Física

Licenciatura em Engenharia Informática

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Aula 18

Sumário

- ✓ Circuitos eléctricos
- ✓ Força electromotriz
- ✓ Associação de resistências em série e em paralelo
- ✓ Leis de Kirchhoff

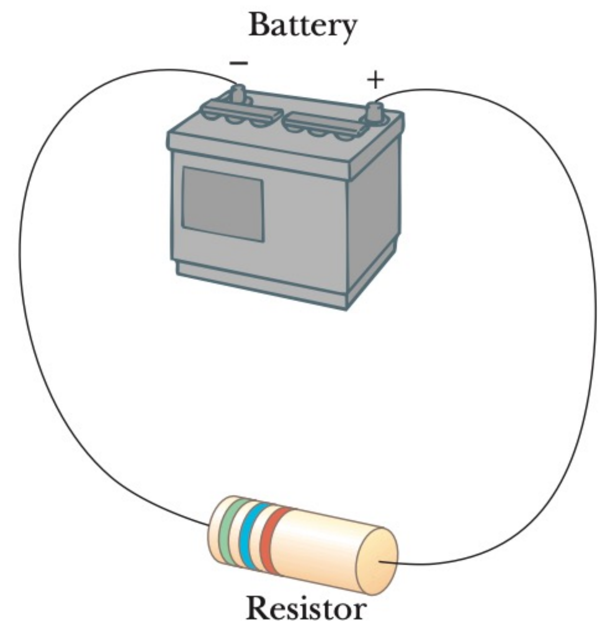
Circuito eléctrico

Num circuito eléctrico precisamos de uma bateria, que é a fonte de tensão, ou força electromotriz, ε

No circuito temos ainda vários outros elementos: resistências, condensadores, etc. que são os elementos dissipativos.

A **força electromotriz** (fem) ε, é a tensão máxima que a bateria pode dar aos seus terminais, desprezando a sua resistência interna.

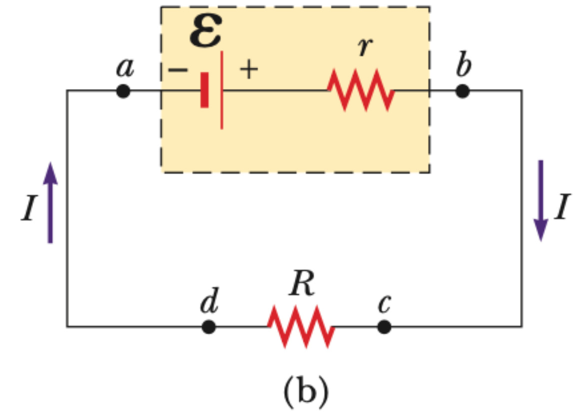
A fem é uma fonte de energia e não de força.



Força Electromotriz

- Numa bateria há uma resistência interna, que podemos considerar desprezável, sendo a tensão aos terminais igual a ε
- Num circuito eléctrico consideramos o sentido da corrente o das cargas positivas
- Uma resistência R onde passa a corrente I apresenta aos seus terminais a ddp V

V=RI



ACTIVE FIGURE 18.1
(a) A circuit consisting of a resistor connected to the terminals of a battery. (b) A circuit diagram of a source of emf $\mathcal{E}$ having internal resistance r connected to an external resistor R.

