

UNIVERSITÉ DE GENÈVE

IMAGERIE NUMÉRIQUE

13X004

TP 10: 1D Discrete Fourier Transform

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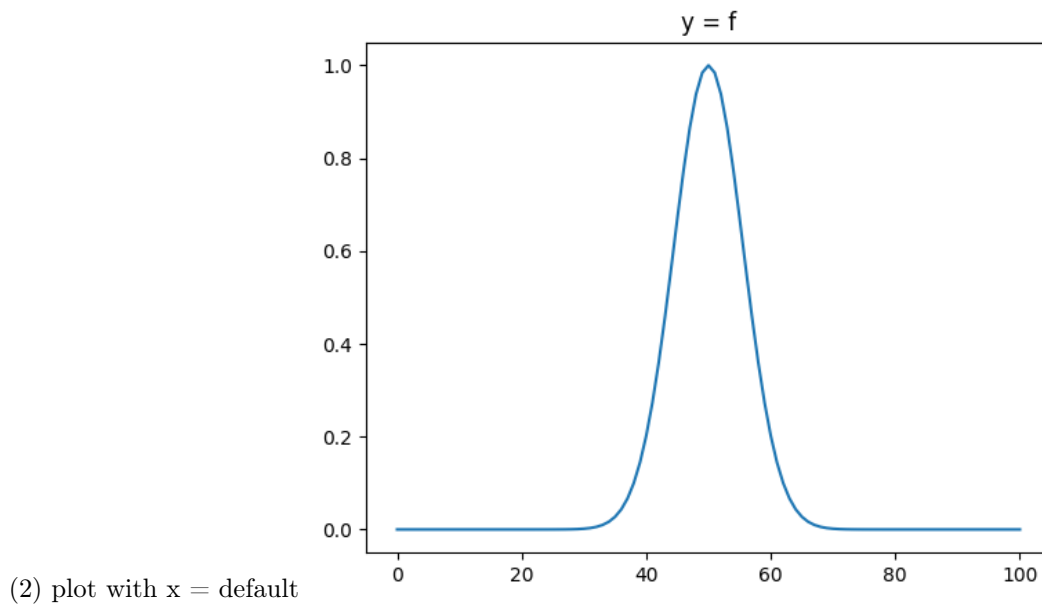
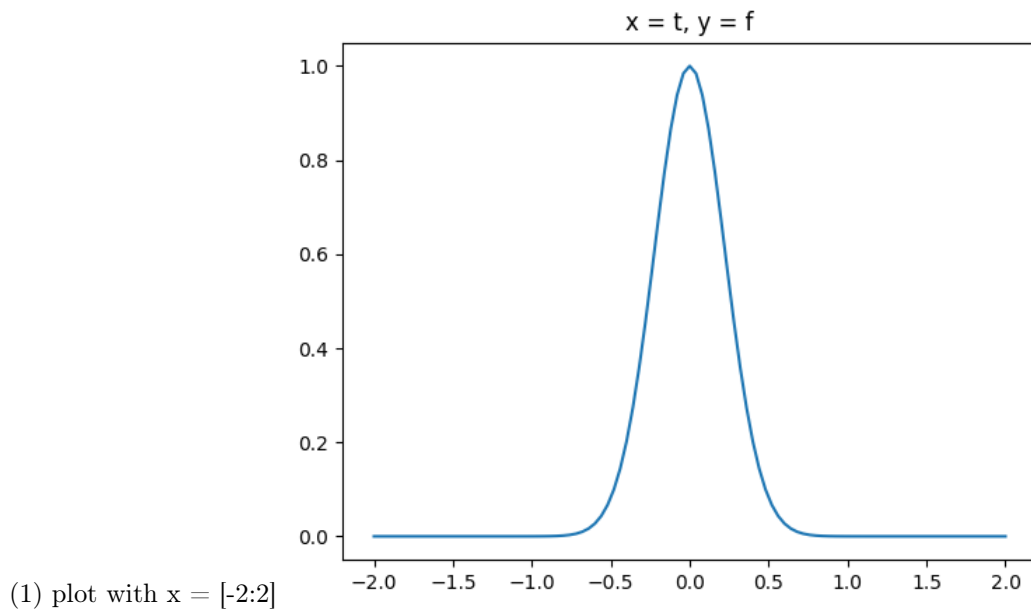
