TESTE MODELO I

① a) $P_{12}A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & -1 & -1 \end{bmatrix} \xrightarrow[E_{31}(-1)]{\ell_3 \leftarrow \ell_3 - \ell_1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix}$

$\xrightarrow[E_{32}(1)]{\ell_3 \leftarrow \ell_3 + \ell_2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} = E_{32}(1)E_{31}(-1)PA = L^{-1}PA = U$

Idêntico para PM = LU.
(o L é o mesmo.)

$L = E_{31}(1)\,E_{32}(-1)$

$P_{12}A = LU$

$E_{31}(1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $\quad E_{32}(-1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$L = E_{31}(1)\,E_{32}(-1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$

b) $Ax = b$

$\boxed{A|b} = \left(\begin{array}{ccc|c} 0 & 1 & 2 & -2 \\ 1 & 0 & 1 & 1 \\ 1 & -1 & 1 & 3 \end{array}\right) \xrightarrow{\ell_1 \leftrightarrow \ell_2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & -2 \\ 1 & -1 & 1 & 3 \end{array}\right)$

$\xrightarrow{\ell_3 \leftarrow \ell_3 - \ell_1} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & -2 \\ 0 & -1 & -2 & 2 \end{array}\right) \xrightarrow{\ell_3 \leftarrow \ell_3 + \ell_2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\begin{cases} x + z = 1 \\ y + 2z = -2 \end{cases} \Rightarrow \begin{cases} x = 1 - z \\ y = -2 - 2z \end{cases}$

O sistema é possível e indeterminado.

$(x, y, z) = (1 - z, -2 - 2z, z)$
$= (1, -2, 0) + z(-1, -2, 1)$

c) $\begin{cases} x + 0 + z + w = 0 \\ y + 2z - 2w = 0 \end{cases} \Rightarrow \begin{cases} x = -z - w \\ y = -2z + 2w \end{cases}$

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -z-w \\ -2z+2w \\ z \\ w \end{pmatrix} = z\begin{pmatrix} -1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + w\begin{pmatrix} -1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$$

$\underline{\underline{B_{N(M)}}} = \langle(-1,-2,1,0),(-1,2,0,1)\rangle$

d) $\dim CS(A) = \text{car}(A) = 2$

$N(\underbrace{[1\ -1\ 1]}_{B})$ $\dim \underbrace{N([1\ -1\ 1])}_{} = \text{nul} = 3 - 1 = 2$

$x - y + z = 0 \Rightarrow x = y - z \Rightarrow$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y-z \\ y \\ z \end{bmatrix} = y\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$B_{N(B)} = \langle \underbrace{(1,1,0)}_{v_1}, \underbrace{(-1,0,1)}_{v_2} \rangle$

$B_{CS(A)} = \langle (0,1,1), (1,0,-1) \rangle$

↳ [São as colunas com variáveis básicas.]

$Ax = v_1$ é possível?

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$Ax = v_2$ é possível?

$$\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$\text{car}\,A = \text{car}(A|v_2)$

logo $v_2 \in CS(A)$.

Uma vez que $\text{car}(A) = \text{car}(A|v_1)$, o sistema é possível, logo $v_1 \in CS(A)$

Como $\dim CS(A) = \dim N(B)$ e $v_1, v_2 \in CS(A)$, então $CS(A) = N(B)$.

② a) $|\lambda I - A| = \begin{vmatrix} \lambda+1 & 0 & 0 \\ 0 & \lambda+1 & 0 \\ 1 & 1 & \lambda \end{vmatrix} =$

$= (-1)^6 * \lambda * ((\lambda+1)(\lambda+1))$

$= (\lambda - 0)(\lambda+1)^2$

$\sigma(A) = \{-1, 0\}$

$\nu_0 = 1 = m_0$

$\nu_{-1} = 2 = m_{-1}$

$N(-1I - A) = \begin{bmatrix} -1+1 & 0 & 0 \\ 0 & -1+1 & 0 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & -1 \end{bmatrix}$

$\dim(N(-1-A)) = 2$

Uma vez que $\nu_0 = m_0$ e $\nu_{(-1)} = m_{(-1)}$, a matriz é diagonalizável.

$D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$x + y - z = 0 \Leftrightarrow x = -y + z$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y+z \\ y \\ z \end{pmatrix} = y\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$B_{N(-1-A)} = \langle (-1, 1, 0), (1, 0, 1) \rangle$

$N(-A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \xrightarrow{\ell_3 - \ell_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{\ell_3 - \ell_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$B_{N(-A)} = \langle (0, 0, 1) \rangle$

$U = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

② a) $|\lambda I - A| = \begin{vmatrix} \lambda+1 & 0 & 0 \\ 0 & \lambda+1 & 0 \\ 1 & 1 & \lambda \end{vmatrix} =$

$$= (-1)^6 * \lambda * ((\lambda+1)(\lambda+1))$$
$$= (\lambda - 0)(\lambda+1)^2$$
$$\sigma(A) = \{-1, 0\}$$

$$\nu_0 = 1 = m_0$$
$$\nu_{-1} = 2 = m_{-1}$$

$$N(-1I - A) = \begin{bmatrix} -1+1 & 0 & 0 \\ 0 & -1+1 & 0 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\dim(N(-1-A)) = 2$$

Uma vez que $\nu_0 = m_0$ e $\nu_{(-1)} = m_{(-1)}$, a matriz é diagonalizável.

