[14X030] Introduction to Computational Finance

Exercise series 8

April 25, 2023

General instructions

Each student is expected to upload on Moodle a **zip** file containing:

- A report in **pdf** format to answer exercise questions.
- The code used to generate the results of the report.

Deadline

Upload on Moodle due by : **May 1, 2023 at 11:59 pm**.

Minority Game

- Let N be the number of agents, M the number of bits of history and S the number of strategies available to each agent among the 2^{2^M} possible strategies.

- Each agent initializes the utilities of its strategies to zero.

- Initialize the history $\mu(0)$ to a random list of M bits.

- For t in $1, ..., T$:

  – Each agent $i \in \{1, \ldots, N\}$ samples a strategy $s_i(t)$ according to the softmax distribution of utilities: $\frac{\exp(\Gamma_i u_s(t))}{\sum_{s'} \exp(\Gamma_i u_{s'}(t))}$ where $\Gamma_i > 0$.

  – Given the current history $\mu(t)$, each agent uses its chosen strategy $s_i(t)$ to pick an action $a_i(t) \in \{+1, -1\}$.

  – Compute the attendance $A(t) = \sum_{i=1}^{N} a_i(t)$.

  – Update the utility of the chosen strategies with a linear payoff:

$$u_{s_i(t)} = u_{s_i(t-1)} - a_i(t)\frac{A(t)}{\beta}$$

  – Remove the oldest bit of history and add a new one.

1. (a) Why is the above procedure called a minority game?

   (b) What is the role of Γ_i? In particular, what does a large or a small value of Γ_i means?

2. Implement a minority game. You can use $\beta = 1$ and $\Gamma_i = 0.01, \forall i \in \{1, ..., N\}$. To add a new bit of history, you can either pick it at random or from some function of the attendance (1 if positive attendance, 0 if negative attendance for instance).

3. Simulate a minority game with $S = 2$ strategies for $T = 100$ steps for values of N in $\{51, 101, 251, 501, 1001\}$ and values of M in $\{0, 1, ..., 18\}$. On a log-log plot, represent $\frac{\sigma^2}{N}$, the scaled variance of the attendance, against $\alpha = \frac{2^M}{N}$.

4. What is the critical value α_c for which the volatility reaches a minimum?

5. Optional: Define an initial price $p(0)$ (for instance 100) and then update it as follows: $p(t) = p(t-1)\exp(\frac{A(t)}{\lambda})$ with λ some positive constant. What is the intuition behind this update rule? Plot price curves for different values of α and λ and comment.