

$$9.) \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \in A: \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} \alpha - \beta \\ 2\alpha + \beta \\ \beta \\ \alpha + \beta \end{pmatrix}$$

$$\begin{pmatrix} \alpha & \beta \\ 1 & -1 \\ 2 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & x \\ 0 & 3 & y-2x \\ 0 & 1 & z \\ 0 & 2 & t-x \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & x \\ 0 & 1 & z \\ 0 & 3 & y-2x \\ 0 & 2 & t-x \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & x \\ 0 & 1 & z \\ 0 & 0 & y-5z-3x \\ 0 & 0 & t-2z-2x \end{pmatrix}$$

$C(A) = 2$ para que o sistema seja possível, temos que ter $C(A|b) = 2$

$$y - 2x - 3z = 0 \Rightarrow y = 2x + 3z$$

$$t - x - 2z = 0 \Rightarrow t = x + 2z$$

$$\begin{cases} \alpha - \beta = x \\ \beta = z \end{cases} \Rightarrow \begin{cases} \alpha = x + z \\ \beta = z \end{cases}$$

$$\begin{pmatrix} x + z \\ 2x + 2z \\ 0 \\ x + z \end{pmatrix} + \begin{pmatrix} -z \\ z \\ z \\ z \end{pmatrix} = \begin{pmatrix} x \\ 2x + 3z \\ z \\ x + 2z \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \in A: A = \left\{ \begin{pmatrix} x \\ 2x + 3z \\ z \\ x + 2z \end{pmatrix} : x, z \in \mathbb{R} \right\}$$

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \in B: \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \alpha \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 2\beta \\ 2\beta \\ \alpha + 2\beta \\ \alpha + 2\beta \end{pmatrix}$$

$$\Rightarrow \begin{cases} 2\beta = x \\ 2\beta = y \\ \alpha + 2\beta = z \\ \alpha + 2\beta = t \end{cases} \quad B = \left\{ \begin{pmatrix} x \\ x \\ z \\ z \end{pmatrix} : x, z \in \mathbb{R} \right\}$$

$$\boxed{A \cap B} \quad a \in A: a = \begin{pmatrix} x \\ 2x + 3z \\ z \\ x + 2z \end{pmatrix}$$

$$a \in B: a = \begin{pmatrix} x \\ x \\ z \\ z \end{pmatrix}$$

$$\Rightarrow \begin{cases} x = 2x + 3z \\ z = x + 2z \end{cases} \Rightarrow \begin{cases} -x = 3z \\ z = -3z + 2z \end{cases} \Rightarrow \begin{cases} x = 0 \\ 2z = 0 \end{cases} \quad A \cap B = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$