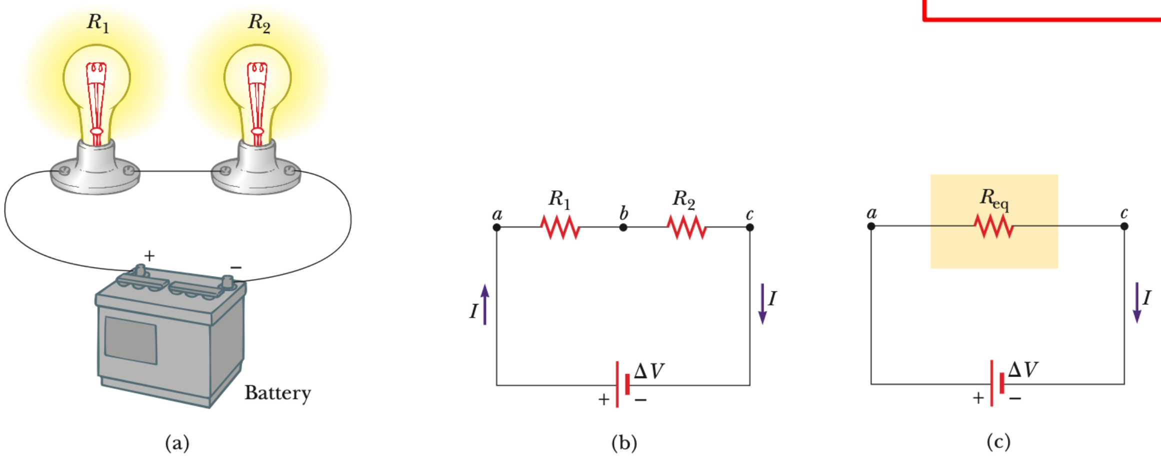
Associação de resistências em série

- ✓ A intensidade da corrente é a mesma em todos os elementos do circuito
- ✓ A ddp é igual à soma das ddp em cada elemento do circuito

$$V_T = V_1 + V_2$$

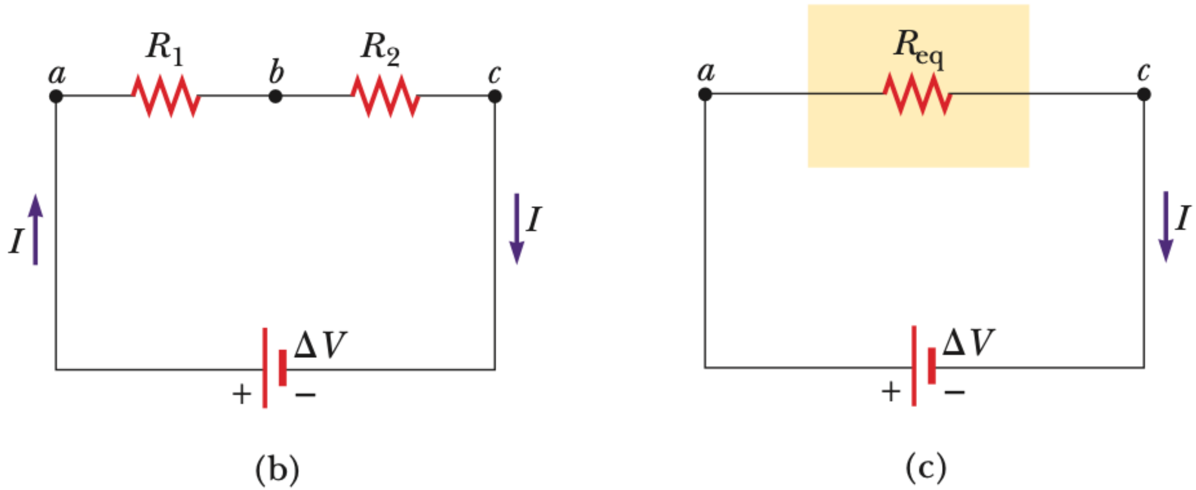
$$V_T = I\,R_1 + I\,R_2 = I\,(R_1 + R_2)$$

$$\boxed{R_{eq} = R_1 + R_2}$$



ACTIVE FIGURE 18.2
A series connection of two resistors, R_1 and R_2. The currents in the resistors are the same, and the equivalent resistance of the combination is given by $R_{eq} = R_1 + R_2$.

