BST 140.752 Final exam

printed name

Questions

1. Let $E[Y_i] = f(x_i) = \sum_{j=1}^{J} \beta_j x_i^j + \sum_{k=1}^{K} \psi_k (x_i - \xi_k)_+^J$. Here ξ_k are known knot points and $(a)_+ = a$ if $a > 0$ and 0 otherwise. Characterize f in terms of its continuity and differentiability.

2. Let $E[Y_{ij}] = \mu + \beta_i + \gamma x_{ij}$ where $i = 1, 2$ and $j = 1, \ldots, J_i$. Argue that $\theta = \beta_1 - \beta_2$ is estimable. Given $\hat{\theta}$ and $\hat{\gamma}$, give the estimates of μ, β_1 and β_2 under the assumptions $\mu = 0$, $\beta_1 = 0$, $\beta_2 = 0$ and $\beta_1 + \beta_2 = 0$.

3. Consider the previous problem. Derive the ordinary least squares (OLS) estimate of $\beta_1 - \beta_2$. Hint, it is of the form $(a - b) - \hat{\gamma}(c - d)$, where a, b, c, d and $\hat{\gamma}$ are functions of the data.

4. Consider Problem 2 again. Consider two estimates of θ, one as $\tilde{\theta} = \bar{Y}_1 - \bar{Y}_2$ and the other as the regression adjusted estimates from the previous problem, $\hat{\theta}$. Draw scatterplots where the model clearly holds and i) $\hat{\theta} = \tilde{\theta}$, ii) $\hat{\theta} > \tilde{\theta}$, iii) $\hat{\theta} < \tilde{\theta}$, iv) $\hat{\theta} = 0$ and $\tilde{\theta} > 0$ and v) $\hat{\theta} > 0$ and $\tilde{\theta} = 0$.

5. Consider a mean model $E[Y_{ij}] = \beta_0 + \beta_i + \gamma_i x_{ij}$ for $i = 1, 2$ and $j = 1, \ldots, J_i$. Argue that $\gamma_1 - \gamma_2$ is estimable.