

1 Noise-free Wheel Odometry

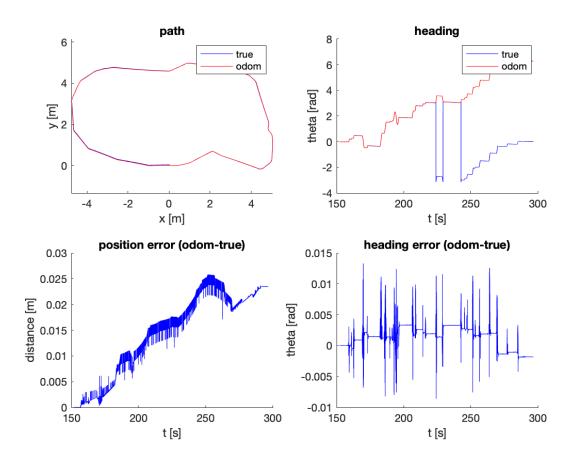


Figure 1: Output plot from to Question 1

The algorithm estimates the pose of robot using wheel odometry data, which is considered **noise-free** (i.e., no slipping and no measurement precision error in the encoder) in this part.

- Position. From the path plot, the ground truth and odometry are nearly identical. The position error ranging from 0 in the beginning to about 2.6 cm at worst and grows (roughly) proportionally. That being said, the error in centimeters is negligible compared to our scale of position in meters.
- Heading. The heading is quite accurate in the beginning as well, until around 230 seconds where there is a small period of offset. The same offset (i.e., $\sim 2\pi$) occurred after 250 as well. Since the offset is essentially a full round, the offset is unlikely to be caused by an error; instead, it is likely due to the way that ROS/Gazebo/Turtlebot is designed, which seems to favor smaller angle changes (i.e., increment by $\delta\theta$ instead of $\delta\theta 2\pi$ if $|\delta\theta| < |\delta\theta 2\pi|$). However, none of such offset (or any offset of an integer number of full circles) would have an impact when we determine the pose of robot, as θ and $\theta \pm 2\pi$ are practically equal to each other.

From the error plot, we can see that the 2π offset was indeed not taken into account. Instead, we get a bunch of small errors. Through comparison with the heading plot, we can see that these small errors occur at sudden direction changes - the larger the increment/decrement in heading, the larger the error. This totally makes sense as the robot might needs to overshoot a little bit, physically, when as it changes its heading.

2 Noisy Wheel Odometry

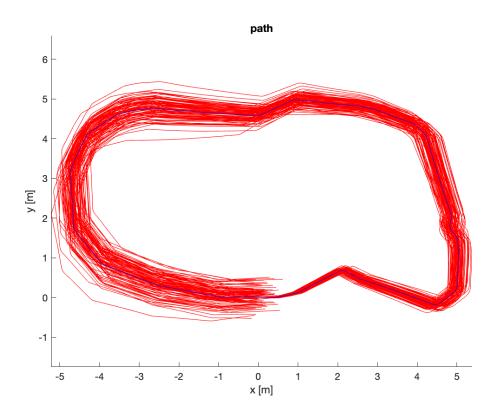


Figure 2: Output plot from to Question 2

The algorithm in this part remains the same as that in the previous part, with an addition of noise that simulates the real wheel odometry. We randomly sample 100 instances with from such noise and plot the 100 resulting paths in Figure 2. A few observations are worth being discussed:

- Obviously, the paths starts at the same point as the ground truth and continues to diverge away from it, while the general trend still follows the ground truth.
- Mathematically speaking, as randn retunes scalars drawn from the standard normal distribution, the random instances we generated have the noise-free velocities (i.e., same as ground truth) as mean values. Therefore, the expected (i.e., mean) path should indeed match the ground truth one, while the error (i.e., variance) error propagates.
- Unlike part 1, this the result produced in this part doens't exactly match the reference solution, which makes sense since the samples are drawn randomly.

3 Map from Odometry

In this part, we compare the performance in both the noise-free (i.e., ground truth) and noisy odometry data. The post-processing of odometry data contains three patches:

• Interpolation. We first interpolate the odometry data to align with the laser timestamps (code provided already).

