharger l'ensemble du note
%load_ext autoreload
%autoreload 2
```

Arbres de décision

1. **Base de donnée**

In [2]:
```
data = np.load("TP1a.npz")
X_train, y_train, X_test, y_test = (data[key] for key in ["X_train"

plt.scatter(X_train[:, 0], X_train[:, 1], c=y_train, s=1, cmap='rai
plt.show()

print(X_train.shape, X_test.shape)
```



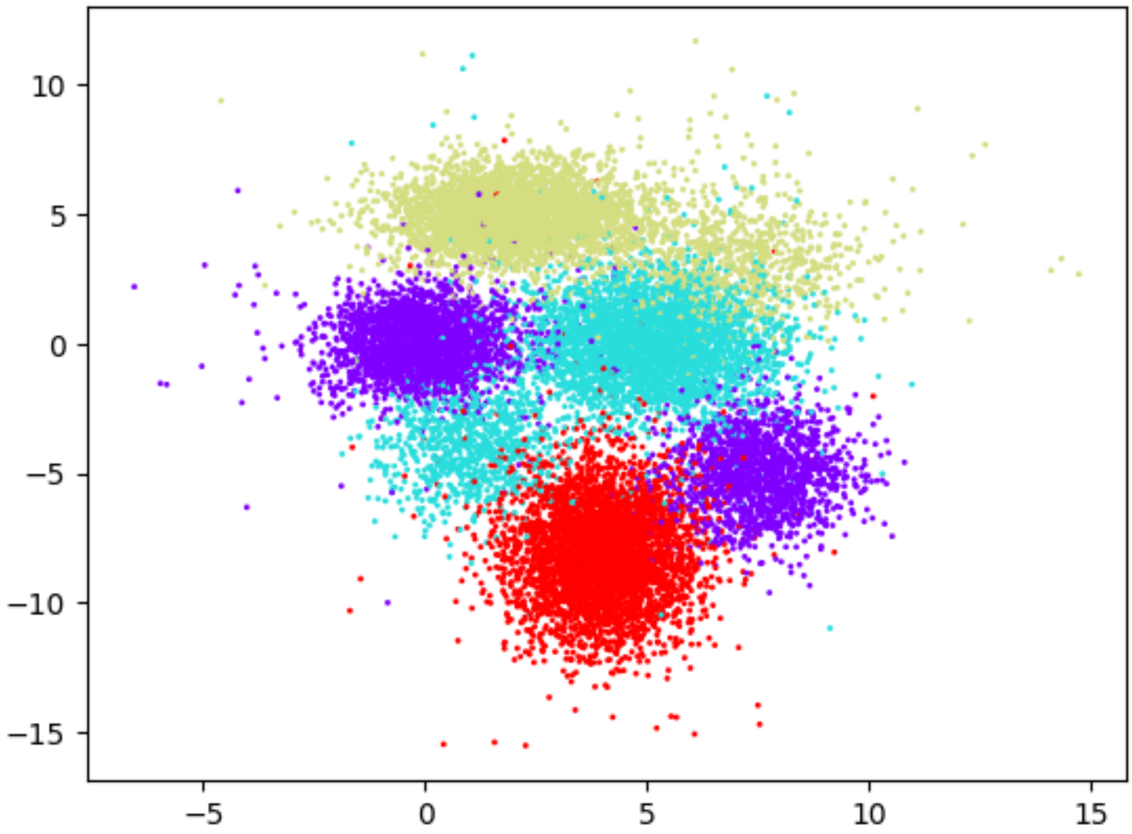

