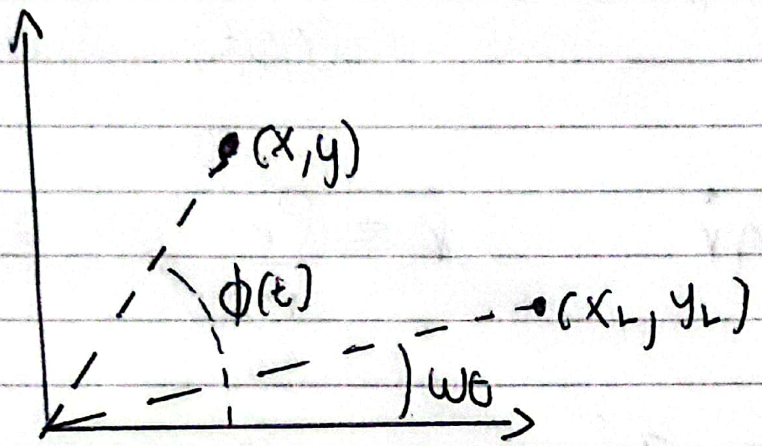
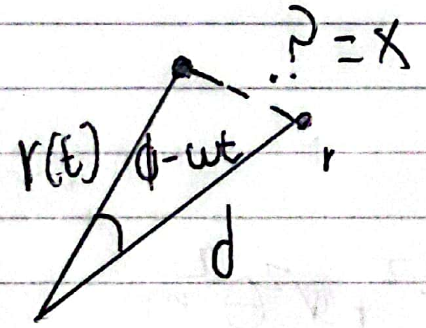
Ejercicio 1.c



Tenemos que el triangulo esta dado por



Por Teorema del coseno

$$x^2 = a^2 + b^2 - 2ab\,\cos C$$

$$x^2 = r^2 + d^2 - 2r(t)d\cos(\phi - \omega t)$$

$$\boxed{x = \sqrt{r^2 + d^2 - 2r(t)d\cos(\phi - \omega t)}}$$

d)

$$L = T - U = \frac{1}{2} m v^2 = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\phi}^2) - \left(\frac{G m m_T}{r} + \frac{G m m_L}{r_L} \right)$$

$$p_r = \frac{\partial L}{\partial \dot{r}} = m \dot{r} \quad ; \quad p_\phi = \frac{\partial L}{\partial \dot{\phi}} = m r^2 \dot{\phi}$$

$$; \quad \frac{p_\phi}{m r^2} = \dot{\phi} \quad ; \quad \frac{p_r}{m} = \dot{r}$$

Reemplazando:

$$\frac{1}{2} m \left(\frac{p_r^2}{m^2} \right) = \frac{1}{2m} p_r^2 \quad ; \quad \frac{1}{2} m r^2 \dot{\phi}^2 = \frac{1}{2} m r^2 \left(\frac{p_\phi^2}{m^2 r^4} \right)$$

$$= \frac{1}{2} \frac{p_\phi^2}{m r^2}$$

Finalmente

$$\boxed{H = \frac{1}{2m} p_r^2 + \frac{1\, p_\phi^2}{2 m r^2} - \frac{G m m_T}{r(t)} - \frac{G m m_L}{r_L}}$$

e) Tenemos que las ecuaciones de Hamilton

$$\frac{\partial H}{\partial p_i} = \dot{q}_i \quad y \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Donde $\dot{q}_i = \dot{r} + \dot{\phi}$ $\quad \dot{p}_i = \dot{p}_r + \dot{p}_\phi$

Entonces sustituyendo para $\dot{r}$

$\dot{r} = \frac{\partial H}{\partial p_r} \rightarrow$ derivando el termino $\left(\frac{p_r^2}{2m}\right)$ los otros son ctes el la derivada parcial

$$= \frac{p_r}{m}$$

Para $\dot{\phi}$

$$\dot{\phi} = \frac{\partial H}{\partial p_\phi} \rightarrow \frac{\partial}{\partial p_\phi}\left(\frac{p_\phi^2}{2mr^2}\right) = \boxed{\frac{p_\phi}{mr^2}}$$

