Université de Genève

Imagerie Numérique
13X004

TP 7: DSP and Complex Numbers

Author: Joao Filipe Costa da Quinta

E-mail: Joao.Costa@etu.unige.ch

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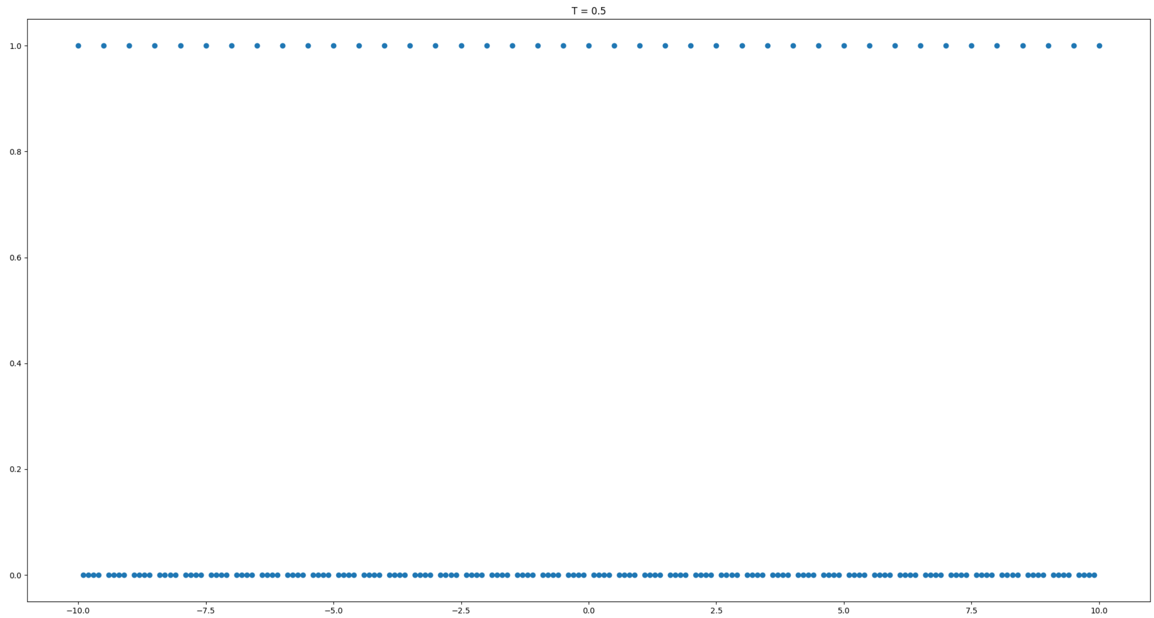
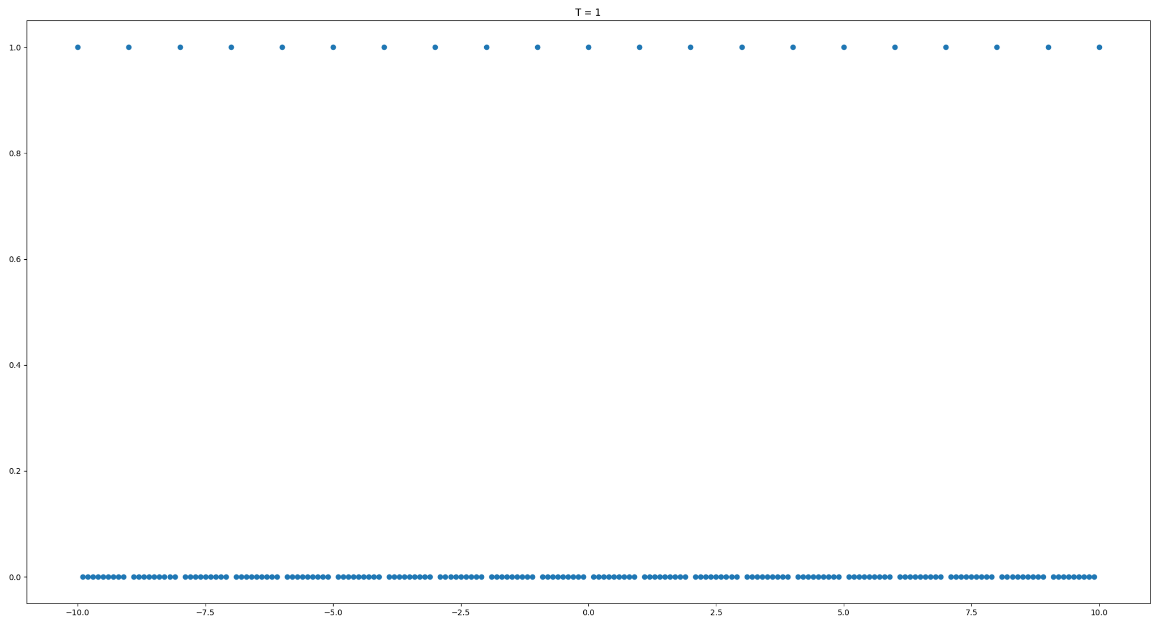
UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES
Département d'informatique

