12- Convergence of simulated annealing: how to formulate it, how to compute it.

➔ Does simulated annealing find the global optimum and under which conditions?

Simulated annealing can be analysed mathematically, it converges in probability.

It gives a solution arbitrary close to the global optimum with a probability arbitrary close to 1

➔ The conditions are:
- Movements are inversible
- Any point in the search space can be reached from any other point in a finite number of movements
- The initial temperature must be large enough (do exploitation before exploration)
- The temperature should not decrease too fast, at iteration t:

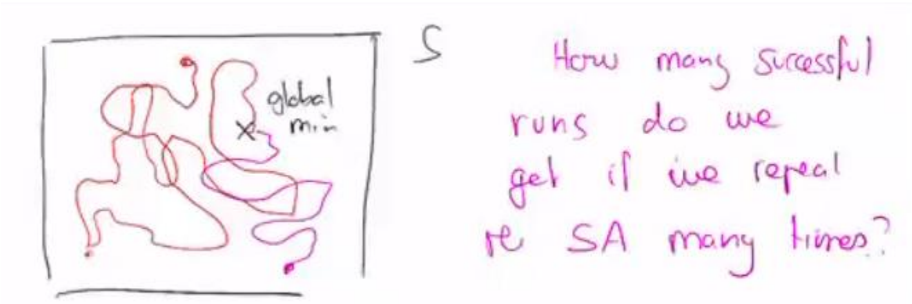
$$T(t) \sim \frac{C}{\log t} \quad \text{for } t \gg 1$$

C is an unknown constant that reflects the variation of E in S.

!!! If temperature is decreased too fast, we risk not converging (quenching) !!!

➔ In practice, log t is too slow. So, temperature will decrease faster

How to compute it:



If we get 100% of success there is convergence, in practice it may end up sub optimal

We want to compute the probability $P(t, i)$ that the SA is at point $i \in S$ at iteration t

$$P(t+1, j) = \sum_i P(t, i)\, W_{ij}(t)$$

W_{ij} is the transition probability from i to j. Since we have 100 possible values of i an j, W_{ij} is a

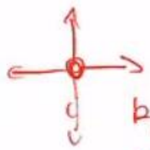
$$\begin{array}{ccl} 100 & \times 100 & \text{matrix.} \\ \uparrow & \uparrow & \\ |S| & |S| & \end{array}$$

For SA, we have:

$$\boxed{W_{ij}} = \begin{cases} 0 & \text{if } j \text{ is not a neighbor of } i \\ \dfrac{1}{k_{out}(i)} \times P_{metropolis}(E_i, E_j, T(t)) & \end{cases}$$

$k_{out}(i)$: number of neighbors of i

$P_{metropolis}(E_i, E_j, T(t))$: $\min\left(e^{-(E_j - E_i)/T(t)}\right)$



$k_{out}(i) = 4$ in our example.

We also add the following

$$W_{ii} = 1 - \sum_{j \text{ neighbour of } i} W_{ij}$$