```
(16640, 2) (4160, 2)
```

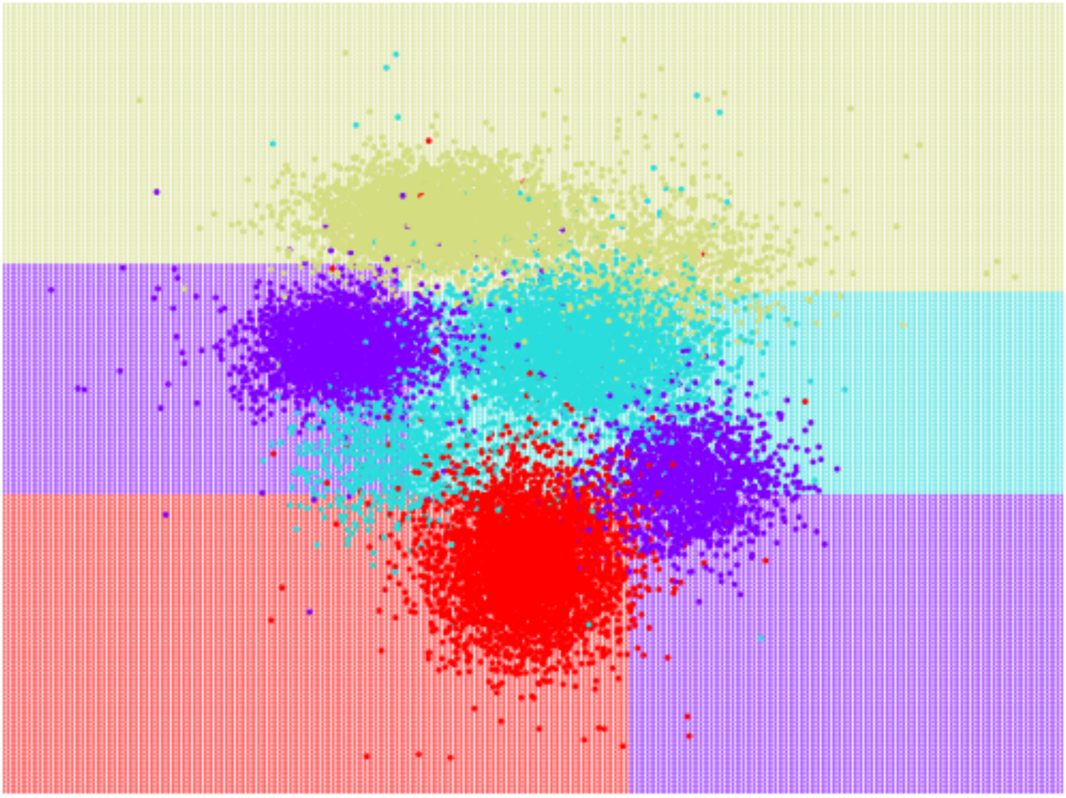
Le script ci-dessus permet de charger et visualiser les différentes classes en 2D. En affichant la forme de X_train et X_test on a `(16640, 2) (4160, 2)`, indiquant des examples en deux dimensions avec 16640 examples d'entrainement contre 4160 en base de test (80%/20%)

2. **Arbre de decision**

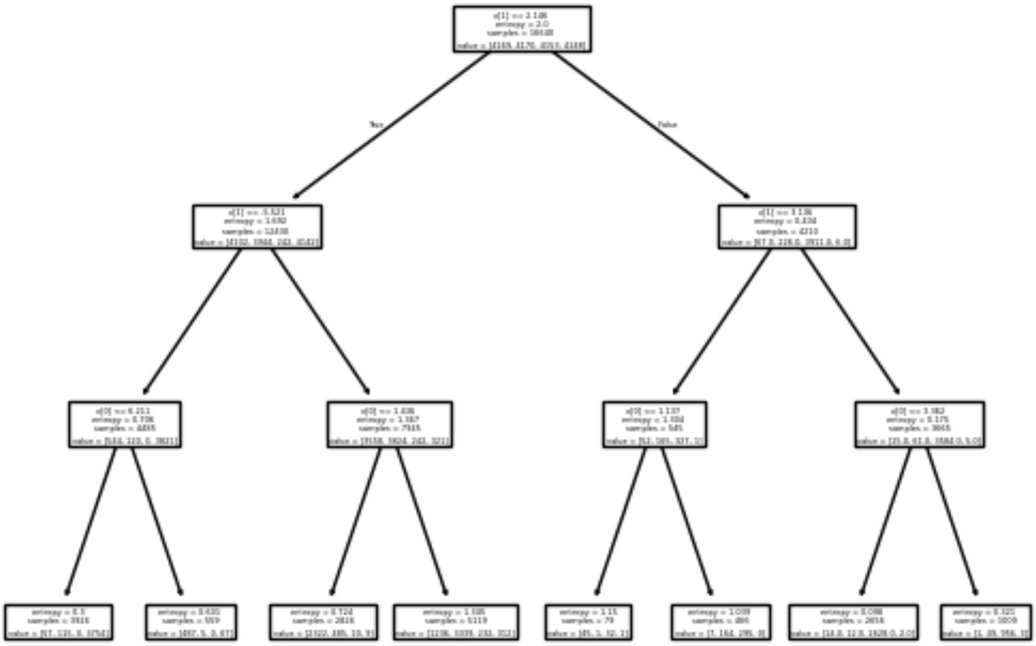
In [3]:
```python
from sklearn import tree
from sklearn.tree import DecisionTreeClassifier
from TP1a_ETU import visualize_classifier

accuracies = []
tree_max = None
max_acc = 0
max_depth_best = None

for i in range(3, 21):
    treei = DecisionTreeClassifier(criterion='entropy', max_depth=i
    treei.fit(X_train, y_train)

    acc = treei.score(X_test, y_test)
    accuracies.append(acc)
    if acc > max_acc:
        max_acc = acc
        tree_max = treei
        max_depth_best = i

    # if i == 8 or i == 20:
    #     plt.figure()
```

```python
    #     visualize_classifier(treei, X_train, y_train)

    if i == 3:
        visualize_classifier(treei, X_train, y_train)
        tree.plot_tree(treei)
        text_representation1 = tree.export_text(treei)
        print(text_representation1)

print(f"Max accuracy: {max_acc:.4f} at max_depth = {max_depth_best}
```





```
|--- feature_1 <= 2.15
|   |--- feature_1 <= -5.52
|   |   |--- feature_0 <= 6.21
|   |   |   |--- class: 4.0
|   |   |--- feature_0 >  6.21
|   |   |   |--- class: 1.0
|   |--- feature_1 >  -5.52
|   |   |--- feature_0 <= 1.44
|   |   |   |--- class: 1.0
|   |   |--- feature_0 >  1.44
|   |   |   |--- class: 2.0
|--- feature_1 >  2.15
|   |--- feature_1 <= 3.14
|   |   |--- feature_0 <= 1.14
|   |   |   |--- class: 1.0
|   |   |--- feature_0 >  1.14
|   |   |   |--- class: 3.0
|   |--- feature_1 >  3.14
|   |   |--- feature_0 <= 3.36
|   |   |   |--- class: 3.0
|   |   |--- feature_0 >  3.36
|   |   |   |--- class: 3.0

Max accuracy: 0.9303 at max_depth = 8
```



On constate que `max_depth` est la profondeur de l'arbre, soit le nombre de discriminateur binaire maximum pour atteindre une feuille. `visualize_classifier` permet de mapper le découpage en zones de classe induite par l'arbre. Quant à `tree.plot_tree` et `tree.export_text`, ils montrent tous deux la structure de l'arbre, avec pour chaques neuds les paramètres importants

In [4]:
```
from sklearn.metrics import classification_report, confusion_matrix

y_pred = tree_max.predict(X_test)

C=confusion_matrix(y_test, y_pred)
print(classification_report(y_test, y_pred))
print("Accuracy:", accuracy_score(y_test, y_pred))
```

