BST 140.651 Final Exam
Notes:

- Please use only the basic mathematical functions on your calculator.
- Show your work on all questions. Simple "yes" or "no" answers will be graded as if blank.
- Please be neat and write legibly. Use the back of the pages if necessary.
- Good luck!

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signature and **printed name**

1. The next three questions (A., B., C.) deal with the following setting. Forced expiratory volume, FEV_1, is a measure of lung function that is often expressed as a proportion of lung capacity called forced vital capacity, FVC. Suppose that the population distribution of FEV_1/FVC of asthmatics adults in the US has mean of $.55$ and standard deviation of $.10$.

   A. Suppose a random sample of 100 people are drawn from this population. What is the probability that their average FEV_1/FVC is larger than $.565$?

B. Suppose the population of non-asthmatics adults in the US have a mean FEV_1/FVC of $.8$ and a standard deviation of $.05$. You sample 100 people from the asthmatic population and 100 people from the non-asthmatic population and take the difference in sample means. You repeat this process $10,000$ times to obtain $10,000$ differences in sample means. What would you guess the mean and standard deviation of these $10,000$ numbers would be?

C. Moderate or severe lung dysfunction is defined as $FEV_1/FVC \leq .40$. A colleague tells you that 60% of asthmatics in the US have moderate or severe lung dysfunction. To verify this, you take a random sample of 5 subjects, only one of which has moderate or severe lung dysfunction. What is the probability of obtaining only one or fewer if your friend's assertion is correct? What does your result suggest about their assertion?

2. Consider three sample variances, S_1^2, S_2^2 and S_3^2. Suppose that the sample variances are comprised of n_1, n_2 and n_3 iid draws from normal populations $N(\mu_1, \sigma^2)$, $N(\mu_2, \sigma^2)$ and $N(\mu_3, \sigma^2)$, respectively. Argue that

$$\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2 + (n_3 - 1)S_3^2}{n_1 + n_2 + n_3 - 3}$$

is an unbiased estimate of σ^2.

3. You need to calculate the probability that a normally distributed random variable is less than 1.25 standard deviations below the mean. However, you only have an oddly shaped coin with a known probability of heads of $.6$. Describe how you could estimate this probability using this coin. (Do not actually carry out the experiment, just describe how you would do it.)

4. In an effort to improve efficiency, hospital administrators are evaluating a new triage system for their emergency room.

   A. In an validation study of the system, 5 patients were tracked in a mock ER under the new and old triage system. Their waiting times in natural log of hours were:

Subject	New	Old	Diff (New - Old)
1	0.929	2.233	-1.304
2	-1.745	-2.513	4.258
3	1.677	1.204	0.473
4	0.701	1.938	-1.237
5	0.128	2.533	-2.405
Mean	1.036	1.079	-0.043
Std. Dev.	0.682	2.068	2.615

   Create and interpret a relevant confidence interval to evaluate whether or not the new system seems to be working.

B. To further test the system, administrators selected 20 nights and randomly assigned the new triage system to be used on 10 nights and the standard system on the remaining 10 nights. They calculated the nightly median waiting time (MWT) to see a physician. The average MWT for the new system was 4 hours with a standard deviation of $.5$ hours while the average MWT for the old system was 6 hours with a standard deviation of 2 hours. Give a 95% confidence interval estimate for the decrease in the mean MWT associated with the new treatment. Does it appear to be effective?

5. After the new system has been put in place, the administrators would like to estimate the percentage of patients who approved of the new system. They plan to sample n subjects and simply ask them "Did you approve of the ER triage system? (Yes/No)" and calculate a 95% Wald confidence interval. How large does n need to be to guarantee a confidence interval width of $.02$ or smaller?