Université de Genève

Imagerie Numérique

13X004

TP 8: FS and FT

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Exercise 1

(a) To compute a_k we use the following formula seen in class:

$$\frac{2}{T}\int_{-T/2}^{T/2} \tilde{f}(t)\cos(k w_0 t)\,dt \Leftrightarrow$$

$$\Leftrightarrow \frac{2}{T}\int_{-1}^{3} \tilde{f}(t)\cos(k w_0 t)\,dt \Leftrightarrow$$

$$\Leftrightarrow \frac{2}{T}\left(\int_{-1}^{1} \tilde{f}(t)\cos(k w_0 t)\,dt + \int_{1}^{3} \tilde{f}(t)\cos(k w_0 t)\,dt\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{2}\left(\frac{\sin\left(k\frac{\pi}{2}\cdot 1\right) - \sin\left(-k\frac{\pi}{2}1\right) - \sin\left(k\frac{\pi}{2}\cdot 3\right) + \sin\left(k\cdot\frac{\pi}{2}1\right)}{k\pi/2}\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{\sin\left(k\frac{\pi}{2}\cdot 1\right) - \sin\left(-k\frac{\pi}{2}1\right) - \sin\left(k\frac{\pi}{2}\cdot 3\right) + \sin\left(k\cdot\frac{\pi}{2}1\right)}{k\pi} \Leftrightarrow$$

$$\Leftrightarrow \frac{4}{k\pi}\cdot\left(\sin\left(k\frac{\pi}{2}\right)\right) \longrightarrow$$

1. if k is even: $\sin\left(k\frac{\pi}{2}\right) = 0$
2. if k is odd: its either -1 or 1

$$\rightarrow (-1)^{(k-1)/2}\cdot\frac{4}{k\pi}$$

(b) To compute b_k we use the following formula seen in class, I skipped some steps, but its the same idea as exercise (a) :

$$b_k = \frac{2}{T} \int_{-T/2}^{T/2} \tilde{f}(t) \sin(k w_0 t)\, dt$$

(skip steps, but same idea as ⓐ)

$$= \frac{\overbrace{\cos\left(k\frac{\pi}{2}1\right) - \cos\left(-k\frac{\pi}{2}1\right)}^{① = 0} - \overbrace{\cos\left(k\frac{\pi}{2}\cdot 3\right) + \cos\left(k\frac{\pi}{2}1\right)}^{② = 0}}{k\pi}$$

① since: $\cos\left(k\frac{\pi}{2}\right) = \cos\left(-k\frac{\pi}{2}\right)$

② and : $\cos\left(k\frac{\pi}{2}\right) = \cos\left(k\frac{\pi}{2}\cdot 3\right)$

to compute F_k we can use the formula seen in class

$$F_k = \frac{1}{2}\left(a_k - j b_k\right)$$

$$\rightarrow \tilde{f}(t) = \sum_k F_k e^{jk\cdot\frac{\pi}{2}\cdot t} \qquad (3)$$

