Lógica Computacional

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FCT UNL

Aula Prática 7

Semântica da Lógica de Primeira Ordem.

1. Interpretação de termos e fórmulas.

   Considere a assinatura $\Sigma = (SF, SP)$ onde:

   - $SF_0 = \{zero\},\ SF_1 = \{suc\},\ SF_2 = \{\oplus, \otimes\}$ e;
   - $SP_1 = \{Par, Impar\},\ SP_2 = \{Eq, Meq\}$.

   Considere também a estrutura de interpretação $\mathsf{Nat} = (\mathbb{N}_0, I)$, sendo:

   - $\underline{zero}_I = 0$;
   - $\underline{suc}_I :\ \mathbb{N}_0 \to \mathbb{N}_0$ tal que $\underline{suc}_I(n) = n + 1$;
   - $\underline{\oplus}_I :\ \mathbb{N}_0^2 \to \mathbb{N}_0$ tal que $\underline{\oplus}_I(n, m) = n + m$;
   - $\underline{\otimes}_I :\ \mathbb{N}_0^2 \to \mathbb{N}_0$ tal que $\underline{\otimes}_I(n, m) = n \times m$;
   - $\underline{Par}_I :\ \mathbb{N}_0 \to \{0, 1\}$ tal que $\underline{Par}_I(n) = 1$ sse n é par;
   - $\underline{Impar}_I :\ \mathbb{N}_0 \to \{0, 1\}$ tal que $\underline{Impar}_I(n) = 1$ sse n é ímpar;
   - $Eq :\ \mathbb{N}_0^2 \to \{0, 1\}$ tal que $\underline{Eq}_I(n, m) = 1$ sse $n = m$;
   - $\underline{Meq}_I :\ \mathbb{N}_0^2 \to \{0, 1\}$ tal que $\underline{Meq}_I(n, m) = 1$ sse $n \leq m$.

   Assuma a atribuição $\rho :\ X \to \mathbb{N}_0$ tal que $\rho(n) = 3$ e $\rho(m) = 2$.

   (a) Determine a interpretação dos seguintes termos em Nat.

      i. $[\![zero]\!]^{\rho}_{\mathsf{Nat}}$
      ii. $[\![n]\!]^{\rho}_{\mathsf{Nat}}$
      iii. $[\![suc(n)]\!]^{\rho}_{\mathsf{Nat}}$
      iv. $[\![\oplus(suc(zero), m)]\!]^{\rho}_{\mathsf{Nat}}$
      v. $[\![\otimes(\oplus(m, suc(n)), \oplus(suc(zero), m))]\!]^{\rho}_{\mathsf{Nat}}$

   (b) Determine se são verdadeiras as afirmações seguintes.

      i. $\mathsf{Nat}, \rho \Vdash Meq(zero, n)$;
      ii. $\mathsf{Nat}, \rho \Vdash Meq(m, \oplus(suc(zero), m))$;
      iii. $\mathsf{Nat}, \rho \Vdash Eq(n, m) \wedge Meq(n, suc(n))$;
      iv. $\mathsf{Nat}, \rho \Vdash Par(n) \to Impar(n)$;
      v. $\mathsf{Nat}, \rho \Vdash \exists n\, Impar(suc(n))$;
      vi. $\mathsf{Nat}, \rho \Vdash \exists n\, Eq(suc(n), zero)$;
      vii. $\mathsf{Nat}, \rho \Vdash \forall n\, Eq(suc(n), m)$;
      viii. $\mathsf{Nat}, \rho \Vdash \forall n\, \neg Eq(suc(n), zero)$.

2. Consequência semântica
   Verifique se são verdadeiras as seguintes afirmações.

   (a) $\{P(y)\} \models \forall x\, P(x)$
   (b) $\{\exists x\, P(x)\} \models \forall x\, P(x)$
   (c) $\{\exists x\, P\} \models \forall x\, P$
   (d) $\{\forall x\, P(x) \to \forall x\, Q(x)\} \models \forall x\, (P(x) \to Q(x))$
   (e) $\{\forall x\, (P(x) \vee Q(x))\} \models \forall x\, P(x) \vee \forall x\, Q(x)$
   (f) $\{\exists x\, P(x) \wedge \exists x\, Q(x)\} \models \exists x\, (P(x) \wedge Q(x))$
   (g) $\{\exists x\, (P(x) \wedge Q(x))\} \models \exists x\, P(x) \wedge \exists x\, Q(x)$
   (h) $\{\forall x\, (\neg(P(x) \wedge Q(x)))\} \models \forall x\, \neg P(x) \wedge \forall x\, \neg Q(x)$
   (i) $\{\forall x\, (P(x) \to \neg Q(x)), \neg Q(a)\} \models P(a)$
   (j) $\{\forall x\, (P(x) \to \neg Q(x)), P(a)\} \models \neg Q(a)$
   (k) $\{\forall x\, (P(x) \to Q(x)), \forall x\, (Q(x) \to R(x))\} \models \forall x\, (P(x) \to R(x))$
   (l) $\{\exists x\, (\varphi \wedge \psi)\} \models \exists x\, \varphi \wedge \exists x\, \psi$
   (m) $\{\forall x\, \varphi \wedge \forall x\, \psi\} \models \forall x\, (\varphi \wedge \psi)$
   (n) $\{\exists x\, \varphi \vee \exists x\, \psi\} \models \exists x\, (\varphi \vee \psi)$
   (o) $\{\exists x\, \neg\varphi\} \models \neg\forall x\, \varphi$
   (p) $\{\forall x\, \neg\varphi\} \models \neg\exists x\, \varphi$

   Se $x \notin \mathsf{VL}(\psi)$:

   (q) $\{\forall_x\, (\varphi \wedge \psi)\} \models \forall_x\, \varphi \wedge \psi$
   (r) $\{\forall_x\, (\varphi \vee \psi)\} \models \forall_x\, \varphi \vee \psi$
   (s) $\{\exists_x\, (\varphi \wedge \psi)\} \models \exists_x\, \varphi \wedge \psi$
   (t) $\{\exists_x\, (\varphi \vee \psi)\} \models \exists_x\, \varphi \vee \psi$