快乐舟教育

HappyBoat Education

Physics

Lecture Note

Module 7 Light and Special Relativity

Name: ________________________

Term: ________ Week: ________

Contents

1 Overview

Light and Special Relativity

Inquiry question: How does the behaviour of light affect concepts of time, space and matter?

Students:

- analyse and evaluate the evidence confirming or denying Einstein's two postulates:
  - the speed of light in a vacuum is an absolute constant
  - all inertial frames of reference are equivalent (ACSPH131)
- investigate the evidence, from Einstein's thought experiments and subsequent experimental validation, for time dilation $t = \frac{t_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$ and length contraction $l = l_0\sqrt{\left(1-\frac{v^2}{c^2}\right)}$, and analyse quantitatively situations in which these are observed, for example:
  - observations of cosmic-origin muons at the Earth's surface
  - atomic clocks (Hafele–Keating experiment)
  - evidence from particle accelerators
  - evidence from cosmological studies
- describe the consequences and applications of relativistic momentum with reference to:
  - $p_v = \frac{m_0 v}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$
  - the limitation on the maximum velocity of a particle imposed by special relativity (ACSPH133)
- Use Einstein's mass–energy equivalence relationship $E = mc^2$ to calculate the energy released by processes in which mass is converted to energy, for example: (ACSPH134)
  - production of energy by the sun
  - particle–antiparticle interactions, eg positron–electron annihilation
  - combustion of conventional fuel

2 The Aether Model

It was thought that **energy required a medium for transmission**. This led to a **hypothetical aether**: a medium for light propagates.

It was believed that light waves disturbed this invisible medium which **filled all of space**. It was termed the **"absolute reference frame"** because all objects in space move relative to this medium.

Definition 2.1 Aether

A **Aether** is is the material that fills the region of the universe above the terrestrial sphere. Also it is the medium for light to propagate.

2.1 The Characteristic of the Aether

- Perfectly Transparent
- Permeated ALL matter
- Stationary
- All objects in space move relative towards it
- Filled ALL of space (Because light travels everywhere, the aether must also exist anywhere)
- Low to No density (massless)
- Rigid AND/OR Elastic to support these high velocity waves
- No detectable physical or chemical properties – zero resistance

Definition 2.2 Absolute Frame of Reference

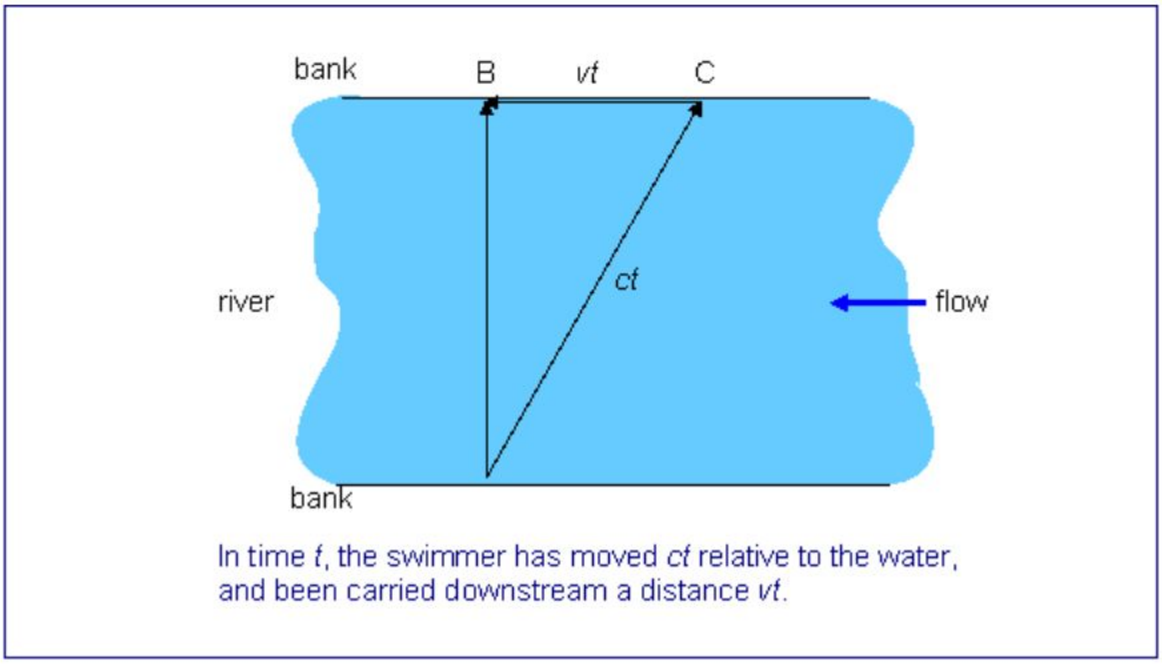
An **absolute frame of reference** is some fixed reference frame that every observer in the universe would agree is at rest at all times.

