

# Hw5 Report (b07801004 陳佳雯)

## Environment:

Visual studio code; Windows 10

## Langage:

C++11

## Execute:

1. Make sure the test data exist.
2. Execute cpp file “hw5.cpp”
3. Output will be on the screen, such as “Case 1: 5”

## Discription:

Definition of the border:

Let say the  $F(n)$ 's border means the connection point between  $F(n-1)$  and  $F(n-2)$ .

Ex:  $F(2) = 10$ ,  $F(3) = 101$ ,  $F(4) = 10101$ , according to  $F(4)$ , if we have a query 010, then it is on the border.

$n$	$F(n)$
0	0
1	1
2	10
3	101
4	10110
5	10110101
6	1011010110110
7	101101011011010110101
8	10110101101101011010110110110110
9	1011010110110101101011011010110101101101011010110101

The Fibonacci words have 3 types of case as follow:

**Case1: if the given query first occurred in  $F(n)$ , it will also occur on the border of  $F(n + 1)$**

Take  $n = 96$ , query = 10110101101101 as an example.

The query first occurs in  $F(7)$ , and it also occurs on the border of  $F(8)$ .

**Case2: if the given query first occurred in  $F(n)$ , it will also occur on the border of  $F(n + 2)$ , but not on the border of  $F(n+1)$ .**

Take  $n = 6$ , query = 01 as an example.

The query first occurs in  $F(3)$ , and it also occurs on the border of  $F(5)$ .

**Case3: The given query first occurred in  $F(n)$ , but it never occur on the border in**

**F(n+1) and F(n+2).**

Take  $n = 6$ , query = 10 as an example.

The query first occurs in  $F(2)$ , and it doesn't show on the border afterward.

We denote firstOccur as the first occurrence of the query at  $F(x)$ ,  $x \leq n$ .

Then, we can compute the number of occurrences now!

From observation, we find that the Fibonacci words obey the following rules:

Case1:  $F(n - \text{firstOccur} + 1) + F(n - \text{firstOccur} + 1 - 2) + F(n - \text{firstOccur} + 1 - 2*2) + \dots$

Case2:  $F(n - \text{firstOccur} + 1) + F(n - \text{firstOccur} + 1 - 1) + F(n - \text{firstOccur} + 1 - 1*2) + \dots$

Case3:  $F(n - \text{firstOccur} + 1)$

\*  $F(n)$  represents the Fibonacci numbers.

However, if the query is unreasonable, such as "00", "111", "01010"..., we can detect and exclude them from the beginning.