UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES
Département d'informatique

Université de Genève

Département d'informatique

Imagerie Numérique 2D Discrete Fourier transform (**DFT**)

TP Class N° 11 April 30, 2021

Basic shapes

Exercise 1. *(1.5 point)*

(a) Create a two-dimensional image of size 256×256 as show in Figure 1 using the sinusoidal function with 10 periods. Display the image and its DFT magnitude. Explain the obtained result.

(b) Create a two-dimensional image by flipping the horizontal and vertical directions in the image from the task (a) and by dividing the frequency by a factor of 2 as shown in Figure 2. Display the image and its DFT magnitude. Compare with (a) and explain the obtained result.

(c) Sum up the images from (a) and (b) together. Display the DFT magnitude of the sum image and explain the obtained results.

(d) Using the image created in (c) that is the sum of two sinusoids, rotate this image by 25° and crop the central part of the image. Display the magnitude part of the DFT of the rotated image and explain the obtained result.

(e) Repeat the task (c), except multiply the two images instead of adding them. Explain the obtained result using the product theorem.

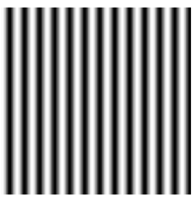

Figure 1: Vertical sin-wave

Figure 2: Horizontal sin-wave

Image Phase and Magnitude

Exercise 2. *(0.33 point)*

(a) Take any grayscale image. Display this image and its DFT *phase* and *magnitude*.

(b) Reconstruct the image via the inverse DFT twice:

  (a) using the image phase and constant magnitude equal to 1;
  (b) using the image magnitude and zero phase;

  What can you conclude from the two reconstructions?

Exercise 3. *(0.33 point)*

(a) Take any two grayscale images. Display these images and their DFT *phases* and *magnitudes.*

(b) Pair the magnitude of one image with the phase of the other and vice versa. Invert new pairs and display the results. What can you conclude from these reconstructions?

Exercise 4. *(0.33 point)*

(a) Take any grayscale image. Display this image and its DFT *phase* and *magnitude.*

(b) Mirror the image in the coordinate domain in

    (a) the horizontal direction;
    (b) the vertical direction;
    (c) the horizontal and vertical directions

    Can you predict what happens with the Fourier spectra of this image?

(c) Flip the phase in the Fourier domain by multiplying by -1. Display and explain the result in the direct domain.

Filtering in the Fourier Domain

Exercise 5. *(1 point + 0.5 extra point (sub-ex. 5.d))*

(a) Take any grayscale image. Display this image and its DFT *phase* and *magnitude.*

(b) Downsample this image by a factor of 2 in the coordinate domain without interpolation by keeping each second element of the image. Display the Fourier spectrum of the dowsampled image. Explain how it differs from the spectrum of the original image. What kind of effects do you observe?

(c) Upsample the original image by a factor of 2 in the coordinate domain without interpolation by adding zeroes in between each original pixels. Display the Fourier spectrum of the upsampled image. Explain how it differs from the spectrum of the original image. What kind of effects do you observe?

Optional sub-exercise:

(d) Estimate and explain what kind of filtering is needed in the Fourier domain to interpolate, i.e., fill the zero samples in the upsampled image, in the coordinate domain. Implement the needed filtering in the Fourier domain. Convert image back to the coordinate domain. Display the obtained result and compare the quality of new image to the image in (c). Explain the obtained result.

Multiplication and Convolution

Exercise 6. *(1 point)*

(a) Program a python function (by yourself) that implement the 2D cross-correlation between two images in the Fourier domain. Pay attention to the zero padding problem. Test your code on the images `img_001.jpg` and `img_002.jpg`. Display the obtained result and describe what you see on the resulting image.

(b) Implement the cross-correlation again but now in the coordinate domain using the Python function *scipy.signal.correlate2d*. Display the obtained result and compare it with the result in (a).

(c) Measure the time differences (Python `datetime`) between two approaches ((a) and (b)) to compute cross-correlation. Explain which approach is more efficient and why.

Image segmentation

Exercise 7. *(1.5 point)*

The goal of this part is to apply the DFT to the image shown in Figure 3 and to extract the elements that compose this image. Intuitively, it is easy to see that the image *img_003.png* can be decomposed into two periodical signals (vertical and horizontal orientations).

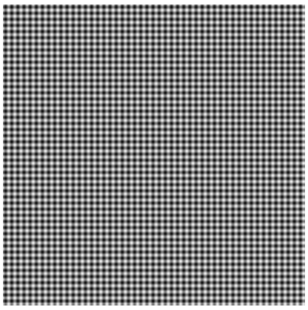

Figure 3: img_003.png.

(a) Display the original image and its DFT *phase* and *magnitude*.

(b) Decompose the image `img_003.png` into composed signals by the next steps:

- Perform image filtering in the Fourier domain by modifying the magnitude of the DFT as follows:
  - filter the magnitude of the DFT, e.g. put some components to 0 (that might be done by the element-wise multiplication with the corresponding matrix of 0 and 1);
  - calculate the inverse DFT from the modified magnitude and the original phase.
- It is important that the filters you use are symmetric otherwise there can be some odd artefacts.

(c) Display the obtained composed signals in the coordinate domain.

(d) Sum up the composed signals in the coordinate domain. Compare the result with the original image and explain the difference, if any.

Submission

Please archive your report and codes in "Name_Surname.zip" (replace "Name" and "Surname" with your real name), and upload to "`Assignments/TP11 2D Discrete Fourier transform`" on `https://moodle.unige.ch` before Thursday, May 20 2021, 23:59 PM. Note, **the assessment is based not only on your code, but also on your report, which should include your answers to all questions and the experimental results**.