Associação de resistências em série



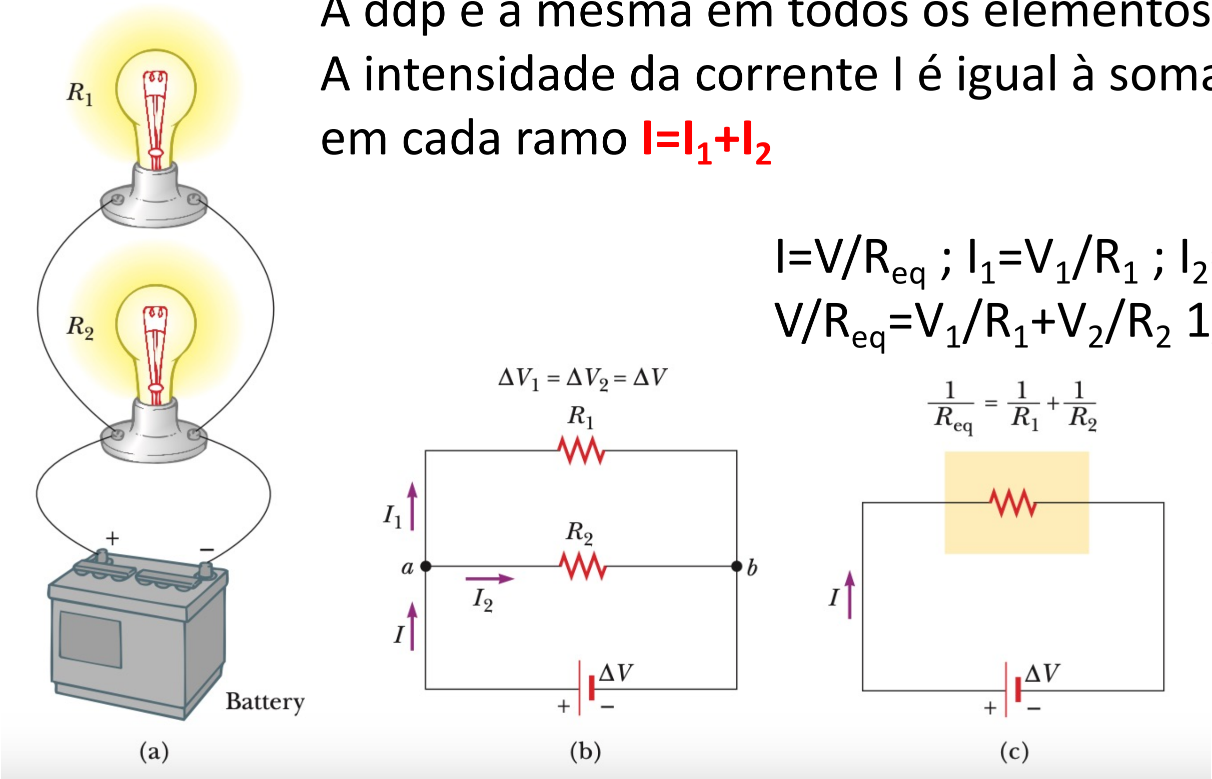
A intensidade da corrente é a mesma em todos os elementos do circuito e é dada por

$$I = V/R_{eq}$$

Associação de resistências em paralelo

A ddp é a mesma em todos os elementos do circuito **$V=V_1=V_2$**
A intensidade da corrente I é igual à soma das intensidades em cada ramo **$I=I_1+I_2$**

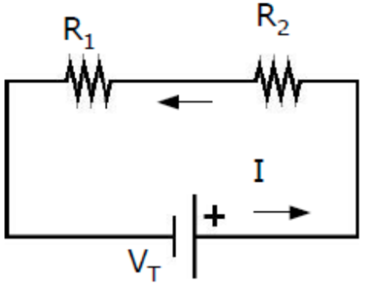
$I=V/R_{eq}$; $I_1=V_1/R_1$; $I_2=V_2/R_2$
$V/R_{eq}=V_1/R_1+V_2/R_2$ $1/R_{eq}=1/R_1+1/R_2$



