Calcular det(A) usando Eliminação Gaussiana:

$$\det\begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 4 & -1 \\ 1 & 0 & 2 & 4 \end{pmatrix} = \det\begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 4 & -1 \\ 0 & -2 & 3 & 1 \end{pmatrix} = \det\begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 4 & -2 \\ 0 & 0 & 3 & 3 \end{pmatrix} = \det\begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 4 & -2 \\ 0 & 0 & 0 & 9/2 \end{pmatrix}$$

pelo que, $\det(A) = 1 \times 1 \times 4 \times \frac{9}{2} = 18$

Teorema: Seja $A\underset{\sim}{x} = \underset{\sim}{b}$ um sistema de $\underline{\underline{n}}$ equações em $\underline{\underline{n}}$ incógnitas. Então,

(i) Se $\det(A) \neq 0$, o sistema $A\underset{\sim}{x} = \underset{\sim}{b}$ tem solução única

(ii) Se $\det(A) \neq 0$, a solução $\underset{\sim}{x} = (x_i)$ pode ser obtida de

$$x_i = \frac{\det(A^{(i)})}{\det(A)}, \quad (i = 1, \dots, n) \qquad \text{(Regra de Cramer)}$$

onde $A^{(i)}$ denota a matriz que resulta de A substituindo a coluna i pelo vector $\underset{\sim}{b}$ dos termos independentes.

Ex: Resolver $\begin{cases} x_1 + x_3 = 1 \\ 2x_2 + x_3 = 3/2 \\ x_1 - x_2 = 1/4 \end{cases}$ pela Regra de Cramer.

Sendo $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix}$ então $A^{(1)} = \begin{pmatrix} 1 & 0 & 1 \\ 3/2 & 2 & 1 \\ 1/4 & -1 & 0 \end{pmatrix}$, $A^{(2)} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3/2 & 1 \\ 1 & 1/4 & 0 \end{pmatrix}$, $A^{(3)} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 3/2 \\ 1 & -1 & 1/4 \end{pmatrix}$

Tem-se, $\det(A) = 1 \times \det\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} + \det\begin{pmatrix} 0 & 2 \\ 1 & -1 \end{pmatrix} = 1 - 2 = -1$

$\det(A^{(1)}) = 1 \det\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} + 1 \times \det\begin{pmatrix} 3/2 & 2 \\ 1/4 & -1 \end{pmatrix} = 1 + (-3/2 - 2/4) = 1 - 2 = -1$

$\det(A^{(2)}) = 1 \det\begin{pmatrix} 3/2 & 1 \\ 1/4 & 0 \end{pmatrix} + 1 \times \det\begin{pmatrix} 1 & 1 \\ 3/2 & 1 \end{pmatrix} = -1/4 + (1 - 3/2) = -1/4 - 1/2 = -3/4$

$\det(A^{(3)}) = 1 \times \det\begin{pmatrix} 2 & 3/2 \\ -1 & 1/4 \end{pmatrix} + 1 \times \det\begin{pmatrix} 0 & 2 \\ 1 & -1 \end{pmatrix} = 1/2 + 3/2 + 1 \times (-2) = 0$

Logo, $x_1 = \frac{-1}{-1} = 1$, $x_2 = \frac{-3/4}{-1} = \frac{3}{4}$ e $x_3 = \frac{0}{-1} = 0$