In relativity, no such reference frame exists.
The aether model provides an absolute stationary frame of reference.

2.2 Michelson-Morley Experiment

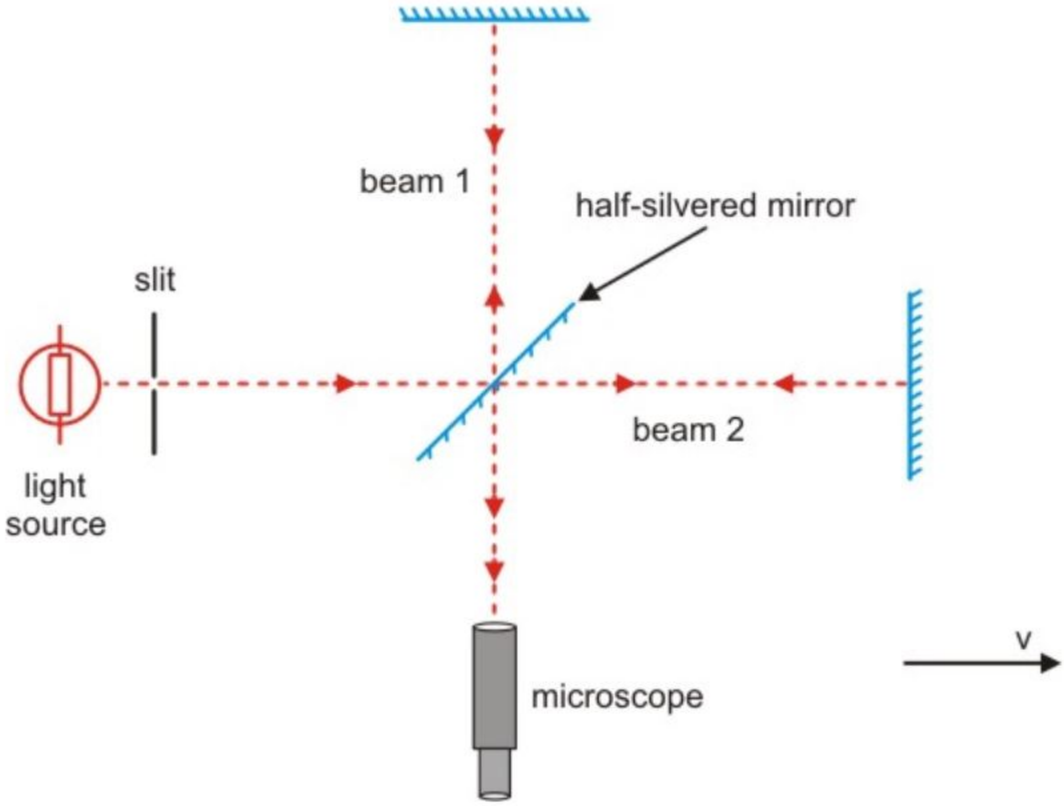
Just as the speed of sound is relative to the air, so the speed of light must be relative to the aether. This must mean, if you could measure the speed of light accurately enough, you could measure the speed of light travelling upwind, and compare it with the speed of light travelling downwind, and the difference of the two measurements should be twice the wind speed.

2.2.1 Premise



2.2.2 Experiment

A pulse of light is directed at an angle of 45 degrees at a half-silvered, half transparent mirror. In this case, half the pulse goes on through the glass, half is reflected. They both go on to distant mirrors which reflect them back to the half-silvered mirror. At this point, they are again half reflected and half transmitted, but a telescope is placed behind the half-silvered mirror as shown in the figure so that half of each half-pulse will arrive in this telescope. Now, if there is an aether wind blowing, someone looking through the telescope should see the halves of the two half-pulses to arrive at slightly different times, since one would have gone more upstream and back, one more across stream in general. To maximize the effect, the whole apparatus, including the distant mirrors, was placed on a large turntable so it could be swung around.



