Université de Genève

Imagerie Numérique

13X004

TP 9: FT of typical signals

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Exercise 1

(a) I couldn't finish the exercise.

exo 1.a

$$\frac{d}{dt}\frac{d}{dt} e^{-t^2/2\sigma^2} \cdot \frac{1}{\sqrt{2\pi\sigma^2}}$$ is what we want to compute

$$\rightsquigarrow \frac{\partial}{\partial t}\left(\frac{\frac{\partial}{\partial t}\left(e^{-t^2/2\sigma^2}\right)}{\sqrt{2\pi}\sqrt{\sigma^2}}\right) = \frac{\partial}{\partial t}\left(\frac{e^{-t^2/2\sigma^2}\,-2t}{\sqrt{2\pi}\sqrt{\sigma^2}\,(2\sigma^2)}\right)$$

$$= -\frac{\frac{\partial}{\partial t}\left(e^{-t^2/2\sigma^2}\,t\right)}{\sqrt{2\pi}\,(\sigma^2)^{3/2}}$$

$$= -\frac{e^{-t^2/2\sigma^2}\left(\frac{\partial}{\partial t}(t)\right) + e^{-t^2/2\sigma^2}\underbrace{\left(\frac{\partial}{\partial t}\left(-\frac{t^2}{2\sigma^2}\right)\right)}_{\frac{-2t}{2\sigma^2}}t}{\sqrt{2\pi}\,(\sigma^2)^{3/2}}$$

$$= -\frac{e^{-t^2/2\sigma^2} - \frac{e^{-t^2/2\sigma^2}}{\sigma^2}t^2}{\sqrt{2\pi}\,(\sigma^2)^{3/2}}$$

$$= \frac{e^{-t^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}\,\sigma^4} - \frac{e^{-t^2/2\sigma^2}\cdot t^2}{\sqrt{2\pi\sigma^2}\,\sigma^6}$$ put in evidence

$$= \underbrace{g(t,\sigma)\left(\frac{1}{\sigma^4} - \frac{t^2}{\sigma^6}\right)}_{g(t,\sigma)\left(\frac{t^2}{\sigma^4} - \frac{1}{\sigma^2}\right)}$$ $\rightarrow$ I couldn't find my mistake

(b) We use Product theorem to compute FT :

product theorem: $\mathfrak{F}\{f(t)\cdot h(t)\} = \frac{1}{2\pi} F(\omega)\cdot H(\omega)$

we have: $k(t) = \underbrace{\cos(\omega_0 t)}_{f(t)} \cdot \underbrace{g(t,\sigma)}_{h(t)}$

$$F(t) \rightsquigarrow \pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)$$

$$H(t) \rightsquigarrow \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{\left(-\frac{t^2}{2\sigma^2}\right)} \cdot \frac{\sqrt{2\pi}}{\sigma}$$

① multiply $F(t)$ by $1/2\pi$

$$\rightsquigarrow \frac{1}{2\pi}\cdot F(t) = \frac{\pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)}{2\pi}$$

$$= \frac{\cancel{\pi}\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)}{2\cancel{\pi}}$$

② multiply the result by $\sqrt{2\pi}/\sigma$

$$\rightsquigarrow \frac{\sqrt{2\pi}}{2\sigma}\cdot\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)$$

③ multiply the result by $g(\omega,\sigma)$

$$\rightsquigarrow \frac{\sqrt{2\pi}}{2\sigma}\cdot\left(g(\omega,\sigma)\cdot\delta(\omega-\omega_0) + g(\omega,\sigma)\cdot\delta(\omega+\omega_0)\right)$$

$$= \frac{\sqrt{2\pi}}{2\sigma}\cdot\left(g(\omega-\omega_0,\sigma^{-1}) + g(\omega+\omega_0,\sigma^{-1})\right)$$

$\square$

(c) We use Linearity property to compute FT :

linearity theorem: $\mathfrak{F}\{a f_1(t) + b f_2(t)\} = aF_1(w) + bF_2(w)$

$u(t)$: $\underbrace{(1+\gamma)}_{a}\underbrace{\delta(t)}_{f_1(t)} - \underbrace{\gamma}_{b}\underbrace{g(t,\sigma)}_{f_2(t)}$

$F_1(w) = 1$

$F_2(w) = \frac{\sqrt{2\pi}}{\sigma} \cdot g(w, \sigma^{-1})$

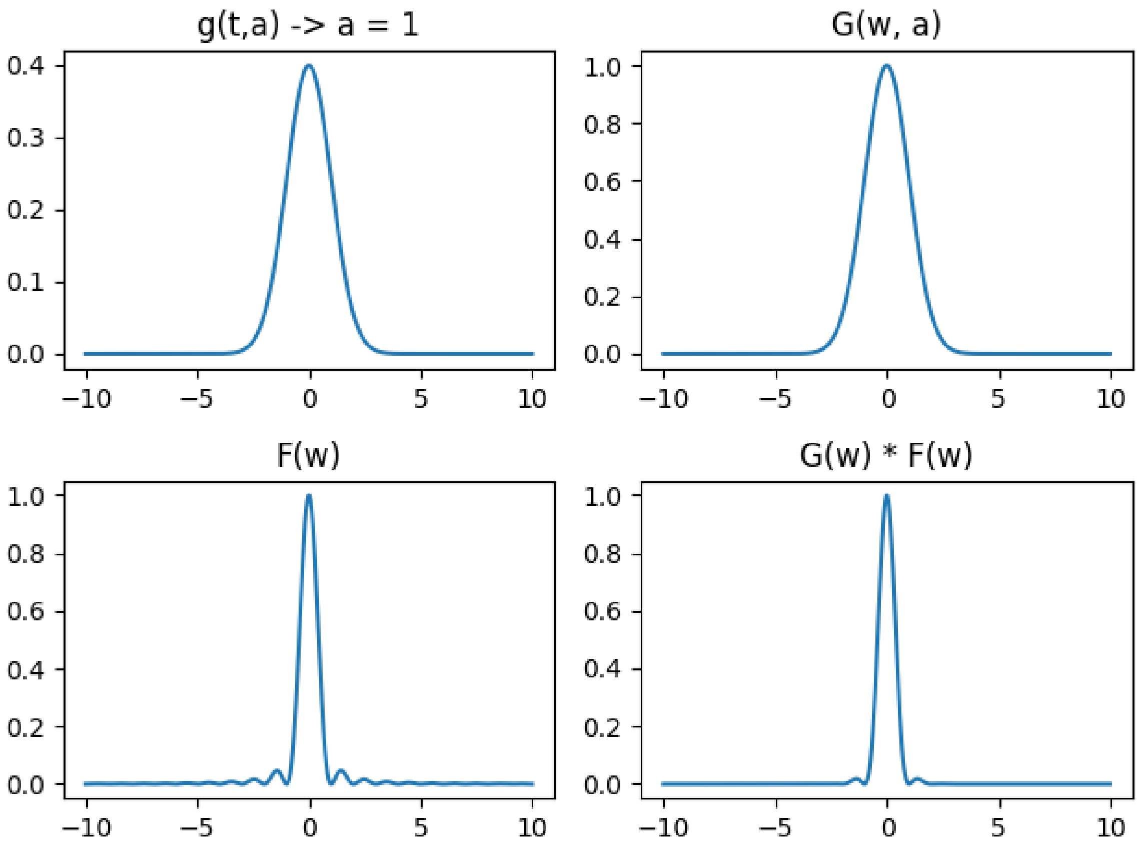
① a and b multiply F_1 and F_2 respectively

$\rightsquigarrow (1+\gamma) - \gamma \cdot \frac{\sqrt{2\pi}}{\sigma} \cdot g(w,\sigma^{-1})$ $\square$

