

TP 8

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In [3]: import numpy as np
import matplotlib.pyplot as plt
from datetime import datetime
import matplotlib.dates as mdates
from tqdm import tqdm
```

EX 1

.1

(a)

The procedure is called a minority game because it is a strategic decision-making game where we (the agent) aim to choose the action that will put us in the minority group.

There are N agents with S strategies available --> each agent n tries to predict the strategy s that will be less popular.

(b)

Γ_i represents the "inverse temperature" parameter for agent i

Large Γ_i means the agent will be more sensitive to the differences in utilities among its strategies.

Small Γ_i means that the agent will be less sensitive to the differences in utilities.

The smaller the Γ_i the more stochastic the choice becomes

.2

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In [4]: def run_minority_game(N, M, S, T=100, gamma=0.01, beta=1):
    # Initialize history with random values
    hist = np.random.randint(0, 2**M)

    # Initialize the utilities matrix for each agent and strategy
    util = np.zeros((N, S))

    # Create random strategies for each agent
    strats = np.random.randint(0, 2, (N, S, 2**M))
    attendance_list = []

    for t in range(T):
        # Compute softmax for each agent's strategy
        softmax_values = np.exp(gamma * util) / np.sum(np.exp(gamma * util), axis=1, keepdims=True)
        random_value = np.random.random()
        cum_softmax = np.cumsum(softmax_values, axis=1)
        chosen_strat = np.argmax(np.array(np.where(cum_softmax > random_value, cum_softmax)))

        # Get the action for each agent based on the chosen strategy
        actions = np.array([strats[i, s, hist] for i, s in enumerate(chosen_strat)])
        mod_actions = np.where(actions != 0, actions, -1)

        # Calculate the attendance
        total_attendance = np.sum(mod_actions)
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# Update utilities for the chosen strategies
for i, s in enumerate(chosen_strat):
    util[i, s] -= mod_actions[i] * (total_attendance / beta)

# Update history
hist = ((hist << 1) + np.random.randint(0, 2)) % (2**M)

attendance_list.append(total_attendance)

return attendance_list

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.3

```

In [5]: np.random.seed(42)
all_M = np.arange(0, 19)
all_N = [51, 101, 251, 501, 1001]
S = 2

alphas = [[2**m_i/n_i for n_i in all_N] for m_i in all_M]
res_sigmas_m = []

for M in tqdm(all_M):
    res_sigmas = []
    for N in all_N:
        res = run_minority_game(N, M, S)
        res_sigmas.append(np.var(res) / N)
    res_sigmas_m.append(res_sigmas)

```

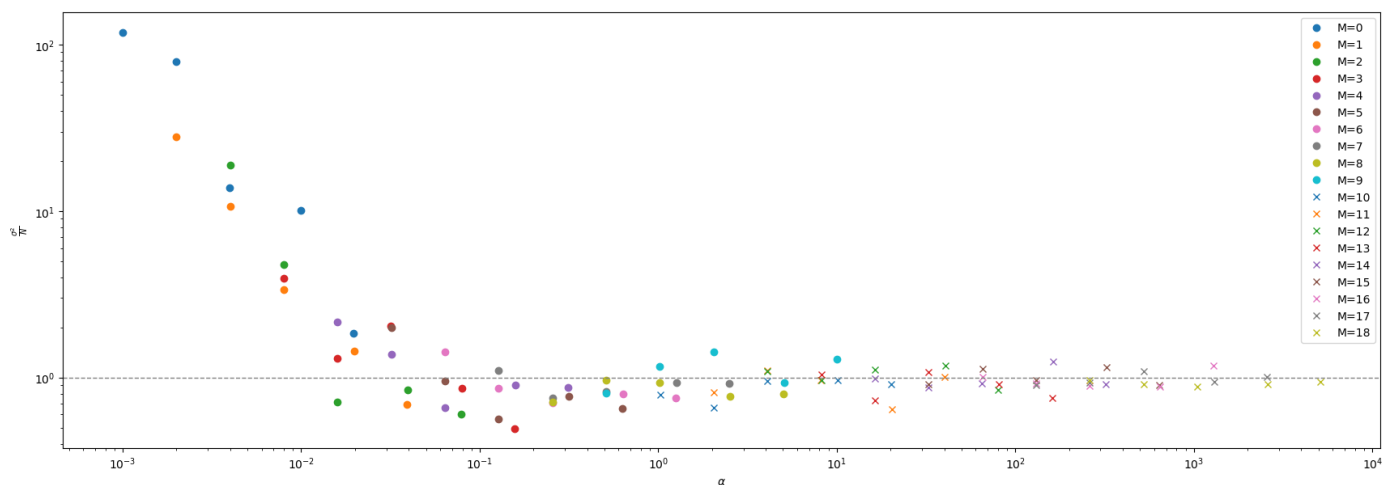
100%|██████████| 19/19 [00:14<00:00, 1.34it/s]

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In [6]: plt.figure(figsize=(21, 7))
for m_i in range(len(all_M)):
    if m_i < 10:
        plt.loglog(alphas[m_i], res_sigmas_m[m_i], "o", label=f"M={all_M[m_i]}")
    else:
        plt.loglog(alphas[m_i], res_sigmas_m[m_i], "x", label=f"M={all_M[m_i]}")

plt.axhline(y=1, color='gray', linestyle='--', lw=1)
plt.xlabel(r"$\alpha$")
plt.ylabel(r"$\frac{\sigma^2}{N}$")
plt.legend()
plt.show()

```



.4

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In [7]: alpha_critical_idx = np.argmin(res_sigmas_m[3])
alpha_critical_value = alphas[3][alpha_critical_idx]

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print(f"Alpha critical when the volatility is minimal is {alpha_critical_value}, with M=
```

```
Alpha critical when the volatility is minimal is 0.1568627450980392, with M=3
```

In []: