

BST 140.651 Midterm Exam Make Up

Notes:

- Please use only the basic mathematical functions on your calculator.
- Show your work on all questions. Simple “yes” or “no” answers will be graded as if blank.
- Please be neat and write legibly. Use the back of the pages if necessary.
- There are 8 questions.
- Good luck!

signature and **printed name**

1. If two events A and B are independent, argue that $P(A \mid B) = P(A \mid B^c) = P(A)$.

2. A basketball player makes 90% of his second free throws if he's made the first and 80% if he's missed the first. He makes 85% of his first free throws. What is the probability that he's made a first free throw *given that he's made the second*? (Show some work.)

3. Suppose that $f_1(x)$ and $f_2(x)$ are densities with associated distribution functions $F_1(x)$ and $F_2(x)$. Derive the distribution function of $\pi f_1(x) + (1 - \pi)f_2(x)$ where $0 \leq \pi \leq 1$. (Show some work.)

4. Let X be a random variable from a distribution with mean μ and variance σ^2 . Let $Z = (X - \mu)/\sigma$. Show that Z has mean 0 and variance 1. (Show some work.)

5. You and your friend are playing a game where you toss a coin with probability of heads of .4. If it comes up heads, you have to give him a dollar. If it comes up tails, you win a dollar. You play the game 5 times. Let X be your total earnings. What is the variance of X ? (Show some work.)

6. What value of c makes this function a valid density? (Show your work.)
7. Is this a reasonable density for the proportion of a person's body that is covered in freckles? (Explain briefly.)
8. The mean of this density is $1/3$ and the variance is $1/18$. You simulated 100,000 sample variances, each comprised of 100 draws from this density. You then took the mean of those 100,000 numbers. Approximately what number did you obtain? (Explain briefly.)