26- Phase transition in optimization problems: problem description and properties.

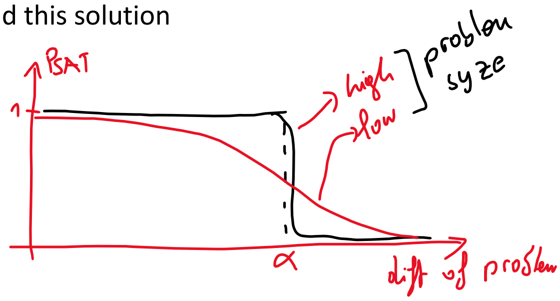
We will analyse statistically a metaheuristic for solving problems of increasing difficulty

We will consider a sub-class of satisfaction problems

We will analyse this problem analytically, we will define the probability that a random instance of the problem has a solution and whether the metaheuristic will find this solution

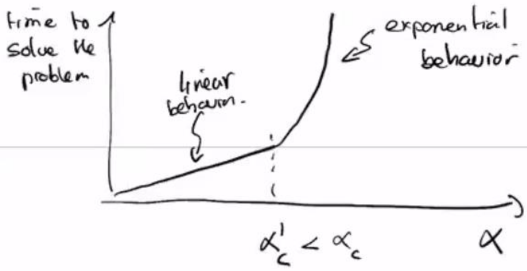
The specificity of the problems we will consider is that:

- if the difficulty parameter is low, the probability that a random instance has a solution is one.
- If this parameter increases up to the critical value the probability drops abruptly to zero.



This abrupt behaviour is called phase transition

We will also see that the metaheuristics used to find solution has another phase transition in the CPU time needed to solve the problem:



SAT Problem has N Boolean variables and M Boolean equations

We want to find an assignment of these N variables that satisfy all M equations

The goal is to find an assignment of these N variables that satisfy all M equations, if it exists, we say the problem is satisfiable

These problems can be turned into an optimization problem, the goal is to minimize the energy E, which is the number of unsatisfied equations

➔ Example with only XOR operation

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_2 + x_4 = 0 \\ x_1 + x_4 = 1 \end{cases}$$

$N = 4$ (4 variables $x_1\ x_2\ x_3$ and x_4)

$M = 3$ (3 equation)

$$(x_1\ x_2\ x_3\ x_4) = \begin{cases} (1\ 0\ 0\ 0) \\ \cancel{(0\ 1\ 1\ 0)} \end{cases}$$

➔ This problem has a solution, so the minimal energy is 0, if a problem can't be satisfied, then its energy optimal is 1