&0>0 10,&0>0 1 &0>0 10,&0>0 1

# Assignment1 kNN vs Linear Regression

February 10, 2019

```
In [4]: from data_utils import load_dataset
        from sklearn.neighbors import KDTree
        import matplotlib.pyplot as plt
        import numpy as np
        import pandas as pd # Only for formatting and plotting
        import math

        _COLORS = ['#d6616b', '#e6550d', '#fdae6b', '#ffbb78', '#e7ba52', '#dbdb8d']

        def loadData(datasetName, d=2, n=1000):
            '''
            Loads the dataset and normalize the x_ sets
            INPUT: datasetName: a string of the name of file to be loaded. Note that this file m
            OUTPUT: 6 datasets in array form, 3 of which are normalized x data
            '''
            if datasetName == 'rosenbrock':
                x_train, x_valid, x_test, y_train, y_valid, y_test = load_dataset(datasetName, n
            else:
                x_train, x_valid, x_test, y_train, y_valid, y_test = load_dataset(datasetName)

            x_all = np.concatenate([x_train, x_valid])
            y_all = np.concatenate([y_train, y_valid])
            index_all = list(range(np.shape(x_all)[0]))
            np.random.seed(99)
            np.random.shuffle(index_all)

            # Normalizetion of each x data
            mean = x_all.mean(axis=0, keepdims=True)
            stddev = x_all.std(axis=0, keepdims=True)
            x_all = normalization(x_all, mean, stddev)
            x_test = normalization(x_test, mean, stddev)
            return index_all, x_all, x_test, y_all, y_test

        def foldDataset(allIndex, x_all, y_all, foldIndex):
            '''
            Split data into two sets of ratio 4:1 according to the foldIndex
            INPUT: allData: concatenate dataset
```

```
        INPUT: foldIndex: from 1 to 5, decides how to partition the dataset (must be from ou
        OUTPUT: train, set: the 4:1 ratio datasets
        '''
        total = len(allIndex)
        oneFifth = round(total/5)
        if foldIndex in [1, 2, 3, 4]:
            index_train = allIndex[:oneFifth*(foldIndex-1)] + allIndex[oneFifth*foldIndex:]
            index_valid = allIndex[oneFifth*(foldIndex-1) : oneFifth*foldIndex]
        elif foldIndex == 5: # for the last fold, cound backwards so that it has the same nu
            index_train = allIndex[:(total-oneFifth)]
            index_valid = allIndex[(total-oneFifth):]
        x_train, x_valid = x_all[index_train], x_all[index_valid]
        y_train, y_valid = y_all[index_train], y_all[index_valid]
        return x_train, x_valid, y_train, y_valid

    def normalization(x, mean, stddev):
        '''
        Returned a matrix of x data normalized against x_train's mean and stddev
        '''
        return (x - mean)/stddev
```

In [21]:
```
class kNNTraining:
    def __init__(self, datasetName, distanceHeuristic='l2', k=3, modificationIndex=1, d
        '''
        To run kNNTraining, please declare a class with the desired parameters and then
        "kNNtest1.kNNRegression(kNNtest1.x_test[i], kNNtest1.y_test[i])" in a loop pf d
        '''
        self.distanceHeuristic = distanceHeuristic # distance calculation distanceHeuri
        self.k = k  # number of nearest neighbours required
        self.modificationIndex = modificationIndex # chooses which method to use to mod

        # Extraxt datasets associated with the dataset's name
        # x/y_train: the training sets, must be a N-by-D matrix for x_train and N-by-(#
        self.index_all, self.x_all, self.x_test, self.y_all, self.y_test = loadData(dat
        self.num_dimension = np.shape(self.x_test)[1]
        self.num_validSet = round(np.shape(self.x_all)[0]/5)
        self.num_trainSet = np.shape(self.x_all)[0] - self.num_validSet
        self.num_testSet = np.shape(self.x_test)[0]

    def foldDataset(self, foldIndex):
        self.x_train, self.x_valid, self.y_train, self.y_valid = foldDataset(self.index


    def kNNClassification(self, x, y):
        '''
        Classify which class this x is in and compare to its actual value
        INOUT: x, y: 1-dimensional vectors, typically a row from x/y_test or x/y_valid
        OUTPUT: kNNClass: a classification result of class y
```

```
            correctness: a boolean indicating if the prediction is the same as labe
        '''
        actualClass = y
        iNN = self.getNeighbours_2(x, y)
        # print('Selected', self.k, "nearest neighbours' classes:\n", self.y_train[iNN]

        vote, count = np.unique(self.y_train[iNN], axis=0, return_counts=True) # Find t
        kNNClass = vote[np.argmax(count)]
        # print('Classified in class', list(kNNClass).index(True), 'and it is actually

        correctness = np.unique(kNNClass == actualClass)[0] # Compare to the actual cla
        # print('Classified in class', list(kNNClass).index(True), '\nResult is', corre
        return kNNClass, correctness


    def kNNRegression(self, x, y, modificationIndex):
        '''
        Predict the output value of given x and compare to its actual label y
        INOUT: x, y: 1-dimensional vectors, typically a row from x/y_test or x/y_valid
        INPUT: modificationIndex: one of 1, 2, 3
        OUTPUT: kNNClass: a classification result of class y
                error: absolute difference between predicted and given y's
                correctness: a boolean indicating if the prediction is within a certain
        '''
        actualValue = y[0]
        if modificationIndex == 1: iNN = self.getNeighbours(x, y)
        elif modificationIndex == 2: iNN = self.getNeighbours_2(x, y)
        yNN = self.y_train[iNN]
        # print('Selected', self.k, "nearest neighbours' values:\n", yNN)

        kNNValue = (sum(yNN)/len(yNN))[0]
        # print('Classified in class', list(kNNClass).index(True), 'and it is actually

        error = kNNValue - actualValue # Compare to the actual class
        percent_error =  abs(error/actualValue) # Compare to the actual class
        correctness = (percent_error < 0.25)
        # print('Predicted value is', kNNValue, '\nError is', error*100, '%', 'and cons
        return kNNValue, error, correctness


    def kNNRegression_4(self, x_set, y_set):
        '''
        Predict the output values of ALL x's and compare to their actual label y's -- k
        INOUT: x_set, y_set: either x_test and y_test or x_valind and y_valid
        OUTPUT: kNNValues: results of class y (dimension = num_testSet or num_validSet)
                errorList: array of error
        '''
        tree = KDTree(self.x_train, leaf_size=2)
```

```python
        distances, iNN = tree.query(x_set, self.k)

        y_train = np.broadcast_to(self.y_train,(len(x_set),)+self.y_train.shape)
        yNN = y_train[0, iNN]
        kNNValues = np.mean(yNN, axis=1)

        errorList = kNNValues - y_set # Compare to the actual value
        return kNNValues, errorList # Both should be arrays
        errorList = kNNValues - y_set # Compare to the actual value
        return kNNValues, errorList # Both should be arrays


    def kNNRegression_3(self, x_set, y_set):
        '''
        Predict the output values of ALL x's and compare to their actual label y's -- f
        INOUT: x_set, y_set: either x_test and y_test or x_valind and y_valid
        OUTPUT: kNNValues: results of class y (dimension = num_testSet or num_validSet)
                errorList: array of error
        '''
        x_train = np.broadcast_to(self.x_train,(len(x_set),)+self.x_train.shape)
        y_train = np.broadcast_to(self.y_train,(len(x_set),)+self.y_train.shape)
        x_set = np.expand_dims(x_set, axis=1)

        if self.distanceHeuristic == 'l2': distances = np.sqrt(np.sum(np.square(x_train
        elif self.distanceHeuristic == 'linf': distances = np.max(np.absolute(x_train -
        elif self.distanceHeuristic == 'l1': distances = np.sum(np.absolute(x_train - x

        iNN = np.argpartition(distances, range(self.k), axis = 1)[:, :self.k]
        yNN = y_train[0, iNN]
        kNNValues = np.mean(yNN, axis=1)

        errorList = kNNValues - y_set # Compare to the actual value
        return kNNValues, errorList # Both should be arrays


    def getNeighbours_2(self, x, y):
        '''
        Get k nearest neighbours for a given x using vectorized python code instead of
        INOUT: x, y: 1-dimensional vectors, typically a row from x/y_test or x/y_valid
        OUTPUT: a list of indexes of data in x_train that are the k nearest neighbours
        '''
        if self.distanceHeuristic == 'l2': distances = np.sqrt(np.sum(np.square(self.x_
        elif self.distanceHeuristic == 'linf': distances = np.max(np.absolute(self.x_tr
        elif self.distanceHeuristic == 'l1': distances = np.sum(np.absolute(self.x_trai
        iNN = np.argpartition(distances, range(self.k))[:self.k]
        return iNN
```

```
    def getNeighbours(self, x, y):
        '''
        Get k nearest neighbours for a given x
        INOUT: x, y: 1-dimensional vectors, typically a row from x/y_test or x/y_valid
        OUTPUT: a list of indexes of data in x_train that are the k nearest neighbours
        '''
        distances = [self.getDistance(self.x_train[i], self.y_train[i], x, y) for i in
        iNN = np.argpartition(distances, range(self.k))[:self.k]
        return iNN


    def getDistance(self, x1, y1, x2, y2):
        '''
        Calculates the distance with specified distanceHeuristic (default is 'l2')
        INPUT: xy1 and xy2: 1-dimensional vectors (two rows in a dataset)
        INPUT: distanceHeuristic: 'l1', 'l2', 'linf'
        OUTPUT: a numeric value of the distance
        '''
        try:
            # print('Label of x1:', y1, '\nLabel of x2:', y2)
            sum_distance = 0 # Initiate the distance
            if self.distanceHeuristic == 'l1': return np.linalg.norm(x1 - x2, 1)
            elif self.distanceHeuristic == 'l2': return np.linalg.norm(x1 - x2)
            elif self.distanceHeuristic == 'linf': return np.linalg.norm(x1 - x2, 'inf'
            else: print("Error! Input 'distanceHeuristic' must be one of 'l1', 'l2', an
        except:
            print("Error! xy1 and xy2 must be 1-dimensional vectors.")
            print("x1 is now a", type(x1), 'in shape', np.shape(x1))
```

```
In [22]: def RMSEComparison(datasetName, model, set, kRange, modificationIndex=3):
             '''
             Record the performance (RMSE) on each k value and distance metric
             INPUT: model: one of 'classification' or 'regression'
             INPUT: set: must be one of 'validation' or 'test'
             INPUT: kRange: a range of k to be tested on
             '''
             print('::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::')
             dict = {} # with keys: k and distance distanceHeuristic
             optimal = {} # with key: distance distanceHeuristic
             for distanceHeuristic in ['l1', 'l2', 'linf']:
                 dict[distanceHeuristic] = {}
                 for k in kRange:
                     if model == 'regression': dict[distanceHeuristic][k] = RMSELoss_Regression(
                     else: dict[distanceHeuristic][k] = Accuracy_Classification(datasetName, set
                 if model == 'regression': rmseOptimal = min(dict[distanceHeuristic].values())
                 else: rmseOptimal = max(dict[distanceHeuristic].values())
                 kOptimal = [k for k, rmse in dict[distanceHeuristic].items() if rmse == rmseOpt
                 optimal[distanceHeuristic] = kOptimal
```

```python
    df = pd.DataFrame(dict)
    print('For dataset '+datasetName+', the test results are:\n', df, '\nThe optimal k
    print('::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::')
#     return df


def RMSELoss_Regression(datasetName, set, distanceHeuristic, k, modificationIndex=3):
    '''
    Takes a dataset and calculates the root-mean-square-error(RMSE) loss of the regress
    INPUT: databaseName: must be one of the regression datasets, can't be classificatio
    INPUT: distanceHeuristic: distance calculation distanceHeuristic, 'l1', 'l2', or 'l
    INPUT: k: number of nearest neighbours required
    OUTPUT: a value of the average RMSE loss across 5 folds
    '''
    # print('::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::')
    # print('Processing', datasetName, 'with', k, 'nearest neighbours using', distanceH
    kNNtest = kNNTraining(datasetName, distanceHeuristic, k, modificationIndex)
    rmseValues = [] # 5 rmse values from each fold
    if set == 'validation':
        for foldIndex in range(1,6):
            kNNtest.foldDataset(foldIndex)
            errorList = []
            if modificationIndex in [1, 2]:
                for i in range(kNNtest.num_validSet):
                    kNNValue, error, correctness = kNNtest.kNNRegression(kNNtest.x_vali
                    errorList.append(error)
                    # print(kNNtest.y_valid[i], kNNValue)
            elif modificationIndex == 3:
                kNNValue, errorList = kNNtest.kNNRegression_3(kNNtest.x_valid, kNNtest.