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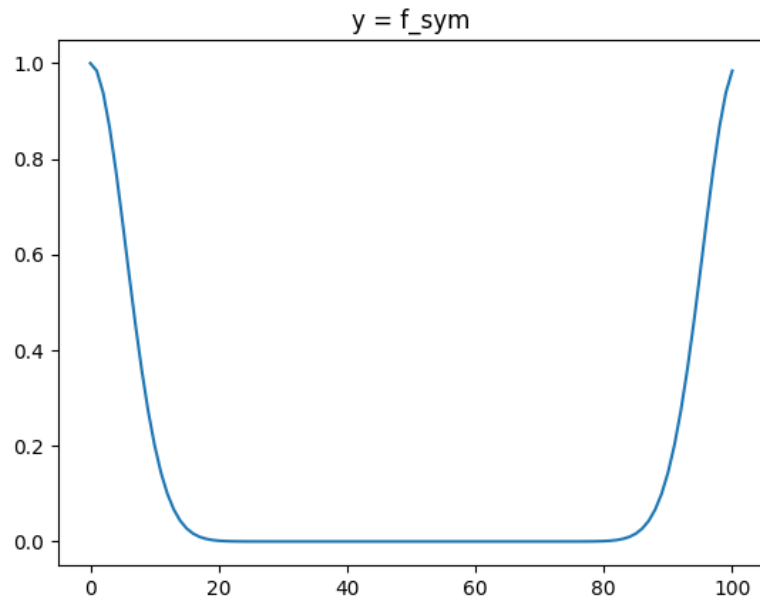
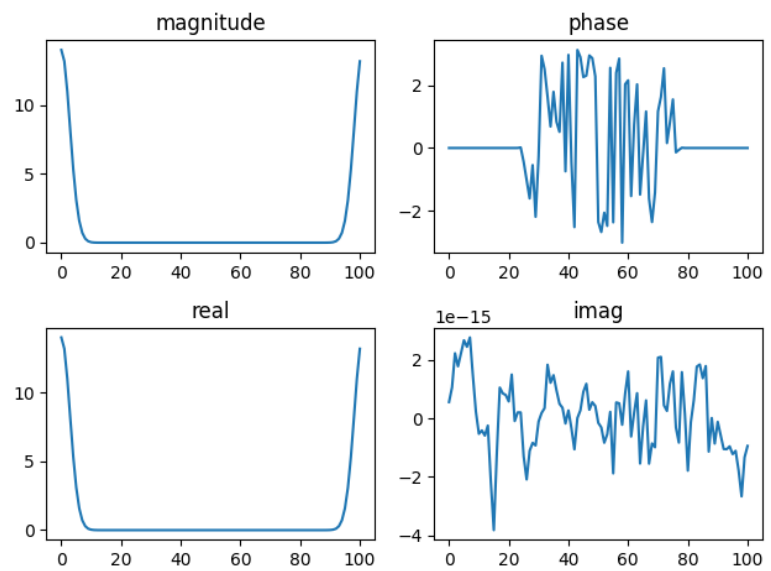
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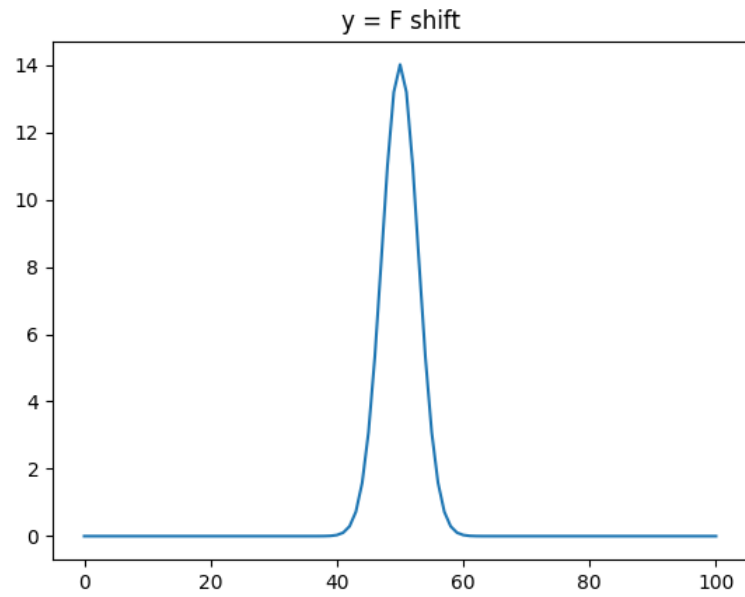
Exercise 1

