BST 140.651 Midterm Exam
Notes:

- Please use only the basic mathematical functions on your calculator.
- Show your work on all questions. Simple "yes" or "no" answers will be graded as if blank.
- Please be neat and write legibly. Use the back of the pages if necessary.
- There are 8 questions.
- Good luck!

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signature and **printed name**

1. A basketball player makes 90% of his second free throws if he's made the first and 80% if he's missed the first. Do his second free throws appear to be independent of the first? (Explain using probability notation.)

2. Continuing the previous question. He makes 85% of his first free throws. What is the probability that he's made a first free throw *given that he's made the second?* (Show some work.)

3. Suppose that $f_1(x)$ and $f_2(x)$ are densities with associated distribution functions $F_1(x)$ and $F_2(x)$. Derive the distribution function of $\pi f_1(x) + (1-\pi) f_2(x)$ where $0 \leq \pi \leq 1$. (Show some work.)

4. Let X_1, X_2 and X_3 be iid draws from a population with mean μ and variance σ^2. Derive the mean and variance of $.3X_1 + .2X_2 + .5X_3$. (Show some work.)

5. You and your friend are playing a game where you roll a fair die. If it comes up a 1, 2, 3 or 4 he gives you a dollar. If it comes up a 5 or 6, you have to give him a dollar. How many times do you have to play the game for you to have an expected winnings greater than $\$5$? (Show some work.)

The next three questions involve the following scenario. Consider a density for the proportion of a person's body that is covered in freckles, X, given by $f(x) = cx$ for $0 \leq x \leq 1$ and some constant c.

6. What value of c makes this function a valid density? (Show your work.)

7. According to this density, what is the population median proportion of a person's body that is covered in freckles? (Show your work.)

8. The mean of this density is $1/3$ and the variance is $1/18$. You simulated $100,000$ sample means, each comprised of 100 draws from this density. You then took the variance of those $100,000$ numbers. Approximately what number did you obtain? (Explain briefly.)