```
              precision    recall  f1-score   support

         1.0       0.93      0.94      0.93      1031
         2.0       0.89      0.88      0.88      1030
         3.0       0.94      0.94      0.94      1047
         4.0       0.96      0.96      0.96      1052

    accuracy                           0.93      4160
   macro avg       0.93      0.93      0.93      4160
weighted avg       0.93      0.93      0.93      4160

Accuracy: 0.9302884615384616
```

Les valeurs obtenue représentent:

- **précision**: proportion de prédictions positives correctes parmi toutes les

prédictions positives.
$precision_i = \frac{TP_i}{TP_i + FP_i}$

- **rappel**: proportion de vrais positifs détectés parmi tous les vrais positifs.
  $recall_i = \frac{TP_i}{TP_i + FN_i}$

- **F1-score**: moyenne harmonique de la précision et du rappel, équilibrant les deux mesures.
  $f1score_i = 2 \cdot \frac{precision_i \cdot recall_i}{precision_i + recall_i}$

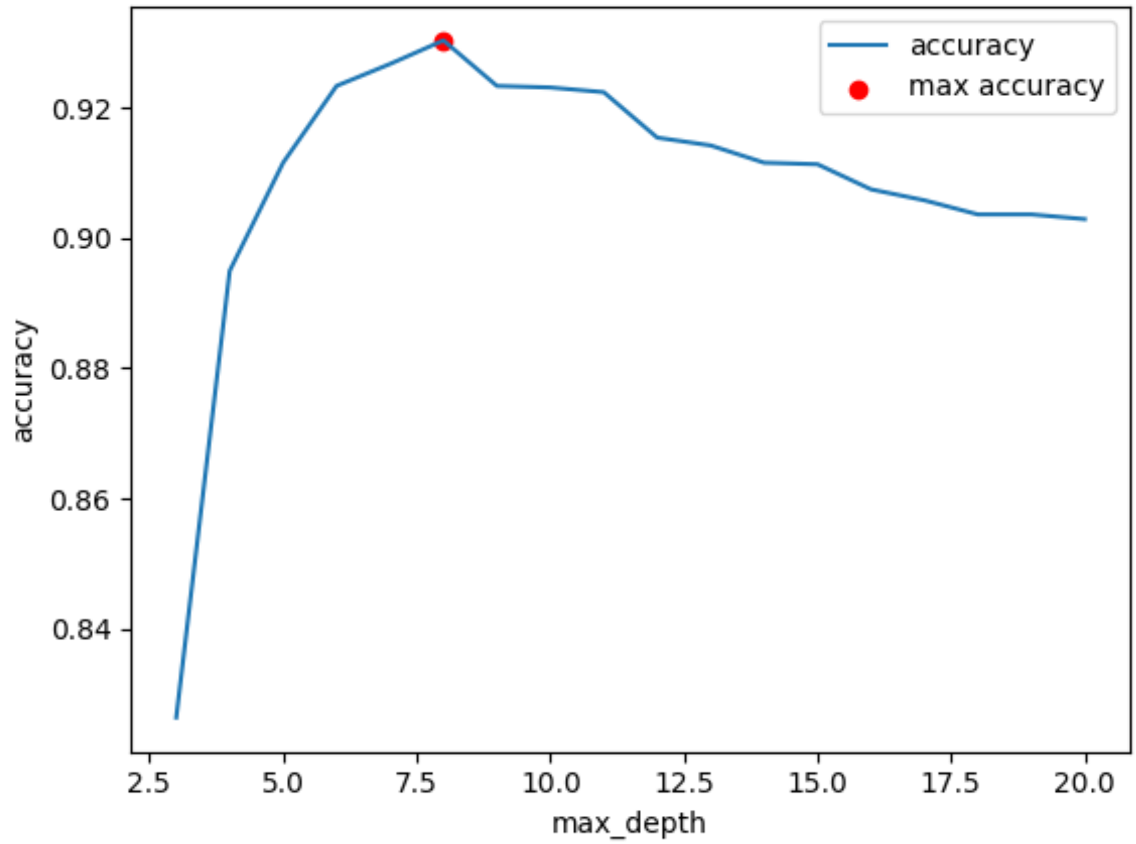
D'où:

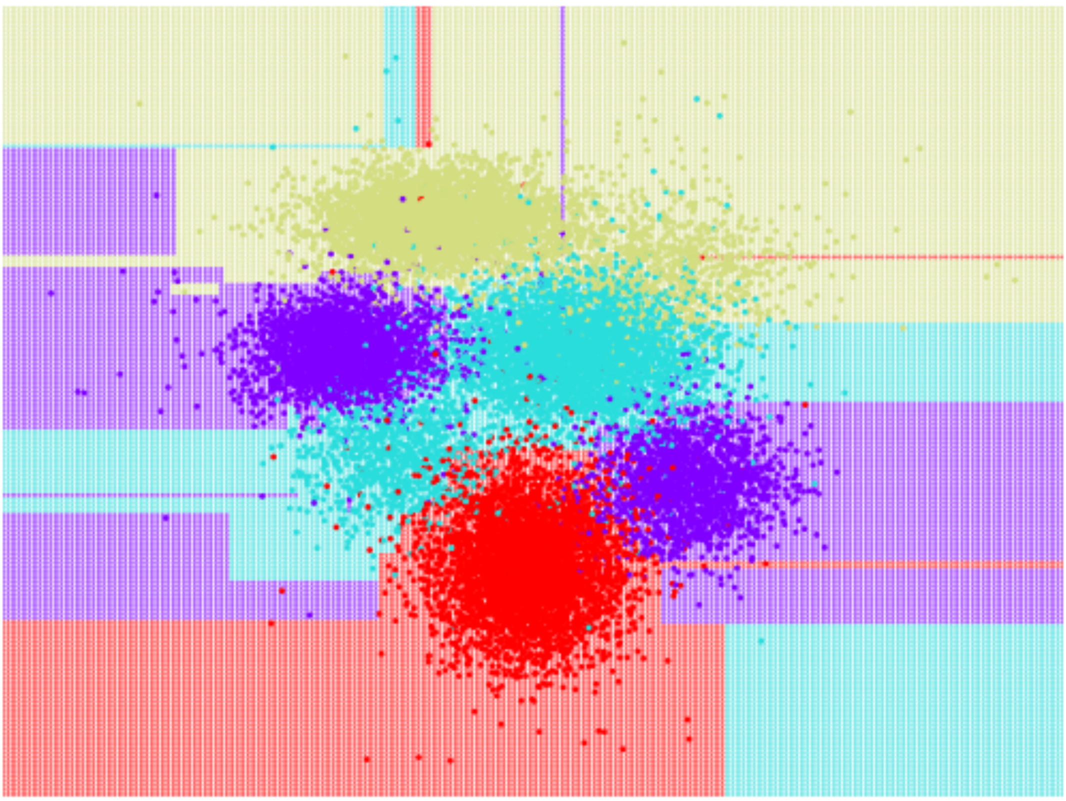
$$precision_1 = \frac{TP_1}{TP_1 + FP_1} = \frac{680}{680+139} \approx 0.83$$

$$recall_1 = \frac{TP_1}{TP_1 + FN_1} = \frac{680}{680+351} \approx 0.66$$

$$F1_1 = 2 \cdot \frac{precision_1 \cdot recall_1}{precision_1 + recall_1} = 2 \cdot \frac{0.83 \cdot 0.66}{0.83+0.66} \approx 0.73$$

In [5]:
```
plt.figure()
plt.plot(range(3, 21), accuracies, label='accuracy')
plt.xlabel('max_depth')
plt.ylabel('accuracy')
plt.scatter(max_depth_best, max_acc, color='red', label='max accura
plt.legend()
plt.show()

