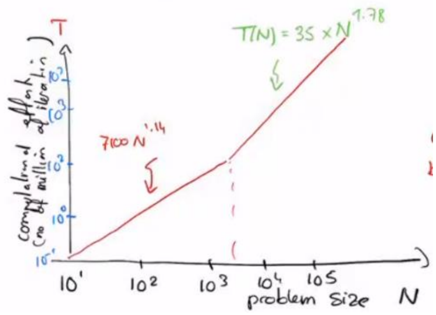
25- Performance of metaheuristics: examples, specificities of the performance evaluation, metrics, approach, "No Free Lunch" theorem

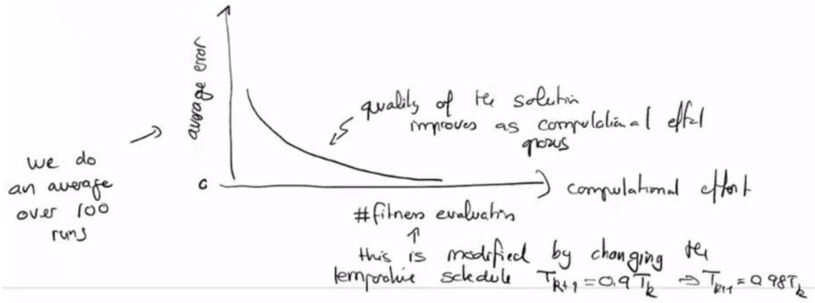
- ➔ No guarantee of quality of solution
- ➔ They are stochastic, so the behaviour is always different
- ➔ To determine the success of an algorithm in a problem, we must use statistics

EXAMPLE:

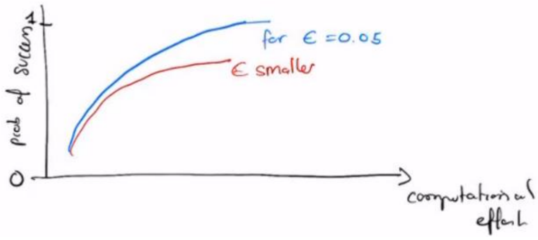
- ➔ Let's consider SA for TSP, we have n towns randomly placed in a 2x2 spatial region
- ➔ We will generate many such problems, varying N
- ➔ Our question is: how long does it take does it take for SA to give an answer? (This does not mean that the answer is the global optimum, but when SA stops)
- ➔ Performance speed:
  - With $N \in [20,2000]$, the difficulty is linear $O(N^{1.14})$
  - With $N \in [5000,50000]$, difficulty is $O(N^{1.78})$ which is smaller than quadratic $< O(N^2)$
  - It is also much better than exhaustive search $O(N!)$



- ➔ Performance quality (1):
  - To compute the quality of a solution one must know the best solution for a given problem
  - We will place 50 cities in a circle and see how SA finds an accurate solution



- ➔ Performance quality (2):
  - We can also compute the probability of success, out of 100 runs, how many gave us the global optimum? (accuracy of ϵ)



The metrics used to evaluate a metaheuristic are:

- The complexity in time to get a solution
- Average error as a function of computational effort
- Probability of success
- Statistics are needed, between 100 and 1000 runs

Question: if metaheuristics A is better than B on a given problem, can we conclude that A is always better?

Let us consider a finite search space S of size $|S|$

we consider fitness function

$f : S \to Y$ where Y is a finite subset of $\mathbb{R}$

All possible problems are then specified by a given f sampled from a number $|Y|^{|S|}$ of possibility.

One considers a computational effort m, meaning that we consider a trajectory of exploration of m points] - m iterations

$(x_i, f(x_i)) \quad i = 1, \dots m$

Formulation of NFL:

- A metaheuristics cannot be better than another one on all possible problems.
- For any performance metric, no algorithm will be better if all discrete fitness function are considered
- If A is better than B on a given class of problems, there is another class where B will be better.

NFL Theorem

Let $P(d_m | f, m, A)$ the probability that trajectory $d_m = \{(x_1, f(x_1)), \dots, (x_m, f(x_m))\}$ generated by metaheuristics A contains the optimal value of f.

$$\sum_f P(d_m | f, m, \underline{A}) = \sum_f P(d_m | f, m, \underline{B})$$

Thus, on average all metaheuristics behaves the same when compared on all possible problems.

But in practice, not all possible problems have the same probability and some are highly pathological and not realistics.