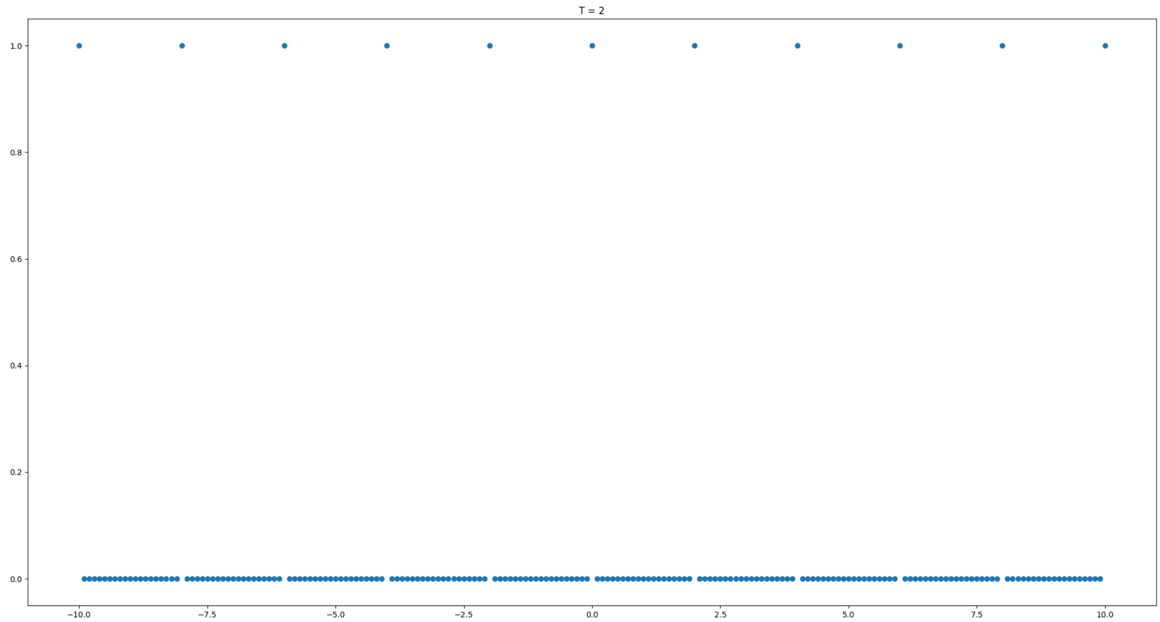
Exercise 1

(a) Sha function is implemented in lib.py file, and is named computeSha().
We have to use num=201, because both edges of the interval are taken into account, if we were to add the argument -> endpoint=False, we would only need num=200, however, the right edge wouldn't be in the array.
Because the length of the interval is 20, dividing by 200 is 0.1. But because 10 value is in the final array as well, we need 201.

In these 3 images below, we can see that depending on T we a different number of values x such that f(x)=1.