tree_best = DecisionTreeClassifier(criterion='entropy', max_depth=m
tree_best.fit(X_train, y_train)
visualize_classifier(tree_best, X_train, y_train)
```





On remarque qu'initialement, la précision augmente avec la profondeur de l'arbre. Cependant au bout d'un certain rang la précision diminue. En effet, il s'agit d'un exemple d'overfitting, où l'arbre est bien trop habitué aux données d'entrainement. On comprends ce comportement en regardant le partitionnement, ou des points outliers occupent une zone qui ne devrait pas être associé à leur classe.

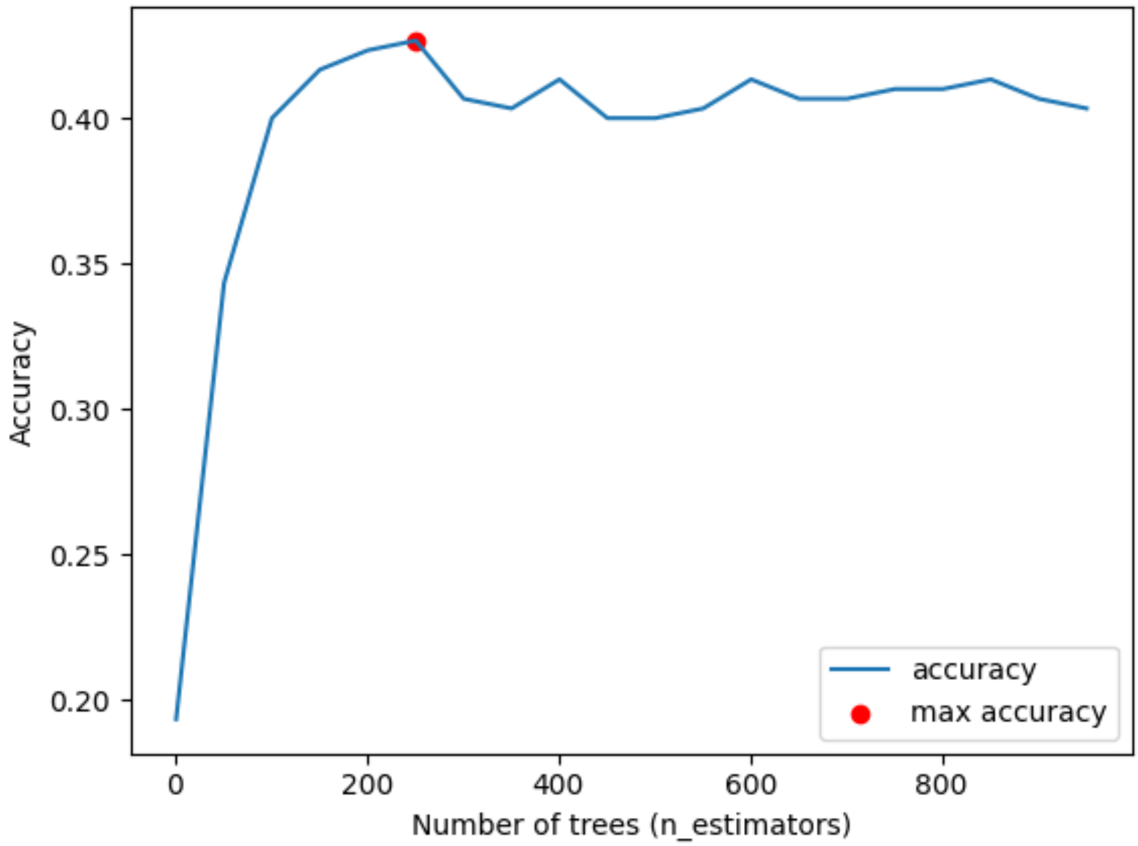
In [21]:
```
data = np.load("TP1b.npz")
X_train, y_train, X_test, y_test = (data[key] for key in ["X_train"

plt.show()
print(X_train.shape, X_test.shape)
```

```
(700, 100) (300, 100)
```

Les examples sont maintenant de dimension 100, pour 700 examples d'entrainement et 300 de test. Le classifieur précédent ne peux plus fonctionner ici.

Forêt d'arbres aléatoire

In [7]:
```
from sklearn.ensemble import RandomForestClassifier

accuracies_rf = []
rf_max_acc = 0
rf_max_n = None
rf_best_model = None
epsilon = 1e-3

test_range = range(1, 1000, 50)

for n_trees in test_range:
    RFi = RandomForestClassifier(criterion='entropy', n_estimators=
    RFi.fit(X_train, y_train.ravel())
    acc = RFi.score(X_test, y_test)
    accuracies_rf.append(acc)