            elif modificationIndex == 4:
                kNNValue, errorList = kNNtest.kNNRegression_4(kNNtest.x_valid, kNNtest.
            rmse = np.sqrt(pow(np.array(errorList), 2).mean())
            # print('RMSE is', rmse, 'for fold', foldIndex)
            rmseValues.append(rmse)
    else:
        kNNtest.foldDataset(1)
        kNNtest.x_train, kNNtest.y_train = kNNtest.x_all, kNNtest.y_all
        errorList = []
        if modificationIndex in [1, 2]:
            for i in range(kNNtest.num_testSet):
                kNNValue, error, correctness = kNNtest.kNNRegression(kNNtest.x_test[i],
                errorList.append(error)
        elif modificationIndex == 3:
            kNNValues, errorList = kNNtest.kNNRegression_3(kNNtest.x_test, kNNtest.y_te
        elif modificationIndex == 4:
            kNNValue, errorList = kNNtest.kNNRegression_4(kNNtest.x_test, kNNtest.y_tes
        rmseValues.append(np.sqrt(pow(np.array(errorList), 2).mean()))
    # print('::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::')
```

```
        return np.mean(rmseValues)


    def Accuracy_Classification(datasetName, set, distanceHeuristic, k):
        '''
        Takes a dataset and calculates the root-mean-square-error(RMSE) loss of the regress
        INPUT: databaseName: must be one of the regression datasets, can't be classificatio
        INPUT: 'set': must be one of 'validation' or 'test'
        INPUT: distanceHeuristic: distance calculation distanceHeuristic, 'l1', 'l2', or 'l
        INPUT: k: number of nearest neighbours required
        OUTPUT: a value of the RMSE loss
        '''
        np.random.seed(20192019)
        # print('::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::')
        # print('Processing', datasetName, 'with', k, 'nearest neighbours using', distanceH
        kNNtest = kNNTraining(datasetName, distanceHeuristic, k)
        Accuracies = [] # 5 rmse values from each fold
        if set == 'validation':
            for foldIndex in range(1,6):
                kNNtest.foldDataset(foldIndex)
                accuracies = []
                for i in range(kNNtest.num_validSet):
                    kNNValue, correctness = kNNtest.kNNClassification(kNNtest.x_valid[i], k
                    accuracies.append(correctness)
                avg = np.array(accuracies).mean()
                Accuracies.append(avg)
        else:
            kNNtest.foldDataset(1)
            kNNtest.x_train, kNNtest.y_train = kNNtest.x_all, kNNtest.y_all
            accuracies = []
            for i in range(kNNtest.num_testSet):
                kNNValue, correctness = kNNtest.kNNClassification(kNNtest.x_test[i], kNNtes
                accuracies.append(correctness)
            Accuracies.append(np.array(accuracies).mean())
        # print('::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::')
        return np.mean(Accuracies)
```

In [12]:
```
# np.random.seed(20192019)
# kNNtest = kNNTraining('mauna_loa', 'l2', 2, 4)
# kNNtest.foldDataset(1)
# kNNValue, errorList = kNNtest.kNNRegression_4(kNNtest.x_valid, kNNtest.y_valid)
```

In [16]:
```
Q1_Regression = {}
print('Processing dataset mauna_loa...')
Q1_Regression['mauna_loa'] = RMSEComparison('mauna_loa', 'regression', 'validation', ra
```

```
Processing dataset mauna_loa...
:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
```

```
For dataset mauna_loa, the test results are:
  & l1  & l2  & linf
1   & 0.048766  & 0.048766  & 0.048766
2   & 0.034748  & 0.034748  & 0.034748
3   & 0.042465  & 0.042465  & 0.042465
4   & 0.047779  & 0.047779  & 0.047779
5   & 0.056833  & 0.056833  & 0.056833
6   & 0.064124  & 0.064124  & 0.064124
7   & 0.072076  & 0.072076  & 0.072076
8   & 0.078499  & 0.078499  & 0.078499
9   & 0.084152  & 0.084152  & 0.084152
10  & 0.088429  & 0.088429  & 0.088429
11  & 0.091492  & 0.091492  & 0.091492
12  & 0.093084  & 0.093084  & 0.093084
13  & 0.093264  & 0.093264  & 0.093264
14  & 0.092027  & 0.092027  & 0.092027
15  & 0.090025  & 0.090025  & 0.090025
16  & 0.086828  & 0.086828  & 0.086828
17  & 0.083502  & 0.083502  & 0.083502
18  & 0.080067  & 0.080067  & 0.080067
19  & 0.076951  & 0.076951  & 0.076951
20  & 0.074499  & 0.074499  & 0.074499
21  & 0.072748  & 0.072748  & 0.072748
22  & 0.072042  & 0.072042  & 0.072042
23  & 0.071830  & 0.071830  & 0.071830
24  & 0.072513  & 0.072513  & 0.072513
25  & 0.073488  & 0.073488  & 0.073488
26  & 0.074957  & 0.074957  & 0.074957
27  & 0.076719  & 0.076719  & 0.076719
28  & 0.078367  & 0.078367  & 0.078367
29  & 0.080033  & 0.080033  & 0.080033
30  & 0.081457  & 0.081457  & 0.081457
The optimal k for each distance heuristic is:
 {'l1': [2], 'l2': [2], 'linf': [2]}
:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
```

```
In [27]: print('Processing dataset pumadyn32nm...')
         Q1_Regression['pumadyn32nm'] = RMSEComparison('pumadyn32nm', 'regression', 'validation'
```

```
Processing dataset pumadyn32nm...
:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
For dataset pumadyn32nm, the test results are:
  & l1  & l2  & linf
11  & 0.883386  & 0.901126  & 0.983084
12  & 0.882108  & 0.899225  & 0.980124
13  & 0.879611  & 0.899332  & 0.979474
14  & 0.878278  & 0.898026  & 0.978867
```

```
15  & 0.876904  & 0.896746  & 0.977092
16  & 0.875445  & 0.896835  & 0.975236
17  & 0.875945  & 0.896979  & 0.974749
18  & 0.874893  & 0.895930  & 0.973623
19  & 0.874223  & 0.895664  & 0.973290
20  & 0.874857  & 0.895344  & 0.972799
21  & 0.874745  & 0.895177  & 0.971743
22  & 0.873958  & 0.894352  & 0.971673
23  & 0.873104  & 0.893947  & 0.971126
24  & 0.873535  & 0.894846  & 0.970855
25  & 0.872985  & 0.894633  & 0.970556
26  & 0.872973  & 0.894929  & 0.970842
27  & 0.873117  & 0.895326  & 0.970808
28  & 0.873294  & 0.895341  & 0.970788
29  & 0.874463  & 0.895456  & 0.970688
30  & 0.874884  & 0.896378  & 0.970625
31  & 0.875496  & 0.896785  & 0.970095
32  & 0.875848  & 0.896747  & 0.969583
33  & 0.875969  & 0.897263  & 0.969211
34  & 0.875786  & 0.897464  & 0.968942
35  & 0.875514  & 0.897795  & 0.969062
36  & 0.876018  & 0.897742  & 0.969312
37  & 0.876163  & 0.897870  & 0.968781
38  & 0.875949  & 0.898411  & 0.968615
39  & 0.876167  & 0.898847  & 0.968634
40  & 0.876426  & 0.899129  & 0.968676
The optimal k for each distance heuristic is:
 {'l1': [26], 'l2': [23], 'linf': [38]}
:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
```

```
In [28]: print('Processing dataset rosenbrock...')
         Q1_Regression['rosenbrock'] = RMSEComparison('rosenbrock', 'regression', 'validation',
```

```
Processing dataset rosenbrock...
:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
For dataset rosenbrock, the test results are:
  & l1  & l2  & linf
1   & 0.332775  & 0.302071  & 0.284103
2   & 0.324385  & 0.313196  & 0.306085
3   & 0.326387  & 0.314938  & 0.316781
4   & 0.340917  & 0.340087  & 0.324859
5   & 0.358044  & 0.341096  & 0.346290
6   & 0.364532  & 0.350955  & 0.359211
7   & 0.384731  & 0.363403  & 0.371243
8   & 0.387628  & 0.374365  & 0.381987
9   & 0.388331  & 0.382530  & 0.399302
10  & 0.395729  & 0.393533  & 0.405465
```

```
11  & 0.411511  & 0.404733  & 0.415856
12  & 0.422927  & 0.417982  & 0.422297
13  & 0.431890  & 0.431304  & 0.430378
14  & 0.440227  & 0.437629  & 0.445694
15  & 0.446847  & 0.448048  & 0.453156
16  & 0.455490  & 0.456542  & 0.464046
17  & 0.463999  & 0.467643  & 0.474001
18  & 0.471164  & 0.468807  & 0.481155
19  & 0.478940  & 0.477989  & 0.488723
20  & 0.486524  & 0.486030  & 0.494573
21  & 0.492681  & 0.494561  & 0.502887
22  & 0.503494  & 0.502339  & 0.507402
23  & 0.507977  & 0.506800  & 0.513355
24  & 0.516067  & 0.514839  & 0.520491
25  & 0.521659  & 0.516910  & 0.524363
26  & 0.526170  & 0.522010  & 0.531439
27  & 0.533785  & 0.525756  & 0.538876
28  & 0.540525  & 0.532321  & 0.543211
29  & 0.545939  & 0.534342  & 0.546516
30  & 0.550650  & 0.536537  & 0.552435
The optimal k for each distance heuristic is:
 {'l1': [2], 'l2': [1], 'linf': [1]}
:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
```

In [24]:
```
Q1_Classification = {}
print('Processing dataset iris...')
Q1_Classification['iris'] = RMSEComparison('iris', 'classification', 'validation', rang
```

```
Processing dataset iris...
:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
For dataset iris, the test results are:
  & l1  & l2  & linf
1   & 0.925926  & 0.925926  & 0.925926
2   & 0.940741  & 0.918519  & 0.911111
3   & 0.940741  & 0.948148  & 0.925926
4   & 0.948148  & 0.940741  & 0.911111
5   & 0.948148  & 0.955556  & 0.918519
6   & 0.940741  & 0.955556  & 0.911111
7   & 0.940741  & 0.962963  & 0.933333
8   & 0.940741  & 0.948148  & 0.940741
9   & 0.940741  & 0.962963  & 0.955556
10  & 0.948148  & 0.962963  & 0.948148
11  & 0.948148  & 0.962963  & 0.940741
12  & 0.955556  & 0.962963  & 0.940741
13  & 0.948148  & 0.962963  & 0.933333
14  & 0.955556  & 0.955556  & 0.933333
15  & 0.948148  & 0.962963  & 0.933333
```

```
16  & 0.948148  & 0.933333  & 0.918519
17  & 0.948148  & 0.925926  & 0.903704
18  & 0.940741  & 0.918519  & 0.903704
19  & 0.948148  & 0.940741  & 0.896296
20  & 0.940741  & 0.933333  & 0.881481
21  & 0.933333  & 0.933333  & 0.881481
22  & 0.933333  & 0.940741  & 0.888889
23  & 0.925926  & 0.940741  & 0.881481
24  & 0.925926  & 0.940741  & 0.888889
25  & 0.925926  & 0.918519  & 0.881481
26  & 0.925926  & 0.933333  & 0.874074
27  & 0.918519  & 0.911111  & 0.874074
28  & 0.918519  & 0.918519  & 0.874074
29  & 0.911111  & 0.911111  & 0.859259
30  & 0.911111  & 0.896296  & 0.866667
The optimal k for each distance heuristic is:
 {'l1': [12, 14], 'l2': [12, 13, 15], 'linf': [9]}
:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
```

In [27]:
```
import os
tic = os.times()[0]
print('Processing dataset mnist_small...')
Q1_Classification['mnist_small'] = RMSEComparison('mnist_small', 'classification', 'val
print('Process time: '+str(os.times()[0] - tic)+' sec')
```

```
Processing dataset mnist_small...
:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
For dataset mnist_small, the test results are:
  & l1  & l2  & linf
1  & 0.925000  & 0.908727  & 0.678545
2  & 0.918273  & 0.895364  & 0.648909
3  & 0.929091  & 0.911636  & 0.670182
4  & 0.925636  & 0.908636  & 0.671727
5  & 0.927818  & 0.911636  & 0.669727
6   & 0.925545  & 0.908273  & 0.665364
7   & 0.925909  & 0.910273  & 0.664000
8   & 0.924182  & 0.906909  & 0.659364
9   & 0.923273  & 0.906364  & 0.660182
10  & 0.919455  & 0.904273  & 0.657455
The optimal k for each distance heuristic is:
 {'l1': [3], 'l2': [3], 'linf': [1]}
:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
Process time: 7530.089999999999 sec
```

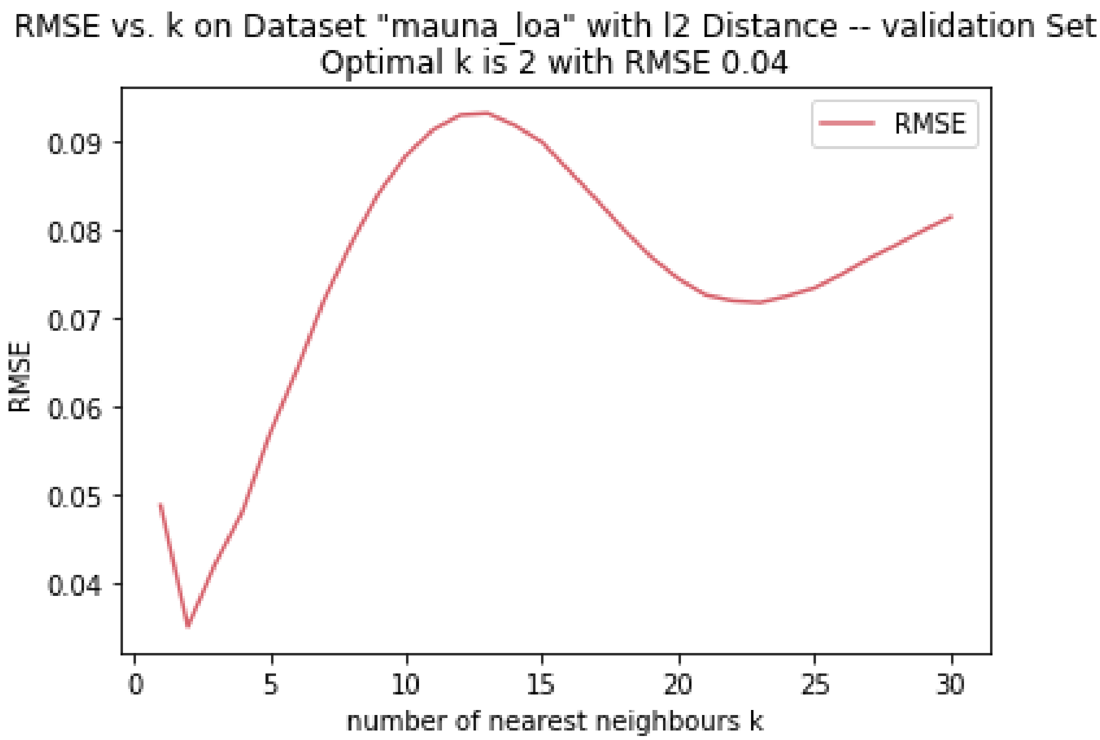
In [ ]:
```
Q1_Classification['mnist_small'] = RMSEComparison('mnist_small', 'classification', 'vali
```

```
:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
```

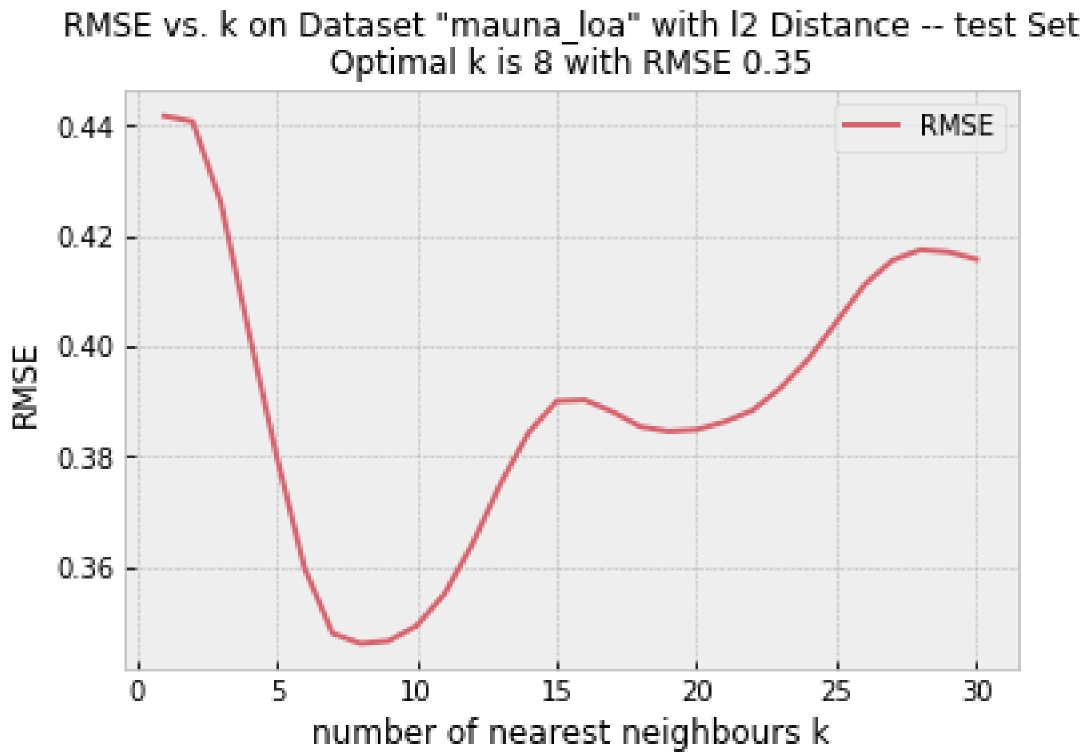
In [28]:
```
def plotPredictionCurves(datasetName, set, kRange, distanceHeuristic='l2', modification
    '''
    Plot the prediction curve of RMSE(%) vs. k
    '''
    data = {'k':[], 'RMSE':[]}
    for k in kRange:
        data['k'].append(k)
        data['RMSE'].append(RMSELoss_Regression(datasetName, set, distanceHeuristic, k,
    rmseOptimal = min(data['RMSE'])
    kOptimalIndex = list(data['RMSE']).index(rmseOptimal)
    kOptimal = data['k'][kOptimalIndex]
    curve = pd.DataFrame(data)
    curve.plot(x='k', y='RMSE', color=_COLORS)
    plt.style.use('bmh')
    plt.xlabel('number of nearest neighbours k')
    plt.ylabel('RMSE')
    plt.title('RMSE vs. k on Dataset "%s" with %s Distance -- %s Set\nOptimal k is %d w
    # plt.savefig('kNNRegression-ValidCurve-mauna_loa.png')
    plt.show()
    return kOptimal


def plotPrediction(datasetName, set, k, distanceHeuristic='l2', modificationIndex=2):
    '''
    Plot the test set's predicted y-values vs. labelled y-values
    '''
    print(k, 'nearest neighbours using', distanceHeuristic, 'distance:')
    kNNtest = kNNTraining(datasetName, distanceHeuristic, k, modificationIndex)
    kNNtest.foldDataset(1)
    kNNtest.x_train, kNNtest.y_train = kNNtest.x_all, kNNtest.y_all
    if set == 'validation': x_set, y_set = kNNtest.x_valid, kNNtest.y_valid
