Univ-× DeF-fst DeF-snd DeF-△	CANCEL-X REFLEX-X FUSION-X DEF-X ABSOR-X FUNCTOR-X FUNCTOR-ID-X NAT-SIG	Univ+ $DeF-inI$ $DeF-inr$ $C \rightarrow g \ z \} DEF-\nabla$ $CANCEL-+$ $REFLEX-+$ $REFLEX-+$ $ROSON-+$ $ABSON-+$ $ABSON-+$ $ABSON-+$ $FUNCTOR-+$ $FUNCTOR-ID-+$ $EQUAL-\nabla$ $NAT-inI$ $NAT-inI$
$h = f \triangle g \Leftrightarrow fst \circ h = f \land snd \circ h = g$ $fst(x,y) = x$ $snd(x,y) = y$ $(f \triangle g) \ x = (f x, g x)$	fst o $(f \triangle g) = f \wedge \operatorname{snd} \circ (f \triangle g) = g$ fst \(\triangle \text{ snd} = \operatorname{id} \) $(f \triangle g) \circ h = f \circ h \triangle g \circ h$ $f \times g = f \circ \operatorname{fst} \triangle g \circ \operatorname{snd} $ $(f \times g) \circ (h \triangle i) = f \circ h \times g \circ i$ $(f \times g) \circ (h \times i) = f \circ h \times g \circ i$ $\operatorname{id} \times \operatorname{id} = \operatorname{id} $ $\operatorname{fst} \circ (f \times g) = f \circ \operatorname{fst} $ $\operatorname{snd} \circ (f \times g) = g \circ \operatorname{snd} $	$h = f \lor g \Leftrightarrow h \circ \text{inl} = f \land h \circ \text{inr} = g$ $\text{inl } x = Left x$ $\text{inr } x = Right x$ $(f \lor g) x = \mathbf{case} x \text{ of } \{Left y \to f y; Right z \to g z\}$ $(f \lor g) \circ \text{inl} = f \land (f \lor g) \circ \text{inr} = g$ $\text{inl } \lor \text{inr} = \text{id}$ $f \circ (g \lor h) = f \circ g \lor f \circ h$ $f + g = \text{inl } \circ f \circ \text{inr } \circ g$ $(f \lor g) \circ (h + i) = f \circ h \lor g \circ i$ $(f + g) \circ (h + i) = f \circ h \lor g \circ i$ $(f + g) \circ (h + i) = f \circ h \lor g \circ i$ $(f + g) \circ (h + i) = f \circ h \lor g \circ i$ $(f + g) \circ (h + i) = f \circ h \lor g = i$ $(f + g) \circ (h + i) = f \circ h \lor g = i$ $(f + g) \circ (h + i) = f \circ h \lor g = i$ $(f + g) \circ (h + i) = f \circ h \lor g = i$ $(f + g) \circ (h + i) = f \circ h \lor g = i$ $(f + g) \circ (h + i) = f \circ h \lor g = i$ $(f + g) \circ (h + i) = f \circ h \lor g = i$
Ext-= Leibniz Iso	ELIM-let ELIM-pair snd z] ELM- \times DEF- \circ ASSOC- \circ DEF-id NAT-id	Univ-1 Der-bang Reflex-1 Fusion-1 Der-const Der-guard Fusion-guard Der-Cond Der-Cond Trision-L-cond Fusion-L-cond Fusion-R-cond
$f = g \Leftrightarrow \forall x . f x = g x$ $f \circ h = g \circ h \Leftrightarrow f = g$ $f \circ h = g \circ h \Leftrightarrow f = g \text{ se } h \text{ é um iso}$	let $x = a$ in $b = b [x/a]$ a = a [(x,y)/z,x/fst z,y/snd z] $f a = b \Leftrightarrow f a [(x,y)/z] = b [x/\text{fst }z,y/\text{ snd }z]$ $(f \circ g) x = f (g x)$ $(f \circ g) \circ h = f \circ (g \circ h)$ id $x = x$ $f \circ \text{id} = f = \text{id} \circ f$	f = bang bang x = () $\text{bang } \circ f = \text{bang}$ \underline{x} () = x $p \ge 0$ (snd + snd) \circ distl \circ (out _B \circ $p \triangle$ id) $p \ge 0$ $p \ge 0$ ($p \ge 0$) $p \ge 0$ $p \to f$, $p \to$

