BST 140.651 Midterm Exam
Notes:

- Please use only the basic mathematical functions on your calculator.
- Show your work on all questions. Simple "yes" or "no" answers will be graded as if blank.
- Please be neat and write legibly. Use the back of the pages if necessary.
- There are 8 questions.
- Good luck!

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signature and **printed name**

$A =$ 1st comp fails $\quad C =$ 3rd
$B =$ 2nd

1. A nuclear test site fail-safe system tests three components. The first component fails 6% of the time. If the first component has failed, the second component fails 10% of the time. If the first two components have failed, the third fails 5% of the time. The components are known to be dependent. What is the probability of all three failing? [Hint, first argue that $P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$]

$$P(A)P(B|A)P(C|A\cap B) = P(A)\frac{P(A\cap B)}{P(A)}\frac{P(A\cap B\cap C)}{P(A\cap B)}$$
$$= P(A\cap B\cap C)$$

$$\Rightarrow P(A\cap B\cap C) = (.06)\times(.10)\times(.05)$$

2. Let $X_1, \ldots, X_n$ be iid random variables from a population with mean μ_1 and variance σ_1^2 and $Y_1, \ldots, Y_n$ be random variables from a population with mean μ_2 and variance σ_2^2. What is the expected value of $\bar{X}^2 + \bar{Y}^2$ (notice the squares).

$$E[\bar{x}^2 + \bar{y}^2] = E[\bar{x}^2] + E[\bar{y}^2]$$
$$= Var(\bar{x}) + E[\bar{x}]^2 + Var(\bar{y}) + E[\bar{y}]^2$$
$$= \frac{\sigma_1^2}{n} + \mu_1^2 + \frac{\sigma_2^2}{n} + \mu_2^2$$
$$= \mu_1^2 + \mu_2^2 + \frac{1}{n}(\sigma_1^2 + \sigma_2^2)$$

+ not -

3. Let X_1 and X_2 be **independent** random variables with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2. What is the variance of $\frac{1}{2}(X_1 - X_2)$?

$$Var\left(\frac{1}{2}(x_1 - x_2)\right) = \frac{1}{4} Var(x_1 - x_2)$$

$$= \frac{1}{4}\left\{Var(x_1) + Var(x_2)\right\}$$

$$= \frac{1}{4}(\sigma_1^2 + \sigma_2^2)$$

4. A nasal wash test is known to be 90% sensitive and 70% specific for detecting the H1N1 flu strain among patients with some variant of the flu. It is predicted that 40% of flu cases this year will be H1N1. Suppose a patient with the flu has a negative nasal wash test for H1N1; what is the probability that the test was correct? (Show some work.)

$P(+|D) = .90 \Rightarrow P(-|D) = .10$

$P(-|D^c) = .70 \quad P(+|D^c) = .3$

$P(D) = .40$

$$P(D^c|-) = \frac{P(-|D^c)P(D^c)}{P(-|D^c)P(D^c) + P(-|D)P(D)}$$

$$= \frac{.7 \times .6}{.7 \times .6 + .1 \times .4}$$

5. Refer to the previous problem. What are the odds of disease without knowledge of the test result? By what factor are these odds increased with a positive test result? What are the odds of disease in the presence of a positive test result?

odds of disease (H1N1) $\frac{.4}{.6} = \frac{2}{3}$

factor = DLR+ = $\frac{.9}{.3} = 3$

odds in presence of + = $3 \times \frac{2}{3} = 2$

The next three questions involve the following scenario. Suppose that the time until death for successful kidney transplant recipients follows a density

$$ce^{-\frac{x}{10}}$$

for $x > 0$. (General math hints for this problem: $\frac{d}{dt}e^{tk} = ke^{tk}$ and $\int e^{tk}dt = \frac{1}{k}e^{tk}+$constant.)

6. What value of c makes this function a valid density? (Show your work.)

$$1 = \int_0^\infty c e^{-t/10} dt = c\left(-10 e^{-\frac{t}{10}} \Big|_\emptyset^\infty\right)$$

$$= c(10)$$

$$\Rightarrow \quad c = \frac{1}{10}$$

7. What's the survival function for this population? Use your answer to calculate the probability a subject from this population survives more than 15 years?

$$S(x) = \int_x^\infty \tfrac{1}{10} e^{-t/10}\, dt = -e^{-t/10} \Big|_x^\infty$$

$$= e^{-x/10}$$

$$P(X \geq 15) = e^{-15/10} = 22\%$$

8. What is the median survival time for this population?

Find m so that

$$P(X \geq m) = .5 = e^{-m/10}$$

$$\Rightarrow \quad \log(.5) = -\frac{m}{10}$$

$$\Rightarrow \quad -10 \log(.5) = 6.93 = m$$