Grelha de respostas certas

Versão A

Grupo	1		2					3			4			5			
	a)	b)	a) i.	a) ii.	b) i.	b) ii.	c)	a)	b)	c)	a)	b)	c)	a)	b)	c)	d)
	D	B	C	B	D	D	B	A	D	B	C	C	A	C	C	A	A

Versão B

Grupo	1		2					3			4			5			
	a)	b)	a) i.	a) ii.	b) i.	b) ii.	c)	a)	b)	c)	a)	b)	c)	a)	b)	c)	d)
	A	C	B	A	C	B	D	B	C	A	A	B	C	D	A	B	C

Resolução abreviada do Exame de Recurso

1. Considerem-se os acontecimentos: Um doente
   - A-ter tomado o medicamento A
   - B-ter tomado o medicamento B
   - C-ter tomado o medicamento C
   - L-perder peso

   $P(A) = 0.3 \quad P(B) = 0.3 \quad P(C) = 0.4 \qquad P(L|A) = 0.9 \quad P(L|B) = 0.8 \quad P(L) = 0.75$

   (a) $P(L) = P(L|A)\,P(A) + P(L|B)\,P(B) + P(L|C)\,P(C)$

   $\Leftrightarrow 0.75 = 0.51 + 0.4 \times P(L|C) \Leftrightarrow P(L|C) = 0.6$

   (b) $P(A|L) = \dfrac{P(L|A)\,P(A)}{P(L)} = 0.36$

2. (a) i. Seja W - n.º de dados vermelhos obtidos numa amostra de 4 dados (seleccionadas ao acaso e sem reposição). $\quad W \sim H(20, 12, 4)$

   ii. $P(W = 3) = \dfrac{\binom{12}{3}\binom{8}{1}}{\binom{20}{4}} = 0.363261094$

   (b) $X \sim B\left(5, \frac{8}{20}\right) \equiv B(5, 0.4)$

   i. $P(X > 4) = P(X = 5) = \binom{5}{5} 0.4^5 0.6^{5-5} = 0.01024$

   ii. $E(X) = 5 \times 0.4 = 2 \qquad V(X) = 5 \times 0.4 \times (1 - 0.4) = 1.2$

   $P(X \in]E(X) - 1.5\sigma(X)\ ,\ E(X) + 1.5\sigma(X)[) = P(X \in]0.356832327\ ,\ 3.643167673[)$

   $= \sum_{k=1}^{3} P(X = k) = \sum_{k=1}^{3} \binom{5}{k} 0.4^k 0.6^{5-k} = 0.8352$

   (c) $Y \sim B(100, 0.4) \qquad E(Y) = 40 \qquad V(Y) = 24$

   Pelo TLC, e porque $np = 100 \times 0.4 > 5$ e $n(1-p) = 100 \times 0.6 > 5$

   $\dfrac{Y - E(Y)}{\sqrt{V(Y)}} = \dfrac{Y - 40}{\sqrt{24}} \overset{a}{\sim} N(0,1)$

   $P(33 < Y \leq 47) = P(Y \leq 47) - P(Y \leq 33) = P\left(\dfrac{Y-40}{\sqrt{24}} \leq \dfrac{47-40}{\sqrt{24}}\right) - P\left(\dfrac{Y-40}{\sqrt{24}} \leq \dfrac{33-40}{\sqrt{24}}\right)$

   $\approx P(Z \leq 1.43) - P(Z \leq -1.43) = P(Z \leq 1.43) - 1 + P(Z \leq 1.43) = 2\,P(Z \leq 1.43) - 1$

   $= 2 \times 0.9236 - 1 = 0.8472$

3. $X \sim U(\theta, 0)\,,\ \theta \in \mathbb{R}^-$ $\qquad E(X) = \dfrac{\theta}{2},\quad V(X) = \dfrac{\theta^2}{12}$

   (a) $E(X) = \overline{X} \Leftrightarrow \dfrac{\theta}{2} = \overline{X} \Leftrightarrow \theta = 2\,\overline{X}$ $\qquad$ sendo $\overline{X} = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} X_i$

   O estimador dos momentos para θ é, $\theta^* = 2\,\overline{X}$

   (b) $E\left(\tilde{\theta}\right) = \theta \Leftrightarrow E(a\,T_n + b) = \theta \Leftrightarrow a\,E(T_n) + b = \theta \Leftrightarrow a\dfrac{2\theta}{n} + b = \theta \Rightarrow b = 0;\quad a\dfrac{2}{n} = 1 \Rightarrow b = 0;\quad a = \dfrac{n}{2}$

   (c) $V\left(\hat{\theta}\right) = V\left(2\overline{X}\right) = 4\dfrac{V(X)}{n} = \dfrac{4}{n}\dfrac{\theta^2}{12} = \dfrac{\theta^2}{3n}$

   $$\frac{V\left(\ddot{\theta}\right)}{V\left(\hat{\theta}\right)} = \frac{\dfrac{\theta^2}{n(n+2)}}{\dfrac{\theta^2}{3n}} = \frac{3}{n+2} < 1 \quad \text{porque } n \geq 2$$

   $\ddot{\theta}$ é o estimador mais eficiente porque os dois estimadores são centrados e $V\left(\ddot{\theta}\right) < V\left(\hat{\theta}\right)$