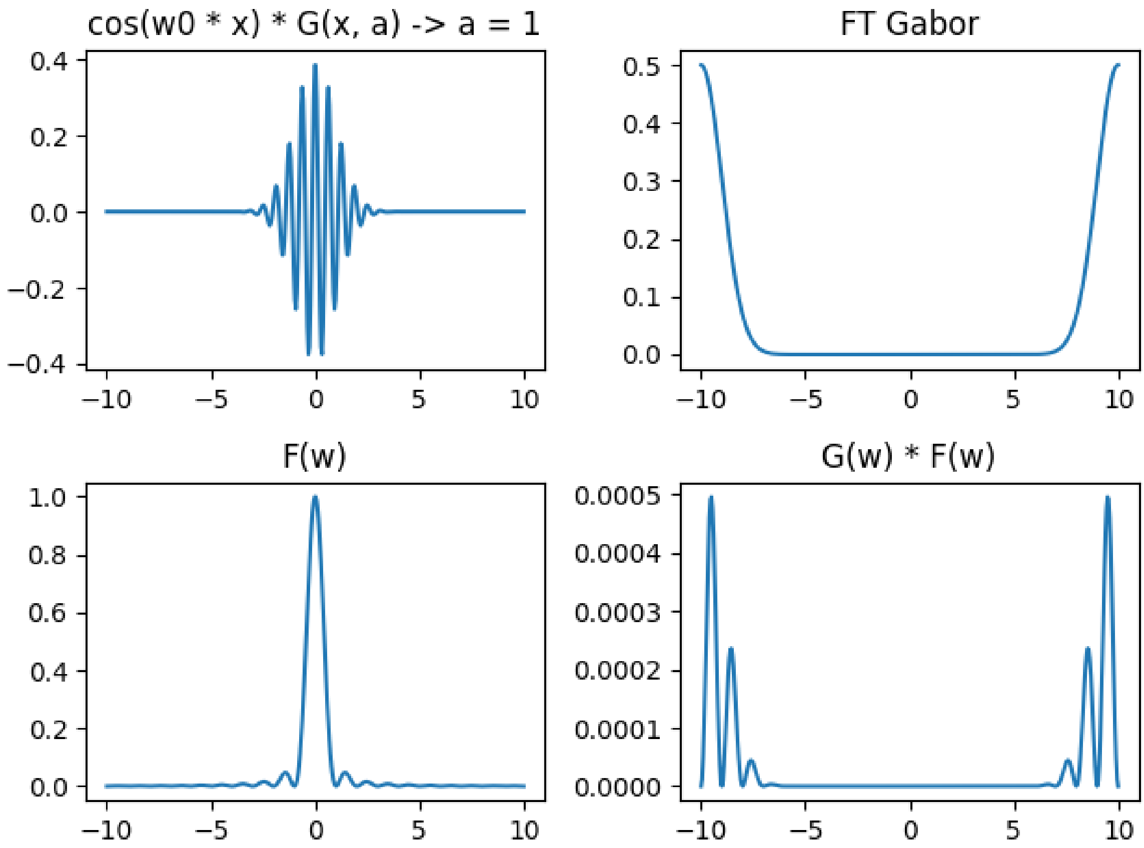
(d)