    # Track the best RF model
    if acc > rf_max_acc + epsilon:
        rf_max_acc = acc
        rf_max_n = n_trees
        rf_best_model = RFi

# Plot accuracy vs number of trees
plt.figure()
plt.plot(test_range, accuracies_rf, label='accuracy')
plt.xlabel('Number of trees (n_estimators)')
plt.ylabel('Accuracy')
plt.scatter(rf_max_n, rf_max_acc, color='red', label='max accuracy'
plt.legend()
plt.show()

print(f"Best compromise accuracy/n_tree: {rf_max_acc:.4f} at n_esti
```




```
Best compromise accuracy/n_tree: 0.4267 at n_estimators = 251
```

In [13]:
```
from sklearn.metrics import classification_report, confusion_matrix

y_pred = rf_best_model.predict(X_test)
C=confusion_matrix(y_test, y_pred)
print(classification_report(y_test, y_pred))
print("Accuracy:", accuracy_score(y_test, y_pred))
```

```
              precision    recall  f1-score   support

           0       0.36      0.32      0.34        25
           1       0.50      0.61      0.55        31
           2       0.65      0.31      0.42        35
           3       0.35      0.52      0.42        23
           4       0.33      0.41      0.37        32
           5       0.67      0.46      0.54        35
           6       0.32      0.62      0.42        26
           7       0.62      0.14      0.23        36
           8       0.36      0.35      0.35        26
           9       0.44      0.61      0.51        31

    accuracy                           0.43       300
   macro avg       0.46      0.43      0.42       300
weighted avg       0.48      0.43      0.42       300

Accuracy: 0.4266666666666667
```

In [16]:
```
from sklearn.model_selection import GridSearchCV

grid_search_rf = GridSearchCV(
    estimator=RandomForestClassifier(random_state=61),
```

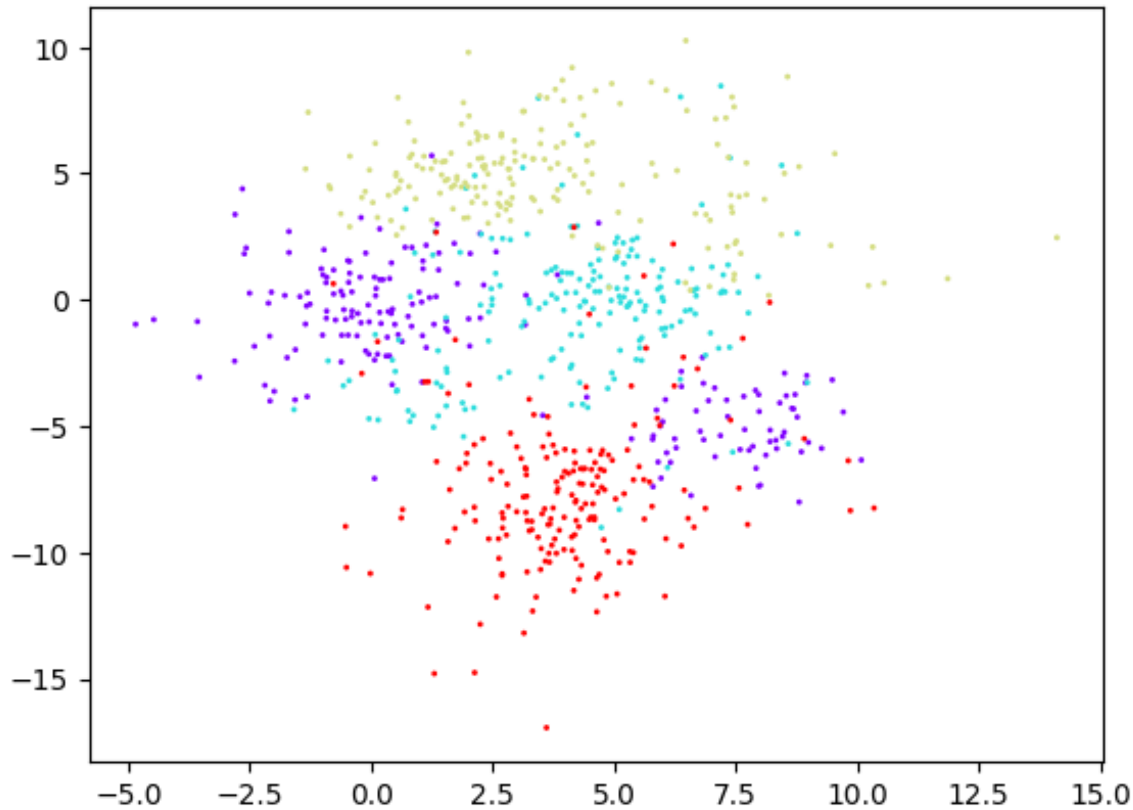
```
    param_grid={
        'n_estimators': [1, 2, 4, 8, 16, 32, 64, 128, 256, 512],
        'max_depth': [1, 2, 4, 8, 16]
    }
)

grid_search_rf.fit(X_train, y_train.ravel())
print(grid_search_rf.best_params_)

