Université de Genève

Imagerie Numérique

13X004

TP 8: FS and FT

Author: Joao Filipe Costa da Quinta

E-mail: Joao.Costa@etu.unige.ch

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UNIVERSITÉ DE GENÈVE

FACULTÉ DES SCIENCES

Département d'informatique

Exercise 1

(a) To compute a_k we use the following formula seen in class:

$$\frac{2}{T}\int_{-T/2}^{T/2} \tilde{f}(t)\cos(k w_0 t)\, dt \Leftrightarrow$$

$$\Leftrightarrow \frac{2}{T}\int_{-1}^{3} \tilde{f}(t)\cos(k w_0 t)\, dt \Leftrightarrow$$

$$\Leftrightarrow \frac{2}{T}\left(\int_{-1}^{1} \tilde{f}(t)\cos(k w_0 t)\, dt + \int_{1}^{3} \tilde{f}(t)\cos(k w_0 t)\, dt\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{2}\left(\frac{\sin(k\frac{\pi}{2}\cdot 1) - \sin(-k\frac{\pi}{2}1) - \sin(k\frac{\pi}{2}\cdot 3) + \sin(k\cdot\frac{\pi}{2}1)}{k\pi/2}\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{\sin(k\frac{\pi}{2}\cdot 1) - \sin(-k\frac{\pi}{2}1) - \sin(k\frac{\pi}{2}\cdot 3) + \sin(k\cdot\frac{\pi}{2}1)}{k\pi} \Leftrightarrow$$

$$\Leftrightarrow \frac{4}{k\pi}\cdot\left(\sin\left(k\frac{\pi}{2}\right)\right) \rightarrow$$

① if k is even:

$\sin\left(k\frac{\pi}{2}\right) = 0$

② if k is odd:

its either -1 or 1

$\rightarrow (-1)^{(k-1)/2}\cdot\frac{4}{k\pi}$

(b) To compute b_k we use the following formula seen in class, I skipped some steps, but its the same idea as exercise (a) :

$$b_k = \frac{2}{T}\int_{-T/2}^{T/2} \tilde{f}(t) \sin(k w_0 t)\, dt$$

(skip steps, but same idea as (a))

$$= \frac{\underbrace{\cos\left(k\frac{\pi}{2}1\right) - \cos\left(-k\frac{\pi}{2}1\right)}_{① = 0} - \underbrace{\cos\left(k\frac{\pi}{2}\cdot 3\right) + \cos\left(k\frac{\pi}{2}1\right)}_{② = 0}}{k\pi}$$

① since: $\cos\left(k\frac{\pi}{2}\right) = \cos\left(-k\frac{\pi}{2}\right)$

② and: $\cos\left(k\frac{\pi}{2}\right) = \cos\left(k\frac{\pi}{2}\cdot 3\right)$

to compute F_k we can use the formula seen in class

$$F_k = \frac{1}{2}(a_k - j b_k)$$

$$\rightarrow \tilde{f}(t) = \sum_k F_k e^{jk\cdot\frac{\pi}{2}\cdot t} \qquad (3)$$

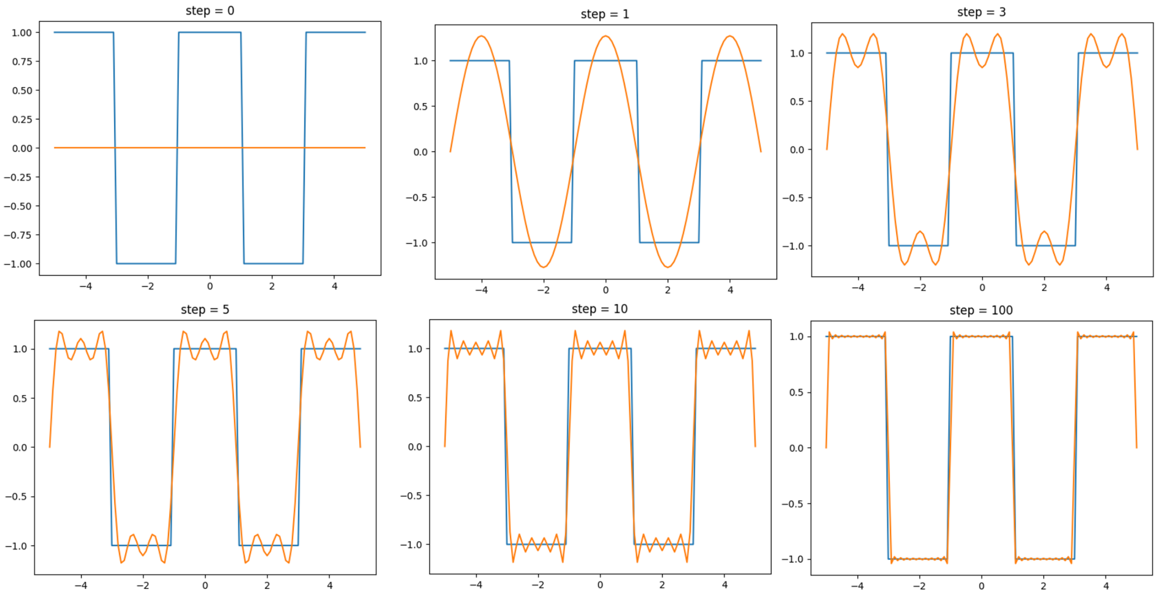
$$\rightarrow \tilde{f}(t) = \frac{1}{2}a_0 + \sum_k A_k \sin\left(k\frac{\pi}{2}t + \phi_k\right) \qquad (2)$$