Exercise 2

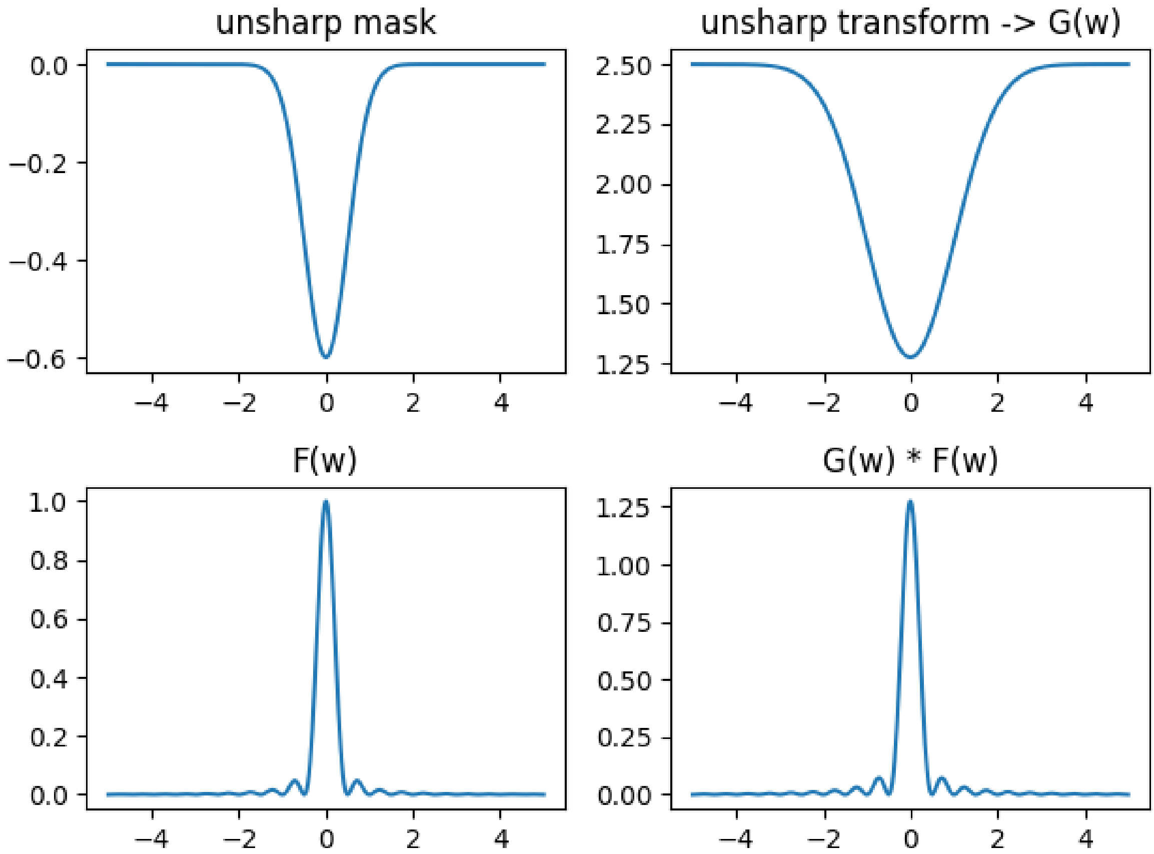
(a) Apart from where Gauss is at its maximum (at x=0), we killed all the other frequencies, which is what we wanted to do, we finish with a gaussian like result.



(b) We can see that everything from the signal that wasn't close to w0, was completely killed, since it is a simple multiplication, FT Gabor is equal to 0 everywhere other than around w0, so everything is killed apart for the values around w0.



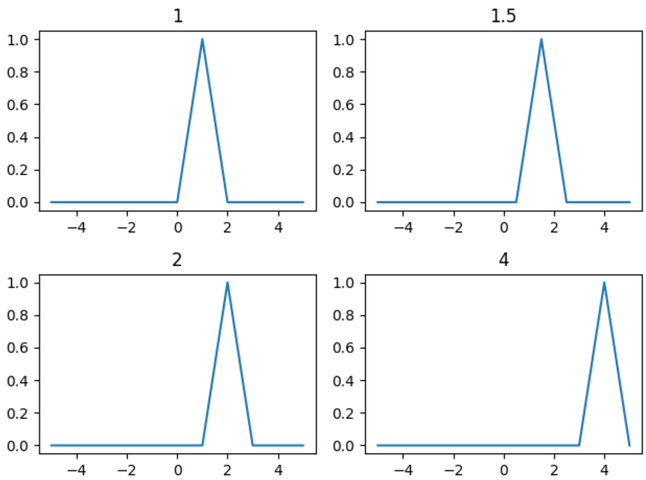
(c) With unsharp mask we amplify the small frequency values and so we end up having less low frequency components.

