14- Ant System: description of the algorithm for the TSP problem

Algorithm 1

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for all t = 1, ..., t_max do
    for all ant k = 1, ..., m do
        choose a city at random
        while there exists a city not visited do
            choose a city j according to (1)
        end while
        mark a path according to (3)
    end for
    update all paths according to (2)
    Keep the best of solutions obtained at last iteration
end for
```

(1)

$$p_{ij}^k(t) = \begin{cases} \frac{(\tau_{ij}(t))^\alpha (\eta_{ij})^\beta}{\sum_{l \in J} (\tau_{il}(t))^\alpha (\eta_{il})^\beta} & \text{if } j \in J \\ 0 & \text{otherwise} \end{cases}$$

First off, J is a set of cities not yet visited in the ant's path, so only cities unvisited can be the next city chosen.

Each unvisited city has a probability of being chosen, this probability is a function of:

(Pheromones in a path to a city * visibility of the city)/

(Sum of all pheromones*visibility for all cities unvisited)

(2)

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}^k(t)$$

This function allows us to update the pheromones in each path at each iteration, there is an evaporation rate ρ, so each path has some of its pheromones that evaporates + an integer that is a function of how many ants took this path -> this value is given by formula (3)

(3)

$$\Delta\tau_{ij}^k(t) = \begin{cases} \frac{Q}{L^k(t)} & \text{if ant } k \text{ used edge } (i,j) \text{ in its tour} \\ 0 & \text{otherwise} \end{cases}$$

Q is a constant, Lk is the length of the path taken by the ant -> the longer the path -> the smaller is the result -> less pheromones in this path during the next iteration

Choice of Parameters

- $\alpha = 1$, $\beta = 5$, $\rho = 0.1$
- $Q = L_{nn}$, $\tau_0 = \frac{1}{L_{nn}}$
- m , t_{max}