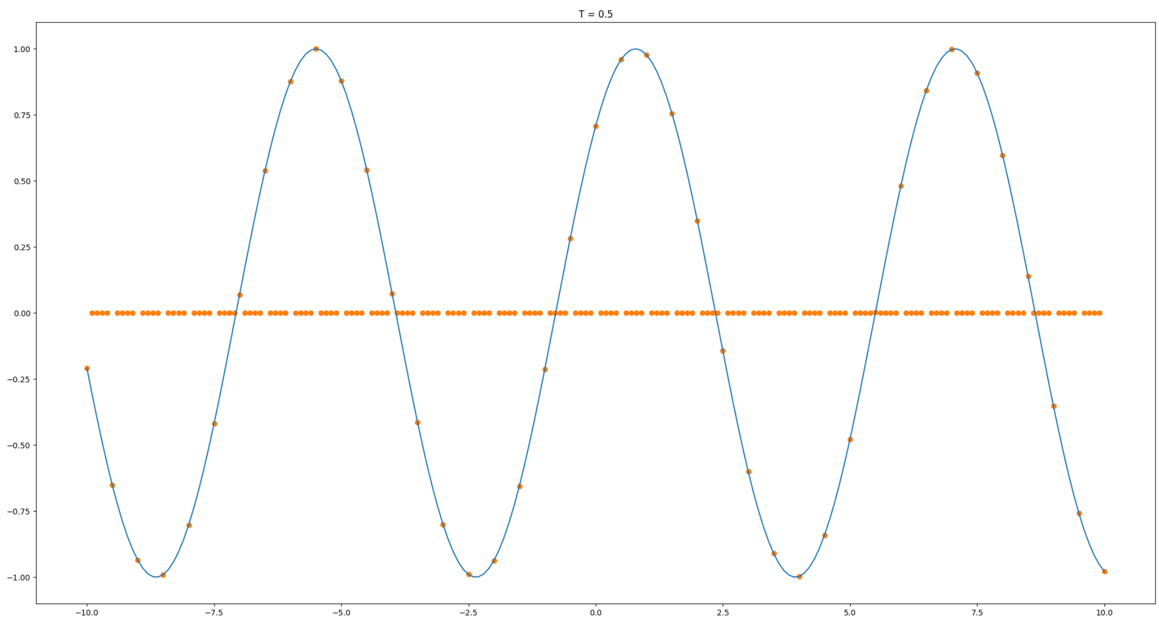


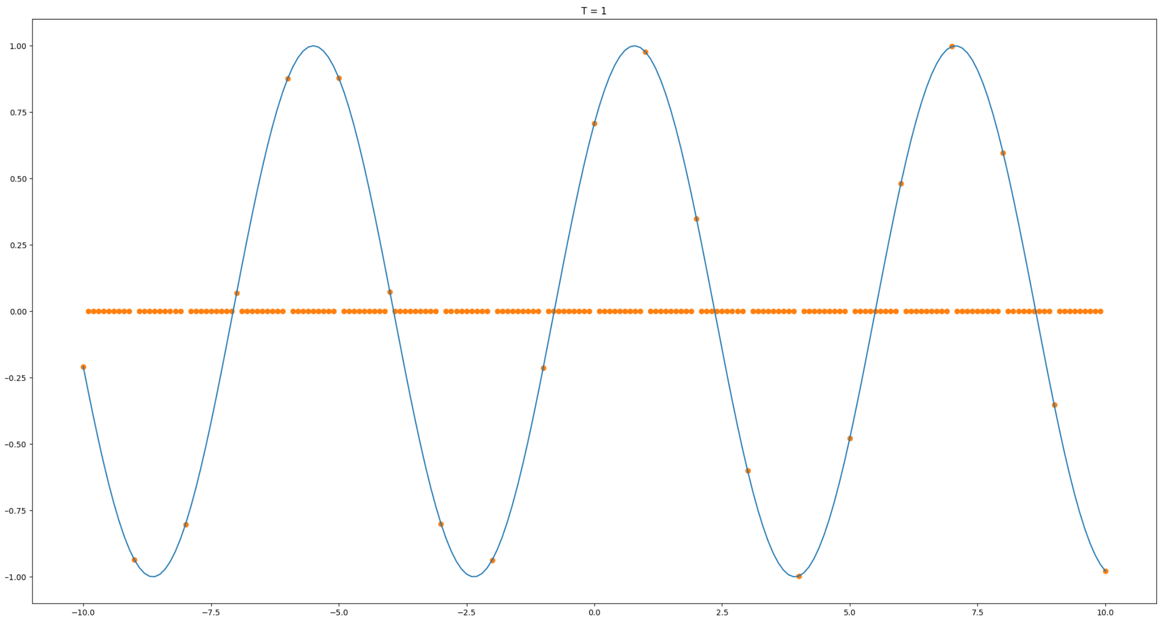


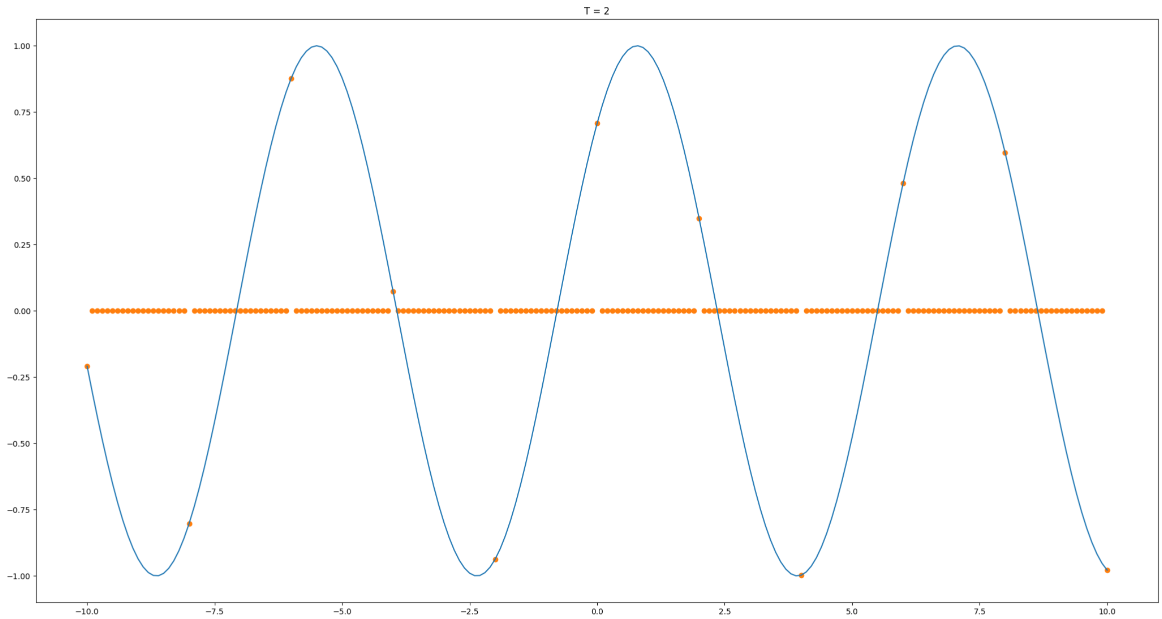


(b) Because the rate at which we want to sample the function is the same as the rate at which sha will have an x such that sha(x) = 1, we can simply multiply both functions, and get the desired sampling