Grelha de respostas certas

Versão A

Grupo	1				2							3	
	a)	b)	c)i.	c)ii.	a)	b)i.	b)ii.	b)iii.	b)iv.	c)i.	c)ii.	a)	b)
	D	B	C	A	A	C	B	C	B	B	A	A	C

Versão B

Grupo	1				2							3	
	a)	b)	c)i.	c)ii.	a)	b)i.	b)ii.	b)iii.	b)iv.	c)i.	c)ii.	a)	b)
	B	A	A	C	B	D	D	A	D	C	B	E	D

Resolução abreviada do 3º Teste

1. $X \sim N\left(\mu, \sigma^2\right)$

   (a) Informação populacional: $\sigma = 2.5$ Informação amostral: $n = 25$, $\overline{x} = 31$

   - $Z = \sqrt{25}\,\dfrac{\overline{X} - \mu}{2.5} \sim N(0,1)$
   - $1 - \alpha = 0.8, \quad z_{0.1} = 1.28$
   - $-1.28 \leq 5\dfrac{\overline{X} - \mu}{2.5} \leq 1.28 \Leftrightarrow \overline{X} - 1.28\dfrac{1}{2} \leq \mu \leq \overline{X} + 1.28\dfrac{1}{2}$
   - $IC_{80\%}(\mu) \equiv \left[\overline{X} - 1.28\dfrac{1}{2}\, ,\, \overline{X} + 1.28\dfrac{1}{2}\right]$
   - $IC_{80\%}(\mu) = \left[31 - 1.28\dfrac{1}{2}\, ,\, 31 + 1.28\dfrac{1}{2}\right] = [30.36\, ,\, 31.64]$

   (b) Informação amostral: $n = 9$, $s^2 = 9.61$

   - $W = (9-1)\,\dfrac{S^2}{\sigma^2} \sim \chi^2_{9-1}$, ou seja $W = 8\dfrac{S^2}{\sigma^2} \sim \chi^2_8$
   - $P(a \leq W \leq b) = 1 - \alpha$ e $P(W \leq a) = P(W > b) = \alpha/2$

     $a = \chi^2_{8:1-\alpha/2}$ e $b = \chi^2_{8:\alpha/2}$
   - $\chi^2_{8:1-\alpha/2} \leq 8\dfrac{S^2}{\sigma^2} \leq \chi^2_{8:\alpha/2} \Leftrightarrow \dfrac{8S^2}{\chi^2_{8:\alpha/2}} \leq \sigma^2 \leq \dfrac{8S^2}{\chi^2_{8:1-\alpha/2}}$
   - $IC_{100(1-\alpha)\%}\left(\sigma^2\right) \equiv \left[\dfrac{8S^2}{\chi^2_{8:\alpha/2}}\, ,\, \dfrac{8S^2}{\chi^2_{8:1-\alpha/2}}\right]$
   - Para $s^2 = 9.61$, $IC_{100(1-\alpha)\%}\left(\sigma^2\right) = \left[\dfrac{76.88}{\chi^2_{8:\alpha/2}}\, ,\, \dfrac{76.88}{\chi^2_{8:1-\alpha/2}}\right]$
   - $\dfrac{76.88}{\chi^2_{8:1-\alpha/2}} = 35.266055 \Leftrightarrow \chi^2_{8:1-\alpha/2} = \dfrac{76.88}{35.266055} \approx 2.18$

     $1 - \alpha/2 = 0.975 \Leftrightarrow 1 - \alpha = 0.95$

   (c) Considere a amostra de registos da velocidade em 36 unidades de memória.

3	3	2	1	4	3	4	2	5	6	1	8
1	5	3	3	4	4	1	2	2	7	7	2
9	2	2	1	3	3	4	5	6	7	6	6

   i. $\hat{p} = \dfrac{9}{36} = 0.25$

   ii. Informação amostral: $n = 36$, $\hat{p} = 0.75$

   - $W = \sqrt{36}\dfrac{\hat{P} - p}{\sqrt{\hat{P}\left(1 - \hat{P}\right)}} \overset{a}{\sim} N(0,1)$

- $P(-a \leq W \leq a) \approx 0.95 \Rightarrow a \approx z_{0.025} = 1.96$
- $-1.96 \leq \sqrt{36}\dfrac{\hat{P}-p}{\sqrt{\hat{P}\left(1-\hat{P}\right)}} \leq 1.96 \Leftrightarrow \hat{P} - 1.96\sqrt{\dfrac{\hat{P}\left(1-\hat{P}\right)}{36}} \leq p \leq \hat{P} + 1.96\sqrt{\dfrac{\hat{P}\left(1-\hat{P}\right)}{36}}$
- $IC_{95\%}(p) \equiv \left[\hat{P} - 1.96\sqrt{\dfrac{\hat{P}\left(1-\hat{P}\right)}{36}}\ ,\ \hat{P} + 1.96\sqrt{\dfrac{\hat{P}\left(1-\hat{P}\right)}{36}}\right]$
- Estimativa de $IC_{95\%}(p)$

  $IC_{95\%}(p) = \left[075 - 1.96\sqrt{\dfrac{0.75 \times 0.25}{36}}\ ,\ 075 + 1.96\sqrt{\dfrac{0.75 \times 0.25}{36}}\right] = [0.608549184\ ,\ 0.8914550816]$

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2. População: X-tempo de resposta (em ms) $\qquad X \sim N\left(\mu, \sigma^2\right)$

   Informação amostral: $n = 9, \quad \overline{x} = 101, \quad s^2 = \dfrac{72}{9-1} = 9$

