Ejercicio 2

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = w$$

$$\frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{(\Delta x)^2} + \frac{U_{i,j+1} \cancel{+} 2U_{i,j} + U_{i,j-1}}{(\Delta y)^2}$$

$$= w$$

$(\Delta x)^2 = h$, $(\Delta y)^2 = h$

En ~~x~~ y y las pasos son iguales

$$\Rightarrow \frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j} + U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{h^2} = w$$

Multiplicando h^2 com w y despejando $U_{i,j}$

$$\boxed{U_{i,j} = \frac{1}{4}\left(U_{i+1,j} + U_{i-1,j} + U_{i,j+1} + U_{i,j-1} + w_{ij} h^2 \right)}$$

Ahora

$$\nu\left(\frac{\partial^2 \omega}{\partial x^2}+\frac{\partial^2 \omega}{\partial y^2}\right)=\frac{\partial U}{\partial y}\frac{\partial \omega}{\partial x}-\frac{\partial U}{\partial x}\frac{\partial \omega}{\partial y}$$

Al igual que antes

$$\frac{\partial^2 \omega}{\partial x^2}+\frac{\partial^2 \omega}{\partial y^2}=\frac{\omega_{i+1,j}-4\omega_{i,j}+\omega_{i-1,j}+\omega_{i,j+1}+\omega_{i,j-1}}{h^2}$$

$$\frac{\partial U}{\partial y}\frac{\partial \omega}{\partial x}=\frac{U_{i,j+1}-U_{i,j-1}}{\Delta x}\;\frac{\omega_{i+1,j}-\omega_{i-1,j}}{\Delta y}$$

$$\Delta x=\Delta y=2h$$

$$\frac{\partial U}{\partial x}\frac{\partial \omega}{\partial y}=\frac{U_{i+1,j}-U_{i-1,j}}{2h}\cdot\frac{\omega_{i,j+1}-\omega_{i,j-1}}{2h}$$

$$U_{i+1,j}-U_{i-1,j}\cdot\omega_{i,j+1}-\omega_{i,j-1}=b$$

Para el denominador del otro término llamemoslo a

$$w_{i,j+1}+w_{i,j-1}+w_{i+1,j}+w_{i-1,j}-4w_{i,j}$$

$$=\frac{1h^2}{r}\left(\frac{a}{4h^2}-\frac{b}{4h^2}\right)=\frac{1}{r}\left(\frac{a}{4}-\frac{b}{4}\right)$$

Despejando $w_{i,j}$ $\qquad \frac{1}{r}=R$

$$w_{ij}=\frac{w_{i,j+1}+w_{i,j-1}+w_{i+1,j}+w_{i-1,j}}{4}+\frac{b}{r16}-\frac{a}{r16}$$

$$w_{ij}=\frac{1}{4}\left(w_{i+1,j}+w_{i-1,j}+w_{i,j+1}+w_{i,j-1}\right)$$

$$-\frac{R}{16}\left((U_{i,j+1}-U_{i,j-1})(w_{i+1,j}-w_{i-1,j}\right)$$

$$+\frac{R}{16}\left(\left(U_{i+1,j}-U_{i-1,j}\right)\left(w_{i,j+1}-w_{i,j-1}\right)\right)$$