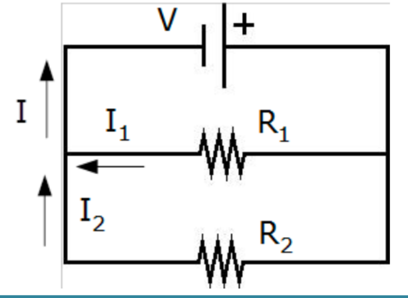
Associação de resistências

- ✓ Numa associação de resistências em série a resistência do circuito aumenta
- ✓ Numa associação de resistências em paralelo a resistência do circuito diminui

- Ex. $R_1=R_2=100\ \Omega$



- Série: Req=200 Ω $\quad R_{eq}=R_1+R_2$

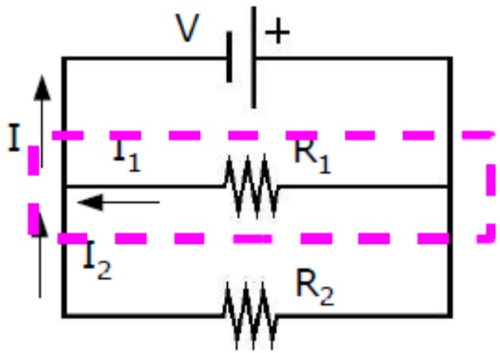


- Paralelo: Req=50 Ω

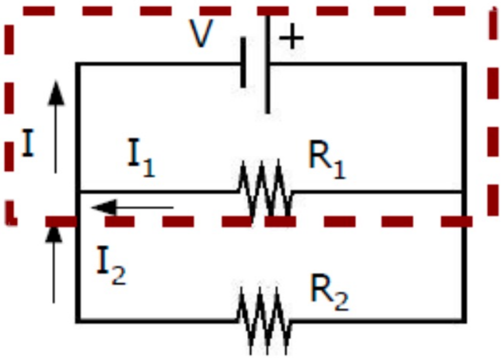
$$\boxed{\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}}$$

Leis de Kirchhoff

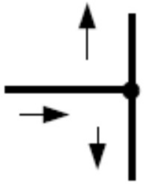
- Vamos definir num circuito eléctrico o conceito de ramo, malha e nodo



Ramo



Malha



Nodo

Leis de Kirchhoff

Lei dos nodos: A soma das intensidades das correntes que entram num nodo é igual à soma das intensidades de corrente que saem desse nodo. Esta lei traduz a conservação da carga eléctrica.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

Lei das malhas: A soma das diferenças de potencial em todos os elementos de uma malha (percurso fechado) é igual a zero.
Esta lei traduz a conservação da energia.

$$\sum \epsilon_j = \sum R_j I_J$$

EXAMPLE 18.3 Equivalent Resistance

Goal Solve a problem involving both series and parallel resistors.

Problem Four resistors are connected as shown in Figure 18.11a. **(a)** Find the equivalent resistance between points *a* and *c*. **(b)** What is the current in each resistor if a 42-V battery is connected between *a* and *c*?

Strategy Reduce the circuit in steps, as shown in Figures 18.11b and 18.11c, using the sum rule for resistors in series and the reciprocal-sum rule for resistors in parallel. Finding the currents is a matter of applying Ohm's law while working backwards through the diagrams.

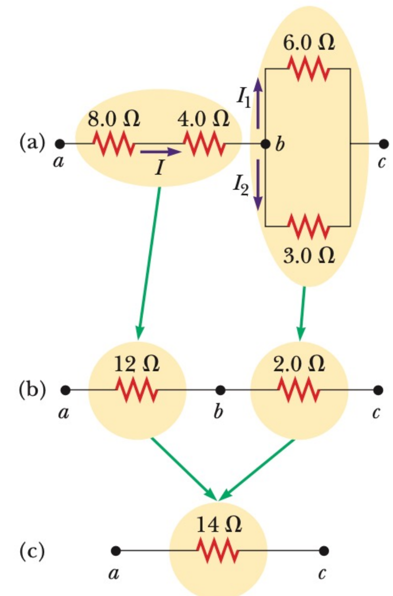


Figure 18.11 (Example 18.3) The four resistors shown in (a) can be reduced in steps to an equivalent 14-Ω resistor.