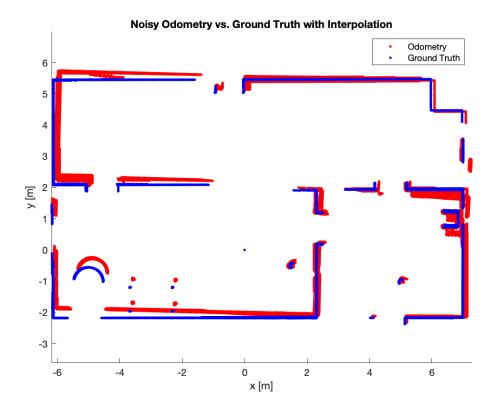


Figure 3: Output plot from to Question 3

- Filtering. Only the laser scans with an angular velocity $\omega < 0.1 \text{rad/s}$ were concerned, as our interpolation wouldn't be accurate at sharp turns.
- Transformation. We first apply a purely rotational transformation to the laser scan to turn them into the vehicle frame. Then, we apply a homogeneous transformation (i.e., rotation & linear displacement including the laser sensor's 10 cm offset) to have everything in the inertial frame.

From the resulting plot, we can see that the odometry share the same general shape as the ground truth but with some offset, as expected. The offset didn't effect the detection of obstacles, corners, or doors, etc. This indicates both that the odometry algorithm works correctly and that the uncertainty/error indeed propagates. In fact, without prior knowledge of the starting point, we can still make a good guess on where the robot started - somewhere around (4, -2) - and even how it traversed the space, purely based on how accurately and precisely our odometry predicts the pose.