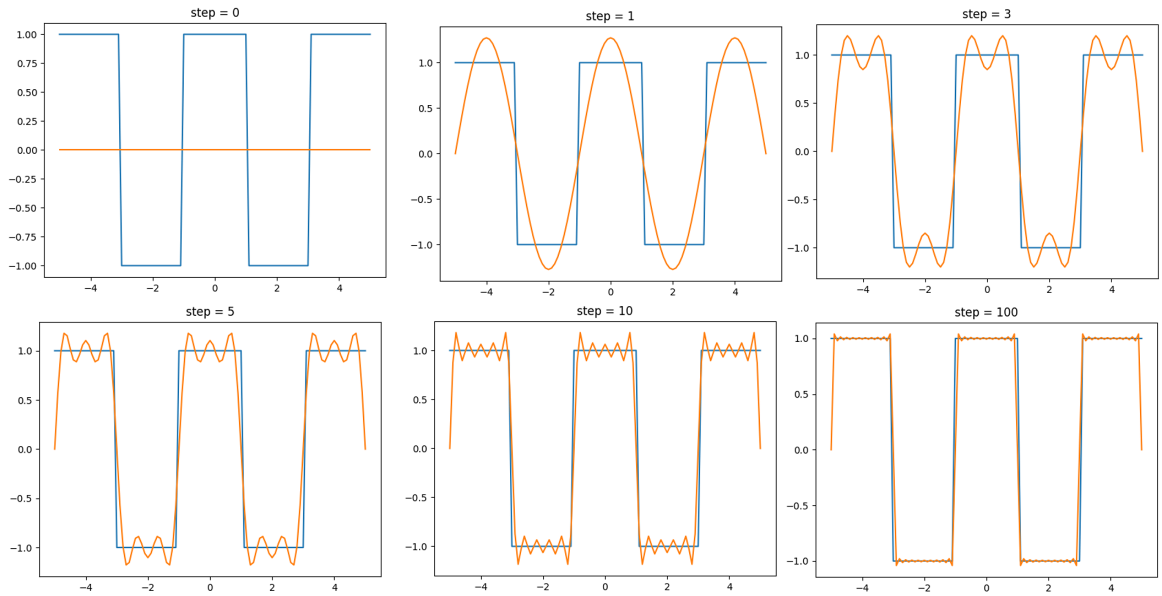
$$\rightarrow \tilde{f}(t) = \frac{1}{2}a_0 + \sum_k A_k \sin\left(k\frac{\pi}{2}t + \phi_k\right) \quad (2)$$

$$A_k = \sqrt{a_k^2 + b_k^2}$$

$$\phi_k = \tan^{-1}\left(\frac{a_k}{b_k}\right)$$

for every possible k, we end up with 0 in the numerator, so bk = 0 for every k.

(c) Code in python file



(d) The Gibbs phenomenon says that it is hard to approximate a discontinuous function by adding a series of sine and cosine waves. Which is what we are doing with FS, we can see in our point (c), with step = 10, the edges of the square are of higher amplitude, this is the called overshooting, and as we increase the step, it doesn't fix this overshooting, as we can see with step = 100, it is still visible.

Exercise 2

(a) a :

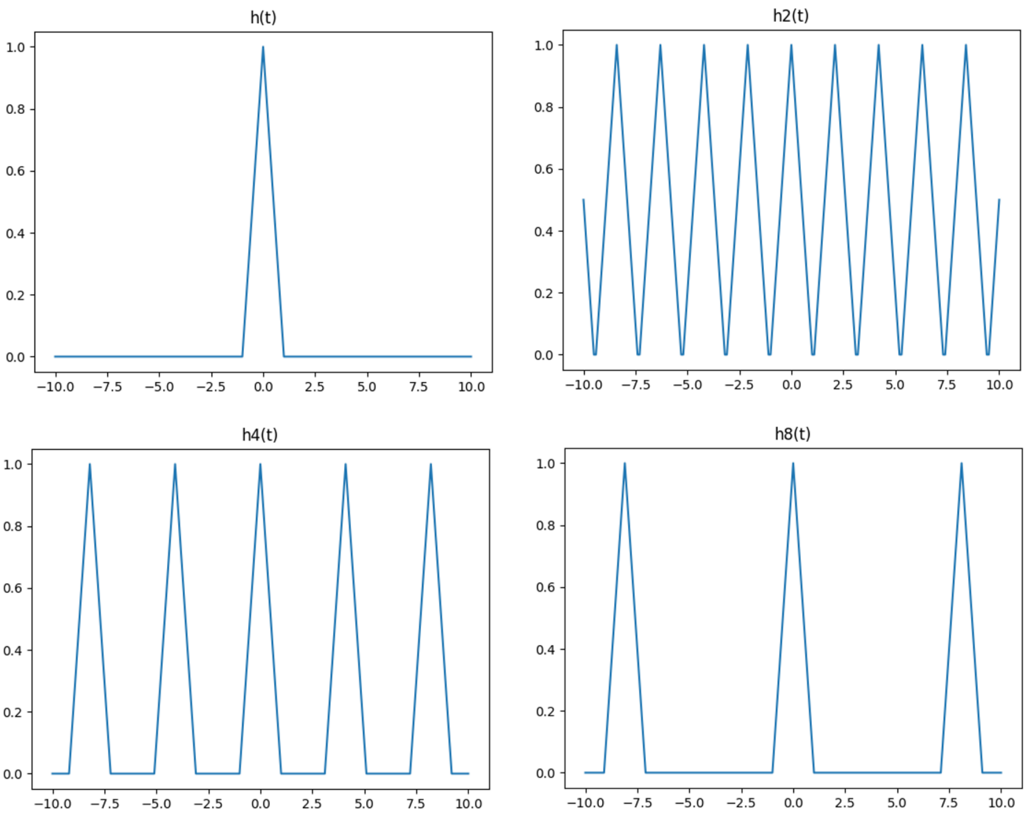
$$\hat{h}(\omega) = 2 \cdot \int_0^1 (1-t)\cos(t\,\omega)\,dt \Leftrightarrow$$