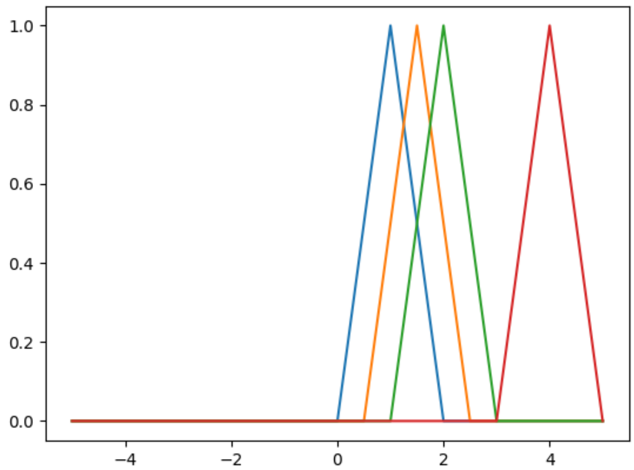


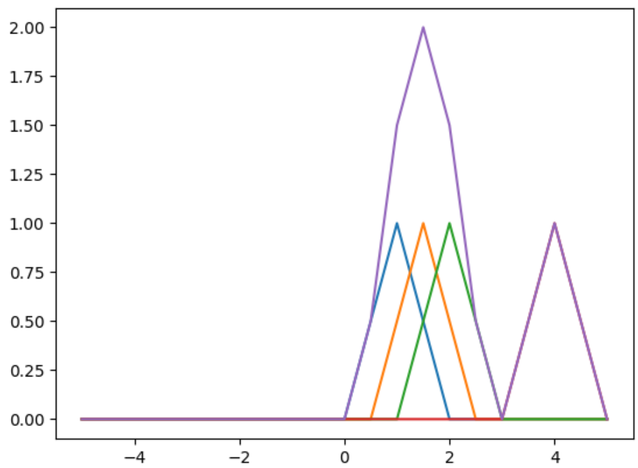
Exercise 3

(a)

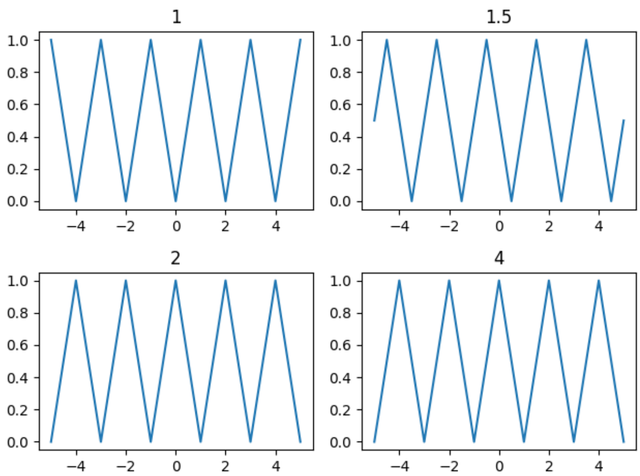
(b) First we have to compute the hat function, and then shift it for each value T = [1, 1.5, 2, 4]. Then we have to add all the values, from (a) we know we will get a periodic signal.

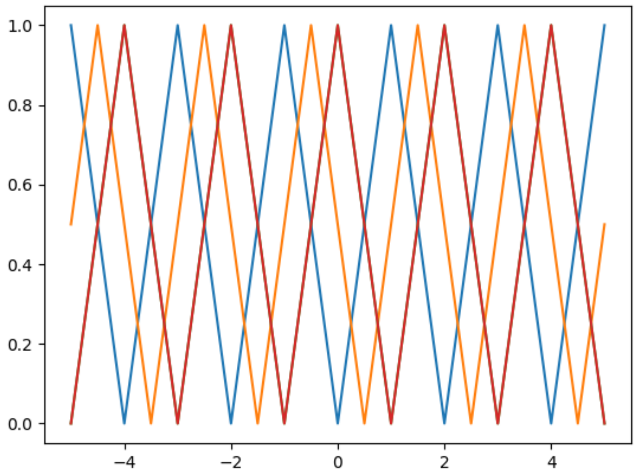


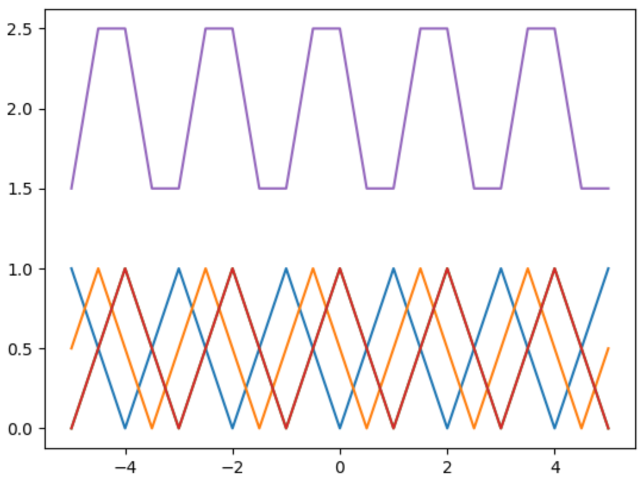




We can now do the same experiment, but instead of using the hat function as reference, we use its periodic representation







Because the shift by T=2 is the same as T=4, the sum is of period T=2.

(c)