Appendix: Source Code

```
1 % =====
2 % ass1.m
3 % =====
_{5} % This assignment will introduce you to the idea of estimating the motion
_{6} % of a mobile robot using wheel odometry, and then also using that wheel
_{7} % odometry to make a simple map. It uses a dataset previously gathered in
_{8} % a mobile robot simulation environment called Gazebo. Watch the video,
_{9} % 'gazebo.mp4' to visualize what the robot did, what its environment
10 % looks like, and what its sensor stream looks like.
12 % There are three questions to complete (5 marks each):
      Question 1: code (noise-free) wheel odometry algorithm
14 %
      Question 2: add noise to data and re-run wheel odometry algorithm
       Question 3: build a map from ground truth and noisy wheel odometry
16 %
_{18} % Fill in the required sections of this script with your code, run it to
_{19} % generate the requested plots, then paste the plots into a short report
_{20} % that includes a few comments about what you've observed. Append your
_{21} % version of this script to the report. Hand in the report as a PDF file.
22 %
23 % requires: basic Matlab, 'gazebo.mat'
25 % T D Barfoot, December 2015
27 clear all;
29 % set random seed for repeatability
30 rng(1);
33 % load the dataset from file
34 % =======
35 %
       ground truth poses: t_true x_true y_true theta_true
37 % odometry measurements: t_odom v_odom omega_odom
38 %
             laser scans: t_laser y_laser
      laser range limits: r_min_laser r_max_laser
      laser angle limits: phi_min_laser phi_max_laser
40 %
41 %
42 load gazebo.mat;
44 % -----
_{45} % Question 1: code (noise-free) wheel odometry algorithm
46 % ============
47 %
```

```
_{48} % Write an algorithm to estimate the pose of the robot throughout motion
_{49} % using the wheel odometry data (t_odom, v_odom, omega_odom) and assuming
_{50} % a differential-drive robot model. Save your estimate in the variables
_{51} % (x_odom y_odom theta_odom) so that the comparison plots can be generated
_{52} % below. See the plot 'ass1_q1_soln.png' for what your results should look
53 % like.
_{55} % variables to store wheel odometry pose estimates
56 numodom = size(t_odom,1);
s7 x_odom = zeros(numodom,1);
58 y_odom = zeros(numodom,1);
theta_odom = zeros(numodom,1);
61 % set the initial wheel odometry pose to ground truth
x_{odom}(1) = x_{true}(1);
63 y_odom(1) = y_true(1);
64 theta_odom(1) = theta_true(1);
66 % -----insert your wheel odometry algorithm here-----
67 for i=2:numodom
      % Note: we update the ith as current using (i-1) as previous
      h = t_odom(i) - t_odom(i-1);
      theta_odom(i) = theta_odom(i-1) + omega_odom(i) * h;
      x_{odom(i)} = x_{odom(i-1)} + v_{odom(i)} * cos(theta_{odom(i)}) * h;
      y_odom(i) = y_odom(i-1) + v_odom(i) * sin(theta_odom(i)) * h;
73 end
_{74} % -----end of your wheel odometry algorithm------
_{76} % plot the results for verification
77 figure (1)
78 clf;
80 subplot(2,2,1);
81 hold on;
82 plot(x_true,y_true,'b');
83 plot(x_odom, y_odom, 'r');
84 legend('true', 'odom');
85 xlabel('x [m]');
86 ylabel('y [m]');
87 title('path');
88 axis equal;
90 subplot(2,2,2);
91 hold on;
plot(t_true,theta_true,'b');
93 plot(t_odom, theta_odom, 'r');
94 legend('true', 'odom');
95 xlabel('t [s]');
96 ylabel('theta [rad]');
```

```
97 title('heading');
99 subplot(2,2,3);
100 hold on;
pos_err = zeros(numodom,1);
102 for i=1:numodom
     pos_{err}(i) = \frac{sqrt}{(x_odom(i)-x_true(i))^2} + (y_odom(i)-y_true(i))^2);
plot(t_odom,pos_err,'b');
106 xlabel('t [s]');
ylabel('distance [m]');
title('position error (odom-true)');
subplot(2,2,4);
111 hold on;
theta_err = zeros(numodom,1);
113 for i=1:numodom
114
      phi = theta_odom(i) - theta_true(i);
      while phi > pi
         phi = phi - 2*pi;
117
      end
     while phi < -pi
118
       phi = phi + 2*pi;
119
120
      end
      theta_err(i) = phi;
121
122 end
plot(t_odom,theta_err,'b');
124 xlabel('t [s]');
ylabel('theta [rad]');
title('heading error (odom-true)');
print -dpng ass1_q1.png
128
129 %%
130 % -----
_{131} % Question 2: add noise to data and re-run wheel odometry algorithm
_{134} % Now we're going to deliberately add some noise to the linear and
135 % angular velocities to simulate what real wheel odometry is like. Copy
_{136} % your wheel odometry algorithm from above into the indicated place below
_{137} % to see what this does. The below loops 100 times with different random
_{\rm 138} % noise. See the plot 'ass1_q2_soln.pdf' for what your results should look
139 % like.
_{141} % save the original odometry variables for later use
v_odom_noisefree = v_odom;
omega_odom_noisefree = omega_odom;
145 % set up plot
```

```
146 figure (2);
147 clf;
148 hold on;
150 % loop over random trials
151 for n=1:100
       \% add noise to wheel odometry measurements (yes, on purpose to see effect)
       v_odom = v_odom_noisefree + 0.2*randn(numodom,1);
154
       omega_odom = omega_odom_noisefree + 0.04*randn(numodom,1);