Solution

(a) Find the equivalent resistance of the circuit.

The 8.0-Ω and 4.0-Ω resistors are in series, so use the sum rule to find the equivalent resistance between *a* and *b*:

$$R_{eq} = R_1 + R_2 = 8.0\ \Omega + 4.0\ \Omega = 12\ \Omega$$

The 6.0-Ω and 3.0-Ω resistors are in parallel, so use the reciprocal-sum rule to find the equivalent resistance between *b* and *c* (don't forget to invert!):

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6.0\ \Omega} + \frac{1}{3.0\ \Omega} = \frac{1}{2.0\ \Omega}$$

$$R_{eq} = 2.0\ \Omega$$

EXAMPLE 18.4 Applying Kirchhoff's Rules

Goal Use Kirchhoff's rules to find currents in a circuit with three currents and one battery.

Problem Find the currents in the circuit shown in Figure 18.14 by using Kirchhoff's rules.

Strategy There are three unknown currents in this circuit, so we must obtain three independent equations, which then can be solved by substitution. We can find the equations with one application of the junction rule and two applications of the loop rule. We choose junction *c*. (Junction *d* gives the same equation.) For the loops, we choose the bottom loop and the top loop, both shown by blue arrows, which indicate the direction we are going to traverse the circuit mathematically (not necessarily the direction of the current). The third loop gives an equation that can be obtained by a linear combination of the other two, so it provides no additional information.

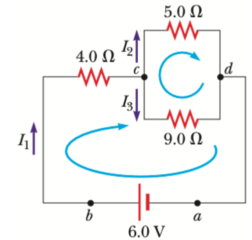


Figure 18.14 (Example 18.4)

Solution

Apply the junction rule to point *c*. I_1 is directed into the junction, I_2 and I_3 are directed out of the junction.

$$I_1 = I_2 + I_3$$

Select the bottom loop, and traverse it clockwise starting at point *a*, generating an equation with the loop rule:

$$\Sigma\Delta V = \Delta V_{\text{bat}} + \Delta V_{4.0\Omega} + \Delta V_{9.0\Omega} = 0$$

$$6.0\text{ V} - (4.0\ \Omega)I_1 - (9.0\ \Omega)I_3 = 0$$

Select the top loop, and traverse it clockwise from point *c*. Notice the gain across the 9.0-Ω resistor, because it is traversed *against* the direction of the current!

$$\Sigma\Delta V = \Delta V_{5.0\Omega} + \Delta V_{9.0\Omega} = 0$$

$$-(5.0\ \Omega)I_2 + (9.0\ \Omega)I_3 = 0$$

Rewrite the three equations, rearranging terms and dropping units for the moment, for convenience:

$$(1)\quad I_1 = I_2 + I_3$$

$$(2)\quad 4.0I_1 + 9.0I_3 = 6.0$$

$$(3)\quad -5.0I_2 + 9.0I_3 = 0$$

Solve Equation 3 for I_2 and substitute into Equation 1:

$$I_2 = 1.8I_3$$

$$I_1 = I_2 + I_3 = 1.8I_3 + I_3 = 2.8I_3$$

Substitute the latter expression into Equation 2 and solve for I_3:

$$4.0(2.8I_3) + 9.0I_3 = 6.0 \quad\rightarrow\quad I_3 = \boxed{0.30\text{ A}}$$

Substitute I_3 back into Equation 3 to get I_2:

$$-5.0I_2 + 9.0(0.30\text{ A}) = 0 \quad\rightarrow\quad I_2 = \boxed{0.54\text{ A}}$$

Substitute I_3 into Equation 2 to get I_1:

$$4.0I_1 + 9.0(0.30\text{ A}) = 6.0 \quad\rightarrow\quad I_1 = \boxed{0.83\text{ A}}$$