The apparatus would be placed in a dark room, so that there is no other light being observed.

2.2.3 The Null Result

There is no observation of any change in the interference pattern meant Michelson and Morley were unable to detect the aether. The failure of such a highly accurate apparatus to detect the aether was a blow to this belief, although it **did not disprove the existence of aether**. It opens the door for other thoughts.
One is **Einstein's Theory of Special Relativity**. One of the main postulates is that the **speed of light is constant** regardless of the velocity of the source or the observer.

3 Principle of Relativity (Galileo)

Definition 3.1 Inertial Frame of Reference

In classical physics and special relativity, an **inertial frame of reference** is a frame of reference that is not undergoing acceleration.

3.1 Principle of Relativity

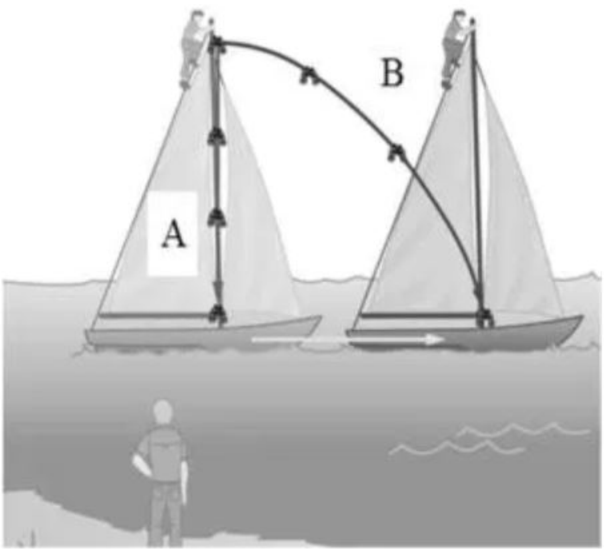
Galileo was the first to express the principle of relativity for mechanical motion.

Within an inertial frame of reference:

- The laws of mechanics are applicable to any experiments performed.
- It is not possible to determine the velocity of the frame of the reference using any mechanical experiments without an external frame of reference

3.2 Galileo's Thought Experiment

He proposed a thought experiment in which a ship is at rest first and then at constant velocity. Inside the hold of the ship all objects, including the air, have the same velocity as the ship and so there is no relative velocity between them. Relative to an observer on a boat, a dropped boat would fall vertically as there is no relative motion between the two as the ball is dropped and there is no lateral acceleration on the inertial frame of reference. However, for an observer on the shore, the initial velocity of the ball would depend on the ship's velocity which would appear to not fall vertically.

