[14X030] Introduction to Computational Finance

Exercise series 7

April 18, 2023

General instructions

Each student is expected to upload on Moodle a **zip** file containing:

- A report in **pdf** format to answer exercise questions.
- The code used to generate the results of the report.

Deadline

Upload on Moodle due by : **April 24, 2023 at 11:59 pm**.

Optimal portfolios

In this series we look at the closing prices of Macdonald's, Bank of America, IBM, Chevron, Coca-Cola, Novartis and AT&T, over a one-year time span extending from 2013-05-01 to 2014-05-01.

Download the file `closing_prices.csv` on Moodle. You can load it with the following Python code:

```
import pandas as pd

df = pd.read_csv('closing_prices.csv')
data = df.to_numpy() # if you prefer working with np array
```

We define the weight vector of the portfolio $w = \{w_1, ..., w_7\}$ such that $\sum_{i=1}^{7} w_i = 1$.

1. Write a function that estimates the expected return and the risk (standard deviation of returns) for a given weight vector. Then, plot the return against the risk for 100000 randomly chosen weights vectors (Monte-Carlo simulation). What do you observe?

2. We introduced the following analytical expression during the course to compute the weight vector that minimizes the risk given a desired portfolio return μ_p:

$$\begin{aligned} a &= \mathbb{1}^T C^{-1} \mathbb{1} \\ b &= \mathbb{1}^T C^{-1} \mu \\ c &= \mu^T C^{-1} \mu \\ d &= ac - b^2 \\ \lambda_1 &= \frac{c - b\mu_p}{d} \\ \lambda_2 &= \frac{a\mu_p - b}{d} \\ w &= C^{-1}(\lambda_1 \mathbb{1} + \lambda_2 \mu) \end{aligned} \tag{1}$$

   With $\mu = \{\mu_1, ..., \mu_7\}$ the expected returns of each stock and C the covariance matrix of returns.

   Using this expression, draw Markowitz's efficient frontier for portfolio return $\mu_p \in [-0.0006, +0.0004]$.

3. Using the efficient frontier, find the weight of the portfolio with the minimal volatility. What can you say about the return of this portfolio?