

## 9 - Quadratic Assignment Problems

Combinatorial optimization problem

N objects and n possible locations -> must find the best permutation

- ➔ We have n objects and n locations
- ➔ We know the distance  $d_{rs}$  between locations r and s
- ➔ We are given flows  $f_{ij}$  between all pairs ij of objects

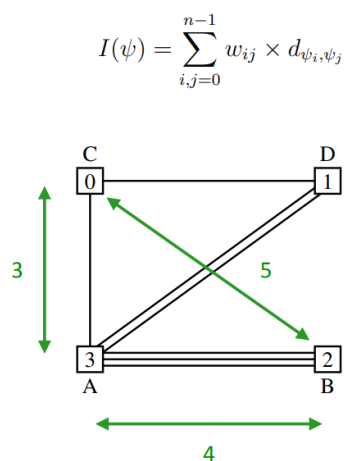
➔ GOAL: find the optimal location for each object -> minimize distance flow -> sum of products

$$S = [11, 9, 6, 5, 0, 1, 10, 8, 3, 4, 2, 4]$$

$$\begin{array}{l} \tau(0,1) \rightarrow [9, 11, 6, 5, 0, 1, 10, 8, 3, 4, 2, 4] \\ \tau(0,2) \rightarrow [6, 9, 11, 5, 0, 1, 10, 8, 3, 4, 2, 4] \\ \vdots \end{array}$$

$$f = \sum_{i,j} f_{ij} d_{r_i r_j}$$

- Example: Find the best location (A, B, C, D) for each facility (0, 1, 2, 3) in order to minimize



$$I(\psi) = \sum_{i,j=0}^{n-1} w_{ij} \times d_{\psi_i, \psi_j}$$

distances

$$\begin{aligned} d_{AB} &= d_{CD} = 4 \\ d_{AC} &= d_{BD} = 3 \\ d_{AD} &= d_{BC} = 5 \end{aligned}$$

flows

$$\begin{aligned} w_{13} &= 2 \\ w_{01} &= w_{03} = 1 \\ w_{23} &= 3 \end{aligned}$$

Fitness  $I(\psi) = w_{01} \times d_{\psi_0 \psi_1} + w_{03} \times d_{\psi_0 \psi_3} + w_{13} \times d_{\psi_1 \psi_3} + w_{23} \times d_{\psi_2 \psi_3}$

Here  $\psi = (C, D, B, A)$

Hence

$$I(\psi) = d_{CD} + d_{AC} + 2d_{AD} + 3d_{AB} = 29$$