BST 140.651 Midterm Exam
Notes:

- You may use your one 8.5 by 11 formula sheet.
- Please use only the basic mathematical functions on your calculator.
- Show your work on all questions. Simple "yes" or "no" answers will be graded as if blank.
- Please be neat and write legibly. Use the back of the pages if necessary.
- There are 9 questions, giving you 12 minutes per question. Use your time wisely.
- Good luck!

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signature and **printed name**

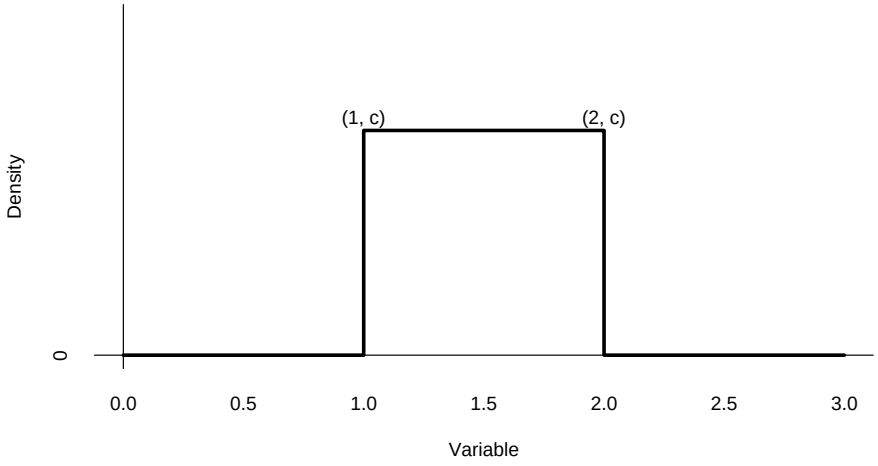
1. A well known soccer player historically scores a goal in 90% of his games. What is the probability that the player scores in two consecutive games? Explain your answer briefly stating any assumptions made.

2. Suppose that f_1, f_2 and f_3 are all densities and let π_1,π_2,π_3 be positive numbers so that $\pi_1 + \pi_2 + \pi_3 = 1$. Show that $g(x) = \pi_1 f_1(x) + \pi_2 f_2(x) + \pi_3 f_3(x)$ is a valid density.

3. Consider n iid observations drawn from a population with mean μ and variance σ^2. What is the expected value of $\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n}$? (Hint, relate to the unbiased estimator of σ^2. Show your work.)

4. An HIV-antibody test has a sensitivity of $.80$ and a specificity of $.90$. What is the diagnostic likelihood ratio for a positive test? Interpret this number.

5. Let X_1, X_2 be independent, identically distributed coin flips (taking values $0 =$ failure or $1 =$ success) having success probability π. Give and interpret the likelihood ratio comparing the hypothesis that $\pi = .5$ (the coin is fair) versus $\pi = 1$ (the coin always gives successes) when both coin flips result in successes.



The above picture displays a density for the population of increases in wages for assistant professors being promoted to associates ($1 =$ no increase, $2 =$ salary has doubled).

6. What value of c makes this function a valid density? (Show your work.)

7. What is the probability that a newly promoted professor has a new salary is more than one and one half times as large as their old salary? (Show your work.)

8. Is this a reasonable density for this setting? (Explain briefly.)

9. The variance of this density is $1/12$. Suppose that the sample variance of 10 observations from this density was sampled say $10,000$ times. What number would we expect the average value from these $10,000$ variances to be near? (Explain your answer briefly.)