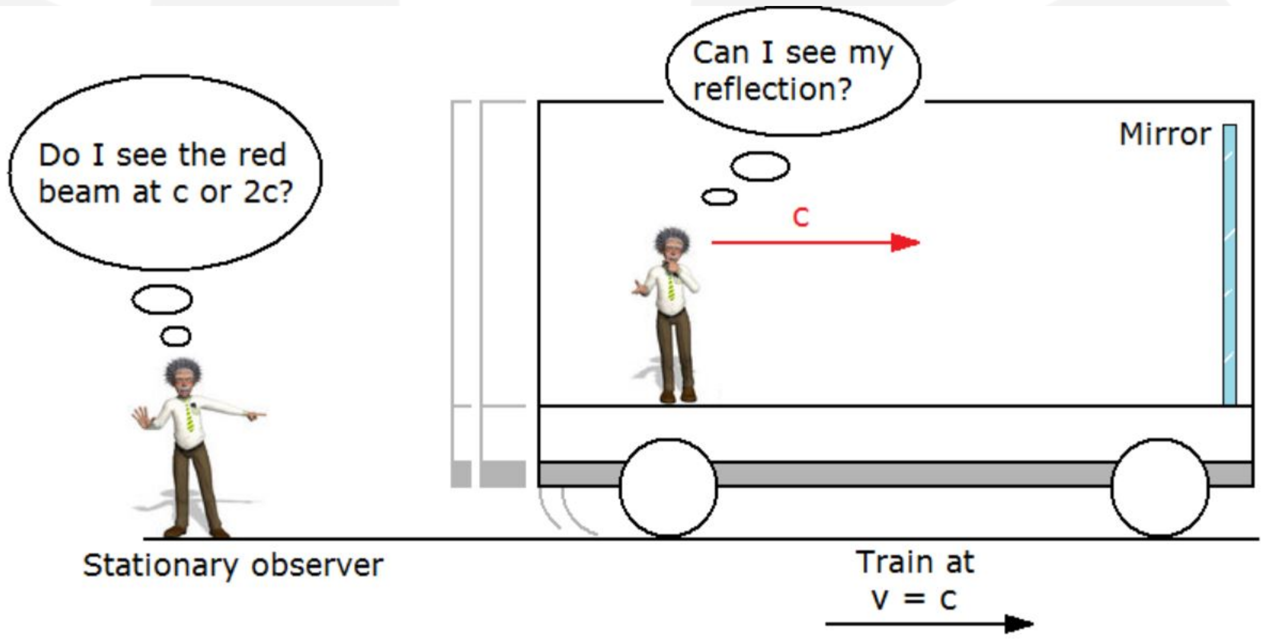


Velocity is only able to be obtained by measuring displacements to other objects outside of the frame of reference.
All movement can be characterised by relative velocities, and that there is no **absolute frame of reference** which is preferred for all motion calculation.

4 Special Relativity

4.1 The mirror thought experiment

Einstein had a thought experiment in which he imagined he was travelling at the speed of light while holding a mirror stretched out in front of him. He was trying to work out whether he would still see his reflection in the mirror.



No
Because light From his face will not reach the mirror and back. However he will therefore be able to work out that he is travelling at the speed of light which violates our foundational definition of inertia frame of reference. **Yes**
If he does see his refraction this means that a stationary observer will measure the speed of light travelling from his face to mirror and back at 2 times the speed of light, which the beam of light on which he is travelling was at a speed of c.

Einstein preferred YES
To address this issue, Einstein proposed that the stationary observer will see the normal speed of light regardless of the relative velocity between two frames of reference. Their observation of everything else on the train will change to ensure the laws of physics still hold.

4.2 The Theory of Special Relativity

Einstein's special relativity contains two postulates:

4.2.1 The principle of Relativity

Within an inertial frame of reference:

- The laws of mechanics are applicable to any experiments performed.
- It is not possible to determine the velocity of the frame of the reference using any mechanical experiments without an external frame of reference

4.2.2 The principle of Invariant Light Speed

Light in a vacuum is always observed to have a velocity of c in empty space regardless of the speed of the source or the observer.

Newtonian physics had considered the passage of time to be progressing at a constant rate. The proposal of constancy of the speed of light means that time and length become relative value and will change depending on the relative velocity between object and the observer.

4.3 The relativity of Simultaneity

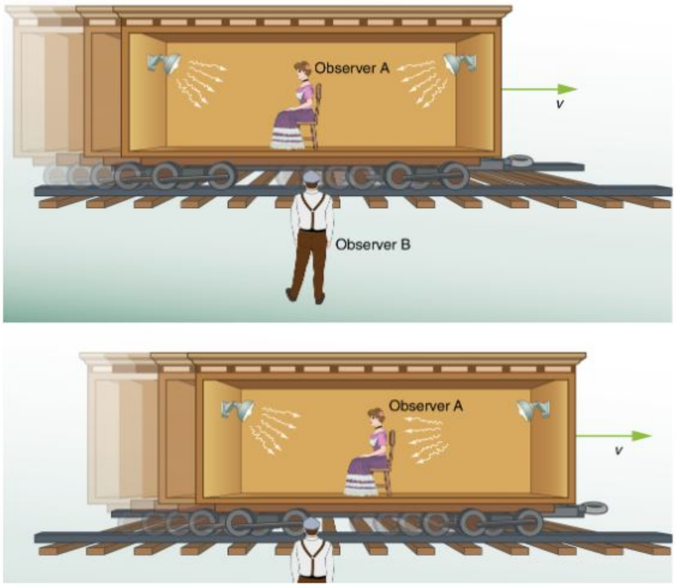
In physics, the relativity of simultaneity is the concept that distant simultaneity:

Definition 4.1 The relativity of Simultaneity

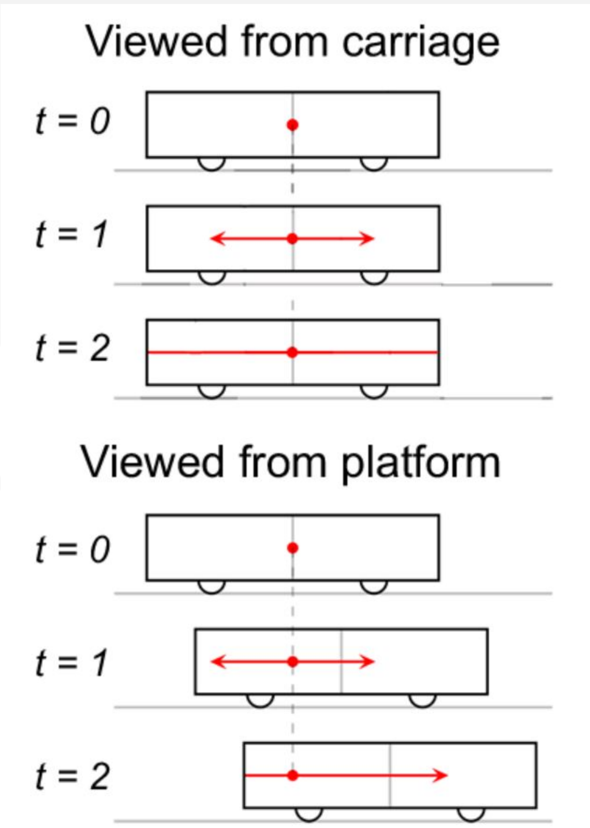
> Whether two spatially separated events occur at the same time – is not absolute, but depends on the observer's reference frame.

4.3.1 Einstein's train

Einstein's version of the experiment presumed that one observer was sitting midway inside a speeding train and another was standing on a platform as the train moved past. As measured by the standing observer, the train is struck by two bolts of lightning simultaneously, but each at the back and front of the train.


Observer A
v
Observer B
Observer A
v

Analysis

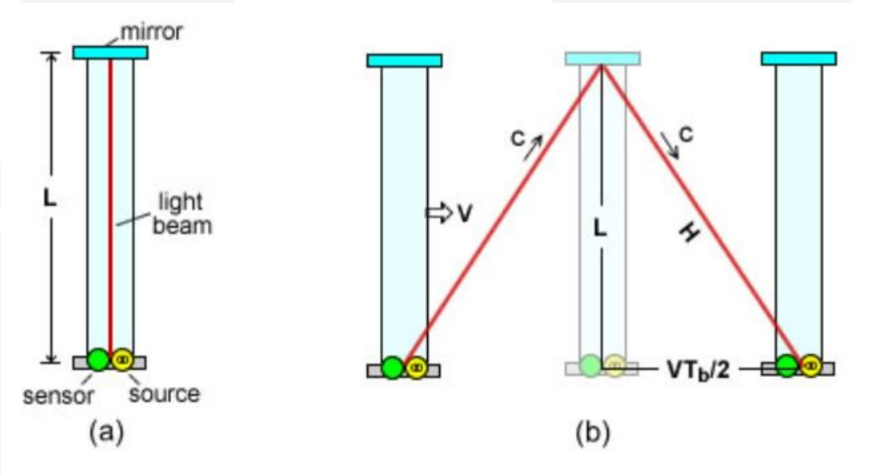

Viewed from carriage
t = 0
t = 1
t = 2
Viewed from platform
t = 0
t = 1
t = 2

4.4 Consequences of the Theory of Special Relativity

It has been established that the speed of light is not addictive which means it is impossible to make a speed of light greater via relative motion from the source to all the observer.

4.4.1 Time dilation

Slowing of time at the speed of light Time dilation is a difference in the elapsed time measured by two clocks due to them having a velocity relative to each other.



