**BST 140.752 Final exam**

Notes:

- You may not use a calculator for this exam.
- Please be neat and write legibly. Use the back of the pages if necessary.
- Good luck!

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**printed name**

Define the following:

Model $(1)$ $Y = X_1\beta_1 + \epsilon$.

Model $(2)$ $Y = X_1\beta_1 + X_2\beta_2 + \epsilon$.

where $Y$ is $n \times 1$, $X_1$ is $n \times p_1$ and $X_2$ is $n \times p_2$.

1. Let $S^2_{(i)}$ be the residual variance estimate obtained from fitting Model $(i)$.

   A. Given the expected value of $S^2_{(1)}$ and $S^2_{(2)}$ given that Model $(1)$ is true.

   B. Given the expected value of $S^2_{(1)}$ and $S^2_{(2)}$ given that Model $(2)$ is true.

   C. Suppose that Model $(1)$ is true. Show that the variance of $S^2_{(2)}$ is larger than that of $S^2_{(1)}$. (Hint, the variance of a Chi-Squared with $df$ degrees of freedom is $2df$.)

2. Consider a linear model $Y = X\beta + \sum_{j=1}^{J} \delta_{i_j} \Delta_j + \epsilon$ where $\delta_{i_j}$ is an $n \times 1$ vector with a $1$ at position $i_j$ and $0$ elsewhere, $Y$ is $n \times 1$, and $X$ is $n \times p$.

   A. Fix $\beta$. Show that the least squares estimate of $\Delta_j$ is $(y_{i_j} - x_{i_j}'\beta)$ where $x_{i_j}$ is row $i_j$ from $X$ as a column vector.

   B. Given the prevous answer, prove that the $i_j$ residual from the least squares fit of this model (where $\beta$ is estimated and not held fixed) is $0$.

3. Consider a model $E[Y_i] = f(x_i) = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 (x_i - k)_+^2$ where $(a)_+$ is $a$ if $a > 0$ and $0$ otherwise and $k$ is a known knot point.

   A. Show that $f$ is continous at point $k$.

   B. Show that $f$ has exactly one continuous derivative.