of the function, and they don't scale up or down.

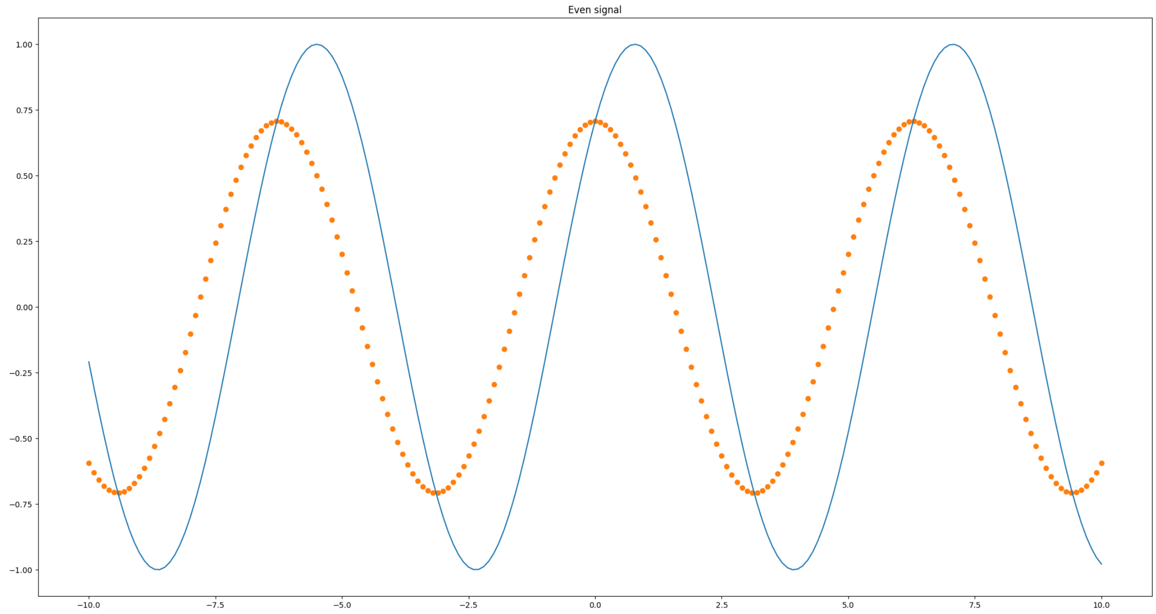
(c) To sample the function at the desired rate, I used the technique explained in the question (b). We will be able to see that for every value T, there will be the same amount of x such that f(x) != 0, as there was in the answer (a).