4. Informação populacional: X-peso/saco (em kg) $\qquad X \sim N\left(\mu, \sigma^2\right)$

   Informação amostral: $n = 25,\quad \overline{x} = \dfrac{1255}{25} = 50.2,\quad s^2 = \dfrac{245.76}{25-1} = 10.24,\quad s = \sqrt{s^2} = 3.2$

   (a)
   - $W = \dfrac{(25-1)\,S^2}{\sigma^2} \sim \chi^2_{25-1} \equiv \chi^2_{24}$
   - $P(a \leq W \leq b) = 0.9$, com $P(W < a) = 0.05$ e $P(W > b) = 0.05$

     $a = \chi^2_{24:0.95} = 13.8 \qquad b = \chi^2_{24:0.05} = 36.4$
   - $13.8 \leq \dfrac{24\,S^2}{\sigma^2} \leq 36.4 \quad \Leftrightarrow \quad \dfrac{24\,S^2}{36.4} \leq \sigma^2 \leq \dfrac{24\,S^2}{13.8}$
   - $IC_{90\%}\left(\sigma^2\right) \equiv \left[\dfrac{24\,S^2}{36.4}\,,\ \dfrac{24\,S^2}{13.8}\right]$

     $IC_{90\%}(\sigma) \equiv \left[\sqrt{\dfrac{24\,S^2}{36.4}}\,,\ \sqrt{\dfrac{24\,S^2}{13.8}}\right]$

   Estimativa por intervalo de 90% de confiança para σ

   $$IC_{90\%}(\sigma) = \left[\sqrt{\frac{245.76}{36.4}}\,,\ \sqrt{\frac{245.76}{13.8}}\right] = [2.598393417\,,\ 4.22003503]$$

   (b)
   - $T = \sqrt{25}\dfrac{\overline{X} - \mu}{S} \sim t_{25-1} \equiv t_{24}$
   - $P(-a \leq T \leq a) = 0.9$, com $P(T > a) = 0.05,\quad a = t_{24:0.05} = 1.71$
   - $-1.71 \leq 5\dfrac{\overline{X} - \mu}{S} \leq 1.71 \ \Leftrightarrow\ \overline{X} - 1.71\dfrac{S}{5} \leq \mu \leq \overline{X} + 1.71\dfrac{S}{5}$
   - $IC_{90\%}(\mu) \equiv \left[\overline{X} - 1.71\dfrac{S}{5}\,,\ \overline{X} + 1.71\dfrac{S}{5}\right]$, cuja amplitude é $A_{90\%}(\mu) \equiv 2 \times 1.71\dfrac{S}{5}$

     A estimativa da amplitude do $IC_{90\%}(\mu)$ é: $A_{90\%}(\mu) = 2 \times 1.71\dfrac{3.2}{5} = 2.1888$

   (c) $p = P(\text{saco com peso superior a 51 kg})$

   Informação amostral: $n = 100,\quad \hat{p} = 20/100 = 0.2$

   - $W = \sqrt{100}\dfrac{\hat{P} - p}{\sqrt{\hat{P}\left(1 - \hat{P}\right)}} \overset{a}{\sim} N(0,1)$
   - $P(-a \leq W \leq a) = 0.95 \Rightarrow a \approx z_{0.025} = 1.96$
   - $-1.96 \leq \sqrt{100}\dfrac{\hat{P} - p}{\sqrt{\hat{P}\left(1 - \hat{P}\right)}} \leq 1.96 \ \Leftrightarrow\ \hat{P} - 1.96\sqrt{\dfrac{\hat{P}\left(1 - \hat{P}\right)}{100}} \leq \hat{P} + 1.96\sqrt{\dfrac{\hat{P}\left(1 - \hat{P}\right)}{100}}$
   - $IC_{95\%}(p) \equiv \left[\hat{P} - 1.96\sqrt{\dfrac{\hat{P}\left(1 - \hat{P}\right)}{100}}\,,\ \hat{P} + 1.96\sqrt{\dfrac{\hat{P}\left(1 - \hat{P}\right)}{100}}\right]$

- Estimativa: $IC_{95\%}(p) = \left[0.2 - 1.96\sqrt{\frac{0.2\,(1-0.2)}{100}}\ ,\ 0.2 + 1.96\sqrt{\frac{0.2\,(1-0.2)}{100}}\right] = [0.1216\ ,\ 0.2784]$

5. Informação populacional: X-tempo máximo/conjunto $\quad X \sim N\left(\mu, \sigma^2\right)$
   (a) $H_0\ : \sigma \geq \sigma_0$
   (b) $W = (25-1)\,\dfrac{S^2}{0.64} = 37.5\,S^2 \underset{\sigma^2=0.64}{\sim} \chi^2_{25-1} \equiv \chi^2_{24}$
   (c) A. $W = 37.5\,S^2 \underset{\sigma^2=0.64}{\sim} \chi^2_{24}$
       B. $R_{obs} =]w_{obs}, +\infty[\, =]29.6, +\infty[$

       $p - value = P\left(W \in R_{obs}\right) = P\left(W > 29.6\right) = 0.2$
   (d) $p - value = 0.15$

   Rejeitamos a hipótese H_0 ao nível α de significância, se $p - value < \alpha$.