(a) in .py

(b) s

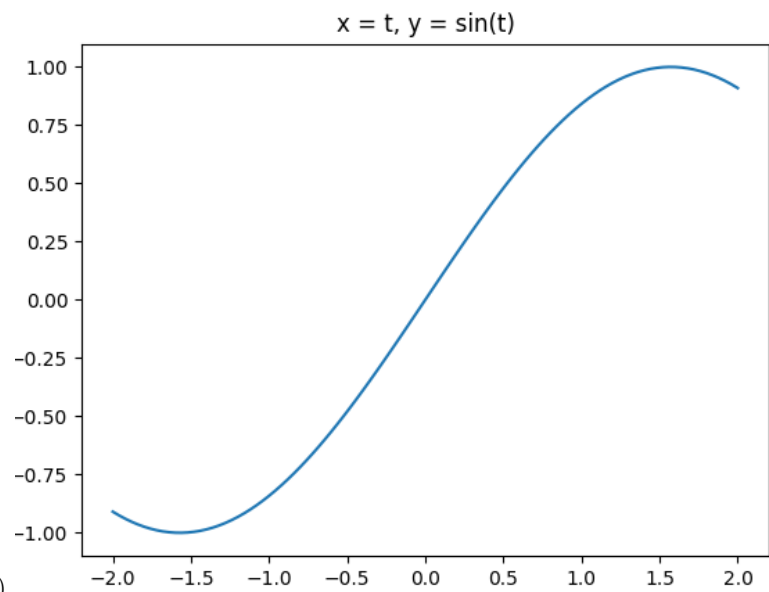


(3) plot $y = f_{\text{sym}}$ (4) plot $y = F = \text{np.fft.fft}(f_{\text{sym}})$