       % -----insert your wheel odometry algorithm here-----
157
       for i=2:numodom
158
           % Same as Q1
159
           h = t_odom(i) - t_odom(i-1);
160
           theta_odom(i) = theta_odom(i-1) + omega_odom(i) * h;
161
           x_{odom(i)} = x_{odom(i-1)} + v_{odom(i)} * cos(theta_{odom(i)}) * h;
           y_{odom(i)} = y_{odom(i-1)} + v_{odom(i)} * sin(theta_odom(i)) * h;
       end
       % -----end of your wheel odometry algorithm------
166
167
       % add the results to the plot
169
       plot(x_odom, y_odom, 'r');
170 end
171
172 % plot ground truth on top and label
plot(x_true,y_true,'b');
174 xlabel('x [m]');
175 ylabel('y [m]');
title('path');
177 axis equal;
178 print -dpng ass1_q2.png
180 %%
182 % Question 3: build a map from noisy and noise-free wheel odometry
183 % -----
184 %
_{185} % Now we're going to try to plot all the points from our laser scans in the
_{186} % robot's initial reference frame. This will involve first figuring out
_{187} % how to plot the points in the current frame, then transforming them back
_{\rm 188} % to the initial frame and plotting them. Do this for both the ground
_{
m 189} % truth pose (blue) and also the last noisy odometry that you calculated in
_{190} % Question 2 (red). At first even the map based on the ground truth may
_{191} % not look too good. This is because the laser timestamps and odometry
192 % timestamps do not line up perfectly and you'll need to interpolate. Even
_{193} % after this, two additional patches will make your map based on ground
194 % truth look as crisp as the one in 'ass1_q3_soln.png'. The first patch is
```

```
_{195} % to only plot the laser scans if the angular velocity is less than
_{196} % 0.1 rad/s; this is because the timestamp interpolation errors have more
_{197} % of an effect when the robot is turning quickly. The second patch is to
_{198} % account for the fact that the origin of the laser scans is about 10 cm
_{199} % behind the origin of the robot. Once your ground truth map looks crisp,
_{200} % compare it to the one based on the odometry poses, which should be far
201 % less crisp, even with the two patches applied.
203 % set up plot
204 figure (3);
205 clf;
206 hold on;
207
208 % precalculate some quantities
209 npoints = size(y_laser,2);
angles = linspace(phi_min_laser, phi_max_laser, npoints);
cos_angles = cos(angles);
212 sin_angles = sin(angles);
214 % initialize some quantities for future use
one = ones(1, npoints);
x_i = zeros(1, size(y_laser, 1)*size(y_laser, 2));
y_i = zeros(1, size(y_laser,1)*size(y_laser,2));
218
219 for n=1:2
       if n==1
           \% interpolate the noisy odometry at the laser timestamps
222
           t_interp = linspace(t_odom(1),t_odom(numodom),numodom);
           x_interp = interp1(t_interp,x_odom,t_laser);
           y_interp = interp1(t_interp,y_odom,t_laser);
           theta_interp = interp1(t_interp,theta_odom,t_laser);
           omega_interp = interp1(t_interp,omega_odom,t_laser);
       else
           % interpolate the noise-free odometry at the laser timestamps
229
           t_interp = linspace(t_true(1),t_true(numodom),numodom);
230
           x_interp = interp1(t_interp,x_true,t_laser);
231
           y_interp = interp1(t_interp,y_true,t_laser);
232
           theta_interp = interp1(t_interp,theta_true,t_laser);
           omega_interp = interp1(t_interp,omega_odom,t_laser);
234
       end
236
       % loop over laser scans
       for i=1:size(t_laser,1)
           % -----insert your point transformation algorithm here-----
           \% only plot the laser scans if angular velocity < 0.1 rad/s
           if abs(omega_interp(i)) < 0.1</pre>
242
               % Transform from laser frame -> vehicle frame
```

```
x_v = y_laser(i,:) .* cos_angles;
244
                y_v = y_laser(i,:) .* sin_angles;
245
246
                % Define homogeneous transformation H_{iv: vehicle -> inertial}
                sin_ang = sin(theta_interp(i));
                cos_ang = cos(theta_interp(i));
                H_iv = [cos_ang -sin_ang x_interp(i)-0.1*cos_ang;
                         sin_ang cos_ang y_interp(i)-0.1*sin_ang;
251
                              0
                                      1];
252
253
                % Transform from vehicle frame -> inertial frame
254
                xy_i = H_iv*[x_v; y_v; one];
255
256
                % Record into the plot
257
                x_i((i-1)*npoints+1 : i*npoints) = xy_i(1, :);
258
                y_i((i-1)*npoints+1 : i*npoints) = xy_i(2, :);
259
           \mbox{\ensuremath{\mbox{\%}}} -----end of your point transformation algorithm-----
       end
       if n == 1
264
           plot(x_i, y_i, 'r.')
265
       else
266
           plot(x_i, y_i, 'b.')
267
       end
268
269 end
270
271 % Plot laser data
272 xlabel('x [m]');
273 ylabel('y [m]');
10 legend('Odometry','Ground Truth')
title('Noisy Odometry vs. Ground Truth with Interpolation');
277 axis equal;
278 print -dpng ass1_q3.png
```