   (a) $H_0:\ \mu = 100\ \ H_1:\ \mu \neq 100$

   (b) $H_0:\ \mu = 102\ \ H_1:\ \mu \neq 102$ $\qquad$ Estatística de teste: $T = \sqrt{9}\dfrac{\overline{X}-102}{S} \underset{\mu=102}{\sim} t_{9-1}$

   i. $R_{0.02} =]-\infty, -2.90[\cup]2.90, +\infty[\qquad a = t_{8:0.01} = 2.90$

   ii. $t_{obs} = \sqrt{9}\dfrac{101-102}{\sqrt{9}} = -1$

   iii. $p-value = P\left(T \in]-\infty, -1.86[\cup]1.86, +\infty[\right) = 2P(T > 1.86) = 2(1-0.05) = 0.1$

   iv. Rejeitamos H_0 se $p-value < \alpha$, ou seja, se $\alpha > 0.08$

   (c) $H_0:\ \mu \leq 100\ \ H_1:\ \mu > 100 \qquad \sigma = 3$

   i. Estatística de teste: $Z = \sqrt{n}\dfrac{\overline{X}-100}{\sigma} \underset{\mu=100}{\sim} N(0,1)$

   ii.
   - Rejeitamos H_0 se $\overline{X} > c$
   - $\alpha = \max\limits_{\mu \leq 100} P\left(\overline{X} > c\right) = P\left(\sqrt{n}\dfrac{\overline{X}-100}{\sigma} > \sqrt{n}\dfrac{c-100}{\sigma}\right) = P\left(Z > \sqrt{n}\dfrac{c-100}{\sigma}\right)$

     Se $n = 9$ e $\alpha = 0.0228$

     $0.0228 = P(Z > c - 100) \Leftrightarrow c - 100 = z_{0.0228} = 2.00 \Leftrightarrow c = 102.00$

     Rejeitamos H_0 ao nível de 0.0228 de significância se $\overline{X} > 102.00$

   - Se $\mu = 101$, $n = 9$ e $\sigma = 3$, $\quad Z = \overline{X} - 101 \underset{\mu=101}{\sim} N(0,1)$

     $P(\text{erro tipo II}\,|\mu = 101) = P(\text{Não rejeitarmos } H_0\,|\mu = 101) = P\left(\overline{X} \leq 102.00\,|\mu = 101\right) =$

     $= P\left(\overline{X} - 101 \leq 102.00 - 101\,|\mu = 101\right) = P(Z \leq 1) = 0.8413$

3. (a)
   - $W = \sqrt{\dfrac{n\pi}{4-\pi}}\dfrac{\theta^*-\theta}{\theta} \overset{a}{\sim} N(0,1)$
   - Para $1-\alpha = 0.95$, $P(-a \leq W \leq a) \approx 0.95 \Rightarrow a \approx z_{0.025} = 1.96$
   - $-1.96 \leq \sqrt{\dfrac{n\pi}{4-\pi}}\dfrac{\theta^*-\theta}{\theta} \leq 1.96 \Leftrightarrow -1.96\sqrt{\dfrac{4-\pi}{n\pi}} \leq \dfrac{\theta^*}{\theta} - 1 \leq 1.96\sqrt{\dfrac{4-\pi}{n\pi}} \Leftrightarrow$

     $\Leftrightarrow 1 - 1.96\sqrt{\dfrac{4-\pi}{n\pi}} \leq \dfrac{\theta^*}{\theta} \leq 1 + 1.96\sqrt{\dfrac{4-\pi}{n\pi}} \Leftrightarrow \dfrac{\theta^*}{1+1.96\sqrt{\dfrac{4-\pi}{n\pi}}} \leq \theta \leq \dfrac{\theta^*}{1-1.96\sqrt{\dfrac{4-\pi}{n\pi}}}$
   - $IC_{95\%}(\theta) \equiv \left[\dfrac{\theta^*}{1+1.96\sqrt{\dfrac{4-\pi}{n\pi}}}\ ,\ \dfrac{\theta^*}{1-1.96\sqrt{\dfrac{4-\pi}{n\pi}}}\right]$

   (b)
   - $W = 2 \times 5\dfrac{\hat{\beta}}{\beta} \sim \chi^2_{2\times 5} \Leftrightarrow W = 10\dfrac{\hat{\beta}}{\beta} \sim \chi^2_{10}$

- $P\left(a \leq W \leq b\right) = 1 - \alpha \quad \text{e} \quad P\left(W \leq a\right) = P\left(W > b\right) = \alpha/2$

  $a = \chi^2_{10:1-\alpha/2} \quad \text{e} \quad b = \chi^2_{10:\alpha/2}$

- $\chi^2_{10:1-\alpha/2} \leq 10\dfrac{\hat{\beta}}{\beta} \leq \chi^2_{10:\alpha/2} \Leftrightarrow \dfrac{\chi^2_{10:1-\alpha/2}}{10\hat{\beta}} \leq \dfrac{1}{\beta} \leq \dfrac{\chi^2_{10:\alpha/2}}{10\hat{\beta}} \Leftrightarrow \dfrac{10\hat{\beta}}{\chi^2_{10:\alpha/2}} \leq \beta \leq \dfrac{10\hat{\beta}}{\chi^2_{10:1-\alpha/2}}$

- $IC_{100(1-\alpha)\%}\left(\beta\right) \equiv \left[\dfrac{10\hat{\beta}}{\chi^2_{10:\alpha/2}}\,,\,\dfrac{10\hat{\beta}}{\chi^2_{10:1-\alpha/2}}\right]$