$$\Leftrightarrow -\underbrace{\int_0^1 t\cdot\cos(t\,\omega)\,dt}_{1} + \underbrace{\int_0^1 \cos(t\cdot\omega)\,dt}_{2}$$

$$\left.\begin{array}{l} 1 = \dfrac{1-\cos(\omega)}{\omega^2} - \dfrac{\sin(\omega)}{\omega} \\[2em] 2 = \dfrac{\sin(\omega)}{\omega} \end{array}\right\} \quad \frac{1-\cos(\omega)}{\omega^2}$$

$$\boxed{\hat{h}(\omega) = 2\cdot\left(\frac{1-\cos(\omega)}{\omega^2}\right)}$$

(b) $h(t), h_2(t), h_4(t)$ and $h_8(t)$

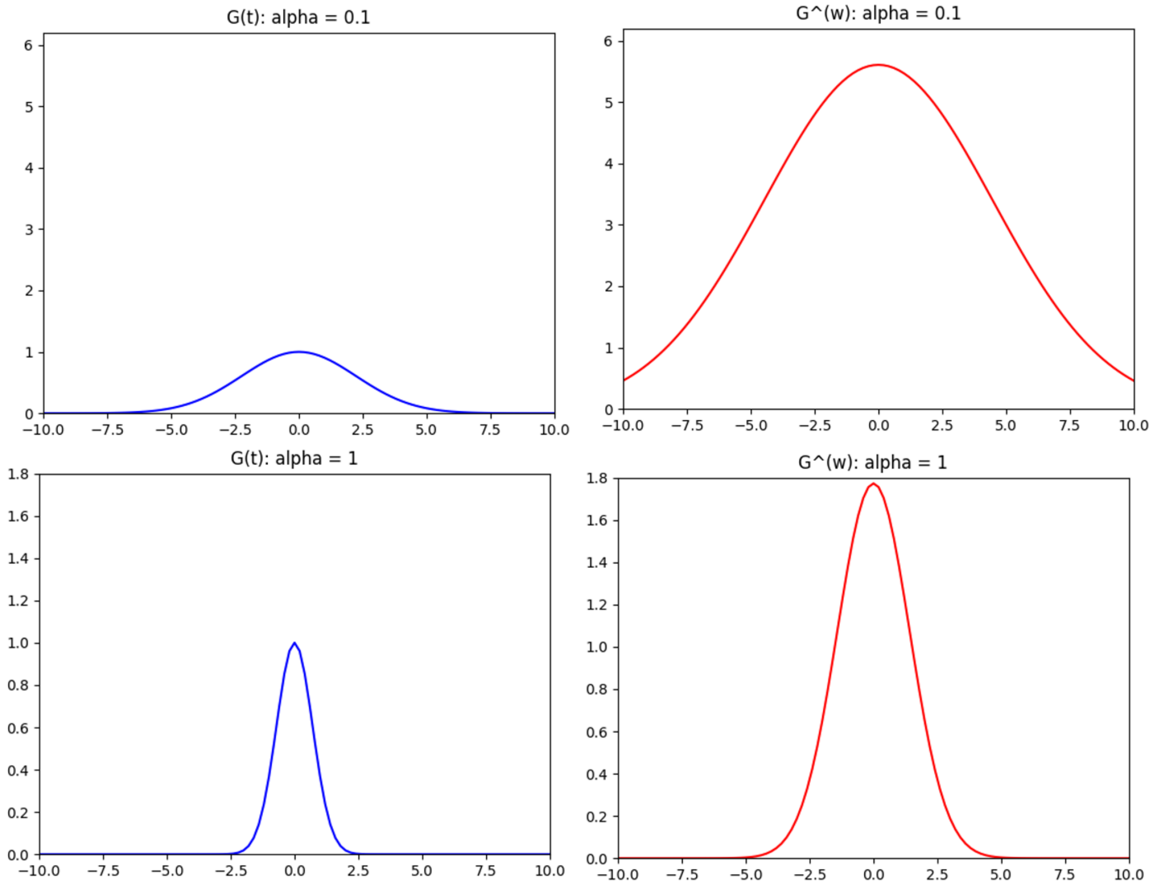


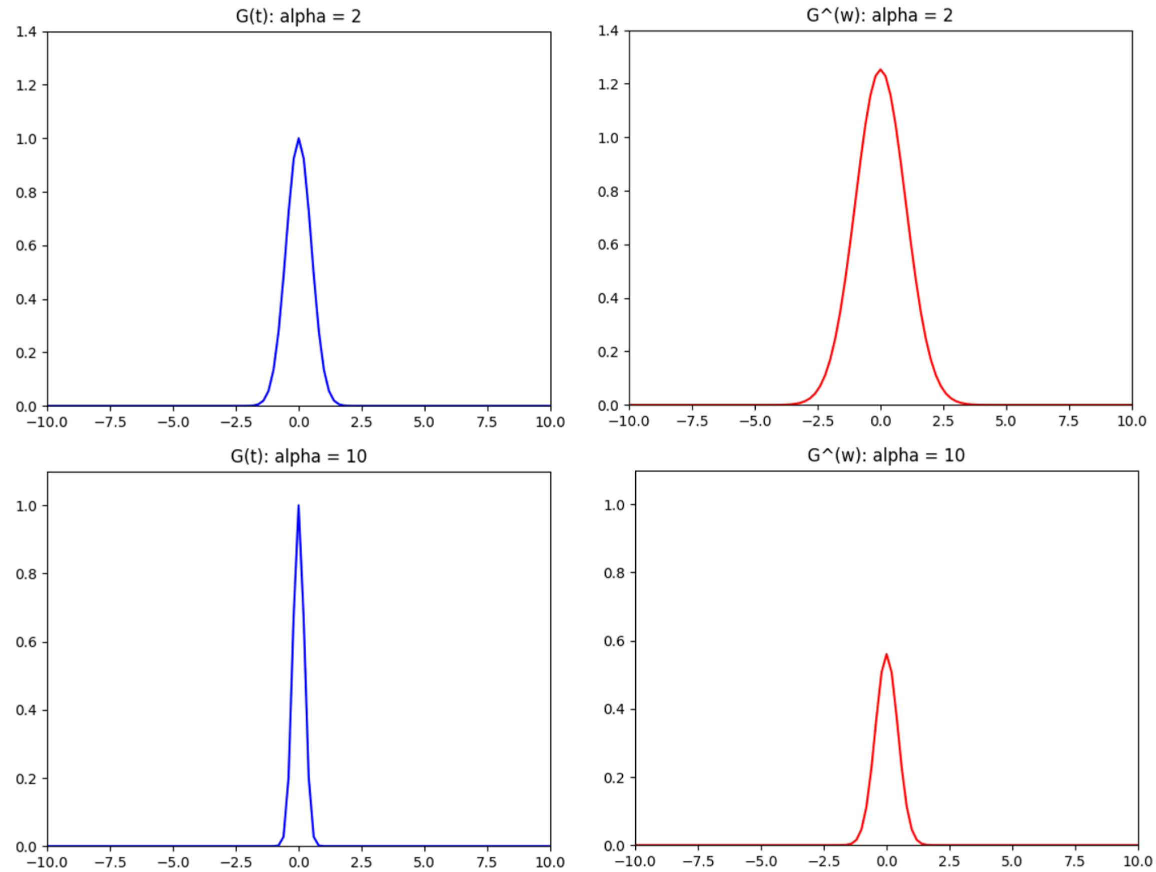
(c)

