Grelha de respostas certas

Versão A

Grupo	1			2 a)	2 b)			2 c)		
	a)	b)	c)		i.	ii.	iii.	i.	ii.	iii.
	D	F	C	D	B	C	A	B	C	C

Versão B

Grupo	1			2 a)	2 b)			2 c)		
	a)	b)	c)		i.	ii.	iii.	i.	ii.	iii.
	B	V	A	B	A	B	C	C	A	B

Resolução abreviada do 3º Teste

Versão A

1. Informação populacional: X-intensidade/componente (Amperes) $\quad X \sim N\left(\mu, \sigma^2\right)$

   Informação amostral: $n = 20$, $\overline{x} = 10.43$, $s = 1.98$

   (a) A. Variável pivot para μ: $\quad T = \sqrt{20}\,\dfrac{\overline{X}-\mu}{S} \sim t_{19}$

   B. Determinação da constante $a \in \mathbb{R}^+$ que satisfaz $P\left(-a \le T \le a\right) = 0.99$: $\quad a = t_{19:0.005} = 2.86$

   C. $-2.86 \le \sqrt{20}\,\dfrac{\overline{X}-\mu}{S} \le 2.86 \Leftrightarrow \overline{X} - 2.86\dfrac{S}{\sqrt{20}} \le \mu \le \overline{X} + 2.86\dfrac{S}{\sqrt{20}}$

   D. $IC_{99\%}\left(\mu\right) \equiv \left[\overline{X} - 2.86\dfrac{S}{\sqrt{20}}\, ,\, \overline{X} + 2.86\dfrac{S}{\sqrt{20}}\right]$

   $$IC_{99\%}\left(\mu\right) = \left[10.43 - 2.86\frac{1.98}{\sqrt{20}}\, ,\, 10.43 + 2.86\frac{1.98}{\sqrt{20}}\right] = [9.163759426\, ,\, 11.69624057]$$

   (b) Falsa porque $IC_{90\%}\left(\mu\right) \subset IC_{99\%}\left(\mu\right)$

   (c) A. Variável pivot para σ^2: $\quad X^2 = \dfrac{19\,S^2}{\sigma^2} \sim \chi^2_{19}$

   B. Determinação das constantes $a < b$ $\left(a, b \in \mathbb{R}^+\right)$ que satisfazem:

   $P\left(a \le X^2 \le b\right) = 0.9 \quad$ e $\quad P\left(X^2 < a\right) = P\left(X^2 > b\right) = 0.05$

   $a = \chi^2_{19:0.95} = 10.1 \qquad b = \chi^2_{19:0.05} = 30.1$

   C. $10.1 \le \dfrac{19\,S^2}{\sigma^2} \le 30.1 \Leftrightarrow \dfrac{19\,S^2}{30.1} \le \sigma^2 \le \dfrac{19\,S^2}{10.1}$

   D. $IC_{90\%}\left(\sigma^2\right) \equiv \left[\dfrac{19\,S^2}{30.1}\, ,\, \dfrac{19\,S^2}{10.1}\right] \qquad IC_{90\%}\left(\sigma\right) \equiv \left[\sqrt{\dfrac{19\,S^2}{30.1}}\, ,\, \sqrt{\dfrac{19\,S^2}{10.1}}\right]$

   $$IC_{90\%}\left(\sigma\right) \equiv \left[\sqrt{\frac{19 \times 1.98^2}{30.1}}\, ,\, \sqrt{\frac{19 \times 1.98^2}{10.1}}\right] = [1.573108736\, ,\, 2.715696946]$$

2. Informação amostral: $n = 196$, $\overline{x} = 3.2$, $s^2 = 0.64$, $s = 0.8$

   (a) $H_0 :\ \mu = 3.5 \ \ vs \ \ H_1 :\ \mu \neq 3.5$

   (b) i. Estatística de teste: $\quad W = \sqrt{196}\,\dfrac{\overline{X}-3.0}{S} \underset{\mu=3.0}{\overset{a}{\sim}} N\left(0, 1\right)$

   $0.03 = P\left(W > a\right) \Rightarrow a \approx z_{0.03} = 1.88 \qquad R_{0.03} =]1.88, +\infty[$

ii. $w_{obs} = \sqrt{196}\,\dfrac{3.2-3}{0.8} = 3.5$

iii. $w_{obs} = 1.45, \quad R_{obs} =]1.45, +\infty[$

$$p-value = P(W \in R_{obs}) \approx P(Z > 1.45) = 1 - 0.9265 = 0.0735$$

(c) Seja $p = P(X > 3.5)$ $\qquad H_0 : \ p \geq 0.5 \ vs \ H_1 : \ p < 0.5$

i. $W = \sqrt{196}\,\dfrac{\hat{P}-0.5}{\sqrt{0.5(1-0.5)}} \underset{p=0.5}{\overset{a}{\sim}} N(0,1) \Leftrightarrow W = 28\left(\hat{P}-0.5\right) \underset{p=0.5}{\overset{a}{\sim}} N(0,1)$

ii. $R_{obs} =]-\infty, w_{obs}[$

$$0.07656 = P(W \in R_{obs}) \Leftrightarrow 0.07656 \approx P(Z < w_{obs}) \Leftrightarrow w_{obs} = -1.43$$

iii. Para $\alpha \geq 0.08$, rejeitamos H_0 porque $p - value < \alpha$.