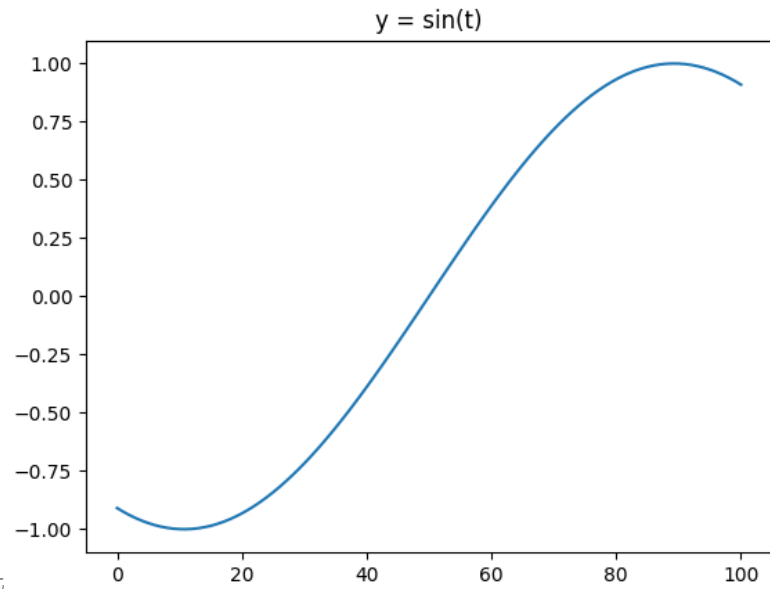


(5) plot $y = \text{np.fft.fftshift}(F)$

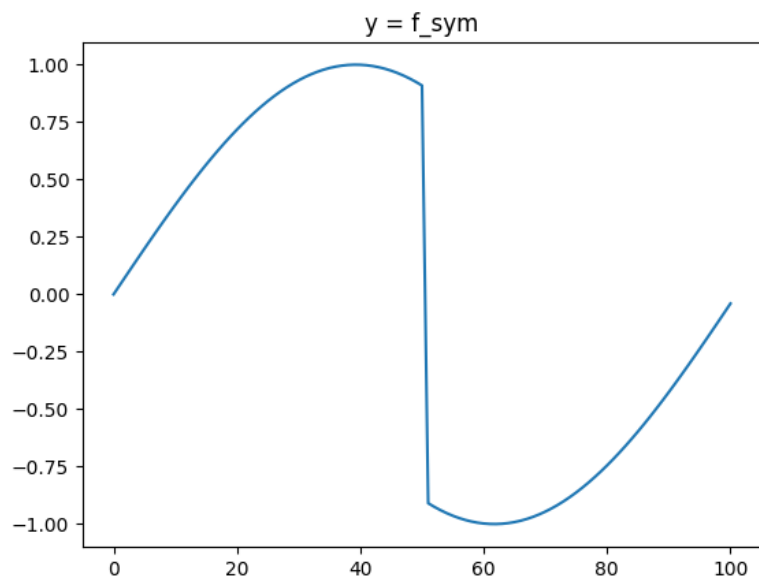
(c) s



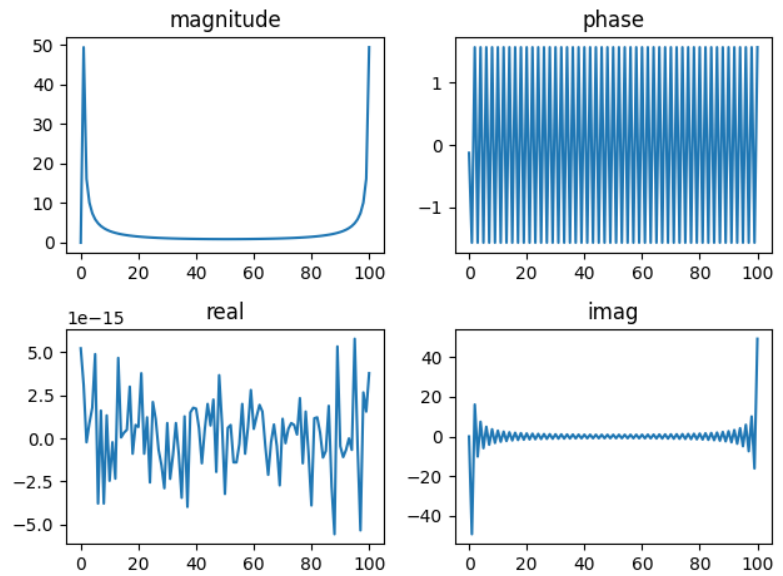
(1) plot with $x = [-2:2]$ and $f = \sin(x)$



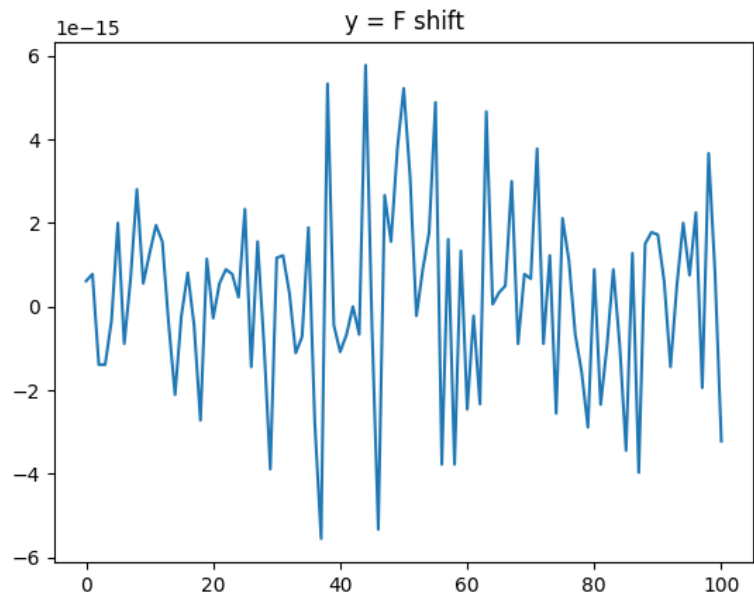
(2) plot with $x = \text{default}$



(3) plot $y = f_{\text{sym}}$



(4) plot $y = F = \text{np.fft.fft}(\text{fsym})$



(5) plot $y = \text{np.fft.fftshift}(F)$

Exercise 2

(a) normal convolution :