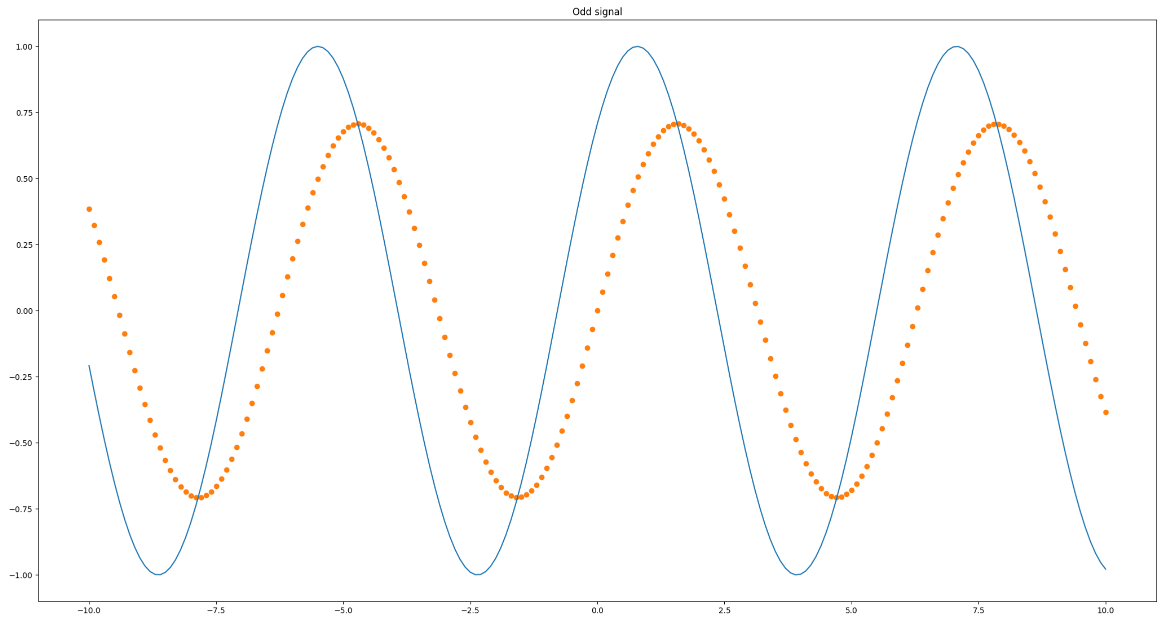


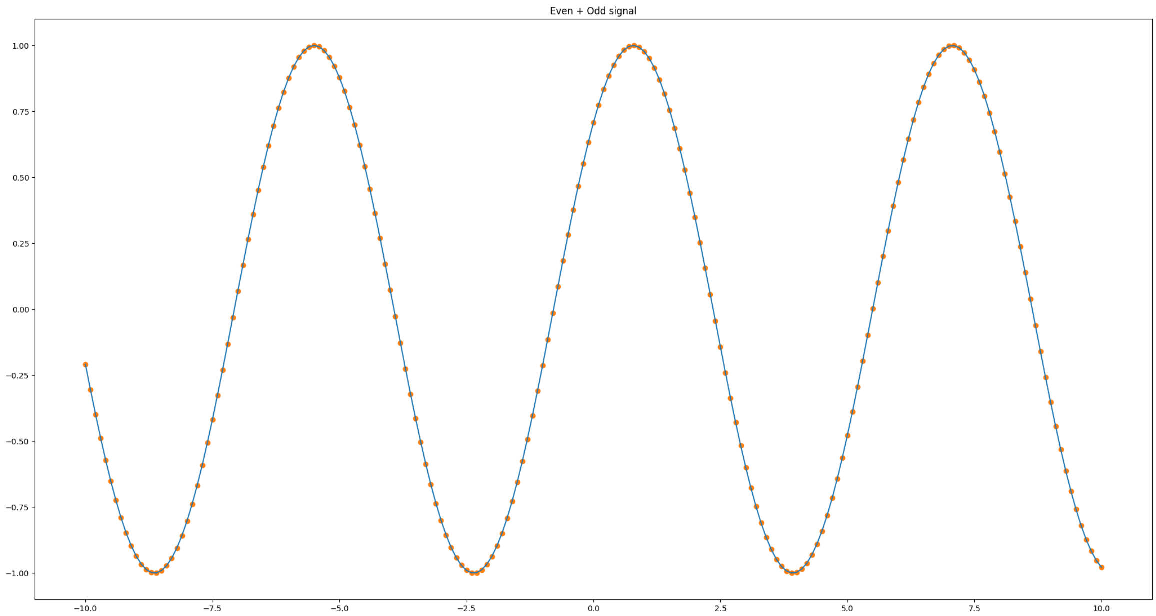




(d) To compute the even and odd part of a function I simply used the formula given in class. from the signal S, we can split it into the even and odd part, this operation is an encoder, we can later add the two separated signals to find the signal S again, this would be the corresponding decoder. This is what we will witness in the 3 images below.


Even signal
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0


Odd signal
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0



(e)

Exercise 2

(a) :

$f(z) = z^3 - 1$ $\qquad z^3 = 1$

module : $|z^3| = |1| \iff |z|^3 = 1 \iff |z| = 1$

$\arg(z^3) = \arg(1) \iff 3 \cdot \arg(z) = 0 \bmod 2\pi$

$\iff \arg(z) = \frac{0}{3} \bmod \frac{2}{3}\pi$

$\iff \arg(z) = 0 + k \cdot \frac{2}{3}\pi$

$z^3 \rightarrow 3$ solutions

$z_0 = |z| \cdot (\cos(\arg(z)) + i \cdot \sin(\arg(z))$, $k=0$

$\rightarrow z_0 = 1 \cdot (\underbrace{\cos(0)}_{1} + i \cdot \underbrace{\sin(0)}_{0}) = 1$

$z_1 = 1 \left(\cos\left(\frac{2\pi}{3}\right) + i \cdot \sin\left(\frac{2\pi}{3}\right)\right) = \cos\left(\frac{2\pi}{3}\right) + i \cdot \sin\left(\frac{2\pi}{3}\right)$

