BST 140.651 Midterm Exam
Notes:

- Show your work on all questions. Simple "yes" or "no" answers will be graded as if blank.
- Please be neat and write legibly. Use the back of the pages if necessary.
- There are eight questions.
- Good luck!

____
signature and **printed name**

1. Yearly medical expenditures (in 1000 dollars) for a particular demographic follow the density $f(x) = 2x \exp\left(-x^2\right)$ for $x > 0$. Show mathematically that this is a valid density.

2. Refer to the previous problem. What is the median level of expenditure in this population? (Note, do not solve for a final number, just carry the mathematics until the point of solving for a final number.)

3. Refer to the two previous problems. The **mode** of a density is the point at which density is the highest; that is, the point x_m so that $f(x_m) > f(x)$ for any other x. Calculate the modal medical expenditure for this population. (Hint, take a log before differentiating.)

4. Let X_1 and X_2 be iid random variables from a population with mean μ_1 and variance σ_1^2 and Y_1 and Y_2 be iid random variables from a population with mean μ_2 and variance σ_2^2. What is the **expected value** of $(X_1 + X_2)^2 + (Y_1 + Y_2)^2$?

5. Let X be a random variable, draw the density $f(x) = 2x$ for $0 \leq x \leq 1$. Calculate $E[X]$ and $E[X^2]$.

6. An HIV antibody test is known to be 90% sensitive and 80% specific. By what factor are the prior odds of disease increased with a positive test result? By what fraction are the prior odds of disease decreased with a negative test result?

7. Three events, A, B and C all have the same probability of occurrence; call this probability p. Argue that the probability of at least one occurring is less than or equal to $3p$. (You may assume any result proved in class or in the homework.)

8. Consider a diagnostic test for tumors that yields two possible diagnoses, T for tumor and T^c for no tumor. Potential tumors for this cancer are always one of: malignant (D_M), benign (D_B) or not tumors (D_n). Argue the generalized version of Bayes rule:

$$P(D_M \mid T) = \frac{P(T \mid D_M)P(D_M)}{P(T \mid D_M)P(D_M) + P(T \mid D_B)P(D_B) + P(T \mid D_N)P(D_N)}.$$

You may use any result proved in class or in the homework.