BST 140.752
Problem Set 4

1 Residuals

1. Consider a linear model $Y = X\beta + \delta\Delta + \epsilon$ where δ is a vector with a 1 at position i_0 and 0 elsewhere. Argue the following.

   A. The i_0 residual is 0 for this model.

   B. The fitted value for β using all of the data and this model is equivalent to that using only the data with the i_0 observation deleted.

   C. Argue that the standardized Press residuals are a test statistic for $\Delta = 0$.

2. Consider the residuals for the ordinary linear model. Derive their mean and variance.

3. Carefully write up the proof that relates the Press residuals to the ordinary residuals. Derive the mean and variance/covariance of the Press residuals.

4. Prove the Sherman/Morrison/Woodburry theorem.

5. Prove that the hat matrix diagonals are between 0 and 1.

6. Why are the studentized residuals not exactly distributed as t statistics?

2 Inference under incorrectly specified models

For all of this section, let Model 1 be $Y = X_1\beta_1 + \epsilon$ and Model 2 be $Y = X_1\beta_1 + X_2\beta_2 + \tilde{\epsilon}$.

1. Suppose that Model 1 is fit while Model 2 represents the actual truth. Give the bias and variance of β_1. Give the expected value of S^2.

2. Suppose that Model 2 is fit while Model 1 is true. Give the bias and variance of the estimted β. Give the expected value of S^2.