## BST 140.752 Problem Set 5

## 1 Residuals

- 1. Let  $Y_{ij} = \mu + u_i + \epsilon_{ij}$  for  $u_i \sim N(0, \sigma_u^2)$  and  $\epsilon_{ij} \sim N(0, \sigma^2)$ . Calculate the BLUP for  $u_i$
- 2. Let  $Y = X\beta + Zu + \epsilon$  for  $uN(0, \Sigma_u)$  and  $\epsilon \sim N(0, \sigma^2 I)$ . Calculate the BLUP for u.
- 3. Load the Rail data set in R. Fit a mixed model of the form from question 1. Compare a the estimates of the mean for each rail with the empirical mean.
- 4. Load the pixel data set in R. Fit a linear mixed effect model where you have  $Y_{ijk} = \beta_0 + \beta_1 x_k + u_i + u_{ij} + \epsilon_{ijk}$  where  $Y_{ijk}$  is pixel, i is dog, j is side and k is day index and  $x_k$  is day. Fit the model and interpret the results.
- 5. Consider the model  $Y_i = \mu + \epsilon_{ij}$ . Consider putting a so-called "flat" prior on  $\mu$ . That is acting like a distribution that is 1 from  $-\infty$  to  $+\infty$  is a valid density. Calculate the distribution marginalized over  $\mu$  and show that it is the same likelihood used to obtain the REML estimates.