BST 140.651 Midterm Exam
Notes:

- Please use only the basic mathematical functions on your calculator.
- Show your work on all questions. Simple "yes" or "no" answers will be graded as if blank.
- Please be neat and write legibly. Use the back of the pages if necessary.
- There are 8 questions.
- Good luck!

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signature and **printed name**

1. A nuclear test site fail-safe system tests three components. The first component fails 6% of the time. If the first component has failed, the second component fails 10% of the time. If the first two components have failed, the third fails 5% of the time. The components are known to be dependent. What is the probability of all three failing? [Hint, first argue that $P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$]

2. Let $X_1, \ldots, X_n$ be iid random variables from a population with mean μ_1 and variance σ_1^2 and $Y_1, \ldots, Y_n$ be random variables from a population with mean μ_2 and variance σ_2^2. What is the expected value of $\bar{X}^2 + \bar{Y}^2$ (notice the squares).

3. Let X_1 and X_2 be **independent** random variables with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2. What is the variance of $\frac{1}{2}(X_1 - X_2)$?

4. A nasal wash test is known to be 90% sensitive and 70% specific for detecting the H1N1 flu strain among patients with some variant of the flu. It is predicted that 40% of flu cases this year will be H1N1. Suppose a patient with the flu has a negative nasal wash test for H1N1; what is the probability that the test was correct? (Show some work.)

5. Refer to the previous problem. What are the odds of disease without knowledge of the test result? By what factor are these odds increased with a positive test result? What are the odds of disease in the presence of a positive test result?

The next three questions involve the following scenario. Suppose that the time until death for successful kidney transplant recipients follows a density

$$ce^{-\frac{x}{10}}$$

for $x > 0$. (General math hints for this problem: $\frac{d}{dt}e^{tk} = ke^{tk}$ and $\int e^{tk}dt = \frac{1}{k}e^{tk} +$ constant.)

6. What value of c makes this function a valid density? (Show your work.)

7. What's the survival function for this population? Use your answer to calculate the probability a subject from this population survives more than 15 years?

8. What is the median survival time for this population?