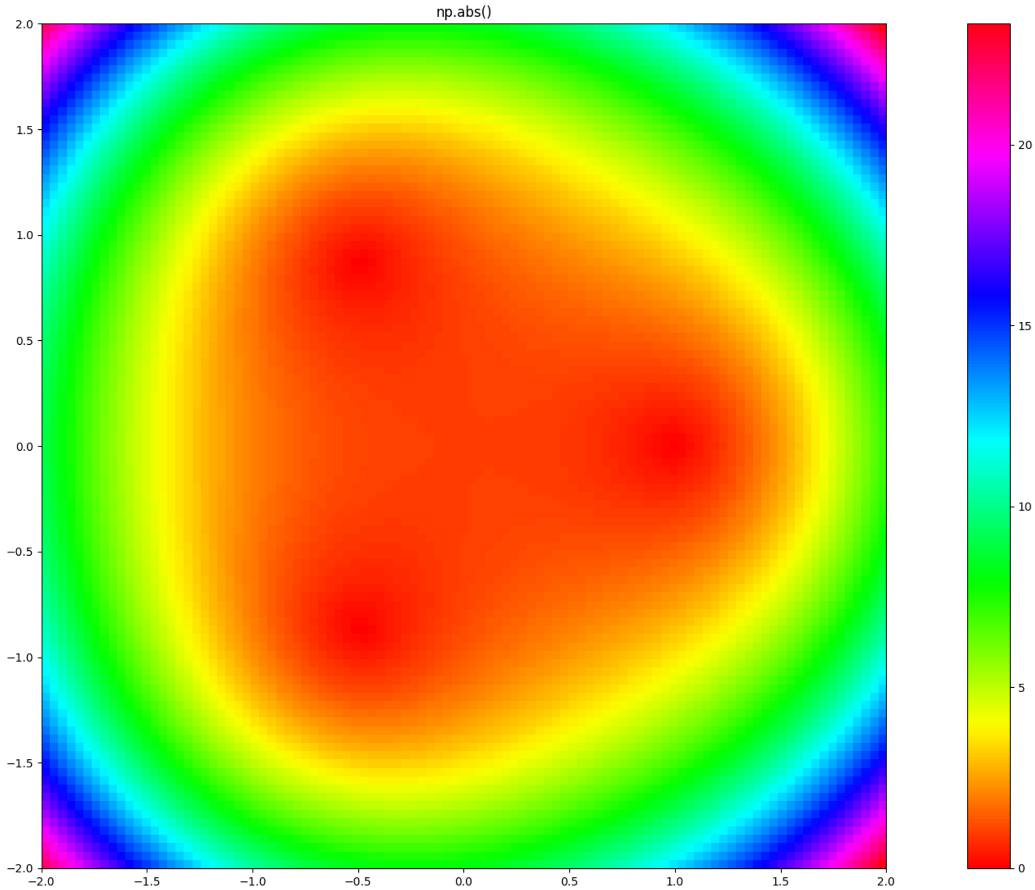
$= e^{i\frac{2}{3}\pi}$, $k=1$

$z_2 = 1 \cdot \left(\cos\left(\frac{4}{3}\pi\right) + i \cdot \sin\left(\frac{4\pi}{3}\right)\right) = e^{i\frac{4}{3}\pi}$, $k=2$

