**BST 140.752 Midterm exam**

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Assumptions

- $Y = \alpha + X_1\beta_1 + X_2\beta_2 + \epsilon = X\beta + \epsilon$ where $X = [J \; X_1 \; X_2]$ and $J$ is a vector of ones.
- $Y$ is $n \times 1$.
- $X_1$ is a full column rank $n \times p_1$ matrix.
- $X_2$ is a full column rank $n \times p_2$ matrix.
- $\epsilon \sim N(0, \sigma^2 I)$.
- $X_1'X_2 = 0$ and $X_1'J = X_2'J = 0$.

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Questions

1. Argue that the empirical correlation between any column of $X_1$ and any column of $X_2$ is 0 and that the mean of the columns of $X_1$ are zero.
2. Show that $\hat{\alpha}$, $\hat{\beta}_1$ and $\hat{\beta}_2$ are all independent of one and another and derive their variances.
3. Assume that $\sigma^2$ is known. Derive a Chi-squared test that $\beta_1 = 0$. Show that it doesn't depend on $X_2$.
4. An ellipse centered at the point $v \in \mathbb{R}^k$ is defined as the solutions $x \in \mathbb{R}^k$ to the equation

   $$(x - v)'A^{-1}(x - v) = 1$$

   The eigenvalues of $A$ determine the length of the axes, by the way. Use the F-test of the general linear hypothesis $H_0 : K\beta_1 = m$ to derive a confidence ellipse for $\beta_1$.
5. Suppose that you had a full rank design matrix $W = [J \; W_1 \; W_2]$ where $W_1$ and $W_2$ are $n \times p_1$ and $n \times p_2$ respectively. Derive a linear transformation that gets you from $W$ to a design matrix satisfying all of the qualities of $X$. Specifically that $X_1'J = X_2'J = 0$ and $X_1'X_2 = 0$.
6. Give a setting where, by virtue of the design of an experiment, that $X_1$ would be exactly or nearly uncorrelated with $X_2$.

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