20- The takeover time in genetic algorithms: goal, definition, value for tournament selection.

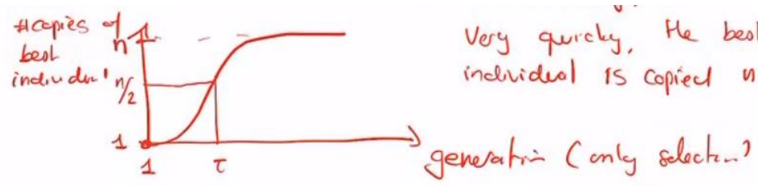
Quantify how fast a selection mechanism eliminates all but the best individuals

Let's define τ the takeover time as the number of selection rounds (no crossover and mutation), to have only the best individuals

It measures the intensity of selection:

- τ Is small -> strong selection
- τ Is large -> weak selection

"large" and "small" is proportional to size of population n -> we will show that $\tau = O(\log n)$



Very quickly, the best individual is copied n times

➔ Selection is often exponential

Theory:

➔ The number o copies m(t) of the best individual at generation t goes as:

$$\frac{dm}{dt} = \underbrace{\alpha m}_{\text{prop. to the current number}} \left(1 - \frac{m}{n}\right) \leftarrow \text{limitation due to a finite population of size } n$$

This solution for tournament selection:

At each selection we have a probability of choosing the best individual:

$$\underbrace{\left(1 - \frac{m}{n}\right)}_{\text{prob not to take the best}}{}^{k} \leftarrow \text{repeated } k \text{ times}$$

$\uparrow$ prob to take the best

As well as the probability of choosing the individual at least once:

$$\left[1 - \left(1 - \frac{m}{n}\right)^k\right]$$

$$m(t+1) = n\left(1 - \left(1 - \frac{m}{n}\right)^k\right)$$
$$\geq n\left(1 - \left(1 - \frac{m}{n}\right)^2\right)$$
$$= n\left(1 - \left(1 - \frac{2m}{n} + \frac{m^2}{n^2}\right)\right)$$
$$= n\left(\frac{2m}{n} - \frac{m^2}{n^2}\right)$$
$$= m\left(2 - \frac{m}{n}\right)$$

with $t = \underbrace{\ln(n-1)}$ $\Rightarrow$ $m(t) = \frac{n}{2}$

To have half of the population filled with the best individual, one has to perform $O(\log n)$ selections.