$$D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y+z \\ y \\ z \end{pmatrix} = y\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$B_{N(-1-A)} = \langle (-1, 1, 0), (1, 0, 1) \rangle$$

$$N(-A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \xrightarrow{l_3 - l_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{l_3 - l_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_{N(-A)} = \langle (0, 0, 1) \rangle$$

$$U = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

b) $|B| = \begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & -1 & 0 \end{vmatrix} = -(-1 * -1 * 1) = -1 \neq 0 \Rightarrow$

a matriz é invertível

$$\left(\begin{array}{ccc|ccc} -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\ell_3 - \ell_1]{}$$

$$\left(\begin{array}{ccc|ccc} -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 & 1 \end{array}\right) \xrightarrow[\ell_3 - \ell_2]{}$$

$$\left(\begin{array}{ccc|ccc} -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 & 1 \end{array}\right) \xrightarrow[\ell_1 + \ell_3]{}$$

$$\left(\begin{array}{ccc|ccc} -1 & 0 & 0 & 0 & -1 & +1 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 & 1 \end{array}\right) \xrightarrow[\ell_1 \to -\ell_1 \quad \ell_2 \to -\ell_2 \quad \ell_3 \to -\ell_3]{}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & +1 & +1 & -1 \end{array}\right) \qquad B^{-1} = \begin{pmatrix} 0 & 1 & -1 \\ 0 & -1 & 0 \\ 1 & 1 & -1 \end{pmatrix}$$

II

① e) C.A.

$$A = \begin{array}{c} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \end{pmatrix} \\ \begin{array}{ccc} v_1 & \quad & v_2 \end{array} \end{array} \xrightarrow[\ell_2 - 2\ell_1]{} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad car(A) = 2$$

$\dim CS(A) = 2$ $\qquad A^T = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 0 & 1 \end{pmatrix} \xrightarrow[\ell_2 - 2\ell_1]{} \begin{pmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$

$\dim CS(A^T) = 2$

$\dim CS(A) = \dim \mathbb{R}^2$ $\qquad car(A^T) = 2$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

logo (como v_1 e $v_2 \in \mathbb{R}^2$) $CS(A) = \mathbb{R}^2$

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 2 & 2 & -1 \end{pmatrix} \xrightarrow[\ell_3 - 2\ell_1]{} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$ logo é singular

(2) A

(3) $|A| = 0$ (linha nula)

$\boxed{d)}$

Como 2 são verdadeiras, a resposta tem de ser a d)

(4) $\boxed{b)}$

$$|\lambda I - A| = \begin{vmatrix} \lambda-2 & -1 & 0 \\ 1 & \lambda & 0 \\ 0 & 0 & \lambda-1 \end{vmatrix} = (\lambda-1)(\lambda-2)(\lambda-0)+1$$

$$= (\lambda^2 - \lambda - 2\lambda - \lambda + 2 + 1)\lambda$$

$$= (\lambda^2 - 4\lambda + 3)\lambda$$

$$\lambda = \frac{4 \pm \sqrt{16 - 4 \times 3}}{2}$$

$$\lambda = \frac{4 \pm \sqrt{4}}{2} = 3 \vee \lambda = 1$$

$$= (\lambda-3)(\lambda-1)(\lambda-0)$$

$\sigma(A) = \{0, 1, 3\}$

como são distintos (ν de cada um é 1), a matriz é diagonizável.

(5) $\boxed{b)}$

$$A|b = \left(\begin{array}{cc|c} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 2 & 0 \end{array}\right) \xrightarrow[\ell_3 - 2\ell_1]{\ell_2 - 2\ell_1} \left(\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & -3 & -3 \\ 0 & -2 & -4 \end{array}\right)$$

$$\xrightarrow[\ell_3 - \frac{2}{3}\ell_2]{} \left(\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & -3 & -3 \\ 0 & 0 & -2 \end{array}\right)$$ logo $Ax = b$ é impossível

$$N(A) = \begin{cases} x + 2y = 0 \\ -3y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$N(A) = \{(0, 0)\}$

⑥ $\boxed{d)}$

a) ver combinação linear

b) pelo teorema

c) dimensões diferentes $\dim \mathbb{R}^n > \dim V$

⑦ c) pois $\dim CS(X) = \text{car}\, X$, e portanto $\underline{\text{car}(A) = \text{car}(U)}$, o que é sempre verdadeiro.

⑧ $\boxed{d)}$ (?)

$$|\lambda I - A| = \begin{vmatrix} \lambda & -1 & 0 \\ 1 & \lambda+2 & 0 \\ 1 & 1 & \lambda \end{vmatrix} = (\lambda - 0)\left[\lambda(\lambda+2)+1 \right] - = \lambda(\lambda^2 + 2\lambda + 1) = \lambda(\lambda+1)^2$$

$$\sigma(A) = \{0, -1\}$$

$$N(-I - A) = \begin{bmatrix} -1 & -1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{matrix} \\ \\ \ell_3 + \ell_1 \end{matrix} \longrightarrow \begin{bmatrix} -1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{cases} -x - y = 0 \\ -z = 0 \end{cases} \Rightarrow \begin{cases} x = -y \\ z = 0 \end{cases} \Downarrow$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = y \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \Rightarrow m_g(-1) = 3 - 2 = 1$$

9) $\boxed{d)}$ Ver exercícios do prof.; na e), $\overset{AB=0}{\text{é}}$ possível mesmo que nem A nem B sejam 0.

10) $\boxed{e)}$

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 0 \end{pmatrix} \xrightarrow[\ell_3-\ell_1]{\ell_2-2\ell_1} \begin{pmatrix} 1 & 3 \\ 0 & -5 \\ 0 & -3 \end{pmatrix} \xrightarrow{\ell_3-\frac{3}{5}\ell_2} \begin{pmatrix} 1 & 3 \\ 0 & -5 \\ 0 & 0 \end{pmatrix}$$

$(x,y)=(0,0)$

car(A) < 3, pois é sempre $\leq n \vee \leq m$