Versão B

1. Informação populacional: X-intensidade/componente (Amperes) $\qquad X \sim N\left(\mu, \sigma^2\right)$

Informação amostral: $n = 20$, $\overline{x} = 10.43$, $s = 1.98$

(a) A. Variável pivot para μ: $\quad T = \sqrt{20}\,\dfrac{\overline{X}-\mu}{S} \sim t_{19}$

B. Determinação da constante $a \in \mathbb{R}^+$ que satisfaz $P(-a \leq T \leq a) = 0.99$: $\quad a = t_{19:0.05} = 1.73$

C. $-1.73 \leq \sqrt{20}\,\dfrac{\overline{X}-\mu}{S} \leq 1.73 \ \Leftrightarrow \ \overline{X} - 1.73\dfrac{S}{\sqrt{20}} \leq \mu \leq \overline{X} + 1.73\dfrac{S}{\sqrt{20}}$

D. $IC_{90\%}(\mu) \equiv \left[\overline{X} - 1.73\dfrac{S}{\sqrt{20}}\ ,\ \overline{X} + 1.73\dfrac{S}{\sqrt{20}}\right]$

$$IC_{90\%}(\mu) = \left[10.43 - 1.73\frac{1.98}{\sqrt{20}}\ ,\ 10.43 + 1.73\frac{1.98}{\sqrt{20}}\right] = [9.664057275\ ,\ 11.19594273]$$

(b) Verdadeira porque $IC_{90\%}(\mu) \subset IC_{99\%}(\mu)$

(c) A. Variável pivot para σ^2: $\quad X^2 = \dfrac{19\,S^2}{\sigma^2} \sim \chi^2_{19}$

B. Determinação das constantes $a < b$ $(a, b \in \mathbb{R}^+)$ que satisfazem:

$$P\left(a \leq X^2 \leq b\right) = 0.99 \quad \text{e} \quad P\left(X^2 < a\right) = P\left(X^2 > b\right) = 0.005$$

$$a = \chi^2_{19:0.995} = 6.84 \qquad b = \chi^2_{19:0.005} = 38.6$$

C. $6.84 \leq \dfrac{19\,S^2}{\sigma^2} \leq 38.6 \ \Leftrightarrow \ \dfrac{19\,S^2}{38.6} \leq \sigma^2 \leq \dfrac{19\,S^2}{6.84}$

D. $IC_{99\%}\left(\sigma^2\right) \equiv \left[\dfrac{19\,S^2}{38.6}\ ,\ \dfrac{19\,S^2}{6.84}\right] \qquad IC_{99\%}(\sigma) \equiv \left[\sqrt{\dfrac{19\,S^2}{38.6}}\ ,\ \sqrt{\dfrac{19\,S^2}{6.84}}\right]$

$$IC_{90\%}(\sigma) \equiv \left[\sqrt{\frac{19 \times 1.98^2}{38.6}}\ ,\ \sqrt{\frac{19 \times 1.98^2}{6.84}}\right] = [1.389147426\ ,\ 3.3]$$

2. Informação amostral: $n = 144$, $\overline{x} = 3.7$, $s^2 = 0.81$, $s = 0.9$

(a) $H_0 : \ \mu = 3.2 \ vs \ H_1 : \ \mu \neq 3.2$

(b) i. Estatística de teste: $\quad W = \sqrt{144}\,\dfrac{\overline{X}-3.5}{S} \underset{\mu=3.5}{\overset{a}{\sim}} N(0,1)$

$$0.02 = P(W > a) \Rightarrow a \approx z_{0.02} = 2.05 \qquad R_{0.02} =]2.05, +\infty[$$

ii. $w_{obs} = \sqrt{144}\,\dfrac{3.7-3.5}{0.9} \approx 2.667$

iii. $w_{obs} = 1.38, \quad R_{obs} =]1.38, +\infty[$

$$p-value = P(W \in R_{obs}) \approx P(Z > 1.38) = 1 - 0.9162 = 0.0838$$

(c) Seja $p = P(X > 3.2)$ $\qquad H_0 : \ p \geq 0.5 \ vs \ H_1 : \ p < 0.5$

i. $W = \sqrt{144}\,\dfrac{\hat{P}-0.5}{\sqrt{0.5(1-0.5)}} \underset{p=0.5}{\overset{a}{\sim}} N(0,1) \Leftrightarrow W = 24\left(\hat{P}-0.5\right) \underset{p=0.5}{\overset{a}{\sim}} N(0,1)$

ii. $R_{obs} =]-\infty, w_{obs}[$

$0.09121 = P\left(W \in R_{obs}\right) \Leftrightarrow 0.09121 \approx P\left(Z < w_{obs}\right) \Leftrightarrow w_{obs} = -1.33$

iii. Para $\alpha \geq 0.1$, rejeitamos H_0 porque $p - value < \alpha$.