

6 - NK-problems: motivation, definition, goal

Inspired by biological problems -> genetic regulatory networks

NK problems belong to np-hard class !

Lets give an example of nk landscape problems:

- We consider N persons or agents, labelled with an index $i, i = 1, \dots, N$
- Each agent i acts according to two possible strategies, denoted $x_i = 0$ or $x_i = 1$
- The success of an agent depends on the strategy it chooses and the type of relation it has (competition or collaboration) with the other persons it interacts with
- If we assume that each agent i depends on K other agents, we may define a function $f_i(x_i, \dots, x_{i+K})$ which gives the profit resulting from the chosen strategy and that of the connected agents

The problem is specified once the f_i are given, as well as K, N

We want to maximize

$$F = \sum_{i=1}^N f_i \quad \begin{array}{l} \text{the profit of} \\ \text{the population.} \end{array}$$

Our main interest with NK-problems is to be able to generate synthetic problems of increasing difficulty. (as N and K increase, it become more and more difficult to solve)

Example MaxOne problem.

Find a chain of N bits that maximize the number of ones.

Obvious solution: $X = (x_1 x_2 \dots x_N) = \underbrace{111 \dots 1}_{N \text{ times.}}$

$$f_i(x_i, \dots) = x_i \quad K=1$$

$$F = \sum_{i=1}^N f_i = \sum_{i=1}^N x_i$$

$$F = N$$

Another example

$$x = \underline{x_1 x_2} \dots x_N \quad f_i(x_{i-1}, x_i, x_{i+1}) \quad K=3$$

f_i is max for 111 then it is a trivial solent
But if f_i is large for 101 and small for 010
then the chain $\overline{101010101}$