Deriving the equation

$$(ct)^2 = (vt)^2 + (ct_0)^2$$

$$c^2t^2 - v^2t^2 = c^2t_0^2$$

$$t^2(c^2 - v^2) = c^2t_0^2$$

$$t^2\left(1 - \left(\frac{v^2}{c^2}\right)\right) = t_0^2$$

$$t^2 = \frac{t_0^2}{\left(1 - \left(\frac{v^2}{c^2}\right)\right)}$$

$$t = \frac{t_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

where:
t = Time measured from the observer outside the frame of reference

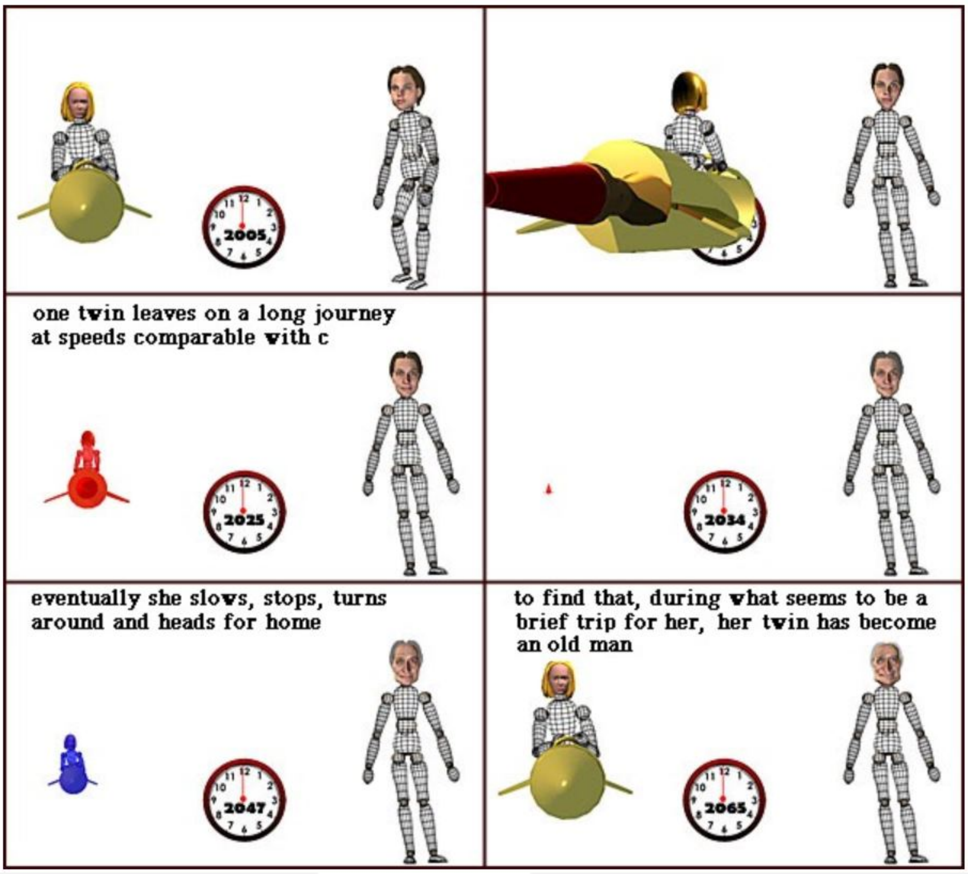
t_0 = Time measured from the observer inside the frame of reference

v = relative speed between two object

c = speed of the light

4.4.2 The twins Paradox

In physics, the twin paradox is a thought experiment in special relativity involving identical twins, one of whom makes a journey into space in a high-speed rocket and returns home to find that the twin who remained on Earth has aged more.



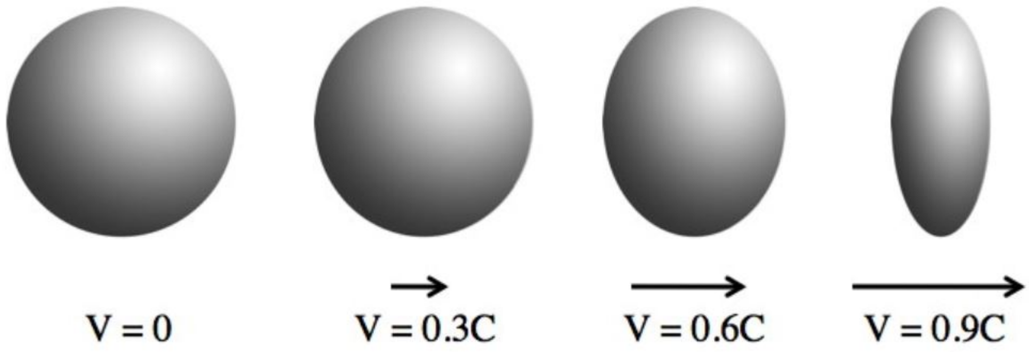
This result appears puzzling because each twin sees the other twin as moving. So according to an incorrect and naïve application of time dilation and the principle of relativity, each should paradoxically find the other to have aged less.

