

3 - The space of permutations

S is the set of permutation of n objects

$$n=3 \quad S = \{(1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1)\}$$

$$|S| = n! = 6$$

What kind of transformation can we define on a permutation

The simple way is to use transposition or swap of two entries in the permutation.

$T_i = (i \ j)$ which will swap entry i with entry j

$$\text{For instance: } T_{(2,3)}((1,2,3)) = (1,3,2)$$

How many transposition do we have?

$$n=5$$

$$T_i \in \{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\}$$

How many are there? $O(n^2)$



The neighborhood is of size n^2
whereas the search space is of size $n!$

Can we reach any point in S starting from any initial condition, using a finite number of these transformations?

Answer: Yes see math:

It means that the entire space of permutations is accessible