Ahora usando la segunda ecuación tenemos que:

$$\dot{p}_r = -\frac{\partial H}{\partial r} = -\frac{\partial H}{\partial r}\left(\frac{p_r^2}{2m}\right) + \frac{\partial H}{\partial r}\left(\frac{p_\phi^2}{2mr^2}\right)$$

$$+ \frac{\partial H}{\partial r}\left(-\frac{Gm M_T}{r}\right) + \left(\frac{\partial H}{\partial r}\left(-\frac{G m M_L}{r_L(r,\phi,t)}\right)\right)$$

$$\frac{\partial H}{\partial r}\left(\frac{p_r^2}{2m}\right) = 0 \; ; \; \frac{\partial H}{\partial r}\left(\frac{p_\phi^2 r^{-2}}{2m}\right) = -\frac{p_\phi^2}{2mr^3}$$

$$\frac{\partial H}{\partial r}\left(-GmM_T r^{-1}\right) = \frac{GmM_T}{r^2}$$

interna

$$\frac{\partial H}{\partial r}\left(-\frac{GmM_L}{r_L(r,\phi,t)}\right) = \frac{+GmM_L}{2((r(t))^2+d^2-2rd\cos(\phi-\omega t))^{3/2}} \cdot \frac{d(r_L)}{dr}$$

$$= + \frac{GmM_L}{2(r^2+d^2-2rd\cos(\phi-\omega t))^{3/2}}(2r-2d\cos(\phi-\omega t))$$

$$= \boxed{\frac{GmM_L}{(r_L(r,\phi,t))^3}(r-d\cos(\phi-\omega t))}$$

Entonces

$$\dot{p}_r = \frac{p^2_\phi}{mr^3} - \frac{GmM_T}{r^2} - \frac{GmM_L}{r_L(r,\phi,t)^3}(r - d\cos(\phi - \omega t))$$

Ahora con $\dot{p}_\phi$

$$\dot{p}_\phi = -\frac{\partial H}{\partial \phi} = -\frac{\partial}{\partial \phi}\left(-\frac{GmM_L}{r_L(r,\phi,t)}\right)$$

$$= -\frac{GmM_L}{2r_L(r,\phi,t)}\frac{\partial}{\partial \phi}\left(r^2+d^2-2rd\cos(\phi-\omega t)\right)$$

$$= -\frac{GmM_L}{2r_L(r,\phi,t)}(2rd\,\text{sen}(\phi-\omega t))$$

$$= -\frac{GmM_L}{r_L(r,\phi,t)}rd\,\text{sen}(\phi-\omega t)$$

f) Primero tenemos que:

$$\tilde{r} = \frac{r}{d} \rightarrow \tilde{r}d = r \rightarrow \dot{\tilde{r}}d = \dot{r} \quad ; \quad p_r = md\tilde{p}_r$$

Sustituyendo:

$$\dot{\tilde{r}}\cancel{d} = \frac{m\cancel{d}\tilde{p}_r}{\cancel{m}} \rightarrow \dot{\tilde{r}} = \tilde{p}_r$$

Ahora

$$\dot{\phi} = \frac{p_\phi}{mr^2} \rightarrow \dot{\phi} = \frac{\tilde{p}_\phi md^2}{\cancel{m}(d\tilde{r})^2} = \boxed{\frac{\tilde{p}_\phi}{\tilde{r}^2}}$$

$$\dot{\tilde{p}}_r md = \frac{\tilde{p}_\phi \cancel{m} d^{2\cancel{4}}}{\cancel{m}(d\tilde{r})^3} - \frac{Gmm_T}{d^2\tilde{r}^2} - \frac{Gmm_L d}{d^3 \tilde{r}_L'(r,\phi,t)^3}[\tilde{r} - \cos(\phi - \omega t)]$$

$$\dot{\tilde{p}}_r = \frac{\tilde{p}_\phi}{\tilde{r}^3} - \frac{Gmm_T}{md^3\tilde{r}^2} - \frac{Gm_L m_T}{d^3 m_T \tilde{r}'^3}[\tilde{r} - \cos(\phi - \omega t)]$$

$$\Delta = \frac{Gm_T}{d^3} \quad , \quad \mu = \frac{m_L}{m_T}$$

$$\dot{\tilde{p}}_r = \frac{\tilde{p}_\phi}{\tilde{r}^3} - \Delta\left(\frac{1}{\tilde{r}^2} + \frac{\mu}{\tilde{r}'^3}(\tilde{r} - \cos(\phi - \omega t))\right)$$