    else: x_set, y_set = kNNtest.x_test, kNNtest.y_test
    y, errorList, rmse =[], [], 0
    if modificationIndex in [1, 2]:
        for i in range(x_set.shape[0]):
            kNNValue, error, correctness = kNNtest.kNNRegression(x_set[i], y_set[i], mo
            y.append(kNNValue)
            errorList.append(error)
    elif modificationIndex == 3:
        y, errorList = kNNtest.kNNRegression_3(kNNtest.x_valid, kNNtest.y_valid)
    rmse = np.sqrt(pow(np.array(errorList), 2).mean())

    data = {'x': np.transpose(x_set)[0], 'y_Labelled': np.transpose(y_set)[0], 'y_Predi
    df = pd.DataFrame(data)
    df.plot(kind='scatter',x='x',y='y_Labelled', ax=plt.gca(), marker='o',  color=_COLO
    df.plot(kind='scatter',x='x',y='y_Predicted', ax=plt.gca(), marker='*', color=_COLO
    plt.style.use('bmh')
    plt.legend(('y_Labelled', 'y_Predicted'))
```

```
        plt.title('Predictions of Dataset "%s"\nwith %s Distance and Optimal k = %d -- %s S
        # plt.savefig('kNNRegression-ValidSet-mauna_loa.png')
        plt.show()
        return rmse
```

```
In [29]: print('Optimal k:', plotPredictionCurves('mauna_loa', 'validation', range(1, 31)))
         print('Optimal k:', plotPredictionCurves('mauna_loa', 'test', range(1, 31)))
```



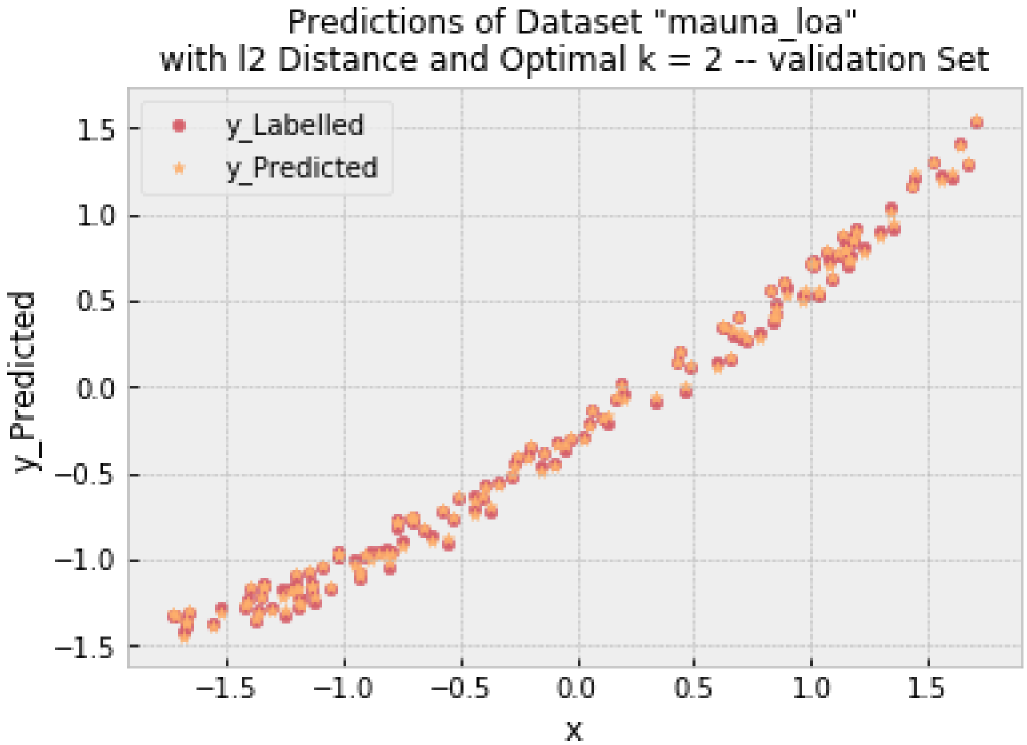

```
Optimal k: 2
```



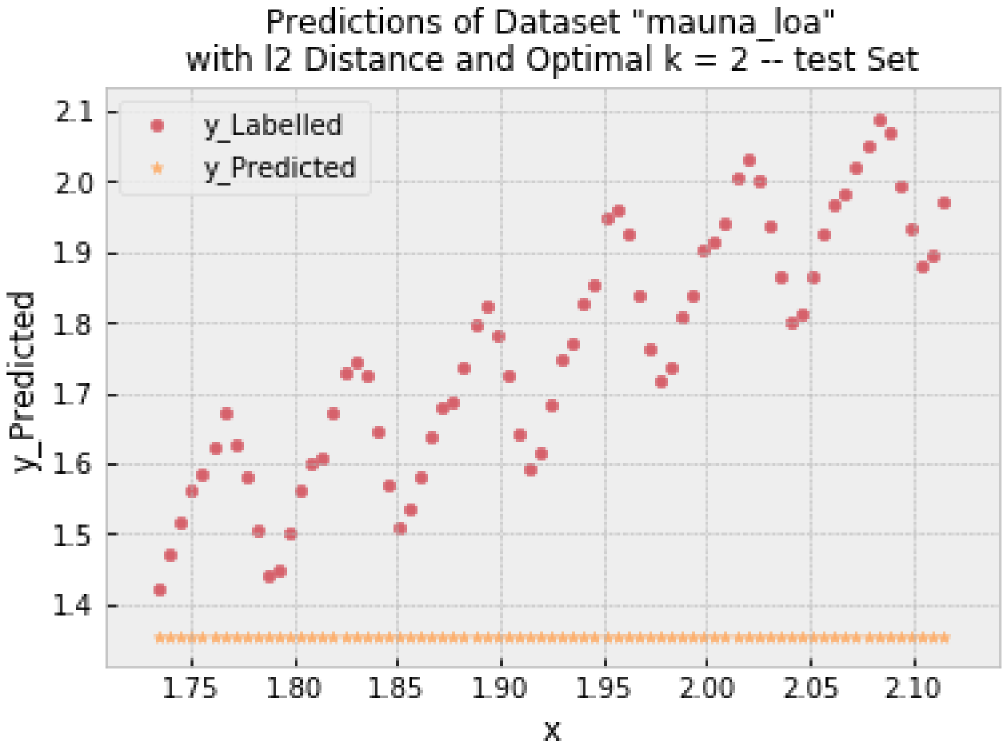

```
Optimal k: 8
```

```
In [44]: plotPrediction('mauna_loa', 'validation', 2)
         plotPrediction('mauna_loa', 'test', 2)
```

```
2 nearest neighbours using l2 distance:
```




```
2 nearest neighbours using l2 distance:
```



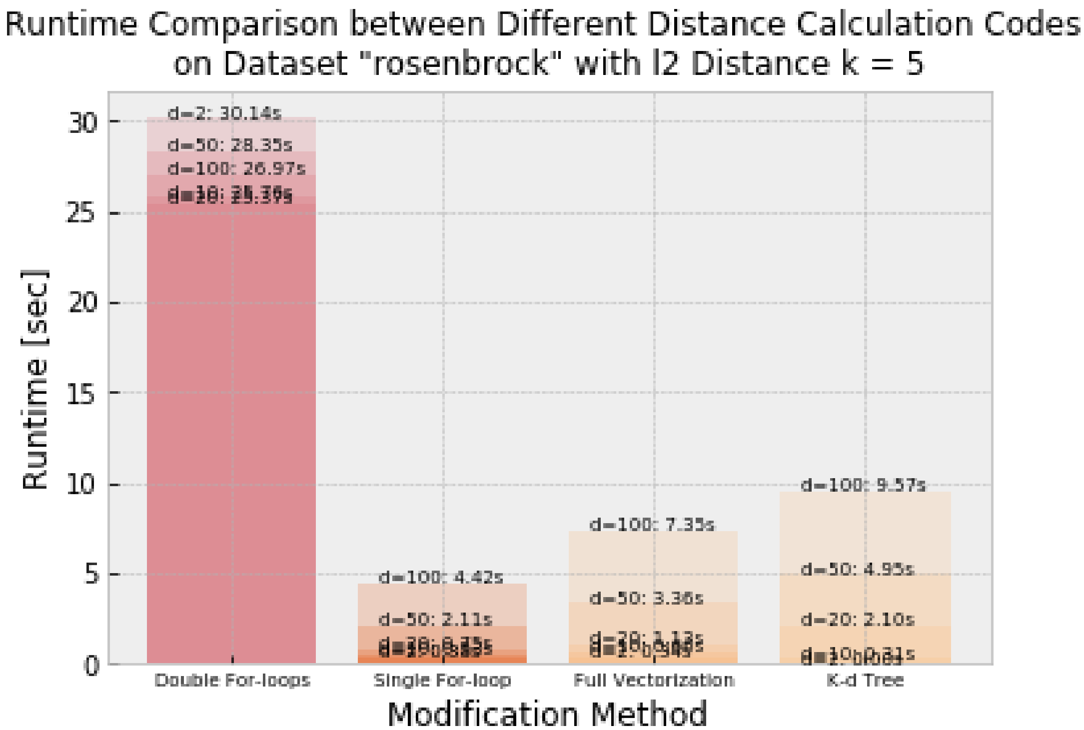

Out[44]: & 0.44070489035463933

```
In [19]: import os

         def kNNPerformance(datasetName, modificationRange, distanceHeuristic='l2', k=5, dRange=
             '''
             Implements kNN in 4 different data structures/computation methods, and compare them
             INPUT: databaseName: must be one of the regression datasets, can't be classificatio
             INPUT: modificationRange: a range of modification indices corresponding to a, b, c
             INPUT: distanceHeuristic: distance calculation distanceHeuristic, here we use 'l2'
             INPUT: k: number of nearest neighbours required, here we use 5