$\operatorname{Iso-in}_L$	$DeF ext{-in}_L$	$\mathrm{Der} ext{-}out_L$	$\operatorname{Univ-cata}_L$	$Cancel ext{-}cata_L$	R ег Ex - $cata_L$	Fusion-cata $_L$	$SPLIT ext{-}cata_L$	
$in_L \circ out_L = id = out_L \circ in_L$	$in_L = [\underline{\hspace{0.1cm}}] \triangledown cons$	$out_L = (bang + (head \triangle tail)) \circ null$?	$h = (f)_L \Leftrightarrow h \circ \operatorname{in}_L = f \circ (\operatorname{id} + \operatorname{id} \times h)$	$(f)_L \circ in_L = f \circ (id + id \times (f)_L)$	$(\operatorname{lin}_L)_L = \operatorname{id}$	$f \circ (g)_L = (h)_L \Leftrightarrow f \circ g = h \circ (id + id \times f)$	$(f)_L \triangle (g)_L = (f \circ (id + id \times fst) \triangle g \circ (id + id \times snd))_L$	

$$h = [\Gamma]_L \Leftrightarrow \text{out}_L \circ h = (\text{id} + \text{id} \times h) \circ f$$

$$\text{out}_L \circ [\Gamma]_L = (\text{id} + \text{id} \times [\Gamma]_L) \circ f$$

$$\text{Cancel-ana}_L$$

$$[\text{out}_L]_L = \text{id}$$

$$\text{Reflex-ana}_L$$

$$\text{Reflex-ana}_L$$

$$\text{Reflex-ana}_L$$

$$\text{Reflex-ana}_L$$

$$\text{Reflex-ana}_L$$

$$\text{Fusion-ana}_L$$

$$\text{in}_{\mu F} \circ \text{out}_{\mu F} = \text{id} = \text{out}_{\mu F} \circ \text{in}_{\mu F}$$

$$\text{Iso-in}$$

$$h = (f)_{\mu F} \circ \text{in}_{\mu F} = f \circ F \text{ in}$$

$$\text{Iso-in}$$

$$h = (f)_{\mu F} \circ \text{in}_{\mu F} = f \circ F \text{ in}_{\mu F}$$

$$\text{Cancel-cata}$$

$$\text{(in}_{\mu F})_{\mu F} \circ \text{in}_{\mu F} = \text{id}$$

$$\text{Reflex-cata}$$

$$h = [f]_{\mu F} \Leftrightarrow \operatorname{out}_{\mu F} h = \mathsf{F} \, h \circ f$$
 Univ-ana $\operatorname{out}_{\mu F} \circ [f]_{\mu F} = \mathsf{F} \, [f]_{\mu F} \circ f$ CanceL-ana $[\operatorname{out}_{\mu F}]_{\mu F} = \operatorname{id}$ Reflex-ana $[f]_{\mu F} \circ g = [f]_{\mu F} \Leftrightarrow f \circ g = \mathsf{F} \, g \circ h$ Fusion-ana

$join \circ join = join \circ F join$	Join-join
$join \circ return = id = join \circ F return$	Join-return
join \circ F F f = F f \circ join	Nar-join
return $\circ f = F f \circ return$	Nar-return

$$f \bullet g = \mathrm{join} \circ \mathsf{F} f \circ g$$
 Defined by $f \bullet (g \bullet h) = (f \circ g) \bullet h$ Associated and $f \circ (g \circ h) = f \circ (g \circ h)$ Associated by $f \circ (g \circ h) = f \circ (f \circ \circ) \bullet h = f \circ (f \circ \circ) \bullet h$

$$(f \circ g) \bullet h = f \bullet (F g \circ h)$$
 Assoc-o•
 $x \bowtie f = (\text{join o F} f) x$ DeF- \bowtie

$$\mathbf{do} \{x \leftarrow a; b\} = x \succ (\lambda a \rightarrow b) \qquad \text{DeF-do}$$

Fusion-cata

SPLIT-cata

 $(f)_{\mu F} \triangle (g)_{\mu F} = (f \circ \mathsf{F} \mathsf{ fst} \triangle g \circ \mathsf{F} \mathsf{ snd})_{\mu F}$

 $f\circ (g)_{\mu F}=(h)_{\mu F}\Longleftrightarrow f\circ g=h\circ \mathsf{F}\, f$

$$h = [f]_{\mu F} \Leftrightarrow \operatorname{out}_{\mu F} h = \mathsf{F} \, h \circ f$$
 Univ-ana $\operatorname{out}_{\mu F} \circ [f]_{\mu F} = \mathsf{F} \, [f]_{\mu F} \circ f$ Cancet-ana $[\operatorname{out}_{\mu F}]_{\mu F} = \operatorname{id}$ Reflex-ana $[f]_{\mu F} \circ g = [h]_{\mu F} \Leftrightarrow f \circ g = \mathsf{F} \, g \circ h$ Fusion-ana