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Analyse et traitement de l'information

TP5 : Probabilities and communications

Exercice 1 : Chain rules for probabilities

The idea of the chain rules is that to compute a given probability we check the dependencies (ie an arrow (direct or not) from X to Y means that Y depends of X).

a.

p(X, Y, Z) = p(X) * p(Y|X) * p(Z|X,Y) (by applying the rule)

p(X, Z) = p(X) * p(Z|X)

p(X, Y) = p(X) * p(Y|X)

p(Y, Z) = p(Y) * p(Z|Y)

b.

P(X1 ,X2,…, X7) = p(X1)p(X2)p(X3|X1, X2)p(X4)p(X5)p(X6|X3, X4, X5)p(X7|X6)

P(X1, X3, X5, X7) = p(X1)p(X3|X1)p(X5)p(X7|X3, X5)

P(X2, X4, X6, X7) = p(X2)p(X4)p(X6|X2, X4)p(X7|X6)

P(X3, X6, X7) = p(X3)p(X6|X3)p(X7|X6)

P(X1, X2, X4, X5) = p(X1)p(X2)p(X4)p(X5)

c.

P(X1, X2,…, X7)= p(X1)p(X2)p(X3|X1, X2)p(X4)p(X5|X2)p(X6|X3, X4, X5)p(X7|X6, X4)

P(X1, X3, X5, X7)= p(X1)p(X3|X1)p(X5)p(X7|X3, X5)

P(X2, X4, X6, X7)= p(X2)p(X4)p(X6|X2, X4)p(X7|X6, X4)

P(X3, X6, X7)= p(X3)p(X6|X3)p(X7|X6)

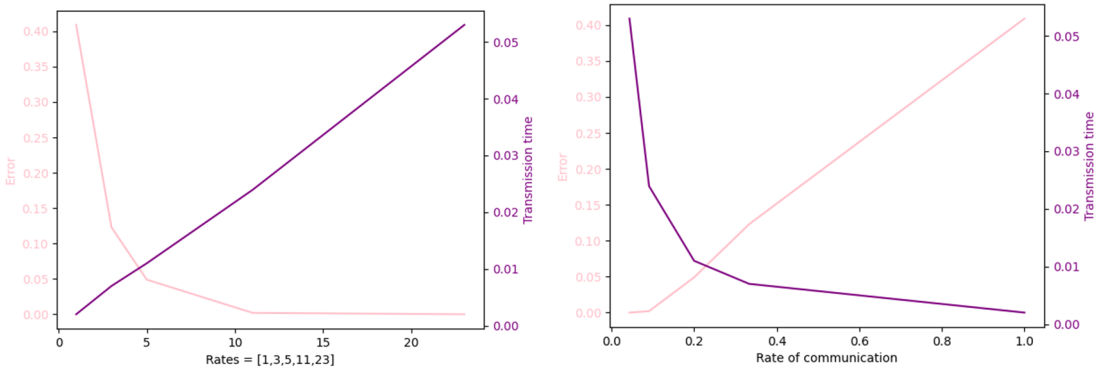
P(X1, X2, X4, X5)= (X1)p(X2)p(X4)p(X5|X2)

Exercice 2 : Communication through noisy channels

a. Main part of the code:

```
def startCommunication(self):

    start = time.time()
    # Add the redondancy, for example if msg is [0110] and rate=3 => [000111111000]
    preprocessed_message = np.repeat(self.input_message, self.rate)
    output_bsc = self.bsc(preprocessed_message)

    # Decoding the output message
    output_message = np.ndarray.astype(np.sum(output_bsc.reshape((-1, self.rate)), 1) > rate / 2, 'int')
    transmission_time = time.time() - start


    # Convert bits to words for the input and the output message
    input_words = np.dot(self.input_message.reshape((-1, self.M)), 2 ** np.arange(self.M)[::-1])
    output_words = np.dot(output_message.reshape((-1, self.M)), 2 ** np.arange(self.M)[::-1])

    # Fraction of Error
    nbrError = np.sum(input_words != output_words) / len(input_words)

    return nbrError, transmission_time, self.communicationRate()
```

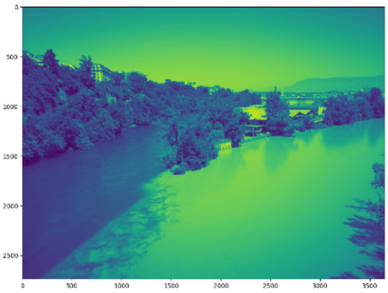
b. Results:



	Rates = [1,3,5,11,23]	Error	Transmission time	Rate of communication
0	1	0.409	0.002021	1.000000
1	3	0.123	0.007001	0.333333
2	5	0.049	0.011000	0.200000
3	11	0.002	0.023942	0.090909
4	23	0.000	0.052997	0.043478