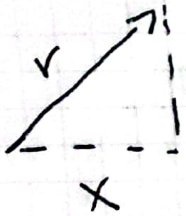
$$\dot{\tilde{p}}_\phi \, m d^2 = -\frac{G m M_L}{d^3 \tilde{r}'^3} \tilde{r} d^2 \, \mathrm{sen}(\phi - \omega t)$$

$$\dot{\tilde{p}}_\phi = -\frac{G M_L}{d^3 \tilde{r}'^3} \frac{M_T}{M_T} \tilde{r} \, \mathrm{sen}(\phi - \omega t)$$

$$\boxed{\dot{\tilde{p}}_\phi = -\frac{\Delta \mu \tilde{r}}{\tilde{r}'^3} \, \mathrm{sen}(\phi - \omega t)}$$

g)

$$\tilde{\rho}_r^{\circ} = \frac{\rho_r}{md} = \frac{\cancel{m}V_r}{\cancel{m}d} = \frac{dr}{d\cdot dt} = \frac{d(\sqrt{x^2+y^2})}{d\cdot dt}$$



$\to$ Pitágoras

Ahora con regla de la cadena:

$$= \frac{1}{\cancel{2}\sqrt{x^2+y^2}}\cdot(2x\,dx+2y\,dy) = \frac{x\,dx+y\,dy}{\sqrt{x^2+y^2}}$$

Sustituyendo

$$= \frac{x\,dx+y\,dy}{d\cdot r} = \frac{x\dot{x}+y\dot{y}}{d\cdot r}$$

; $\dot{x}\to V_0$ en x

$\dot{y}\to V_0$ en y

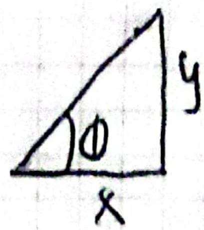
$\cos(x-y)=\cos x\cos y+\operatorname{sen}x\operatorname{sen}y$

$x\to r_{0x}$; $y\to r_{0y}$

$$= \frac{x V_{0x}+y V_{0y}}{d\cdot r} = \frac{r_{0x} V_{0x}+r_{0y} V_{0y}}{d\cdot r_0} = \frac{\cancel{r_0}\cos\phi\, V_0\cos\theta + r_0\operatorname{sen}\phi\cos\theta}{d\,r_0}$$

$$= \frac{V_0(\cos\phi\cos\theta+\operatorname{sen}\phi\operatorname{sen}\theta)}{d} = \boxed{\frac{V_0\cos(\theta-\phi)}{d}}$$

Ahora $p_\phi^\circ = \frac{p_\phi}{md^2}$; $\frac{m\dot{\phi}}{md^2}$



$$\frac{\tilde{p}_\phi \cancel{md^2}}{\cancel{md^2}} = \dot{\phi}\tilde{r}^2 = \tilde{r}^2 \frac{d\phi}{dt}$$

$$\frac{d}{dt}(\arctan(x)) = \frac{1}{1+x^2}$$

$$= \tilde{r}^2 \frac{d(\arctan(y/x))}{dt} = \frac{\tilde{r}^2}{1+y^2/x^2}\frac{d}{dt}\left(\frac{y}{x}\right)$$

$$= \frac{\tilde{r}^2}{1+\frac{y^2}{x^2}}\left(\frac{\dot{y}}{x} - \frac{y}{x^2}\dot{x}\right) = \frac{\tilde{r}^2}{1+\frac{y^2}{x^2}}\left(\frac{\dot{y}x - y\dot{x}}{x^2}\right)$$

$$= \frac{\tilde{r}^2}{\frac{x^2+y^2}{\cancel{x^2}}}\left(\frac{\dot{y}x - y\dot{x}}{\cancel{x^2}}\right) = \frac{\tilde{r}^2}{r^2}(\dot{y}x - y\dot{x})$$

$$= \frac{\tilde{r}^2}{r^2} Vor(\operatorname{sen}\theta\cos\phi - \operatorname{sen}\phi\cos\theta)$$

$$= \bar{r}_0\bar{V}_0 \operatorname{sen}(\theta-\phi)$$