             OUTPUT: a value of the average RMSE loss across 5 folds
             '''
             for d in dRange:
                 print('Processing d =', d, '...')
                 kNNtest = kNNTraining(datasetName, distanceHeuristic, k, 1, d, n=5000)
                 runtimes = []
                 for modificationIndex in modificationRange:
                     # print('::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::')
                     # print('5 nearest neighbours using l2 distance with method', modificationI
                     kNNtest.modificationIndex = modificationIndex
                     tic = os.times()[0] # Record starting time
                     kNNtest.x_train, kNNtest.y_train = kNNtest.x_all, kNNtest.y_all
```

```
                y, errorList, rmse =[], [], 0
                if modificationIndex in [1, 2]:
                    for i in range(kNNtest.num_testSet):
                        kNNValue, error, correctness = kNNtest.kNNRegression(kNNtest.x_test
                        y.append(kNNValue)
                        errorList.append(error)
                elif modificationIndex == 3:
                    kNNValue, errorList = kNNtest.kNNRegression_3(kNNtest.x_test, kNNtest.y
                elif modificationIndex == 4:
                    kNNValue, errorList = kNNtest.kNNRegression_4(kNNtest.x_test, kNNtest.y
                runtimes.append(os.times()[0] - tic)
                # print(':::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::')
            plt.style.use('bmh') # plt.style.use('ggplot')
            xAxis = np.arange(len(modificationRange))
            plt.bar(xAxis, height= runtimes, alpha=0.2, color=_COLORS)
            for x in xAxis:
                plt.text(x=x-0.3, y=runtimes[x]+0.001, s='d=%d: %1.2fs'%(d, runtimes[x]), s
        plt.xticks(xAxis, ['Double For-loops', 'Single For-loop', 'Full Vectorization', 'K-
        plt.xlabel('Modification Method')
        plt.ylabel('Runtime [sec]')
        plt.title('Runtime Comparison between Different Distance Calculation Codes \non Dat
        plt.savefig('kNNPerformance-TestSet-rosenbrock.png')
        plt.show()
        return runtimes
```

```
In [20]: runtimes = kNNPerformance('rosenbrock', modificationRange=range(1,5))
```

```
Processing d = 2 ...