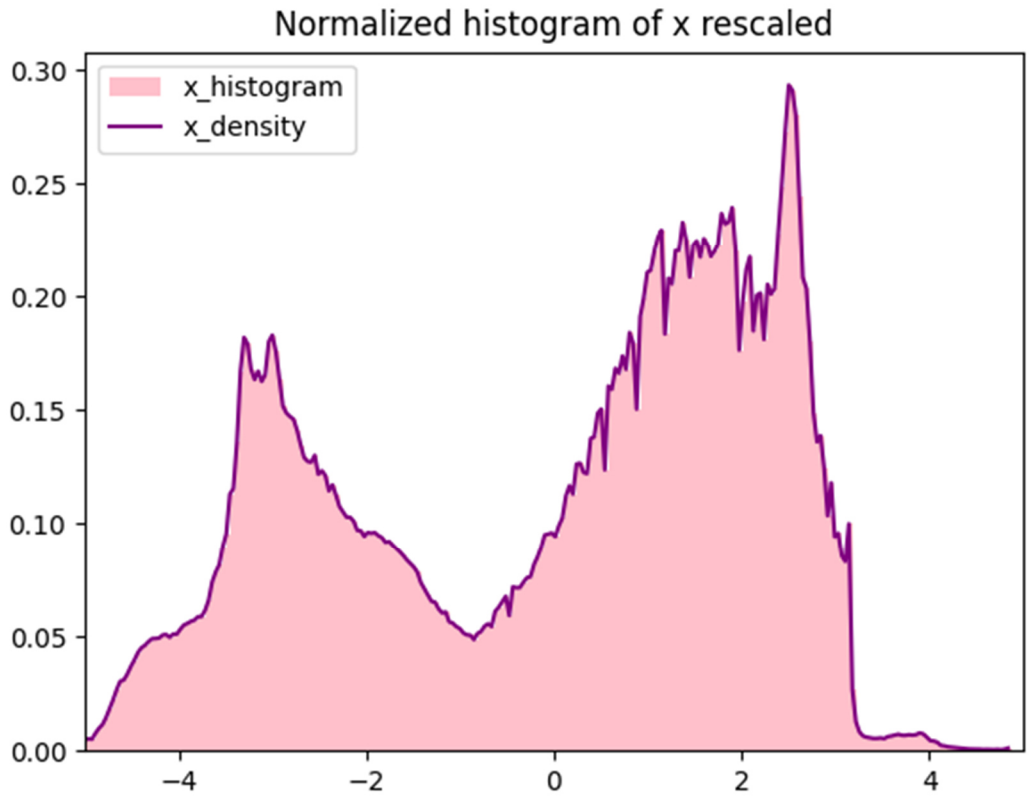
c. Interpretations:

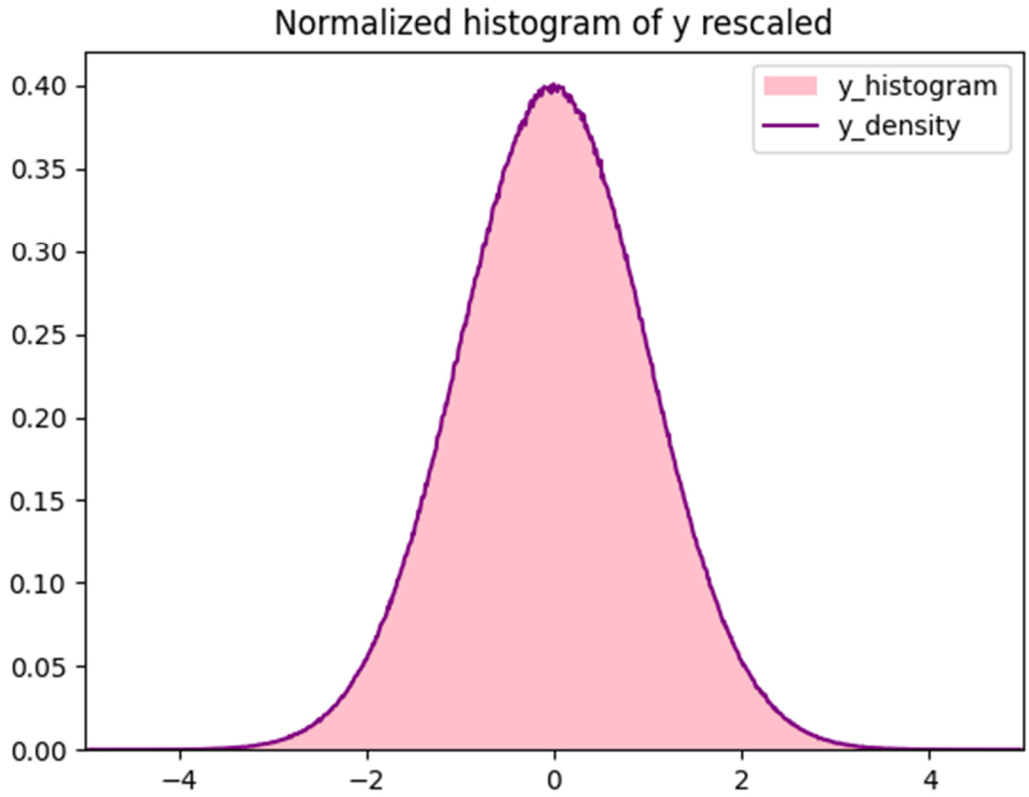
- We can see that as the rate increase, the error decrease till reaching an error of 0, the rate of communication also decreases.
- On the other hand, the transmission time increase which is logical because the length of the message being sent get bigger.
- The error decreases because the probability of each bit being wrong get lower because there are more bits.
- The communication rate decreases because we send more bits for each message with the length of the alphabet being fixed. (Redundant bits)



