BST 140.651 Final Exam
Notes:

- Please use only the basic mathematical functions on your calculator.
- Show your work on all questions. Simple "yes" or "no" answers will be graded as if blank.
- Please be neat and write legibly. Use the back of the pages if necessary.
- There are three pages containing 8 questions.
- Good luck!

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signature and **printed name**

1. Suppose that DBPs drawn from a certain population are normally distributed with a mean of $90\ mmHg$ and standard deviation of $5\ mmHg$. Suppose that $1,000$ people are drawn from this population. If you had to guess the number of people people in having DBPs less than $80\ mmHg$ what would you guess? (Argue your case briefly.)

Let X be a person drawn from this pop.

$P(X < 80) = P\left(Z < \frac{80-90}{5}\right) = P(Z < -2) \approx 2.5\%$

So about $1{,}000 \times .025 = 25$ people

2. Consider the setting for the previous problem. You draw 25 people from this population. What's the probability tha the sample average is larger than $92\ mmHg$?

$P(\bar{X} > 92) = P\left(Z > \frac{92-90}{5/\sqrt{25}}\right) = P(Z > 2) \approx 2.5\%$

3. Consider the setting from the previous two questions. You select 5 people from this population. What's the probability that 4 or more of them have a DBP larger than $100\ mmHg$? (Show some work.)

Let X be the bp of one person selected.

$P(X > 100) = P\left(Z > \frac{100-90}{5}\right) = P(Z > 2) \approx 2.5\%$
$= p$

$P(4 \text{ or more}) = \binom{5}{4} p^4 (1-p)^1 + \binom{5}{5} p^5 (1-p)^0$

p= true prop. who would report an improvement in symptoms

4. In a random sample of 100 subjects with low back pain, 27 reported an improvement in symptoms after execise therapy. Give and interpret an interval estimate for the true proportion of subjects who respond to exercise therapy. (Show some work.)

$\hat{p} = \frac{27}{100} = .27$ ival $= \hat{p} \pm 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{100}}$

Say this ival is [a,b]. It is interpretted as "We are 95% confident that p lies in [a,b]" or "In repeated sampling [a,b] is constructed so that 95% of the intervals would contain p."

5. Let X be binomial with success probability p_1 and n_1 trials and Y be an independent binomial with success probability p_2 and n_2 trials. Let $\hat{p}_1 = X/n_1$ and $\hat{p}_2 = Y/n_2$ be the associated sample proportions. Suppose that it is known that

I'll show the start and end steps.

$$Z = \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_2)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

is standard normally distributed for large n_1 and n_2. Use this fact to derive a 95% confidence interval for $p_1 - p_2$. (Do not just give an answer, show some steps.)

Start with this: $1-\alpha = P\left(-Z_{1-\frac{\alpha}{2}} \le \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}} \le Z_{1-\frac{\alpha}{2}}\right)$

End with this $= P\left(\hat{p}_1 - \hat{p}_2 - Z_{1-\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} < p_1 - p_2 \le \hat{p}_1 - \hat{p}_2 + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}\right)$

6. Consider the setting from problem 4. Suppose that in a second sample of 100 subjects with lower back pain who received no treatment, 10 reported an improvement in symptoms. Using your answer to question 5, answer the question of whether or not the treatment appear to be effective?

This is simply plugging into the interval $\hat{p}_1 - \hat{p}_2 \pm Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$. Note that, when interpreting, if 1 is trt & 2 is ctrl, then efficacy is demonstrated if the interval is above 0.

If the interval is below Ø, then the treatment appears to be harmful. If it contains Ø then their appears to be no difference.

7. You need to calculate the probability that a *standard normal* is larger than 2.20, but have nothing available other than a regular coin. Describe how you could estimate this probability using only your coin. (Do not actually carry out the experiment, just describe how you would do it.)

① Flip the coin 25 times (say). Let $\hat{p}$ = prop of heads out of 10

② calculate $z = \frac{\hat{p} - .5}{\sqrt{\frac{.5(1-.5)}{25}}} = \frac{\hat{p} - .5}{.1}$

③ Repeat steps 1 + 2 several times to get lots of z variables.

④ Calculate the %-age of z variables larger than 2.2.

8. Suppose that 18 obese subjects were randomized, 9 each, to a new diet pill and a placebo. Subjects' body mass indices (BMIs) were measured at a baseline and again after having received the treatment or placebo for four weeks. The average difference from follow-up to the baseline (followup - baseline) was $-3\ kg/m^2$ for the treated group and $1\ kg/m^2$ for the placebo group. The corresponding standard deviations of the differences was $1.5\ kg/m^2$ for the treatment group and $1.8\ kg/m^2$ for the placebo group. Does the change in BMI over the two year period appear to differ between the treated and placebo groups? (Show some work and interpret your results.) Assume normality and a common variance.

Pretty much everyone got this one.