y_pred = grid_search_rf.best_estimator_.predict(X_test)
C=confusion_matrix(y_test, y_pred)
print(classification_report(y_test, y_pred))
print("Accuracy:", accuracy_score(y_test, y_pred))
```

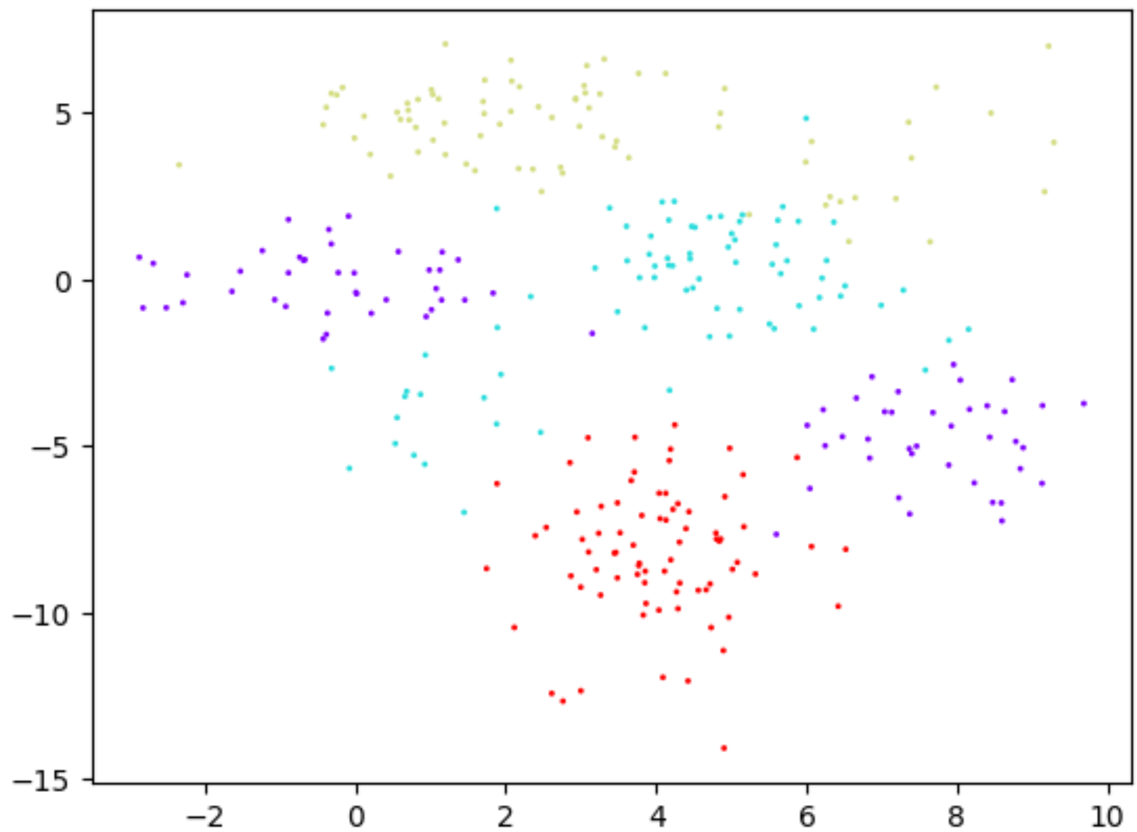
```
{'max_depth': 16, 'n_estimators': 512}
              precision    recall  f1-score   support

           0       0.52      0.52      0.52        25
           1       0.56      0.71      0.63        31
           2       0.53      0.23      0.32        35
           3       0.32      0.43      0.37        23
           4       0.36      0.44      0.39        32
           5       0.64      0.51      0.57        35
           6       0.33      0.54      0.41        26
           7       0.54      0.19      0.29        36
           8       0.48      0.46      0.47        26
           9       0.42      0.58      0.49        31

    accuracy                           0.45       300
   macro avg       0.47      0.46      0.45       300
weighted avg       0.48      0.45      0.44       300

Accuracy: 0.4533333333333333
```

---

2 - Biais / Variance dilema

In [17]:
```
data = np.load("TP1c.npz")
X_train, y_train, X_test, y_test = (data[key] for key in ["X_train"
plt.scatter(X_train[:, 0], X_train[:, 1], c=y_train, s=1, cmap='rai
plt.show()
plt.scatter(X_test[:, 0], X_test[:, 1], c=y_test, s=1, cmap='rainbo
plt.show()

print(X_train.shape, X_test.shape)
# print(y_train.shape, y_test.shape)
```



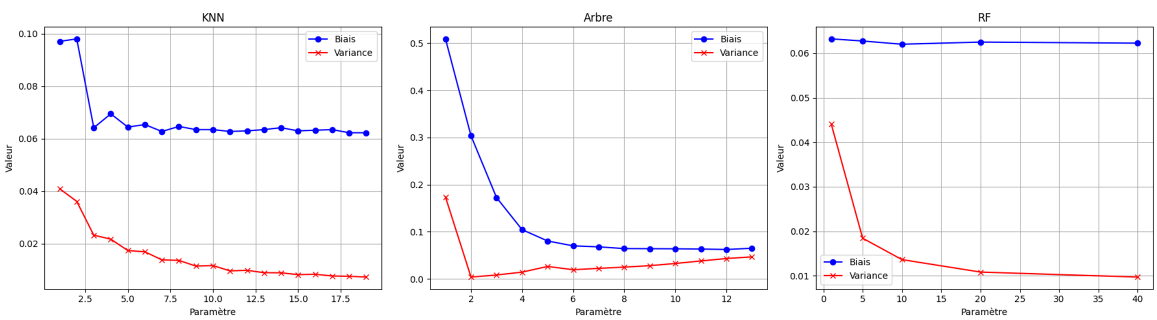



```
(748, 2) (312, 2)
```

In [18]:

```
import matplotlib.pyplot as plt
from TP1a_ETU import predict_multiple, get_results

Nrun = 30
proportion = 0.6

# range de parametres
```

```
knn_k_values = range(1, 20)
tree_depth_values = range(1, 14)
rf_n_estimators_values = [1, 5, 10, 20, 40]