(d)

Exercise 3

(a) G(t) and G(w) for the different alpha values.
We can notice that G(w) decreases in magnitude, and becomes narrower as we increase alpha, this is due to the formula, the signal is is not multiplied by alpha, but rather divided by alpha, so as it increases the values become smaller, which means that alpha in this formula is inversely proportional.

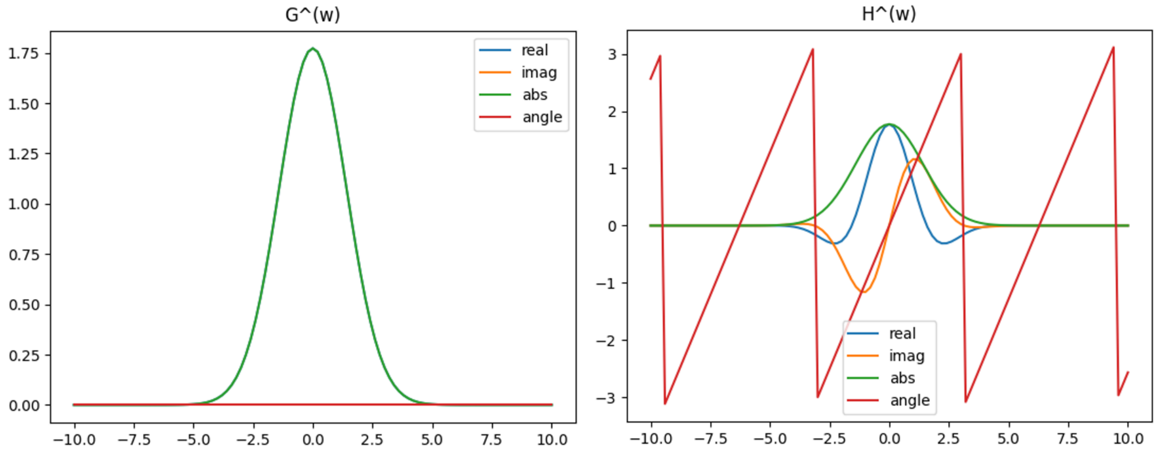

G(t): alpha = 0.1
G^(w): alpha = 0.1
G(t): alpha = 1
G^(w): alpha = 1


G(t): alpha = 2
G^(w): alpha = 2
G(t): alpha = 10
G^(w): alpha = 10

(b)

(c) in python file

(d) Time shift property : when we do a time shift on a signal, the resulting magnitude stays the same, only the phase is shifted, because the phase is shifted, we can clearly see in the graph the "consequences" of such a shift.



(e) Time reverse property : when we do a time reverse, we only change the sign in the imaginary part of the complex, so the real part and the magnitude don't change, however, we can notice that since we changed the sign, the imaginary part changed, and the angle had to change accordingly, its value is the same, only changing the sing as well.