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Imagerie Numérique

DSP and Complex Numbers

TP Class N° 7

February 18, 2021

Exercise 1. The Dirac comb *(3 points)*
The Dirac comb or train of impulses is the following "function":

$$Ш_T(t) = \sum_{k \in \mathbb{Z}} \delta(t - k \cdot T)$$

(a) In Python, implement a function

$$\text{sha}(t, T) = \begin{cases} 1 & \text{if t is a multiple of T} \\ 0 & \text{otherwise.} \end{cases}$$

Using *np.linspace()* and *plt.scatter()*, represent $\text{sha}(t, 1)$, $\text{sha}(t, 0.5)$ and $\text{sha}(t, 2)$ on a range $t \in [-10, 10]$.
Remark: Use $num = 201$ in $np.linspace()$. Why is this value important ?

(b) Explain why sampling a function at rate $1/T$ is equivalent to multiplying it with $Ш_T(t)$.

(c) Illustrate this fact by sampling $f(t) = \sin(t + \pi/4)$ with $T = 1,\ 2,\ 0.5$. Display the results on separate graphs. On each graph, plot the original function $f(t)$ using *plt.plot()* and the sampled version using *plt.scatter()*. Comment the results.

(d) Implement the two functions $Even(S)$ and $Odd(S)$, that compute the even and odd parts of a signal S. Apply them to the signal $f(t)$ and visualise $Even(S)$, $Odd(S)$ and $Even(S){+}Odd(S)$ on three different graphs. Comment the results.

(e) Using trigonometric formulas, compute by hand the odd and even parts of $f(t)$. Compare your results with part (d).

Exercise 2. Complex function visualization *(3 points)*

In this exercise, you will visualize a complex polynomial function using complex numpy arrays. Consider

$$f(z) = z^3 - 1$$

(a) Show that 1, $e^{j2\pi/3}$ and $e^{j4\pi/3}$ are the complex roots of f by computing by hand. Write them in rectangular coordinates.

(b) Create a numpy complex matrix z of size 100×100 whose entries range from $[-2, 2]$ both in real and imaginary parts.
Hint : Use *np.linspace()*, *np.meshgrid()* and define $z = xx + yy * 1j$

(c) Apply the function f to the matrix z, using pointwise operations, giving you a new complex matrix w of same size.

(d) Use the function *np.abs()* on w and visualize the result as an image, with its colorbar.
Hint : To have interpretable coordinates, use parameter *extent* of *plt.imshow()*.
We also recommand to use the 'hsv' colormap which has stronger constrast than the default one.

(e) To further enhance the visualization, we will use a logarithmic scale.
Instead of visualizing directly $|w|$, first apply the transformation $m = \log(1 + |w|)$ and do *plt.imshow()* on m. This trick will come very often when we will investigate the magnitude of complex images. Comment the resulting image.

(f) Visualise now the real and imaginary parts of w on two different plots. When is $f(z)$ a real number ?

(g) Prove point (f) by computing explicitely the developpement of $f(a + jb)$ and finding a relation between a and b.

(h) Finally, visualise the phase of w using $np.angle()$ and plot the colorbar as well. Give a new interpretation on when is $f(z)$ a real number based on the value of the phase.

Submission

Please archive your report and codes in "Name_Surname.zip" (replace "Name" and "Surname" with your real name), and upload to "`Assignments/TP7: DSP and Complex Numbers`" on `https://moodle.unige.ch` before Thursday, March 11, 2021, 23:59 PM. Note, **the assessment is based not only on your code, but also on your report, which should include your answers to all questions and the experimental results**.