Explanation
However, this scenario can be resolved within the standard framework of special relativity: the travelling twin's trajectory involves two different inertial frames, one for the outbound journey and one for the inbound journey.

Another way of looking at it is by realizing that the travelling twin is undergoing acceleration, which makes him a non-inertial observer. In both views there is no symmetry between the spacetime paths of the twins. Therefore, the twin paradox is not a paradox in the sense of a logical contradiction.

4.4.3 Length contraction

Length contraction is the phenomenon that a moving object's length is measured to be shorter than its proper length, which is the length as measured in the object's own rest frame. **The one travelling near the speed of light would appear shorter.**



It can be calculated by:

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

where:

L = Time measured from the observer outside the frame of reference

L_0 = Time measured from the observer inside the frame of reference

v = relative speed between two object

c = speed of the light

4.4.4 Mass dilation/Relativistic Momentum

As an object accelerates, the mass of the object increases compared with its mass when at rest. This results in the energy put into accelerating the object becoming progressively less effective in terms of increasing the object's velocity as the velocity of the object increases.

Relativistic Momentum

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Mass Dilation

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where: m_v = Relativistic mass

m_0 = Rest mass

v = relative speed between two object

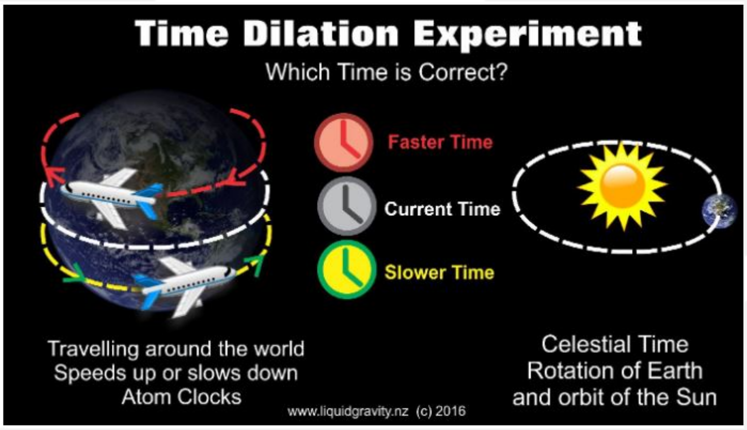
c = speed of the light

4.5 Evidences

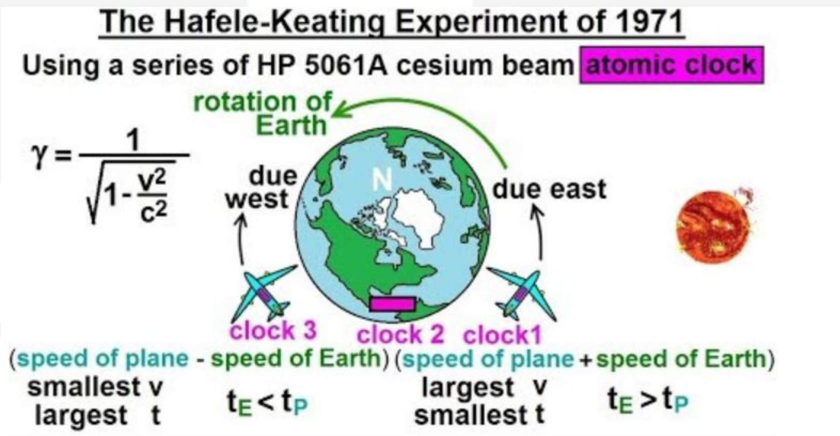
4.5.1 Time dilation

Caesium Atomic Clocks

Caesium atomic clocks are highly accurate clocks which can measure time down to one billionth of a second.



In 1971, Hafele and Keating placed four caesium atomic clocks on board a commercial airline and flew them twice around the world, one eastwards and one westwards and compared the time with an Earth-based reference clock.