Processing d = 10 ...
Processing d = 20 ...
Processing d = 50 ...
Processing d = 100 ...
```




```
In [66]: from numpy import dot
         from matplotlib import pyplot

         class LinearRegression:
             def __init__(self, datasetName, d=2):
                 '''
                 Takes in a dataset and calculates the weight w according to the training set
                 '''
                 if datasetName == 'rosenbrock':
                     self.x_train, self.x_valid, self.x_test, self.y_train, self.y_valid, self.y
                 else:
                     self.x_train, self.x_valid, self.x_test, self.y_train, self.y_valid, self.y

                 self.dataset = datasetName

                 # Normalizetion of each x data
                 mean = self.x_train.mean(axis=0, keepdims=True)
                 stddev = self.x_train.std(axis=0, keepdims=True)
                 self.x_train = (self.x_train - mean)/stddev
                 self.x_valid = (self.x_valid - mean)/stddev
                 self.x_test = (self.x_test - mean)/stddev
                 self.num_dimension = np.shape(self.x_test)[1]
                 self.num_classes = np.shape(self.y_test)[1]
```

```
        # Add x0 = 1 to each x vector
        x0 = np.ones((np.shape(self.x_train)[0], 1))
        self.x_train = np.concatenate((x0, self.x_train), axis=1)
        x0 = np.ones((np.shape(self.x_valid)[0], 1))
        self.x_valid = np.concatenate((x0, self.x_valid), axis=1)
        x0 = np.ones((np.shape(self.x_test)[0], 1))
        self.x_test = np.concatenate((x0, self.x_test), axis=1)

        self.w = self.optimalWeight()


    def optimalWeight(self):
        '''
        Uses the economic SVD method to compute vector w for f(X, w) = (X^T)*w
        '''
        U, s, VT = np.linalg.svd(self.x_train, full_matrices=False) # The economy SVD (
        S = np.zeros((U.shape[1], VT.shape[0]))
        S[:VT.shape[0], :VT.shape[0]] = np.diag(s)
        w = VT.T.dot(np.linalg.inv(S)).dot(U.T).dot(self.y_train)
        return w


    def plotRegression(self, set):
        if set == 'validation': x, y_actual = self.x_valid, self.y_valid
        else: x, y_actual = self.x_test, self.y_test
        y_predicted = x.dot(self.w)
        pyplot.scatter(x[:, 1], y_actual[:, 0])
        pyplot.plot(x[:, 1], y_predicted[:, 0], color='red')
        pyplot.grid()
        pyplot.show()


    def linRegRegression(self, set):
        '''
        Uses the weight w to predict the y values for a given set of x
        '''
        if set == 'validation': x, y_actual = self.x_valid, self.y_valid
        else: x, y_actual = self.x_test, self.y_test
        y_predicted = x.dot(self.w)
        rmse = np.sqrt(pow(np.array(y_predicted-y_actual), 2).mean())
#         print('RMSE is', rmse, 'for data', self.dataset, 'with linear regression on t
        return rmse


    def linRegClassification(self, set):
        '''
        Uses the weight w to predict the values of each class for a given set of x
```

```
            These valuesa re interpreted as the likelihood of the classes, the maximum of u
            '''
            if set == 'validation': x, y_actual = self.x_valid, self.y_valid
            else: x, y_actual = self.x_test, self.y_test
            f_predicted = x.dot(self.w)
            y_predicted = []
            for f in f_predicted:
                maxClass = list(f).index(max(f))
                y_base = np.zeros(self.num_classes)
                y_base[maxClass] = 1
                y_predicted.append(y_base)
#         print(y_predicted)
#         print(y_actual)
            accuracies = [sum(list(y_predicted)[i]!=list(y_actual)[i])/2 for i in range(np.
            accuracy = 1-sum(accuracies)/len(accuracies)
            print('Accuracy is', str(accuracy*100)+'% for data', self.dataset, 'with linear
            return accuracy
```

```
In [62]: for dataSet in ['iris', 'mnist_small']:
             LRtest = LinearRegression(dataSet)
             LRtest.linRegClassification('test')
```

```
Accuracy is 86.66666666666667% for data iris with linear regression on the test set.
Accuracy is 85.1% for data mnist_small with linear regression on the test set.
```

```
In [64]: for dataSet in ['mauna_loa', 'pumadyn32nm']:
             LRtest = LinearRegression(dataSet)
             LRtest.linRegRegression('test')
```

```
RMSE is & 0.249432445189863 for data mauna_loa with linear regression on the test set.
RMSE is & 0.8630385189324261 for data pumadyn32nm with linear regression on the test set.
```

```
In [77]: dRange=[2, 10, 20, 50, 100]
         performance = {}
         for d in dRange:
             performance['d='+str(d)] = {}
             LRtest = LinearRegression('rosenbrock', d)
             tic = os.times()[0]
             performance['d='+str(d)]['RMSE'] = LRtest.linRegRegression('test')
             performance['d='+str(d)]['Runtime'] = str(os.times()[0] - tic)+' sec'
         print('Linear regression on Rosenbrock:\n', pd.DataFrame(performance))
```

```
Linear regression on Rosenbrock:
               d=2       d=10       d=20      d=50     d=100
RMSE     & 0.98527  & 0.997958  & 0.989245  1.03334  1.03846
Runtime  & 0.0 sec   & 0.0 sec   & 0.0 sec  & 0.0 sec  & 0.0 sec
```

```
In [ ]:
```