## 24- Evolution Strategy: individual and population versions.

Single individual metaheuristic $x \in \mathbb{R}^d$ evolving by mutation only

Population version also including crossover

Metaheuristic for continuous optimization $S \subseteq \mathbb{R}^d$

**(1+1) ALGORITHM -> 1 parent and 1 child**

$$x' = x(t) + N(0, \sigma)$$

$$x(t+1) = \begin{cases} x' & \to f(x') > f(x) \\ x & \to f(x') < f(x) \end{cases}$$

- The child is produced by a gaussian mutation of the parent:
- X' is the child, x(t) the parent, and N(0,$\sigma$) is the mutation
- The mutation follows a normal distribution
- The child is only accepted if its fitness is better than the parents

- ➔ The $x \in \mathbb{R}^N$ which means $\sigma$ is also a vector
- ➔ $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_d)$
- ➔ Each dimension can have a different $\sigma$
- ➔ (1+1)-ES is much like a random walk search with hill climbing strategy
- ➔ If we are accepting too many children, we increase $\sigma$, to make larger jumps
- ➔ If we accept too few children, we decrease $\sigma$
- ➔ We should want to be around 1/5 acceptance rate

**POPULATION ALGORITHM**

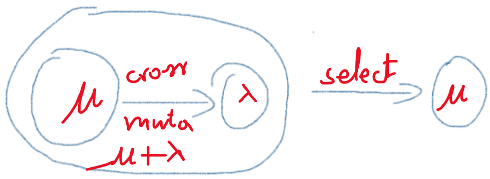
The size of the population (nb of solutions) is denoted $\mu$

Each individual is represented as $(x^i, \sigma^i)$ $i = 1, \ldots, \mu$

$x^i \in S$, $\sigma^i$ Is the associated mutation parameter

There are 2 variants of the population approach:

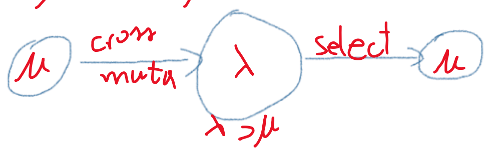
① ($\mu + \lambda$)-ES: $\lambda$ children generated from $\mu$ parents




② ($\mu, \lambda$)-ES: in this case $\lambda > \mu$ children are generated from the $\mu$ parents, $\mu$ are selected

This last selection is deterministic, we choose the $\mu$

best individuals




**CHILDREN GENERATION**

MUTATION

- ➔ Choose 2 parents among the $\mu$ possible parents
- ➔ Apply mutation to that child
- ➔ Repeat $\lambda$ times to generate $\lambda$ children

$$\begin{cases} x' = x + N(0, \sigma) \\ \sigma' = \sigma + e^{N(0, \Delta\sigma)} \end{cases}$$

$\Delta\sigma = 1/\sqrt{d}$

Crossover

The crossover is performed both on x and $\sigma$

- The uniform crossover is the following

$$(x_\ell^{child}, \sigma_\ell^{child}) = \left(x_\ell^{\begin{cases} parent_1 & prob\ 1/2 \\ parent_2 & prob\ 1/2 \end{cases}}, \sigma_\ell^{\begin{cases} parent_1 & prob\ 1/2 \\ parent_2 & prob\ 1/2 \end{cases}}\right)$$

or one can also use the arithmetic crossover. Let us consider two parents, i and j

$$(x^{child}, \sigma^{child}) = \frac{1}{2}(x^i, \sigma^i) + \frac{1}{2}(x^j, \sigma^i)$$