The difference in time was measured and within the experimental error, it coincided with the calculated results using the relativistic equation. Thus it provides the evidence for the Theory of Relativity.

Cosmological Studies

The stars observed in Earth's sky are at different distances away. The time it takes for light to be observed depends on the distance between the source and the observer.

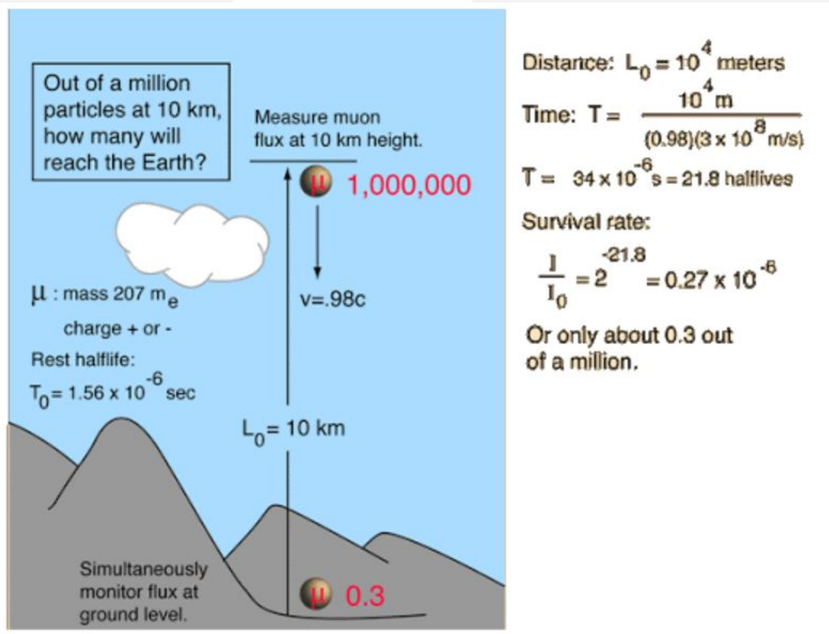
Consider a rapidly retreating star that emits pulses of light each year, in the time between two pulses, the star will have moved further from Earth and the second beam of light will have to travel further to reach Earth. Thus this time between two pulses will be longer than one year.

4.5.2 Length contraction

Muon's experiment

Muons are subatomic particles which are formed in the upper atmosphere. Muons have relatively short half-lives (about 2 ms) and it is because of this short half-life that it would be impossible to detect a large number of muons reaching the surface of the Earth from the upper atmosphere since most of them would have decayed.

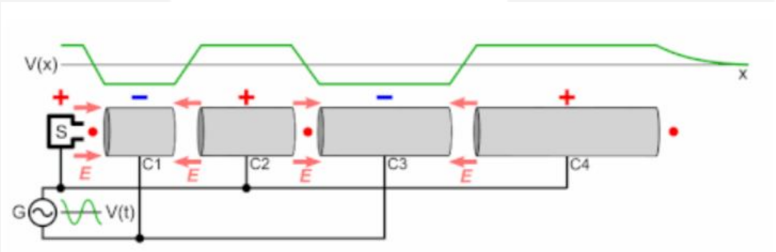
However, In 1941 scientists Rossi and Hall record the number muons at a particular altitude per hour. It travelled at 0.994c. By calculation, it was deduced that only 25 muons per hour would reach the sea level. However, it was measured that there exists over 400 muons per hour at sea level.



The only explanation for this is time dilation and length contraction. From our (Earth) perspective, the half-life of the muons increased, however, from the muon's perspective, the length of the journey has decreased.

4.5.3 Mass Dilation

Particle Accelerator Relativistic mass increase has been tested on elementary particles. Electrons can be accelerated to over 0.9994c inside a particle accelerator. These electrons are allowed to collide with a target. We can calculate the mass from the data. The increases in the mass agree with the calculation provided by Einstein (within experimental error).



5 Limitation of Maximum Velocity as Imposed by Special Relativity

According to

$$a = \frac{F}{m}$$

acceleration will approach zero when mass approaches infinity near the speed of light. Because acceleration becomes zero at the point where speed equals the speed of light for an object. This means the maximum speed of any particle is the speed of light. So we cannot exceed the speed of light.