exo2 @ $f = [1, 2, 3, 4]$, $g = [1, -1]$

$$h(x) = f(x) * g(x) = \sum_{-\infty}^{\infty} f(m) \cdot g(x-m)$$

$$\hat{g}(x) = [-1, 1] \rightarrow g(x) \text{ flipped}$$

$f: \begin{array}{|c|} \hline 1 \\ \hline \end{array} 2 \ 3 \ 4$
 $g: -1 \begin{array}{|c|} \hline 1 \\ \hline \end{array}$
 $\hookrightarrow 1 \cdot 1 = 1 = h_1$

$f: \begin{array}{|c|} \hline 1 \\ \hline \end{array} \begin{array}{|c|} \hline 2 \\ \hline \end{array} 3 \ 4$
 $g: \begin{array}{|c|} \hline -1 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array}$
 $\left. \begin{array}{l} \hookrightarrow 2 \cdot 1 = 2 \\ \hookrightarrow 1 \cdot -1 = -1 \end{array} \right\} 2 - 1 = 1 = h_2$

$$h_3 = 2 \cdot -1 + 3 \cdot 1 = 1$$

$$h_4 = 3 \cdot -1 + 4 \cdot 1 = 1$$

$$h_5 = -1 \cdot 4 = -4$$

$$h(x) = [1, 1, 1, 1, -4]$$

(b) circular convolution :

h_1 → means h is moved by 1 position

1 position

$$\begin{array}{r}
 f: \begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 4 \\ \hline \end{array} \\
 g: \begin{array}{cccc} 0 & 0 & -1 & 1 \end{array}
 \end{array}
 \quad
 \begin{array}{r}
 \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
 \begin{array}{cccc} 1 & 0 & 0 & -1 \end{array}
 \end{array}
 \quad
 \begin{array}{l}
 \downarrow \text{multiply} \\
 \begin{array}{cccc} 1 & 0 & 0 & -4 \end{array} \\
 \hline
 \text{add} \rightarrow \boxed{-3}
 \end{array}$$

h_2 → 2 positions

$$\begin{array}{r}
 f: \begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 4 \\ \hline \end{array} \\
 g: \begin{array}{cccc} 0 & 0 & -1 & 1 \end{array}
 \end{array}
 \quad
 \begin{array}{r}
 \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
 \begin{array}{cccc} -1 & 1 & 0 & 0 \end{array}
 \end{array}
 \quad
 \begin{array}{l}
 \downarrow \\
 \begin{array}{cccc} -1 & 2 & 0 & 0 \end{array} \\
 \hline
 + \\
 \boxed{1}
 \end{array}$$

h_3

$$\begin{array}{r}
 f: \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
 g: \begin{array}{cccc} 0 & 0 & -1 & 1 \end{array}
 \end{array}
 \quad
 \begin{array}{l}
 \downarrow \\
 \begin{array}{cccc} 0 & -2 & 3 & 0 \end{array} \\
 \hline
 + \\
 \boxed{1}
 \end{array}$$

h_4

$$\begin{array}{r}
 f: \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
 g: \begin{array}{cccc} 0 & 0 & -1 & 1 \end{array}
 \end{array}
 \quad
 \begin{array}{l}
 \downarrow \\
 \begin{array}{cccc} 0 & 0 & -3 & 4 \end{array} \\
 \hline
 + \\
 \boxed{1}
 \end{array}$$

$$h(x) = [-3, 1, 1, 1]$$

(c) in .py → results were the same

(d) in .py → results were the same