Lógica Computacional

LEI, 2023/2024
FCT UNL

Aula Prática 3

Dedução Natural em Lógica Proposicional.

Pergunta 1. Prove as afirmações seguintes.

1. $\vdash \varphi \to (\varphi \vee \psi)$
2. $\vdash (\varphi \vee \varphi) \to \varphi$
3. $\vdash (\varphi \wedge \psi) \to \varphi$
4. $\vdash \varphi \to (\psi \to \varphi)$
5. $\vdash ((\varphi \to \psi) \wedge (\psi \to \gamma)) \to (\varphi \to \gamma)$
6. $\vdash (\varphi \to (\psi \to \gamma)) \to ((\varphi \to \psi) \to (\varphi \to \gamma))$
7. $\vdash (\varphi \to \psi) \to (\varphi \to (\psi \vee \gamma))$
8. $\vdash (\psi \to \gamma) \to ((\varphi \wedge \psi) \to \gamma)$
9. $\vdash \neg(\varphi \vee \psi) \to \neg\varphi$
10. $\vdash (\psi \to \gamma) \to ((\varphi \wedge \psi) \to (\varphi \wedge \gamma))$

Pergunta 2. Prove as afirmações seguintes usando o sistema $\mathcal{N}$.

1. $\models \neg\varphi \to (\varphi \to \psi)$
2. $\models ((\varphi \to \psi) \wedge \neg\psi) \to \neg\varphi$
3. $\models \varphi \leftrightarrow (\neg\neg\varphi)$
4. $\models ((\varphi \to \psi) \leftrightarrow (\neg\psi \to \neg\varphi)$
5. $\models \bot \to \varphi$
6. $\models \top$
7. $\models \varphi \vee \neg\varphi$
8. $\models ((\varphi \to \delta) \to \varphi) \to \varphi$
9. $\models \varphi \vee (\varphi \to \psi)$
10. $\models (\varphi \to \psi) \vee (\psi \to \delta)$

Pergunta 3. Prove as afirmações seguintes usando o sistema $\mathcal{N}$.

1. $\{\neg\varphi \vee \psi\} \models \varphi \to \psi$
2. $\{\varphi \to \psi\} \models \neg\varphi \vee \psi$
3. $\{\neg(\varphi \wedge \psi)\} \models (\neg\varphi \vee \neg\psi)$
4. $\{\neg\varphi \vee \neg\psi\} \models \neg(\varphi \wedge \psi)$
5. $\{\neg(\varphi \vee \psi)\} \models \neg\varphi \wedge \neg\psi$
6. $\{\neg\varphi \wedge \neg\psi\} \models \neg(\varphi \vee \psi)$
7. $\{\varphi \vee (\psi \wedge \delta)\} \models (\varphi \vee \psi) \wedge (\varphi \vee \delta)$
8. $\{(\varphi \vee \psi) \wedge (\varphi \vee \delta)\} \models \varphi \vee (\psi \wedge \delta)$
9. $\{\varphi \wedge (\psi \vee \delta)\} \models (\varphi \wedge \psi) \vee (\varphi \wedge \delta)$
10. $\{(\varphi \wedge \psi) \vee (\varphi \wedge \delta)\} \models \varphi \wedge (\psi \vee \delta)$
11. $\{\varphi \leftrightarrow \psi\} \models (\varphi \wedge \delta) \leftrightarrow (\psi \wedge \delta))$
12. $\{\neg(\neg\varphi \vee \neg\psi)\} \models \varphi \wedge \psi$