Costa da Quinta, João Filipe

Problem set 8 06/05/2020

Problem ①

We have the Markov chain $X \to Y \to Z$

ⓐ Prove that $I(X;Z) \leq I(X;Y)$

we start by decomposing $I(X;Y,Z)$

$$I(X;Y,Z) = I(X;Z) + I(X;Y|Z)$$
$$= I(X;Y) + I(X;Z|Y)$$

X, Z indep $| Y \Rightarrow I(X;Z|Y) = 0$ et $I(X;Y|Z) \geq 0$

Donc $\Rightarrow I(X;Z) + I(X;Y|Z) = I(X;Y) \Leftrightarrow$

$I(X;Z) \leq I(X;Y)$

ⓑ Prove that $I(X;Y|Z) \leq I(X;Y)$

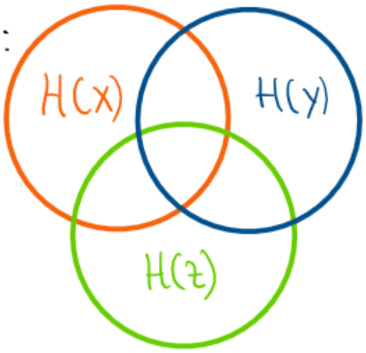
Like before we decompose $I(X;Y,Z)$
(same as before)

$\Rightarrow I(X;Z) + I(X;Y|Z) = I(X;Y) \Leftrightarrow$

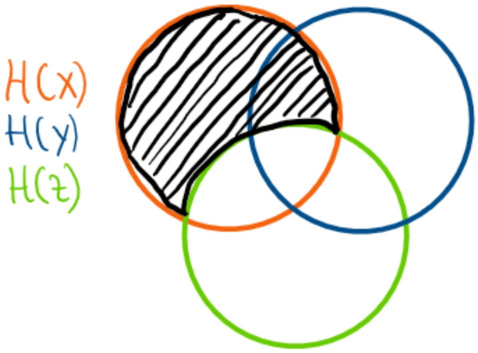
$I(X;Y|Z) \leq I(X;Y)$

Problem ②

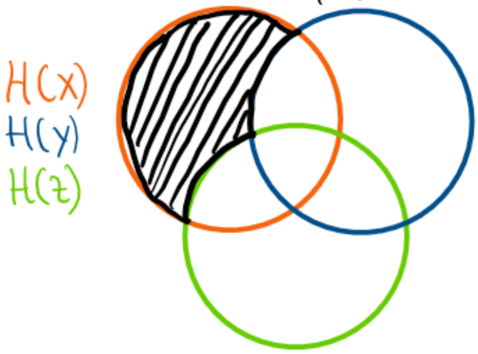
Van diagram:



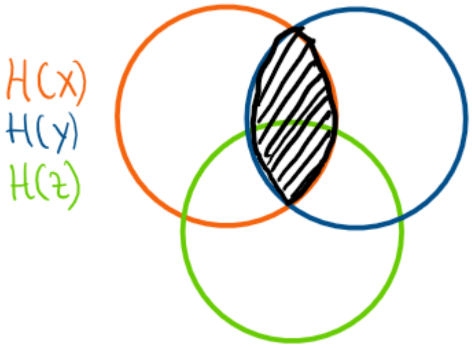
$H(X|Y)$



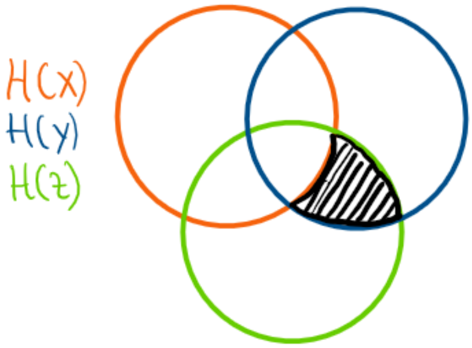
$H(X|Y,Z)$



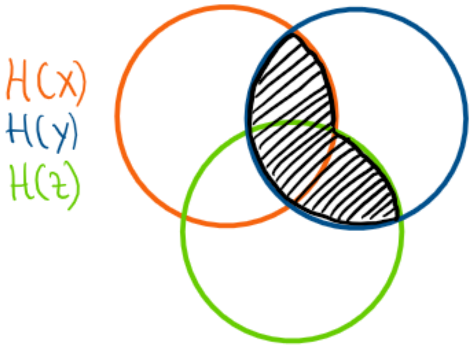
$I(X;Y)$



$I(Y;Z|X)$



$I(Y;X,Z)$



Problem ③

We have the Markov chain $X \to Y \to Z$

$X = \{1,2,\dots,n\}$, $Y = \{1,2,\dots,k\}$, $Z = \{1,2,\dots,m\}$

$k < n$, $k < m$

ⓑ show that : if $k=1$ $\to$ X et Z are independent

$I \geq 0$ (we have proven that mutual information is ≥ 0)

$\rightsquigarrow$ $k=1$ $\Leftrightarrow$ $I(X;Y) \leq \log_2(1) = 0$

$\Rightarrow$ $I(X;Y) = 0$

so X and Y are independent

$\rightsquigarrow$ X and Z are aswell.