$$A_k = \sqrt{a_k^2 + b_k^2}$$

$$\phi_k = \tan^{-1}\left(\frac{a_k}{b_k}\right)$$

for every possible k, we end up with 0 in the numerator, so bk = 0 for every k.

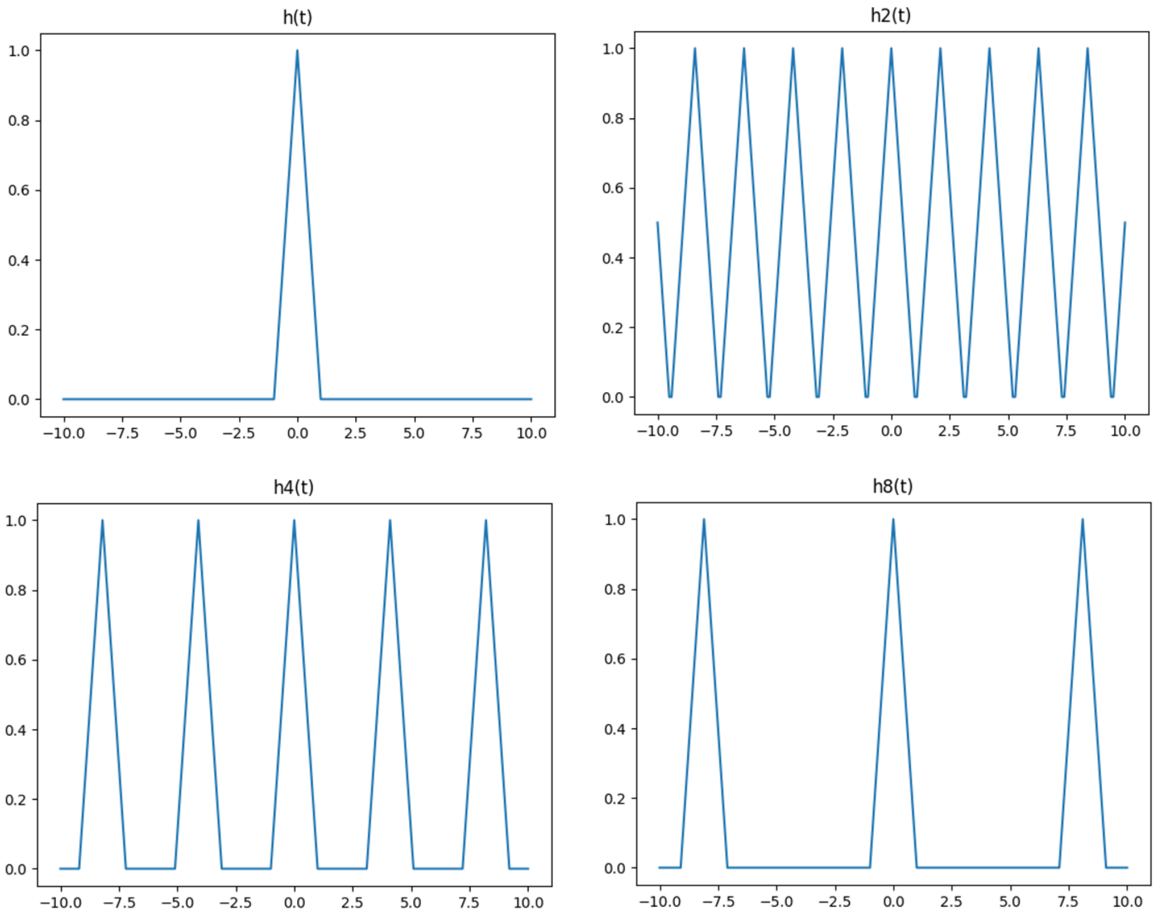
(c) Code in python file



(d) The Gibbs phenomenon says that it is hard to approximate a discontinuous function by adding a series of sine and cosine waves. Which is what we are doing with FS, we can see in our point (c), with step = 10, the edges of the square are of higher amplitude, this is the called overshooting, and as we increase the step, it doesn't fix this overshooting, as we can see with step = 100, it is still visible.

Exercise 2

(a)



(b)