# dictionnaire pour ranger les résultats
results = {
    'KNN': {'param': knn_k_values, 'biais': [], 'variance': [], "la
    'Arbre': {'param': tree_depth_values, 'biais': [], 'variance':
    'RF': {'param': rf_n_estimators_values, 'biais': [], 'variance'
}

# Prédiction et calcul de biais/variance pour chacuns
for k in knn_k_values:
    preds = predict_multiple(Nrun, proportion, model_type='knn', k=
    biais, variance = get_results(Nrun, preds)
    results['KNN']['biais'].append(biais)
    results['KNN']['variance'].append(variance)

for depth in tree_depth_values:
    preds = predict_multiple(Nrun, proportion, model_type='tree', k
    biais, variance = get_results(Nrun, preds)
    results['Arbre']['biais'].append(biais)
    results['Arbre']['variance'].append(variance)

for n in rf_n_estimators_values:
    preds = predict_multiple(Nrun, proportion, model_type='rf', k=n
    biais, variance = get_results(Nrun, preds)
    results['RF']['biais'].append(biais)
    results['RF']['variance'].append(variance)

# visualisation des résultats
fig, axes = plt.subplots(1, 3, figsize=(18, 5))

for ax, method in zip(axes, ['KNN', 'Arbre', 'RF']):
    param = results[method]['param']
    ax.plot(param, results[method]['biais'], label='Biais', color='
    ax.plot(param, results[method]['variance'], label='Variance', c
    ax.set_xlabel('Paramètre')
    ax.set_ylabel('Valeur')
    ax.set_title(method)
    ax.legend()
    ax.grid(True)

plt.tight_layout()
plt.show()
```



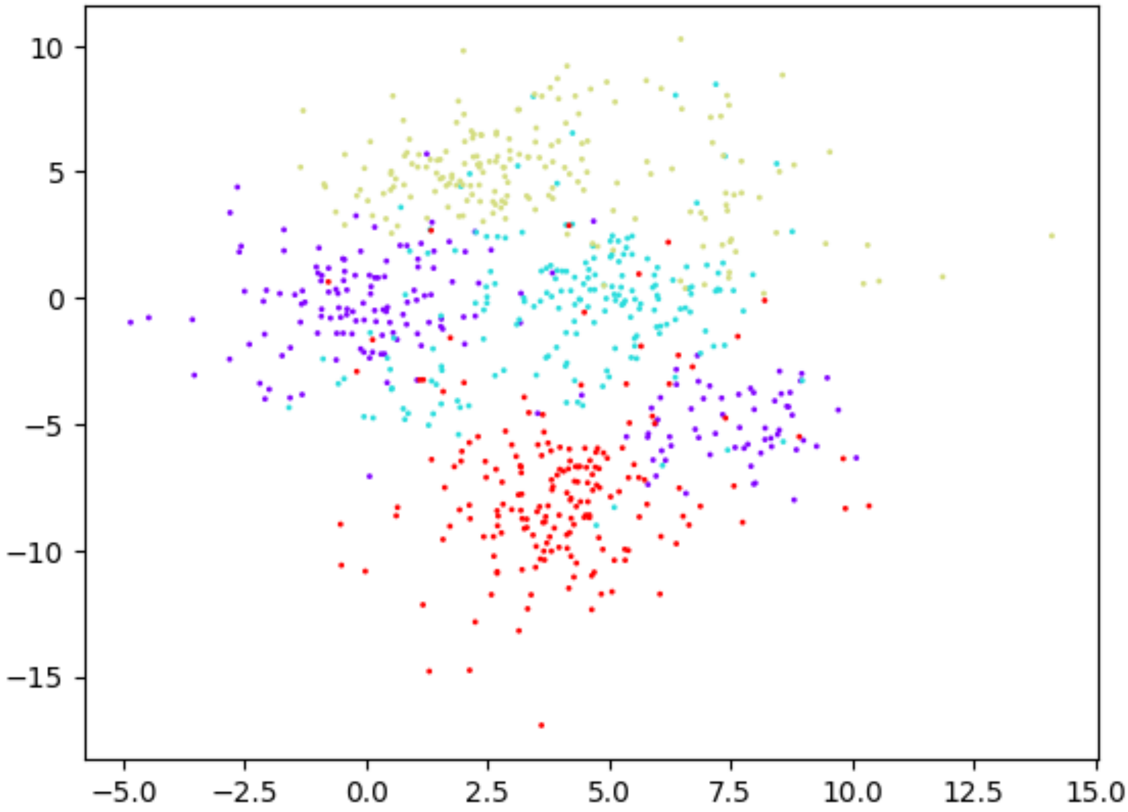
1. **KPPV**
   On observe que lorsque k augmente, le biais croît tandis que la variance décroît. Ce comportement correspond bien à la théorie : un petit k donne un modèle très flexible avec faible biais mais forte variance, alors qu'un grand k lisse les prédictions et augmente le biais mais réduit la variance. Le meilleur compromis est obtenu pour une valeur intermédiaire de k.

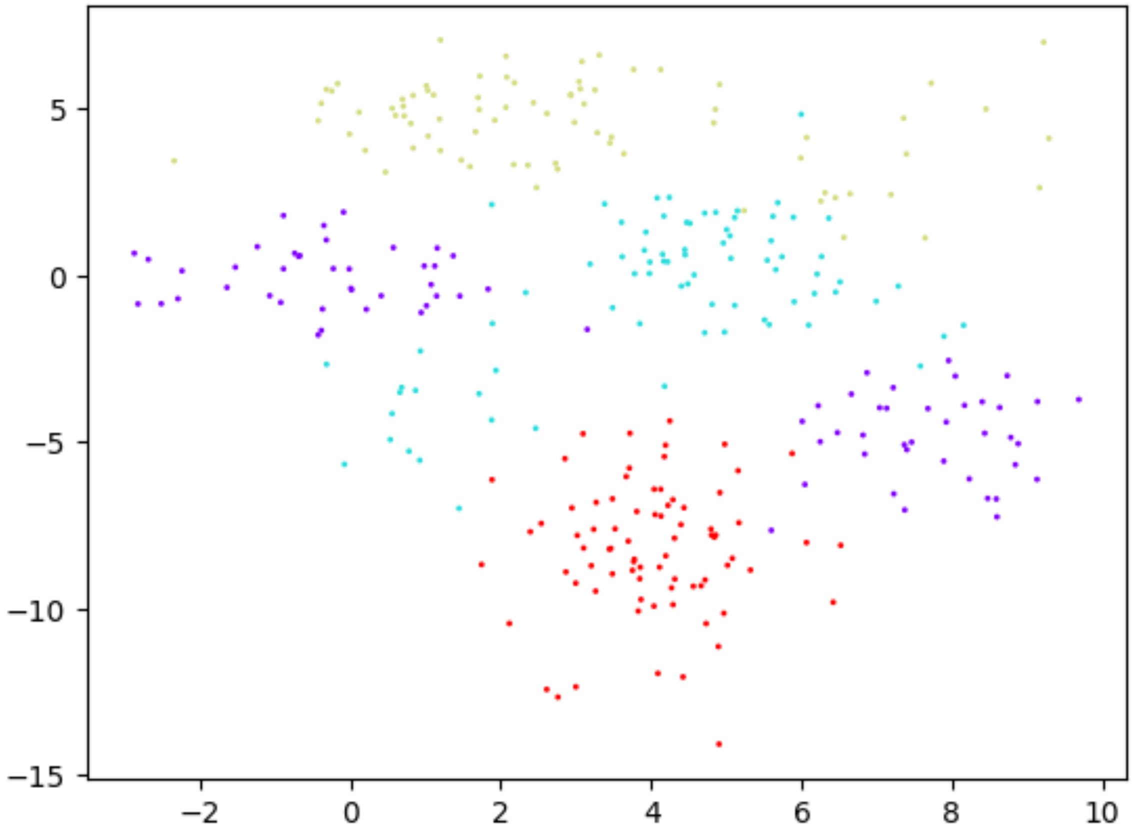
2. **Arbre de décision**
   Lorsque la profondeur maximale augmente, le biais diminue mais la variance augmente fortement, ce qui illustre l'overfitting. Le compromis optimal est atteint pour une profondeur intermédiaire, où l'arbre n'est ni trop simple ni trop spécifique.

3. **Forêt aléatoire**
   L'augmentation du nombre d'arbres réduit la variance grâce à l'agrégation des prédictions, tandis que le biais reste globalement stable. Le compromis biais/variance est donc amélioré par les forêts, qui offrent un classifieur plus robuste que les arbres seuls.

In [19]:
```
from sklearn.neighbors import KNeighborsClassifier
from sklearn.tree import DecisionTreeClassifier
from sklearn.ensemble import RandomForestClassifier