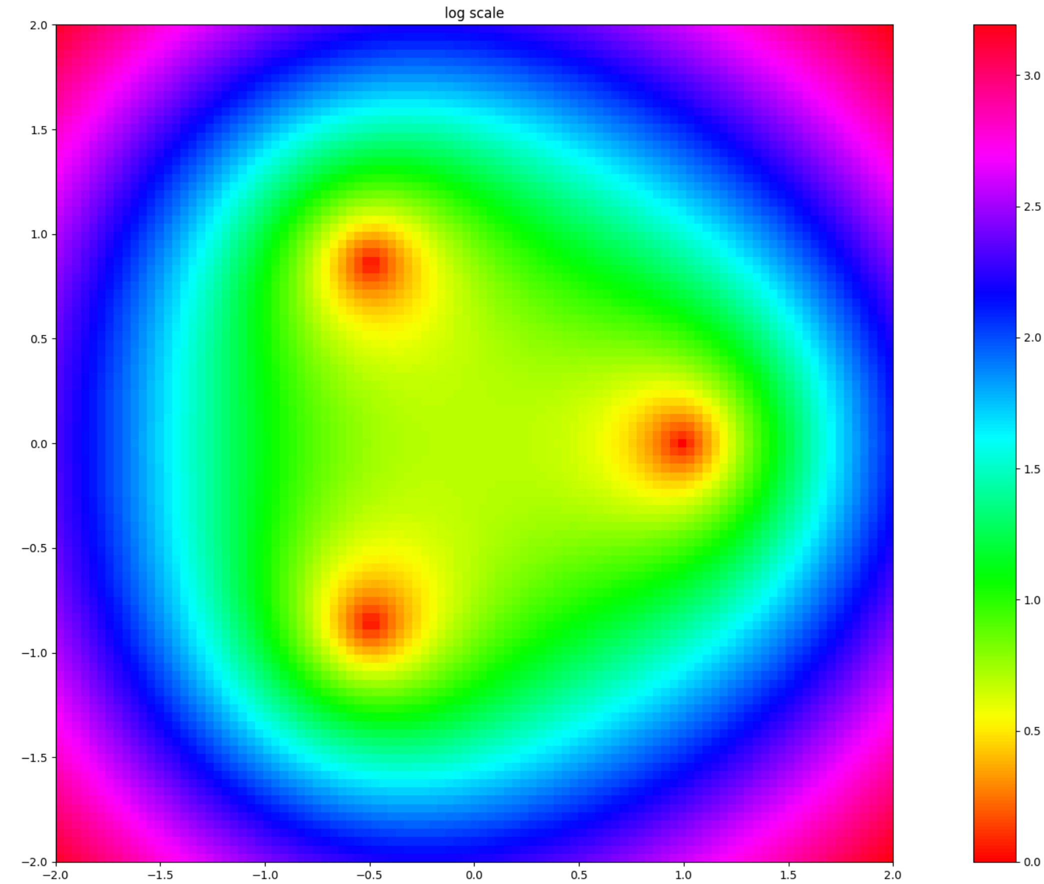
(b) in .py file

(c) in .py file

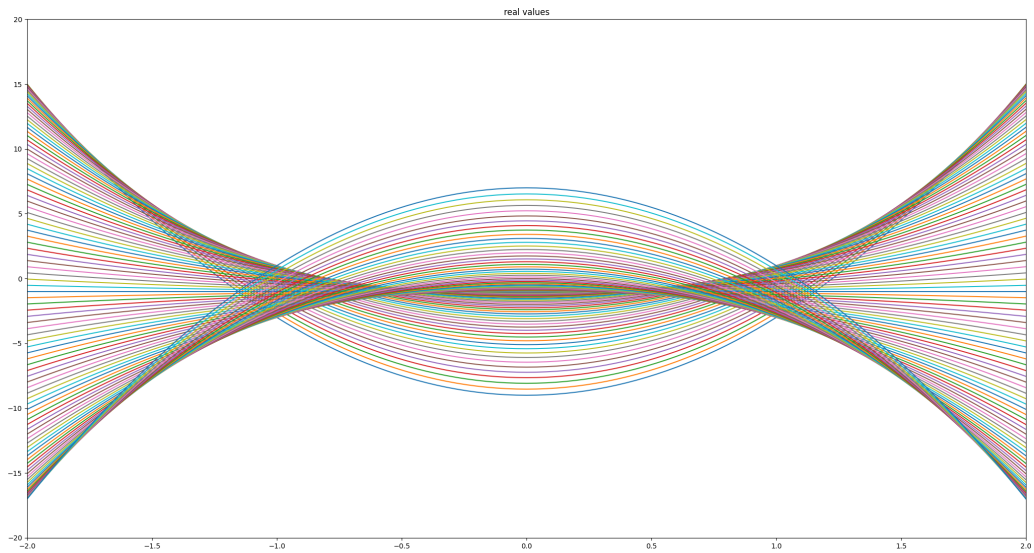
(d) At this point we have a matrix that represents the np.abs(), of our polynome $P(z) = z^3 - 1$, where z is a complex number. with real values between [-2,2] and imag values between [-2,2].
For example, for z = 0 + 0i, $z^3 - 1$ = -1, and np.abs(-1) = 1, which is what we can see in our visualisation below, we can also see that the 3 areas in which np.abs() is the lowest, is where our polynome P has a root, where P(z) = 0 and so np.abs(0) = 0.

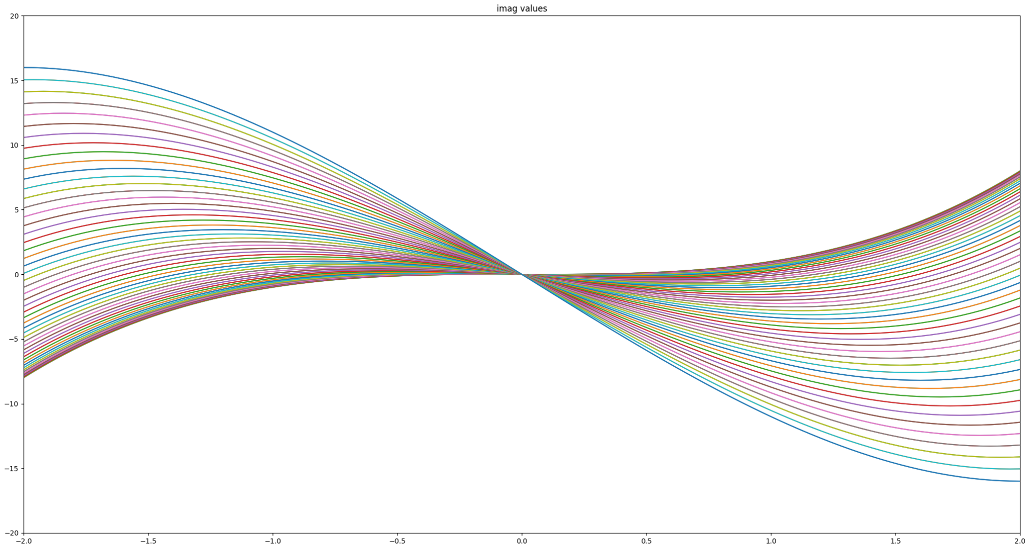


(e) The difference between the visualisation in (d) and (e) is basically that it is much easier too see the results, since $\log(1) = 0$, we now have to add 1 to the previous matrix, so that where we had the 0 value, we now have 1.



(f) Let a be a real number, we can also rewrite the real a as a complex number z, z = a + 0i = a, this is the same as saying that for every complex number z of form z = a + bi, is a real number <=> b = 0. Which is what we see in the graph that represents the imag values.





(g)

(h) When we compute np.angle(), and we find 0 as a result, that means that the angle between the complex number and the real axe, is equal to 0, which also means the complex is represented only by the real axis. Which means that that complex number can be represented as a real number.