Exercice 3 : Moments

2. The normalized histograms of the data:



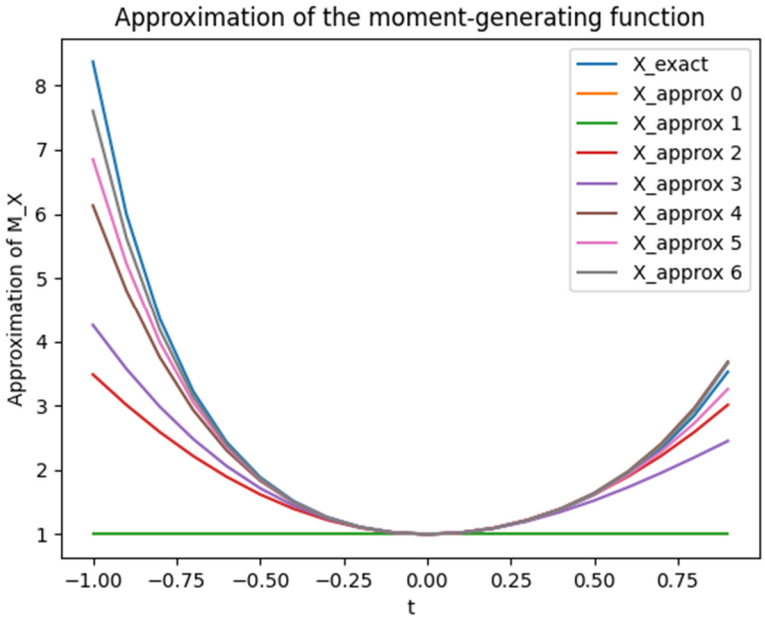


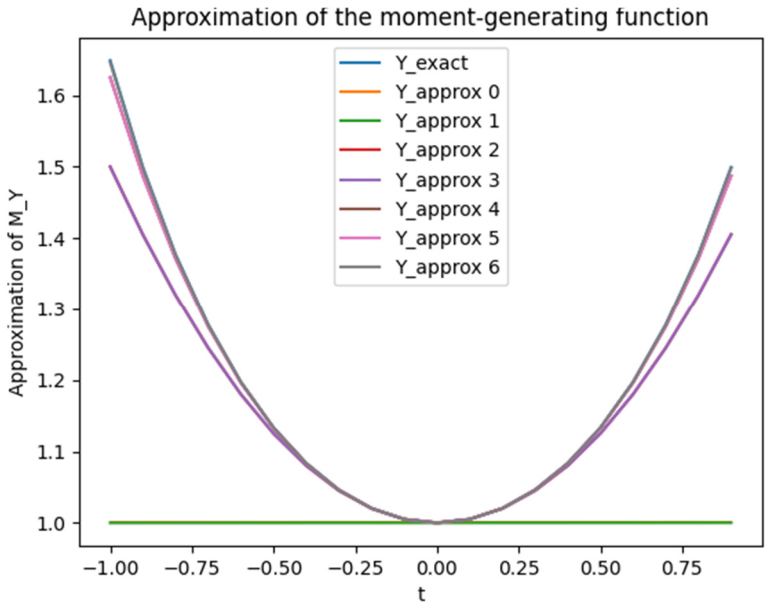
Since Y values were randomly sampled from the gaussian distribution of mean 0 and variance 5, the plot looks like a gaussian centered at 0 having a variance of 5.

4. The 7 first moments of the centered distribution X and Y.

```
Moments of X :  [1.0, 5.484435677284444e-15, 4.969513782353428, -4.623405633388906, 45.058973938097395, -86.27514065057433, 542.9180664370957]
Moments of Y :  [1.0, 0.00011921914798030496, 1.0001850805456336, 0.0005921135005593106, 2.9990887216169533, -0.002201655712986162, 14.978629542287985]
```

5. For each N = [0,1,2,3,4,5,6] here are the plots of the moment-generating function between [-1,1]. We can see in the following pictures that, for the gaussian distribution Y, the moments of order 1 and 2 are sufficient to capture the distribution, but for a distribution more “real” like X we need much more moments to capture the distribution. In particular, we see that even with the 7 first moments we do not obtain an approximation as good as the 2 moments for the gaussian.


Approximation of the moment-generating function
X_exact
X_approx 0
X_approx 1
X_approx 2
X_approx 3
X_approx 4
X_approx 5
X_approx 6
Approximation of M_X
t


Approximation of the moment-generating function
Y_exact
Y_approx 0
Y_approx 1
Y_approx 2
Y_approx 3
Y_approx 4
Y_approx 5
Y_approx 6
Approximation of M_Y
t