data = np.load("TP1c.npz")
X_train, y_train, X_test, y_test = (data[key] for key in ["X_train"
plt.scatter(X_train[:, 0], X_train[:, 1], c=y_train, s=1, cmap='rai
plt.show()
plt.scatter(X_test[:, 0], X_test[:, 1], c=y_test, s=1, cmap='rainbo
plt.show()

print(X_train.shape, X_test.shape)

best_knn = KNeighborsClassifier(n_neighbors=5)
best_tree = DecisionTreeClassifier(criterion='entropy', max_depth=5
best_rf = RandomForestClassifier(criterion='entropy', n_estimators=
best_knn.fit(X_train, y_train)
best_tree.fit(X_train, y_train)
best_rf.fit(X_train, y_train)

y_pred = best_knn.predict(X_test)
print(" KNN Results:")
C=confusion_matrix(y_test, y_pred)
print(classification_report(y_test, y_pred))
print("Accuracy:", accuracy_score(y_test, y_pred))

y_pred = best_tree.predict(X_test)
print(" Decision tree Results:")
C=confusion_matrix(y_test, y_pred)
print(classification_report(y_test, y_pred))
print("Accuracy:", accuracy_score(y_test, y_pred))
```

```
y_pred = best_rf.predict(X_test)
print(" Random Forest Results:")
C=confusion_matrix(y_test, y_pred)
print(classification_report(y_test, y_pred))
print("Accuracy:", accuracy_score(y_test, y_pred))
```





```
(748, 2) (312, 2)
 KNN Results:
              precision    recall  f1-score   support

         0.0       0.90      0.97      0.94        78
         1.0       0.90      0.88      0.89        78
         2.0       0.97      0.91      0.94        78
         3.0       0.97      0.97      0.97        78

    accuracy                           0.94       312
   macro avg       0.94      0.94      0.94       312
weighted avg       0.94      0.94      0.94       312

Accuracy: 0.9358974358974359
 Decision tree Results:
              precision    recall  f1-score   support

         0.0       0.90      0.79      0.84        78
         1.0       0.69      0.92      0.79        78
         2.0       0.99      0.86      0.92        78
         3.0       0.97      0.87      0.92        78

    accuracy                           0.86       312
   macro avg       0.89      0.86      0.87       312
weighted avg       0.89      0.86      0.87       312

Accuracy: 0.8621794871794872
 Random Forest Results:
              precision    recall  f1-score   support

         0.0       0.90      0.96      0.93        78
         1.0       0.87      0.88      0.88        78
         2.0       0.99      0.90      0.94        78
         3.0       0.96      0.97      0.97        78

    accuracy                           0.93       312
   macro avg       0.93      0.93      0.93       312
weighted avg       0.93      0.93      0.93       312

Accuracy: 0.9294871794871795
```